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## A macroeconomic disequilibrium model of the German credit market

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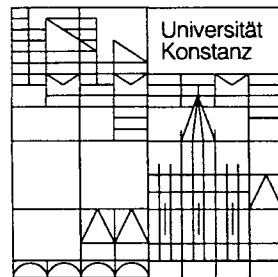
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**Sonderforschungsbereich 178**  
**„Internationalisierung der Wirtschaft“**

Diskussionsbeiträge



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Peter Winker

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# **A Macroeconomic Disequilibrium Model of the German Credit Market\***

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### **Abstract**

Not all markets are always characterized by immediate market clearing, for example due to a delayed adjustment of quantities and prices. Macroeconometric disequilibrium models proved to be a valuable tool in assessing the relevance of such phenomena on goods and labour markets.

Asymmetric information is the central ingredient of most theoretic models of credit markets allowing for a non market clearing price setting. However, often the empirical analysis is either restricted to the micro level or does not allow for disequilibrium at the macro level.

In this paper a macromodel of the German credit market is constructed starting with a microeconomic theory of asymmetric information. The resulting macroeconometric disequilibrium model is estimated for the market of loans to the private sector in West Germany 1975 – 1989. The results indicate that rationing on the credit market exhibited a relevant extent for several subperiods.

# 1 Introduction

“So far, however, as bank loans are concerned, lending does not – in Great Britain at least – take place according to the principles of a perfect market. There is apt to be an unsatisfied fringe of borrowers, the size of which can be expanded or contracted, so that banks can influence the volume of investment by expanding or contracting the volume of their loans, without there being necessarily any change in the level of bank-rate, in the demand-schedule of borrowers, or in the volume of lending otherwise than through the banks. **This phenomenon is capable, when it exists, of having great practical importance.**”<sup>1</sup>

The analysis of credit rationing or at least financing constraints regained interest through the availability of a sound theoretic basis given by the theory of asymmetric information. It was up to the seminal paper of Akerlof (1973) to supply this base for the more recent literature on credit rationing.<sup>2</sup> Maybe the best known theoretic model at the micro level is the one provided by Stiglitz and Weiss (1981).

In the Stiglitz and Weiss model, banks possess only limited information about the projects of their potential borrowers. Then, the loan rate can have an adverse selection and adverse incentive effects resulting in an eventually negative relationship between the interest rate and the expected returns for the banks. Consequently, the profit maximizing interest rates set by the banks might – at least for some groups of potential borrowers – be below the market clearing level. Excess demand arises on the credit market as the outcome of the decisions of profit maximizing agents.<sup>3</sup>

Although there is a growing literature on the theoretical base of credit rationing both at the micro and the aggregate level, the empirical evidence is limited. Three different approaches of the empirical analysis might be distinguished.

The first one concerns the micro level. Firm panel data are used to test if the theory helps to explain why certain firms face credit constraints. As most panel data sets do not include explicit statements for financing constraints,<sup>4</sup> the impact of such constraints is tested by introducing proxy

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<sup>1</sup>Keynes (1930) I, p. 212.

<sup>2</sup>Cf. Jaffee and Stiglitz (1990) for a survey.

<sup>3</sup>This central conclusion persists in almost all extensions of the Stiglitz and Weiss framework. Cf. Gertler (1992) for some examples.

<sup>4</sup>The ifo data used in Winker (1993, 1996) seem to be an exception.

variables in investment equations.<sup>5</sup>

Osano and Tsutsui (1985) and Winker (1995) derive hypotheses on the adjustment speed of different interest rates with regard to changes on the money market. The differences are explained as the outcome of asymmetric information on the credit market. Hence, the empirical testing is directed towards an assesement of the adjustment speed.

The third approach, and this will be the one followed in this paper, consists in specifying and estimating aggregate models of the credit market. One source of this approach can be found in the modelling of loan supply and demand by Melitz and Pardue (1973). However, they still used an equilibrium model, whereas in later work Laffont and Garcia (1977), Ito and Ueda (1981), Artus (1984), King (1986) and Martin (1990) among others choose specifications allowing for disequilibrium on the loan market. In most specifications identification of the rationed market side is based on using the loan rate movements as an instrument. Following the argumentation in Stiglitz and Weiss (1981) it is at least not a straightforward consequence of asymmetric information that interest rate movements allow for this interpretation.

Here, a modified modelling strategy is used. Both, the aggregate supply of and the demand for loans are based on microeconomic models. As risk is one of the central ingredients of the theory of asymmetric information the loan market is disaggregated in high and low risk loans. Consequently, different reactions of the market segments to changes in interest rates or overall risk become possible. Since in Germany, there are no data available for different loan risk categories, the resulting disequilibrium model was estimated for aggregate data. Nevertheless, the assumption that loan rate movements indicate rationing is not necessary any more.

The paper is organized in four sections. The first two sections describe the modelling of the loan supply and demand, respectively. In section 4 the aggregate disequilibrium model is derived. Section 5 provides the results of the estimation for the German market for industrial loans for the time span 1975 – 1989. A conclusion summarizes the main findings and gives an outlook to further research.

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<sup>5</sup>Cf. Fazzari, Hubbard and Petersen (1988) and the more recent papers cited in Winker (1996).

## 2 The supply of business loans

Supply of loans is almost always used synonymously for bank supply of loans. In fact, while analyzing the supply of business loans in this section a model of bank behaviour is developed. The justification for this approach is that external financing in Germany is dominated by banks. In order to derive the loan supply, a theory of the financial firm has to be developed.<sup>6</sup> As models based purely on portfolio theoretic approaches seem unsuitable for the quantitative derivation of the loan supply,<sup>7</sup> banks will be modelled like other productive firms.<sup>8</sup>

The production of business loans is based on three main inputs, namely qualified labour, capital, and deposits. Since labour and physical capital may impose restrictions rather on the number of loans than on the total amount of the loan supply and in order to keep the model tractable, a simplification using only the two inputs equity ( $EQ$ ) and deposits ( $D$ ) is used.

The output of banks might be divided into the holding of money, central bank reserves, securities, and loans to private customers. Both securities and loans might be further disaggregated with regard to maturity or risk. How many assets are necessary to model bank behaviour under asymmetric information? The number of three might be regarded as a good first guess. Besides money and reserves with high liquidity and securities with very low risk and lower liquidity than money there would be a loan aggregate with higher risk, again lower liquidity due to missing secondary markets, but higher (expected) returns than for the other assets.

However, loans to debtors with high ratings such as large or public enterprises are not subject to a relevant default risk. Consequently, the effects of asymmetric information might be minor on this credit market segment. Moreover, these debtors can switch between several banks without facing negative signalling effects. Therefore, banks can only react as price taker on this market segment and adjust the quantity they want to offer to these customers.

The picture is quite different for loans with a non-negligible default risk: loans to small and young firms for which information is not publicly

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<sup>6</sup>Klein (1971), p. 205, wrote: "In spite of the importance of commercial banking both as a major financial intermediary and as an important link in the monetary transmission process, there is little consensus as to what constitutes a workable and productive theory of the financial firm."

<sup>7</sup>Cf. Sealey and Lindley (1977).

