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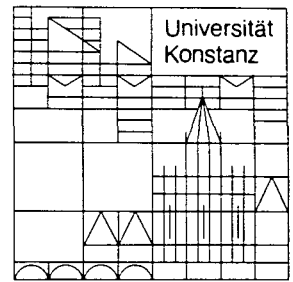
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„Internationalisierung der Wirtschaft“

Diskussionsbeiträge



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**Trade, Investment and Debt
in a Two-Country Growth Model**

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**TRADE, INVESTMENT AND DEBT
IN A TWO-COUNTRY GROWTH MODEL**

Karl-Josef Koch



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Trade, Investment and Debt in a Two-Country Growth Model

Karl-Josef Koch*

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Abstract

We study a dynamic version of a Heckscher–Ohlin model with two countries, two factors and two sectors of production. It is based on the neoclassical growth model by Oniki and Uzawa (1965). We remove their balance of payments restriction by introducing an international market for equity shares of the type used by Hori and Stein (1977). We solve the indeterminacy problem of the capital market in case of factor price equalization by making explicit assumptions on the investment behavior. Two extreme cases are considered which correspond to different attitudes towards domestic versus foreign investment. The model has a unique and globally stable steady state with factor price equalization. Along the adjustment path international debt serves the purpose of increasing efficiency. In the long run holding foreign equity shares can bridge the gap between two possibly conflicting goals: efficiency requires similar factor endowments whereas different consumer preferences establish the need of an uneven income distribution. If consumers are different enough, there will be unbalanced trade and international debt in the long run.

1. Introduction

The Heckscher–Ohlin theory relates volume and direction of international trade in commodities in a specific way to national comparative advantages. The distribution of factors of production among countries plays a decisive role. If na-

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tional factor endowments differ substantially the unconstrained Pareto-efficient integrated world equilibrium cannot emerge. The world is forced into efficiency losses as factor allocations cannot equate marginal value products across countries. These inefficiencies reflected in factor price differentials constitute an incentive for international factor movements. This is the theme the factor price equalization theorem deals with. (See Dixit and Norman (1980) as a general reference.)

Dynamic extensions of the so far static theory pick up this theme and try to model time consuming processes of international reallocation and accumulation of factors. Interesting questions concerning the relation between trade and factor movements in intertemporal settings arise. (See Vosgerau (1989) for an overview.) They dispense with the exogenous restriction of fixed factor endowments in favor of an endogenous determination of factor endowments in a long run equilibrium. Different forms and degrees of international mobility of factors like capital or labor can be considered. In this paper we will confine the analysis to mobility of capital.

As a common feature of such models an investment good can be produced and turned into new capital, one of the factors of production. We will assume there are no costs of adjustment. The transformation of the investment good into new capital takes place instantaneously and one unit of investment yields one unit of capital. (See Albert (1989) for a thorough discussion of the relation between partial mobility of capital and adjustment costs.)

In this paper we extend a special version of the two-country growth model of Oniki and Uzawa (1965). It is constructed on the basis of the two factors, two goods and two countries Heckscher-Ohlin model. There the disappearance of national comparative advantages is the natural consequence of the endogenous accumulation of capital if technological prerequisites and consumer preferences are equal across countries. Otherwise, if preferences differ, the long run endogenous solution to the factor allocation problem may still exclude worldwide efficiency of production.

Very much in the tradition of static pure trade theory Oniki and Uzawa assume that trade between countries must be balanced at any point of time. Clearly, international differences in capital rentals create demand for capital movements beyond the balanced trade in commodities.

One of the first to take this observation into account was Kemp (1968). He separates location and ownership of capital across the borders and assumes that the allocation of capital always allows for worldwide efficiency of production regardless of the ownership situation. So Kemp solves this problem in a pure but

slightly artificial way. He ignores the question how the distribution of physical capital across countries arises.

An international capital market where equity shares are traded can serve to describe capital movements more appropriately. But it causes a new problem: the role of this market is undetermined if there are no factor price differentials and if the system of commodity markets is complete at the same time.

Two ways to overcome this indeterminacy are treated in the literature. Fisher and Frenkel (1974) remove the international market for the investment good. The remaining market structure is rich enough to allow for capital movements beyond Oniki and Uzawa's model but it changes the former Heckscher-Ohlin type temporary equilibrium substantially. Undoubtedly there are good reasons to consider untradable goods; yet one has to find the reasons for this change in the basic structure independent of the above indeterminacy problem. Under these conditions Fisher and Frenkel's approach is useful and enlightening.

Hori and Stein (1977) provide an alternative giving up a different fundamental element of Heckscher-Ohlin theory, they assume that technologies are country specific. As a consequence, generically the equalization of marginal productivities is excluded. Of course this describes a world where global efficiency of production is impossible even in the long run. The persistent exclusion of a country from the use of a technology is against the interest of the country using this technology and therefore certainly needs a specific justification. In one particular sense the approach of Hori and Stein shows a natural way out of the dilemma – factor price equalization is exceptional in the large variety of uneven models where the number of factors differs from the number of commodities produced. So the indeterminacy problem occurs rather by chance than systematically.

There is another extreme element in Hori and Stein's approach. Whenever factor price differentials occur the capital market directs all new investments into the one country with higher capital rentals. As far as capital rentals are the only parameters in such a model investors can base their decision on this is the natural functioning of the capital market. On the other hand it leads to an extremely onesided accumulation of capital. Nevertheless it is the consequence of the fact that the model is kept free from all ingredients making a capital market more realistic.

In this paper we will formulate a synthesis of Oniki and Uzawa's, Kemp's and

Hori and Stein's paper.¹ The pattern of specialization of production will be endogenous as in Oniki and Uzawa. The physical accumulation of capital as well as the accumulation of wealth will follow the lines designed by Hori and Stein. In the strongest possible form the capital market supports the increase of world-wide efficiency of production; thereby international debt is built up. As soon as efficiency is reached the capital market loses its initial rôle. We overcome the resulting indeterminacy by making two alternative assumptions on the investors' behavior. Either they invest their savings from income they earn on foreign shares abroad again or they spend all their savings on the domestic capital market. Both of these extreme versions of investors' behavior lead to the same unique and globally stable long run equilibrium. There will be a persistent uneven distribution of income or wealth, in accordance with the difference in average savings per country. In extreme cases this asymmetry enforces a gap between capital ownership and capital usage. The thriftier country will own more capital than is used by its industries. It will be a long run creditor.

