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Sonderforschungsbereich 178 "Internationalisierung der Wirtschaft"

Diskussionsbeiträge



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Strategic Vertical Foreign Investment Under Exchange Rate Uncertainty

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Strategic Vertical Foreign Investment Under Exchange Rate Uncertainty*

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Abstract: This paper investigates the strategic incentives for vertical foreign investment by risk-neutral oligopolistic firms and the effect of exchange rate uncertainty on such investment. Firms competing in a domestic final good market meet their input requirements through import. They have the option of investing abroad in a subsidiary and producing their input requirement through such subsidiary, partly or entirely. The other option is to purchase inputs at the market price in the oligopolistic intermediate good market abroad. Exchange rate uncertainty affects the cost of procuring input in both cases. Firms investing upstream can bid up the input price faced by their rivals which do not invest, through strategic purchase. Assuming linear market demand and constant returns technology, we show that an increase in foreign exchange variability has a positive effect on vertical foreign direct investment and on trade in the intermediate good. Further, the incentive to undertake foreign investment may increase when the number of rival firms undertaking such investment increases. This leads to the possibility of multiple equilibria as well as herding in investment decisions; competing firms invest upstream because their rivals do. A small decrease in investment cost can trigger a large increase in investment. (JEL-code: D43, F2, F31)

Keywords: vertical foreign investment, foreclosure, exchange rate uncertainty, intra-firm trade

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1.Introduction

The importance of multinational firms has grown rapidly over the last two decades. Currently, they account for about one-third of world output. A large number of parent firms, mostly based in rich industrialized countries, control an even larger number of foreign affiliates, about half of which are located in less industrialized poor countries. Activities of multinational firms are a powerful determinant of the allocation of productive resources across the world and affect the market structure of many industries. Not surprisingly, about one-third of world trade takes place in the form of intra-firm trade, that is, trade between parent firms and their subsidiaries. Intermediates and manufactured goods constitute the major components of such trade flows (see UNCTAD (1994)).

The collapse of the Bretton Woods regime of fixed exchange rates has led to a surge of fluctuations in the major national currencies of the world. These fluctuations, illustrated by the behavior of the U.S. dollar in the 80's and the 90's, have drawn attention to trade and investment strategies of multinational firms operating under exchange rate uncertainty. Central to the issue is the popular conjecture that the floating exchange rate regime has led to a decrease in the volume of direct foreign investment by multinational firms¹. This view has been repeatedly put forward and even officially accepted by various international organizations and governments (see, for example, UNCTAD (1993, pp. 224, Table XI.2)).

The objective of this paper is to investigate the incentives for strategic vertical foreign investment by firms and the effect of exchange rate uncertainty on such investment. In particular, we examine a framework where firms compete in an oligopolistic downstream (or final goods) market and meet their intermediate good requirements through import. These firms have the option of either investing abroad (by incurring a fixed cost) and importing their inputs through intra-firm trade at transfer prices, or of not investing and importing their input requirements at arm's length prices. In both cases, they are affected by exchange rate movements. The market for the intermediate good abroad is also an oligopoly. We assume that the market demand for the final good is linear and that production technology at both final and intermediate stages exhibit constant returns. All firms are risk-neutral expected profit maximizers. After investment decisions are made, exchange rate uncertainty

¹A related issue is the conjectured negative effect of exchange rate variability on international trade flows. The numerous empirical investigations into this issue have, on the whole, yielded no conclusive evidence about such relationship. Recent theoretical models have offered several explanations for this empirical ambiguity (see, for example, Dellas and Zilberfarb (1993) and references contained therein).

is resolved and the firms strategically determine their net supply in each market they participate in. Firms investing upstream can bid up the input price faced by their rivals which do not invest, through strategic purchase.

The existing literature on foreign direct investment has largely concentrated on the investment decisions of exporting firms.² Vertical foreign investment which enables firms to directly manufacture their foreign input requirements, has not attracted as much research attention despite the notable tendency of multinationals to increasingly locate various stages of production in developed and developing countries that are liberalizing their trade [Markusen (1995)]. There is, however, a fairly well developed literature on vertical integration and vertical foreclosure in industrial organization theory to which our analysis is closely related³. Unlike most of this literature, however, the decision to undertake vertical investment is endogenous in our model. This enables us to characterize the amount of strategic vertical investment for different ranges of the fixed cost of investment.

The strategic incentive to invest for a domestic final good firm in our model is the expected additional profit that it can obtain through vertical investment. This (reduced form) gain from investment for a firm can be derived by solving for the market equilibrium corresponding to each possible configuration of investment decisions. Among other things, it depends on the random exchange rate and the number of rival firms which invest. There are two interesting features about the gain from investment for an individual firm.

The first feature is that it is a *strictly convex* function of the uncertain exchange rate. Therefore, an increase in foreign exchange variability (more precisely, a second-order decrease in the distribution of the exchange rate) has a positive effect on vertical foreign direct investment and the flow of international trade.

The second feature is that, for a large class of environments, the gain from investment may increase as more and more rival firms invest; in other words, vertical investment decisions may exhibit strategic complementarity. This feature is a direct consequence of the fact that investing final good firms raise the input price faced by their rivals through strategic purchase in the upstream market which magnifies the relative disadvantage of not investing. In fact, this motivation for strategic

²See, for example, Broll and Zilcha (1992), Goldberg and Kolstad (1995) and other references contained therein.

³See, among others, Grossman and Hart (1986), Salinger (1988), Hart and Tirole (1990), Ordover et al. (1990), Bolton and Whinston (1991), Schrader and Martin (1995).

investment becomes dominant when the intermediate market is more "competitive" in terms of the number of foreign producers active in the market. In such situations, there are typically multiple equilibria: some equilibria involve many firms investing and others involve very few firms investing, sometimes, all or none. In other words, we can observe *herding* in investment decisions; competing firms invest upstream because their rivals do. Another consequence of strategic complementarity is that a small decrease in the fixed cost of investment can trigger a big jump in the volume of investment.

