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## Asset pricing with cognitive dissonance

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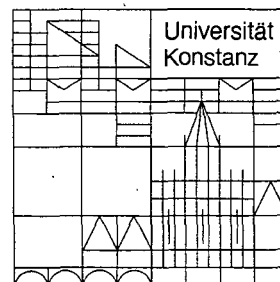
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Diskussionsbeiträge



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**Asset Pricing  
with Cognitive Dissonance**

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# ASSET PRICING WITH COGNITIVE DISSONANCE

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### **Abstract**

The behavior of asset prices is analyzed in a general equilibrium model where agents not only have preferences over consumption but also (implicitly) over their beliefs. Agents endogenously choose to disregard information contained in a signal if it conflicts with their desired beliefs. In this way, systematic overvaluation and undervaluation of shares arise, as well as regions of excessive price volatility and regions of excessive price stability. We find that the distortion of the asset pricing process is closely related to the precision of the information provided by the signal. The latter result might contribute to an understanding of the genesis of sunspots which occur in the limit when the signal becomes completely uninformative.

## 1 Introduction

Most economic analyses are based on the axiom that investors act rationally in the sense that they use all information available to them as long as this information is costless. However, sociologists and in particular psychologists claim that agents are not objective in processing and interpreting information. Rather information is selected and processed in such a way that earlier decisions tend to be confirmed. Information that could cause cognitive dissonance because it is somehow in conflict with previous decisions taken by the agent might simply be ignored or inaccurately interpreted in a more favorable light.<sup>1</sup>

Applied to the theory of asset pricing, modelling cognitive dissonance would mean that after having invested into a portfolio of assets the agents' perceptions of the economic environment become biased in favor of the investment decision just made. Thus an agent might possibly ignore adverse information about his asset portfolio – in particular if he is unable to process all the information which is available to him so that *some* selection of information will take place anyway.

In this paper, we will attempt to apply the cognitive dissonance approach to a theoretical model of asset pricing. The traditional behavioral approach to asset pricing leads to the Efficient Capital Markets Theory<sup>2</sup>, which says that the price of an asset is entirely determined by the investors' unbiased expectations of its fundamental value. For a share of common stocks, for instance, the fundamental value is the discounted future stream of dividends and earnings. According to the Efficient Capital Markets Hypothesis, all publicly available information is at any time reflected in the asset price. In other words, assets are priced correctly at all times.

Of course, this rules out persistent over- and undervaluation of stocks and other assets. Yet several empirical studies have found that assets are not priced as the theory would predict (e.g. the 'equity premium puzzle' of Mehra and Prescott [1985]).

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<sup>1</sup>The only formal application of the theory of cognitive dissonance that we are aware of is Akerlof and Dickens [1982]. This paper deals with the behavior of workers in hazardous industries.

<sup>2</sup>The concept of Efficient Capital Markets is reviewed in Hess and Reinganum [1979], West [1988] and LeRoy [1989].

Furthermore some studies have concluded that stock prices are excessively volatile. Dividends and earnings are much too smooth over time to explain the observed volatility of stock prices ('excess volatility puzzle', e.g. Shiller [1981]). Moreover stock market crashes are very difficult to reconcile with the Efficient Markets Hypothesis. The new information publicized just prior to stock market crashes does not usually warrant the huge drop in stock prices. All of these issues remain largely unresolved based on the current state of financial theory, which is almost entirely based on the rational behavior model where investors maximize exclusively their material wealth and regard their information sets as exogenous.

In recent years theorists have constructed models with naive agents (so-called noise traders or liquidity traders) coexisting alongside professional investors (De-Long et. al. [1990]; Campbell, Kyle [1993]). These naive traders are assumed to trade randomly, and they earn less than the market return. Such behavior seems hard to rationalize. Why do these agents trade if they make losses on average? Why don't they simply hold the market portfolio and increase their wealth? In the existing literature it is hard to understand the larger environment which would motivate these agents to participate in the market.