2. Temporary Equilibrium on the Commodity Market

Two countries, H and F , have access to two constant return to scale technologies. With the help of two factors, labor L and capital K , a consumption good C and an investment good I can be produced. For the sake of simplicity we assume that the production can be described by Cobb-Douglas production functions with production elasticities of capital being equal to α_C and α_I , respectively. Furthermore we assume that the production of C is more capital intensive than the production of I , i.e. $\alpha_C > \alpha_I$. Since the production is subject to constant returns to scale it is appropriate to reduce the formal description to per capita terms. Let k_j^i denote the capital labor ratio used in industry j in country i , and f_j the per capita production function of sector j :

$$f_j(k_j^i) = (k_j^i)^{\alpha_j}. \quad (1)$$

Factors are immobile between countries but perfectly mobile between the firms of a country. Under perfect competition firms take world commodity prices p_I and p_C as given. In equilibrium domestic factor prices w and r will be equal to the corresponding marginal value products. Within each country we can aggregate over firms producing the same good. Efficiency conditions on the factor usage

¹In a preceding paper (Koch (1989)) we already analyzed the case of identical savings ratios.

give us the following well known relations:

Let γ be the technological constant defined by

$$\gamma := \frac{\alpha_I}{1 - \alpha_I} \cdot \frac{1 - \alpha_C}{\alpha_C}. \quad (2)$$

Notice that $0 < \gamma < 1$. Whenever a country produces both commodities the following relations hold:

$$k_I^i = \gamma k_C^i \quad (3)$$

and

$$\lambda_I^i = \frac{k_C^i - k^i}{k_C^i - k_I^i}, \quad \lambda_C^i = \frac{k^i - k_I^i}{k_C^i - k_I^i} \quad (4)$$

where λ_j^i denotes the share of country i 's labor force allocated to sector j .

The relations between commodity prices and factor allocations are given by the following functions of (p_I/p_C) :

$$\begin{aligned} k_C &= \left(\frac{p_I}{p_C} \frac{\alpha_I}{\alpha_C} \gamma^{\alpha_I-1} \right)^{\frac{1}{\alpha_C-\alpha_I}}, \\ k_I &= \left(\frac{p_I}{p_C} \frac{\alpha_I}{\alpha_C} \gamma^{\alpha_C-1} \right)^{\frac{1}{\alpha_C-\alpha_I}}. \end{aligned} \quad (5)$$

If $k_C < k^i < k_I$, then k_C and k_I are the efficient capital intensities in country i . Otherwise it is efficient to specialize on the production of only one good. Inverting the functions k_I and k_C we get the following description of the pattern of specialization and diversification of production:

Country i produces:

1). both commodities, if

$$\frac{\alpha_C}{\alpha_I} \gamma^{1-\alpha_I} (k^i)^{\alpha_C-\alpha_I} < \frac{p_I}{p_C} < \frac{\alpha_C}{\alpha_I} \gamma^{1-\alpha_C} (k^i)^{\alpha_C-\alpha_I}, \quad (6)$$

2). only commodity C, if

$$\frac{p_I}{p_C} \leq \frac{\alpha_C}{\alpha_I} \gamma^{1-\alpha_I} (k^i)^{\alpha_C-\alpha_I}, \quad (7)$$

3). only commodity I, if

$$\frac{\alpha_C}{\alpha_I} \gamma^{1-\alpha_C} (k^i)^{\alpha_C-\alpha_I} \leq \frac{p_I}{p_C}. \quad (8)$$

In case 2. and 3. of course all resources are allocated to the one sector which is active.

In all cases factor prices are equal to the corresponding marginal value products. They can be written in the following form:

$$r^i = p_j \partial f_j(k_j^i) = p_j \frac{\alpha_j}{k_j^i} (k_j^i)^{\alpha_j} \quad (9)$$

and

$$w^i = p_j f_j(k_j^i) - p_j k_j^i \partial f_j(k_j^i) = p_j (1 - \alpha_j) (k_j^i)^{\alpha_j}. \quad (10)$$

For further reference we define the wage rental ratio :

$$\omega^i = \frac{w^i}{r^i} = \frac{f_j(k_j^i)}{\partial f_j(k_j^i)} - k_j^i = \frac{1 - \alpha_j}{\alpha_j} k_j^i. \quad (11)$$

So far, for any vector of world commodity prices, (p_I, p_C) , we have a description of factor allocation, commodity supply, and factor prices for each country. Together with the distribution of capital ownership the latter determine the income of individuals. A consumer h of country H who owns k units of domestic capital and u units of foreign capital earns the following income π^h :

$$\pi^h = r^H k + r^F u + w^H \quad (12)$$

or equivalently

$$\pi^h = r^H (k + \omega^H) + r^F u. \quad (13)$$

To derive the consumer demand for commodities we assume constant average savings ratios s^H and s^F per country. Without loss of generality we assume that $s^H \geq s^F$. For the rôle of international debt in the steady state it will matter whether we keep to the classical Heckscher-Ohlin assumption of identical preferences in both countries or allow the two savings ratios to be different. Nevertheless, only the average per capita income of each of the countries, π^H and π^F , respectively, will enter the demand functions. Let u be country H 's per capita net holdings of foreign shares. Then

$$\rho := \begin{cases} r^F u & \text{if } u \geq 0 \\ r^H u & \text{if } u \leq 0 \end{cases} \quad (14)$$

denotes the per capita capital services received by country H , if country H is a creditor (u is positive), and paid by country H , if H is a debtor (u is negative).

Let ϵ be H 's (constant) share of the world population. This gives the following formula for per capita income :

$$\begin{aligned}\pi^H &= r^H (\omega^H + k^H) + \rho \\ \pi^F &= r^F (\omega^F + k^F) - \frac{\epsilon}{1 - \epsilon} \rho\end{aligned}\tag{15}$$

Then the per capita income of the world population is

$$\pi = \epsilon \pi^H + (1 - \epsilon) \pi^F\tag{16}$$

which is independent of ρ .

Finally the market clearing condition will complete the characterization of the commodity market equilibrium² :

$$p_I y_I = \epsilon s^H \pi^H + (1 - \epsilon) s^F \pi^F\tag{17}$$

where y_j is the world production of commodity j per head of the world population.

For convenience we define the income weighted world average savings ratio s_π which in general will vary with the distribution of income across countries.

$$s_\pi := \frac{\epsilon s^H \pi^H}{\pi} + \frac{(1 - \epsilon) s^F \pi^F}{\pi}.\tag{18}$$

Using this notation the market clearing condition can be reduced to

$$p_I y_I = s_\pi \pi\tag{19}$$

As world income is equal to the value of world production this is equivalent to :

$$\frac{p_I}{p_C} = \frac{s_\pi}{1 - s_\pi} \frac{y_C}{y_I}.\tag{20}$$

y_C and y_I are functions of k^H, k^F and p_I/p_C , and s_π is a function of these variables and of u . The following lemma implies that the solution to this equation is unique in relative prices³:

²Due to Walras's law we consider only one market

³For arbitrary production functions there may be multiple equilibria in relative prices on the commodity market . Indeed, the world supply function always is monotonically increasing in p_I/p_C , but as long as $s^H \neq s^F$ the world average savings ratio s_π is not a constant and the world demand is not necessarily decreasing (compare Kemp (1968)).

