

Koch, Karl-Josef

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## International capital movements and human capital growth

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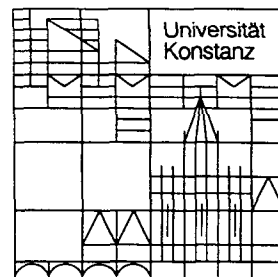
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**Sonderforschungsbereich 178**  
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Karl-Josef Koch

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**Karl-Josef Koch**

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# International Capital Movements and Human Capital Growth

Karl-Josef Koch  
University of Konstanz  
Germany

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## **Abstract**

The paper analyses aspects of international capital movements and formation of human capital in a three sector two country growth model. We assume that direct foreign investment from a country with a high level of knowledge goes along with a positive external effect on human capital growth in the capital importing country. Substantially different long run distributions of wealth and human capital can occur in case of small or large externalities.

In general it is in the interest of capital exporting and importing countries to encourage foreign direct investment. Although the major positive welfare effect can be observed on the side of the debtor, there still is an incentive in the creditor country to encourage foreign direct investment. International commodity trade allows for a more efficient use of comparative advantages, if the gap of efficiency of labor is smaller.

The short and long run distribution of benefits of such strategies is studied analytically and in numerical examples.

## 1. Introduction

Since the pioneering work of Romer (1986) and Lucas (1988) there is a revival of interest in the neoclassical model of growth. Some well known elements of economic modelling were used as additional fundamentals driving the 'mechanics' of growth. They opened up an ongoing scientific discussion and established the basis for a flood of publications. The framework of traditional neoclassical growth theory was broken up by the implementation of increasing returns, externalities, monopolistic competition and other features.

The major motivation for this new approach was the well known critique of some implications of traditional models, the keywords being endogeneity of technical progress and the question of international convergence. It was the positive aspect of theory which had to stand these attacks. Without convincing all their readers, that they followed a positive approach too, the contributors to New Growth Theory succeeded to satisfy the demand for alternative answers. Models with endogenous growth rates and endogenous rates of technical progress were developed. Here, a change of technology could either be interpreted in the vague sense of creation of human capital or in a more specific meaning of industrial economics. Nevertheless, it still seems to be one of the predominant, serious challenges to clarify the specific relations between characteristic features of modelling and certain positive results. What is necessary and appropriate to establish these results, and what are the relevant, empirically supported stylized facts which are to be explained still is a point of discussion. (Cf. Backus, Kehoe and Kehoe 1992, Gundlach 1993, Mankiw, Romer and Weil 1990 for example).

On the background of these considerations our paper deals with the international distribution of human capital and wealth. Alternative specifications of the transfer of knowledge, the propensity towards foreign direct investment, and the share of resources devoted to the development of human capital lead to different long run distributions. From these distributions we derive growth paths of income, consumption and other variables and study their international comparison. The degree of convergence is endogenous and heavily depends on the respective specifications.

A key feature of the model is the positive external effect foreign direct investment has on the growth of knowledge in the capital importing country. This idea goes back at least to arguments of Findlay (1978). We assume that the knowledge transferred this way is public good in nature. As we concentrate our efforts

on positive aspects of modelling we avoid to specify this process of transfer via internationally operating firms (which is done by Wang and Blomström 1992). Instead, we stick to an ad hoc specification of the type provided by Wang (1990). We assume that the competitive pressure within the capital importing country leads to instantaneous diffusion of technology or knowledge. The reason behind foreign direct investment is not the technological advantage as such, but simply the possible chance of greater marginal returns to capital.

The main results of our paper characterize the set of possible international distributions of wealth and income which satisfy two constraints: they are compatible with efficient allocations of labor and physical capital to two sectors of production, a consumption and an investment sector. At the same time a certain share of labor per country is spent on the development of human capital. The second constraint is the steady state condition of the capital accumulation process. In general these allocations require international capital movements within certain bounds which are determined endogenously. The dynamic process of human capital formation may select a particular one of these allocations. We describe the generic quality of the corresponding dynamics without claiming to have a solid economic foundation to choose a particular specification.

Most of the authors of recent contributions to growth theory assume that agents have rational expectations. This allows for a high degree of endogeneity in the sense that short run decisions are derived from intertemporal utility maximization. In the context of this model this would determine the share of labor in the human capital sector, the average savings ratios and the amount of foreign direct investment as results of this optimization. Deliberately we don't go that way. We consider the coordination problem behind these decisions to be too important to be ignored. Our results support this view, as they show how sensitive variables which measure welfare react on changes of parameters. A normative approach could not do without solving this problem by providing a model to describe the dynamic strategic behavior of countries, firms and individuals. In order to simplify the analysis and in direct correspondence to the positive interests we have in mind we don't use any intertemporal optimization.

## **2. The Short Run Equilibrium**

We consider a two-factor, two-good, two-country perfectly competitive general

equilibrium setting.<sup>1</sup> The production not only depends on the two private factors, labor and capital, but also on the level of technical knowledge of the country, where the production takes place. We assume that this technical knowledge can be interpreted as efficiency of labor, regardless of the sector this labor is allocated to. A certain percentage of each country's labor resources is devoted to education or development of human capital. The relative size of the respective educational sector is constant over time and exogenously given.

$$K = K_C + K_I \quad \text{the allocation of capital to sector } C \text{ and } I.$$

$$\tau L = L_C + L_I \quad \text{the allocation of labor in production to sector } C \text{ and } I.$$

$$L = \tau L + (1 - \tau)L \quad \text{the allocation of labor to production and education.}$$

Let  $k := K/L$  be economy's ratio of capital over total labor and  $k_C := K_C/L_C$  and  $k_I := K_I/L_I$  denote the capital intensities of the respective sectors of production. Full employment of capital and labor relates the capital intensities of both sectors and the capital intensity of total production,  $k/\tau$ , to the labor shares (relative to total labor in the production of commodities):

$$\lambda_C = \frac{k/\tau - k_I}{k_C - k_I}, \quad \lambda_I = \frac{k_C - k/\tau}{k_C - k_I}. \quad (1)$$

Obviously  $\lambda_C$  and  $\lambda_I$  add up to 1. The production of the consumption and the investment good is described by Cobb-Douglas production functions with production elasticities of capital  $\alpha_C$  and  $\alpha_I$ , respectively. Let  $h$  denote the efficiency of labor or, synonymously, the stock of human capital.

$$Y_C = K_C^{\alpha_C} (hL_C)^{(1-\alpha_C)}, \quad Y_I = K_I^{\alpha_I} (hL_I)^{(1-\alpha_I)} \quad (2)$$

is the total production of commodities and the production of the respective sectors per head of the population is

$$y_C = \tau \lambda_C h (k_C/h)^{\alpha_C}, \quad y_I = \tau \lambda_I h (k_I/h)^{\alpha_I}, \quad (3)$$

Under perfect competitive conditions factors are payed according to their marginal value product. If  $p_C$  and  $p_I$  are the commodity prices, the rental rate of capital and the wage rate are

$$r = p_C \frac{\alpha_C}{\tau \lambda_C k_C} y_C = p_I \frac{\alpha_I}{\tau \lambda_I k_I} y_I \quad (4)$$

$$w = p_C \frac{1 - \alpha_C}{\tau \lambda_C} y_C = p_I \frac{1 - \alpha_I}{\tau \lambda_I} y_I. \quad (5)$$

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<sup>1</sup>Models of this type are common in the literature. See for example Oniki and Uzawa 1965, Hori and Stein 1977 or Koch 1992.

