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Tax coordination in a cross-hauling model: Conflict or harmony of interest?

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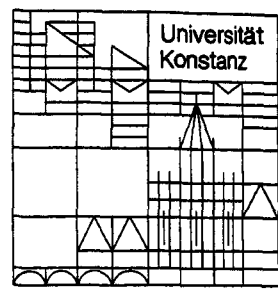
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**Sonderforschungsbereich 178
„Internationalisierung der Wirtschaft“**

Diskussionsbeiträge

Juristische
Fakultät

Fakultät für Wirtschafts-
wissenschaften und Statistik

Andreas Haufler

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Tax Coordination in a Cross-Hauling Model: Conflict or Harmony of Interest?

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Revised Version, October 1994

Abstract

The paper analyzes strategic commodity taxation in a model with trade in a single private good which is simultaneously imported by consumers of a high-tax region and exported by its producers. Conditions for the existence of a Nash equilibrium are given and an asymmetry is introduced through different preferences for public goods. Two tax coordination measures are discussed, a minimum tax rate and a coordinated increase in the costs of cross-border shopping. It is shown that tax coordination generally benefits the high-tax country while the low-tax region will gain only if the intensity of tax competition is high in the initial equilibrium or if governments are price-sensitive towards the effective marginal costs of public good supply.

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1 Introduction

It is a well-known problem of regional integration that tax or tariff differentials can give rise to trade flows which are exclusively based on tax considerations rather than comparative advantage. In the European Union (EU) the abolition of internal border controls has led to different tax treatments for imports of the same good, depending on whether the purchaser is a registered trader (destination principle) or a final consumer (origin principle), and has opened up the possibility that identical products are simultaneously exported and imported by a single country. As an example, British producers export beer to France while British consumers re-import their own beer via the ferry trade in order to save taxes.

Similar arbitrage possibilities have long existed for consumers in the United States where the enforcement of use taxes levied on out-of-state purchases is relatively difficult and mail ordering has become a growing source of concern [cf. Trandel (1992)]. Furthermore, even when the destination principle is technically in place for consumer purchases it may be severely undermined in practice. A recent example is the forced reversal of Canada's high-tax policy for cigarettes which proved unsustainable in view of the large tax differential to the United States. Canadian tobacco manufacturers exported their products net of tax to the United States from where they illegally re-entered the Canadian market, circumventing existing rules of destination¹.

Cross-border shopping simultaneously redistributes tax revenues from the high-tax to the low-tax region and affects overall efficiency. An obvious source of welfare losses lies in the real resources used up in shipping goods abroad and re-importing them via consumer purchases. Further (domestic) inefficiencies arise when both countries engage in strategic tax setting as a result of the fiscal externality. Possible tax coordination measures to reduce these welfare losses include minimum tax requirements imposed on the low-tax region and a tightening of rules of destination, for example in the field of mail-ordering. Both of these measures have been taken in the European Union and the last policy is also under active consideration in the United States [ACIR (1986)]. Since the viability of such coordination measures depends, in large part, on the consent of the low-tax region an important policy question is whether they can benefit both countries simultaneously.

¹According to estimates by the government of Quebec, two thirds of all cigarettes consumed in this province were bought illegally before tobacco tax reductions took effect in February 1994 (*The Globe and Mail*, January 29, 1994).

To discuss these issues, the literature on strategic commodity taxation has taken two rather different approaches. Mintz and Tulkens (1986) use a model with a general government objective and balanced trade in two goods. In their model, however, reaction functions are discontinuous and a Nash equilibrium cannot in general be proven to exist. Furthermore, while it can be shown under some additional restrictions that a tax rise in either country benefits the trading partner [de Crombrughe and Tulkens (1990)] a more detailed discussion of tax coordination measures is precluded by the complexity of the model. Alternatively, Kanbur and Keen (1993) use a one good partial equilibrium model and assume that governments behave as revenue maximizers. In this setting a rather optimistic result for the prospects of tax coordination is obtained: both a minimum tax requirement and an increase in the costs of cross-border shopping raise welfare in both countries since the mutual gains from reduced tax competition dominate purely redistributive effects. It is not clear, however, whether these results continue to hold under more general government objectives.

The present paper combines elements of both analyses. It simplifies the Mintz/Tulkens model in two respects: first, as suggested by the above examples, trade occurs in a single private good which is simultaneously imported by consumers of the high-tax country and exported by its producers. Thus cross-hauling of a homogeneous commodity occurs in the present model. Second, the level of cross-border shopping is directly determined from a consumer arbitrage condition. With this specification sufficient conditions for the continuity of reaction functions can be specified and a Nash equilibrium can be shown to exist. Furthermore, the simplicity of the model allows to derive comparative static results on the effects of alternative tax coordinations measures when governments care about both private and public consumption.

Much of the recent work on both commodity and capital tax competition [e.g. Bucovetsky (1991), Wilson (1991), Kanbur and Keen (1993), Trandel (1994)] has focused on the role of country size as a source of tax differentials. While many tax havens actually tend to be small countries the reverse is not true in general, i.e., not all small countries have low tax rates. In the EU context Denmark may serve as a counterexample that points to the relevance of economic “fundamentals” in the shaping of tax policy, in addition to country size. The present analysis assumes that different preferences for public versus private consumption are the underlying reason for interregional tax differentials. However, the main results derived in this paper carry over to other sources of tax asymmetries.

The paper is organized as follows: section 2 presents the model of tax competition and

analyzes the conditions under which a Nash equilibrium must exist. Section 3 introduces different preferences for public goods in the trading countries. Section 4 analyzes the effects of a minimum tax policy and a coordinated increase in the costs of cross-border shopping on optimal tax rates and welfare in each country. Section 5 briefly summarizes the results and suggests possible extensions.

2 Model Description and Nash Equilibrium

There are two countries H and L, which will later be identified as the high-tax and the low-tax country (or region). However, for the description of the Nash equilibrium in this section it is important that each region can, in principle, have the higher or the lower tax rate. This requires the additional use of indices $i \in [H, L]$ and $j \in [H, L]$, $i \neq j$. In each region there is a single representative consumer which is endowed with one unit of output (i.e., countries are of equal size). Output can either be used for private consumption c^i or for local public consumption g^i so that the marginal rate of transformation between these two goods is equal to one. Preferences are described by the strictly quasi-concave utility function

$$u^i(c^i, g^i) \quad \forall \quad i \in [H, L]. \quad (1)$$

To provide the public good, the government of region i purchases some fraction of total output, which is financed by a tax on private consumption². Interdependencies between the two country's tax choices arise through international trade in the private commodity. Producer trade follows the destination principle, as is true under both the European value-added tax and a retail sales tax as applied in the United States. In the absence of transportation costs producer arbitrage will equalize net-of-tax prices in the two countries, which can jointly be normalized $p^H = p^L \equiv 1$.

In contrast, cross-border purchases by final consumers are taxed at the rate of the origin country. Therefore, residents of the high-tax region have an incentive to purchase at least part of their demand for the private good abroad. This leads to a trade deficit in the high-tax country, which is balanced by producer exports. In the aggregate, each country will then import and export the same (homogeneous) good as a result of different

²The assumption that governments cannot impose other (e.g. income) taxes to finance the public good is, of course, restrictive. We will argue in section 4, however, that our results can be *interpreted* in a way that allows for alternative tax instruments.

tax arbitrage conditions for producers on the one hand and consumers on the other³. In the following we will use the term ‘cross-hauling’ to refer to the overall trade pattern in this model while ‘cross-border shopping’ is reserved for the consumer side of the exchange.

There are obvious constraints on consumer purchases abroad since residents of the high-tax country must either physically cross the border or place mail orders in order to take advantage of the lower tax rate. These constraints are captured by a strictly convex transaction (transportation) cost function. In partial equilibrium models convex transportation costs for cross-border shopping are often rationalized by assuming heterogeneous consumers which live at varying distances from the border [Kanbur and Keen (1993), Christiansen (1993)]⁴. In a representative consumer model one can alternatively think of a continuum of consumer goods, some of which are easily transported (or purchased through mail orders) while others impose virtually prohibitive transaction costs.

