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Exchange rate volatility and international trade: The option approach

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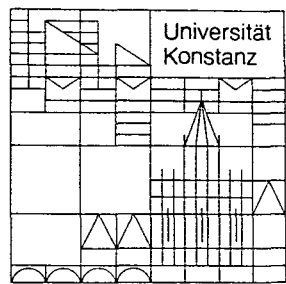
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Diskussionsbeiträge

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and International Trade:
The Option Approach**

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ABSTRACT

Usually it is argued that an increase in exchange rate volatility reduces the volume of international trade since trading firms are risk averse. This paper shows for risk neutral firms that the expected international trade volume in standardized commodities grows with exchange rate volatility. The firms adjust their trade volume to the exchange rate level. The more favorable the exchange rate is, the higher is the export volume. If the rate drops below some level, exports are stopped. Thus international trading represents an option for firms. Its value increases with exchange rate volatility.

1 Introduction^{*}

Since the collapse of the Bretton Woods agreement in 1973 various papers have investigated the effects of exchange rate volatility on international trade. Many of the early theoretical papers (cf. Baron (1976), Clark (1973), Ethier (1973)) show that an increase in exchange rate volatility should reduce the volume of international trade. The basic reasoning of these papers can be summarized as follows: Exporters are assumed to be risk averse. Financial markets are incomplete so that perfect hedging against exchange rate risk is impossible; moreover, hedging is costly. The exchange rate risk is the main source of the exporter's profit risk. If the exchange rate volatility increases, then profit risk increases. Since exporters are risk averse, the increase in profit risk reduces the benefits and therefore the volume of international trade. Hence an increase in exchange rate volatility reduces the volume of international trade.

This proposition has been tested frequently (e.g. Hooper and Kohlhagen (1978), Cushman (1983), Akhtar and Hilton (1984), Gotur (1985), Thursby and Thursby (1985)). Most findings do not support the proposition. Perhaps the strongest support is given by Cushman (1983) who finds a significant effect in 6 out of 14 bilateral trade analyses. The overall evidence is weak, at best.

* I acknowledge the helpful comments of John Chipman and Horst Siebert. Any errors are mine, of course.

This motivates another theoretical attempt to look again into the effects of exchange rate volatility on international trade. The focus of this paper is completely different from previous papers. Risk aversion of firms will be ignored. Administrative constraints of the export volume do not exist. Contrary to the literature quoted above, exporters adjust their trade volume to the current exchange rate level. If the exchange rate is low, exports will be low or stopped completely. If the exchange rate surges to high levels, exports will increase. Thus exporters adjust their export volume to the exchange rate level. Therefore exporting will be regarded as an option of exporters. Whenever the exchange rate permits profitable exports, the exporter will exercise the option, i.e., he will export.¹⁾ The exercise of this option is associated with transaction costs. As is well known from the finance literature, the value of an option increases with the risk of the underlying price. Similarly, the option to export will be shown to be more valuable when exchange rate volatility increases, despite an increase in transaction costs. Therefore, the volume of international trade grows given a sufficiently large increase in exchange rate volatility. This result is precisely opposite to the existing literature and may help to explain the empirical findings.

The paper is organized as follows. The economic setting will be defined in the next section. Then some characteristics of the optimal strategy of an internationally trading firm will be de-

1) Brennan and Schwartz (1985) use the option framework to analyse the optimal policy for extracting natural resources.

rived for a given level of exchange rate volatility (section 3). Section 4 shows the effects of volatility growth on the firm's trading strategy. Section 5 aggregates across all firms to show the effects on international trade volume. In addition, an interpretation of the results is offered to show that the analysis is consistent with notions of equilibrium.

2 The Economic Setting

2.1 Cash Flows From International Trading

This section presents the assumptions of the paper. There exist two countries, the "domestic" and the "foreign" country. Firms in both countries produce internationally traded commodities. The international trade volume of these commodities is not restricted by either government. The firms are risk neutral. The internationally traded commodities are standardized products such that their quality is independent of the producer. Therefore a producer's sales volume in either country depends on his sales prices relative to those of his competitors. Each firm sells its products in its home country and exports them whenever the export is expected to be profitable. Sales in the home country are exposed to competition from imports. The strength of this competition depends on the exchange rate. Therefore a firm's sales volume and sales prices in its home market depend on the exchange rate.

Now consider a firm being located in the "domestic" country. Whenever the firm exports, it earns at date t from exports the

(expected) domestic currency-cash flow x_t which depends on the exchange rate w_t (units of domestic currency per unit of foreign currency). Hence $x_t = x_t(w_t)$. x_t is defined as the incremental cash flow which the firm derives from its exports. As depicted in figure 1, the cash flow strictly increases with the exchange rate at a decreasing rate.