The paper is organized as follows. In Section 2, we lay out the general framework, describe the extensive form of the game as well as the solution concept. In Section 3, we derive the equilibrium in the final and intermediate good markets following any profile of investment decisions by domestic firms. In Section 4, we consider the reduced form foreign investment game where firms simultaneously decide whether or not to invest, prior to the resolution of exchange rate uncertainty, and characterize the equilibria of this game. In particular, we derive a set of interesting results about the strategic incentive to invest and analyze the effects of decline in investment barriers. We illustrate the possibility of herding in investment decisions, existence of multiple equilibria and of jumps in strategic investment. In Section 5, we outline the effects of change in exchange rate risk on investment. We conclude in Section 6. The Appendix contains a glossary of symbols.

2. The Model

We analyze the issue of vertical foreign investment in the context of markets in two countries which we shall call "domestic" and "foreign". A homogenous final good is produced by N firms ($N \ge I$) in the domestic economy by using a (homogenous) intermediate good as input. The demand curve for the final good is linear; in particular, the inverse demand function is given by:

$$p = a - bQ, \ a > 0, \ b > 0, \ Q < (a/b)$$
$$= 0, \ Q \ge (a/b) \tag{1}$$

where p denotes the price and Q is the total output/quantity demanded in the final good market. We assume that the input must be imported by domestic firms from the foreign economy⁴ and that the

⁴This could reflect a situation where production of the input requires some foreign country specific factor which is non-tradeable or, more simply, the unit cost of producing the input domestically is relatively very high (for example higher than the largest possible domestic currency cost of importing it at monopoly price from abroad).

final good is produced and consumed in the domestic market only⁵. Further, apart from the domestic final good producers mentioned above, there are no other firms which demand the intermediate good produced in the foreign market. Assume that production of one unit of the final good requires only one unit of the imported intermediate good⁶. There are m^* existing foreign firms (indexed by k) which produce the intermediate good in the foreign economy. The unit foreign currency cost of producing the input abroad is denoted by c^* .

Domestic final good firms have the option of making vertical foreign investment in order to set up subsidiary units in the foreign economy which can directly produce the intermediate good for own use or for sale to other users. Let f > 0 denote the fixed cost of setting up such a subsidiary unit, measured in domestic currency. The domestic resource cost of importing the intermediate good, whether from own subsidiary or from other suppliers, depends on the exchange rate, defined as domestic price of the foreign currency and denoted by \tilde{e} . Final good firms face uncertainty about \tilde{e} when they make vertical investment decisions. All firms are risk-neutral, maximize expected net profit and have rational expectations.

The market evolves in two stages; decisions made in the first stage being irreversible and fully observable in the second stage. First, N domestic final good firms decide simultaneously on whether or not to make vertical foreign investments in intermediate production. Once firms have decided on vertical investment, uncertainty about the exchange rate is resolved. In the second stage, firms operating in both domestic final good as well as foreign intermediate good markets make their supply decisions rationally. Producers of the intermediate good, consisting of the pre-existing m^* foreign firms as well as newly set-up subsidiaries, anticipate the derived demand for the intermediate good and determine their net supply. Each of them holds Cournot conjecture about the decisions of other producers in the intermediate good market. Subsidiaries of investing firms can bid up the market price of the intermediate good by actually buying some amount of the intermediate good, even though they can produce it at a unit cost less than the market price. This has the effect of raising the input cost of

⁵Allowing export of final output would make the revenue side subject to exchange rate variability; in this paper we wish to confine ourselves to the effect of exchange rate fluctuations on the import/input supply side.

⁶This is a simplifying assumption which is representative of the technology of trading houses, for example, but can be easily generalized to any constant factor requirement technology. Allowing for possibility of factor substitution would complicate the analysis without adding to the basic story.

rival final good firms which do not invest⁷. The derived demand for intermediates from the final good market emanates entirely from domestic firms which do not undertake vertical investment and these firms act as price-taking buyers in the intermediate good market. The net supply decisions of the foreign firms and the subsidiaries, together with the derived demand function, determine the market price for the intermediate good. This price is anticipated by firms in the final good market where, again, each firm determines its final output holding Cournot conjecture about the behaviour of the other firms in the same market. Firms from each country evaluate their net profit in terms of their own currency. A final good firm which invests and its subsidiary maximize the sum of their profits in both markets (evaluated in domestic currency).

We solve the model by backward induction. First, we consider the final and intermediate good markets and determine the equilibrium in these markets for any given profile of investment decisions and for any specific realization of the exchange rate. This allows us to determine the expected net profit for each final good firm corresponding to any profile of investment decisions across the *N* firms. Next, we consider the reduced form game where each firm decides whether or not to make vertical foreign investment; the payoff to each firm is its expected profit, net of any fixed cost of investment. The Nash equilibrium of this reduced form game determines the solution of the model; in particular, the number of investing firms.

Suppose that firms have made their vertical investment decisions and that a particular value of \tilde{e} is realized. Let m denote the number of final good firms (indexed by i) which invest in setting up subsidiaries abroad, $0 \le m \le N$. Thus there are (N - m) non-investing firms (indexed by j). In the next section, we work out the market equilibrium for each value of m = 0, 1, ...N and each possible realization of \tilde{e} .