The transaction cost function is assumed to be continuously differentiable and there are zero marginal costs for purchasing the first unit abroad. Any tax differential will then lead to positive levels of cross-border shopping. Furthermore, we initially allow for country-specific differences in the transaction cost function. Thus

$$\tau_i' > 0, \tau_i'' > 0, \tau_i(0) = \tau_i'(0) = 0, \tau_i''(0) > 0 \quad \forall i. \quad (2)$$

Overall private consumption in country i consists of domestic purchases, c_i^i , and cross-border purchases, c_j^i (where $i, j \in [H, L]$ and $i \neq j$). If country i is the low-tax country its residents have no incentive to shop abroad so that

$$c^i \equiv c_i^i + c_j^i, \quad \begin{aligned} c_j^i &> 0 && \text{if } t^i > t^j \\ c_j^i &= 0 && \text{if } t^i \leq t^j. \end{aligned} \quad (3)$$

Since producer prices are equal to one in both countries residents of the high-tax country will cross-border shop until the tax savings are just compensated by the marginal transportation cost incurred. Therefore the following consumer arbitrage condition must hold in equilibrium

$$\tau_i'(c_j^i) = t^i - t^j \quad \text{if } t^i > t^j. \quad (4)$$

³A similar trade pattern underlies the analysis of trade deflection under the restricted origin principle [e.g. Georgakopoulos and Hitiris (1992)].

⁴Similarly, Mintz and Tulkens (1986, p. 137) argue that “individual consumers are responsible for their own transportation of commodities”.

Given the marginal transaction cost schedule equation (4) implicitly defines the level of cross-border shopping. Inverting gives

$$c_j^i(t^i - t^j) = (\tau_i)^{-1} \quad \text{if } t^i > t^j. \quad (5)$$

Cross-border shopping thus depends exclusively on the international tax *differential*, but not on the level of taxes in the two countries. In other words, an equal rise in t^i and t^j does not affect c_j^i . Equation (5) is consistent with the partial equilibrium treatment in Christiansen (1993) and Kanbur/Keen (1993). It differs, however, from the model of Mintz and Tulkens [1986, p. 139, eq. (7)], who determine c_i^i and c_j^i simultaneously as a function of the two tax *levels* (and public good supply). This difference will turn out to be crucial for the continuity of reaction functions.

Budget constraints for country i depend on whether its tax rate is higher or lower than that of the union partner. Let $t^i \geq t^j$ in regime I whereas $t^i \leq t^j$ holds in regime II. The regime-specific constraints for *private* consumption are given by

$$R I: \quad (1 + t^i) c_i^i + (1 + t^j) c_j^i + \tau_i(c_j^i) = 1, \quad (6)$$

$$R II: \quad (1 + t^i) c^i = 1. \quad (7)$$

In regime I, total expenditures by residents of country i include the resources used up in the process of cross-border shopping. These constitute a “pure waste” from a perspective of overall efficiency since producer trade has zero transportation costs by assumption⁵. However, the *private* budget constraint faced by country i 's residents is relaxed through cross-border shopping: from the convexity of the transaction cost function the sum of country j 's taxes and transaction costs is always less in an arbitrage equilibrium than if the same amount of goods were purchased in country i . In contrast, if country i is in regime II then its residents have no incentive to shop abroad and all private purchases occur in country i .

The *government* budget constraints are easily derived: in regime I, the tax base of country i is given by the domestic purchases of its own residents, which equals total private consumption less the amount purchased abroad. In regime II country i 's tax base consists of domestic purchases plus cross-border shopping by residents of country j

$$R I: \quad g^i = t^i c_i^i = t^i (c^i - c_j^i), \quad (8)$$

$$R II: \quad g^i = t^i (c^i + c_j^i). \quad (9)$$

⁵Similar efficiency losses arise in more conventional multi-good models when selective tariffs cause a substitution of higher (transportation) cost suppliers for lower cost suppliers [e.g. Melvin (1985)].

It is worth pointing out how trade redistributes tax revenues in this model. If country i is in regime I its private and government budget constraints [eqs. (6) and (8)] can be combined to

$$1 - c_j^i - g^i - \tau_i = (1 + t^j) c_j^i.$$

The left-hand side of this expression gives country i 's producer exports to country j , which equals the value of consumer imports on the right-hand side. Tax revenues are redistributed between countries since residents of the high-tax country i purchase their imports from country j on a gross-of-tax basis while balancing producer exports from i to j occur net of tax. These tax transfers increase the resource base of country j (which is in regime II), as can be seen by inserting (9) in (7).

In comparison to the Mintz/Tulkens (1986) model of trade in two private goods (a taxed commodity and leisure) the cross-hauling assumption employed here allows a much simpler representation of the consumer's optimization problem. More specifically, for a given transportation cost schedule private consumption depends only on the two tax rates t^i and t^j . In regime I, cross-border shopping $c_j^i(t^i - t^j)$ is obtained from (5). This in turn determines $c_i^i(t^i, t^j)$ from (6) and total private consumption $c^i(t^i, t^j)$ from (3). In regime II overall private consumption $c^i(t^i)$ is given from (7). Using the inverse function rule and $\tau_i' = t^i - t^j$ to differentiate (5) the effects of changes in t^i and t^j on the components of private consumption are summarized by

$$\begin{aligned} R I : \quad & \frac{\partial c_j^i}{\partial t^i} = \frac{1}{\tau_i''} > 0, \quad \frac{\partial c_i^i}{\partial t^i} = \frac{-c_i^i}{(1+t^i)} - \frac{1}{\tau_i''} < 0, \quad \frac{\partial c^i}{\partial t^i} = \frac{-c_i^i}{(1+t^i)} < 0, \\ & \frac{\partial c_j^i}{\partial t^j} = \frac{-1}{\tau_i''} < 0, \quad \frac{\partial c_i^i}{\partial t^j} = \frac{-c_j^i}{(1+t^i)} + \frac{1}{\tau_i''} < > 0, \quad \frac{\partial c^i}{\partial t^j} = \frac{-c_j^i}{(1+t^i)} < 0, \\ R II : \quad & \frac{\partial c^i}{\partial t^i} = \frac{-c^i}{(1+t^i)} < 0, \quad \frac{\partial c^i}{\partial t^j} = 0. \end{aligned} \quad (10)$$

In regime I, a tax rise in country i increases cross border shopping so that purchases in country i fall by more than overall consumption. In contrast, a rise in t^j reduces both cross-border shopping and overall private consumption. Therefore, an ambiguity arises for the derivative $\partial c_i^i / \partial t^j$, which determines the change in country i 's tax base in regime I. This effect is further discussed below. If country i is in regime II, a tax rise in country j has no effect on private consumption since no purchases are made in this country.

Given the effects of taxes on private consumption welfare-maximizing governments decide simultaneously on public good supply and the domestic tax rate. These decision variables are linked by the government budget constraint. Since each country $i \in [H, L]$

may, in equilibrium, be either in regime I or in regime II and optimization problems differ across regimes we first solve each government's problem conditional on a particular regime. In a second step the regime-specific solutions are compared to obtain the unconstrained solution to each country's optimization problem.

In regime I country i is constrained to set its tax rate at least as high as the exogenously fixed tax rate of country j . Incorporating the regime-specific government budget constraint (8) it maximizes the indirect utility function

$$\max_{t^i} V_I^i(t^i, t^j) \equiv u^i[c^i(t^i, t^j), t^i c^i(t^i, t^j) - t^i c_j^i(t^i - t^j)] \quad s.t. \quad t^i - t^j \geq 0. \quad (11)$$

Note that the tax differential in $c_j^i(t^i - t^j)$ is a functional argument, not a multiplication. If country i is in regime II it is constrained to set its tax rate below or equal to t^j . Incorporating (9) its optimization problem is

$$\max_{t^i} V_{II}^i(t^i, t^j) \equiv u^i[c^i(t^i), t^i c^i(t^i) + t^i c_j^i(t^j - t^i)] \quad s.t. \quad t^i - t^j \leq 0. \quad (12)$$

To solve the problems (11) and (12) we differentiate V^i with respect to t^i and use (10). This gives regime-specific reaction functions which implicitly define optimal tax rates. Note, however, that these tax rates are a solution to the regime-specific problems only if they fulfill the inequality constraints $t^i \geq t^j$ and $t^i \leq t^j$, respectively. Denoting marginal utilities by subscript letters the solution to (11) is thus given by

$$\frac{\partial V_I^i}{\partial t^i} \equiv R_I^i(t^{i*}, t^j) = \begin{cases} \frac{-t^{i*}}{\tau_{i''}} + \frac{c_i^i}{(1+t^{i*})} \left(1 - \frac{u_c^i}{u_g^i}\right) = 0 & \text{if } t^{i*} \geq t^j \\ t^{i*} = t^j & \text{otherwise.} \end{cases} \quad (13)$$

