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International Investments and Exchange Rate Risk

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Abstract

In this paper we consider a risk averse multinational firm under exchange rate risk. We analyze the impact of exchange rate risk and of the use of currency forwards upon the firm's global market decisions with respect to international firm-specific capital allocation and direct foreign investment. A rise in exchange risk lowers the holdings of foreign real assets provided that there are no external hedging instruments. However, when forward markets exist, the firm's optimal holdings of foreign assets are independent of its attitude towards risk. Furthermore we show that direct foreign investments are increasing with the introduction of forward markets if firm-specific capital and direct investments are complementary factors.

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1 Introduction

Multinational firms are perhaps the most important actors in international trade and capital movements. Their importance as exporters and importers is significant, and multinational firms are responsible for the vast majority of direct foreign investment and production abroad (see UNTNC (1988)).

An international firm considering a direct foreign investment may experience considerable uncertainty over the future profit from that investment. For instance, exchange rate risk may cause the uncertainty of the future profit. A risk averse firm will respond to that uncertainty by adjusting its investment decisions and by taking hedging opportunities into account.\(^1\)

The framework of this paper relies upon the approach by Williamson (1989), who argues that a firm has to adapt capital in order to make it suitable for the production of its final good. Once adapted, this capital becomes a firm–specific asset. Hence the firm possesses capital, which is intangible, i.e., the capital asset benefits can be made available at little extra cost to domestic and foreign plants, but the asset itself cannot easily be sold in the market. Besides the firm–specific factor is not abundant and has to be allocated internationally.\(^2\)

\(^1\) Broll and Zilcha (1990).
ITAGAKI (1981), CALDERON–ROSSELL (1987), and CUSHMAN (1988) indicated economic situations in which exchange rate risk could increase direct foreign investments as a substitute for reduced exports, for example. But independent of considerations of international trade we argue that direct foreign investments may be stimulated by increased exchange rate risk as a consequence of the nature of the investments’ relationship to the multinational firm’s intangible capital. Especially when hedging markets are present the importance of this point will be revealed.

The paper presents a model of a risk averse multinational firm which owns and controls assets in two countries. The firm chooses the international allocation of firm–specific capital, foreign investments and forward contracts (whenever available) before the exchange rate is known. The model provides a framework within which the impacts of introducing risk sharing markets such as forward markets on the global market decisions of a multinational firm can be analyzed.

We structure the article as follows: In section 2 we present the model. In section 3 we examine the effect of uncertainty on international capital allocation when no risk sharing markets exist. In section 4 we introduce forward markets and we show that a separation theorem holds in this multinational firm setting: the optimal international allocation of firm–specific ca-
pital is independent of the firm's attitude towards risk and its probability beliefs. If firm-specific capital and direct foreign investments are complementary factors, then, as we show in section 5, foreign investments will increase when forward markets become available. After giving an example of optimal forward contracting we end the paper with some conclusions.

2 The Model

Consider a multinational firm producing and selling at home and abroad. Both markets are separated. We denote the deterministic operation profits in both countries by $\Pi$ at the domestic market and by $\Pi^*$ at the foreign market. Both $\Pi(K)$ and $\Pi^*(K^*)$ are strictly increasing and concave functions in firm-specific capital $K$ and $K^*$, respectively.

The stochastic global profit from home and foreign activities in domestic currency, $\tilde{\Pi}^g$, can be written as

$$\tilde{\Pi}^g = \Pi(K) + \tilde{e}\Pi^*(K^*) + Z^*(e_f - \tilde{e}),$$  \hspace{1cm} (1)

where $\tilde{e}$ is the uncertain spot exchange rate, $e_f$ the forward exchange rate and $Z^*$ the forward position of the multinational firm. $Z^* > (<)0$ indicates a long (short) position.

