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## Tax competition in a Bertrand-Edgeworth model

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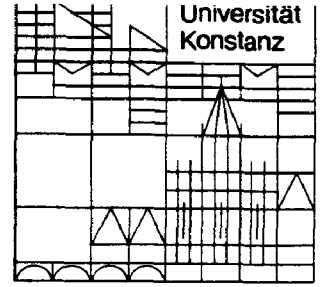
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Bertrand-Edgeworth Model**

# **TAX COMPETITION IN A BERTRAND-EDGEWORTH MODEL**

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### **Abstract**

Tax competition of two countries for foreign direct investment is analyzed in a Bertrand–Edgeworth dyopoly model. In the symmetric case zero-taxation is the unique equilibrium in pure strategies. If asymmetries are introduced only  $\epsilon$ -equilibria will exist. However, if the tax rate applies to foreign direct investment as well as to domestic sources (i.e. players have an outside option) there is no equilibrium in pure strategies.

# 1 Introduction

We consider two countries competing for foreign direct investment (fdi) via their tax policy. Governments seek to maximize tax revenue by attracting fdi: they set income tax rates which are assumed to be the sole policy variables under their control. If gross profit possibilities do not differ across countries, firms decide to invest in the country with the lowest income tax rate after both tax rates have been set.

As known from the literature, this symmetric version of the Bertrand–Edgeworth model<sup>1</sup> has at most one unique Nash equilibrium in pure strategies (see e.g. Beckmann (1965)). Since we do not assume any capacity constraint, i.e. each government is prepared to allow the entire amount of fdi to be invested in its country, zero-taxation will be the equilibrium in pure strategies. If we drop the assumption of identical gross profits only a sequence of  $\epsilon$ -equilibria converging to a unique point will exist.

These results hinge on the assumption that tax rates for newly established foreign firms can be set independently of tax rates for already existing (domestic) firms. If we remove this assumption, it turns out that no equilibrium exists in pure strategies. For convenience (but without loss of generality) we assume that tax rates for newly attracted firms must be equal to the prevailing tax rate.

We concentrate on the analysis of *pure* strategy equilibria since it is difficult to imagine that governments randomize over possible tax rates. For the interpretation of tax setting behavior it is hence reasonable to consider pure strategies instead of probability measures on the tax space. Note, however, that Dasgupta & Maskin (1986a, 1986b) and Allen & Hellwig (1986) have shown that for this type of game

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<sup>1</sup>The original references are Bertrand (1883) and Edgeworth (1925).

equilibria in mixed strategies do exist, even if equilibria in pure strategies do not.

Bertrand–Edgeworth competition is often recognized in economic modelling. The emphasis of this paper is therefore on the game-theoretical consequences of outside options which players might have on the existence of equilibria. We do not claim to model tax competition in the most realistic way but we do hope to provide some useful insights into the game theoretical structure of this and related problems.

This paper is organized as follows. Section two describes the equilibrium in pure strategies if the tax rate faced by newly attracted firms is independent of the prevailing rate, whereas section three contains the case of identical tax rates for fdi and domestic economic activities. Concluding remarks follow in section four.

## 2 Tax Competition with Tax Discrimination

A finite number of  $n$  identical firms face the decision in which of the two countries ( $i = 1, 2$ ) to invest a fixed amount of capital. This investment yields a fixed and finite gross profit  $\Pi_i$  and is taxed at the constant rate  $\tau_i$ .<sup>2</sup> Investment decisions are made after tax rates have been set simultaneously by the two governments. In other words, we deal with a two-stage game in which governments have a Cournot-Nash relationship moving first, while firms react under perfect information.

The firms will invest in the country in which net profits are maximized. If net profits are equal in both countries the firms are indifferent, and we assume that the attracted investment is shared equally between both countries.

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<sup>2</sup>Since we focus on the underlying game-theoretic structure of the model we disregard important features such as uncertainty and different forms of capital income taxation.