Lemma 1

If both countries diversify their production the world excess demand for the investment good is equal to:

$$r (a_1 \omega + a_2 (k^H + u) + a_3 (k^F - \frac{\epsilon}{1 - \epsilon} u))$$

where

$$\begin{aligned} a_1 &= \epsilon s^H + (1 - \epsilon) s^F - \frac{\alpha_C}{\alpha_C - \alpha_I} < 0, \\ a_2 &= \epsilon s^H + \epsilon \frac{1 - \alpha_C}{\alpha_C - \alpha_I} > 0, \\ a_3 &= (1 - \epsilon) s^F + (1 - \epsilon) \frac{1 - \alpha_C}{\alpha_C - \alpha_I} > 0. \end{aligned}$$

The proof of lemma 1 is given in appendix A.

As a consequence the equilibrium wage rental ratio ω in case of complete diversification is a linear function of k^H, k^F and u :

$$\omega = a (k^H + u) + b (k^F - \frac{\epsilon}{1 - \epsilon} u) \quad (21)$$

where $a = -a_2/a_1$ and $b = -a_3/a_1$.

The above equation allows for the explicit solution of the model for the temporary equilibrium values of all real variables. Recall from equation (11), that the capital intensities k_I and k_C which are used if both countries diversify their production are given by:

$$k_j = \frac{\alpha_j}{1 - \alpha_j} \omega \quad (22)$$

whenever

$$k_I < k^i < k_C, \quad i = H, F. \quad (23)$$

By construction this system of inequalities describes the set of (k^H, k^F, u) where both countries diversify their production. The closure of this set is characterized by factor price equalization (FPE). Geometrically this set is a closed convex cone. For short we call it FPE-cone. Indeed, if we substitute the equations (21) and (22) into the system of inequalities, we see that this system is homogeneous of degree zero in k^H, k^F and u . In addition the feasibility constraints $k^H + u \geq 0$

and $k^F - \epsilon/(1 - \epsilon)u \geq 0$ are binding. They describe two non-parallel hyperplanes. Hence the FPE-set is a cone. The intersection of this cone with any plane where ω is constant is bounded by three pairs of line segments: two pairs of parallel lines corresponding to the diversification conditions and two non-parallel lines due to the feasibility constraints. Figure 1 shows the FPE-cone. The edges of the cone and the intersection with the plane of a particular constant ω are drawn with thick lines.

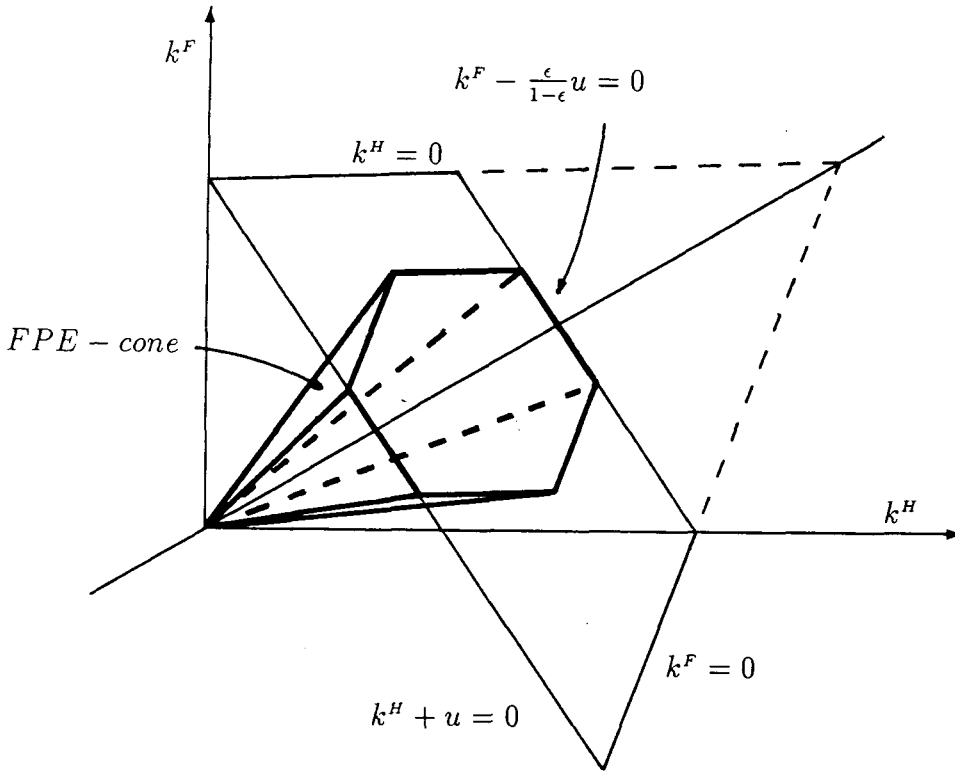


Figure 1.

Notice that the cone contains the set of all feasible points with $k^H = k^F$, because identical capital intensities necessarily lead to factor price equalization. Hence the cone is convex.

For further reference we now prove the following lemma:

Lemma 2 Assume $r^H \neq r^F$. Then $k^H < k^F$ iff $r^H > r^F$.

Proof

Assume $k^H < k^F$. Since $r^H \neq r^F$ at least one country specializes its production. First assume that this only happens in one country. The diversifying country produces according to some diversification cone. Clearly, the specializing country's factor endowment cannot lie within this cone. So we get : $k^H \leq k_I^F < k^F < k_C^H$ or $k_I^H < k^H < k_C^H \leq k^F$. This implies $r^H > r^F$. Now assume that both countries are specialized. Then H specializes on I and F on C . Let k_C and k_I be the capital intensities which maximize revenue of the integrated world economy. This gives : $r^H = p_I \partial f_I(k^H) > p_I \partial f_I(k_I) = p_C \partial f_C(k_C) > p_C \partial f_C(k^F) = r^F$. The reverse implication follows by symmetry. □

This temporary equilibrium is part of a dynamic model we discuss later. In context with the accumulation of capital and the growth of the labor force we will give an illustration of the pattern of specialization of production for $s^H = s^F$ in section 5 (Figure 3).

3. Temporary Equilibrium on the Capital Market

Whereas the consumption good is used up instantaneously the investment good is turned into new capital (one to one) which is perfectly durable. Consumers spend their savings on equity shares issued by firms. As all firms in one country pay the same capital rentals, r^H or r^F , the price for all newly issued shares of one country must be the same, say Q^H or Q^F , respectively. This gives interest rates

$$i^H = \frac{r^H}{Q^H}, \quad i^F = \frac{r^F}{Q^F} \quad (24)$$

for the two countries.

No firm will offer any share below the price of the investment good because otherwise they must expect to make losses in the future. As firms are competitors on the capital market those whose share price exceeds the price of the investment good are driven out of the market. Hence

$$Q^H = Q^F = p_I. \quad (25)$$

In a perfect capital market the interest rate must be equal for all capital assets. Let i denote this world interest rate. In equilibrium we have

$$i = \max\left\{\frac{r^H}{p_I}, \frac{r^F}{p_I}\right\}. \quad (26)$$

These equilibrium considerations for the capital market imply that firms if they acquire new capital do it by selling an equal amount of equity shares at the price of the investment good. One country will acquire all of the new investment if it can offer a higher capital rental. If capital rentals are equal wherever the capital is used there is no financial reason in our model explaining the allocation of new investments. To make the model tractable we introduce ad hoc "preferences" of investors over domestic versus foreign shares.⁴

The financial equivalence of all assets occurs, as explained above, if and only if the distribution of resources over countries implies factor price equalization. This, in turn, implies that marginal redistributions of capital across the borders neither affect the equilibrium world production nor the income distribution — provided FPE is preserved. Therefore, the financial indeterminacy of our model does not have real consequences as it happens in the factor price equalization area only.