This implies for the wage rental ratio

$$\omega = \frac{1 - \alpha_C}{\alpha_C} k_C = \frac{1 - \alpha_I}{\alpha_I} k_I \quad (6)$$

and therefore there is a constant ratio

$$\gamma = \frac{1 - \alpha_C}{\alpha_C} \frac{\alpha_I}{1 - \alpha_I} = \frac{k_I}{k_C} \quad , \quad (7)$$

if factors are allocated efficiently among sectors. At given prices this information is sufficient to compute the allocation of resources between the production sectors. For further reference, we add the relation between relative factor prices and commodity prices, which follows from the equations above

$$\frac{p_I}{p_C} = \frac{1 - \alpha_C}{1 - \alpha_I} \left( \frac{\alpha_C}{1 - \alpha_C} \right)^{\alpha_C} \left( \frac{1 - \alpha_I}{\alpha_I} \right)^{\alpha_I} \left( \frac{\omega}{h} \right)^{\alpha_C - \alpha_I} \quad (8)$$

To close the short run model we have to specify the demand side. As we are not going to analyze distributional aspects within each country, we only compute the average income across the whole population. The average supply of factors is  $(1 - \tau)$  units of labor for the production of human capital,  $\tau$  units of labor for commodity production and  $k$  units of capital. In our context we neglect all problems concerning the personal distribution of income. As human capital production does not yield any direct income, we may either assume that this sector receives transfers from the other sectors or that each individual shares its labor supply between education and commodity production. In any case, the income of a representative consumer is

$$\pi = r(\tau\omega + k) \quad . \quad (9)$$

We assume that the consumer has an exogenous saving ratio  $s$ . He spends a fixed proportion  $(1 - s)$  of his income on consumption and the remainder on investment goods.

Combining demand and supply we could now compute the equilibrium allocation of an isolated economy. Instead of doing so we add a second country with identical production functions. The two economies may differ with respect to their factor endowments, the level of technical knowledge and the share of labor in the educational sector. In the following superscripts will always refer to country  $H$  or  $F$ . From now on  $k$  denotes the worldwide capital intensity. The relative size of the countries is measured by the share of the world labor force situated in country  $H$  and is denoted by  $\epsilon$ . Labor is internationally immobile, whereas capital is



distributed in such proportions that the rental rate  $r$  is the same over all sectors and countries.<sup>2</sup> Both commodities are traded on international markets without any distortions. The uniform commodity price ratio  $p_C/p_I$  determines the local allocations. Provided both countries diversify their production, this yields different wage rates in  $H$  and  $F$ , if the knowledge differs across countries. In the short run the ratio of wages,  $w^H/w^F$ , is determined by levels of human capital. Only in the long run the amount of resources devoted to education can change this ratio. The wage rate in efficiency units, and consequently  $\tilde{\omega}$  is the same (compare equation (8)):

$$r^H = r^F \quad \text{and} \quad \tilde{\omega} = \frac{\omega^H}{h^H} = \frac{\omega^F}{h^F}. \quad (10)$$

We use the term factor price equalization (*FPE*) whenever these relations hold. By equation (6) we get:

$$\frac{\alpha_C}{1 - \alpha_C} \tilde{\omega} = \frac{k_C^H}{h^H} = \frac{k_C^F}{h^F} \quad \text{and} \quad \frac{\alpha_I}{1 - \alpha_I} \tilde{\omega} = \frac{k_I^H}{h^H} = \frac{k_I^F}{h^F}. \quad (11)$$

This places lower and upper bounds on the capital intensities of each country for any given value of  $\tilde{\omega}$ .

**Notation:** Let  $\alpha_{min}$  and  $\alpha_{max}$  denote the minimum and the maximum of  $\{\alpha_C, \alpha_I\}$ .

Then the resources of the countries can be allocated in accordance with the *FPE* condition (11), if

$$\tilde{\omega} \frac{\alpha_{min}}{1 - \alpha_{min}} < \frac{k_I}{h^i \tau^i} < \tilde{\omega} \frac{\alpha_{max}}{1 - \alpha_{max}}, \quad \text{for } i = H, F. \quad (12)$$

Capital mobility of some form may be needed to satisfy these conditions in equilibrium. Part of a country's physical capital may be owned by foreigners. Let  $u$  be the amount of foreign capital the average inhabitant of  $H$  owns abroad, if  $u$  is positive. If  $u$  is negative, its absolute value denotes his indebtedness to country  $F$ .

Summing up the relations concerning capital ownership and endowments we have:

$$x^H = k^H + u, \quad (13)$$

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<sup>2</sup>This ad hoc assumption of perfect capital mobility can be justified by an extensive analysis of the dynamic behavior of this type of model (compare Koch 1992).

$$x^F = k^F - \frac{\epsilon}{1-\epsilon}u \quad \text{and} \quad (14)$$

$$k = \epsilon k^H + (1-\epsilon)k^F = \epsilon x^H + (1-\epsilon)x^F \quad . \quad (15)$$

Apart from endowments the countries may differ with respect to the saving ratios  $s^H$  and  $s^F$ , and their educational efforts  $\tau^H$  and  $\tau^F$ . Throughout the paper we make use of the asymmetry of saving ratios. Without loss of generality we let  $H$  be the more thrifty country.

**Assumption 1**  $s^H \geq s^F$

For every temporary *FPE* – equilibrium we can explicitly calculate the value of  $\tilde{\omega}$ :

**Lemma 1** *If both countries diversify their production, then*

$$\tilde{\omega} = \frac{\epsilon(1-a^H)x^H + (1-\epsilon)(1-a^F)x^F}{\epsilon a^H \tau^H h^H + (1-\epsilon)a^F \tau^F h^F} \quad ,$$

where

$$a^H = (1-s^H)\alpha_C + s^H\alpha_I \quad , \quad a^F = (1-s^F)\alpha_C + s^F\alpha_I \quad .$$

Furthermore

$$r = \alpha_I \left( \frac{\alpha_I}{1-\alpha_I} \right)^{\alpha_I-1} \tilde{\omega}^{\alpha_I-1} \quad .$$

A proof of this lemma can be found in appendix A.

A cut in the supply of labor to the production sectors by an increase of  $\tau^H$  or  $\tau^F$  raises the wage rental ratio. If  $\alpha_C > \alpha_I$ , an increase of the savings ratio in either country qualitatively has the same result – although by different means. Turning the consumption sector into the labor intensive sector reverses the effect on relative factor prices.

Notice two special cases concerning the savings ratios and the capital elasticities of production: if  $s^H = s^F$ , then  $a^H = a^F =: a$  and the expression for  $\tilde{\omega}$  reduces to

$$\tilde{\omega} = \frac{1-a}{a} \frac{\epsilon x^H + (1-\epsilon)x^F}{\epsilon \tau^H h^H + (1-\epsilon)\tau^F h^F} \quad , \quad (16)$$

i.e.: the savings and production parameters and the factor distribution are separated.<sup>3</sup> If there is no relevant distinction between the two commodities, i.e.:  $\alpha_C = \alpha_I =: \alpha$ ,

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<sup>3</sup>In a way  $\tau^H$ ,  $\tau^F$  and  $\epsilon$  are parameters of the factor distribution.

then  $a^H = a^F = \alpha$  and the savings parameters don't affect the wage rental ratio anymore.

The following lemma shows that world resources always can be allocated efficiently among both countries at any given distribution of capital ownership.

**Lemma 2** *Consider any positive values of  $x^H, x^F, h^H$  and  $h^F$ . Then there is a nonempty open interval  $U$  such that for  $u \in U$  the corresponding  $k^H = x^H - u$  and  $k^F = x^F - \frac{\epsilon}{1-\epsilon}u$  together with the equilibrium  $\tilde{\omega}$  satisfy the FPE condition (12) and the feasibility constraint  $\frac{1-\epsilon}{\epsilon}x^F > u > -x^H$ .*

A proof of this lemma can be found in appendix A.