The firm maximizes the expected utility of profits with respect to a firm-specific capital constraint $\bar{K} = K + K^*$. Given
the von Neumann–Morgenstern utility function $U$, the decision problem reads

$$\max_{K,Z^*} EU(\tilde{\Pi}^g),$$

where

$$\tilde{\Pi}^g = \Pi(K) + \tilde{e}\Pi^*(\tilde{K} - K) + Z^*(e_f - \tilde{e}).$$

The optimal solution requires the following conditions:

$$EU'(\tilde{\Pi}^g)(\Pi_K(K) - \tilde{e}\Pi^*_K(K^*)) = 0,$$
$$EU'(\tilde{\Pi}^g)(e_f - \tilde{e}) = 0,$$

where $\Pi_K, \Pi^*_K$ denote marginal profit functions and $U'$ denotes marginal utility. Owing to the assumed properties of the profit and utility function, these are also sufficient conditions for a unique maximum.

If there exists no forward foreign exchange market we set $Z^* \equiv 0$ in definition (1). In the following we consider the multinational firm's decision with respect to international capital allocation when there are no markets for risk sharing (section 3). Then in section 4 we introduce forward markets.

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3 The objective function $U$ is assumed to be increasing, continuous, and strictly concave: $U' > 0, U'' < 0$. 
3 Exchange Rate Risk

Let us first analyze the behavior of the multinational firm, when forward markets for foreign exchange are not available ($Z^* \equiv 0$). What is the impact of exchange risk on the levels of international capital allocations and foreign investments?

**Internalization of missing forward markets.**

The investigation proceeds as follows: (a) It is common to compare the firm's optimal decision under uncertainty with the *certainty (equivalent) case*, i.e., with the case where $\bar{e}$ is replaced by its expected value, i.e., $E\bar{e}$ (see LELAND (1972), ELDOR and ZILCHA (1987)). Denote by $K_c$ and $K^*_c$ the optimal capital allocation levels for this case. (b) In order to explore the impact of uncertainty we use equation (3). Since $\bar{\Pi}^g$ increases in $\bar{e}$ and $U'$ is a decreasing function, we get $Cov(\bar{e}, U'(\bar{\Pi}^g)) < 0$.

Now we can state the impact of exchange rate risk on the firm-specific capital allocation of the multinational firm.

**Proposition 1 (Effects of Uncertainty).** A mean preserving spread at $E\bar{e}$ in the foreign exchange rate leads to higher allocation of firm-specific capital at home and lower firm-specific capital allocation abroad, i.e., $K > K_c$, $K^* < K^*_c$.

**Proof.** It follows from the optimality condition (3) that $\Pi^*_K(K^*)\theta = \Pi_K(K)$, where $\theta \equiv E\bar{e} + Cov(\bar{e}, U'(\bar{\Pi}^g))/EU'(\bar{\Pi}^g)$. Hence in comparison with the certainty case $\Pi^*_K(K^*_c)\theta_c = \Pi^*_c(K^*_c)\theta_c$.
\[ \Pi_K(K_c) \text{ and } \theta < \theta_c = E\bar{e} \] we get the result due to concave profit functions and the firm’s capital constraint. ||

4 Forward Markets and Capital Allocation

Now we consider the allocation of firm-specific capital in the presence of currency forwards. In this case the multinational firm can choose its optimal contract \( \{e_f, Z^*\} \) for currency sale in the forward market.

Let \( K_f \) and \( K_J^* \) be the optimal international capital allocation in the presence of risk sharing markets, then from equations (3) and (4) we obtain a parity condition:

\[ \Pi_K(K_f) = e_f \Pi_{K^*}(K^*_f). \]

Hence the forward rate adjusted marginal profits are equal in both markets. This leads to

**Proposition 2 (Separation).** When currency forwards are available the multinational firm’s optimal capital allocation is independent of its attitude towards risk and the distribution function of the random spot rate of foreign exchange.

The proof is a direct result of equation (5).