3. Equilibrium in Goods Markets:

Let e denote a specific realization of \tilde{e} . Fix e and m. We now proceed to derive the equilibrium outcome in stage 2. Let q_i and r_j be respectively (investing) firm i's and (non-investing) firm j's final good production, so that:

⁷The literature on vertical foreclosure contains models where such input cost raising is pursued by oligopolistic vertically integrated downstream firms [see Salop and Scheffman (1983, 1987), Schrader and Martin (1995)].

$$\sum_{i=1}^{m} q_{i} + \sum_{j=1}^{N-m} r_{j} = Q$$

Let p^* denote the anticipated market price of the intermediate good in foreign currency. The anticipated profit (in domestic currency) of a final good firm j which does not have a foreign subsidiary and produces output r_j , is given by:

$$\Pi_{j} = r_{j}(a - bQ - ep^{*})$$
 $j = 1, 2, ... N - m.$ (2)

On the other hand, a firm i which undertakes investment in a foreign subsidiary has to take into account its revenues and costs in both the final and the intermediate good markets. As stated in the previous section, such a firm and its subsidiary have identical objective functions. Let x_i denote the net *purchase* of the intermediate good by a subsidiary of firm i. Its anticipated domestic currency profit from combined participation in the downstream and upstream markets is given by:

$$\Pi_i = q_i(a - bQ - ec^*) - x_i(ep^* - ec^*) \qquad i = 1, 2, ..., m; \ m \ge 1.$$
(3)

When x_i is positive, the subsidiary of firm i actually buys a strictly positive amount of the intermediate good in the foreign market. Then, out of the total input need q_i which is imported from its subsidiary, a quantity $(q_i - x_i) \ge 0^8$ is internally produced by the subsidiary (and imported at transfer price ec^* in terms of domestic currency), while an amount x_i is bought by the subsidiary from the intermediate good market at price p^* (imported at arm's length price ep^* in terms of domestic currency). Being a net buyer of the intermediate good, the investing firm (or, equivalently, its subsidiary) earns a negative profit upstream in order to gain strategic advantage by pushing up the market price of the intermediate good faced by its non-investing rivals. When x_i is negative, firm i's subsidiary actually sells on the intermediate good market and its total production is $(q_i - x_i) > q_i$.

Lastly, a foreign intermediate good firm k produces output q_k * which leads to anticipated profit (in foreign currency):

$$\Pi_{\nu}^{*} = q_{\nu}^{*} [p^{*} - c^{*}], \quad k = 1, ..., m^{*}. \tag{4}$$

The expressions (2) - (4) define the profit for each type of firm.

 $^{^{8}\}mathrm{It}$ can checked that x_{i} never exceeds q_{i} in equilibrium.

In the final good market, firms i and j simultaneously choose q_i and r_j (for given anticipated p^*). Imposing symmetry within each group of domestic firms ($q_i = q$, $\forall i; r_j = r$, $\forall j$), we obtain the Cournot-Nash equilibrium:

$$q = \frac{1}{b(N+1)} [a - ec^* + (N-m)(ep^* - ec^*)], \tag{5}$$

$$r = \frac{1}{b(N+1)} [a - ec^* - (m+1)(ep^* - ec^*)], \tag{6}$$

with final good price:

$$p = \frac{1}{(N+1)} (a + (N-m)ep^* + mec^*).$$
 (7)

Profits are given by:

$$\Pi_i = bq^2 - x_i(ep^* - ec^*),$$
 (8)

$$\Pi_i = br^2. \tag{9}$$

Now, consider the intermediate good market abroad. As assumed earlier, the market demand for intermediate good is generated by non-investing firms, which act as price-taking buyers in this market. Pre-multiplying (6) by (N-m), we can obtain the total derived demand for the intermediate good. The inverse demand for the intermediate good is then given by:

$$p^* = c^* + \frac{1}{e(m+1)} \left[a - ec^* - \frac{b(N+1)Q^*}{(N-m)} \right], \tag{10}$$

where Q^* is the total quantity of the intermediate good demanded. Given this, foreign producers of intermediates and investing domestic firms simultaneously choose their actions: foreign firms choose the quantity $q_k^* \geq 0$ they wish to supply and subsidiaries of investing firms decide on x_i . The total output available for satisfaction of intermediate good demand (by non-investing final good firms) is:

$$Q^* = \sum_{k=1}^{m} q_k^* - \sum_{i=1}^{m} x_i. \tag{11}$$

A foreign producer k sets q_k^* so as to maximize its foreign currency profit given by (4), while the (subsidiary of) investing firm i sets x_i so as to maximize its reduced form profit given by (3). Using (10) and (11), the best response of firm k satisfies:

$$a - ec^* + \frac{b(N+1)}{(N-m)} (mx - (m^*+1)q^*) = 0,$$
(12)

where symmetry is imposed $(q_k^* = q^*, \forall k, x_i = x, \forall i)$. From (9), (10) and (11), we can calculate the net reduced profit of firm i from any choice of x_i . The best response for firm i satisfies (imposing symmetry of actions chosen):

$$\left[\frac{2}{(m+1)}(a-ec^*+bmx-bm^*q^*)\right] - \left[a-ec^*+\frac{b(N+1)}{(N-m)}((m+1)x-m^*q^*)\right] = 0.$$
 (13)

The first expression in square brackets represents the marginal effect on firm i's profits derived from the sales of the final good as a result of raising rivals' $costs^9$. The symmetric best response x must be such that this marginal gain is just offset by the additional resources lost in buying the intermediate good at a higher price p^* (compared to its own constant marginal cost c^*). The Nash equilibrium values of x and q are readily obtained from (12) and (13):

$$q^* = \alpha(m)(a - ec^*), \tag{14}$$

$$x = \beta(m)(a - ec^*), \tag{15}$$

and total output Q^* available for non-investing firms is given by:

$$O^* = m^*q^* - mx = (N - m)r = \gamma(m)(a - ec^*), \tag{16}$$

where the terms $\alpha(m)$, $\beta(m)$ and $\gamma(m)$ are given by:

$$\alpha(m) = \frac{(N-m)(N+2m+1)(m+1)}{b(N+1)[(m+1)^3+(N-m)(m^2+1)+((m+1)^2+(N-m)(m+1))m^*]},$$

$$\beta(m) = \frac{\alpha(m)bm^*[(m+1)^2 + (N-m)(m-1)] - (N-m)(m-1)}{b[(m+1)^3 + (N-m)(m^2+1)]},$$

⁹Note that if, instead of domestic firms setting up their own subsidiaries, we allowed for vertical mergers, then the input-price-raising effect of vertical investment would be even sharper as the total number of firms willing to supply positive quantity in the input market would be smaller.