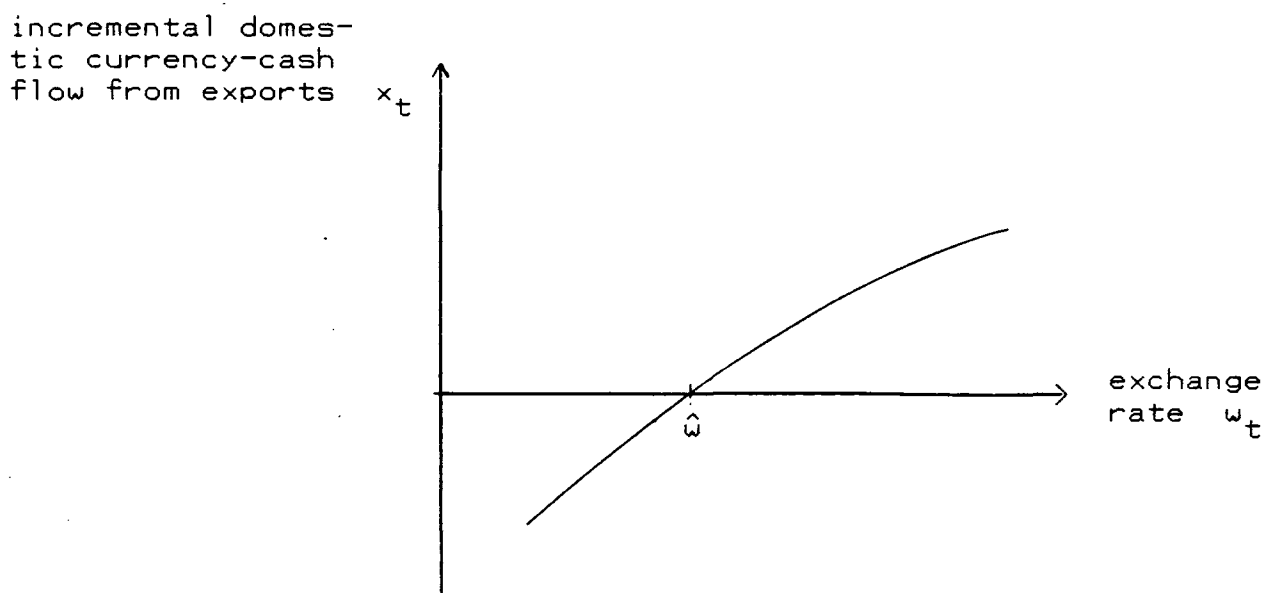


Figure 1 : The incremental domestic currency-cash flow from exports is strictly increasing and strictly concave in the exchange rate.

In order to retain symmetry for an exporting firm located in the domestic country, and an exporting firm located in the foreign country, the cash flow is assumed to be linear in the logarithm of the exchange rate.

$$x_t = c (\ln w_t - \ln \hat{w}) \quad \text{with } c > 0 .$$

\hat{w} is the break-even exchange rate which is deterministic and constant over time. Define $e = \ln w$. Then the equation above yields

$$x_t = c (e_t - \hat{e}) . \quad (1)$$

Equation (1) needs some explanation. First, any effects of inflation are neutralized by deflating all prices, costs and cash flows, denominated in domestic currency, by the domestic price index for internationally traded commodities. Similarly, items denominated in foreign currency are deflated by the foreign price index for internationally traded commodities. The exchange rate w is then defined as the real rate, i.e. the nominal rate adjusted for differential price movements in both countries. There exist two reasons for analyzing real terms. The first is that nominal exchange rate changes are irrelevant for the exporting firm if they only offset differential price movements in both countries (Balassa (1980, p.75)). The second reason is that the constancy of c and \hat{e} over time can only be justified if they are defined in real terms.

Second, the model is cast in continuous time so that x_t is the instantaneous cash flow at date t . Third, whenever the firm is active in the foreign market, it exports at date t the instantaneous quantity of commodities $y(e_t)$. This quantity strictly increases with the exchange rate, it is not bounded from above. At the same time, a higher exchange rate induces other domestic firms to start exporting or to expand exports. Therefore the supply of commodities in the foreign country increases with the exchange rate so that the foreign currency-denominated

commodity prices in the foreign country go down. The effect of both, price and quantity adjustments, is that the cash flow x_t increases with the exchange rate w_t , but at a decreasing rate.

Fourth, define \bar{w} as the parity exchange rate for internationally traded commodities, i.e. \bar{w} is defined by the domestic price index for those commodities, divided by the foreign price index. Then equation (1) may be rewritten as ($\bar{e} = \ln \bar{w}$)

$$x_t = c(e_t - \bar{e}) - c(\hat{e} - \bar{e}). \quad (2)$$

$\hat{e} > \bar{e}$ is assumed so that the second term $c(\hat{e} - \bar{e})$ is positive. It may be interpreted as the instantaneous additional transaction costs of international trading as compared to national trading if the instantaneous export quantity is $y(\bar{e})$. These costs include transportation costs, customs duties, information costs etc.

Sixth, the cash flow at date t depends only on the exchange rate at the same date. This appears reasonable if sales contracting and payments are simultaneous and sales revenues are converted immediately into domestic currency. This condition rules out any transaction risk.

Next, entry and exit costs will be specified. When the firm starts exporting, it enters the foreign market. Then it has to set up a sales office, inform potential customers etc. The corresponding domestic currency-entry costs are $n \alpha(a)$. n de-

notes the expected level of the entry cost, a is the log exchange rate at which the firm enters and $\alpha(a)$ denotes the function relating the entry cost to the "entry rate" a . $\alpha(a)$ is a lump sum-payment at the entry date.

When the firm stops exports, it exits from the foreign market. Then it has to close the sales office ²⁾. Therefore the firm has to pay the lump sum $N \zeta(z)$ at the exit date. N denotes the expected level of the exit cost, $\zeta(z)$ the function relating the exit cost to the log exit-exchange rate z . Usually one would expect the entry costs to be much higher than the exit costs.

As part of the entry and exit costs is generated in the foreign country, $d\alpha/da > 0$ and $d\zeta/dz > 0$. Therefore a higher rate motivates the firm to substitute entry and exit activities in the foreign country by activities in the domestic country. This justifies the assumption that $\alpha(a)$ is concave in the interval $a \in [\bar{e}, \infty)$ and $\zeta(z)$ is concave in the interval $z \in [\bar{e}, \infty)$.

The firm has an infinite time horizon and operates in a stationary world. Therefore, in equation (2), all terms apart from x_t and e_t , are constants. Moreover, the firm faces no constraints when it maximizes the benefits from exporting. This facilitates the characterization of the optimal strategy considerably. The optimal strategy maximizes the net present value of the expected incremental cash flows from exports. As the firm is risk neutral,

2) For a discussion of exit costs see Siebert (1985).

the wealth accumulated in the past does not affect the choice of the optimal strategy. The infinity of the time horizon and the stationarity of the world then suggest that the firm adopts a stationary bang-bang policy, i.e. the non-exporting firm enters the foreign market whenever the exchange rate reaches level a and the exporting firm exits from the foreign market whenever the exchange rate drops to level z . a and z are constant over time. This policy is illustrated in figure 2, assuming that $a > z > 0$ and that the firm does not export at date 0.