Similarly, the solution to (12) is

$$\frac{\partial V_{II}^i}{\partial t^i} \equiv R_{II}^i(t^{i*}, t^j) = \begin{cases} \frac{-t^{i*}}{\tau_{j''}} + c_j^i + \frac{c^i}{(1+t^{i*})} \left(1 - \frac{u_c^i}{u_g^i}\right) = 0 & \text{if } t^{i*} \leq t^j \\ t^{i*} = t^j & \text{otherwise.} \end{cases} \quad (14)$$

The best response functions (13)–(14) are conditional on a particular regime. Country i 's unconstrained best response function consists of those branches in (13)–(14) which yield the higher indirect utility level for any given t^j . We proceed by the following simple argument: in regime I, if the constraint $t^i \geq t^j$ were binding for country i then it would wish to undercut the given level of t^j and switch to regime II. Symmetrically, if the constraint $t^i \leq t^j$ were binding in regime II, optimizing behavior by country i 's government implies a switch to regime I. Therefore, the overall (regime-independent) best response function for

each country $i \in [H, L]$ consists of the implicitly defined upper parts of the regime-specific reaction functions (13)–(14).

Following Mintz and Tulkens (1986, p. 148) we denote the first term in (the relevant branches of) (13) and (14) as a *public consumption effect*, or tax base effect⁶. It isolates the reduction in country i 's tax base due to changes in the level of cross-border shopping. In both regimes this effect is negative for an increase in t^i by the convexity of the transaction cost function. Note, however, that in regime I it is the response of country i 's own residents and thus the marginal transportation cost schedule $\tau_i/(c_j^i)$ which determines the size of this effect while country j 's marginal cost schedule matters in regime II. In regime I the sign of the last term is thus unambiguously determined: in a non-cooperative equilibrium, the marginal rate of substitution (of g^i for c^i) must be less than one, thus indicating an undersupply of public goods in the high-tax country. A second fiscal externality, labelled a *private consumption effect*, occurs in regime II only: if residents of country j shop in country i there is an incentive for country i 's government to raise its tax rate in order to extract more revenues from foreigners⁷. Therefore, public goods may be under- or oversupplied in the low-tax country, depending on whether the public or the private consumption effect dominates in the Nash equilibrium.

The multi-valuedness of best response functions, and the resulting possibility that no Nash equilibrium exists, plays a major role in the Mintz/Tulkens (1986) analysis. Clearly, one critical point is where $t^{i*} = t^j$ and country i switches from regime I to regime II or vice versa. Country i 's reaction function will be single-valued in this point if and only if the relevant branches of (13)–(14) imply the same optimal tax rate $t^{i*} = t^j$.

Proposition 1: *The reaction function for each country can be continuous only if the slopes of the country-specific marginal transaction cost schedules are equal when the volume of cross-border shopping is zero, i.e. $\tau_i''(0) = \tau_j''(0)$.*

Proof: See the appendix.

The intuition for this result is straightforward: if conditions on both sides of the border

⁶To be precise, Mintz/Tulkens (1986) derive the fiscal externalities discussed here by differentiating country i 's objective with respect to t^i . However, they also note (p. 148) that the same effects are present in the optimal tax formulae.

⁷While there are obvious similarities to the more familiar terms of trade effects we avoid the latter term here because private consumption effects and terms of trade effects may simultaneously occur in the same model [e.g. Lockwood (1993)].

are asymmetric then country i 's government will perceive a different change in its tax base, depending on whether t^i is slightly above or below t^j . To stay within a framework of homogeneous consumers, if country j 's residents live close to the border while residents of country i live relatively far away, then the benefits of marginally undercutting country j 's tax rate are higher in absolute terms than the losses incurred by setting the tax rate slightly above t^j . As a consequence, the trade-off between domestic efficiency and a favorable international distribution of tax revenues changes discretely at this point, leading to a "jump" in the reaction function⁸.

Proposition 1 is, however, in contrast to the result by Mintz and Tulkens (1986, Proposition 5) that reaction functions *cannot* be continuous as country i moves from one regime to another. This can be traced to the private consumption effect as a further source of asymmetry in (13) and (14): if c_i^j is a function of the tax differential as in (5), and if $\tau_j'(0) = 0$ as assumed in (2), then cross-border shopping and the private consumption effect will both go to zero as t^i approaches t^j in regime II. In contrast, Mintz/Tulkens (1986) model cross-border shopping as a regular demand function $c_i^j(t^i, t^j)$ which "sets in" with a discrete value at the switch of regimes. In this case the private consumption effect will not disappear as t^i approaches t^j from below and there is a discrete gain for each country from moving slightly into regime II [cf. Mintz/Tulkens (1986, p. 152)]⁹.

In the following we assume identical transaction cost functions in both countries. This is sufficient to ensure continuity of the reaction functions in $t^i = t^j$ and in the special case of quadratic costs (which imply $\tau_i'' = \text{const.}$) it is also necessary.

Assumption 1: *Transaction cost functions are identical in both countries so that*

$$\tau_i'(c_j^i) = \tau_j'(c_i^j) \quad \forall i, j \in [H, L], i \neq j.$$

Note that Proposition 1 gives only a necessary but not a sufficient condition for *overall* continuity of the reaction functions since there may be discontinuities within each regime.

⁸In Kanbur and Keen (1993) differences in population densities ("country size") have very similar effects, and are responsible for the discontinuity of the small region's reaction function (which disappears when countries are equally large).

⁹Mintz and Tulkens (1986, p. 151, footnote 12) suggest that cross-hauling of *differentiated* goods may eliminate the discontinuity of best response functions in their model but argue that this would also make the model more complex. In contrast, cross-hauling of a homogeneous good occurs in the present model, simplifying rather than complicating the Mintz/Tulkens framework. Furthermore, as we have emphasized above it is not the cross-hauling assumption but the derivation of cross-border shopping in (5) which makes the continuity of best response functions possible in the present analysis.

In the following we will, however, be mainly concerned with small deviations from a symmetric equilibrium $t^i = t^j$, and assume best responses to be continuous in this range. We further assume that the second-order conditions of the constrained maximization problems (11)–(12) are fulfilled, i.e., the payoff functions V_I^i, V_{II}^i are quasiconcave in t^i . The second-order conditions are derived and discussed in the appendix.

Under these assumptions at least one Nash equilibrium must exist. If preferences are identical it follows from Assumption 1 that $t^i = t^j$ must be a solution to each country's optimization problem. Therefore, a symmetric Nash equilibrium exists in this case with the public good being undersupplied in both countries¹⁰. This symmetric Nash equilibrium can *not* occur in the analysis of Mintz and Tulkens (1986, p. 156), demonstrating that the continuity of reaction functions is not merely a technical problem but has important implications for model results. On the other hand, the existence of a symmetric Nash equilibrium with an undersupply of public goods in both countries is well known from the analysis of source-based taxes on capital income [e.g. Wilson (1991, pp. 429-430)].

3 Different Preferences for Public Goods

In this section we introduce different preferences for public vs. private consumption in the trading countries. In particular, assume that preferences for public goods are higher in country H as compared to country L. Starting from a symmetric equilibrium this can be modelled as a shift in country H's marginal rate of substitution, $\partial(u_c^H/u_g^H)/\partial\alpha < 0$, while preferences in country L are unchanged. To derive comparative static results in our model we must assume either uniqueness or stability of the initial Nash equilibrium [cf. Wilson (1991, p. 436) for a similar argument]. It is then straightforward to show

Proposition 2: *If differences between the countries' preferences for the public good are small, then there exists a Nash equilibrium in which the country with the higher preference for the public good has a higher tax rate.*

Proof: See the appendix.

¹⁰Note, however, that this symmetric equilibrium need not be the only one. A counterexample, which I owe to David de Meza, can be constructed by assuming fixed revenue requirements in both countries. In addition to the symmetric solution $t^i = t^j$ it is consistent with the two governments' objectives that country i has a high tax base and a low tax rate while the reverse is true in country j .