$$\gamma(m) = m^* \alpha(m) - m \beta(m),$$

for m=0, 1,...N-1. Note that $\alpha(m)$, $\beta(m)$ and $\gamma(m)$ are independent of e. It can be checked that $\alpha(m)$ and $\gamma(m)$ are strictly positive for all values of m < N, while $\beta(m)$ can be positive or negative which suggests that x may take positive or negative values in equilibrium. If all firms invest that is, m=N, the intermediate good market disappears. Thus, we may set $\alpha(N)=\beta(N)=\gamma(N)=0$ (the game reduces to a N-firm symmetric Cournot game where the marginal cost of each firm is given by $\tilde{e}c^*$). From (10) and (16):

$$ep^* - ec^* = \frac{1}{(m+1)} \left[1 - \frac{(N+1)b\gamma(m)}{(N-m)} \right] (a - ec^*) \ge 0,$$
 (17)

and, using (6), (9) and (17), the profit of firm j from the reduced form game in the second stage (with m investing firms and given e) is:

$$\Pi_{j}(m|e) = \frac{(\gamma(m))^{2}}{(N-m)^{2}} (a-ec^{*})^{2} > 0.$$
(18)

Using (5), (8), (15) and (17), the reduced form profit of firm i is:

$$\Pi_i(m|e) = \Omega(m) (a - ec^*)^2 > 0,$$
 (19)

where $\Omega(m)$ is given by:

$$\Omega(m) = \left[\frac{1}{b(m+1)}\right] \left[\frac{(1-b\gamma(m))^2}{(m+1)} - b\beta(m)\left(1 - \frac{b\gamma(m)(N+1)}{(N-m)}\right)\right],$$

for m = 1,...N. Note that $\Omega(m)$ is independent of e. It can be checked that $\Omega(m)$ is strictly positive. If m = 0 that is, no firm invests, then the profit of an investing firm is not defined. This complete our derivation of the equilibrium in stage 2.

4. Equilibrium of Reduced Form Investment Game

In this section, we analyze the strategic foreign investment decisions of the N final good firms, where the payoff from each profile of investment decisions is the expected profit from stage 2 equilibrium, as derived in the previous section, net of investment cost. At the point of time in which

the N domestic final good firms make their investment decisions, exchange rate uncertainty is not yet resolved. Suppose that \bar{m} ($0 < \bar{m} < N$) firms invest. Then, using (19), the payoff to the investing firms $i = 1, 2, ... \bar{m}$, denoted by $R_i(\bar{m})$ is given by:

$$R(\overline{m}) = E\Pi(\overline{m}|\tilde{e}) - f = \Omega(\overline{m})E[(a-\tilde{e}c^*)^2] - f, \tag{20}$$

where E is the expectation operator with respect to the uncertainty caused by exchange rate \tilde{e} . Using (18), the payoff to a non-investing firm $j = 1, 2, ..., N - \bar{m}$, denoted by $R_j(\bar{m})$, is given by:

$$R_{j}(\overline{m}) = E \prod_{j} (\overline{m} | \overline{e}) = \frac{(\gamma(\overline{m}))^{2}}{(N - \overline{m})^{2}} E[(a - \overline{e}c^{*})^{2}]. \tag{21}$$

If $\overline{m} = N$, that is, all firms invest, then the payoff to each firm i is also obtained from (20). If $\overline{m} = 0$, that is, no firm invests, the payoff to each firm j is given by (21).

A Nash equilibrium of this game where exactly \bar{m} firms, $\bar{m} = 1, 2, ...N-1$, undertake foreign investment exists if and only if, given the investment decisions of other firms, two conditions hold:

(a) a firm which invests cannot gain by deviating, that is, not investing (in which case there would be only $(\bar{m}-1)$ investing firms):

$$R_i(\overline{m}) \ge R_i(\overline{m}-1), \tag{22}$$

(b) a firm which does not invest cannot gain by deviating, that is, by undertaking to invest (in which case there would be $(\bar{m}+1)$ investing firms):

$$R_i(\overline{m}) \ge R_i(\overline{m}+1). \tag{23}$$

An equilibrium where all N firms invest exists if and only if (22) holds at $\bar{m} = N$. An equilibrium where no firm invests exists if and only if (23) holds at $\bar{m} = 0$.

For m = 0,1,...N-1, let $\phi(m)$ denote the "gain from investment" that is, the increase in expected profit (gross of the fixed cost of investment) of a firm which changes its decision from "no investment" to "investment", in a situation where there are exactly m other investing firms. More formally:

$$\phi(m) = E(\prod_{i} (m+1|\tilde{e})) - E(\prod_{j} (m|\tilde{e})), \qquad (24)$$

which, using (18) and (19), implies:

$$\phi(m) = \left[\Omega(m+1) - \frac{(\gamma(m))^2}{(N-m)^2}\right] E\left[(a-\tilde{e}c^*)^2\right]. \tag{25}$$

It can be verified that $\phi(m)$ is strictly positive for m = 0,1,...N-1. The necessary and sufficient conditions (22) and (23) for Nash equilibrium can now be rewritten in terms of a relation between the gain from investment $\phi(m)$ and the fixed cost of investment f:

Proposition 1. In a Nash equilibrium of the reduced form investment game, exactly \bar{m} firms, $\bar{m} = 1,2,...N-1$, undertake foreign investment if and only if:

$$\phi(\overline{m}) \leq f \leq \phi(\overline{m}-1) \tag{26}$$

There exists an equilibrium where all firms invest $(\bar{m} = N)$ if and only if.

$$f \leq \phi(N-1) \tag{27}$$

There exists an equilibrium where no firm invests $(\bar{m} = 0)$ if and only if:

$$f \geq \phi(0) \tag{28}$$

In our model, the number of firms which invest in equilibrium reflects the flow of vertical foreign investment. The fixed cost of investment, in part, reflects barriers to foreign direct investment. It is of interest to see how one can characterize the flow of foreign investment as a function of the level of fixed cost f, given the function $\phi(m)$. Note that if $\phi(m-1) < \phi(m)$ for some m, then (26) cannot hold at $\overline{m} = m$, for any level of f. This means that there is no equilibrium with exactly m firms investing. In such situation, what can we say about the amount of investment if we know that f exceeds or lies below $\phi(m)$ for a certain m? The following result is useful in this context:

Proposition 2. (i) For m = 1, 2, ...N, if $f \ge \phi(m-1)$, then there exists at least one equilibrium where strictly less than m firms invest;

(ii) For, m = 0, 1, 2, ...N-1, if $f \le \phi(m)$, then there exists at least one equilibrium where strictly greater than m firms invest.