The objective function of a single enterprise is given by

$$\max_{i=1,2} V(i) := \Pi_i(1 - \tau_i) . \quad (1)$$

Governments seek to maximize tax revenue which stems from two sources. (Domestic) enterprises already established are taxed at the rate  $\tau_i^d$ , whereas taxes on investment income from newly attracted capital are levied at the rate  $\tau_i$  (which for convenience carries no superscript). Thus, the objective function of government  $i$  can be written as:

$$\max_{\tau_i^d, \tau_i} \tilde{U}_i(\tau_i^d, \tau_i, \tau_j) := \tau_i^d \Sigma_i + \alpha_i \tau_i n \Pi_i , \quad i \neq j \quad (2)$$

with  $\Sigma_i$  being the given sum of profits generated by already existing firms. The parameter  $\alpha_i$  describes the Bertrand–Edgeworth competition for fdi.

$$\alpha_i = \begin{cases} 1 & < \\ \frac{1}{2} & \iff \tau_i = 1 - \Pi_j / \Pi_i(1 - \tau_j) , \\ 0 & > \end{cases} \quad i \neq j . \quad (3)$$

For further reference we define

$$\rho_i(\tau_j) := 1 - \Pi_j / \Pi_i(1 - \tau_j) , \quad i \neq j . \quad (4)$$

If  $\rho_i(\tau_j) > 0$  country  $i$  has the opportunity to compete for the entire fdi. Given  $\tau_j$  the taxrate  $\tau_i = \rho_i(\tau_j)$  is the supremum of all  $\tau_i$  by which country  $i$  can attract the fdi. Nevertheless, if  $\tau_i$  equals  $\rho_i(\tau_j)$  country  $i$  will loose half of the fdi to country  $j$ . Country  $i$  will be better off slightly undercutting  $\rho_i(\tau_j)$ .

If, on the other hand,  $\rho_i(\tau_j) < 0$ , then country  $i$  cannot attract any fdi even by setting its own tax rate equal to zero.



Each government has a continuum of pure strategies, i.e. the tax rates it can fix. This continuum is the compact and convex unit square.

**Assumption 1:**  $\tau_i, \tau_i^d \in [0, 1]$ ,  $i = 1, 2$ .

For the first step, we assume that governments are able to set  $\tau_i$  independently of  $\tau_i^d$ . The game theoretical problem is thereby reduced to the competition for fdi<sup>3</sup> and the set of strategies from which the player  $i$  may choose simplifies to

$$T_i := \{\tau_i \mid \tau_i \in [0, 1]\}. \quad (5)$$

Hence, (2) is modified to

$$\max_{\tau_i} U_i(\tau_i, \tau_j) = \begin{cases} \tau_i \cdot n \Pi_i & \iff \tau_i < \rho_i(\tau_j) \\ \tau_i \cdot n/2 \Pi_i & \iff \tau_i = \rho_i(\tau_j), \quad i \neq j \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

From (6) we see that the line

$$L := \{(\tau_1, \tau_2) \in T_1 \times T_2 \mid \tau_1 = \rho_1(\tau_2)\} \quad (7)$$

divides the unit square into two regions, where country one and country two attract the entire fdi, respectively. Let  $S$  denote the point on  $L$  where at least one tax rate is equal to zero.

$$S := (\max(0, \rho_1(0)), \max(0, \rho_2(0))) \quad (8)$$

$S$  is equal to  $(0, 0)$  if  $\Pi_1 = \Pi_2$ . In figure 1 below,  $L$  is shown for the asymmetric

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<sup>3</sup>Since we assume  $\Sigma_i$  to be fixed it will be taxed at the maximum rate of  $\tau_i^d = 1$ .

case  $\Pi_1 > \Pi_2$ . We get  $S = (1 - \Pi_2/\Pi_1, 0)$ .

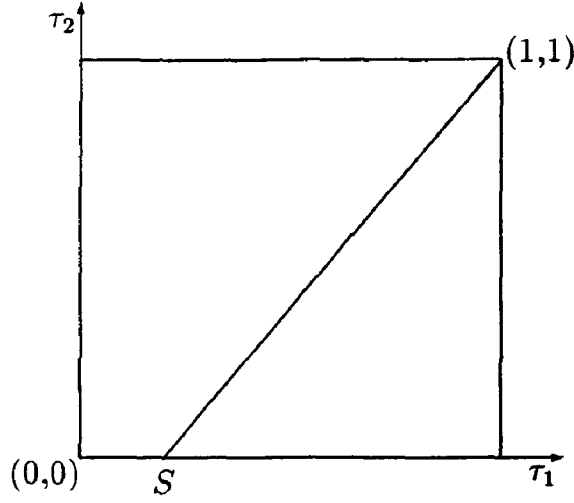


Figure 1

**Definition 1** A pair of strategies  $(\tau_1^*, \tau_2^*) \in [0, 1] \times [0, 1]$  is a **Nash equilibrium** in pure strategies of the game  $[(T_i, U_i); i = 1, 2]$  if  $U_i(\tau_i^*, \tau_j^*) = \max_{\tau_i \in T_i} U_i(\tau_i, \tau_j^*)$   $\forall i \neq j$ .