Our approach goes further, since in our model the investors' beliefs and information sets are to some extent endogenously determined, rather than exogenously given. We assume that investors are not as neutral and objective in processing and interpreting information as the traditional theory asserts. In particular, they like to view themselves as smart investors. Thus after having invested into a portfolio of assets they might ignore adverse information or they might interpret information in a more favorable light with respect to their current asset position. This type of behavior is found in some psychological studies. Knox and Inkster [1968] for instance observed that just after placing a bet at a race track people put much higher odds on 'their horse' than just prior to placing the bet. It is also well documented that the perceptions of people who have bought a new car typically become biased towards the advertisement of the car type they have purchased.

Our approach of modelling cognitive dissonance is based on a trade-off between the psychological costs of processing an unfavorable information signal and the

utility costs of ignoring the signal. Roughly speaking, an information signal is unfavorable if it strongly deviates from what the investor expected. Such a signal suggests that the investor's portfolio choice might have been based on an unwise prediction of the future market conditions. E.g. if an agent observes a signal which carries bad information about his asset portfolio (and adds the signal to his information set), then he is forced to revise downward his expected portfolio payoffs. In our approach the psychological costs of acknowledging the bad news is simply measured by the loss of utility resulting from the lower expected return on the portfolio. The agent can avoid these psychological costs if he disregards the information signal. In this case, however, he will be exploited to some extent by other agents who have included the signal into their information sets and can therefore trade on superior information. Depending on whether the psychological costs outweigh the utility costs of obtaining a suboptimal return, an agent will either ignore the information (to avoid cognitive dissonance) or heed the information.

The economy lasts for three periods and has two assets; one safe asset and shares of common stocks. At time 1, each investor (out of a continuum of identical investors) chooses an initial portfolio. Then at the beginning of time 2, a new piece of information about the shares' future payoff in period 3 is released. Now the investor faces the question of how to respond to this noisy signal. Should he include the signal into his information set and rearrange his portfolio? The range of realizations of the signal can be divided into five disjoint regions. Considering Nash equilibria, we find that if the realization is not too far from its *ex ante* expected value, then no one disregards the information. As the information becomes more unfavorable it will be disregarded by more and more investors and eventually it is ignored by everybody. But for sufficiently negative information some agents start heeding the signal again because they can exploit the fact that the shares are overpriced (judged on the basis of their superior information) and realize arbitrage profits that compensates them for the psychological costs of acknowledging the bad news. A similar situation occurs if a strongly positive information signal is released.

Finally we analyze how the information content of the signal affects the shape of the pricing function under cognitive dissonance. We find that the distortion of the pricing process is closely related to the precision of the information provided by the



signal. In the limit, as the information content of the signal goes to zero, the pricing process and the allocation in the economy depend on an irrelevant variable which is reminiscent of a sunspot variable in the rational expectations literature (See the extensive literature on sunspots, e.g. Azariadis [1981], Cass and Shell [1983]).

## 2 A Model of Cognitive Dissonance: Two Benchmark Cases

The economy lasts for three periods and is populated by a continuum of identical agents,  $I = [0, 1]$ , who start out with some initial endowment,  $e > 0$ , at time 1 and who want to consume only at time 3. Preferences are described by the utility function  $U(c) = -\exp(-\beta c)$ , where  $\beta$  is the constant degree of absolute risk aversion.  $c$  is the final wealth to be consumed by the agent at time 3 (we allow for negative values of  $c$ ). Two assets are available: a risky asset (shares), and a safe asset the price of which is normalized to one (numeraire). At time 1, each agent is endowed with one unit of the risky asset and  $e > 0$  units of the safe asset. The safe asset pays a fixed rate of return of  $r$  in periods 2 and 3, and is in perfectly elastic supply. The second asset pays a random dividend,  $\tilde{d} = \tilde{\gamma} + \tilde{\varepsilon}$ , in period 3.  $\tilde{\gamma}$  and  $\tilde{\varepsilon}$  are stochastically independent random variables with a multivariate normal distribution and  $E\{\tilde{\varepsilon}\} = 0$ . The realization of  $\tilde{\varepsilon}$  can be observed at time 2, while  $\tilde{\gamma}$  will not be revealed prior to time 3. Thus the agents get some information about the risky asset's payoff before the dividend,  $d$ , actually materializes. We denote the realizations of the random variables  $\tilde{\gamma}$  and  $\tilde{\varepsilon}$  by  $\gamma$  and  $\varepsilon$ . In light of the advance information he receives at time 2, an investor may rethink his portfolio decision made at time 1.