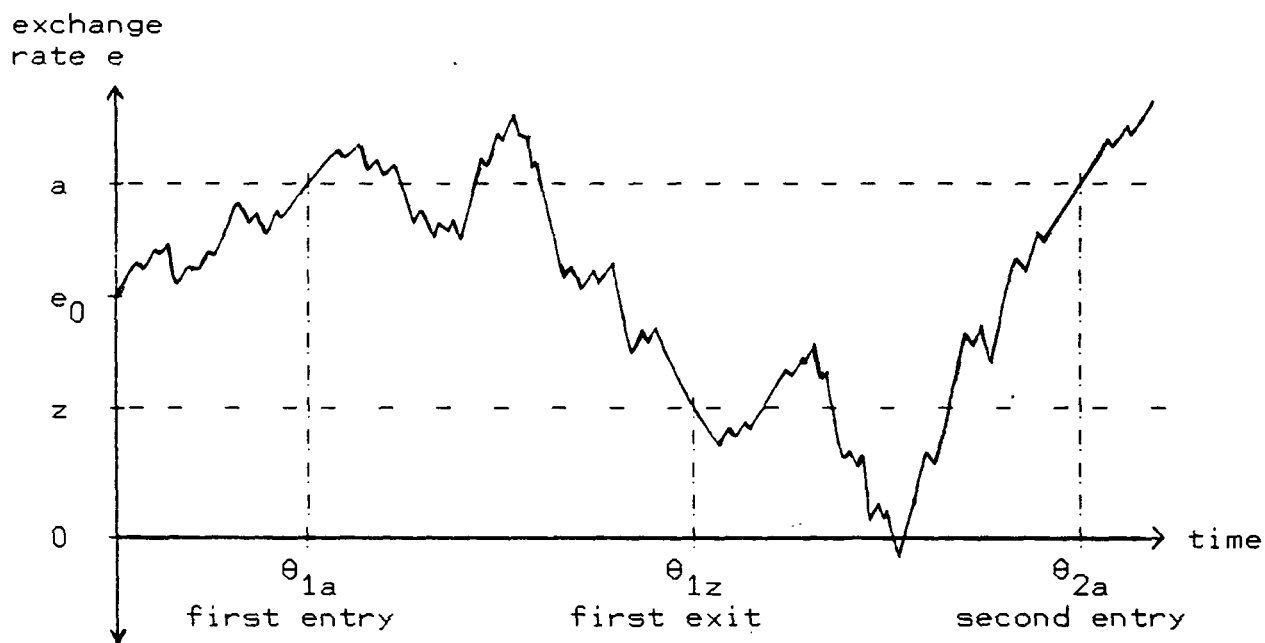


Figure 2 : A stationary bang-bang policy is defined by the entry-rate a and the exit-rate z .

Although the author does not know a proof that a stationary bang-bang policy is the best strategy under the previously stated assumptions, no better strategy is known. Therefore the analysis will be restricted to stationary bang-bang policies.

The preceding assumptions specify a very simple world. Any international trade due to comparative cost advantages is ruled out. Direct investments do not exist in this world. These simplifications permit a clear exposition of the option approach.

2.2 Exchange Rate Movements

Now consider exchange rate movements. The firm operates under a flexible exchange rate regime. The log of the real exchange rate follows a time-homogeneous mean reverting Ito-process with stationary independent increments in the noise

$$de_t = \gamma (e_t - \bar{e})dt + \delta dz(t) \quad ; \quad \gamma < 0. \quad (3)$$

de_t is the instantaneous change of e at date t , γ is a negative constant, the increment $dz(t)$ is normally distributed with expectation 0 and standard deviation \sqrt{dt} , the increments $dz(t)$ and $dz(\tau)$ ($t \neq \tau$) are independent. δ is the instantaneous standard deviation of e_t . The drift of the Ito-process is $\gamma (e_t - \bar{e})$. This term is negative whenever $e_t > \bar{e}$ and positive whenever $e_t < \bar{e}$. Therefore the drift moves e_t back towards the parity rate \bar{e} . The larger $|\gamma|$ is, the stronger is the drift.

The implications of the process (3) become more transparent by looking at the solution of the process. Let e_0 denote the known log exchange rate at date 0. Then < Wymer (1972) >

$$e_t = (e_0 - \bar{e}) \exp(\gamma t) + \bar{e} + \delta \int_0^t \exp[\gamma(t-s)] dz(s) \quad (4)$$

Hence e_t is normally distributed with expectation $(e_0 - \bar{e}) \exp(\gamma t) + \bar{e}$ and variance $(\exp(2\gamma t) - 1) \delta^2 / (2\gamma)$. Therefore, over time, the expected rate approaches asymptotically the parity rate \bar{e} and the variance the upper limit $\delta^2 / (-2\gamma)$. Hence, the exchange rate is expected to move back to the parity rate \bar{e} in the long run, i.e. internationally traded commodities are expected to be equally expensive in both countries. The actual rate may, of course, deviate strongly from the parity rate.

The Ito-process is assumed to hold for the log of the exchange rate w_t . This means that (1) the instantaneous standard deviation of w_t is proportional to the level of w_t , (2) $\text{Prob}(w_t > 0) = 1$ and (3) $\ln w_t$ and $\ln(1/w_t)$ follow an Ito-process, by Ito's lemma.

The rationale behind this process is that both international capital movements and international trade determine exchange rate movements. International capital movements may drive the exchange rate away from the parity rate. Sufficiently large differences between both rates create opportunities for international commodity arbitrage. This arbitrage drives the exchange rate back towards the parity level. Hence, the mean reversion is caused by this arbitrage. The smaller the obstacles to commodity arbitrage are, the more intensive this arbitrage is, the faster the rate moves back towards the parity level, i.e. the higher is $|\gamma|$.