<sup>8</sup>Cf. Stiglitz (1987), p. 40.

available.<sup>9</sup> These customers depend much more on business relations to specific banks. If such a firm wants to change its business relations it would have to transmit information to the new bank. Consequently, the most risky firms will have the highest incentives to change banking connections when facing financing constraints. Due to this adverse selection effect the “new bank” will prefer its old customers. Even if the new bank faces an excess supply of credits, in general it could be preferable to transfer this credit volume to other banks instead of facing the risk of adverse selection. One can conclude that banks act as monopolists on the market for risky loans which is segmented in many micromarkets defined by the availability of information.<sup>10</sup>

As the markets for high risk and low risk loans differ fundamentally with regard to the possible effects of asymmetric information, a macromodel of bank behaviour should treat these markets separately. Therefore, the model of bank production used in this paper includes four assets, namely money and reserves ( $R$ ), securities ( $B$ ), low risk loans ( $L_l$ ) and loans with higher inherent risks ( $L_h$ ). Table 1 shows the resulting balance-sheet of the bank.

Table 1: Banks' Balance-Sheet

Assets	Liabilities
<b>Money and Reserves <math>R</math></b>	<b>Deposits <math>D</math></b>
– Cash balances	– Credits by other banks
– Reserves	– Sight and term deposits, savings
<b>Securities <math>B</math></b>	– Holding of own securities
– Bonds	
– Credits to other banks	<b>Equity <math>EQ</math></b>
– Credits to public sector	– liable equity
<b>Loans with low risk <math>L_l</math></b>	– Reserves
<b>Loans with higher risk <math>L_h</math></b>	

Of course, one could achieve a more realistic model of the credit market by further subdividing the risk classes. However, the observed behaviour of

<sup>9</sup>Cf. Stiglitz (1987), p. 50: “Because of sunk costs associated with the acquisition of information, these loan markets are inherently imperfectly competitive. Only a few lenders (banks) will have information relevant to judging the riskiness of any particular borrower.”

<sup>10</sup>Cf. Petersen and Rajan (1994), p. 6.

banks including the German central bank indicates that a low number of risk classes already represents the optimum due to high information costs for the classification.<sup>11</sup>

Banks as other private enterprises will try to maximize their expected profit. For given interest rates and prices, the expected profit is given by<sup>12</sup>

$$E(\pi) = r^B B + \rho_l L_l + \rho_h L_h - (uEQ + r^D D + \Omega), \quad (1)$$

where  $r^B$  and  $r^D$  denote the interest rates on bonds and deposits,  $\rho_l$  and  $\rho_h$  the expected returns on loans with low and high risk, respectively,<sup>13</sup> and  $u$  the user costs of equity. Finally,  $\Omega$  includes all other costs such as costs of illiquidity.

The maximization of the expected profit is subject to technical, institutional and budget constraints. The technical constraint is the outcome of the technical production function using qualified labour and physical capital as inputs. The institutional constraints include minimum reserve requirements or interest rate ceilings.

Finally, the budget constraint is given by

$$B + L_l + L_h = EQ + D - R \quad (2)$$

or, using  $d$  as mean reserve fraction and the assumption of no relevant holding of excess reserves,

$$B + L_l + L_h \leq EQ + (1 - d)D. \quad (3)$$

This budget constraint combined with the assumption of exogenously given deposits or reserves, respectively, might be the reason for studying the liability side rather than the asset side of banks' balances. However, if a functioning interbank money market exists, the availability of deposits might influence the cost structure of the banks, but can hardly be a limiting factor for their loan supply.

In addition to technical constraints and the budget inequality, banks are subject to regulatory constraints. For Germany, regulation is based on the financial market law ("Gesetz über das Kreditwesen") enacted in 1962

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<sup>11</sup>Cf. Deutsche Bundesbank (1992), p. 35.

<sup>12</sup>Cf. Klein (1971), p. 208, and Hofmann (1994), p. 49f.

<sup>13</sup>As  $\rho_l$  and  $\rho_h$  are calculated from the nominal interest rates  $r^{L_l}$  and  $r^{L_h}$ , respectively, taking into account the default risk, which itself depends on the nominal interest rates due to adverse selection and negative incentives, these variables are non monotonic functions of  $r^{L_l}$  and  $r^{L_h}$ .

and changed repeatedly afterwards. Prior to the proposition of the Cooke Committee<sup>14</sup> it imposed ceilings for the risk weighted asset volume of a bank as a multiple  $\lambda$  of its equity.<sup>15</sup>

Although equity requirements used to be one of the central monitoring tools of the German central bank even prior to the enactment of the financial market law, the aggregate loan market models did not seem to pay much attention to this issue. This might be due to the fact that equity requirements comparable to those of the German law were imposed in the United States only after the propositions of the Cooke Committee in the late 1980s. Interest in the effects of these equity requirements increased following the New England credit crunch in the early 1990s.<sup>16</sup>

Using the expected profit function (1), the constraints given by (3) and the ceilings defined by available equity, a loan supply function similar to the one provided by Melitz and Pardue (1973) can be derived. Following their approach, loan supply depends on a scale variable, the expected return of loans, the return to other assets and the costs of deposits.

Whereas the theoretical base of the scale variable, which is represented by monetary aggregates or deposit volume, remains rather weak in the Melitz and Pardue approach, it can be based on equity requirements given the German institutional framework and the assumption that equity adjustment is expensive and time consuming. The expected return on loans should exhibit a positive influence on the loan supply, whereas the return to other assets reduces the loan supply. An increase in the costs of deposits, however, leads to an increase of the loan supply in the Melitz and Pardue framework as the induced decrease in expected profit is partly compensated by a shift to the riskier asset loan supply.

Turning back to the bank loan supply model developed so far, the ceiling imposed by the equity requirements is not absolutely binding. Short term ex post violations are possible. Therefore, the strictly binding constraint

$$L_l + L_h \leq \lambda EQ$$

is extended by including an additional term in the specification of expected costs of illiquidity.<sup>17</sup> Consequently, these costs  $\Omega$  are given by

<sup>14</sup>Cf. Bank für Internationalen Zahlungsausgleich (1988), p. 116.

<sup>15</sup>For more details on this institutional aspect cf. Winker (1996), pp. 159ff.

<sup>16</sup>Cf. Bernanke and Lown (1991). In an empirical study based on bank balance sheet data, Hancock and Wilcox (1992) reported a negative impact of missing equity on the supply of some loan categories.

<sup>17</sup>Cf. Hofmann (1994), p. 49f, for a standard formulation of illiquidity costs.

$$\Omega = \Omega(L_l, L_h, \lambda EQ). \quad (4)$$

Including this term and the budget constraint into the equation for the expected profit (1) leads to the following optimization problem

$$\begin{aligned} \max E(\pi) = & r^B(EQ + (1 - d)D - L_l - L_h) + \rho_l(r_l^L, ir)L_l + \rho_h(r_h^L, ir)L_h \\ & - (uEQ + r^D D + \Omega(L_l, L_h, \lambda EQ)), \end{aligned} \quad (5)$$

where  $r^B, r_l^L, r^D, d$ , the user cost of equity  $u$  and the rate of insolvencies  $ir$  as indicator of the loan repayment probability are given for a single (small) bank. Consequently, the expected return to low and high risk loans  $\rho_l$  and  $\rho_h$  do not depend on the volume of these loans. Equity  $EQ$  is also a fixed factor in the short run, whereas  $D, L_l, L_h$ , and  $r_h^L$  are the decision variables. Furthermore, it is assumed that the rate of insolvencies  $ir$  is independent of the loan volume, but differs by a constant factor between the two risk groups. Then, the first order conditions for (5) are given by

$$\frac{\partial E(\pi)}{\partial D} = r^B(1 - d) - r^D = 0 \quad (6)$$

$$\frac{\partial E(\pi)}{\partial L_l} = -r^B + \rho_l(r_l^L, ir) - \frac{\partial \Omega}{\partial L_l} = 0 \quad (7)$$

$$\frac{\partial E(\pi)}{\partial L_h} = -r^B + \rho_h(r_h^L, ir) - \frac{\partial \Omega}{\partial L_h} = 0 \quad (8)$$

$$\frac{\partial E(\pi)}{\partial r_h^L} = \frac{\partial \rho_h}{\partial r_h^L} L_h = 0. \quad (9)$$

Equation (6) implies that equilibrium on the deposit side of the banking system is given if and only if the deposit rate  $r^D$  corrected by the costs imposed by reserve requirements equals the risk free rate  $r^B$ .