As we are mainly interested in aspects of distribution between countries we reduce the dimension of the system by the following definitions:

$$\kappa := \frac{x^F}{x^H}, \quad \rho := \frac{h^F}{h^H}, \quad \xi := \frac{x^H}{h^H} = \frac{x^F}{h^F \kappa / \rho}. \quad (17)$$

Furthermore we use the following notation:

$$\sigma := \frac{s^F}{s^H}, \quad \eta := \begin{cases} \frac{\epsilon u}{(1-\epsilon)k^F} & \text{if } u \geq 0 \\ \frac{u}{k^H} & \text{if } u \leq 0 \end{cases}. \quad (18)$$

Lemma 2 tells us that every vector  $(\kappa, \rho, \xi) \in R_{++}^3$  can result from a feasible state of the two economies, and vice versa, completed by an appropriate  $u$  and a level variable, say  $h^H$ , such a vector determines a feasible state.

From Lemma 1 we get with this notation in terms of country  $H$ :

$$\tilde{\omega} = R^H(\kappa, \rho)\xi \quad \text{and} \quad r = \alpha_I \left( \frac{\alpha_I}{1-\alpha_I} \right)^{\alpha_I-1} (R^H(\kappa, \rho)\xi)^{\alpha_I-1}, \quad (19)$$

where

$$R^H(\kappa, \rho) = \frac{\epsilon(1-a^H) + (1-\epsilon)(1-a^F)\kappa}{\epsilon a^H \tau^H + (1-\epsilon)a^F \tau^F \rho}. \quad (20)$$

Seen from the perspective of country  $F$  this translates into  $\tilde{\omega} = R^F(\kappa, \rho)\xi\kappa/\rho$ , an analogous expression for the interest rate  $r$ , and:

$$R^F(\kappa, \rho) = R^H(\kappa, \rho)\rho/\kappa = \frac{\epsilon(1-a^H)/\kappa + (1-\epsilon)(1-a^F)}{\epsilon a^H \tau^H / \rho + (1-\epsilon)a^F \tau^F}. \quad (21)$$

Notice that  $\tau^H R^H(\kappa, \rho)$  and  $\tau^F R^F(\kappa, \rho)$  are equal to the ratio of average wage income over capital income:

$$\tau^H R^H(\kappa, \rho) = \frac{\tau^H \omega^H}{r x^H}, \quad \tau^F R^F(\kappa, \rho) = \frac{\tau^F \omega^F}{r x^F}. \quad (22)$$

Now we can express *FPE* conditions with the help of  $R^H(\kappa, \rho)$  and  $R^F(\kappa, \rho)$  in order to find out whether a distribution of factors given by  $k^H, k^F, \kappa$  and  $\rho$  allows for or even requires international debt formally represented by a positive or negative value of  $u$ :

*FPE* can be reached, if for  $i = H, F$ :

$$\begin{aligned} (1 - \alpha_{max})/\alpha_{max} &\leq \tau^H R^H(\kappa, \rho) \frac{k^H + u}{k^H} \leq (1 - \alpha_{min})/\alpha_{min} \quad \text{and} \\ (1 - \alpha_{max})/\alpha_{max} &\leq \tau^F R^F(\kappa, \rho) \frac{k^F + \frac{\epsilon}{1-\epsilon} u}{k^F} \leq (1 - \alpha_{min})/\alpha_{min}. \end{aligned} \quad (23)$$

From these equations we can derive the domains in the  $(\kappa, \rho)$ -plane where  $u$  is bound to be positive or negative, respectively.

Figure 1

### 3. Dynamic Behavior

#### 3.1 The Distribution of Capital Ownership

Now let us turn to the dynamic specification of the model. All endowment variables will change over time. Partly the forces driving this growth process are exogenous, partly they are induced, endogenous.

The supply of labor grows with a uniform exogenous growth rate  $n$ .<sup>4</sup> The change of capital stock is purely endogenous. Investment creates new capital. Adjustment costs to transform investment goods into new capital are neglected. Hence savings are equal to the change in capital. This implies for the growth rates of capital per head owned by the respective population

$$\begin{aligned} \hat{x}^H &= s^H \pi^H / x^H - n \\ &= r s^H (\tau^H R^H(\kappa, \rho) + 1) - n \quad , \end{aligned} \quad (24)$$

$$\begin{aligned} \hat{x}^F &= s^F \pi^F / x^F - n \\ &= r s^F (\tau^F R^F(\kappa, \rho) + 1) - n \quad \text{and} \end{aligned} \quad (25)$$

$$\hat{\kappa} = \hat{x}^F - \hat{x}^H = \Phi(\kappa, \rho) r \quad , \quad (26)$$

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<sup>4</sup>As usual,  $n$  can cover a constant rate of depreciation as well.

where

$$\Phi(\kappa, \rho) = s^F(\tau^F R^F(\kappa, \rho) + 1) - s^H(\tau^H R^H(\kappa, \rho) + 1) \quad . \quad (27)$$

We see that

$$\hat{\kappa} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \iff \quad \Phi(\kappa, \rho) \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad . \quad (28)$$

Let  $\phi$  for short denote the curve  $\Phi(\kappa, \rho) = 0$ . As an immediate consequence of (24) and (25)  $\Phi$  is the locus where the international ratio of profit ratios equals the reverse ratio of saving ratios. On the other hand this means that the ratio of social products of  $H$  over  $F$  is proportional to  $\kappa$ , i.e. it changes along  $\Phi$ .

Notice that the dynamics of capital depend on the distribution of human capital,  $\rho$ , and the respective educational effort, but not on the specification of the dynamic process changing human capital. This is the advantage we bought by taking the parameters  $\tau^H$  and  $\tau^F$  as given exogenously.

### 3.2 Growth of Knowledge

Human capital or technical knowledge always changes over time, partly due to reasons endogenous to a country, partly to exogenous ones. Disregarding catastrophes which can erase cultural achievements the endogenous change should be accumulative. The domestic forces developing human capital are characterized by a constant  $A$  measuring the effectivity of the educational system and the amount of labor  $\delta$  devoted to education. On the other hand, knowledge may be expanded by influence from abroad. Various kinds of international relations may appear to be the vehicle of foreign knowledge. In our model we focus on the transfer of knowledge which comes together with foreign investment.<sup>5</sup> This effect on domestic human capital is measured by a function  $\theta$  which depends on the degree of superiority of foreign human capital and on the relative amount of imported capital. For simplicity, we assume that the change of human capital is linear with respect to its level. To avoid extra notational complications we restrict the analysis to the case where country  $H$  is the capital exporting one. This is induced by

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<sup>5</sup>As we want to point out, how sensitive growth models are with respect to the specification of such a process, we avoid a detailed microeconomic elaboration of it. An attempt to incorporate such a microeconomic foundation can be found in Wang and Blomström 1992. The rather ad hoc approach we use here goes back to Wang 1990.

assumption 1 and

$$\textbf{Assumption 2} \quad A^H(1 - \tau^H) \geq A^F(1 - \tau^F)$$

In this asymmetric model the dynamics of human capital are described by:

$$\dot{h}^H = A^H(1 - \tau^H)h^H \quad \text{and} \quad \dot{h}^F = A^F(1 - \tau^F)h^F\theta^F(\rho, \eta). \quad (29)$$

We want to interpret  $\theta^F$  as the spillover factor, which captures the external effect foreign capital has on the growth of domestic human capital, if foreign technical knowledge is superior to domestic. The following properties of  $\theta^F$  reflect these ideas:

$$\theta^F \geq 1 \quad \text{and} \quad \theta^F > 1 \iff \rho < 1 \quad \text{and} \quad 0 < \eta \leq 1. \quad (30)$$

Furthermore

$$\theta^F > 1 \implies \theta_\rho^F < 0 \quad \text{and} \quad \theta_\eta^F > 0. \quad (31)$$

For our numerical calculations we specify  $\theta^F$  to be of Cobb–Douglas type

$$\theta^F(\rho, \eta) = \theta^0(1/\rho - 1)^\delta \left(\frac{\eta}{1 - \eta}\right)^{1-\delta} + 1, \quad (32)$$

if  $\rho < 1$  and  $0 < \eta \leq 1$ , and  $\theta^F(\rho, \eta) = 1$  otherwise.