**Definition 2** <sup>4</sup> In a game  $[(T_i, U_i), i = 1, 2]$  the strategy  $\tau_i^* \in T_i$  for player  $i$  is an  $\epsilon$ -best reply to  $\tau_j^*$  for  $\epsilon \geq 0$  if  $U_i(\tau_i^*, \tau_j^*) \geq U_i(\tau_i, \tau_j^*) - \epsilon$  for all  $\tau_i \in T_i$ .

**Definition 3**  $(\tau_1^*, \tau_2^*, \epsilon) \in T_1 \times T_2 \times \mathbf{R}_+$  is an  $\epsilon$ -equilibrium for the game  $[(T_i, U_i), i = 1, 2]$  if for  $\epsilon \geq 0$   $\tau_i^*$  is an  $\epsilon$ -best reply to  $\tau_j^* \forall i \neq j$ .

**Remark:** If  $\epsilon = 0$  the  $\epsilon$ -equilibrium becomes a Nash equilibrium.

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<sup>4</sup>The following definitions were adopted, however in a slightly modified way, from Friedman 1986: 100.

We modify a basic result, which was first pointed out by Bertrand (1883) and Edgeworth (1925), for our problem.

**Proposition 1** *For the game  $[(T_i, U_i); i = 1, 2]$  defined by (5) and (6) and  $\Pi_1 \neq \Pi_2$  every sequence of  $\epsilon$ - equilibria in pure strategies  $(\tau_1^k, \tau_2^k, \epsilon^k)$  with  $\lim_{k \rightarrow \infty} \epsilon^k = 0$  converges to  $S$ .  $S$  does not establish an  $\epsilon$ -equilibrium.*

**Proof:**

From definition 2 follows that  $\tau_i$  is  $\epsilon$ -best reply to  $\tau_j$  for  $\epsilon > 0$  if

- (i)  $\rho_i(\tau_j) > \tau_i \geq \rho_i(\tau_j) - \epsilon/(n\Pi_i)$  for  $\rho_i(\tau_j) > \epsilon/(n\Pi_i)$
- (ii)  $\tau_i \in T_i$  for  $\rho_i(\tau_j) \leq \epsilon/(n\Pi_i)$ .

W.l.o.g. we assume  $\Pi_1 > \Pi_2$ . No  $\epsilon$ -equilibrium will occur for  $\tau_1 > \tilde{\tau}_1 := \rho_1(0) + \epsilon/(n\Pi_1)$  since (i) applies for  $i = 1, 2$ . Players will underbid each other thereby attracting all fdi. For sufficiently small  $\epsilon$  the set of  $\epsilon$ -equilibria is the set given by  $\tau_1 \leq \tilde{\tau}_1$  and  $\rho_1(\tau_2) > \tau_1 \geq \rho_1(\tau_2) - \epsilon/(n\Pi_1)$  (i\*). For  $\tau_1 \leq \tilde{\tau}_1$  player 2 is indifferent towards his tax rate  $\tau_2$ , and (i\*) ensures that player 1 cannot gain more than  $\epsilon$  if he deviates. Because of the strict inequality in (i\*) the set of  $\epsilon$ -equilibria is not closed. There are no  $\epsilon$ -equilibria on the line  $L$ . Hence the point  $S = (\rho_1(0), 0)$  on  $L$  is no  $\epsilon$ -equilibrium.

For  $\epsilon \rightarrow 0$  the set of  $\epsilon$ -equilibria shrinks towards  $S$ . Its limit is the empty set.

Q.E.D.

Figure 2 illustrates the  $\epsilon$ -best replies of both players and thereby the proof of Proposition 1.