The optimal rate on the low risk loans is given by (7). The expected return from low risk loans should equal the risk free rate  $r^B$ , augmented by the marginal costs of expected illiquidity. The latter is determined mainly by the fixed equity. The optimal rate  $r_l^L$  is given for a single bank due to the assumed competition on this market segment. Therefore, a single bank can only adapt its volume of low risk loans  $L_l$  in order to satisfy condition (7), which defines implicitly the supply of low risk loans

$$L_l = L_l(r_l^L, ir, r^B, \lambda EQ). \quad (10)$$

+   -   -   +

As the information asymmetries lead to a quasi monopolistic situation of a bank towards its high risk customers it can vary both the nominal interest rate  $r_h^L$  on and the volume  $L_h$  of high risk loans. Following (9), banks will choose the optimal rate  $r_{opt}^L$  as in Stiglitz and Weiss (1981),<sup>18</sup> and the loan volume is chosen again in a way to obtain the equality of the risk free rate and the expected return on high risk loans less the expected costs of illiquidity. Finally, the supply of high risk loans is given by

$$L_h = L_h(r_{opt}^L(ir), ir, r^B, \lambda EQ), \quad (11)$$

+                      -                      -                      +

where the negative impact of  $ir$  is larger than for low risk loans due to the assumed risk difference. Note that  $ir$  has an additional negative influence via  $r_{opt}^L$ .

The result of this rather simple model of bank behaviour is an endogenous split of bank production into the holding of securities, low and high risk loans. The portfolio composition becomes non-trivial without necessarily assuming risk aversion. Rather it is the outcome of the impact of equity requirements.

Since the German central bank does not publish information about rates or volumes for different loan risk categories, the fraction of high risk loans  $\Theta \equiv L_h/(L_l + L_h)$  is determined endogenously. Then, the aggregate loan supply  $L^s$  and the observed aggregate loan rate  $r^L$  are given by

$$L^s = L_l + L_h \quad (12)$$

$$r^L = (1 - \Theta)r_l^L + \Theta r_h^L \quad (13)$$

$$= (1 - \Theta)r_l^L + \Theta r_{opt}^L. \quad (14)$$

As  $\Theta$  is implicitly defined by the supply equations for low and high risk loans (10) and (11), it is possible to deduce some hypotheses on the dependence of the portfolio composition on variables such as  $ir$ ,  $r^B$  and  $\lambda EQ$ . An increase of the insolvency rate  $ir$  will ceteris paribus increase the expected costs of illiquidity and have a negative impact on the optimal rate for high risk loans,

$$\frac{\partial L_h}{\partial ir} < \frac{\partial L_l}{\partial ir} < 0. \quad (15)$$

It follows

$$\frac{\partial \Theta}{\partial ir} < 0 \iff \frac{\partial L_h}{\partial ir} \frac{ir}{L_h} > \frac{\partial L_l}{\partial ir} \frac{ir}{L_l}, \quad (16)$$

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<sup>18</sup>Cf. section 4 for the econometric specification of  $r_{opt}^L$ .

i.e. as long as the elasticity of the supply of high risk loans with regard to the insolvency rate is larger than for low risk loans, an increase of the overall default risk will increase the fraction of low risk loans in the portfolio. The same holds if the low risk loans only are affected by increasing risk. As the expected cost of illiquidity  $\Omega$  are mainly determined by the fraction of high risk loans equations (7) and (8) lead to

$$0 > \frac{\partial L_h}{\partial r^B} > \frac{\partial L_l}{\partial r^B} \quad (17)$$

and consequently

$$\frac{\partial \Theta}{\partial r^B} > 0 \iff \frac{\partial L_h}{\partial r^B} \frac{r^B}{L_h} < \frac{\partial L_l}{\partial r^B} \frac{r^B}{L_l}. \quad (18)$$

Under reasonable circumstances ( $\Theta > 0.5$  is a sufficient condition) an increase of the risk free rate  $r^B$  will lead to a reduction of the aggregate loan supply and to a shift towards high risk loans in order to compensate partially the lower expected profit.

Finally, an increase of equity  $EQ$  has an expansive effect on the total loan supply. If the relation of equity to the balance volume increases also, a shift towards high risk loans may result as the reduction in expected illiquidity costs has a more crucial impact on the supply of high risk loans.<sup>19</sup>

### 3 The demand for business loans

Abstracting from constraints on goods and labour markets, the firms' intertemporal profit maximizing will depend solely on current and expected future relative prices for factors and products. The resulting optimal production and investment plans determine the financing needs. If there are no frictions on the financial markets either, then the single market interest rate will influence the investment decision.

However, neither goods nor labour markets are characterized by Walrasian' equilibrium at all times. Furthermore, the arguments given by the theory of asymmetric information cast doubts about the assumption of frictionless markets for equity and bank loans. Therefore, the modelling of the demand for business loans should take into account those limitations. Then, optimal production and investment plans should be derived taking account

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<sup>19</sup>Cf. Stiglitz (1987), p. 40, for a similar argument: "Accordingly, when the bank's working capital is reduced, its willingness to make risky loans is reduced."

for these constraints. Since the explicit incorporation of all constraints in a formal optimization problem is beyond the scope of this paper, it is assumed that firms optimize their production and investment plans regardless of possible financial constraints and adapt them when they become binding.

In order to avoid inconsistencies, some aspects have to be taken into account. In particular, it is necessary to distinguish between production and investment plans and the ex post realized values which might differ. Consequently, loan demand depends on variables reflecting costs and returns and – in contrast to standard macro models – not on production or investment but on their planned figures.

Due to the time lag between investment decisions and their realization the decision has to be made under uncertainty. Therefore, optimal investment is determined by the expected values of relevant variables and possible rationing on goods and labour markets. The optimization program results in an optimal investment plan depending on the expected demand ( $E(Y^a)$ ) on the goods market and relative prices of factors and output.

As there exists a time lag between the payment of production factors and sale revenues, firms' financing needs will depend also directly on expected production. This influence is modelled by ( $E(Y^a)$ ) and the expected wage share ( $E(ws)$ ). Finally, the (expected) inflation rate ( $\hat{p}$ ) might influence the financial situation of the firm not only via real interest rates. In particular, the relation of outstanding loans to current revenues is influenced by price changes. The financing needs induced by optimal investment and production plans  $F^o$  is given by

$$F^o = f(E(Y^a), E(r^L - \hat{p}), E(ws), E(\hat{p})). \quad (19)$$

Finally, the composition of firms' finance has to be taken into account. Assuming a financing hierarchy with different costs and rationing probabilities, the demand for business loans  $L^d$  can be given by

$$L^d = \theta F^o, \quad (20)$$

where  $\theta$  is a function of retained profits, relative costs of financing by equity or debt, and of the inflation rate  $\hat{p}$ .