The general form of the growth rate of  $\rho$  is

$$\hat{\rho} = \hat{h}^F - \hat{h}^H = A^F(1 - \tau^F)\theta^F(\rho, \eta) - A^H(1 - \tau^H) \quad (33)$$

Dynamic equilibrium of  $\rho$  requires

$$\theta^F = \frac{A^H(1 - \tau^H)}{A^F(1 - \tau^F)} \quad (34)$$

The interpretation of (34) is clear and simple: in the long run capital mobility can equate the growth rates of human capital to the maximum of the autarcic growth rates  $A^H(1 - \tau^H)$  and  $A^F(1 - \tau^F)$ , which is larger in the capital exporting country.

Every value of  $\kappa$  and  $\rho$  defines lower and upper bounds for  $\eta$  presuming *FPE*. These bounds correspond to minimal or maximal foreign investment. The model provides no intrinsic economic reasons for investors to choose a particular amount of foreign investment within these bounds. The agents don't foresee the external effect their investment may have in the long run on foreign human capital. Their indifference between all alternatives which don't violate short run efficiency constraints is the consequence. For our further considerations we will restrict

ourselves to the most extreme ad hoc assumptions one can think of. Either the agents are reluctant to invest abroad. They prefer domestic capital shares unless the return on foreign capital exceeds the domestic return. Or they are in favor of foreign shares unless the domestic return is higher. In both cases the resulting debt ratio  $\eta$  is a function of  $\kappa$  and  $\rho$ :

$$\eta = \eta(\kappa, \rho) . \quad (35)$$

This transforms  $\hat{\rho}$  according to (33) into a function

$$\hat{\rho} = \Psi(\kappa, \rho) . \quad (36)$$

### 3.2 Growth of Capital

The description of the dynamics is completed by the law of movement of  $\xi$ . The growth rate of  $\xi$  is

$$\hat{\xi} = \hat{x}^H - \hat{h}^H = \hat{x}^H - A^H(1 - \tau^H) . \quad (37)$$

Substituting from (24) we get:

$$\hat{\xi} = rs^H(\tau^H R^H(\kappa, \rho) + 1) - n - A^H(1 - \tau^H) . \quad (38)$$

Using  $I$  as numeraire we compute the rental rate of to be

$$r = \alpha_I \left( \frac{k_I^H}{h^H} \right)^{\alpha_I - 1} = \alpha_I \left( \frac{\alpha_I}{1 - \alpha_I} \right)^{\alpha_I - 1} (\tilde{\omega})^{\alpha_I - 1} \quad (39)$$

$$= \alpha_I \left( \frac{\alpha_I}{1 - \alpha_I} \right)^{\alpha_I - 1} (R^H(\kappa, \rho))^{\alpha_I - 1} \xi^{\alpha_I - 1} , \quad (40)$$

and find that  $\hat{\xi}$  is a function of  $\kappa$ ,  $\rho$  and  $\xi$ :

$$\hat{\xi} = \Gamma(\kappa, \rho, \xi) . \quad (41)$$

$\Gamma$  is strictly increasing in  $\xi$  and therefore there is a  $\bar{\xi}$  for every  $(\kappa, \rho)$  such that  $\hat{\xi} = 0$ . Indeed, we get:

$$\bar{\xi} = \frac{1 - \alpha_I}{\alpha_I} \left( \frac{\alpha_I s^H}{n + A^H(1 - \tau^H)} \right)^{\frac{1}{1 - \alpha_I}} \frac{(\tau^H R^H(\kappa, \rho) + 1)^{\frac{1}{1 - \alpha_I}}}{R^H(\kappa, \rho)} . \quad (42)$$

Hereby the formal definition of the dynamical system in  $\kappa$ ,  $\rho$  and  $\xi$  by the dynamical equations (26), (33) and (41) is complete. It is left to the reader to verify that is represents a proper reduction of the underlying basic system in the countries' resources capital and labor, and the human capital factors. Figure 2 shows curves  $\Phi$  and  $\Psi$  for both extreme cases of foreign investment.

Figure 2

#### 4. Long Run Equilibria

The analysis of long run behavior of the model shows three aspects: existence of equilibrium, growth rates of economic variables and the dependency of levels of long run growth paths on the specification of investment behavior together with its external effect on foreign human capital. The existence of a steady state in  $(\kappa, \rho, \xi)$  with finite values crucially depends on the power of the external effect of capital exports. If it is too small, country  $F$  pays for its low intrinsic effort and ability to improve its efficiency of labor. The countries will drift apart for ever with respect to wealth and knowledge. But in general this will not be to the advantage of the capital exporting country. In its own interest it should extend its effort to improve the foreign country's productivity. A higher degree of efficiency will most likely enlarge its possibilities of consumption.

To calculate sufficient or even necessary conditions for the existence of positive, finite steady states is not a very useful exercise in the context of our analysis. The only way we are going to deal with this problem is by providing examples. Anyway, the examples rather than the pure analysis will illustrate to what extent and in which direction parameter changes influence the outcome. The computations of long run growth rates in a steady state on the other hand can easily be done. Nevertheless, we leave it to the reader to do these calculations on top of the results provided in appendices A and B. Just in order state these unproven facts in a concise form we give the following

**Observation 3** *If the spillover is large enough, there is a finite long run equilibrium of the dynamical system in  $(\kappa, \rho, \xi)$ . It is characterized by the following growth rates: the rental rate of capital is constant. Labor grows with the exogenous rate  $n$ . Human capital, capital ownership per head, wage rates, and hence social products, consumption and investment per head grow with rate  $\{A^H(1 - \tau^H)\}$ .*

Typically, the more thrifty country will be creditor to the other country in the long run. Every long run equilibrium can be represented by a point in the  $(\kappa, \rho)$ -plane. The corresponding value of  $\xi$  can then be computed by equation (42). Two dynamic equations determine the equilibrium. The growth equation for the capital ownership ratio  $\kappa$  tells us, that the equilibrium is a point on  $\Phi$ . The dynamics



of the human capital ratio  $\rho$  and of  $\kappa$  jointly determine particular points as long run equilibria on  $\Phi$ . They may be locally stable, saddlepoint stable or unstable depending on the agents' attitude towards foreign investment, the size of external effects foreign investment has on human capital and on the other parameters of the model. Figure 3 show three typical examples.

Figure 3

We want to analyse the model as far as possible before specifying this behavior. Therefore, we compare different points of  $\Phi$  with respect to the value the most interesting economic variables take in these states.

Recall as an economic interpretation of the definition, that  $\Phi$  is the locus, where the ratio of profit shares equals the reverse ratio of saving ratios. Equivalently, the ratio of social products of  $H$  over  $F$ ,  $\pi^H/\pi^F$ , is proportional to  $\kappa$ . With the three propositions below we don't intend to perform a complete dynamical analysis. A proper computation of adjustment processes towards a steady state goes beyond the scope of this paper. The idea behind these propositions is to compare different possible paths of steady growth which may be reached under different presumptions concerning the propensity towards foreign investment.

**Proposition 4** *Consider the curve  $\Phi := \{(\kappa, \rho) | \hat{\kappa} = 0\}$ . Then along  $\Phi$*

- 1)  $\rho$  is a strictly increasing function of  $\kappa$ , i.e.:  $\Phi = \{(\kappa, \phi(\kappa))\}$  and  $\phi' > 0$ ,
- 2) the international income ratio  $\pi^H/\pi^F$  is a strictly decreasing function of  $\kappa$ ,
- 3) the international ratio of capital ownership per efficiency unit of labor  $\kappa/\rho$  is a decreasing function of  $\kappa$ . It is strictly decreasing, iff  $s^H > s^F$  and is bounded between the minimum and the maximum of

$$\frac{\tau^F s^F - s^H a^F}{\tau^H s^H - s^H a^F} \quad \text{and} \quad \frac{\tau^F s^F - s^F a^H}{\tau^H s^H - s^F a^H}. \quad (43)$$

A proof of the proposition can be found in appendix B.