As shown in the appendix the first step in the proof is to demonstrate $dt^H/d\alpha^H > 0$, i.e., the optimal tax rate rises in the high-tax country when its preferences for the public good increase. It then remains to show that the shock to country H's preferences cannot raise t^L by more than t^H . The following relationship is derived in the appendix [eq. (A.8)]

$$\frac{\partial R^L}{\partial t^H} \frac{dt^H}{d\alpha^H} = - \frac{\partial R^L}{\partial t^L} \frac{dt^L}{d\alpha^H}. \quad (15)$$

From the second-order condition $\partial^2 V^i / (\partial t^i)^2 = \partial R^L / \partial t^L < 0$ it follows that $dt^H/d\alpha^H > dt^L/d\alpha^H$ will always be true when $\partial R^L / \partial t^H$ is negative. This corresponds to a negative slope of country L's reaction function in the initial equilibrium [see eq. (24) below]. If the initial equilibrium is symmetric t^L cannot rise by more than t^H , even if the slope of country L's reaction function is positive. In this case, however, the analysis applies strictly only to small parameter changes from a symmetric equilibrium.

Restricting the analysis to small differences in preferences, Proposition 2 gives us an asymmetric Nash equilibrium where country H is the high-tax country (and is in regime I) while country L is in regime II. The next step is to determine the effects of a change in one country's tax rate on welfare in the other country. Differentiating the objective functions V^i in (11) and (12) with respect to t^j and using the partial derivatives in (10) gives

$$\frac{\partial V^H}{\partial t^L} = u_c^H \left[\frac{-c_L^H}{(1+t^H)} \right] + u_g^H t^H \left[\frac{-c_L^H}{(1+t^H)} + \frac{1}{\tau_{II}} \right] < > 0, \quad (16)$$

$$\frac{\partial V^L}{\partial t^H} = u_g^L t^L \left(\frac{1}{\tau_{II}} \right) > 0. \quad (17)$$

By equation (17) a tax increase in the high-tax country increases welfare in the neighboring low-tax region. Private consumption in country L is unaffected while tax revenues rise as a result of increased cross-border shopping by residents of country H. Matters are more complicated for the high-tax region. As shown in (16) private consumption unambiguously falls while the tax base in country H, and thus the consumption of public goods, may rise or fall following the increase in t^L . The last effect depends on the elasticity with which cross-border shopping reacts to the reduced tax differential, and thus on the transportation cost schedule. To sign this effect we introduce

Assumption 2: *The transaction cost function is quadratic in the volume of cross-border shopping and given by $\tau = (1/2) \beta (c_L^H)^2$.*

A quadratic transaction cost function is a standard specification in similar models and underlies, e.g., the analysis in Kanbur and Keen (1993). It implies that cross-border

shopping is a linear function of the tax differential and $\tau'' = \beta$ is a constant. Using this assumption in (5) and (6) gives

$$c_L^H = \frac{(t^H - t^L)}{\beta}, \quad (18)$$

$$c_H^H = \frac{1}{(1 + t^H)} \left[1 - \frac{(t^H - t^L)}{\beta} - \frac{(t^H)^2 - (t^L)^2}{2\beta} \right], \quad (19)$$

$$c^H = c_H^H + c_L^H = \frac{1}{(1 + t^H)} \left[1 + \frac{(t^H - t^L)^2}{2\beta} \right]. \quad (20)$$

The effect of an increase in t^L on welfare in country H can now be signed. Inserting c_H^H from (19) into (16) yields after some manipulations

$$\frac{\partial V^H}{\partial t^L} = \frac{u_g^H}{\beta(1 + t^H)} \left[(t^H - t^L) \left(1 - \frac{u_c^H}{u_g^H} \right) + t^L(1 + t^H) \right] > 0, \quad (21)$$

which must be positive since country H undersupplies the public good in equilibrium. Under a quadratic transaction cost schedule an increase in t^L raises the tax base and public consumption in country H, as is seen from (19). Furthermore, this gain overcompensates the representative consumer in country H for the fall in her private consumption. The above discussion is summarized in

Proposition 3: *In an asymmetric Nash equilibrium the low-tax country gains from a tax increase in the high-tax region. If the transaction cost schedule is quadratic, then the high-tax country also gains from a tax increase in the low-tax region.*

Proposition 3 corresponds to a result by de Crombrugghe and Tulkens (1990, Proposition 1) who show that, in an asymmetric Nash equilibrium, a tax increase in either country benefits its union partner. The result by de Crombrugghe and Tulkens seems to be more general than the one derived here since it requires no assumptions on the shape of the transaction cost function. However, their proof rests on the assumption that the objective function is *concave* within each regime (1990, pp. 347–348). This assumption, which is not made in the original Mintz/Tulkens (1986) analysis, implies $\partial V^H / \partial t^L > 0$ whenever t^L is below the level that would maximize the utility of *country H*. But this will always be the case: if country H could choose both tax rates but were constrained not to leave regime I then it would always set $t^L = t^H$ and save both transportation costs and tax transfers to the union partner¹¹.

¹¹Formally, choosing both t^H and t^L leads to a global maximum of $V_I^H(t^H, t^L)$ [cf. de Crombrugghe/Tulkens (1990, p. 347)]. Solving this problem is equivalent to solving the tax problem of a closed economy, or the problem under a general destination principle which applies to all producer and consumer purchases.

4 Tax Coordination

We are now ready to discuss the effects of alternative tax coordination measures when governments care about both private and public consumption. In this analysis optimal tax rate adjustments to exogenous shocks depend partly on the change in the marginal rate of substitution u_c^i/u_g^i in response to changes in the ratio of public and private consumption g^i/c^i . This effect must always be positive along an indifference curve but ambiguities arise when the change in g^i/c^i simultaneously affects country i 's utility. We therefore introduce

Assumption 3: *Preferences are homothetic in both countries. The inverse of the elasticity of substitution between the public and the private good, σ^i , is then given by*

$$\frac{1}{\sigma^i} \equiv \frac{\partial [u_c^i(g^i, c^i)/u_g^i(g^i, c^i)]}{\partial (g^i/c^i)} \frac{(g^i/c^i)}{(u_c^i/u_g^i)} = \text{const.} \quad \forall \quad i \in [H, L].$$

With homothetic preferences σ^i is constant along each isoquant, and is equal across isoquants. Therefore, an increase in g^i/c^i will always increase u_c^i/u_g^i , even if country i simultaneously reaches a new indifference curve. A low value of σ^i implies that country i 's government prefers a largely fixed ratio of public over private consumption. In contrast, if σ^i is large then g^i/c^i will change substantially in response to exogenous shocks.

Stepping aside the confines of our model σ^i can also be interpreted as the elasticity of substitution between alternative sources of tax revenue. A low σ^i then indicates that the revenue requirement from *commodity* taxation is inflexible since other taxes (e.g. income taxes) cannot readily be adjusted. If σ^i is high the changes in g^i/c^i derived from the model can be interpreted as changes in the relative reliance on commodity taxes vis-a-vis other sources of government revenue.

Throughout this section we assume that the transportation cost function is quadratic and private consumption in country H is given by (18)–(20). For convenient reference we summarize here the effects of changes in t^j and the transportation cost parameter β on public and private consumption in country i :

$$\frac{\partial c_H^H}{\partial t^L} > 0, \quad \frac{\partial g^H}{\partial t^L} = t^H \frac{\partial c_H^H}{\partial t^L} > 0, \quad \frac{\partial c^H}{\partial t^L} < 0, \quad \frac{\partial c^L}{\partial t^H} = 0, \quad \frac{\partial g^L}{\partial t^H} = t^L \frac{\partial c_L^H}{\partial t^H} > 0, \quad (22)$$

$$\frac{\partial c_L^H}{\partial \beta} < 0, \quad \frac{\partial c_H^H}{\partial \beta} > 0, \quad \frac{\partial g^H}{\partial \beta} > 0, \quad \frac{\partial c^H}{\partial \beta} < 0, \quad \frac{\partial c^L}{\partial \beta} = 0, \quad \frac{\partial g^L}{\partial \beta} < 0. \quad (23)$$

4.1 Minimum Tax Rates

A first coordination measure, which has been implemented in the European Union, is a minimum tax requirement imposed on the low-tax country. If this constraint is binding, and if transportation costs are quadratic, then country H will gain by Proposition 3. If the tax increase is small for country L then – by Proposition 3 – the welfare change in the low-tax country depends on whether its tax increase lowers or raises the optimal tax rate in country H. Thus it is the slope of the high-tax country’s reaction function which determines the welfare change in country L. Of course, if the required increase in t^L is discrete rather than marginal then country L will, on impact, lose from the change in its *own* tax rate which forces it to move away unilaterally from the non-cooperative Nash equilibrium.