The separation theorem\(^4\) demonstrates that the decision of the multinational enterprise about the international allocation

\(^4\)See also **Benninga, Eldor and Zilcha** (1985), **Kawai and Zilcha** (1986), **Eldor and Zilcha** (1987).
of its firm-specific capital is independent of the utility function or the probability distribution of the random spot exchange rate. This means that any two multinational firms with identical revenue functions and technologies but with different attitudes towards risk and/or different probability beliefs will choose the same international investment policy. However, the hedging behavior of the firm will depend upon its risk aversion and also upon its assessment of the distribution of the spot rate of foreign exchange, or upon its assessment of some parameters of this distribution, at least\(^5\). This shows Proposition 3.

**Proposition 3 (Optimal Hedging).** With unbiased forward markets (i.e., \(e_f = E\tilde{e}\)), the firm completely hedges its foreign exchange risk (full hedge). If the risk premium is positive, i.e., \(e_f < E\tilde{e}\), then the forward sales of currency are less than the foreign profit (underhedge). If the risk premium is negative, i.e., \(e_f > E\tilde{e}\), the firm sells forward more than its foreign profit (overhedge).

**Proof.** The global profit function may be written as

\[
\tilde{\Pi}^g = \Pi(K) + e_f Z^* + \tilde{e}[\Pi^*(K^*) - Z^*].
\]

Using equation (4), we obtain that

\[
(e_f - E\tilde{e})EU'(\tilde{\Pi}) = Cov(\tilde{e}, U'(\tilde{\Pi}^g)).
\]

\(^5\)This will be illustrated by an example in section 5.
If the LHS of (6) is zero, then $\tilde{\Pi}^g$ must be independent of $\tilde{e}$, i.e., $Z^* = \Pi^*(K^*)$. Similarly, if the LHS of (6) is positive ($e_f > E\tilde{e}$), $\tilde{\Pi}^g$ must be a decreasing function of the spot exchange rate, i.e., $Z^* > \Pi^*(K^*)$ (and similary for normal backwardation, i.e., $e_f < E\tilde{e}$).

From the preceding analysis of the multinational firm’s hedging and investment decisions under exchange rate risk the following should be noted: The separation theorem does not imply, that the investment and hedging decisions of the multinational firm are independent of each other. It means that these decisions can be economically divided, that is taken sequentially. This procedure is correct because when forwards exist, risk elements and risk aversion only affect the level of hedging of the global profit risk but not the quantities of firm–specific capital allocated to different countries.

5 Direct Foreign Investment

Let us include the possibility that the multinational firm invests in the foreign country in a way that improves the marketability of its products. We observe many such cases, for example Japanese manufacturers invest overseas in advertising, dealership, and other services facilities that increase the at-
tractiveness of their products. In order to include such direct foreign investments (DFI) in our model it is assumed that the foreign profit function depends not only upon the capital allocation \( K^* \) but also upon direct foreign investments \( S^* \):

\[
\Pi^* = \Pi^*(K^*, S^*).
\]

Let \( \Pi^* \) be increasing in \( S^* \) at a diminishing rate. The marginal profit \( \Pi_{K^*} \) may be (i) increasing in, (ii) independent of, or (iii) decreasing in \( S^* \). We will refer to these conditions by naming foreign firm-specific capital and direct foreign investment as (1) complementary, (2) independent, or (3) substitutional factors.

The firm’s optimization problem in the presence of currency forwards is given by

\[
\max_{K, S^*} EU(\Pi^g), \tag{7}
\]

where

\[
\Pi^g = \Pi(K) + \tilde{c}[\Pi^*(\bar{K} - K, S^*) - S^*] + Z^*(e_f - \tilde{e}).
\]

Let \( S^*_f \) denote the optimal direct foreign investment in the presence of forward markets. Then the first order conditions are analogous to equations (3)–(4) and in addition we have

\[
[\Pi_{S^*}^*(K^*_f, S^*_f) - 1]EU'(\Pi^g)\tilde{e} = 0. \tag{8}
\]

\(^{6}\text{See for example Shapiro (1989).}\)
The implications are summarized in

**Proposition 4 (Direct Foreign Investment).**

(a) When currency forward markets are available and the forward price $e_f$ is given, the optimal direct foreign investment $S_f^*$ is independent of the multinational firm's attitude towards risk and its probability beliefs about the occurrence of the future spot rate of foreign exchange.