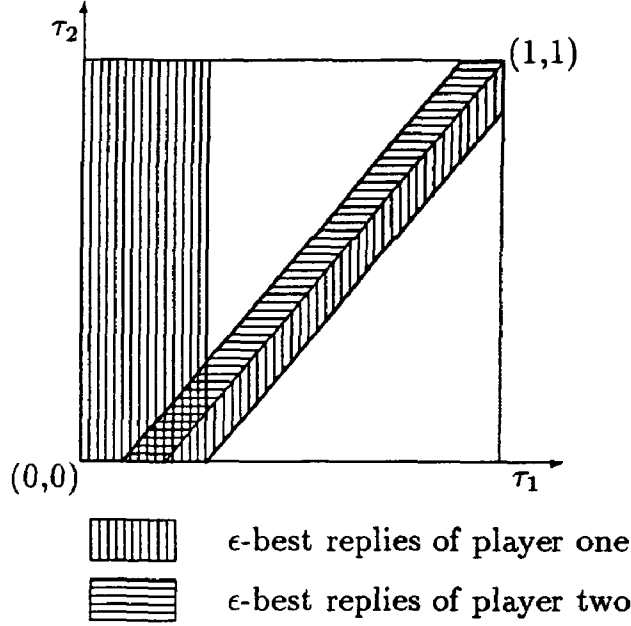


Figure 2:  $\epsilon$ -equilibria for the case  $\Pi_1 > \Pi_2$ .

We now turn to the symmetric case and obtain the well known result:<sup>5</sup>

**Corollary 1** *For  $\Pi_1 = \Pi_2 = \Pi$  the game  $[(T_i, U_i); i = 1, 2]$  defined by (5) and (6) possesses a unique Nash equilibrium in pure strategies given by  $(0, 0)$ .*

**Proof:**

The proof follows straightforwardly from the proof of proposition 1. (i) reduces to (i')  $\tau_j > \tau_i \geq \tau_j - \epsilon/(n\Pi)$  for  $\tau_j > \epsilon/(n\Pi)$ , (ii) is reduced to: (ii')  $\tau_i \in T_i$  for  $\tau_j \leq \epsilon/(n\Pi)$ . As long as  $\tau_i > \epsilon/(n\Pi)$ , for  $i = 1, 2$ , (i') applies for both players. Underbidding is rewarding and  $\epsilon$ -equilibria cannot occur. The set of  $\epsilon$ -equilibria is described by  $\tau_1 + \epsilon/(n\Pi) \geq \tau_2 \geq \tau_1 - \epsilon/(n\Pi)$  and  $\tau_i \leq \epsilon/(n\Pi)$ , for at least one player. The set is

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<sup>5</sup>See e.g. Dasgupta&Maskin (1986b:28-30), Beckmann (1965).

closed and shrinks towards the point  $(0,0)$  for  $\epsilon \rightarrow 0$ .  $(0,0)$  is an  $\epsilon$ -equilibrium for  $\epsilon = 0$ , i. e. a Nash equilibrium.

Q.E.D.

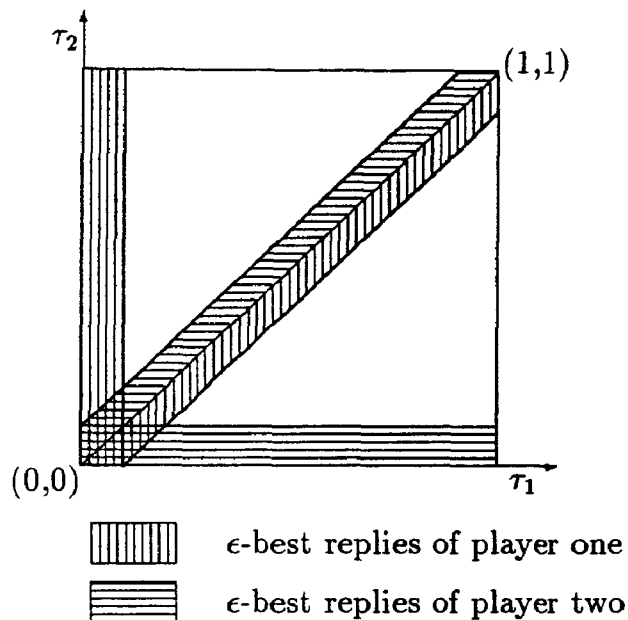


Figure 3:  $\epsilon$ -equilibria for the case  $\Pi_1 = \Pi_2$ :

### 3 Tax Competition without Tax Discrimination

In this section, we show that even in the symmetric case ( $\Pi_1 = \Pi_2 = \Pi$ ) there exists no equilibrium in pure strategies if prevailing tax rates (on domestic economic activities) are a function of taxes on newly established enterprises. Moreover, in the asymmetric case not even  $\epsilon$ -equilibria can be found. For simplicity we assume that  $\tau_i$  and  $\tau_i^d$  must be identical.<sup>6</sup>

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<sup>6</sup>This is, however, not crucial for the analysis. All we need to assume is that  $\tau_i^d = f(\tau_i) \quad \forall i$ .