Proof: (i) If $f \ge \phi(0) = \phi(1-1)$, then $\bar{m} = 0$ is an equilibrium which proves the proposition. So consider the case where $f < \phi(0)$ and suppose that, contrary to the proposition, there is no equilibrium with $\bar{m} = 0,1,2,...$ m-1. Using Proposition 1, as (26) does not hold for $\bar{m} = 1,2,...m-1$, one can show by

induction (starting from $f < \phi(0)$), that $f < \phi(\bar{m}-1)$, $\bar{m} = 1,2,...m$ which, in particular, implies that $f < \phi(m-1)$, a contradiction.

(ii) If $f \le \phi(N-1)$, then $\bar{m} = N$ is an equilibrium which proves the proposition. So consider the case where $\phi(N-1) < f \le \phi(m)$ for some m < N-1 and suppose that, contrary to the proposition, there is no equilibrium with $\bar{m} = m+1, m+2,...N$. Using Proposition 1, as (26) does not hold for $\bar{m} = m+1,...N$, it follows by induction (start from $f > \phi(N-1)$), that $f > \phi(\bar{m})$, $\bar{m} = N-1, N-2,...m$, a contradiction. //

The intuition behind Proposition 2 is simple. As we have said earlier, $\phi(m-1)$, m=1,2,...N, reflects the additional gain from becoming an investor when (m-1) firms are already investing. If f exceeds this additional gain, it appears reasonable that in equilibrium less than m firms should be investing. On the other hand, if f falls below this level, more than (m-1) firms should be investing. It therefore stands to reason that as the barriers to investment go down, the flow of investment should increase. One has to be a bit careful in stating this however, as equilibrium is not necessarily unique. For f > 0, let M(f) be the set defined by:

 $M(f) = \{\bar{m}: \bar{m} \text{ firms undertaking foreign investment is an equilibrium, given } f\}$

Proposition 3. If $f_1 < f_2$, then

- (i) if $m_1 \in M(f_1)$, then there exists $m_2 \in M(f_2)$ such that $m_2 \le m_1$;
- (ii) if $m_2 \in M(f_2)$, then there exists $m_1 \in M(f_1)$ such that $m_1 \ge m_2$;

Proof. (i) If $m_1 = N$, then the statement holds by definition. So consider the case where $m_1 < N$. From Proposition 1, (26) and (28) imply that $f_2 > f_1 \ge \phi(m_1)$. Using Proposition 2, we obtain that there exists at least one equilibrium at $f = f_2$, where less than m_1 firms invest.

(ii) If $m_2 = 0$, the statement holds by definition. So consider the case where $m_2 > 0$. From Proposition (1), (26) and (27), it follows that $f_1 < f_2 \le \phi(m_2 - 1)$. Using Proposition 2, $f_1 < \phi(m_2 - 1)$ implies that there exists at least one equilibrium at $f = f_1$, where more than m_2 firms invest. //

One of the implications of Proposition 3 is that the maximum and the minimum number of firms which invest in equilibrium at any level of f, decreases as f increases. Further, if equilibrium is unique at f_1 and f_2 , the number of firms investing at f_1 is always at least as large as the number of firms investing at f_2 ; for values of f close to certain critical values, an increase in f can lead to a strict decline in the number of firms investing.

Consider the behaviour of the function $\phi(m)$. Suppose that the function ϕ is decreasing over integer values of m going from 0 to (N-1), that is

$$\phi(0) \ge \phi(1) \ge \dots \ge \phi(N-1) \tag{29}$$

As more and more other firms invest, the additional gain from becoming an investor, decreases. In this case, the correspondence M(f) is very well behaved. The range of possible values of f can be divided into a finite chain of intervals $\{[0,\phi(N-1)],[\phi(N-1),\phi(N-2)],...,[\phi(1),\phi(0)],[\phi(0),\infty)\}$, non-intersecting except at end points. The equilibrium number of firms is unique in the interior of each interval. As the value of f moves from a higher to a consecutive lower interval, the number of firms investing in Nash equilibrium goes up by exactly one. ¹⁰

However, $\phi(m)$ is not necessarily decreasing; it can also be increasing in m over a subset of the possible values of m. The behaviour of $\phi(m)$ depends on the expected profit from the product market accruing to investors and non-investors corresponding to various configurations of investment. If $\phi(m)$ is monotonic decreasing as in (29), then the market profits are such that investment by firms are strategic substitutes; more other firms invest, the less worthwhile it is for a non-investor to invest. This is actually something one would normally expect. On the other hand, $\phi(m)$ increasing over a range of values of m implies that there is strategic complementarity between investment decisions of firms. As the number of firms which invest goes up, there might actually be greater incentive for a non-investor to become an investor. The reason behind this is that the strategic actions of the investing firms in the upstream market designed to raise the intermediate good price might become more damaging as the number of investing firms increases, so that the relative disadvantage of being a non-investor increases 11 .

There are two interesting features of foreign investment flows that can occur as a consequence of strategic complementarity of investment that is, of ϕ increasing in m. The first feature is that there can be a relatively large increase in the amount of foreign investment when f falls just below a critical level. To see this, suppose ϕ exhibits the following non-monotonic behaviour:

$$\phi(N-1) < \phi(N-2) < \dots < \phi(\bar{m}) < \phi(\bar{m}-1) > \phi(\bar{m}-2) \dots > \phi(0) \quad (30)$$

¹⁰Note that in this case, for any $\bar{m} = 0,1,2,...N$, there is always a non-empty interval such that if f lies in this interval, it is an equilibrium for exactly \bar{m} firms to invest. This is not necessarily so if ϕ is not decreasing in m.