Finally consider the real interest rates in both countries. The real rate in a country is the nominal rate deflated by the expec-

ted change in the price index of internationally traded commodities. If the capital markets of both countries are well integrated and informationally efficient, the real interest rates in both countries cannot be constant. Suppose, for example, that the dollar is overvalued relative to the Deutsche Mark ($e_t > \bar{e}$). Then the expected exchange rate change is negative. Hence the real \$-interest rate must exceed the real DM-interest rate to compensate the investor for the expected dollar depreciation. Let r_t and r_t^* denote the domestic and the foreign real interest rate. Symmetry between both countries requires $|r_t - \bar{r}| = |r_t^* - \bar{r}|$. \bar{r} denotes the real interest rate in both countries if the actual exchange rate equals the parity rate \bar{e} . Symmetry and the international Fisher-effect imply $r_t - \bar{r} = (1/2) E[de_t]$. From this and equation (3) follows

$$r_t - \bar{r} = (r/2) (e_t - \bar{e}). \quad (5)$$

\bar{r} is required to be sufficiently positive so that the net present value of any perpetual stream of cash flows remains finite.

3 The Optimal Trading Strategy

3.1 The Firm's Objective Function

This section specifies the firm's objective function and derives some properties of the optimal export policy for a given level of exchange rate risk, measured by the instantaneous standard deviation of the Ito-process. The risk neutral firm maximizes the net

present value (NPV) of its expected incremental cash flows from exports.

Let \tilde{v}_1 denote the random NPV of the first engagement in the foreign market with the cash flows being discounted to the date of the first entry, θ_{1a} . The first engagement starts with the first entry and ends with the first exit from the foreign market. Similarly, $\tilde{v}_2, \tilde{v}_3, \dots$ denote the NPVs of the second, third, ... engagement in the foreign market discounted to the respective dates of entry. The random NPV of all these engagements, discounted to the date of the first entry, is

$$\tilde{NPV} = \tilde{v}_1 + \tilde{R}_{12} \left\{ \tilde{v}_2 + \tilde{R}_{23} [\tilde{v}_3 + \dots \right. \quad (6)$$

with

$$\tilde{R}_{j,j+1} = \exp \int_{\tilde{\theta}_{ja}}^{\tilde{\theta}_{j+1,a}} -r_t(\tilde{e}_t) dt ; \quad j = 1, 2, \dots \quad (7)$$

$[\tilde{\theta}_{j+1,a} - \tilde{\theta}_{ja}]$ is the random time between the j -th and the $(j+1)$ -th entry in the foreign market. Therefore, it is the first passage time required by an exchange rate move from the entry rate a to the exit rate z and back to the entry rate a .³⁾ $\tilde{R}_{j,j+1}$ depends on the actual path of the exchange rate between dates $\tilde{\theta}_{ja}$ and $\tilde{\theta}_{j+1,a}$.

3) As the mean reverting Ito-process does not satisfy a reflection principle, we do not know an explicit probability density function of the first passage time.

The risk neutral firm maximizes the expectation of equation (6). As the mean reverting Ito process is a strong Markov process, all factors in equation (6) are conditionally independent. Therefore, by the telescope property of the expected value, the expected value of the NPV is

$$NPV = v_1 + R_{12} \left\{ v_2 + R_{23} [v_3 + \dots \right. \quad (8)$$

Omission of the tilde indicates expected values. Time-homogeneity implies $v_1 = v_2 = \dots = v$ and $R_{12} = R_{23} = \dots = R$ so that

$$\begin{aligned} NPV &= v [1 + R + R^2 + \dots \\ &= \frac{v}{1-R} \end{aligned} \quad (9)$$

v remains to be defined. The first component of v is the entry cost $n \alpha(a)$. After entry the firm receives the random cash flows $x_t(e_t)$ until the exchange rate drops to z . This cash flow has to be multiplied by the discount factor ($\tau = 0$ denotes the entry date)

$$R_t(e_t) = E \left[\exp \int_0^t -r_\tau(\tilde{e}_\tau) d\tau \mid e_0 = a; e_\tau > z \quad \forall \tau \in [0, t) \right] \quad (10)$$

The value of the integral depends on the actual path of the exchange rate moving from a to e_t without dropping to z before date t . Taking expectations across all feasible paths yields the discount factor $R_t(e_t)$. The conditional expectation of the discounted cash flows then is

$$CF = \int_0^{\infty} \int_z^{\infty} x_t(e_t) R_t(e_t) dF(e_t|a) dt \quad (11)$$

$F(e_t|a)$ denotes the conditional probability distribution function of e_t , subject to the conditions $e_0 = a$ and $e_{\tau} > z \quad \forall \tau \in [0, t)$.

When the exchange rate drops to z , the firm incurs the exit cost $N \zeta(z)$. Since the lifetime of the engagement equals the first passage time θ_{az} , i.e. the time between entry and exit, the appropriate discount factor is given by equation (10) with $t = \theta_{az}$ and $e_t = z$. This discount factor is denoted $R_{\theta}(z)$. Let $F(\theta_{az})$ denote the conditional probability distribution function of θ_{az} . Then the conditional expected exit cost is

$$N \zeta(z) \int_0^{\infty} R_{\theta}(z) dF(\theta_{az}). \quad (12)$$

Combining the preceding results yields

$$v = -n\alpha(a) + CF - N \zeta(z) \int_0^{\infty} R_{\theta}(z) dF(\theta_{az}) \quad (13)$$

Equations (9) and (13) define the firm's objective function.

3.2 The Effects of Entry and Exit Costs

In order to gain insight into the optimal bang-bang policy, the effects of entry and exit costs will be investigated. First sup-

pose, that these costs approach zero. Then the optimal policy would be $a^+ \rightarrow \hat{e}$ and $z^+ \rightarrow \hat{e}$, i.e. the optimal entry rate a^+ and the optimal exit rate z^+ approach the break even-rate \hat{e} . Since the cash flow $x_t(e_t) > 0$ for $e_t > \hat{e}$ and $x_t(e_t) < 0$ for $e_t < \hat{e}$, the firm exports whenever $e_t > \hat{e}$, and never exports when $e_t < \hat{e}$. This policy implies uncountably many entries and exits since, for $a \rightarrow z$, the expected first passage time from a to z and the one from z to a approach zero.