## 4 A Disequilibrium Model of the Credit Market

The main novelty of this paper is the modelling of the optimal lending rate based on asymmetric information theory instead of using the observed

values as rationing indicator. The latter approach seems inappropriate for the business loan market as the asymmetric information theory implies that interest rates have not to react in a “natural” way to an excess demand situation.<sup>20</sup> Furthermore, it offers the first aggregate disequilibrium analysis for German data, which is not restricted to interest rate adjustments, but supplies estimates of effective loan supply and demand.

The central assumption of the model derived in the sequel is that banks set both the loan supply and the loan interest rates. The explicit treatment of the effects of asymmetric information allows for the derivation of target values for the loan supply  $L_*^s$  and the interest rate  $r_*^L$ . The transacted volume  $L$  on the loan market is subject to the free trade constraint, i.e.

$$L \leq \min(L^s, L^d),$$

where  $L^d$  denotes effective loan demand. Delayed adjustment with regard to exogenous shocks and the different competition schemes on the credit market segments described in section 2 can result in deviations between  $L^s$  and  $r^L$  and their target values  $L_*^s$  and  $r_*^L$ , respectively. Profit maximization in a static framework results in a loss function weighting solely derivations of actual from target values. However, long term loan contracts, not exhausted credit lines, and menu costs introduce dynamic elements into banks' objective functions. Since the econometric modelling approach used here does not allow for the explicit inclusion of time dependence, an additional term for the deviation between transacted quantities and effective demand is included in the loss function as a compromise solution. The loss function in relative terms is stated as

$$\mathcal{V} = (\log(1 + r^L) - \log(1 + r_*^L))^2 + \nu_1(l - l_*^s)^2 + \nu_2(l - l^d)^2, \quad (21)$$

where lower case letters stand for the logarithms of the respective variable. Additional dynamic aspects are introduced by including lagged exogenous variables for the estimations.

From equation (19) and (20), the effective loan demand is given by

$$L^d = \theta F^o = f(E(Y^a), E(r^L - \hat{p}), E(ws), E(\hat{p}), \varepsilon_D), \quad (22)$$

where  $\varepsilon_D$  is an error with expected value zero. The activity indicator variable  $Y^a$  is instrumented by lagged values of production indices and expectations from business cycle surveys. The other relevant variables are represented by

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<sup>20</sup>Cf. Artus (1984), p. 617.

their actual values, corresponding to a simple adaptive expectations scheme for the expected values of these variables.

The target interest rate for the market segment of high risk loans  $r_{opt}^L$  is deduced from the Stiglitz and Weiss (1981) model. The expected return of a high risk loan of volume  $L^d$  at interest rate  $r^L$  is given by

$$E(\pi_h) = ((1 + r^L)L^d - E^f) \frac{1}{F(p^*)} \int_0^{p^*} pf(p)dp + E^f. \quad (23)$$

$F(p^*)$  denotes the fraction of firms in the high risk segment demanding a loan at the given interest rate  $r^L$ .  $p^*$  is the marginal success probability of these firms, and  $E^f$  is the return of a failed project. In particular,  $p^*$  is a function of  $r^L$ .  $\rho$  is the opportunity rate of return. If  $W$  stands for the lender's equity, indifference between undertaking a project or choosing the save asset with return  $\rho$  leads to

$$p^* = \frac{E - E^f - (1 + \rho)W}{(1 + r^L)L^d - E^f} \quad (24)$$

with

$$\frac{\partial p^*}{\partial r^L} = -\frac{(E - E^f - (1 + \rho)W)L^d}{((1 + r^L)L^d - E^f)^2} < 0. \quad (25)$$

$E = (1 + r^L)L^d$  denotes the project return if successful. The fraction of unsuccessful projects of the last period is given by the rate of insolvencies  $ir$ , i.e.

$$\begin{aligned} ir &= \frac{1}{F_{-1}(p_{-1}^*)} \int_0^{p_{-1}^*} (1 - p)f_{-1}(p)dp \\ &= 1 - \frac{1}{F_{-1}(p_{-1}^*)} \int_0^{p_{-1}^*} pf_{-1}(p)dp. \end{aligned} \quad (26)$$

If the distribution of success probabilities remains stable over time, one obtains

$$E(\pi_h(r^L)) = \frac{((1 + r^L)L^d - E^f)}{F(p^*)} \left[ F(p_{-1}^*)(1 - ir) + \int_{p_{-1}^*}^{p^*} pf(p)dp \right] + E^f. \quad (27)$$

As the optimal interest rate for high risk loans is subject to stochastic shocks in general,  $r_{opt}^L \neq r_{opt,-1}^L$  and, consequently,  $p^* \neq p_{-1}^*$ .

A necessary, and, if the distribution of the success probabilities is unimodal and  $r^L$  finite, sufficient condition for the optimal  $r^L$  is given by

$$\left. \frac{\partial E(\pi_h(r^L))}{\partial r^L} \right|_{r^L=r^L_*} = 0. \quad (28)$$

Partial derivation of banks' expected return (27) gives

$$\begin{aligned} \frac{\partial E(\pi_h(r^L))}{\partial r^L} &= \frac{L^d}{F(p^*)} \left[ F(p_{-1}^*)(1 - ir) + \int_{p_{-1}^*}^{p^*} pf(p)dp \right] \\ &+ \frac{((1 + r^L)L^d - Ef(p^*))}{F(p^*)} \frac{dp^*}{dr^L} \cdot \\ &\cdot \left[ -\frac{F(p_{-1}^*)}{F(p^*)}(1 - ir) - \frac{F(p_{-1}^*)}{F(p^*)} \int_{p_{-1}^*}^{p^*} pf(p)dp + p^* \right]. \end{aligned} \quad (29)$$

Using (24) and (25) for  $p^*$  and its derivative, (29) becomes equivalent to

$$\begin{aligned} &\left( \frac{F(p_{-1}^*)}{F(p^*)}(1 - ir) + \frac{1}{F(p^*)} \int_{p_{-1}^*}^{p^*} pf(p)dp \right) \cdot \\ &\cdot \left( 1 + \frac{p^* f(p^*)}{F(p^*)} \right) - \frac{p^{*2} f(p^*)}{F(p^*)} = 0. \end{aligned} \quad (30)$$

A second order approximation of the integral term in (30) yields

$$\begin{aligned} &\left( \frac{F(p_{-1}^*)}{F(p^*)}(1 - ir) + \frac{p^* f(p^*)}{F(p^*)}(p^* - p_{-1}^*) \right) \cdot \\ &\cdot \left( 1 + \frac{p^* f(p^*)}{F(p^*)} \right) - \frac{p^{*2} f(p^*)}{F(p^*)} = 0. \end{aligned} \quad (31)$$

Using another second order approximation, (31) reduces to

$$(1 - ir) \frac{p_{-1}^* f(p_{-1}^*)}{p^* f(p^*)} + \frac{4p^*}{3} - 2p_{-1}^* = 0. \quad (32)$$

Close to a longrun equilibrium situation, i.e. for small changes in the marginal success probability  $p^*$ , a standard Taylor series expansion argument gives

$$\frac{p_{-1}^* f(p_{-1}^*)}{p^* f(p^*)} \sim 1. \quad (33)$$

Furthermore, the deviation in the marginal success probabilities is small of first order, i.e.