For any agent, the sequence of events can thus be summarized as follows:

- Period 1: the agent chooses a portfolio.
- Period 2: the agent observes the signal  $\varepsilon$ . He then decides to heed the information and to reshape his portfolio, or he decides to disregard the new information to avoid cognitive dissonance.
- Period 3: the agent consumes all asset payoffs.

In period 1, an agent can invest his endowment of  $e$  units of the safe asset and one unit of the risky asset into a portfolio consisting of both assets. His budget constraint in period 1 can thus be expressed as

$$e + p_1 = s_1 + p_1 \lambda_1. \quad (1)$$

$p_t$  is the price of a share at time  $t$ ;  $s_t$  denotes the units of the safe asset held at the end of period  $t$ ;  $\lambda_t$  denotes the number of shares held at the end of time  $t$ .

In period 2, the portfolio can be changed subject to the constraint

$$s_1(1 + r) + p_2 \lambda_1 = s_2 + p_2 \lambda_2. \quad (2)$$

Finally his wealth in period 3 is

$$c = (1 + r)[(1 + r)s_1 + p_2(\lambda_1 - \lambda_2)] + \lambda_2(\gamma + \varepsilon). \quad (3)$$

We will model cognitive dissonance in the following way. At the beginning of period 2, when the signal  $\varepsilon$  can be observed by investors, they might find themselves facing a psychological conflict. On the one hand agents want to feel as being smart investors; on the other hand a realization of the information  $\varepsilon$  that differs significantly from what the agents expected *ex ante* would indicate that they might have chosen the wrong portfolio at time 1. To resolve this internal conflict, investors might be prompted to disregard the signal  $\varepsilon$  if it conflicts too much with their self-image. Here we simply assume that the investor will disregard the new information if his unconditional expected utility (without the information of the signal) is at least as large as the expected utility conditional on the new information. In order to discuss the effect of this kind of cognitive dissonance on asset prices, we will first consider two benchmark cases before we turn to a full-blown description of the equilibrium concept.

#### Benchmark case 1: All agents heed the information signal

We will use a backward solution method to characterize the equilibrium asset prices for the case of fully rational investment behavior. To that end the problem is divided into two stages.

Stage I: Suppose an agent has already chosen his optimal portfolio  $(s_1^*, \lambda_1^*)$  at time 1. Now, at time 2, the investor maximizes

$$\max_{\lambda_2} E_{\tilde{\gamma}} \left\{ U \left( (1+r) \left[ (1+r)s_1^* + p_2(\lambda_1^* - \lambda_2) \right] + \lambda_2(\tilde{\gamma} + \varepsilon) \right) \right\}.$$

The first order condition

$$E_{\tilde{\gamma}} \left\{ U'(c) [-(1+r)p_2 + (\tilde{\gamma} + \varepsilon)] \right\} = 0$$

implies<sup>3</sup>

$$p_2^*(\varepsilon) = \frac{E_{\tilde{\gamma}} \{ U'(c)(\tilde{\gamma} + \varepsilon) \}}{E_{\tilde{\gamma}} \{ U'(c) \} (1+r)}. \quad (4)$$

Note that the expectation is only with respect to  $\tilde{\gamma}$  since  $\varepsilon$  has already been observed.

Since *all* agents behave rational, in equilibrium  $s_1^* = e$  has to hold on the bond market and  $\lambda_1^* = \lambda_2^* = 1$  has to hold on the stock market; hence by (2) the equality  $s_2^* = (1+r)s_1^*$  is satisfied. The equilibrium stock price at time 2 is thus

$$p_2^*(\varepsilon) = \frac{E_{\tilde{\gamma}} \{ U'(c)(\tilde{\gamma} + \varepsilon) \}}{E_{\tilde{\gamma}} \{ U'(c) \} (1+r)} = \frac{1}{1+r} \left[ \varepsilon + E\{\tilde{\gamma}\} + \frac{\text{cov}[U'(c), \tilde{\gamma}]}{E_{\tilde{\gamma}} \{ U'(c) \}} \right] \quad (5)$$

with  $c = (1+r)^2 e + \tilde{\gamma} + \varepsilon$ .