Of course investment decisions affect the balance of payments. Since this is one of the elements of our model we want to focus our attention on, we have to make investors' preferences explicit. We will consider two extreme sorts of preferences :

If foreign and domestic shares carry the same interest then

- A) consumers spend the savings from income they earn on foreign shares on the purchase of new foreign shares and their savings from domestic income⁵ on new domestic shares;
- B) consumers spend all their savings on the purchase of new domestic shares.

There is a clear reason behind this choice of preferences. They represent the most extreme examples one can reasonably think of. In case A) investment of any single unit of account takes place in the country where this unit is earned. As far as

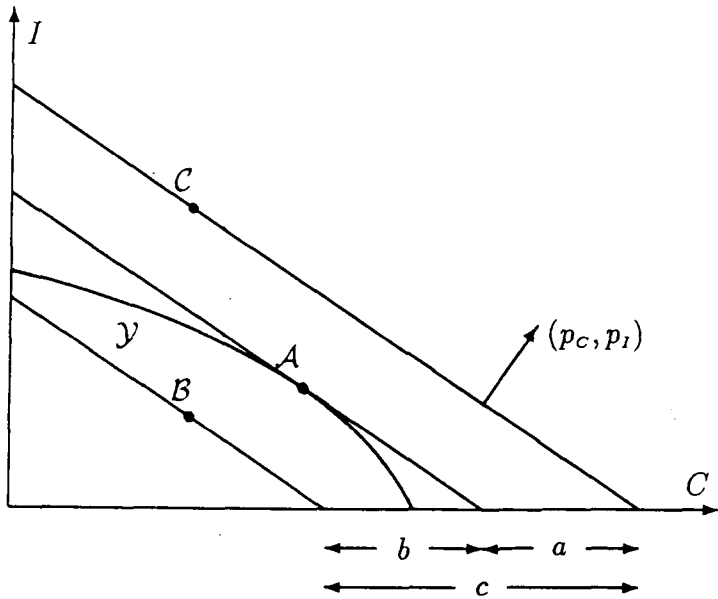
⁴We put the term preferences in quotation marks because we don't use it as in standard consumer theory. Nevertheless we will drop the quotation marks from now on.

⁵Domestic income consists of wage earnings plus, possibly, capital rentals.

capital income is concerned, the *location* of capital matters, not the *ownership*. On the other hand, in case B) it is the *ownership* of capital that determines where capital income is invested.

4. Trade, Investment and International Debt

The following picture illustrates the final allocation of consumption and investment as the outcome of the formation of temporary equilibrium on the commodity and the capital market.



- Y — production possibility set of a country,
- A — point of production of the country,
- B — point of consumption and investment of the population of the country,
- C — point of consumption and investment taking place in the country,
- $-a$ — balance of the trade account,
- $-b$ — balance of the service account, and
- c — balance of the capital account, all in terms of the consumption good.

Figure 2.

At equilibrium prices the production maximizes revenue on the production possibility set in each country. This gives point \mathcal{A} of Figure 2. National income may differ from the value of national production at equilibrium prices due to net holdings of foreign equity shares. Assume the country considered is a net debtor. Then the budget constraint is described by a line with slope p_I/p_C below the point \mathcal{A} . The savings ratio determines the country's consumption and the domestically financed investment. This is point \mathcal{B} . The vector $(\mathcal{B} - \mathcal{A})$ weighted with commodity prices gives the balance of the service account. Now assume the country is not only a net debtor but it also pays a higher capital rental. Then it will attract foreign investment and thereby further increase its debt. Assume \mathcal{C} is the point of consumption and domestic plus foreign investment. Then $(\mathcal{C} - \mathcal{B})$ weighted with commodity prices gives the balance of the capital account. It shows the difference between the value of domestic investment and savings. The balance of payments is completed by the trade account. The difference between domestic production and final domestic consumption and investment is traded across the borders. Weighted with commodity prices $(\mathcal{A} - \mathcal{C})$ gives the balance of trade.

These three entries, the trade account, the service account and the capital account complete the balance of payments of this model. They capture the relation between three simultaneous and interdependent kinds of actions of economic agents: production decisions of domestic firms, demand for consumption and savings of the domestic population and new foreign investments. The resulting entries in the balance of payments can be viewed under two very different aspects: they are outcomes of past decisions on the one hand and the basis of future developments on the other hand. Services are rentals paid on a stock of foreign capital built up in the past; they are an intrinsic consequence of the history of the economy. Only the size of the payments depends on current decisions, not the direction of the flow. The directions of trade and movements of financial capital result from current decisions. The volume of trade determines — apart from the level of consumption — the allocation of new physical capital. The growth rates of the capital stocks endogenously predetermine the future relative factor endowments of the countries. They reflect the increase or possibly decrease of efficiency of future production as far as this efficiency is restricted by the international immobility of physical capital accumulated in the past. International trade can be financed by (net) national income, but there may be the desire for an even greater volume of capital movements in order to accelerate the progress

of efficiency beyond the limits set by balanced trade. Additional trade in investment goods can be financed by movements of financial capital. The greater the inefficiency of production the greater is the incentive to use the financial market in order to extend the intertemporal gains from trade. The evolution of activities on real and financial markets will be considered in the next chapter. We will focus our attention on the rôle of the markets in the steady state.

5. Capital and International Debt in the Long Run

The dynamics of our model will be described by the population growth, the change of capital intensities of capital employed in country H and F per head of population in H and F , respectively, and, finally, by the change of net holdings of foreign shares per head of population in H . The population growth rate n is assumed to be constant and equal in both countries. Recall that ϵ is the share of the world population living in H . For convenience we will normalize commodity prices by $p_I := 1$.

The considerations concerning the capital market made in chapter 3 imply that all new investment will be directed to the country with higher capital rentals as long as capital rentals are different across countries. Using lemma 2 we obtain the following system of differential equations describing the evolution of capital intensities and debt outside the FPE – cone :

Above the FPE – cone : $k^H < k^F$ and $r^H > r^F$

$$\begin{aligned}\dot{k}^H &= s^H \pi^H + \frac{1-\epsilon}{\epsilon} s^F \pi^F - n k^H \\ \dot{k}^F &= -n k^F \\ \dot{u} &= -\frac{1-\epsilon}{\epsilon} s^F \pi^F - n u\end{aligned}\tag{27}$$

Below the FPE – cone : $k^H > k^F$ and $r^H < r^F$

$$\begin{aligned}\dot{k}^H &= -n k^H \\ \dot{k}^F &= \frac{\epsilon}{1-\epsilon} s^H \pi^H + s^F \pi^F - n k^F \\ \dot{u} &= s^H \pi^H - n u\end{aligned}\tag{28}$$

The first observation we make is the following one :

Proposition 3

The vector of capital intensities converges to the FPE – cone in finite time.