Applying the implicit function theorem to the best response function $R^i(t^i, t^j) = 0$ gives

$$\frac{dt^i}{dt^j} = - \frac{\partial R^i / \partial t^j}{\partial R^i / \partial t^i} = \text{sign}(\partial R^i / \partial t^j) \quad (24)$$

from the second-order condition $\partial R^i / \partial t^i < 0$. To determine the derivatives $\partial R^i / \partial t^j$ we differentiate (13)–(14) with respect to the tax rate of the union partner. This gives for country H (which is in regime I)

$$\frac{\partial R^H}{\partial t^L} = \underbrace{\frac{(1 - u_c^H / u_g^H)}{(1 + t^H)} \frac{\partial c_H^H}{\partial t^L}}_{(1) (+)} + \underbrace{\frac{-c_H^H}{(1 + t^H)} \frac{1}{\sigma^H} \frac{(u_c^H / u_g^H)}{(g^H / c^H)} \frac{\partial (g^H / c^H)}{\partial t^L}}_{(2) (-)} <> 0, \quad (25)$$

where individual effects are signed using (22). Similarly we get for country L (which is in regime II)

$$\frac{\partial R^L}{\partial t^H} = \underbrace{\frac{\partial c_L^H}{\partial t^H}}_{(1) (+)} + \underbrace{\frac{-c^L}{(1 + t^L)} \frac{1}{\sigma^L} \frac{(u_c^L / u_g^L)}{(g^L / c^L)} \frac{\partial (g^L / c^L)}{\partial t^H}}_{(2) (-)} <> 0. \quad (26)$$

The slope of both countries’ best response functions is thus ambiguous, in general. By Assumption 2, the second derivative of the transaction cost function is a constant so that, for each region i , the size of the negative public consumption effect (the “marginal costs” of a domestic tax increase) is independent of the other country’s tax rate. This allows to focus on how a change in t^j alters the “marginal benefits” of a tax increase in region i . For country H, the first effect is positive since it undersupplies the public good and its tax base increases from (22). Intuitively a higher t^L increases the amount of extra revenue that country H can collect from marginally raising its tax rate. The second effect is negative, however, since the increase in t^L raises government revenue in country H while reducing

private consumption. By Assumption 3 this will unambiguously raise the marginal rate of substitution (of g^H for c^H) and reduce the marginal benefits from redistributing purchasing power towards the public sector. For country L, the first effect is also positive since a rise in t^H increases the role of the private consumption effect, thus giving an incentive to country L to raise its own tax rate at the (partial) expense of foreigners. However, an increase in t^H simultaneously raises g^L from (22) so that the second effect is negative.

Note that if governments behave as revenue-maximizers the marginal utility of private consumption is zero and the negative second effects are eliminated in (25)–(26). Both reaction functions will then be upward sloping and a small mandated rise in country L’s tax rate will benefit both regions. This corresponds to Proposition 12 in Kanbur and Keen (1993). The same result is obtained when the elasticity of substitution is very high in both countries and the second effect is negligible. More generally, however, when governments care about the “right mix” between private and public consumption, the revenue inflow stemming from an increase in t^j (and, in the case of country H, the simultaneous reduction in private purchasing power) will by itself tend to cause a downward adjustment of country i ’s optimal tax rate, other things being equal. A limiting case is when governments have to raise a fixed level of revenues from commodity taxation, i.e., $\sigma^i \rightarrow 0$. In this case, the negative second effect always dominates and the slope of best response functions is unambiguously negative¹². The discussion is summarized in

Proposition 4: *If the elasticity of substitution is very large (small) in the high-tax country then a small mandated increase in t^L raises (lowers) welfare in the low-tax country.*

Proposition 4 can, informally, be extended to determine the effects of a *discrete* tax increase in country L. We have argued above that this policy will hurt the low-tax country at an unchanged level of t^H . For country L to gain from a discrete increase in its own tax rate the induced tax rise in country H must then be sufficiently large to compensate the negative impact effect. We turn now to a formal analysis of this more complex case in the context of a different coordination policy.

¹²The two polar cases of revenue maximizing governments on the one hand and fixed revenue requirements on the other are also discussed in Mintz/Tulkens (1986, p. 153).

4.2 Tightening Rules of Destination

A second coordination policy consists of legal or administrative measures which tighten the enforcement of the destination principle. Such measures may include stricter controls and penalties for illegal smuggling or the closing of loopholes as in the mail order industry. They are especially important when central governments do not have the legal powers to impose tax rate harmonization, as is the case for retail sales taxes set by U.S. states. The European Union has also tried to maintain rules of destination for consumer purchases wherever this was possible, as in the mail order industry or for purchases of new cars.

A tighter enforcement of the destination principle can be represented by raising the transaction cost parameter β for private consumer purchases abroad. Again, it has been shown by Kanbur and Keen (1993, Proposition 6) that this measure strictly benefits both countries when revenue maximization is the objective. In the present framework the national welfare effects of a change in the transaction cost parameter β can be decomposed into an impact effect (at unchanged tax rates) and the induced change in the tax rate of the other union country [cf. Dixit (1986)]. Algebraically,

$$\frac{dV^i}{d\beta} = \left. \frac{\partial V^i}{\partial \beta} \right|_{t^H, t^L = \text{const.}} + \frac{\partial V^i}{\partial t^j} \frac{dt^j}{d\beta} \quad \forall i, j \in [H, L], i \neq j. \quad (27)$$

It is straightforward to sign the impact effect for each country:

Proposition 5a: *At unchanged tax rates a rise in the marginal transaction cost schedule increases welfare in the high-tax country and lowers welfare in the low-tax country:*

$$\left. \frac{\partial V^H}{\partial \beta} \right|_{t^H, t^L = \text{const.}} > 0, \quad \left. \frac{\partial V^L}{\partial \beta} \right|_{t^H, t^L = \text{const.}} < 0.$$

Proof: See the appendix.

The intuition for this result is obvious for the low-tax country: an upward shift in the marginal transaction cost schedule reduces cross-border shopping by country H's residents and thus the tax base of country L. For the high-tax country, there are counteracting effects similar to the ones discussed in Proposition 3: private consumption falls as residents of country H are redirected towards their more expensive home market. However, with a quadratic transaction cost function the tax base in country H increases and the positive public consumption effect dominates the negative effect on private consumption.

It follows that for country L to gain from a coordinated rise in marginal transaction costs, the tax rate in country H must rise by enough to overcompensate the negative

impact effect. The changes in optimal tax rates in response to a variation in β are derived in the appendix and are given by

$$\frac{dt^i}{d\beta} = \frac{1}{\Delta} \left[-\frac{\partial R^j}{\partial t^j} \frac{\partial R^i}{\partial \beta} + \frac{\partial R^i}{\partial t^j} \frac{\partial R^j}{\partial \beta} \right] \quad \forall i, j \in [H, L], i \neq j, \quad (28)$$

where $\Delta > 0$ is shown in eq. (A.6). The first effect in (28) gives the direct response of country i 's optimal tax rate to the parameter change whereas the second effect describes the response to the induced change in the optimal tax rate of country j . Since $\partial R^i / \partial t^j$ has already been determined in (25)–(26) it remains to differentiate the best response functions (13)–(14) with respect to β . This gives for country H (regime I)

$$\frac{\partial R^H}{\partial \beta} = \underbrace{\frac{t^H}{\beta^2}}_{(1) (+)} + \underbrace{\frac{(1 - u_c^H / u_g^H)}{(1 + t^H)} \frac{\partial c_H^H}{\partial \beta}}_{(2) (+)} + \underbrace{\frac{-c_H^H}{(1 + t^H)} \frac{1}{\sigma^H} \frac{(u_c^H / u_g^H)}{(g^H / c^H)} \frac{\partial (g^H / c^H)}{\partial \beta}}_{(3) (-)} <> 0, \quad (29)$$