Stage II: Working backwards to period 1, each agent chooses an initial portfolio subject to the expectation of the time 2 equilibrium price function  $p_2^*(\varepsilon)$ . He solves the problem

$$\max_{s_1, \lambda_1} E_{\tilde{\gamma}, \tilde{\varepsilon}} \left\{ U \left( (1+r)^2 s_1 + (1+r)p_2^*(\tilde{\varepsilon})(\lambda_1 - 1) + (\tilde{\gamma} + \tilde{\varepsilon}) \right) \right\}$$

$$\text{s.t. } e + p_1 = s_1 + p_1 \lambda_1,$$

where the expectation is with respect to  $\tilde{\gamma}$  and  $\tilde{\varepsilon}$ . A first-order condition for a maximum is

$$E_{\tilde{\gamma}, \tilde{\varepsilon}} \left\{ U'(c) [-(1+r)^2 p_1 + (1+r)p_2^*(\tilde{\varepsilon})] \right\} = 0$$

and we obtain the equilibrium price at time 1

$$p_1^* = \frac{E_{\tilde{\gamma}, \tilde{\varepsilon}} \{ U'(c)p_2^*(\tilde{\varepsilon}) \}}{E_{\tilde{\gamma}, \tilde{\varepsilon}} \{ U'(c) \} (1+r)} = \frac{E\{p_2^*(\tilde{\varepsilon})\}}{1+r} + \frac{\text{cov}[U'(c), p_2^*(\tilde{\varepsilon})]}{E_{\tilde{\gamma}, \tilde{\varepsilon}} \{ U'(c) \} (1+r)}. \quad (6)$$

---

<sup>3</sup>Henceforth we will indicate efficient prices with an upper index \*. These are market clearing prices provided that (almost) all agents heed the information signal  $\varepsilon$  revealed at date 1.

The stock price at time 1 is lower than the discounted expected stock price at time 2, since  $\text{cov}[U'(c), p_2^*(\tilde{\varepsilon})]$  is negative.

Benchmark case 2: All agents ignore the information signal

Again we proceed backwards. A typical agent has chosen his initial portfolio at date 1 (of course in equilibrium  $s_1^* = e$  and  $\lambda_1^* = 1$  has to be satisfied because all agents hold the same portfolio at that time). At the beginning of period 2, the signal  $\varepsilon$  is realized, but now the investor decides to disregard the information. He will then maximize

$$\max_{\lambda_2} E_{\tilde{\gamma}, \tilde{\varepsilon}} \left\{ U \left( (1+r) \left[ (1+r)s_1^* + p_2(\lambda_1^* - \lambda_2) \right] + \lambda_2(\tilde{\gamma} + \tilde{\varepsilon}) \right) \right\}. \quad (7)$$

The first-order condition implies

$$\hat{p}_2(\varepsilon) = \bar{p} = \frac{E_{\tilde{\gamma}, \tilde{\varepsilon}} \{ U'(c)(\tilde{\gamma} + \tilde{\varepsilon}) \}}{E_{\tilde{\gamma}, \tilde{\varepsilon}} \{ U'(c) \} (1+r)} \quad c = (1+r)^2 e + \tilde{\gamma} + \tilde{\varepsilon}. \quad (8)$$

Since no new information enters into the individual portfolio decision between time 1 and time 2,  $\hat{p}_1 = \bar{p}$  holds.

### 3 A Model of Cognitive Dissonance

We now turn to a model in which each investor individually decides whether or not to heed the information signal. Here we apply the simplest method. If the investor achieves a higher expected utility at time 2 by ignoring the signal, he will choose to do so. Otherwise he will act upon the information revealed by the signal  $\varepsilon$ .