The boundedness of  $\kappa/\rho$  tells us that an uneven distribution of human capital always is accompanied by an uneven distribution of wealth. The country with a high level of knowledge will acquire more wealth. This correlation may be relaxed, if the rich country uses more labor in the human capital sector, or may be tied closer, the more thrifty the rich country is. On the other hand, for reasons of efficiency of production capital per efficiency unit of labor must be distributed

fairly equal across countries. Only to a limited amount the uneven distribution of physical capital can be compensated by specialization on the more or less capital intensive sector.

As long as the savings propensities are the same in both countries there are no further differences between the two countries. Further comparisons of possible steady states along  $\Phi$  depend on the countries' savings propensities.

All the interesting long run growth rates of the model are equal either to  $A^H(1 - \tau^H)$  or to zero. The only exception, of course, is the labor force, which grows with the exogenous rate  $n$ . In the proposition below we will compare the respective levels of different growth paths. We therefore normalize the steady growing variables. Under the assumptions made above  $h^H$  grows with constant rate  $A^H(1 - \tau^H)$  and can serve as an appropriate normalizing variable. If we move from one state to another where  $\kappa$  and  $\rho$  are larger, this means that capital ownership is redistributed among the countries and that human capital of  $F$  is lifted to a higher growth path.

**Proposition 5** Consider the curve  $\Phi := \{(\kappa, \rho) | \hat{\kappa} = 0\}$ . If  $s^H > s^F$ , then along  $\Phi$

- 1) the worldwide rental rate of capital  $r$ , is a strictly increasing function of  $\kappa$ ,
- 2) in both countries the labor share is a strictly decreasing function of  $\kappa$ .

Furthermore, if  $s^H = s^F$ , these functions are constant.

**Proposition 6** Consider the curve  $\Phi := \{(\kappa, \rho) | \hat{\kappa} = 0\}$ . If  $s^H > s^F$ , then along  $\Phi$

- 1) in country  $H$ 
  - (a) the normalized capital ownership per head and the normalized social product in terms of the investment good are increasing with  $\kappa$  at some point  $\kappa$ , iff  $\tau^H R^H + 1 < 1/\alpha_I$ .
  - (b) the normalized consumption is increasing with  $\kappa$  at some point  $\kappa$ , iff  $\tau^H R^H + 1 < (1 + \alpha_C - \alpha_I)/\alpha_C$ .

Furthermore, these functions are constant, if  $s^H = s^F$ .

- 2) in country  $F$

- (a) *the normalized capital ownership per head and the normalized social product in terms of the investment good are increasing with  $\kappa$  at some point  $\kappa$ , iff  $(\alpha_I(\tau^H R^H + 1) - 1) \left( \frac{dR^H}{d\kappa} \frac{\kappa}{R^H} \right) + (1 - \alpha_I)(\tau^H R^H + 1) > 0$ .*
- (b) *the normalized consumption is increasing with  $\kappa$  at some point  $\kappa$ , iff  $(\alpha_C(\tau^H R^H + 1) - (1 + \alpha_C - \alpha_I)) \left( \frac{dR^H}{d\kappa} \frac{\kappa}{R^H} \right) + (1 - \alpha_I)(\tau^H R^H + 1) > 0$ .*

*Furthermore, these functions are strictly increasing, if  $s^H = s^F$ .*

A proof of the proposition can be found in appendix B.

We see that welfare comparisons along  $\Phi$  very much depend on the prevailing parameter constellation. Especially the most interesting measure in this model, consumption, may be larger or smaller in one or both countries at different long run values of  $(\kappa, \rho)$ . Figure 4 shows a numerical example illustrating the last proposition.

Figure 4

## 5. Conclusions

Our analysis tried to provide some insight in the role of capital movements with respect to the formation of human capital. Substantially different long run distributions of wealth and human capital can occur. In general it is in the interest of capital exporting and importing countries to encourage foreign direct investment. Although the major positive welfare effect can be observed on the side of the debtor, there still is an incentive in the creditor country to encourage foreign direct investment. International commodity trade allows for a more efficient use of comparative advantages if the gap of efficiency of labor is smaller. Nevertheless, there are many opportunities for strategic behavior in such a situation. For example, a country may increase either its own efforts to develop human capital or may benefit from forms of technology transfer via foreign direct investment. On the other hand, a country with a high level of human capital may try to keep this particular advantage by an ongoing high investment of labor in the educational sector. To look closer into these strategic aspects seem to be an interesting challenge and to some extent the analysis of this paper may give some useful hints to reach this goal.

## Appendix A

### Proof of Lemma 1

Use (4) to (15) to compute supply and demand of commodity  $I$  of a country, say country  $H$ . The share of the labor force employed in sector  $I$  is

$$\begin{aligned}\lambda_I^H &= \frac{k_C^H - k^H/\tau^H}{k_C^H - k_I^H} \\ &= \frac{1}{1-\gamma} \left(1 - \gamma \frac{k^H}{\tau^H k_I^H}\right)\end{aligned}$$

By equation (4) the per capita supply of good  $I$  is equal to

$$\begin{aligned}y_I^H &= r \frac{\tau^H \lambda_I^H k_I^H}{\alpha_I} \\ &= r \frac{1}{1-\gamma} \left(\frac{\tau^H k_I^H}{\alpha_I} - \frac{\gamma}{\alpha_I} k^H\right) \\ &= r \frac{1}{1-\gamma} \left(\frac{1}{1-\alpha_I} \tau^H h^H \tilde{\omega} - \frac{1}{\alpha_I} \gamma k^H\right).\end{aligned}$$

The per capita demand is equal to

$$y_I^H = r s^H (\tau^H h^H \tilde{\omega} + x^H).$$

Adding the weighted demand of country  $H$  and  $F$  yields:

$$y_I = r \{(\epsilon s^H \tau^H h^H + (1-\epsilon) s^F \tau^F h^F) \tilde{\omega} + \epsilon s^H x^H + (1-\epsilon) s^F x^F\}.$$

And world supply of  $I$  adds up to:

$$y_I = r \left\{ \frac{1}{1-\gamma} \left( \frac{1}{1-\alpha_I} (\epsilon \tau^H h^H + (1-\epsilon) \tau^F h^F) \tilde{\omega} - \frac{1}{\alpha_I} \gamma (\epsilon k^H + (1-\epsilon) k^F) \right) \right\}.$$

Replace  $\epsilon k^H + (1-\epsilon) k^F$  by  $\epsilon x^H + (1-\epsilon) x^F$ . The world market clearing condition for commodity  $I$  now is a linear equation in  $\tilde{\omega}$ ,  $x^H$ , and  $x^F$ . We solve this equation for  $\tilde{\omega}$

$$\begin{aligned}\tilde{\omega} &= \frac{(s^H + \frac{1-\alpha_C}{\alpha_C-\alpha_I}) \epsilon x^H + (s^F + \frac{1-\alpha_C}{\alpha_C-\alpha_I}) (1-\epsilon) x^F}{(\frac{\alpha_C}{\alpha_C-\alpha_I} - s^H) \epsilon \tau^H h^H + (\frac{\alpha_C}{\alpha_C-\alpha_I} - s^F) (1-\epsilon) \tau^F h^F} \\ &= \frac{\epsilon (1-a^H) x^H + (1-\epsilon) (1-a^F) x^F}{\epsilon a^H \tau^H h^H + (1-\epsilon) a^F \tau^F h^F}\end{aligned}$$

to complete the proof of the first statement.