(b) Denote $S^*$ as the optimal direct foreign investment without currency forward markets and let us consider a forward rate of foreign exchange that contains a nonpositive risk premium, i.e., $e_f \geq E\bar{e}$. The effect of introducing currency forwards upon the level of direct foreign investment (DFI) will depend upon the impact of firm-specific capital on direct investments:

1. If $K_f^*$ and $S_f^*$ display a complementary relationship, then DFI increases.

2. If $K_f^*$ and $S_f^*$ are independent factors, then DFI remains unchanged.

3. If $K_f^*$ and $S_f^*$ display a substitutional relationship, then DFI decreases.

**Proof.** (a) Since $EU'(\tilde{I})\bar{e} > 0$, we obtain from equation (8)

$$
\Pi_{\tilde{S}^*}(K_f^*, S_f^*) = 1. \tag{9}
$$
With optimality condition $\Pi_K(K_f) = e_f \Pi^*_K(K^*_f, S^*_f)$ for capital allocation it is evident that the separation property holds.

(b) With or without currency forwards, in the optimum marginal profit is equal to marginal costs, i.e., $\Pi^*_S(K^*_f, S^*_f) = \Pi^*_S(K^*, S^*) = 1$. If $e_f \geq E\tilde{e}$ it follows from Proposition 1 that $K^*_f > K^*$. Then we obviously have $\text{sign}\{S^*_f - S^*\} = \text{sign}\{\Pi^*_S(K^*_f, S^*) - \Pi^*_S(K^*, S^*)\}$, since $\Pi^*_S(K^*, S^*)$ is decreasing in $S^*$.

Let us consider the effects of changes in the forward exchange rate $e_f$ on the foreign investment level $S^*_f$.

**Corollary:** In the presence of forward markets for foreign exchange, an increase in the forward rate will result in higher direct foreign investment and higher firm-specific capital allocation abroad if and only if $K^*_f$ and $S^*_f$ are complements.

The *Proof* is provided in the *Appendix*.

If we know how a mean preserving spread in the distribution of the spot exchange rate affects the forward price $e_f$, then we can use the Corollary to analyze the impact of such increase in the volatility of the exchange rate on the foreign investment $S^*_f$. For example, given that $\partial e_f/\partial \sigma^2 < 0$ (where $\sigma^2$ is the variance of $\tilde{e}$) we conclude that in case of *complementary* factors we have $\partial S^*_f/\partial \sigma^2 < 0$. Higher volatility of the exchange rate

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7Such variations may result from changes in the expected value of the exchange rate or as a consequence of changes in the variance of $\tilde{e}$.
will result in a reduction of direct foreign investment because of a less attractive forward rate and a complementary relationship between capital and investments. Furthermore when the market is expecting an appreciation of the foreign currency, i.e., $\bar{E} \bar{e}$ is increasing, the multinational firm will increase the amount of direct foreign investment and capital allocation to the foreign subsidiary.

Forward Contracting: An Example

This example illustrates the firm’s optimal decision on forward contracting under a specific mean–variance function. Let us assume that the firm maximizes $U = E(\tilde{\Pi}^g) - \frac{\alpha}{2} V(\tilde{\Pi}^g)$, where the risk aversion parameter $\alpha$ is positive, $E$ is the expectations operator, $V$ is the variance operator and $\tilde{\Pi}^g$ is defined in equation (7).

Maximizing $U$ with respect to $Z^*$ and rearranging terms the optimal forward contract amount results in

$Z^* = [\Pi^*(K^*_f, S^*_f) - S^*_f] - \frac{E \bar{e} - e_f}{\alpha V(\bar{e})}$.  \hspace{1cm} (10)

Now let us consider an unbiased forward market, i.e., $e_f = E \bar{e}$. Then equation (10) implies that only the net exposure from the firm’s foreign operations is hedged. In other words, since $Z^* + S^*_f = \Pi^*(K^*_f, S^*_f)$, the hedging policy of the multinational firm
has two aspects: Firstly the *internal* hedge $S_f^*$ and secondly the *external* hedge $Z^*$.