$$p_{-1}^* \sim p^*. \quad (34)$$

In order to simplify the exposition, the project size is normalized to 1. Then,  $L^d = dr$  and  $W = 1 - dr$ , where  $dr$  denotes the debt ratio. Furthermore, the return to a failed project  $E^f$  is assumed to be zero. The following condition results, which is approximatively equivalent to (32):

$$(1 + r^L)dr(1 - ir) = \frac{2}{3} [E - (1 + \rho)(1 - dr)] \quad (35)$$

Net returns of investment  $E$  are approximated by the alternative rate of return  $\rho$  interacted with some business cycle indicators ( $bci$ ). Taking logs results in

$$\log(1 + r_{opt}^L) = \log \frac{2}{3} + \log(1 + \rho) - \log(1 - ir) - \log(dr) + \log(bci - (1 - dr)). \quad (36)$$

As pointed out in section 2, the aggregate optimal interest rate  $r_*^L$  is a weighted average of  $r_{opt}^L$  and the interest rate on low risk loans which is mainly determined by refinancing costs. The weighting factor  $\Theta$  is given by the portfolio split which itself is a function of the overall risk proxied by the rate of insolvencies  $ir$ , the alternative rate of return  $\rho$  and the banks' equity. Hence, the marginal costs of refinancing low risk loans given by the money market rate  $r^M$  will influence the optimal interest rate  $r_*^L$ . Furthermore, the parameters  $ir$ ,  $dr$  and  $\rho$  are relevant for high risk loans, and for the endogenous portfolio split. The resulting econometric equation is given by

$$\begin{aligned} \log(1 + r_*^L) = & \alpha_0 + \alpha_1 \log(1 - ir) + \alpha_2 \log(1 + \rho) + \alpha_3 \log dr + \\ & \alpha_4 \log bci + \alpha_5 \log(1 + r^M) + \alpha_6 \log EQr + \varepsilon_R. \end{aligned} \quad (37)$$

$EQr$  denotes the share of equity in banks' balance volume, and  $\varepsilon_R$  is a residual with expected value zero.

It should be noted that the realized interest rate  $r^L$  will exhibit a high correlation with the optimal  $r_*^L$  only in periods of excess demand on the loan market. For other periods,  $r^L$  is determined by minimizing the loss function (21).

The rate of insolvencies  $ir$  has a twofold influence on the optimal interest rate  $r_*^L$ . Firstly, as a measure of overall risk it influences the endogenous portfolio split between high and low risk loans ( $\Theta$ ). An increase of the overall

risk will *ceteris paribus* result in a decrease of the aggregate loan supply and in a decrease of the fraction of high risk loans in the loan portfolio. As  $r_l^L < r_h^L$  holds, a negative impact on the average interest rate  $r_*^L$  results.

Second, the optimal interest rate on high risk loans  $r_{opt}^L$  is directly influenced by  $ir$  proxying the repayment probabilities. As long as the approximations used in the derivation of (35) are exact, a positive effect results despite of adverse selection. This holds true as long as the density function  $f(p)$  for the repayment probabilities shows a “normal” behaviour. In particular, a necessary and sufficient condition is given, if a higher rate of insolvencies  $ir$  is mirrored by a flattening of  $f(p)$  close to the marginal repayment probability  $p^*$ .

The alternative rate of return  $\rho$  has a positive influence on the loan interest rate. It proxies the firms’ rate of return, as only firms with expected rate of return higher than  $\rho$  will ask for a loan. However, it might be difficult to distinguish empirically the effect of  $\rho$  from the likewise positive influence of refinancing costs  $r^M$ . The overall influence of  $dr$  is indeterminated, whereas the business cycle indicator  $bci$  should have a positive impact.

Now, we might come back to the settlement of transacted quantities and market interest rate  $r^L$ . The modelling framework of Goldfeld and Quandt (1986) implies the following course. In a first step, banks minimize the loss function (21) with regard to  $r^L$  and  $L$  under the constraints given by the loan demand (22), the optimal interest rate (37), their loan supply function and the assumption of an undisturbed realization of loan demand, i.e.  $\varepsilon_D \equiv 0$ . Only after setting  $r^L$  the actual realization of  $L^d$  can be observed. Then, the loss function (21) is minimized again for given  $r^L$  and  $L^d$  with regard to  $L$ .

Therefore, the target value for the loan supply  $L_*^s$  does not depend on the target value for  $r_*^L$  in the first step, but on its realization after  $L^d$  is observed. That means the realized value  $r^L$  is used in the loans supply function instead of the latent variable  $r_*^L$ .

Through the endogenous portfolio split in high and low risk loans the target value of the loan supply is influenced besides the volume barrier equity, costs of deposits ( $cd$ ) and/or marginal refinancing costs  $r^M$ , by the insolvency rate  $ir$ . The following econometric function for  $L_*^s$  results:

$$\log L_*^s = \beta_0 + \beta_1 \log EQ + \beta_2 \log(1 + cd) + \beta_3 \log(1 + r^M) + \beta_4 \log(ir) + \varepsilon_S \quad (38)$$

In the appendix a joint likelihood for the system of equations (22), (37) and (38) given the dynamic decision structure is derived. The results of a maximum likelihood estimation are presented below.

## 5 Estimation Results for West Germany

### The Data

The empirical assesement is based on monthly data from the German central bank, the central statistical bureau and business survey data of the ifo institute for economic research. The sample starts in 1974.01 in order to avoid the inclusion of structural breaks caused by the removal of interest rate ceilings in 1967 and the breakdown of the Bretton–Woods–system with the induced change of central bank policy in 1973. The estimation period ends in 1989.10 because of German unification.

The empirical analysis is concentrated on the market for loans to domestic firms and self-employed excluding house-building ( $L^E$ ). Unfortunately, the transacted volume of this credit market segment is published only quarterly. Thus, in order to fit monthly data, an instrument interpolation based on loans to domestic firms and private persons was used.

Loan demand depends on the expected level of economic activity, interest rates on loans, inflation, the wage share and interest rates on alternative assets. The expected level of economic activity is approximated by an instrumental fit based on lagged observations of the index of net production ( $YR$ ) and of business prospect variables from the ifo business cycle survey. The relevant price is the producer price index ( $P$ ). The interest rate on loans ( $r^L$ ) is proxied by the rate on current account credits of 1 to 5 mio. DM.<sup>21</sup> The wage share ( $ws$ ) is calculated from the gross wage and salary incomes corrected for the self-employed effect.

Loan supply depends on the insolvency rate  $ir$  as proxy for the overall repayment probability, equity  $EQ$  due to the institutional framework (in order to avoid biased results due to endogeneity, fits of an instrumental estimation based on lagged values are used instead of  $EQ$ ), and the costs of deposits ( $cd$ ).

Finally, the target value of the interest rate  $r_\star^L$  depends on the insolvency rate ( $ir$ ), the expected return on investment (proxied by a medium term interest rate ( $\rho$ ) and business cycle indicators ( $bci$ )), the debt ratio ( $dr$ ) and the banks' equity ratio ( $EQr$ ).  $\rho$  is approximated by the average return to federal bonds with remaining term of one year. The debt ratio  $dr$  is proxied by the ratio of outstanding loans to the capital stock.

All data have been seasonally adjusted prior to estimation as numerical problems did not allow the inclusion of seasonal dummies in the likelihood

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<sup>21</sup>For the availability of loan interest rates for Germany cf. Winker (1996), section 3.1.1.

maximization. These numerical problems result of the necessity to integrate out the rationing in each iteration step.