The expression for  $r$  is derived straight forwardly combining (4), (3) and (11).  $\square$

### Proof of Lemma 2

Assume we have an equilibrium where the sector capital intensities satisfy equation (11). From the point of view of country  $H$  this requires  $k_I^H \leq k^H \leq k_C^F$ . This in turn is equivalent to

$$\begin{aligned} \frac{\alpha_I}{1 - \alpha_I} \tau^H h^H \tilde{\omega} + u &\leq x^H \leq \frac{\alpha_C}{1 - \alpha_C} \tau^H h^H \tilde{\omega} + u \\ \Leftrightarrow x^H - \frac{\alpha_C}{1 - \alpha_C} \tau^H h^H \tilde{\omega} &\leq u \leq x^H - \frac{\alpha_I}{1 - \alpha_I} \tau^H h^H \tilde{\omega} \quad . \end{aligned} \quad (44)$$

Similarly for country  $F$

$$\frac{1 - \epsilon}{\epsilon} \left( \frac{\alpha_I}{1 - \alpha_I} \tau^F h^F \tilde{\omega} - x^F \right) \leq u \leq \frac{1 - \epsilon}{\epsilon} \left( \frac{\alpha_C}{1 - \alpha_C} \tau^F h^F \tilde{\omega} - x^F \right) \quad . \quad (45)$$

If  $\alpha_C = \alpha_I$ , all these inequalities turn out to be equalities. Otherwise one has to show, that the right hand side of (44) is larger than the left hand side of (45), and that the left hand side of (44) is smaller than the right hand side of (45). But this follows straight away from Lemma 1:

$$\begin{aligned} \frac{\epsilon(1 - \alpha_{\max})x^H + (1 - \epsilon)(1 - \alpha_{\max})x^F}{\epsilon\alpha_{\max}\tau^H h^H + (1 - \epsilon)\alpha_{\max}\tau^F h^F} &\leq \tilde{\omega} \\ &\leq \frac{\epsilon(1 - \alpha_{\min})x^H + (1 - \epsilon)(1 - \alpha_{\min})x^F}{\epsilon\alpha_{\min}\tau^H h^H + (1 - \epsilon)\alpha_{\min}\tau^F h^F} \quad . \end{aligned}$$

$\square$

## Appendix B

Throughout this appendix we assume  $(\kappa, \rho, \xi)$  to be on  $\Phi = \{(\kappa, \rho) | \hat{\kappa} = 0 \text{ and on } \Gamma = \Gamma(\kappa, \rho, \xi) = 0\}$ .

For further reference we prove the following

**Lemma B.1** *Along the curve  $\Phi$*

- 1) *the functions  $R^H(\kappa, \rho)$  and  $R^F(\kappa, \rho)$  are strictly decreasing with  $\kappa$ , if  $s^H > s^F$ , and constant,  $s^H = s^F$ ;*

2)  $\xi$  is constant, iff  $s^H = s^F$ , is increasing at  $\kappa$ , iff  $s^H > s^F$  and  $\tau^H R^H + 1 < 1/\alpha_I$ .

3) if  $s^H > s^F$ , then  $\xi$  considered as a function of  $\kappa$

(a) is a strictly increasing, if  $\alpha_C \geq \alpha_I$ ,

(b) is a strictly decreasing, if  $\frac{s^H(1-s^F)}{s^F(1-s^H)}\alpha_C \leq \alpha_I$ , and

(c) is U-shaped, if  $\frac{s^H(1-s^F)}{s^F(1-s^H)}\alpha_C > \alpha_I > \alpha_C$ .

### Proof of Lemma B.1

1) From the definition of  $\Phi$  in equation (27) and from the relation between  $R^H$  and  $R^F$  in (21) we get

$$\rho = \frac{(\tau^H R^H + 1 - \sigma)\kappa}{\sigma \tau^F R^H}. \quad (46)$$

Substitute this relation into (20) which defines  $R^H(\kappa, \rho)$

$$R^H = \frac{\epsilon(1-a^H) + (1-\epsilon)(1-a^F)\kappa}{\epsilon a^H \tau^H + (1-\epsilon)a^F \frac{(\tau^H R^H + 1 - \sigma)\kappa}{\sigma R^H}} \quad (47)$$

and solve for  $R^H$

$$R^H = \frac{\epsilon\sigma(1-a^H) + (1-\epsilon)(\sigma - a^F)\kappa}{\epsilon\sigma a^H \tau^H + (1-\epsilon)a^F \tau^H \kappa}. \quad (48)$$

Obviously,  $R^H$  is a continuous function of  $\kappa$  (on its domain of definition,  $\kappa \geq 0$ ). It is monotonic and takes on its extreme values  $R_0^H$  and  $R_\infty^H$  at  $\kappa = 0$  and  $\infty$ , respectively

$$R_0^H = \frac{1}{\tau^H} \left( \frac{1}{a^H} - 1 \right) = R^H(0, 0) \quad \text{and} \quad R_\infty^H = \frac{1}{\tau^H} \left( \frac{\sigma}{a^F} - 1 \right). \quad (49)$$

$$\frac{dR^H}{d\kappa} < 0 \iff R_0^H > R_\infty^H \iff \frac{a^F}{a^H} > \sigma \iff s^H > s^F. \quad (50)$$

$R^H$  is constant, if  $s^H = s^F$ . From the definition of  $\Phi$  in (27) we get as well

$$R^F = \frac{\tau^H R^H + 1 - \sigma}{\sigma \tau^F} \quad (51)$$

which rises, if  $R^H$  increases and vice versa.

2) According to (42) the variable  $\xi$  is a function of  $R^H$  along  $\Gamma = 0$

$$\xi = A \frac{(\tau^H R^H + 1)^{\frac{1}{1-\alpha_I}}}{R^H},$$

and implicitly a function of  $\kappa$  along  $\Phi$ . Consider the derivative of  $\xi$  with respect to  $\kappa$ :

$$\begin{aligned} \frac{d\xi}{d\kappa} &= \frac{\xi(\alpha_I \tau^H R^H - (1 - \alpha_I))}{R^H(1 - \alpha_I)(\tau^H R^H + 1)} \frac{dR^H}{d\kappa} \\ &= \frac{\xi(\alpha_I(\tau^H R^H + 1) - 1/\alpha_I)}{R^H(1 - \alpha_I)(\tau^H R^H + 1)} \frac{dR^H}{d\kappa}. \end{aligned} \quad (52)$$

3) Now assume for the rest of this proof, that  $s^H > s^F$  and analyze the expression  $(\tau^H R^H + 1)$ . From 1) we know, that  $(\tau^H R^H + 1)$  decreases monotonically with  $\kappa$ . Its maximum value is  $1/a^H$  and its infimum is  $\sigma/a^F$ .

(a)  $1/a^H < 1/\alpha_I \iff \alpha_C > \alpha_I$

(b)  $\sigma/a^F > 1/a^H \iff s^H(1 - s^F)\alpha_C < s^F(1 - s^H)\alpha_I$

(c) follows from (a), (b) and the fact that  $(\tau^H R^H + 1)$  is decreasing.

□

#### Proof of Proposition 4

1) Differentiate equation (46) to get with the help of Lemma B6:

$$\begin{aligned} \frac{d\rho}{d\kappa} &= \left( \frac{\tau^H}{\sigma\tau^F} + \frac{1 - \sigma}{\sigma\tau^F R^H} \right) - \kappa \left( \frac{1 - \sigma}{\sigma\tau^F (R^H)^2} \right) \frac{dR^H}{d\kappa} \\ &\geq \left( \frac{\tau^H}{\sigma\tau^F} + \frac{1 - \sigma}{\sigma\tau^F R^H} \right) \geq 0. \end{aligned}$$

2) From the definition of  $\Phi$  we get immediately

$$\frac{\pi^F}{\pi^H} = \frac{1}{\sigma} \kappa. \quad (53)$$

3) Follows immediately from (46) and Lemma B6 where the extreme values of  $R^H$  along  $\Phi$  are used to derive the boundaries of  $\kappa/\rho$ .