and for country L (regime II)

$$\frac{\partial R^L}{\partial \beta} = \underbrace{\frac{t^L}{\beta^2}}_{(1) (+)} + \underbrace{\frac{\partial c_L^H}{\partial \beta}}_{(2) (-)} + \underbrace{\frac{-c^L}{(1 + t^L)} \frac{1}{\sigma^L} \frac{(u_c^L / u_g^L)}{(g^L / c^L)} \frac{\partial (g^L / c^L)}{\partial \beta}}_{(3) (+)} <> 0, \quad (30)$$

where individual effects have been signed using (23). The first effect in both (29) and (30) is positive: a higher β makes cross-border shopping less responsive to tax changes, thus lowering the marginal costs of a domestic tax increase in both countries. Note that, for given tax rates t^H and t^L , this effect is strong when the initial value of β is low and tax competition through cross-border shopping is intense in the initial equilibrium. The benefits of a domestic tax rise are again ambiguous in both countries: for country H the reasoning is essentially the same as given in equation (25). The second effect is positive but the rise in β reallocates domestic purchasing power from the private to the public sector and the third effect tends to reduce the optimal tax rate. For country L the signs of the last two effects are now reversed in comparison to (26) because a rise in β lowers cross-border shopping by residents of country H. Thus the private consumption effect becomes less important and tends to reduce the optimal tax rate whereas the reduction in government revenues makes the third effect positive.

Equations (25)–(26) and (29)–(30) provide all the information needed to evaluate (28). Since each of these effects is ambiguous, general results – which hold for all parameter values – can clearly not be expected. However, when substitution elasticities are very low or very large best responses to the parameter change can be signed in both countries:

Table 1: Optimal Tax Changes after a Change in Transaction Costs

		$\partial R^i / \partial \beta$ eq. (29)–(30)	$\partial R^i / \partial t^i$ eq. (25)–(26)	$dt^i / d\beta$ eq. (28)
CASE 1 (low σ^i)	country H	–	–	–
	country L	+	–	+
CASE 2 (high σ^i)	country H	+	+	+
	country L	+	+	+

Proposition 5b: *CASE 1: If substitution elasticities are very low in both countries, then an increase in the transaction cost parameter β lowers the optimal tax rate in the high-tax country and raises it in the low-tax country.*

CASE 2: If substitution elasticities are very high in both countries and the low-tax country undersupplies the public good initially, then an increase in β raises the optimal tax rate in both countries.

Proof: See the appendix.

Optimal tax changes in response to the rise in transaction costs are summarized in Table 1. In case 1 the last effects in in (29)–(30) are very large due to low substitution elasticities and determine the signs $\partial R^H / \partial \beta < 0$ and $\partial R^L / \partial \beta > 0$. The direct effect of the increase in transportation costs [the first term in (28)] is therefore to lower t^H and raise t^L . This is intuitive since the tax base of country H expands and the tax base of country L shrinks following the rise in β and tax rates adjust to meet largely fixed revenue requirements. Furthermore, from Proposition 4 both reaction functions (25)–(26) are downward sloping in this case. Therefore, optimal tax responses to the induced change in the other country's tax rate [the second term in (28)] further reduce t^H (in response to the initial rise in t^L) and further increase t^L (in response to the initial fall in t^H).

In case 2 the last effects in both (29) and (30) are small due to very high substitution elasticities, and the other effects dominate. This is sufficient to sign $\partial R^H / \partial \beta > 0$ in (29). For country L, it is shown in the appendix that the positive first effect dominates the negative second effect if country L undersupplies the public good initially. In this case $\partial R^H / \partial \beta > 0$ and the direct effect of a rise in β will raise optimal tax rates in both (29)–(30). Furthermore, both reaction functions (25) and (26) are upward sloping in this case. Therefore indirect effects are also positive in both countries, reinforcing the direct effects.

Table 2: Welfare Effects of a Rise in Transaction Costs

		impact effect (t^H, t^L const.) $\partial V^i / \partial \beta$	induced change in t^j $(\partial V^i / \partial t^j)(\partial t^j / \partial \beta)$	overall effect $dV^i / d\beta$
CASE 1 (low σ^i)	country H	+	+	+
	country L	-	-	-
CASE 2 (high σ^i)	country H	+	+	+
	country L	-	+	?

Note that this case necessarily applies under the assumption of revenue maximizing governments: with $u_c^i = 0$ the partial effects given in (25)–(26) and (29) are all positive. Furthermore, country L always undersupplies the public good relative to its objective so that (30) will also be positive.

Using (27), the partial results in Propositions 5a and 5b can be combined to give the overall effect of an increase in β on welfare in each country:

Proposition 5c: *CASE 1: If substitution elasticities are very low in both countries, then an increase in the transaction cost parameter β raises welfare in the high-tax country and lowers welfare in the low-tax country.*

CASE 2: If substitution elasticities are very high in both countries and the low-tax country undersupplies the public good initially, then an increase in β raises welfare in the high-tax country. Welfare in the low-tax country rises if the gain from the induced tax increase in the high-tax country overcompensates the negative impact effect, and falls otherwise.

The individual effects underlying this proposition are shown in Table 2. The high-tax country unambiguously gains from the coordination measure in both cases. In case 1 the low-tax country unambiguously loses from a coordinated rise in β since both the impact effect and the effect of the induced change in t^H are negative. Therefore, there is a clear conflict of interest in this case. In case 2, the overall welfare change in country L is ambiguous, in general. The low-tax country will gain from tax coordination, and there is a common interest in raising the costs of cross-border shopping, if and only if the induced rise in t^H is strong enough to compensate the negative impact effect.

4.3 A Numerical Illustration

To illustrate, and to indicate the range of the different cases discussed, we present some simulation results using CES utility functions of the form

$$u^i = \left[s_i^{1/\sigma} (g^i)^{(\sigma-1)/\sigma} + (1 - s_i^{1/\sigma}) (c^i)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \quad \forall i \in [H, L].$$

The simulation is based on the following specification: the weights of the public good are assumed to be $s_H = 1/4$ and $s_L = 1/7$. In a closed economy this implies optimal tax rates of $t^{H*} = 0.333$ and $t^{L*} = 0.166$. Furthermore we exogenously constrain the elasticity of substitution to be equal across countries, i.e., $\sigma^H = \sigma^L$ are always changed simultaneously in the sensitivity analysis. Figure 1 plots optimal tax rates as a function of the transaction cost parameter β for alternative values of σ .

If the elasticity of substitution is low ($\sigma = 0.15$), the tax rate of country H rises initially in response to an increase in β , and falls as β is further increased. This can be explained as follows: for very low initial values of β , which indicate intense fiscal competition¹³, the “marginal cost” effect [the first effect in (29)] dominates and a rise in β will initially raise the optimal t^H . As β is further increased the “marginal benefit” effect [the third effect in (29)] prevails, as described in case 1 of Proposition 5b. In the intermediate case ($\sigma = 0.4$) the optimal t^H is virtually unchanged for all $\beta \geq 3$. Finally, it is monotonously increasing in β if the elasticity of substitution is relatively high ($\sigma = 1.0$). This last example thus corresponds to case 2 in Proposition 5b. It is also seen from Figure 1 that country L’s optimal tax rate is affected very little by changing the elasticity of substitution, and is a rising function of β in all the cases shown.

By Proposition 5c, a rise in country H’s tax rate is only a necessary but not a sufficient condition for the low-tax country to gain from a coordinated increase in the costs of cross-border shopping. Table 3 summarizes the *overall* welfare change in country L for alternative initial values of β and alternative substitution elasticities σ . (The welfare change in country H is positive for all the cases reported and is thus not listed explicitly.) These results demonstrate that the initial value of the transportation cost parameter is itself an important determinant of $dV^L/d\beta$, in addition to the substitution elasticities. Country L is likely to gain from coordination if the intensity of tax competition is high

¹³As an example, $\beta = 1$ implies [from eq. (18)] that a 15 percent tax differential leads to cross-border shopping of 0.15 in absolute value. For $t^H = 0.3$ this corresponds to roughly 20 percent of total private consumption in country H. In contrast, if $\beta = 10$ the ratio of cross-border shopping to overall private consumption induced by the same tax differential is only 2 percent.