## Estimation Results

The algorithm for maximization of the likelihood function was implemented in FORTRAN on a UNIX workstation using a fast sequential quadratic programming procedure of the NAG-library. In order to improve efficiency, analytical derivatives of the likelihood function were used. Furthermore, some of the parameter estimators had to be restricted, for example the weighting factors  $\nu_1$  and  $\nu_2$  have to be non negative. In fact, as the likelihood function is unbounded for  $\nu_2$  tending to zero, it was necessary to bound it away from zero.

In a first step a standard canonical disequilibrium model was estimated for the loan demand and supply equations to provide starting values. Then, the following model was estimated:<sup>22</sup>

$$\begin{aligned} \log L^d = & \gamma_0 + \gamma_1 \log(1 + r^L) + \gamma_2 \log YR + \gamma_3 \hat{p} + \\ & \gamma_4 \log ws + \gamma_5 \log bci + \varepsilon_D \end{aligned} \quad (39)$$

$$\begin{aligned} \log L_*^s = & \beta_0 + \beta_1 \log(1 + r^L) + \beta_2 \log EQ^f + \\ & \beta_3 \log(1 + cd) + \varepsilon_S \end{aligned} \quad (40)$$

$$\begin{aligned} \log(1 + r_*^L) = & \alpha_0 + \alpha_1 \log(1 - ir) + \alpha_2 \log(1 + r^M) + \\ & \alpha_3 \log bci + \alpha_4 \log dr + \alpha_5 \log EQr + \\ & \alpha_6 \log(1 + \rho) + \varepsilon_R \end{aligned} \quad (41)$$

Table 2 reports the parameter estimates for a static version (1) of the model and for a dynamic version (2) including lagged values of the exogenous variables which might be interpreted as instruments for the lagged latent variables. It should be noted that the maximization of the likelihood function resulted in many local optima depending on the chosen starting parameters, the resulting values of the likelihood function being close to the ones of the optimal results presented in the table. As the dimension of the parameter space makes a systematic search for all local optima impossible, the use of heuristic global optimization algorithms might be helpful for future analysis of similar models.

In spite of numerical problems, the qualitative results, i.e. periods and extent of excess loan demand are quite robust to a change of estimation

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<sup>22</sup>The likelihood function is derived in the appendix.

Table 2: Estimation Results for the Disequilibrium Model

Equation	Variable	(1)		(2)	
		Coeff.	Std.dev.	Coeff.	Std.dev.
<b>Weights</b>	$\nu_1$	3.538	0.262	6.325	0.004
	$\nu_2$	0.100	0.300	0.266	0.001
<b>Demand</b>	Constant	-4.260	0.093	-4.097	0.029
	$\log YR$	1.107	0.014	0.481	0.043
	$\log YR_{-1}$			0.611	0.043
	$\log(1 + r^L)$	-0.138	0.008	-0.787	0.107
	$\log(1 + r^L)_{-1}$			0.726	0.130
	$\hat{p}$	-0.169	0.049	-0.214	0.149
	$\hat{p}_{-1}$			0.049	0.120
	$\log ws$	-1.305	0.120	-1.353	0.117
	$\log ws_{-1}$			0.020	0.123
	$\log bci$	0.053	0.005	-0.004	0.004
	$\log bci_{-1}$			0.040	0.006
	$\sigma_D$	0.022	0.001	0.025	0.001
<b>Interest Rate</b>	Constant	0.412	0.074	-0.523	0.011
	$\log(1 - ir)$	-3.060	0.073	-0.574	0.493
	$\log(1 - ir)_{-1}$			-1.135	0.496
	$\log(1 + r^M)$	0.298	0.073	0.300	0.004
	$\log(1 + r^M)_{-1}$			0.158	0.242
	$\log bci$	-0.003	0.111	0.035	0.039
	$\log bci_{-1}$			-0.044	0.039
	$\log dr$	0.087	0.078	0.124	0.259
	$\log dr_{-1}$			-0.192	0.022
	$\log EQr$	0.067	0.113	-0.090	0.126
	$\log EQr_{-1}$			-0.046	0.152
	$\log(1 + \rho)$	0.629	0.073	-0.279	0.190
	$\log(1 + \rho)_{-1}$			0.501	0.025
	$\sigma_R$	0.033	0.003	0.037	0.003
<b>Supply</b>	Constant	2.919	0.071	2.973	0.006
	$\log EQ^f$	0.751	0.016	0.213	0.025
	$\log EQ_{-1}^f$			0.523	0.031
	$\log(1 + r^L)$	0.222	0.073	0.057	0.511
	$\log(1 + r^L)_{-1}$			0.584	0.507
	$\log(1 + cd)$	1.023	0.073	1.812	0.504
	$\log(1 + cd)_{-1}$			-1.180	0.502
	$\sigma_S$	0.014	0.008	0.013	0.002
	Loglikelihood	4.548		4.680	

method (canonical disequilibrium or complete system approach), selection of explanatory variables, different starting values or changes in the restrictions for  $\nu_1$  and  $\nu_2$ . In particular, all reasonable results exhibit a relevant degree of excess demand on the loan market. On average, it amounts to 4.55% for the static (1) and to 4.30% for the dynamic (2) specification.

Proceeding with the necessary caution implied by imperfections of the estimation method, one can nevertheless conclude that estimation results are compatible with the theoretical hypothesis. The order of magnitude of the weighting factors of the loss function  $\nu_1$  and  $\nu_2$  implies that banks weight departures from the target value of loan supply more than departures from the target interest rate, whereas the excess loan demand has the smallest influence.

The effective loan demand is determined mainly by the expected economic activity. The coefficient is highly significant and larger than one, which might be attributed to the fact that firms have to finance current production and investment for future production.

The impact of the loan interest rate is negative and significant. In the dynamic specification (3) it exhibits only a short term impact. Although, a low elasticity of loan demand with respect to the interest rate is found quite often in the empirical literature, it has to be considered with special caution due to the limitation of the estimation procedure.

The high and significant effect of the inflation rate  $\hat{p}$  indicates that it has an influence beyond the difference between nominal and real interest rates as pointed out in section 3. The wage share has a negative influence on credit demand, i.e. the positive effect on financing needs for the current period is outweighed by the scale effect leading to a reduction of investment and production plans. The dynamic specification indicates that this effect is strongest in the short run. Finally, the business cycle indicator measuring the share of firms expecting a deterioration of their business situation exhibits a positive coefficient. This can be attributed to an increasing need for external funds in periods of low cash flow overcompensating the effect on expected activities.

The target value of the loan interest rate depends positively on the insolvency rate  $ir$  as it results from the model for the high risk loan market segment. This effect seems to outweigh the opposite effect of a shift in the loan portfolio towards low risk loans. However, it might be as well due to the fact that the conditions for a normal reaction discussed in section 4 are not fulfilled.

The estimated parameters for  $r^M$  and  $\rho$  give some evidence for the last

argument. For an exogenous portfolio composition, the parameter for  $r^M$  approximates the share of low risk loans in the portfolio  $(1 - \Theta)$ , if it is assumed that the elasticity of the interest rate on low risk loans with respect to changes in the money market rate is close to one. Then, the estimated coefficients indicate, that the share of high risk loans may be larger than 50%. However, the portfolio split itself depends on variables such as the insolvency rate or the equity ratio. Hence, it is not possible to derive reliable estimates of  $\Theta$ . The only other significant coefficients are the lagged value of the debt ratio in the dynamic specification and the alternative rate of return  $\rho$  in both specifications. Both variables exhibit the expected signs.

The specification of the target loan supply volume differs from standard approaches by the scale variable. Instead of deposit or balance volume, the banks' equity is used. It has a highly significant effect although with elasticity less than one. This might be attributed to a change in the asset structure of the banks caused by new financial services. The increase in the required equity ratio following the propositions by the Cooke Committee also limits the elasticity.