Suppose an investor heeds the information  $\varepsilon$  and observes the asset price  $p$  at time 2.<sup>4</sup> Since in equilibrium all agents hold the same asset position at date 1, his current portfolio consists of  $e$  bonds and one share, i.e.  $s_1 = e$ ,  $\lambda_1 = 1$ . Let the value function at time 2,  $\Psi(p, \varepsilon)$ , be defined by

$$\Psi(p, \varepsilon) := \max_{\lambda_2} E_{\tilde{\gamma}} \left\{ U \left( (1+r) \left[ (1+r)e + p(1 - \lambda_2) \right] + \lambda_2(\tilde{\gamma} + \varepsilon) \right) \right\} \quad (9)$$

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<sup>4</sup>From now on we will suppress the time index, since the subsequent analysis focuses exclusively on the asset price *at time 2* when the market has received new information.

and denote the solution to the problem by  $\lambda_2^H(p, \varepsilon)$ ; it is the per capita demand for the risky asset when heeding the information  $\varepsilon$ .<sup>5</sup>

Suppose the investor disregards the information  $\varepsilon$ , but observes the asset price  $p$ . In this case his value function  $\Phi(p)$  is defined as

$$\Phi(p) := \max_{\lambda_2} E_{\tilde{\gamma}, \tilde{\varepsilon}} \left\{ U \left( (1+r)[(1+r)e + p(1-\lambda_2)] + \lambda_2(\tilde{\gamma} + \tilde{\varepsilon}) \right) \right\} \quad (10)$$

It is the maximum expected utility at price  $p$  when the signal is ignored. The demand for the risky asset in period 2 is in this case denoted by  $\lambda_2^D(p)$ .

**Definition 3.1 (equilibrium in pure or mixed strategies)** *Consider a realization of information  $\varepsilon'$ .*

- a)  $\left\{ \lambda_2^H(p^*(\varepsilon'), \varepsilon'), p^*(\varepsilon') \right\}$  is a pure-strategy Nash equilibrium at  $\varepsilon'$  if
- (i)  $p^*(\varepsilon')$  is such that  $\lambda_2^H(p^*(\varepsilon'), \varepsilon') = 1$ ;
  - (ii)  $\Psi(p^*(\varepsilon'), \varepsilon') \geq \Phi(p^*(\varepsilon'))$
- b)  $\left\{ \lambda_2^D(\bar{p}), \bar{p} \right\}$  is a pure-strategy Nash equilibrium at  $\varepsilon'$  if
- (i)  $\bar{p}$  is such that  $\lambda_2^D(\bar{p}) = 1$ ;
  - (ii)  $\Phi(\bar{p}) \geq \Psi(\bar{p}, \varepsilon')$
- c)  $\left\{ \lambda_2^H(\hat{p}(\varepsilon'), \varepsilon'), \lambda_2^D(\hat{p}(\varepsilon')), \hat{p}(\varepsilon') \right\}$  is a mixed-strategy Nash equilibrium at  $\varepsilon'$  if
- (i)  $\exists \mu \in (0, 1)$  such that  $\mu \lambda_2^H(\hat{p}(\varepsilon'), \varepsilon') + (1-\mu) \lambda_2^D(\hat{p}(\varepsilon')) = 1$
  - (ii)  $\Psi(\hat{p}(\varepsilon'), \varepsilon') = \Phi(\hat{p}(\varepsilon'))$

The conditions (ii) in parts (a), (b) and (c) of this definition ensure that in equilibrium an agent will disregard the signal only if he can achieve a higher welfare by ignoring the information signal than by observing and processing it.

First we will prove some properties of the value functions which we will use in the next subsection. (See Figure1).

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<sup>5</sup>The superscript 'H' stands for 'heeding the information'; 'D' will stand for 'disregarding the information'.