□

### Proof of Proposition 5

Along  $\Phi$  the variables considered in Proposition 5 can be expressed in terms of  $R^H$ ,  $\kappa/\rho$ ,  $\kappa$ ,  $\rho$ ,  $\xi$  and two constant:

$$A := \frac{1 - \alpha_I}{\alpha_I} \left( \frac{\alpha_I s^H}{n + A^H(1 - \tau^H)} \right)^{\frac{1}{1 - \alpha_I}} \quad \text{and}$$

$$B_j := (1 - \alpha_j) \left( \frac{\alpha_j}{1 - \alpha_j} \right)^{\alpha_j}, j = C, I.$$

Now we can turn towards the list of statements made in the proposition.

- 1) According to Lemma B.1 the rental rate of capital is

$$r = B_I(\tilde{\omega})^{(\alpha_I - 1)} = B_I(R^H \xi)^{(\alpha_I - 1)} = B_I A^{(\alpha_I - 1)} / (\tau^H R^H + 1) \quad (54)$$

and decreases with  $R^H$ .

- 2) Using (22) and Lemma B.1 the average wage income share of total income in country  $H$  turns out to be

$$\frac{\tau^H w^H}{\pi^H} = \frac{\tau^H r \tilde{\omega}}{\pi^H / h^H} = \frac{r \tau^H \tilde{\omega}}{r(\tau^H \tilde{\omega} + \xi)} = \frac{\tau^H R^H \xi}{\tau^H R^H \xi + \xi} = \frac{\tau^H R^H}{\tau^H R^H + 1} \quad (55)$$

which is increasing with  $R^H$ , hence decreasing with  $\kappa$ . The same occurs to country  $F$ .

□

### Proof of proposition 6

- 1) (a) The normalized capital ownership per head is  $x^H / h^H = \xi$ . The Normalized social product in terms of the investment good turns out to be directly proportional to  $\xi$ :

$$\frac{\pi^H}{h^H} = r(\tau^H \tilde{\omega} + \xi) = r(\tau^H R^H + 1)\xi = A^{(\alpha_I - 1)} B_I \xi.$$

Hence, (52) proves the first part of the statement.



- (b) Taking into account the relative price of the consumption good (equation (8)) yields:

$$\begin{aligned}\frac{p_I \pi^H}{p_C h^H} &= A^{\alpha_I-1} B_C \xi (R^H \xi)^{\alpha_C - \alpha_I} \\ &= A^{\alpha_C} B_C (\tau^H R^H + 1)^{\frac{1+\alpha_C - \alpha_I}{1-\alpha_I}} / R^H.\end{aligned}$$

Differentiating this term with respect to  $R^H$  gives:

$$\frac{d \frac{p_I \pi^H}{p_C h^H}}{d R^H} = \frac{p_I \pi^H}{p_C h^H} \frac{\alpha_C (\tau^H R^H + 1) - (1 + \alpha_C - \alpha_I)}{(1 - \alpha_I) R^H (\tau^H R^H + 1)}.$$

Together with  $dR^H/d\kappa < 0$  this proves the statement .

- 2) (a) The normalized capital ownership per head in country  $F$  is

$$\frac{x^F}{h^H} = \xi \kappa$$

and from the definition of  $\Phi$  we get:

$$\frac{\pi^F}{h^H} = \frac{\kappa \pi^H}{\sigma h^H} = A^{(\alpha_I-1)} B_I \frac{\kappa}{\sigma} \xi.$$

Hence, we can confine the analysis to one of the terms, say the latter one. Differentiating with respect to  $\kappa$  gives:

$$\frac{d \frac{\pi^F}{h^H}}{d \kappa} = \frac{\pi^F}{h^H} \left( \frac{\alpha_I (\tau^H R^H + 1) - 1}{(1 - \alpha_I) (\tau^H R^H + 1)} \left( \frac{dR^H}{d\kappa} \frac{\kappa}{R^H} \right) + 1 \right).$$

Notice, that the elasticity in this expression is negative!

- (b) Finally, differentiating the normalized consumption of country  $F$  with respect to  $\kappa$  we get:

$$\frac{d \frac{p_I \pi^F}{p_C h^H}}{d \kappa} = \frac{p_I \pi^F}{p_C h^H} \left( \frac{\alpha_C (\tau^H R^H + 1) - (1 - \alpha_C + \alpha_I)}{(1 - \alpha_I) (\tau^H R^H + 1)} \left( \frac{dR^H}{d\kappa} \frac{\kappa}{R^H} \right) + 1 \right).$$

This completes the proof of our last proposition.  $\square$

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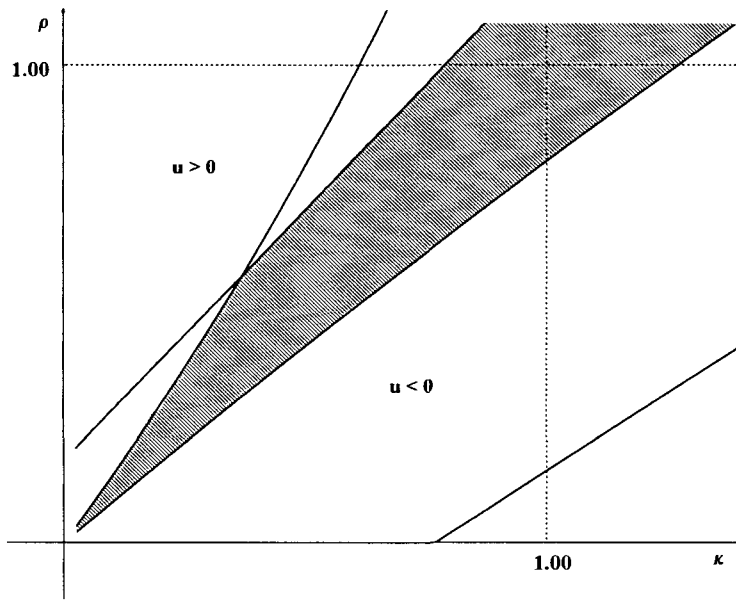


Figure 1

The parameter values used to generate figure 1 are

|              |              |            |            |              |            |
|--------------|--------------|------------|------------|--------------|------------|
| $\alpha_C =$ | $\alpha_I =$ | $s^H =$    | $s^F =$    | $\epsilon =$ | $n =$      |
| 0.35         | 0.20         | 0.23       | 0.18       | 0.625        | 0.01       |
| $A^H =$      | $A^F =$      | $\tau^H =$ | $\tau^F =$ | $\theta^0 =$ | $\delta =$ |
| 0.02         | 0.02         | 0.94       | 0.97       | 42.00        | 0.65       |