The zero entry and exit cost-case is a good starting point for illustrating the option nature of exporting. The firm is not passively exposed to exchange rate changes. Instead it actively pursues a policy of reaping positive and avoiding negative cash flows from exporting. Hence exporting represents an option on positive cash flows.

Next it will be shown that an increase in the level of the entry and /or the exit cost reduces the optimal expected frequency of entry and exit. For given values of n and N , a policy is optimal only if a marginal reduction in the expected frequency generates a marginal reduction in the expected discounted entry and exit costs which equals the marginal reduction in the expected discounted cash flows. Now suppose that n and /or N increase. Then this optimality condition is violated because a marginal reduction in the expected frequency generates a higher reduction in the expected discounted entry and exit costs. Therefore, it is optimal to reduce the expected frequency of entry and exit. Hence $a^+ > z^+$ if entry and /or exit costs are positive.

A positive distance ($a - z$) can be achieved by

- 1. $a > z > \hat{e}$,
- 2. $a > \hat{e} \geq z$ or $a \geq \hat{e} > z$, or
- 3. $\hat{e} \geq a > z$.

The first policy cannot be optimal since lowering z to \hat{e} would yield additional positive cash flows, lower the frequency of entry and exit and the expected exit cost (12). The latter is true since $d\xi/d(-z) < 0$ and since the random lifetime of an engagement increases with $(-z)$ so that the expected discount factor (the integral in equation (12)) declines. This proves $z^+ \leq \hat{e}$.

The third policy can be optimal if the entry cost $n \alpha(a)$ increases sufficiently with the entry rate a . Suppose $n = N \rightarrow 0$ so that $a^+ \rightarrow \hat{e}$ and $z^+ \rightarrow \hat{e}$. Then marginal changes in a and z , da and dz , have a negligible impact on the net present value of cash flows since $x_t = 0$ for $e_t = \hat{e}$ and x_t is linear in e_t . Now raise n by a very small amount. Then, given a sufficiently large slope of $\alpha(a)$, $da^+ < 0$ is preferable to $da^+ > 0$ since it reduces the entry cost instead of raising it. As positive entry costs imply $a^+ > z^+$, $\hat{e} > a^+ > z^+$ follows. This proves that a and $(-z)$ do not monotonically increase with the level of the entry and exit costs. The related simulation results of Eppen and Fama (1968) suggest such a monotonicity; this is explained, however, by their assumption that, in the context of our model, the entry and exit costs are independent of the entry and exit rates.

Finally, suppose that $n \alpha(a) > CF(a,z) \quad \forall$ finite (a,z) with $a \geq z \leq \hat{e}$. This implies $-n \alpha(a) + CF(a,z) < 0$. As $a^+ \geq z^+ \leq \hat{e}$, the firm cannot benefit from exporting so that the optimal policy is $a^+ \rightarrow \infty$. This and the preceding arguments prove proposition 1.

Proposition 1:

- a) If entry and exit costs approach zero, the optimal entry and exit rates approach the break even rate \hat{e} .
- b) The optimal exit rate z^+ never exceeds the break even rate \hat{e} .
- c) If the entry costs $n\alpha(a)$ exceed $CF(a,z) \forall (a,z)$ with $a \geq z \leq \hat{e}$, then the firm never exports.
- d) The optimal policy is not monotonic, i.e. an increase in the level of the entry and/or the exit costs does not necessarily raise the optimal entry rate and lower the optimal exit rate; but it reduces the expected frequency of entry and exit o

4 The Effects of an Increase in Exchange Rate Volatility -----

4.1 The Effects on the Net Present Value of Exporting -----

The literature argues that an increase in exchange rate volatility reduces the volume of international trade because traders are risk averse. This paper regards exporting as an option. A standard result of finance is that the value of an option increases with the risk of the underlying asset's price. The same will be shown for the exporting option. Suppose exchange rate

volatility increases, i.e., the instantaneous standard deviation of the Ito-process, δ , changes to $S\delta$ with $S > 1$. The mean \bar{e} and the break even rate \hat{e} are assumed to be independent of S . In order to find out the effects of S , define a probability-invariant adjustment. For $S = 1$ the optimal entry and exit rates are denoted a_0 and z_0 . Let a_0 and z_0 be finite. When S changes, change the entry and exit rates from a_0 and z_0 to

$$a_1 = \bar{e} + S (a_0 - \bar{e}) ,$$

$$z_1 = \bar{e} + S (z_0 - \bar{e}) .$$

More generally, change any e_t from e_{t0} to

$$e_{t1} = \bar{e} + S (e_{t0} - \bar{e}) .$$

Hence, all distances $(e_{t0} - \bar{e})$ are replaced by $S(e_{t0} - \bar{e})$; in other words, the scale around \bar{e} is expanded by the factor S . Equation (4) yields

$$S(e_{t0} - \bar{e}) = S(e_{00} - \bar{e}) \exp(\gamma t) + S\delta \int_0^t \exp[\gamma(t-s)] dz(s)$$

or

$$e_{t1} - \bar{e} = (e_{01} - \bar{e}) \exp(\gamma t) + S\delta \int_0^t \exp[\gamma(t-s)] dz(s) \quad (14)$$

Therefore the probability distribution function of e_{t1} is identical to that of e_{t0} . In addition, the probability distribution functions of the first passage times remain the same. Thus all probability distribution functions used in the firm's optimization remain the same when S is increased and the scale around \bar{e} is expanded by the factor S . This leads to the following

Definition : Suppose the instantaneous standard deviation of the mean reverting process, δ , changes to $S\delta$. Then a probability-invariant adjustment is defined by expanding the scale around the mean \bar{e} by the factor S .