Proof

Assume $k^H < k^F$ and $r^H \neq r^F$. Let \hat{k}^H and \hat{k}^F denote the rates of change of capital intensities. According to lemma 2 r^H is greater than r^F and (28) gives:

$$\begin{aligned}\hat{k}^H &= \frac{\epsilon s^H \pi^H + (1 - \epsilon) s^F \pi^F}{\epsilon k^H} - n \\ \hat{k}^F &= -n\end{aligned}\tag{29}$$

Hence $\hat{k}^H - \hat{k}^F$ is positive and does not converge to zero if we approach the FPE – cone.

Similarly we get $\hat{k}^H - \hat{k}^F < 0$ if $k^H > k^F$ and $r^H \neq r^F$.

□

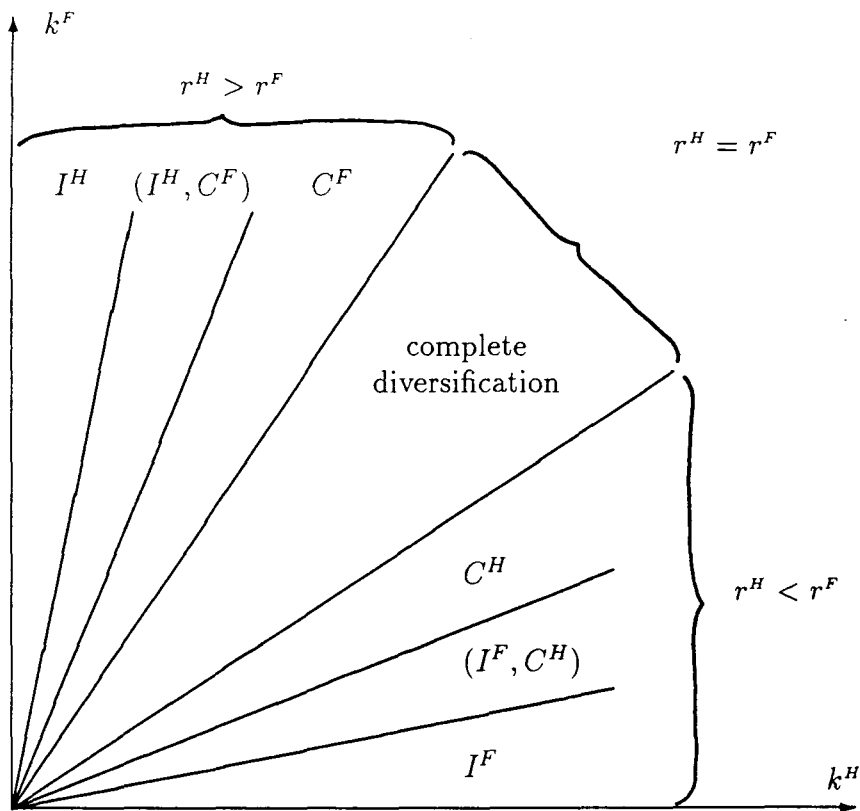
This proposition has an important implication: there cannot exist a steady state with complete specialization⁶.

Inefficiencies reflected by factor price differentials dominate the capital market. The resulting investment in the one country with too small capital intensity relative to the other country increases efficiency. The economies move to the domain of world wide Pareto – efficient production, i.e. factor price equalization, within finite time. Approaching the FPE – cone the economies undergo possibly several changes in the pattern of specialization. If $s^H \neq s^F$ this pattern depends on the level of u . Nevertheless, qualitatively the sequence of structural changes can be deduced from the case $s^H = s^F$. In this case the picture is independent of u . Aggregate supply and demand only depend on the physical capital intensities k^H and k^F . Note that it may well happen that some of the regions of specialization are empty as the distribution of labor is fixed. Indeed, if a capital rich country has a sufficiently large labor force it will dominate the world economy. This country will diversify its production and the capital poor country will be forced into specialization on the less capital intensive sector. Formally, this means that the system of inequalities which characterize complete specialization may have no

⁶Recall that the lemma in fact does not rely on the assumption $\alpha_C > \alpha_I$. This shows that the stability of the dynamic model is reinforced by the capital market.

positive solution in k^H and k^F for some ϵ . In this case all other possible patterns of specialization are excluded.

The following picture illustrates the pattern of specialization. It is drawn in (k^H, k^F) – space under the assumption that $s^H = s^F$.



I^H is the region where H specializes on the production of I whereas F diversifies its production, I^F, C^H and C^F analogously.

(I^H, C^F) is the region where H specializes on the production of I and F on C , (I^F, C^H) analogously.

Figure 3.

Once the economies have reached the FPE – cone the capital market plays a different role. It preserves efficiency and takes care of consumers' portfolio preferences. If necessary, it bridges the gap between two possibly conflicting goals: production efficiency requires similar factor endowments whereas different consumer preferences establish the need of an uneven income distribution.

Within the FPE – cone the evolution of the two economies can be dichotomized: the evolution of $k^H + u$ and $k^F - \frac{\epsilon}{1-\epsilon} u$ is independent of the actual value of u . Indeed, ω, k_C, k_I and r only depend on the distribution of capital ownership. We define:

$$\begin{aligned} x^H &= k^H + u \\ x^F &= k^F - \frac{\epsilon}{1-\epsilon} u \end{aligned} \quad (30)$$

to denote per capita capital ownership in country H and F , respectively. The time path of these variables obeys the following system of differential equations :

$$\begin{aligned} \dot{x}^H &= s^H r (x^H + \omega) - n x^H \\ \dot{x}^F &= s^F r (x^F + \omega) - n x^F . \end{aligned} \quad (31)$$

We get :

$$\hat{x}^H - \hat{x}^F = r \omega \left(\frac{s^H}{x^H} - \frac{s^F}{x^F} \right) + r (s^H - s^F) . \quad (32)$$

Proposition 4

Let s^H be greater than or equal to s^F . Then the capital ownership ratio

$$\frac{x^F}{x^H} = \frac{k^F - \frac{\epsilon}{1-\epsilon} u}{k^H + u}$$

converges monotonically to some constant $\bar{\kappa} \in (0, 1]$. x^H and x^F converge globally to a unique steady state $(\bar{x}^H, \bar{x}^F) \gg 0$.