**Assumption 2:**  $\tau_i = \tau_i^d$ ,  $i = 1, 2$ .

Given assumption 2 player  $i$ 's payoff function (2) is altered to:

$$\max_{\tau_i} U_i^*(\tau_i, \tau_j) = \begin{cases} \tau_i (n\Pi_i + \Sigma_i) & \iff \tau_i < \rho_i(\tau_j) \\ \tau_i (n\Pi_i/2 + \Sigma_i) & \iff \tau_i = \rho_i(\tau_j), \quad i \neq j \\ \tau_i \Sigma_i & \iff \tau_i > \rho_i(\tau_j) \end{cases} \quad (9)$$

From the above formulation of the maximization problem it is seen that there is a tax floor for each country which depends in magnitude on the relative size of the stake ( $n\Pi_i$ ). This is stated in the following lemma.

**Lemma 1** *For  $\epsilon = 0$ , each player of the game  $[(T_i, U_i^*); i = 1, 2]$  defined by equations (5) and (9) has a lower bound for his equilibrium strategies,  $\underline{\tau}_i$ , given by*

$$\underline{\tau}_i = \frac{\Sigma_i}{n\Pi_i + \Sigma_i}, \quad i = 1, 2.$$

**Proof:**

Each player  $i$  faces the alternative either of trying to undercut his opponent's tax rate and generate tax revenue from fdi ( $n\Pi_i$ ) and domestic sources ( $\Sigma_i$ ) or of taxing income from already established (domestic) firms at the maximum rate, i.e.  $\tau_i$  is set to unity. (9) shows that player  $i$  will be indifferent between both alternatives iff

$$\Sigma_i = \tau_i(n\Pi_i + \Sigma_i). \quad (10)$$

We denote the solution to (10) with  $\underline{\tau}_i$ . Player  $i$  will prefer exploitation of domestic economic activity and dispense with tax revenue from fdi if  $\tau_i$  had to fall short of  $\underline{\tau}_i$ . Hence,  $\underline{\tau}_i$  establishes a lower bound for  $\tau_i$ .

Q.E.D.

We can now state our main result:

**Proposition 2** *If  $\Sigma_i > \max\{n(\Pi_i - \Pi_j)\Pi_i/\Pi_j, 0\}$  for at least one  $i$  and  $i \neq j$  the game  $[(T_i, U_i^*); i = 1, 2]$  defined by equations (5) and (9) does neither possess a Nash equilibrium nor an  $\epsilon$ -equilibrium in pure strategies for  $\epsilon$  sufficiently small.*

**Proof:**

W.l.o.g. we assume  $\Pi_1 \geq \Pi_2$ .

We say the outside option  $\Sigma_i$  of player  $i$  is binding if player  $i$  is better off with the outside option than with the supremum of the payoffs of underbidding at least in case player  $j$  sets  $\tau_j$  equal to zero. In terms of  $\underline{\tau}_i$  this means  $\underline{\tau}_1 > \rho_1(0)$  and  $\underline{\tau}_2 > 0$ . These conditions reduce to

$$\Sigma_1 > n(\Pi_1 - \Pi_2)\Pi_1/\Pi_2 \quad \text{and} \quad \Sigma_2 > 0, \quad \text{respectively.} \quad (11)$$