The interest rate on loans is significant in the static specification, whereas no significant effect was found in the dynamic version. This finding corresponds to the high weight  $\nu_1$  banks impose on the realization of their target loan supply volume, which can be explained if banks face binding constraints for their loan supply at least in the short run by equity requirements. Consequently, a low elasticity of the loan supply with regard to the interest rate is sometimes interpreted as the indicator for a rationing situation.<sup>23</sup>

The cost of deposits  $cd$  show the expected positive impact on the loan supply, i.e. higher deposits costs result *ceteris paribus* in a higher loan supply as banks will change their portfolio composition towards more risky assets in order to partially compensate the induced reduction in expected profits.

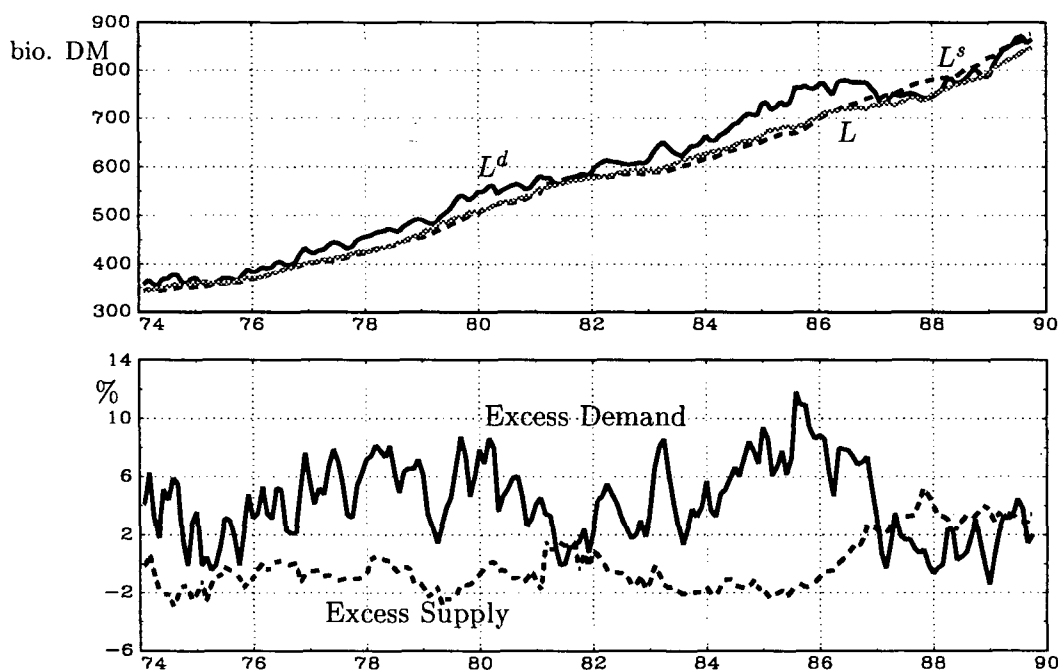
Figure 1 gives a graphical representation of the estimation results for the dynamic specification (2). The upper part of the figure shows effective loan demand  $L^d$  as solid line, the banks' target loans supply  $L_*^s$  as dashed line and the transacted loan volume  $L$  as gray shaded line. The lower part exhibits excess demand as solid line and excess supply as dashed line.

Striking is the high variance of the excess demand for loans over time. After the first OPEC-shock, in 1974 and 1975, no relevant rationing can be observed. In the second half of the 1970s, a large increase to nearly 10% of credit volume could be observed. Caused by the recession and the very

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<sup>23</sup>Cf. King (1986), p. 298.

Figure 1: Loan Demand and Supply



high interest rate level at the beginning of the 1980s, loan demand grows at a smaller rate than loan supply. Consequently, excess demand vanished in 1981. During the following boom, loan demand grows fast whereas loan supply increases along its long run growth path. The result is a renewed increase of excess demand to more than 10% of the transacted volume. The reasons for the slow increase of the loan supply can be found in the binding equity requirements strengthened by stronger regulation following the recommendations of the Cooke Committee.

## 6 Conclusion and Outlook

Based on micromodels of bank behaviour and the loan market under asymmetric information, a macroeconomic disequilibrium model for the German credit market was derived. In the model, the loan interest rate depends both on refinancing costs and risk considerations implied by adverse selection or adverse incentive effects. A simultaneous estimation approach is

used to identify the target values for the interest rate and loan supply together with effective loan demand. The estimation results for the time span 1975 – 1989 indicate that the late 1970s and the period 1983 – 1987 were characterized by excess demand reaching about 10% of the loan volume.

This large amount of excess demand on the credit market raises the question of its real effects. A topic of future research is to extend the model by an explicite treatment of investment demand. Preliminary results indicate that the effects found by some authors on the micro level can be reproduced for the aggregate economy.<sup>24</sup>

Keeping this in mind the presented partial macromodel can be regarded as a first building block for an empirical assesment of alternative monetary transmission mechanisms, which J. M. Keynes might have had in mind with his quote at the beginning of this paper.

## Appendix: Derivation of the Likelihood

In this appendix the ML-estimator for the simultaneous loan market model is derived. For ease of exposition, the time index is dropped. Following the exposition in section 3, the loan demand is estimated by the log-linear approximation

$$l^d = \gamma_1 \log(1 + r^L) + \gamma'_2 x_D + \varepsilon_D. \quad (42)$$

The target values for the interest rate and the loan supply of the banks are given by

$$\log(1 + r_*^L) = \alpha_1 \log(1 - ir) + \alpha'_2 x_R + \varepsilon_R \quad (43)$$

and

$$l_*^s = \beta_1 \log EQ^f + \beta'_2 x_S + \varepsilon_S. \quad (44)$$

The vectors  $x_D, x_R$  and  $x_S$  include all other explanatory variables. The errors  $\varepsilon_D, \varepsilon_R$ , and  $\varepsilon_S$  are assumed to be independently iid normal.

Using these functions, the banks try to minimize the loss function

$$\mathcal{V} = ((\log(1 + r^L) - \log(1 + r_*^L))^2 + \nu_1 (l - l_*^s)^2 + \nu_2 (l - l^d)^2), \quad (45)$$

where the weights  $\nu_1$  and  $\nu_2$  have to be non negative. In fact, for the further derivation, it has to be assumed that the weights are strictly positive. Minimizing of the loss function is subject to the ex post binding constraint

$$L \leq L^d \iff l \leq l^d. \quad (46)$$

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<sup>24</sup>Cf. Winker (1996), pp. 256ff.