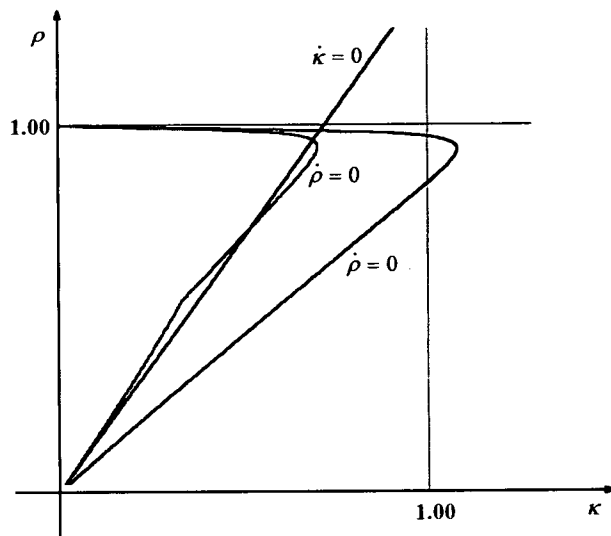


Figure 2

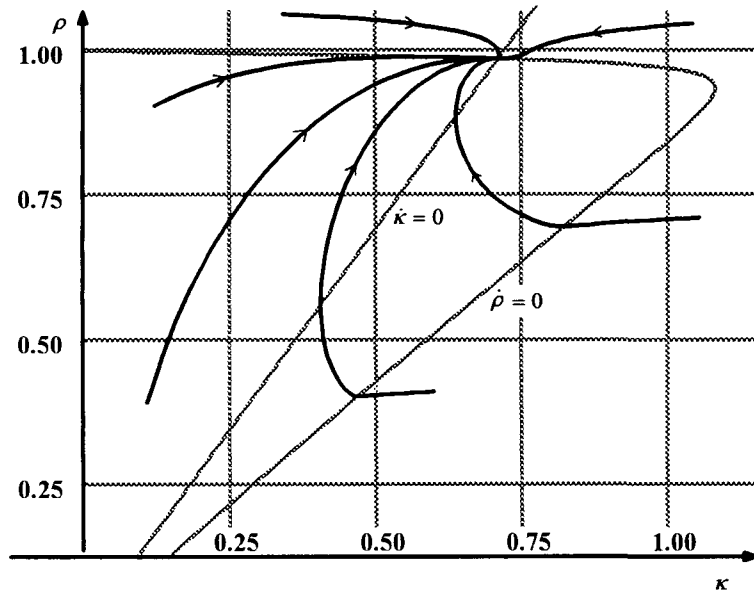


Figure 3a

The parameter values used to generate figure 3a are

|              |              |            |            |              |            |
|--------------|--------------|------------|------------|--------------|------------|
| $\alpha_C =$ | $\alpha_I =$ | $s^H =$    | $s^F =$    | $\epsilon =$ | $n =$      |
| 0.35         | 0.20         | 0.23       | 0.18       | 0.625        | 0.01       |
| $A^H =$      | $A^F =$      | $\tau^H =$ | $\tau^F =$ | $\theta^0 =$ | $\delta =$ |
| 0.02         | 0.02         | 0.94       | 0.97       | 42.00        | 0.65       |

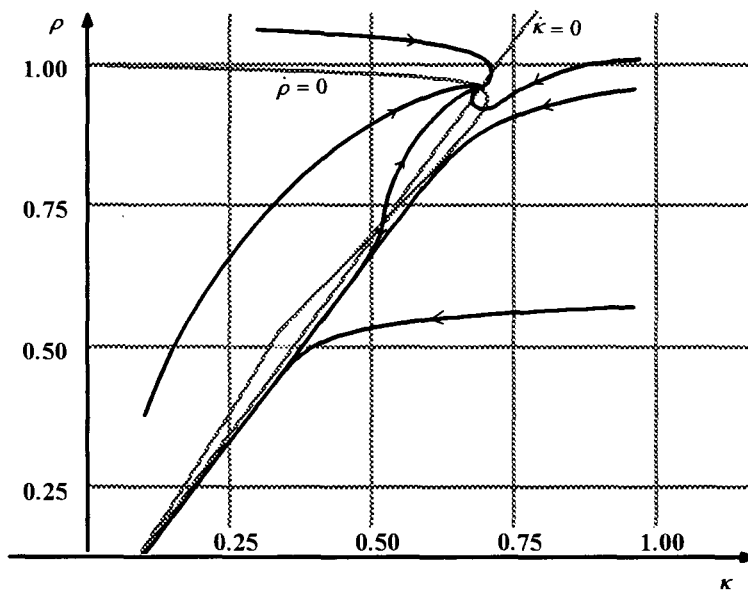
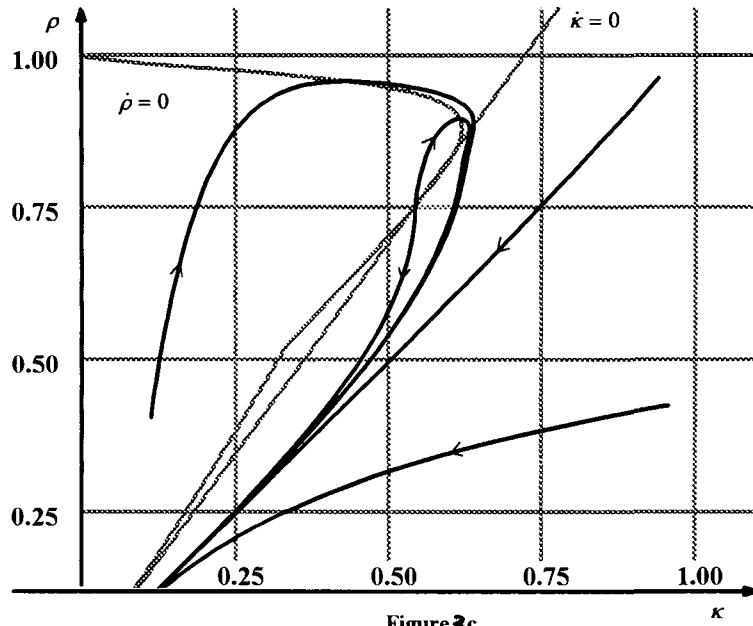


Figure 3b

The parameter values used to generate figure 3b are

|              |              |            |            |              |            |
|--------------|--------------|------------|------------|--------------|------------|
| $\alpha_C =$ | $\alpha_I =$ | $s^H =$    | $s^F =$    | $\epsilon =$ | $n =$      |
| 0.35         | 0.20         | 0.23       | 0.18       | 0.625        | 0.01       |
| $A^H =$      | $A^F =$      | $\tau^H =$ | $\tau^F =$ | $\theta^0 =$ | $\delta =$ |
| 0.02         | 0.02         | 0.94       | 0.97       | 42.00        | 0.65       |



4

The parameter values used to generate figure 3c are

|              |              |            |            |              |            |
|--------------|--------------|------------|------------|--------------|------------|
| $\alpha_C =$ | $\alpha_I =$ | $s^H =$    | $s^F =$    | $\epsilon =$ | $n =$      |
| 0.35         | 0.20         | 0.23       | 0.18       | 0.625        | 0.01       |
| $A^H =$      | $A^F =$      | $\tau^H =$ | $\tau^F =$ | $\theta^0 =$ | $\delta =$ |
| 0.14         | 0.02         | 0.94       | 0.97       | 135.00       | 0.65       |

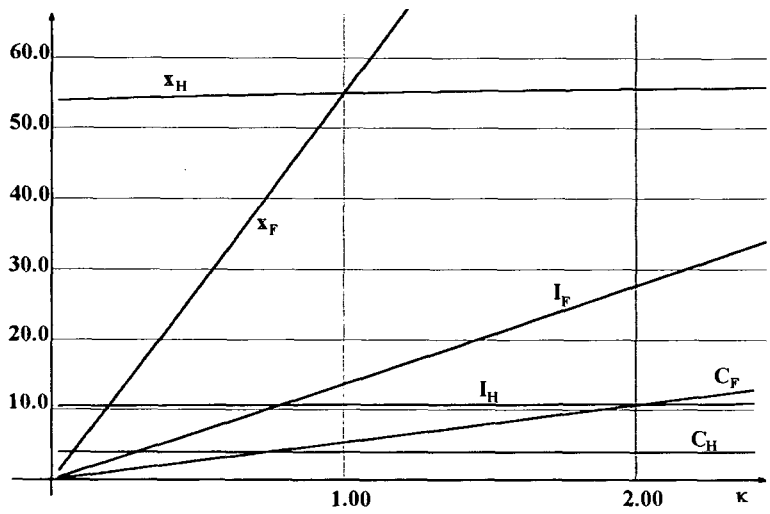


Figure 4