If  $\Sigma_i$  is binding and  $\epsilon$  is sufficiently small then  $\tau_i$  is  $\epsilon$ -best reply to  $\tau_j$  if

$$\begin{aligned} \text{(i'')} \quad & \left\{ \begin{array}{l} \rho_i(\tau_j) > \tau_i \geq \rho_i(\tau_j) - \epsilon/(n\Pi_i + \Sigma_i) \quad \text{or} \\ \tau_i \geq \rho_i(\tau_j)(n\Pi_i + \Sigma_i)/\Sigma_i - \epsilon/\Sigma_i \end{array} \right\} \text{ for } \tau_j \geq \rho_j(\underline{\tau}_i) \\ \text{(ii'')} \quad & \left\{ \begin{array}{l} \rho_i(\tau_j) > \tau_i \geq \underline{\tau}_i - \epsilon/(n\Pi_i + \Sigma_i) \quad \text{or} \\ \tau_i \geq 1 - \epsilon/\Sigma_i \end{array} \right\} \text{ for } \tau_j < \rho_j(\underline{\tau}_i) \quad (\text{Cf. Lemma 1}). \end{aligned}$$

If  $\Sigma_i$  is not binding and  $\epsilon$  is sufficiently small then  $\tau_i$  is  $\epsilon$ -best reply to  $\tau_j$  if

$$\begin{aligned} \text{(i''')} \quad & \rho_i(\tau_j) > \tau_i \geq \rho_i(\tau_j) - \epsilon/(n\Pi_i + \Sigma_i) \quad \text{for } \rho_i(\tau_j) > \epsilon/(n\Pi_i + \Sigma_i) \\ \text{(ii''')} \quad & \tau_i \in T_i \quad \text{for } \rho_i(\tau_j) \leq \epsilon/(n\Pi_i + \Sigma_i). \end{aligned}$$

For  $\epsilon$  sufficiently small the intersection of the set of  $\epsilon$ -best replies of player one and player two is empty. This is still true if  $\epsilon = 0$ . Hence, neither  $\epsilon$ -equilibria nor a Nash equilibrium in pure strategies will exist.

Q.E.D.

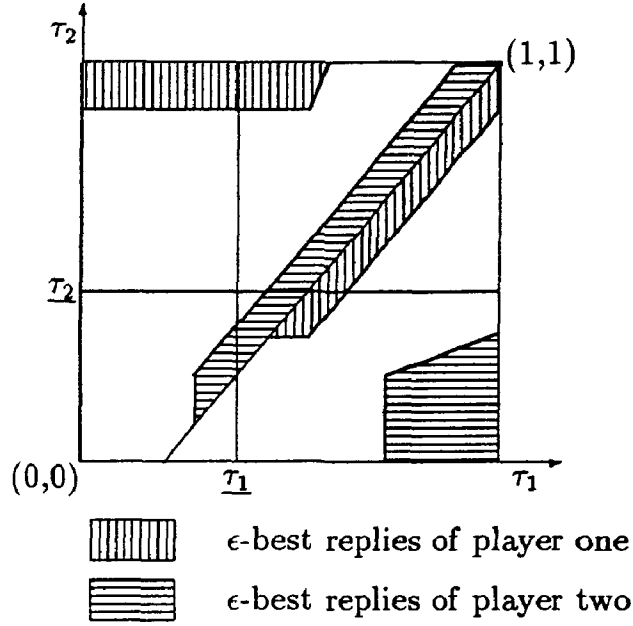


Figure 4:  $\Pi_1 > \Pi_2$  and  $\Sigma_1$  and  $\Sigma_2$  are binding.

In contrast to games without outside options ( $\Sigma_1 = \Sigma_2 = 0$ ) in this game there is no  $\tau_j$  to which player  $i$ 's  $\epsilon$ -best reply is any  $\tau_i \in T_i$  if  $\Sigma_i$  is binding. I.e. indifference cannot occur since player  $i$  can always ensure  $\Sigma_i$  by  $\tau_i = 1$ . Therefore, equilibria close to the point  $S$  are excluded if at least one outside option is binding.

## 4 Concluding Remarks

Bertrand-Edgeworth competition yields a unique pure-strategy equilibrium in the symmetric case if capacity constraints are not binding. Charging the competitive price - or in our application zero-taxation - are the equilibrium strategies. An  $\epsilon$ -equilibrium is obtained if asymmetries are introduced.



If, however, allowance is made for an outside option for the players which is not competed for but affected by the competition, equilibria in pure strategies will no longer exist. In the context of tax competition this outside option could be the taxation of domestically generated profits which have to be taxed at the same rate as the foreign direct investment that governments seek to attract. Various other economic applications are conceivable. E.g. the same structure can be found in a duopoly with Bertrand–Edgeworth competition in which players have clients in segmented markets but they are not allowed to price discrimination.

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