^{I I}If our model is modified such that investing firms are not allowed to make strategic purchase upstream in order to raise non-investing rivals' costs, that is, we put an exogenous constraint $x_i \le 0$, then it can be shown that $\phi(m)$ is always decreasing in m.

For $f > \phi(\bar{m}-1)$, the only equilibrium is that no firm invests. However, if f is reduced to a level just below $\phi(\bar{m}-1)$, there is an equilibrium where the number of firms investing is \bar{m} which implies a jump in the flow of foreign investment, the size of the jump depending on how large \bar{m} is. The second interesting consequence of strategic complementarity is the possibility of multiple equilibria. In the situation described by (30), one can easily check (using Proposition 1) that for $f \in [\phi(0), \phi(\bar{m}-1)]$, there are two equilibria, one in which no firm invests and the other in which at least \bar{m} firms invest. The fact that there can be multiple equilibria, some with large investment flows and others with little or none, brings out the importance of co-ordination devices as an instrument of securing a boost in foreign direct investment. This also illustrates the so-called bandwagon effect or herding in investment behaviour; firms invest when many others decide to do so, but would rather refrain from investing otherwise. If investment decisions exhibit strategic complementarity globally that is, the function $\phi(m)$ is always increasing in m:

$$\phi(0) \le \phi(1) \le \dots \le \phi(N-1) \tag{31}$$

then we have an extreme form of herding. There are just two possible equilibrium outcomes viz., either all firms invest or no firm invests. For $f > \phi(0)$, the only outcome is that no firm invests whereas for $f \le \phi(0)$, $\bar{m} = 0$ and $\bar{m} = N$ are the two equilibrium outcomes. This implies that as f just falls below $\phi(0)$, there can be a really big jump in investment. However, there is a co-ordination problem. Firms will invest only if they think *all* their rivals will do so.

Figures 1(a)-(c) illustrate the function $\phi(m)$ for different parameter value configurations. In all three figures N=10, a=100, b=1, $c^*=1$ and \tilde{e} is taken to be deterministic with value equal to 1. Figure 1(a) depicts the situation when $m^*=5$. Here $\phi(m)$ is decreasing in m everywhere as in (29) and investment decisions are strategic substitutes. Figure 1(b) depicts the case where $m^*=23$. In this case, $\phi(m)$ is increasing for small values of m and decreasing thereafter, as in (30). Figure 1(c) depicts the situation where $m^*=30$. In this case, $\phi(m)$ is strictly increasing in m everywhere so that investment decisions are strategic complements globally.

For the case where N = 2, it is easy to verify that $\phi(0) < \phi(1)$ i.e. the situation depicted in (31) obtains for any configuration of parameter values if $m^* > 4$, while at $m^* = 1$, it is always the case that $\phi(0) < \phi(1)$ i.e. the situation depicted in (29) obtains. The general conclusion obtained by plotting the function $\phi(m)$ for various alternative parameter configurations is that, other things being equal, for low

values of m^* investment decisions are strategic substitutes while for m^* large enough, they are strategic complements.

The intuition behind this is that as m^* increases, the intermediate good market becomes less concentrated so that the purely "cost-reducing" motivation for vertical investment becomes weaker. In such situation, if final good firms were not allowed to raise their rival's cost by strategic purchase through their subsidiaries, there might not be any vertical investment. However, as investing firms are allowed to undertake strategic purchase, the relative disadvantage of not investing increases as more and more rival firms invest upstream and undertake vertical foreclosure.

5. Effect of Exchange Rate Uncertainty

The expected profits of final good firms resulting from any given investment profile (contained in (20) and (21)) depend on the random exchange rate. We have assumed that firms are ex ante risk neutral, that is, maximize expected profits. The expression for expected profit (gross of the fixed cost of investment) of any final good firm, whether or not the firm invests, is multiplicatively separable into two terms - one of which is positive and independent of \tilde{e} and the other is $E[(a-\tilde{e}c^*)^2]$. Observe that $[(a-\tilde{e}c^*)^2]$ is a strictly convex function of \tilde{e} . Therefore, $E[(a-\tilde{e}c^*)^2]$, and hence the expected profit, increases whenever, *ceteris paribus*, there is a second-order stochastic decrease (for example, an increase in the mean-preserving spread) of the distribution of the exchange rate. A second order stochastic decrease is equivalent to an increase in risk (see Rothschild and Stiglitz (1970)). It follows that if the forward exchange rate market is unbiased (forward rate is equal to the expected value of \tilde{e}), final good firms will prefer not to hedge against the exchange rate risk. To summarize:

Proposition 4. Given any profile of investment decisions, an increase in exchange rate risk (a second order decrease in the distribution of exchange rate) leads to an increase in the expected profit for both investing as well as non-investing domestic firms. If a forward market exists and is unbiased, final good firms will not hedge against the exchange rate risk.

The basic intuition behind Proposition 4 is that with an increase in the spread of the exchange rate, the profitability of production increases when there is a favourable realization of the exchange rate and this increase is stronger than the decrease in profitability in case of relatively unfavourable realizations of the exchange rate. Now, an increase in exchange rate variability increases the expected net profit of both investing as well as non-investing firms. Therefore, it is not immediately clear what

the net effect on the equilibrium investment profile is. Obviously, the relevant consideration here is the gain in expected profit when a non-investor becomes an investor, which means that we should be looking at the effect of increase in exchange rate risk on the function $\phi(m)$.