Proof

Define:

$$\psi = \frac{x^F}{x^H} (\hat{x}^H - \hat{x}^F) / r .$$

$\omega = ax^H + bx^F$ with $a, b > 0$ by lemma 1. Replacing ω in (32) and rearranging terms leads to:

$$\psi = bs^H \left(\frac{x^F}{x^H} \right)^2 + ((a+1)s^H - (b+1)s^F) \frac{x^F}{x^H} - as^F .$$

ψ is a quadratic function of x^F/x^H . Recall that a and b are positive. $\psi(0) = -as^F < 0$ and $\psi(1) = (1+a+b)(s^H - s^F) \geq 0$. Hence ψ has exactly one positive root $\bar{\kappa} \in (0, 1]$. On the domain of positive x^F/x^H ψ has the same sign as $\hat{x}^H - \hat{x}^F$. Furthermore, ψ is negative if $0 < x^F/x^H < \bar{\kappa}$ and positive if $\bar{\kappa} < x^F/x^H$. Hence :

$$\begin{array}{ccc} \hat{x}^H - \hat{x}^F & \begin{array}{c} > \\ = \\ < \end{array} 0 & \iff \frac{x^F}{x^H} \begin{array}{c} < \\ = \\ > \end{array} \bar{\kappa}. \end{array}$$

So far we know that the dynamical system (31) converges globally to the ray $x^F = \bar{\kappa}x^H$. Along this ray the system reduces to a simple and familiar form in terms of x^H . We substitute ω according to (21) and x^F by $\bar{\kappa}x^H$ to get:

$$\dot{x}^H = \mu f_I(x^H) - nx^H$$

for some positive constant μ . As f_I is concave and ∂f_I goes to ∞ , if x^H goes to 0, there is a unique and globally stable steady state of (31) with positive values of x^H and x^F . □

In contrast to the capital ownership ratio, k itself does not necessarily converge monotonically, unless $s^H = s^F$. If we redistribute equity shares towards the thriftier country the world wide growth rate of capital will increase. Hence, there is a regime of overshooting of capital adjustment where the capital ownership ratio is too small. Correspondingly there is a regime of undershooting with too large a capital ownership ratio.

The long run equilibrium equations $x^H = \bar{x}^H$ and $x^F = \bar{x}^F$ describe a line in (k^H, k^F, u) - space.

Definition:

Let E be the set of (k^H, k^F, u) which satisfy the long run equilibrium equations for x^H and x^F :

$$E := \{(k^H, k^F, u) | k^H + u = \bar{x}^H, k^F - \frac{\epsilon}{1-\epsilon} u = \bar{x}^F\}.$$

We refer to E as equilibrium line.

The steady state will be a point on the intersection of the equilibrium line with the FPE – cone. All of these points coincide with respect to world production and country specific per capita consumption⁷. They differ in employed physical capital per country and international debt. The efficiency preserving force always will dominate the capital market and guarantee the economies to stay within the FPE – cone. If consumers' preferences stand against that force the boundary of the FPE – cone may constitute a binding constraint. The system will always converge to a long run equilibrium on E in the FPE – cone.

The accumulation of physical capital per country and the evolution of international debt depend on the assumptions made on preferences of consumers.

A) Savings from capital rentals are invested
in the country of origin.

According to this assumption the investment in each country is equal to the savings on the income earned in this country. Hence the debtor's savings ratio has to be applied to the rentals paid for foreign capital. This leads to the following differential equations for k^H, k^F and u :

If H is a creditor ($u \geq 0$) :

$$\begin{aligned}\dot{k}^H &= s^H r (k^H + \omega) - n k^H \\ \dot{k}^F &= s^F r (k^F + \omega) + (s^H - s^F) r \frac{\epsilon}{1 - \epsilon} u - n k^F \\ \dot{u} &= s^H r u - n u\end{aligned}\tag{33}$$

If H is a debtor ($u \leq 0$) :

$$\begin{aligned}\dot{k}^H &= s^H r (k^H + \omega) + (s^H - s^F) r u - n k^H \\ \dot{k}^F &= s^F r (k^F + \omega) - n k^F \\ \dot{u} &= s^F r u - n u\end{aligned}\tag{34}$$

B) All savings are invested at home.

Here we get:

$$\dot{k}^H = s r (k^H + \omega + u) - n k^H$$

⁷Of course, prices and income distribution are also constant along E .

$$\begin{aligned}\dot{k}^F &= s^F r (k^F + \omega - \frac{\epsilon}{1-\epsilon} u) - n k^F \\ \dot{u} &= -n u\end{aligned}\tag{35}$$

Now we can prove the main result which characterizes the long run equilibrium. Two qualitatively different types of long run equilibria can occur: equilibria with or without permanent international debt. Theorem 5 characterizes the possible equilibria and the proposition thereafter states the conditions discriminating between debt and non – debt equilibria.

Theorem 5

There is a unique and globally stable steady state. It satisfies the following conditions :

- 1) $\bar{k}^H + \bar{u} = \bar{x}^H, \bar{k}^F - \frac{\epsilon}{1-\epsilon} \bar{u} = \bar{x}^F,$
- 2) $\bar{u} = 0$ if and only if $\bar{k}^H = \bar{x}^H, \bar{k}^F = \bar{x}^F$ and $\bar{u} = 0$ belongs to the FPE – cone,
- 3) $\bar{u} \geq 0$, and
- 4) $\bar{k}^H \geq \bar{k}^F$ and $\bar{k}^H = \bar{k}^F$ if and only if $s^H = s^F$.

Proof

From proposition (4) we know that any steady state must be on E , which can be parameterized by u . As the system converges to E it is sufficient to examine the evolution of u in order to prove uniqueness and global stability.

Under assumption A) \hat{u} equals $s^H r - n$ or $s^F r - n$, respectively. Due to equation (31) \hat{u} must be negative along E :

$$\begin{aligned}s^H r - n &= -s^H r \omega / x^H < 0 \text{ and} \\ s^F r - n &= -s^F r \omega / x^F < 0 .\end{aligned}$$

By continuity \hat{u} is negative on some neighborhood of E . Under assumption B) \hat{u} equals $-n$. Hence \hat{u} is negative close to E no matter whether we assume A) or B).

Let e^* be the point on E for which $u = 0$. If e^* lies in the FPE – cone it will be the steady state and the economies will globally converge towards e^* . If e^* lies outside the FPE – cone, the efficiency constraint imposed by the capital market will be binding and the system will converge to the point on the intersection of E and the FPE – cone closest to e^* . Call this point \tilde{e} . We complete the proof by checking the characterization of e^* and \tilde{e} , respectively.

- 1) This follows immediately from proposition (4) and the definition of x^H and x^F .
- 2) This is clear from the considerations above.
- 3) We have to show that $\bar{u} \geq 0$ if \tilde{e} is the steady state. Consider the point d on E with $k^H = k^F$. Clearly d belongs to the FPE – cone. $x^H = \bar{\kappa}x^F$ gives $k^H(1 - \bar{\kappa}) = \frac{1}{1-\epsilon}u$ at d . As $0 < \bar{\kappa} \leq 1$ we have $u \geq 0$ at d . \tilde{e} lies on the line segment between d and e^* . Hence $u \geq 0$ at \tilde{e} .
- 4) $\bar{k}^H \geq \bar{x}^H \geq \bar{x}^F \geq \bar{k}^F$. $\bar{k}^H = \bar{k}^F$ if and only if $\bar{\kappa} = 1$, and this is true exactly if $s^H = s^F$.