Note that the external hedging demand $Z^*$ increases in the forward rate ($\partial Z^*/\partial e_f > 0$ using condition (9)). Furthermore a higher level of risk aversion and/or a higher volatility of the future spot rate of foreign exchange will induce more external hedging if and only if normal backwardation holds.\(^8\)

### 6 Conclusions

Forwards and futures are frequently used by international firms for insurance against price and exchange rate risks.\(^9\) Our paper provides a study of a risk averse multinational firm. It elaborates the impact of forward markets of foreign exchange on the capital allocation of the multinational firm under spot exchange rate uncertainty.

The approach contains two factors, firm-specific capital and direct foreign investment, and currency forwards as a hedging instrument. We assume that all the decisions of the firm, such as direct foreign investment and the allocation of firm-specific capital between the separated foreign and domestic markets are determined ex ante, i. e., before the realization of

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\(^8\) $\frac{\partial Z^*}{\partial e} > 0$ and $\frac{\partial Z^*}{\partial e} > 0$ iff $E\tilde{e} \leq e_f$ (normal backwardation).

the random spot exchange rate.

Mainly we investigated the influence of forward markets upon international firm-specific capital allocation, direct foreign investment and the optimal hedging behavior of the multinational firm. It was shown that the adverse impact of uncertainty may be eliminated by establishing currency forward markets or other equivalent arrangements. Furthermore it was demonstrated that direct foreign investment of the multinational firm increases under forward markets if firm-specific capital and direct investments are complementary factors.

From the preceding analysis of a multinational firm under exchange rate risk an important implication can be derived. When forward markets exist, risk elements affect only the level of hedging but not the investment decision of the international firm.
Appendix

Proof of the Corollary. Consider the following optimality conditions:

\[ \Pi_K(K_f) = e_f \Pi_{K^*}(K_f^*, S_f^*), \quad (11) \]
\[ \Pi_{S^*}(K_f^*, S_f^*) = 1. \quad (12) \]

Let us introduce the following abbreviations:
\[ \Pi_{KK} = \Pi_{KK}(K_f), \quad \Pi_{K^*K^*} = \Pi_{K^*K^*}(K_f^*, S_f^*), \]
\[ \Pi_{K^*S^*} = \Pi_{K^*S^*}(K_f^*, S_f^*), \quad \Pi_{S^*S^*} = \Pi_{S^*S^*}(K_f^*, S_f^*), \]
\[ \Pi_{KK}^* = \Pi_{KK}^*(K_f^*, S_f^*), \quad \Pi_{K^*K^*}^* = \Pi_{K^*K^*}^*(K_f^*, S_f^*), \]
Thereby \( \Pi_{ii}(\cdot) \) denotes the second partial derivative \( \frac{\partial^2 \Pi(\cdot)}{\partial i \partial j} \).

By total differentiation of equations (11) and (12), we deduce the following linear system:

\[ \begin{pmatrix}
-\Pi_{KK} - e_f \Pi_{K^*K^*} & -e_f \Pi_{K^*S^*} \\
\Pi_{K^*S^*} & \Pi_{S^*S^*}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial K_f^*}{\partial e_f} \\
\frac{\partial S_f^*}{\partial e_f}
\end{pmatrix}
= \begin{pmatrix}
\Pi_{K^*}^* \\
0
\end{pmatrix}.\]

The determinant of the matrix is

\[ -\Pi_{S^*S^*} \Pi_{KK} - e_f [\Pi_{S^*S^*} \Pi_{K^*K^*} - (\Pi_{K^*S^*})^2]. \]

Assuming that all profit functions are strictly concave, this determinant is negative. Therefore, by Cramer’s rule we obtain

\( \frac{\partial K_f^*}{\partial e_f} > 0 \) and \( sign\{\frac{\partial S_f^*}{\partial e_f}\} = sign\{\Pi_{K^*S^*}(K_f^*, S_f^*)\} \). Hence DFI increases with the forward rate if and only if we have complementary factors. ||
References


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