To look at ϕ as a function of both m and \tilde{e} , we can rewrite (25):

$$\phi(m,\tilde{e}) = \left[\Omega(m+1) - \frac{(\gamma(m))^2}{(N-m)^2}\right] E\left(a - \tilde{e}c^*\right)^2. \tag{32}$$

Note that the term in square bracket on right hand side of (32) is positive and independent of \tilde{e} . So exchange rate uncertainty affects the gain from investment only through the multiplicative term $E[(a-\tilde{e}c^*)^2]$. As discussed at the beginning of the section, an increase in exchange rate risk increases $E[(a-\tilde{e}c^*)^2]$. Thus, for any m < N, the gain from undertaking foreign investment $\phi(m)$ increases with any increase in exchange rate variability. More specifically, consider two possible exchange rate situations and let \tilde{e}_h denote the random exchange rate in these situations (h = 1, 2). Assume that the distribution of the exchange rate in situation 1 has a second-order stochastic dominance over that in situation 2. Situation 2 is one of greater exchange rate risk. Then,

$$\phi(m, \tilde{e}_1) \le \phi(m, \tilde{e}_2), \ m = 0, 1, \dots N-1. \tag{33}$$

It follows intuitively that the equilibrium number of firms investing in situation 2 should be at least as large as that in situation 1, for any given level of f. As discussed in the previous section, our model contains possibility of multiple equilibria and so the exact sense in which the equilibrium number of firms goes up must be carefully specified. Recall that we used the notation M(f) to denote the set of equilibrium outcomes (in terms of the number of firms that undertake investment) when fixed cost of investment equals f. To bring in the effect of exchange rate variability, let us now denote it by $M(f, \tilde{e})$. We want to see how, for any given f, the sets $M(f, \tilde{e})$ and $M(f, \tilde{e})$ compare.

Proposition 5. Suppose \tilde{e}_1 has a second order stochastic dominance over \tilde{e}_2 . Then,

(i) for any $\mu_1 \in M(f, \tilde{e}_1)$, there exists $\mu_2 \in M(f, \tilde{e}_2)$ such that $\mu_2 \ge \mu_1$;

(ii) for any $\mu_2 \in M(f, \tilde{e}_2)$, there exists $\mu_1 \in M(f, \tilde{e}_1)$ such that $\mu_2 \ge \mu_1$.

Proof. (i) Suppose $\mu_1 \in M(f, \tilde{e}_1)$. If $\mu_1 = 0$, then it is clear that there must be some $\mu_2 \in M(f, \tilde{e}_2)$, such that $\mu_2 \ge \mu_1$. If $\mu_1 > 0$, then it follows from (26) and (27) that:

$$f \le \phi(\mu_l - l, \tilde{e}_l) \tag{34}$$

which, using (33), implies that

$$f \le \phi(\mu_1 - l, \tilde{e}_2) \tag{35}$$

Proposition 2 implies then that there exists an equilibrium in situation 2 where at least μ_1 firms invest. (ii) Suppose $\mu_2 \in M(f, \tilde{e}_2)$. If $\mu_2 = N$, then it is clear that there must be some $\mu_1 \in M(f, \tilde{e}_1)$, such that $\mu_2 \ge \mu_1$. If $\mu_2 < N$, then it follows from (26) and (27) that:

$$f \ge \phi(\mu_2 - l, \tilde{e}_2) \tag{36}$$

which, using (33), implies that

$$f \ge \phi(\mu_2 - l, \tilde{e}_l) \tag{37}$$

Proposition 2 implies that there is an equilibrium in situation 1 where less than μ_2 firms invest. //

One of the implications of Proposition 5 is that for any given f, both the maximum as well as the minimum number of firms which invest in equilibrium is non-decreasing in exchange rate risk.

If \tilde{e}_1 has a *strict* second order dominance over \tilde{e}_2 , then (using the fact that $(a-\tilde{e}c^*)^2$ is strictly convex in \tilde{e}) from (32):

$$\phi(m, \tilde{e}_1) < \phi(m, \tilde{e}_2), \ m = 0, 1, ... N-1.$$
 (38)

This implies that, compared to situation 1, there is a strictly greater incentive to invest in situation 2 in terms of the additional gain from investment at any given level of m. It is intuitive, therefore, that for certain intervals of values of f, there will be strictly greater foreign investment under situation 2. **Proposition 6.** Suppose \tilde{e}_1 has a <u>strict</u> second order stochastic dominance over \tilde{e}_2 . Then, for m = 0,1,2,...N-1 and $f \in [\phi(m,\tilde{e}_1), \phi(m,\tilde{e}_2)]$, there always exist μ_1,μ_2 where $\mu_1 \leq m < \mu_2, \mu_1 \in M(f,\tilde{e}_1)$ and $\mu_2 \in M(f,\tilde{e}_2)$, that is, one can always select a pair of equilibria, one for each exchange rate situation, such that not more than m firms invest in the equilibrium in situation 1 and strictly greater than m firms invest in the equilibrium pertaining to situation 2.

Proof. Consider m < N and $f \in [\phi(m, \tilde{e}_l), \phi(m, \tilde{e}_2)]$. From Proposition 2, we can see that as $f \ge \phi(m, \tilde{e}_l)$ $= \phi((m+1)-1, \tilde{e}_l)$, there exists an equilibrium at fixed cost level f in situation 1 where strictly less than m+1 firms invest. Thus there exists $\mu_1 \in M(f, \tilde{e}_l)$ where $\mu_1 \le m$. Similarly, as $f \le \phi(m, \tilde{e}_2)$, from Proposition 2 we have that there exists $\mu_2 \in M(f, \tilde{e}_2)$ where $\mu_2 > m$.

Proposition 5 states that an increase in exchange rate volatility does not decrease the total volume of vertical foreign investment. Proposition 6 indicates that for certain intervals of values for the level of fixed cost, there is a strict increase in the level of foreign investment following a strict increase in exchange rate risk.

It is important to note that in our model, an increase in the number of investing firms is always associated with an increase in the total final good output in domestic market. This is because

as more firms invest upstream, these final good firms are able to produce their output at lower net marginal cost; the weighted marginal cost of production across the industry declines. This, as in other standard Cournot competition models, is passed on to consumers in the form of lower price, greater consumption and increased consumer surplus. Increase in total output of the final good implies a direct increase in the total amount of the intermediate good imported from abroad. Thus, increased exchange rate variability leads to a fall in domestic final good price, an increase in domestic consumer surplus and most importantly, an increase in total volume of trade. The net effect on social surplus is ambiguous as it is influenced (somewhat arbitrarily) by the level of fixed cost of investment.