Figure 1: Optimal Tax Rates

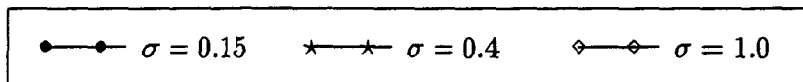
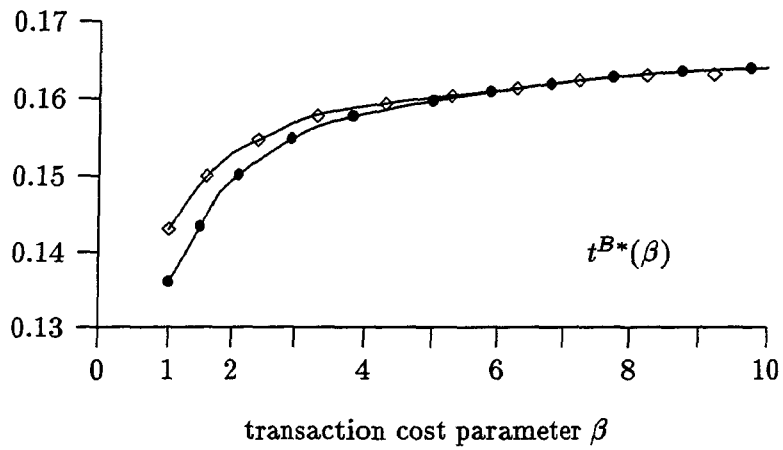
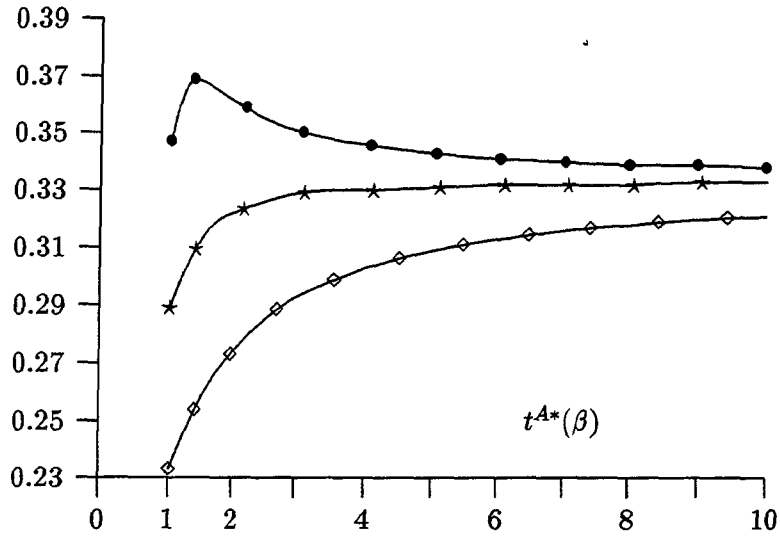


Table 3: Welfare Change in the Low-Tax Country: sign of $d\bar{V}^L/d\beta$

	$\beta = 0.5$	$\beta = 1.0$	$\beta = 2.0$	$\beta = 5.0$	$\beta = 10.0$
$\sigma = 0.15$	+	-	-	-	-
$\sigma = 0.4$	+	-	-	-	-
$\sigma = 1.0$	+	-	-	-	-
$\sigma = 2.0$	+	+	-	-	-
$\sigma = 5.0$	+	+	+	-	-
$\sigma = 10.0$	+	+	+	+	-

in the initial equilibrium, i.e., if β is low. The intuition for this result follows from our theoretical analysis: other things being equal, a low β will induce a strong tax increase in country H by the first effect in (29), which in turn benefits country L by Proposition 3. The second result apparent from Table 3 has already been discussed: the higher is the elasticity of substitution σ the more likely is the low-tax country to gain from an increase in consumer transaction costs.

Clearly, these are only illustrative examples. They indicate, however, under which conditions the mutual gains from reduced tax competition are likely to dominate redistributive effects: tax coordination tends to benefit both countries if fiscal competition is intense before the coordination measure is taken (low- β case) or if governments' tax choices respond elastically to the marginal costs of public good supply in the presence of cross-border shopping (high- σ case). Many observers of both EU and North American conditions have argued, however, that tax competition through cross-border shopping is moderate. If, in addition, governments are price-insensitive due to rigid revenue requirements and lack of flexibility to adjust other taxes the upper right area of Table 3 may be the relevant one. In this case the low-tax country stands to lose not only from a minimum tax requirement (Proposition 4) but also from a coordinated effort to increase the costs of cross-border shopping (Proposition 5). Some (reverse) redistribution is then likely to be involved in efforts to protect the tax base of the country with a high preference for public goods.

This may explain the insistence of the European Commission to achieve an agreement on coordination measures *before* the opening of internal borders. In this case the model also predicts conflicting interests between low- and high-tax states in North America and elsewhere if rules of destination were more tightly enforced for mail ordering and other

forms of cross-border shopping.

5 Concluding Remarks

This paper has introduced cross-hauling of a homogeneous private good to discuss tax competition and tax coordination in a simple trade model which captures the essential elements of cross-border shopping. The analysis has focused on the effects of tax coordination when optimal tax rates differ across countries due to different preferences for public goods. Two coordination measures were considered, a mandated tax increase in the low-tax region and a policy-induced increase in the costs of cross-border shopping. It has been shown by Kanbur and Keen (1993) that these tax coordination measures benefit both countries when revenue maximization is the objective. When governments care about both private and public consumption, however, the results are no longer unambiguous and there is a distinct possibility that the low-tax country loses from tax coordination.

More specifically, both a mandated increase in its own tax rate and a coordinated rise in the costs of cross-border shopping will lower welfare in the low-tax country if the tax rate in the high-tax country falls as a result of tax coordination. This is likely to be the case if one or both of the following conditions are fulfilled: (a) fiscal competition is moderate in the initial equilibrium due to relatively high costs of cross-border shopping and (b) the elasticity of substitution between private and public consumption (or, equivalently, between alternative sources of tax revenues) is low in the high-tax country due to largely fixed revenue requirements. In principle at least, both of these parameters should be accessible to empirical investigation in order to determine whether low-tax countries have an incentive to agree to coordination measures in a given regional and institutional setting.

Finally, a word on the limitations of our analysis is due. It is obvious that the simple model employed here has left out many potentially important effects. Of all the simplifications made the assumption of fixed endowments may be one of the most critical since it rules out all effects of tax competition and tax coordination on the efficiency of production. It is known from previous contributions [Trandel (1992), Keen and Lahiri (1994)] that tax competition may improve *overall* efficiency, and tax coordination may lower it, when firms operate in an imperfectly competitive environment. Whether this change in the basic framework also reverses the effects on *national* welfare – on which the present work has focused – is a question that we leave to future research.

Appendix

Proof of Proposition 1: We determine the limits of the two constrained reaction functions (13)–(14) as t^i approaches t^j . This gives

$$\begin{aligned}\lim_{t^i \rightarrow t^j} R_I^i(t^{i*}, t^j) &= \frac{-t^i}{\tau_i''(0)} + \frac{c^i}{(1+t^i)} \left(1 - \frac{u_c^i}{u_g^i}\right) = 0, \\ \lim_{t^i \rightarrow t^j} R_{II}^i(t^{i*}, t^j) &= \frac{-t^i}{\tau_j''(0)} + \frac{c^i}{(1+t^i)} \left(1 - \frac{u_c^i}{u_g^i}\right) = 0.\end{aligned}\quad (\text{A.1})$$

As the tax differential approaches zero, cross-border shopping goes to zero and the private consumption effect is eliminated. Furthermore, tax bases in both regimes are then identical. The only difference between the two limits in (A.1) then lies in the second derivatives of the country-specific transaction cost functions, evaluated at $t^k - t^l = 0$. \square