**Proposition 3.1** *The value functions  $\Psi(\cdot)$  and  $\Phi(\cdot)$  possess the following properties:*

1. *Let  $p^*(\varepsilon)$  be a pure-strategy equilibrium asset price for the realization  $\varepsilon$  and assume that all agents heed the information. Then, as a function of  $p$ , the value function,  $\Psi(p, \varepsilon)$ , possesses a global minimum at  $p = p^*(\varepsilon)$  and is strictly monotone on  $(-\infty, p^*(\varepsilon))$  and on  $(p^*(\varepsilon), \infty)$ .*
2. *Let  $\bar{p}$  be a pure-strategy equilibrium asset price when all agents ignore the signal  $\varepsilon$ . Then the value function  $\Phi(p)$  possesses a global minimum at  $p = \bar{p}$  and is strictly monotone on  $(-\infty, \bar{p})$  and on  $(\bar{p}, \infty)$ .*

**Proof:** Suppose without loss of generality that the agents observe the signal  $\varepsilon$  (if the signal is ignored we get the same result by a symmetric argument).

Step 1:  $\lambda_2^H$  satisfies the first-order condition

$$E_{\tilde{\gamma}} \left\{ U'(\cdot) [-(1+r)p^*(\varepsilon) + (\tilde{\gamma} + \varepsilon)] \right\} = 0, \quad (11)$$

from which we derive

$$\begin{aligned} \left. \frac{\partial \lambda_2^H}{\partial p} \right|_{p=p^*(\varepsilon)} &= \frac{E_{\tilde{\gamma}} \{ U''(\cdot) (1+r) (1 - \lambda_2^H) [-(1+r)p^*(\varepsilon) + (\tilde{\gamma} + \varepsilon)] + (1+r) U'(\cdot) \}}{E_{\tilde{\gamma}} \{ U''(\cdot) [\tilde{\gamma} + \varepsilon - p^*(\varepsilon)(1+r)]^2 \}} \\ &= \frac{(1+r) E_{\tilde{\gamma}} \{ U'(\cdot) \}}{E_{\tilde{\gamma}} \{ U''(\cdot) [\tilde{\gamma} + \varepsilon - p^*(\varepsilon)(1+r)]^2 \}} < 0, \end{aligned} \quad (12)$$

since  $\lambda_2^H = 1$  holds in equilibrium. In fact, the above inequality is valid for any  $p$  which satisfies  $\lambda_2^H(p, \varepsilon) = 1$ , and thus we conclude

$$\lambda_2^H(p, \varepsilon) \stackrel{(<)}{>} 1 \iff p \stackrel{(>)}{<} p^*(\varepsilon). \quad (13)$$

Step 2: Differentiation of the value function with respect to  $p$  yields

$$\frac{\partial \Psi(p, \varepsilon)}{\partial p} = E_{\tilde{\gamma}} \left\{ U'(\cdot) (1+r) [1 - \lambda_2^H(p, \varepsilon)] \right\}. \quad (14)$$

Combining (13) and (14), we get

$$\frac{\partial \Psi(p, \varepsilon)}{\partial p} \begin{cases} < 0; & p < p(\varepsilon) \\ = 0; & p = p(\varepsilon) \\ > 0; & p > p(\varepsilon) \end{cases}$$

which proves the claim of the proposition. ■

In order to characterize the equilibrium price function, recall that individual preferences are given by  $U(c) = -\exp(-\beta c)$  and that  $\tilde{\gamma}$  and  $\tilde{\varepsilon}$  have a multivariate normal distribution. Thus, if an agent observes the signal  $\varepsilon$ , his expected utility at time 2 is

$$\begin{aligned} E_{\tilde{\gamma}}\{U(c)\} &= E_{\tilde{\gamma}}\left\{U\left([1+r]s_2 + [\tilde{\gamma} + \varepsilon]\lambda_2\right)\right\} \\ &= -\exp\left(-\beta\left[(1+r)s_2 + \lambda_2(\varepsilon + E\{\tilde{\gamma}\}) - \frac{\beta}{2}(\lambda_2)^2\sigma_\gamma\right]\right) \\ &= -\exp\left(-\beta\left[\bar{e} + (1+r)p + \lambda_2(\varepsilon + E\{\tilde{\gamma}\} - (1+r)p) - \frac{\beta}{2}(\lambda_2)^2\sigma_\gamma\right]\right), \end{aligned} \quad (15)$$