□

Proposition 6

$$\bar{u} > 0 \Leftrightarrow \begin{cases} \psi(\frac{\alpha_I a}{1 - \alpha_I - \alpha_I b}) > 0 \\ \psi(\frac{1 - \alpha_C - \alpha_C a}{\alpha_C b}) > 0 \end{cases} \quad \text{or} \quad (36)$$

Proof

The intersection of the boundary of the FPE – cone and the plane with $u = 0$ consists of two rays, one with $k^H < k^F$ and one with $k^H > k^F$. Let \mathcal{H} be the latter one. Let \mathcal{R} be the ray defined by $k^F/k^H = \bar{\kappa}$ and $u = 0$. \mathcal{R} lies outside the FPE – cone if and only if the slope $\eta := k^F/k^H$ of \mathcal{H} is larger than $\bar{\kappa}$. \mathcal{H} can be described by one of two possible conditions:

- 1) $k^F = k_I = \frac{\alpha_I}{1 - \alpha_I}\omega$ and
- 2) $k^H = k_C = \frac{\alpha_C}{1 - \alpha_C}\omega$.

Of course, the two conditions may coincide in exceptional cases. We substitute ω by $ak^H + bk^F$ and get

- 1) $k^F/k^H = (\alpha_I a)/(1 - \alpha_I - \alpha_I b) =: \eta_I$ and
- 2) $k^F/k^H = (1 - \alpha_C - \alpha_C a)/(\alpha_C b) =: \eta_C$,

respectively. As the FPE – cone is convex and contains the ray with $k^F/k^H = 1$ the maximum of η_C and η_I corresponds to the boundary of the cone, i.e. $\eta = \max\{\eta_C, \eta_I\}$. By definition of ψ the slope η is greater than $\bar{\kappa}$ if and only if

$\psi(\eta) > 0$. This completes the proof. □

The case of $\bar{u} \neq 0$ does not require extreme parameter constellations. Roughly speaking, $s^H - s^F$

must be large relative to $\alpha_c - \alpha_I$. In Appendix B we demonstrate by a numerical example that persistent debt occurs for reasonable parameter values. Figure 4 illustrates the case of such an equilibrium. It shows the plane $\omega = a\bar{x}^H + b\bar{x}^F$ with the lines $u = 0$ and E .

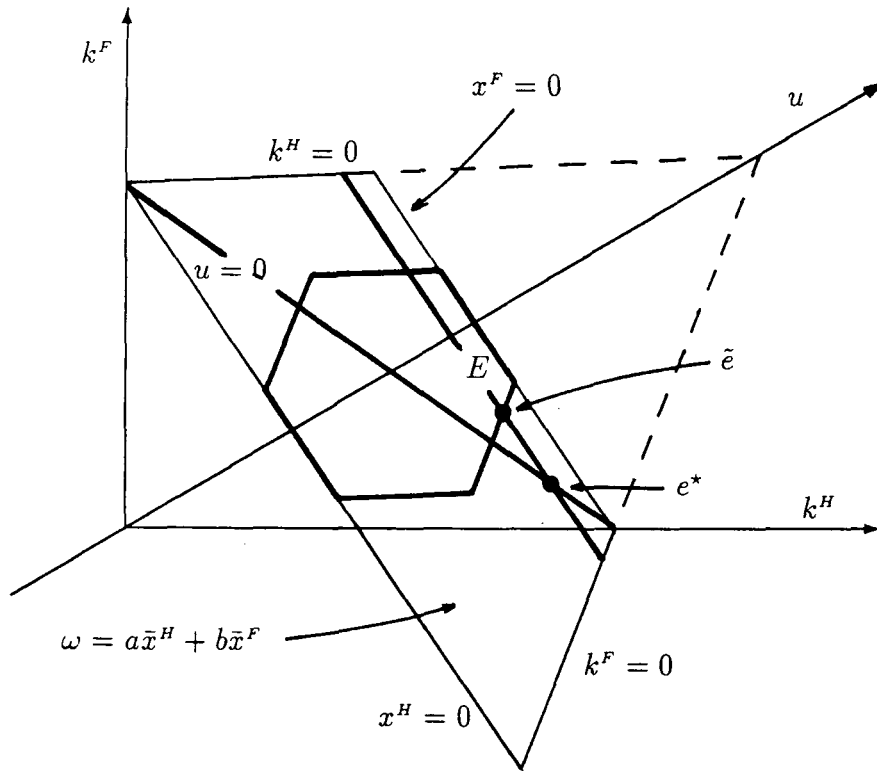


Figure 4.

By proposition 6 the conditions above are sufficient for persistent debt. A complete description of the domain of persistent debt requires a closer look at the properties of the function ψ , because it jointly depends on all exogenous parameters of the model. But through the above example the proposition confirms a very plausible intuition: if the technologies of both sectors are similar, efficiency

of production requires similar capital intensities all over the world. If, on the other hand, the home country is substantially more thrifty than the foreign country, the income distribution must be uneven. This enforces a discrepancy between factor ownership and factor usage. In other words, this requires persistent debt.

6. Concluding Remarks

The paper describes the accumulation of capital in a neoclassical growth model with two countries, two factors and two sectors of production. The distribution of labor across countries is fixed. The distribution of capital is formed via trade in goods and equity shares. This allows for a discrepancy between physical allocation and ownership of capital. In a pure and therefore extreme way the capital market guarantees that new investment is always placed so as to maximize its marginal rate of return. As long as capital rentals differ across countries this evokes changes in the stock of international debt. Unbalanced commodity trade between the countries is compensated by capital services and trade in financial capital to equalize the balance of payments.

The pattern of specialization versus diversification of production is endogenous. Factors are used efficiently subject to the constraint of international immobility. In the short run a substantially asymmetric distribution of factors causes inefficiencies in production which are reflected by international factor price differentials. It is shown that in the long run the capital market resolves these differentials. The economies converge to the domain of factor price equalization or world wide efficient production. This result goes beyond the conclusions drawn in preceding papers: in Oniki and Uzawa model (1965) the economies don't necessarily reach efficiency in the long run, because international debt is excluded; Hori and Stein (1977) generically exclude efficiency by assuming that the pattern of specialization is fixed exogenously.

Once the domain of efficiency is reached the capital market is indeterminate. All equity shares yield the same rate of return. In the absence of uncertainty the investors should be indifferent. To overcome this indeterminacy we consider two alternative types of ad hoc preferences. Either investors in principle prefer domestic equity shares or they reinvest savings of the returns on shares in the country where they are earned. Both extreme assumptions lead to the same long run equilibrium minimizing the volume of international debt subject to the efficiency constraint. As a matter of fact, the ownership of capital in this long

run equilibrium already was computed by Kemp (1968). The missing link was the analysis of the convergence to this equilibrium in a framework including a capital market.