Now consider the effects of a probability-invariant adjustment on the NPV. Rewrite the NPV-equation (9), using (2), (11) and (13),

$$\begin{aligned}
 \text{NPV} = & (1-R)^{-1} \left\{ -n\alpha(a) - N \zeta(z) \int_0^{\infty} R_{\theta}(z) dF(\theta_{az}) \right. \\
 & - \int_0^{\infty} \int_z^{\infty} c(\hat{e}-\bar{e}) R_t(e_t) dF(e_t|a) dt \\
 & \left. + \int_0^{\infty} \int_z^{\infty} c(e_t-\bar{e}) R_t(e_t) dF(e_t|a) dt \right\}. \quad (15)
 \end{aligned}$$

Lemma 1 shows the effects of a probability-invariant adjustment on the terms of equation (15), neglecting the changes in the interest rate r_t .

Lemma 1 : Suppose S is raised above 1, but r_t remains the same. Assume a_0 and z_0 are finite. Then a probability-invariant adjustment implies for equation (15) that

- 1) all probability distribution functions remain the same,
- 2) all discount factors remain the same,
- 3) the entry cost $n\alpha(a_1) \leq S \left[n\alpha(a_0) - (1 - \frac{1}{S})n\alpha(\bar{e}) \right]$ if $a_0 \geq \bar{e}$,
the entry costs goes down if $a_0 < \bar{e}$,

- 4) the exit cost $N \xi(z_1) \leq S \left[N \xi(z_0) - \left(1 - \frac{1}{S}\right) N \xi(\bar{e}) \right]$ if $z_0 \geq \bar{e}$,
the exit cost goes down if $z_0 < \bar{e}$,
- 5) the net present value of the transaction costs $c(\hat{e} - \bar{e})$ remains the same, and
- 6) the net present value of the cash flow $c(e_t - \bar{e})$ grows strictly proportionally to S •

Proof : Statements 1), 2) and 5) follow immediately from the definition of a probability-invariant adjustment and the constancy of \hat{e} . The third statement can be proved as follows. First note that $a_1 > a_0$ iff $a_0 > \bar{e}$. As $d\alpha/da > 0$, the entry cost grows iff $a_0 > \bar{e}$. Suppose $a_0 > \bar{e}$. As $\alpha(a)$ is concave for $a \geq \bar{e}$, it follows for $S > 1$

$$\frac{\alpha \left[a_0 + (S-1)(a_0 - \bar{e}) \right] - \alpha(a_0)}{(S-1)(a_0 - \bar{e})} \leq \frac{\alpha(a_0) - \alpha(\bar{e})}{a_0 - \bar{e}}$$

so that

$$\alpha \left[a_0 + (S-1)(a_0 - \bar{e}) \right] - \alpha(a_0) \leq (S-1) \left[\alpha(a_0) - \alpha(\bar{e}) \right]$$

or

$$\alpha \left[a_0 + (S-1)(a_0 - \bar{e}) \right] = \alpha(a_1) \leq S \left[\alpha(a_0) - \left(1 - \frac{1}{S}\right) \alpha(\bar{e}) \right].$$

The proof of statement 4) is the same. Statement 6) follows since all cash flows $c(e_t - \bar{e})$ grow by the factor S ■

The important result of lemma 1 is: If the firm reacts on the growth of S mechanically by a probability-invariant adjustment,

the net present value of the cash flow $c(e_t - \bar{e})$ grows strictly proportional to S while the net present value of the transaction costs, represented by the first three terms in equation (15), grows at a smaller rate or even declines. Hence, the overall NPV increases with exchange rate volatility even upon the mechanical probability-invariant adjustment. If the firm reacts optimally, the NPV must grow even more. Hence an increase in exchange rate risk raises the value of the exporting option.

Another interpretation of lemma 1 is as follows: When the exchange rate volatility increases, then the firm's decision problem can be viewed equivalently as a problem defined by unchanged volatility, but reduced transaction costs. In other words, the relative magnitude of transaction costs declines when exchange rate volatility grows. Therefore, a growth of exchange rate volatility reduces the importance of frictions in international trading, created by transaction costs. The preceding results are summarized in proposition 2.

Proposition 2 :

Suppose the exchange rate volatility increases. Neglecting changes in interest rates, the net present value of the firm's cash flow $c(e_t - \hat{e})$ then grows faster than the net present value of the entry and exit costs even if the firm reacts mechanically by a probability-invariant instead of optimal adjustment. Therefore the growth of exchange rate volatility renders the exporting option more valuable for a risk neutral firm and reduces the importance of frictions in the international commodity market,

created by entry and exit costs ♦

Proposition 2 has been derived neglecting the effect of changes in interest rates. Now it will be shown that this effect is very unlikely to invalidate the results of proposition 2. Lemma 2 provides some insight into this effect. It is based on the weak assumption $(a_0 - \bar{e}) \geq (\bar{e} - z_0)$. This assumption always holds if $z_0 \geq \bar{e}$. Essentially the assumption requires that a_0 be much above \bar{e} if z_0 is much below \bar{e} .

Lemma 2 : Assume a_0 and z_0 are finite and $a_0 - \bar{e} \geq \bar{e} - z_0$. Then the net present value of cash flows, CF, defined by equation (11), and the discount factor R increase, due to the interest rate change associated with the probability-invariant adjustment. The expected exit cost, defined by equation (12), also increases while the entry cost $\alpha(a)$ remains the same. Hence the interest rate change, associated with the probability-invariant adjustment, raises the net present value of exporting if the level of the exit cost is sufficiently low ♦

Lemma 2 is proved in the appendix. Lemma 2 presents sufficient conditions so that the interest rate change, due to the growth of S, raises the benefits from exporting, i.e., the interest rate change reinforces the positive effects of an increase in exchange rate volatility. As the conditions of lemma 2 appear to be weak, it seems safe to conclude that the results of proposition 2 also hold if interest rate changes are taken into consideration. This enables us to analyse the implications for the firm's expected export volume.