Let  $\tilde{l}^d, \widetilde{lr}_*^L$  and  $\tilde{l}_*^s$  denote the fitted values of equations (42), (43) and (44), respectively, for example

$$\tilde{l}^d = \hat{\gamma}_1 \log(1 + r^L) + \hat{\gamma}_2' x_D.$$

Furthermore, let  $S = l_*^s - E(l^d(lr_*^L))$  be the expected excess supply for a non disturbed realization of loan demand, i.e. for  $\varepsilon_D = 0$ , given the target values of interest rate and loan supply, i.e.

$$S = \tilde{l}_*^s - \gamma_1 \widetilde{lr}_*^L - \gamma_2' x_D + \varepsilon_S - \gamma_1 \varepsilon_R. \quad (47)$$

Then, minimizing of the loss function (45) leads to

$$\log(1 + r^L) = \begin{cases} \frac{\widetilde{lr}_*^L + \nu_1 \gamma_1 \tilde{l}_*^s - \nu_1 \gamma_1 \gamma_2' x_D}{1 + \nu_1 \gamma_1^2} \\ + \frac{\varepsilon_R + \nu_1 \gamma_1 \varepsilon_S}{1 + \nu_1 \gamma_1^2}, \text{ if } S \geq 0 \\ \frac{(\nu_1 + \nu_2) \widetilde{lr}_*^L + \nu_1 \nu_2 \gamma_1 \tilde{l}_*^s - \nu_1 \nu_2 \gamma_1 \gamma_2' x_D}{\nu_1 + \nu_2 + \nu_1 \nu_2 \gamma_1^2} \\ + \frac{(\nu_1 + \nu_2) \varepsilon_R + \nu_1 \nu_2 \gamma_1 \varepsilon_S}{\nu_1 + \nu_2 + \nu_1 \nu_2 \gamma_1^2}, \text{ if } S < 0 \end{cases} \quad (48)$$

and

$$l = \min\{l^d, (\nu_1 l_*^s + \nu_2 l^d)/(\nu_1 + \nu_2)\}. \quad (49)$$

Using  $k_1 = \nu_1/(1 + \nu_1 \gamma_1^2)$  und  $k_2 = \nu_1 \nu_2/(\nu_1 + \nu_2 + \nu_1 \nu_2 \gamma_1^2)$  one obtains

$$\log(1 + r^L) = \begin{cases} \log(1 + r_*^L) + \gamma_1 k_1 S, & \text{if } S \geq 0 \\ \log(1 + r_*^L) + \gamma_1 k_2 S, & \text{if } S < 0. \end{cases} \quad (50)$$

Let  $\varepsilon_D, \varepsilon_R$ , and  $\varepsilon_S$  be iid normal residuals with variance  $\sigma_D^2, \sigma_R^2$  and  $\sigma_S^2$ , respectively. Then, the joint distribution of  $\log(1 + r_*^L)$  and  $S$  is also normal with expected value

$$(\widetilde{lr}_*^L, \tilde{l}_*^s - \gamma_1 \widetilde{lr}_*^L - \gamma_2' x_D)$$

and covariance matrix (cf. Anderson (1958), p. 19f.)

$$\Sigma = \begin{pmatrix} \sigma_R^2 & -\gamma_1 \sigma_R^2 \\ -\gamma_1 \sigma_R^2 & \sigma_S^2 + \gamma_1^2 \sigma_R^2 \end{pmatrix}.$$

Let  $f$  denote this joint density and  $h$  the density for the distribution of  $\varepsilon_D$ . Then, the joint distribution  $g$  of  $\log(1+r^L)$ ,  $S$  and  $l^d$  results to be given by

$$g(\log(1+r^L), S, l^d) = \begin{cases} g_1(\cdot) = f(\log(1+r^L) - \gamma_1 k_1 S, S) h(l^d - \tilde{l}^d), & \text{if } S \geq 0 \\ g_2(\cdot) = f(\log(1+r^L) - \gamma_1 k_2 S, S) h(l^d - \tilde{l}^d), & \text{if } S < 0. \end{cases} \quad (51)$$

Moreover, one finds

$$l_*^s = \begin{cases} S + \gamma_1(\log(1+r^L) - \gamma_1 k_1 S) + \gamma_2' x_D & , \text{if } S \geq 0 \\ S + \gamma_1(\log(1+r^L) - \gamma_1 k_2 S) + \gamma_2' x_D & , \text{if } S < 0. \end{cases} \quad (52)$$

Using  $\delta = \nu_1/(\nu_1 + \nu_2)$  and  $l^c = \delta l_*^s + (1-\delta)l^d$  one obtains

$$\tilde{S} \equiv l^c - (1-\delta)l^d - \delta \tilde{l}^d = \begin{cases} \delta S(1 - \gamma_1^2 k_1) & , \text{if } S \geq 0 \\ \delta S(1 - \gamma_1^2 k_2) & , \text{if } S < 0. \end{cases} \quad (53)$$

$\nu_1 \geq 0$  and  $\nu_2 \geq 0$  leads to  $1 - \gamma_1^2 k_i > 0$  for  $i = 1, 2$ . Consequently, one finds

$$\tilde{S} \geq 0 \iff S \geq 0$$

For  $i = 1, 2$  equation (53) has the following solutions:

$$S_i = (l^c - (1-\delta)l^d - \delta \tilde{l}^d) / \delta(1 - \gamma_1^2 k_i). \quad (54)$$

Now, the joint density  $\psi$  of  $\log(1+r^L)$ ,  $l^c$  and  $l^d$  can be derived from (51). Using  $j_i = 1/\delta(1 - \gamma_1^2 k_i)$  for  $i = 1, 2$  one obtains

$$\psi(\log(1+r^L), l^c, l^d) = \begin{cases} \psi_1(\log(1+r^L), l^c, l^d) = j_1 g_1(\log(1+r^L), S_1, l^d), & \text{if } \tilde{S} \geq 0 \\ \psi_2(\log(1+r^L), l^c, l^d) = j_2 g_2(\log(1+r^L), S_2, l^d), & \text{if } \tilde{S} < 0. \end{cases} \quad (55)$$

Starting with the joint density of  $\log(1+r^L)$ ,  $l^c$  and  $l^d$  as given by (55)  $l^c$  or  $l^d$  has to be integrated out following condition (49) in order to obtain the likelihood function for  $\log(1+r^L)$  and  $l$ .

If  $l^d < l^c$  (and consequently  $l = l^d$ ) and  $l < \tilde{l}^d$ , then  $\tilde{S} = 0$  results for  $l^c = L_1$ , where

$$L_1 = (1-\delta)l + \delta \tilde{l}^d.$$

For  $l \geq \tilde{l}^d$  one always finds  $\tilde{S} > 0$ . Analogous, for  $l^d \geq l^c$  and  $l \geq \tilde{l}^d$  one has

$$L_2 = (l - \delta \tilde{l}^d) / (1 - \delta),$$

such that  $\tilde{S} = 0$  for  $l^c = L_2$ . Then,  $l < \tilde{l}^d$  implies  $\tilde{S} < 0$ . Hence, the joint density of  $\log(1 + r^L)$  and  $l$  is given by

$$\varphi(\log(1 + r^L), l) = \begin{cases} \varphi_1(\cdot) = \int_l^\infty \psi_2(\log(1 + r^L), l, l^d) dl^d \\ \quad + \int_l^{L_1} \psi_2(\log(1 + r^L), l^c, l) dl^c \\ \quad + \int_{L_1}^\infty \psi_1(\log(1 + r^L), l^c, l) dl^c, \text{ if } l < \tilde{l}^d \\ \varphi_2(\cdot) = \int_l^\infty \psi_1(\log(1 + r^L), l^c, l) dl^c \\ \quad + \int_l^{L_2} \psi_1(\log(1 + r^L), l, l^d) dl^d \\ \quad + \int_{L_2}^\infty \psi_2(\log(1 + r^L), l, l^d) dl^d, \text{ if } l \geq \tilde{l}^d \end{cases} \quad (56)$$

and finally the likelihood function

$$\mathcal{L}(\log(1 + r^L), l) = \prod_{l < \tilde{l}^d} \varphi_1(\log(1 + r^L), l) \prod_{l \geq \tilde{l}^d} \varphi_2(\log(1 + r^L), l). \quad (57)$$

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