Another point is worth being considered. All models mentioned — this paper included — work with constant saving ratios and do not consider optimal growth. Introducing a capital market and endogenizing the pattern of specialization only increases efficiency of production. Whether this increases or decreases the production of the consumption good during the adjustment process depends on the current situation. For example, in a situation of complete specialization all new investment will take place in the country producing the investment good. As a consequence consumption per head has to decline temporarily. Optimal growth or individual utility maximization as in overlapping generations models could incorporate this aspect.

Appendix A

Proof of lemma 1

The share of the world labor force employed in sector I is

$$\begin{aligned}\lambda_I &= \frac{k_C - k}{k_C - k_I} \\ &= \frac{\frac{\alpha_C}{1-\alpha_C}\omega - k}{(\frac{\alpha_C}{1-\alpha_C} - \frac{\alpha_I}{1-\alpha_I})\omega} \\ &= \frac{1}{1-\gamma} - \frac{1-\alpha_C}{\alpha_C} \frac{1}{1-\gamma} \frac{k}{\omega}\end{aligned}$$

The world per capita supply is equal to:

$$\begin{aligned}\lambda_I k_I^{\alpha_I} &= \frac{1}{1-\gamma} \left(\frac{\alpha_I}{1-\alpha_I}\right)^{\alpha_I} \omega^{\alpha_I} - \frac{1-\alpha_C}{\alpha_C} \frac{1}{1-\gamma} \left(\frac{\alpha_I}{1-\alpha_I}\right)^{\alpha_I} \omega^{(\alpha_I-1)} k \\ &= r \left\{ \frac{1}{1-\gamma} \frac{1}{\alpha_I} \frac{\alpha_I}{1-\alpha_I} \omega - \frac{1-\alpha_C}{\alpha_C} \frac{1}{1-\gamma} \frac{1}{\alpha_I} \frac{\alpha_I}{1-\alpha_I} k \right\} \\ &= r \left\{ \frac{1}{1-\gamma} \frac{1}{1-\alpha_I} \omega - \frac{\gamma}{1-\gamma} \frac{1}{\alpha_I} k \right\} \\ &= r \left\{ \frac{\alpha_C}{\alpha_C - \alpha_I} \omega - \frac{1-\alpha_C}{\alpha_C - \alpha_I} k \right\}\end{aligned}$$

The world per capita demand is equal to:

$$\epsilon s^H r(\omega + k^H + u) + (1 - \epsilon) s^F r(\omega + k^F - \frac{\epsilon}{1 - \epsilon} u)$$

Hence the world per capita excess demand equals:

$$\begin{aligned} r \quad & \{ (\epsilon s^H + (1 - \epsilon) s^F - \frac{\alpha_C}{\alpha_C - \alpha_I}) \omega \\ & + (\epsilon s^H + \epsilon \frac{1 - \alpha_C}{\alpha_C - \alpha_I}) (k^H + u) \\ & + ((1 - \epsilon) s^F + (1 - \epsilon) \frac{1 - \alpha_C}{\alpha_C - \alpha_I}) (k^F - \frac{\epsilon}{1 - \epsilon} u) \} . \end{aligned}$$

□

Appendix B

By an example we demonstrate the properties of the model. The basic parameters take the following values:

α_C	α_I	ϵ	n	s^H	s^F
0.4	0.2	0.625	0.01	0.23	0.14

We compare four different settings:

1. **Autarchy:** two countries with identical technologies and growth rates of labor reach their steady state without any interaction.
2. **Comparative static gains from trade:** the autarchic countries engage in free trade in commodities starting from their respective autarchic steady states. We observe the comparative static gains from trade due to comparative advantages. Country H specializes on sector C . The utility index $y_I^{s_I} y_C^{1-s_I}$ which is implicitly maximized in the short run is increased.
3. **Steady state with trade only in goods:** the countries reach a new steady state where country H remains specialized. This equilibrium is inefficient as a consequence of the substantially different saving ratios. Compared to autarchy consumption has risen in both countries.
4. **Steady state with trade in goods and equities:** the countries overcome the former inefficiency by building up persistent debt. Country H owns part of the capital stock of country F . But consumption in country H decreases whereas F gains from trade in equities.

Steady States in Autarchy

	H	F
demand for consumption	3.817	3.337
demand for investment	0.541	0.295
utility index	2.435	2.377
supply of consumption	3.817	3.337
supply of investment	0.541	0.295
social product in units of I	2.351	2.110
social product in units of C	4.975	3.880
total asset holding	54.078	29.540
assets held abroad	0.000	0.000
world capital intensity	44.877	
consumption export	0.000	0.000
investment export	0.000	0.000
trade account in units of I	0.000	0.000
trade account in units of C	0.000	0.000
relative price of C	0.474	0.544
wage rate	1.519	1.325
rental rate of capital	0.015	0.027
wage rental ratio	98.685	49.869

Table 1.

Comparative Static Gains form Trade

	H	F
demand for consumption	3.799	3.449
demand for investment	0.566	0.280
utility index	2.452	2.427
supply of consumption	4.934	1.558
supply of investment	0.000	1.223
social product in units of I	2.461	2.000
social product in units of C	4.934	4.011
total asset holding	54.078	29.540
assets held abroad	0.000	0.000
world capital intensity	44.876	
consumption export	1.135	-1.891
investment export	-0.566	0.943
trade account in units of I	0.000	0.000
trade account in units of C	0.000	0.000
relative price of C	0.499	0.499
wage rate	1.476	1.445
rental rate of capital	0.018	0.019
wage rental ratio	81.117	76.840

Table 2.

Steady State with Trade only in Goods

	H	F
demand for consumption	3.937	3.367
demand for investment	0.591	0.276
utility index	2.546	2.372
supply of consumption	5.113	1.407
supply of investment	0.000	1.261
social product in units of I	2.571	1.968
social product in units of C	5.113	3.915
total asset holding	59.128	27.555
assets held abroad	0.000	0.000
world capital intensity	47.288	
consumption export	1.176	-1.960
investment export	-0.591	0.985
trade account in units of I	0.000	0.000
trade account in units of C	0.000	0.000
relative price of C	0.503	0.503
wage rate	1.542	1.433
rental rate of capital	0.017	0.019
wage rental ratio	88.692	73.802

Table 3.

Steady State with Trade in Goods and Equities

	H	F
demand for consumption	3.885	3.408
demand for investment	0.572	0.273
utility index	2.500	2.394
supply of consumption	4.951	1.632
supply of investment	0.000	1.226
social product in units of I	2.485	1.952
social product in units of C	5.046	3.963
total asset holding	57.158	27.322
assets held abroad	2.620	-4.367
world capital intensity	45.970	
consumption export	1.065	-1.776
investment export	-0.572	0.953
trade account in units of I	-0.095	0.159
trade account in units of C	-0.047	0.078
relative price of C	0.492	0.492
wage rate	1.463	1.463
rental rate of capital	0.018	0.018
wage rental ratio	81.807	81.807

Table 4.

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