4.2 The Effects on the Firm's Expected Export Volume

Next the effects of an increase in exchange rate volatility on the firm's expected export volume will be analysed. First note that in our model no international trade exists if the exchange rate always equals the parity rate \bar{e} . This is true since the transaction costs of international trade exceed those of national trade. Second, consider a firm which exports part-time if $S = 1$ and $\delta > 0$. Now assume a modest increase in volatility. As the optimal entry and exit rates may move in either direction, the expected export volume does not necessarily grow, despite of the reduced importance of entry and exit costs.

But a sufficiently large increase in volatility will be shown to raise the firm's expected export volume. It has been shown that an increase in volatility reduces the relative importance of entry and exit costs. Hence, for sufficiently large values of S , the optimal adjustment implies

$$F_t(a^+) \rightarrow F_t(\hat{e}) \quad \text{and} \quad F_t(z^+) \rightarrow F_t(\hat{e}) \quad (16)$$

with $F_t(\cdot)$ being the cumulative probability density of the exchange rate e_t . Let P_t denote the steady state-probability that the firm exports at date t . If it does, then the instantaneous export volume at date t , $y(e_t)$, is positive. The expected instantaneous export volume at date t is

$$E(y_t) = P_t \left[E(y_t \mid y(e_t) > 0 \text{ and } z^+ \leq e_t \leq \bar{e}) + E(y_t \mid y(e_t) > 0 \text{ and } e_t \geq \bar{e}) \right].$$

If the volatility becomes very large, then in the steady state relation (16) implies $P_t \rightarrow 1 - F_t(\hat{e}) \rightarrow .5$ and $E(y_t | y(e_t) > 0$ and $z^+ (e_t \leq \bar{e}) \rightarrow 0$. But $E(y_t | y(e_t) > 0$ and $e_t \geq \bar{e})$ grows monotonically with volatility because the probability mass is shifted to the tails of the distribution of e_t and $dy/de_t > 0$. Moreover, $E(y_t | y(e_t) > 0$ and $e_t \geq \bar{e})$ is not bounded from above because $y(e_t)$ is not. Therefore, $E(y_{t1}) > E(y_{t0})$ if S becomes sufficiently large. The expected steady state-export volume thus has been shown to grow.

Now consider a firm which never exports at $S=1$ because of higher transaction costs. As an increase in exchange rate volatility reduces the importance of entry and exit costs, there exists some finite value of S such that this firm starts exporting at a finite entry rate. Hence the number of firms which engage in exporting grows with S . This proves proposition 3.

Proposition 3 : The expected export volume of a part-time exporting firm grows if exchange rate volatility increases sufficiently. In addition, the number of firms engaged in exporting increases with exchange rate volatility ♦

5 Implications for Equilibrium

5.1 The Volume of International Trade

First, the volume of international trade will be derived by aggregation. Second, an interpretation of the equilibrium implications will be offered. From proposition 3 follows immediately

that the expected aggregate export volume grows if exchange rate volatility grows sufficiently. The same reasoning applies to imports since, by the assumptions of the paper, the firms in both countries face symmetrical choice problems. This establishes proposition 4.

Proposition 4: Assume that each firm's cost functions $n\alpha(a)$ and $N\zeta(z)$ and cash flow function $c(e_t - \hat{e})$ are independent of exchange rate volatility. Then the expected volume of international trade grows if exchange rate volatility increases sufficiently.

Proposition 4 is the central message of this paper. Contrary to the existing literature, a sufficient increase in exchange rate volatility raises the expected volume of international trade instead of reducing it. The intuition behind this result is that a firm can reap large benefits from exporting standardized commodities when the exchange rate is favorable but avoid losses by stopping exports when the exchange rate is unfavorable. The higher the volatility is, the higher are the benefits from exporting. As an increase in volatility raises the probability of high exchange rates which are associated with high domestic export volumes, the expected domestic export volume grows with volatility. An analogous argument holds for imports. Therefore the expected volume of international trade grows with volatility.

Proposition 4 implies that empirical tests of the effects of volatility growth on the volume of international trade may produce any type of result. This is consistent with the mixed empi-

rical evidence published so far. Strong volatility growth, however, should intensify international trade.

5.2 An Interpretation

The reader may ask whether the preceding results are consistent with notions of a general equilibrium. First, the preceding analysis is a partial equilibrium analysis. This is clearly revealed by the assumption of proposition 4. Second, the expected present value of entry and exit costs, aggregated over the domestic and the foreign economy, increases with volatility. This can be seen as follows. If the relative importance of entry and exit costs declines because of an increase in volatility, then the expected frequency of entry and exit increases. In addition, the number of firms trading internationally increases with volatility. Therefore, the aggregate present value of entry and exit costs increases. Third, high exchange rate volatility induces high volatility in exports and imports and thus high, costly volatility in the production volume of individual firms. These three points could raise doubts on the validity of the previously derived results in a general equilibrium.

Let us begin with the third point. Arguing that the value of the exporting option grows with exchange rate volatility does not mean that firms reap a net benefit from an increase in volatility. Firms produce for the domestic and the foreign market. An increase in volatility raises the value of the exporting option, but it may well reduce the benefits from production for the

domestic market since it strengthens potential imports and thereby competition in the domestic market. Thus it may well be true that the overall effect of a volatility increase on firms' values is negative. Despite of this the value of the exporting option grows.

An interpretation of the findings of this paper which is consistent with a general equilibrium might run as follows. Deviations of the exchange rate e from its mean \bar{e} are generated by international capital movements. These deviations indicate differences between the domestic and the foreign price index for internationally traded commodities which are not offset by the exchange rate. Hence, prices of the internationally traded commodities violate the "law of one price", thereby creating welfare losses. The price differences create arbitrage opportunities for international trade. This arbitrage reduces price differences and thereby improves welfare although it cannot eliminate price differences completely because of transaction costs.