6. Conclusion

The effect of flexible exchange rates on investment strategies of multinational firms is often raised in public debates. A popular conjecture, advocated by governments and various international organizations, is that the current system of flexible exchange rates has prevented firms from benefitting from the international division of labour allowed for by the various rounds of multilateral tariff reductions. We show that this is not necessarily true. In our model, increase in exchange rate volatility has a positive effect on vertical foreign investment and on trade in intermediate goods. It is our understanding that models where investment is made by domestic intermediate good firms in foreign final goods production, also yields similar results.

We have also shown that the strategic incentive to undertake vertical foreign investment by oligopolistic downstream firms may increase as more firms invest, leading to possibility of herding in foreign investment decisions. A small decline in the fixed cost of investment can lead to a big jump in the flow of investment. There can be multiple equilibria some involving lot of firms investing and others, very few, thus opening up room for co-ordination efforts.

Our results are derived for the specific case of linear demand and cost functions. Under more general demand and cost structures, the effect of exchange rate uncertainty could be ambiguous or even reversed. Further investigation of this issue in more general models is needed. Another interesting extension, left open for future research, is to allow for two-way strategic vertical investments by international oligopolistic firms, located upstream as well as downstream.

¹²Vertical foreign investment also changes the composition of trade. For $x \ge 0$, total trade in the intermediate good is equal to Q = m(q - x) + (mx + (N - m)r); for x < 0, it is Q = mq + (N-m)r. In both expressions, the first term reflects intra-firm trade, the second arm's length trade.

Appendix

Glossary of Symbols

```
Ν
        = number of firms in the final good market
        = number of investing firms (subscript i)
m
N-m
       = number of non-investing firms (subscript i)
m*
        = number of foreign producers of the intermediate good (subscript k)
        = exchange rate (domestic price of foreign currency)
ē
        = domestic price of the final good
р
p*
        = foreign currency price of the intermediate good
        = foreign currency marginal cost of producing one unit of intermediate
c*
        = final good output of investing firm i
q_i
        = final good output of non-investing firm j
r_{j}
        = aggregate output of the final good
Q
        = firm i's net purchase of the intermediate good
x_i
        = output of foreign intermediate good by firm k
q_k^*
\varrho^*
        = aggregate output of the intermediate good
        = gross profit of investing firm i
Π,
        = net payoff of investing firm i
R_{i}
        = profit of non-investing firm i
\Pi_{i}
R_{i}
        = profit of non-investing firm j
        = gross profit of intermediate producer k
\Pi_{*}^{*}
        = fixed cost of vertical investment
f
M(f,\tilde{e}) = set of equilibrium outcomes in terms of the number of firms investing (also denoted by
           M(f)
```

= expected gain from investment (reduced form), gross of the fixed cost of investment,

 $\phi(m)$

when m rival firms invest.

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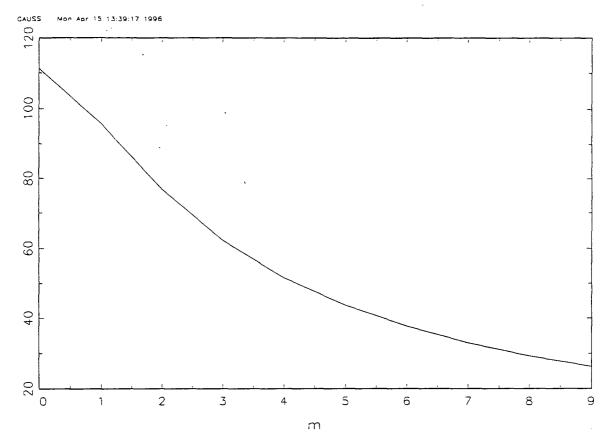


Figure 1(a): Investment Decisions are Strategic Substitutes; the function $\phi(m)$ is globally decreasing in m.

$$[N = 10, a = 100, b = 1, e = 1 \text{ (deterministic)}, c* = 1, m* = 5]$$

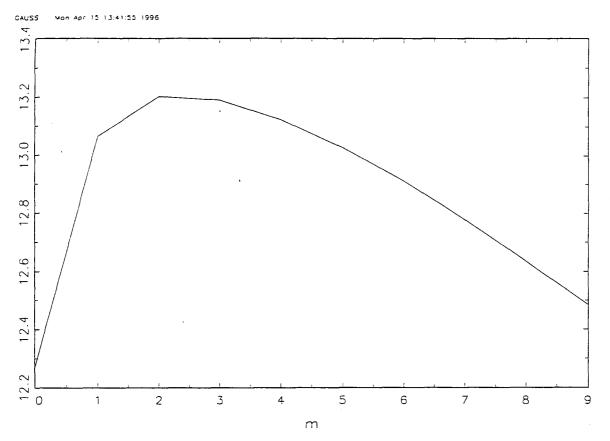


Figure 1(b): Investment Decisions are Strategic Complements when m is small; the function $\phi(m)$ is initially increasing in m. [N=10, a=100, b=1, e=1 (deterministic), c*=1, m*=23]

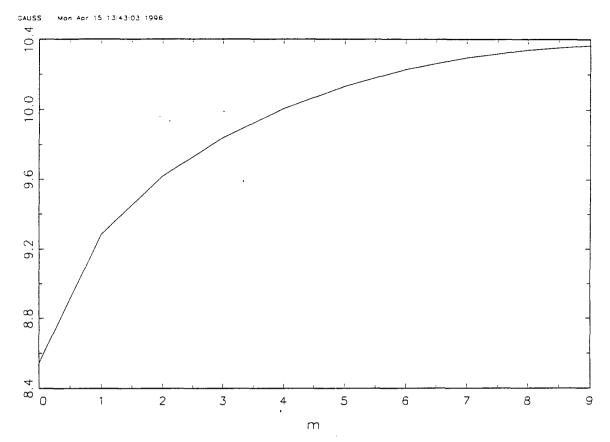


Figure 1(c): Investment Decisions are Strategic Complements Everywhere; the function $\phi(m)$ is globally increasing in m. $[N=10, a=100, b=1, e=1 \text{ (deterministic)}, c^*=1, m^*=30]$