Second-Order Conditions of (11)–(12): Using Assumption 1 the second derivatives of (11)–(12) with respect to t^i are given by

$$\begin{aligned}\frac{\partial^2 V_I^i}{(\partial t^i)^2} \equiv \frac{\partial R_I^i(t^{i*}, t^j)}{\partial t^i} &= \frac{-\tau'' + (t^i \tau''' / \tau'')}{(\tau'')^2} - \frac{2c^i}{(1+t^i)^2} \left(1 - \frac{u_c^i}{u_g^i}\right) \\ &\quad - \frac{c^i}{(1+t^i)} \frac{\partial(u_c^i/u_g^i)}{\partial(g^i/c^i)} \frac{\partial(g^i/c^i)}{\partial t^i}\end{aligned}\quad (\text{A.2})$$

$$\begin{aligned}\frac{\partial^2 V_{II}^i}{(\partial t^i)^2} \equiv \frac{\partial R_{II}^i(t^{i*}, t^j)}{\partial t^i} &= \frac{-\tau'' - (t^i \tau''' / \tau'')}{(\tau'')^2} - \frac{2c^i}{(1+t^i)^2} \left(1 - \frac{u_c^i}{u_g^i}\right) \\ &\quad - \frac{c^i}{(1+t^i)} \frac{\partial(u_c^i/u_g^i)}{\partial(g^i/c^i)} \frac{\partial(g^i/c^i)}{\partial t^i}\end{aligned}\quad (\text{A.3})$$

To ensure that (A.2) and (A.3) are negative the following restrictions are sufficient: the first effect in both (A.2) and (A.3) will be negative if the transaction cost function is quadratic in the volume of cross-border shopping and $\tau''' = 0$. The second effect is negative if country i undersupplies the public good in the Nash equilibrium. This is always the case in regime I but need not be so in regime II. Finally, homothetic preferences ensure $\partial(u_c^i/u_g^i)/\partial(g^i/c^i) > 0$ (cf. Assumption 3) so that the third effects in (A.2) and (A.3) are negative.

Proof of Proposition 2: Let R be an equation system which consists of the two reaction functions (13)–(14) in generalized form

$$\begin{aligned}R^H(t^{H*}, t^L, \theta^H) &= 0, \\ R^L(t^H, t^{L*}, \theta^L) &= 0,\end{aligned}\quad (\text{A.4})$$

where θ^i are exogenous shift parameters. Totally differentiating (A.4) and inverting gives

$$\begin{bmatrix} dt^H \\ dt^L \end{bmatrix} = \frac{1}{|\partial R|} \begin{bmatrix} -\frac{\partial R^L}{\partial t^L} & \frac{\partial R^H}{\partial t^L} \\ \frac{\partial R^L}{\partial t^H} & -\frac{\partial R^H}{\partial t^H} \end{bmatrix} \begin{bmatrix} \frac{\partial R^H}{\partial \theta^H} & d\theta^H \\ \frac{\partial R^L}{\partial \theta^L} & d\theta^L \end{bmatrix}, \quad (\text{A.5})$$

where

$$\partial R = \begin{bmatrix} \partial R^H / \partial t^H & \partial R^H / \partial t^L \\ \partial R^L / \partial t^H & \partial R^L / \partial t^L \end{bmatrix}.$$

Let the set N contain all tax combinations (t^{H*}, t^{L*}) which solve (A.4). Then, by the index theorem [e.g. Mas-Colell (1985, pp. 201–204)], $\sum_{(t^{H*}, t^{L*}) \in N} \text{sign} |-\partial R| = 1$. From the properties of determinants $|-\partial R| = |\partial R|$ in the 2×2 case. Therefore, if the Nash equilibrium is unique the index theorem unambiguously signs

$$|\partial R| = \frac{\partial R^H}{\partial t^H} \frac{\partial R^L}{\partial t^L} - \frac{\partial R^H}{\partial t^L} \frac{\partial R^L}{\partial t^H} \equiv \Delta > 0. \quad (\text{A.6})$$

If multiple Nash equilibria exist $\Delta > 0$ can alternatively be obtained as a ‘stability condition’ [cf. Dixit (1986, p. 110)]. For a symmetric initial equilibrium this stability requirement is equivalent to the familiar condition that the slope of reaction functions must be less than one in absolute value.

To analyze the effects of a change in preferences $\partial(u_c^H/u_g^H)/\partial\alpha^H < 0$ we substitute $d\theta^H = d\alpha^H$ and $d\theta^L = 0$ in (A.5). If the initial equilibrium is symmetric (A.1) can be used to obtain

$$\frac{\partial R^H}{\partial \alpha} = \frac{-c^H}{(1+t^H)} \frac{\partial(u_c^H/u_g^H)}{\partial \alpha} > 0. \quad (\text{A.7})$$

Using (A.6)–(A.7) and the second-order conditions $\partial R^i/\partial t^i < 0 \forall i \in [H, L]$ in the first line of (A.5) gives $dt^H/d\alpha^H > 0$. Combining this with the change in t^L [the second line in (A.5)] gives

$$\frac{\partial R^L}{\partial t^H} \frac{dt^H}{d\alpha^H} = -\frac{\partial R^L}{\partial t^L} \frac{dt^L}{d\alpha^H}, \quad (\text{A.8})$$

which corresponds to eq. (15) in the main text. From $\Delta > 0$ in (A.6) and the symmetry of the initial equilibrium ($\partial R^i/\partial t^i = \partial R^j/\partial t^j$ and $\partial R^i/\partial t^j = \partial R^j/\partial t^i$) it follows that $dt^H/d\alpha^H > dt^L/d\alpha^H$. Since tax rates are equal initially this demonstrates the proposition.

□

Proof of Proposition 5a: As a preliminary step, differentiating (18)–(20) with respect to β gives [cf. (23) in the main text]

$$\frac{\partial c_L^H}{\partial \beta} = \frac{-(t^H - t^L)}{\beta^2} < 0,$$

$$\begin{aligned}\frac{\partial c_H^H}{\partial \beta} &= \frac{2(t^H - t^L) + (t^H)^2 - (t^L)^2}{2\beta^2(1+t^H)} > 0, \\ \frac{\partial c^H}{\partial \beta} &= \frac{-(t^H - t^L)^2}{2\beta^2(1+t^H)} < 0.\end{aligned}\tag{A.9}$$

Differentiating the payoff functions in (11)–(12) with respect to β and using (A.9)

$$\frac{\partial V^H}{\partial \beta} = \frac{u_g^H}{2\beta^2(1+t^H)} \left[(t^H - t^L) \left(1 - \frac{u_c^H}{u_g^H} \right) + 2(t^H)^2 - 2(t^L)^2 \right] > 0,\tag{A.10}$$

$$\frac{\partial V^L}{\partial \beta} = -u_g^L t^L \frac{(t^H - t^L)}{\beta^2} < 0,\tag{A.11}$$

which demonstrates the proposition. \square

Derivation of Equation (28): Equation set (A.5) simultaneously includes the solution for the more general case when best responses in both countries are altered by shift parameters: setting $d\theta^A = d\theta^B = d\beta$ yields eq. (28) in the main text.

Proof of Proposition 5b: The RHS of equations (25), (26) and (29) are all monotonously falling in $(1/\sigma^i)$ whereas eq. (30) is monotonously increasing in $(1/\sigma^L)$. Therefore, there exist lower bounds $\underline{\sigma}^H$ and $\underline{\sigma}^L$ such that for all $\sigma^H < \underline{\sigma}^H$ and $\sigma^L < \underline{\sigma}^L$ it must be true that $\partial R^H/\partial t^L < 0$, $\partial R^L/\partial t^H < 0$, $\partial R^H/\partial \beta < 0$ and $\partial R^L/\partial \beta > 0$. Substituting this combination of partial derivatives into (28) gives case 1 of Proposition 5b.

Similarly there exist upper bounds $\bar{\sigma}^H$ and $\bar{\sigma}^L$ such that for all $\sigma^H > \bar{\sigma}^H$ and $\sigma^L > \bar{\sigma}^L$ it must be true that $\partial R^H/\partial t^L > 0$, $\partial R^L/\partial t^H > 0$ and $\partial R^H/\partial \beta > 0$. It remains to show that $\partial R^L/\partial \beta > 0$ if country L undersupplies the public good initially. Inserting $\partial c_L^H/\partial \beta$ from (A.9) into eq. (30) in the main text shows that the sum of the first two effects in (30) is positive if and only if $2t^L - t^H > 0$. Similarly, using (18) in (14) shows that $2t^L - t^H > 0$ implies $(1 - u_c^L/u_g^L) > 0$. Thus, if country L undersupplies the public good in the initial equilibrium $\partial R^L/\partial \beta > 0$ follows since the last effect in (30) is negligible for very high σ^L . Substituting these results into (28) gives case 2 of Proposition 5b. \square

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