An increase in exchange rate volatility implies an increase in the potential price differences and thus a higher welfare loss. Therefore, more scope for profitable arbitrage through international trade exists. Hence the expected volume of international trade grows. The welfare gain from international trade increases as well, despite of the growth of transaction costs. Therefore a positive relation between exchange rate volatility and international trade is consistent with a general equilibrium. This does not imply, however, that an increase in exchange rate volatility

is socially desirable.

A final remark on the assumptions of proposition 4. Each firm's cost functions and cash flow functions are assumed to be independent of exchange rate volatility. The assumption that the cost functions are independent appears to be innocuous. But the instantaneous cash flows and hence the volume of international trade should be adversely affected by the increase in competition between international traders, associated with an increase in exchange rate volatility. This does not invalidate proposition 4, however. The reason is that the first effect of an increase in exchange rate volatility is the growth in the international trade volume. This growth depresses, second, the instantaneous cash flows which, in turn, depresses the international trade volume. The second effect on the trade volume is smaller than the first one because the first effect induces the second. Therefore proposition 4 remains valid.

6 Conclusions

The literature argues that an increase in exchange rate volatility reduces the volume of international trade as economic agents are risk averse. The empirical evidence is mixed. This paper considers international trade as an option for firms to profit from price differences of internationally traded, standardized commodities. This option is sometimes exercised, sometimes not, depending on the exchange rate level. An increase in exchange rate

volatility means that international price differences grow so that the option becomes more valuable. This is true even if transaction costs are taken into consideration. Therefore the volume of international trade increases with exchange rate volatility.

This paper assumes risk neutrality of firms. Further research should be directed towards incorporating risk aversion. At the same time, hedging opportunities have to be taken into consideration. The final answer to the question whether risk aversion or the option nature dominates the growth effects of exchange rate volatility on international trade, can only be given by empirical studies.

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Appendix: Proof of Lemma 2

In order to prove lemma 2, lemma 3 is useful.

Lemma 3 : Assume a_0 and z_0 are finite, and $(a_0 - \bar{e}) \geq (\bar{e} - z_0)$. Then the discount factors R and $R_t(e_t)$ with $e_t \geq z$ grow with S upon a probability-invariant adjustment. In addition, the relative growth of $R_t(e_t)$ increases with e_t •

Proof : First note that $(a_0 - \bar{e}) \geq (\bar{e} - z_0)$ implies $a_0 > \bar{e}$ since $a_0 > z_0$ for non-zero entry or exit costs. Hence $(a_1 - \bar{e}) \geq (\bar{e} - z_1)$ and $a_1 > \bar{e}$ follow for a probability-invariant adjustment. From equations (5) and (10) follows

$$R_t(e_{t1}) = \exp(-\bar{r}t) E \left[\exp(S\rho_t) \mid e_0 = a_1; e_\tau > z_1 \forall \tau \in [0, t) \right]$$

with

$$\rho_t \equiv -\frac{\gamma}{2} \int_0^t (e_{\tau 0} - \bar{e}) d\tau . \quad (17)$$

If $a_1 > \bar{e}$ and $(a_1 - \bar{e}) \geq (\bar{e} - z_1)$, then $E \left[\rho_t \mid e_0 = a_1; e_\tau > z_1 \forall \tau \in [0, t) \right] > 0$. Hence $E \left[S\rho_t \mid \dots \right] = S E \left[\rho_t \mid \dots \right] > 0$. Then $E \left[\exp(S\rho_t) \mid \dots \right]$ grows with S for two reasons: (1) Holding the mean $E \left[\rho_t \mid \dots \right]$ constant, a growth of S implies a mean-preserving spread which raises the expectation of a strictly convex functional. (2) The mean $E \left[S\rho_t \mid \dots \right]$ grows with S . Hence $R_t(e_{t1})$ grows.

Let $R_t(e_{t1}) / R_t(e_{t0}) \equiv \mu_t(e_{t0})$ denote the relative growth of $R_t(e_t)$. Then $\mu_t(e_{t0})$ may be expressed as

$$\mu_t(e_{t0}) = E \left[\exp(S\rho_t) | \dots \right] / E \left[\exp\rho_t | \dots \right].$$

ρ_t is increasing in e_{t0} . As the exponential function is strictly increasing and strictly convex, μ_t must be increasing in e_{t0} , too.

Now consider R , the expectation of equation (7). The exchange rate starts at a_1 , then drops to z_1 at some future date and then returns to a_1 sometimes later. As $a_1 \geq \bar{e}$ and $(a_1 - \bar{e}) \geq (\bar{e} - z_1)$, $E[\rho_t | \dots] > 0$ for any $t = \theta_{j+1, a} - \theta_{j, a}$. Then the same reasoning as for $R_t(e_t)$ shows that R increases with S . This proves lemma 3 •

As R increases with S , the NPV (equation (9)) increases, if v does. The v -equation (13) shows that the entry cost is independent of interest rates. The expected exit cost grows with S as $R_\theta(z)$ does. The size of this effect depends on $N\zeta(z)$. Hence in order to prove lemma 2, it suffices to show that the net present value of cash flows, CF , grows with S , due to the interest rate change. In other words, it has to be shown that

$$\int_0^\infty \int_{z_1}^\infty c(e_{t1} - \hat{e}) [\mu_t(e_{t0}) - 1] R_t(e_{t0}) dF(e_{t1} | a_1) dt > 0. \quad (18)$$

$\mu_t(e_{t0}) > 1$ and, by lemma 3, $d\mu_t/de_{t0} > 0$. Hence the integral (18) is positive if

$$\int_0^\infty \int_{z_1}^\infty c(e_{t1} - \hat{e}) R_t(e_{t0}) dF(e_{t1} | a_1) dt > 0. \quad (19)$$

By lemma 1, 5) and 6), this integral is greater than

$$CF_0 = \int_0^{\infty} \int_{z_0}^{\infty} c(e_{t0} - \hat{e}) R_t(e_{t0}) dF(e_{t0} | a_0) dt.$$

As a_0 and z_0 define an optimal policy, CF_0 must be positive, therefore (19) and (18) must be positive. This ends the proof.