where  $\sigma_\gamma := \text{var}(\tilde{\gamma})$ . If the agent does not observe the signal  $\varepsilon$ , his expected utility at time 2 is

$$\begin{aligned} E_{\tilde{\gamma}, \tilde{\varepsilon}}\{U(c)\} &= \\ &= -\exp\left(-\beta\left[\bar{e} + (1+r)p + \lambda_2(E\{\tilde{\gamma}\} - (1+r)p) - \frac{\beta}{2}(\lambda_2)^2(\sigma_\gamma + \sigma_\varepsilon)\right]\right), \end{aligned} \quad (16)$$

where  $\sigma_\varepsilon := \text{var}(\tilde{\varepsilon})$ . The first order necessary conditions are

$$\varepsilon + E\{\tilde{\gamma}\} - (1+r)p = \beta\lambda_2\sigma_\gamma \quad (17)$$

and

$$E\{\tilde{\gamma}\} - (1+r)p = \beta\lambda_2(\sigma_\gamma + \sigma_\varepsilon). \quad (18)$$

Thus

$$\lambda_2^H(\varepsilon, p) = \frac{\varepsilon + E\{\tilde{\gamma}\} - (1+r)p}{\beta\sigma_\gamma} \quad (19)$$

$$\lambda_2^D(p) = \frac{E\{\tilde{\gamma}\} - (1+r)p}{\beta(\sigma_\gamma + \sigma_\varepsilon)}. \quad (20)$$

It is useful to divide the range of the random variable  $\tilde{\varepsilon}$  into five intervals,  $(-\infty, \varepsilon_3)$ ,  $[\varepsilon_3, \varepsilon_2]$ ,  $(\varepsilon_2, \varepsilon_1)$ ,  $[\varepsilon_1, \varepsilon_0]$  and  $(\varepsilon_0, \infty)$ , with  $\varepsilon_3 < \varepsilon_2 < \varepsilon_1 < 0 < \varepsilon_0$  (cf. Figure ?). Each interval is characterized by a specific price formation mechanism resulting from the way in which agents treat the new information.

### 3.1 The Equilibrium Pricing Function

In the following we will derive the equilibrium price function  $p(\varepsilon)$  on the above mentioned intervals.

#### 3.1.1 The Pricing Function in the Interval $[\varepsilon_1, \varepsilon_0]$

We will show that for any signal  $\varepsilon \in [\varepsilon_1, \varepsilon_0]$  all agents heed the information and that the equilibrium asset price is the efficient price  $p^*(\varepsilon)$ . In Section 2 we derived the efficient pricing function  $p^*(\varepsilon)$ ; i.e. the market clearing stock price at date 2 as a function of the information  $\varepsilon$  if (almost) all agents heed the information. By (19),

$$p^*(\varepsilon) = \frac{\varepsilon + E\{\tilde{\gamma}\} - \beta\sigma_\gamma}{1+r}. \quad (21)$$

Suppose the signal  $\varepsilon = 0$  realizes and the shares are valued efficiently at  $p^*(0)$ . Then each individual investor will take the information into account, because  $c^D = (1+r)s_2 + (\tilde{\gamma} + \tilde{\varepsilon})\lambda_2$  is dominated by  $c^H = (1+r)s_2 + \tilde{\gamma}\lambda_2$  in the sense of Second Order Stochastic Dominance for all  $(\lambda_2, s_2)$  implying that the maximum value of (15) is larger than the maximum value of (16) (recall that  $\tilde{\gamma}$  and  $\tilde{\varepsilon}$  are stochastically independent). Thus the market clears at the efficient price  $p^*(0)$ .

Let  $\varepsilon$  be a realization of  $\tilde{\varepsilon}$  such that an agent is indifferent between heeding and disregarding the information  $\varepsilon$ , if the shares are valued efficiently, i.e.  $p(\varepsilon) = p^*(\varepsilon)$ . To determine the value(s) of  $\varepsilon$ , note that an agent's expected utility is

$$- \exp \left( -\beta \left[ \bar{e} + (1+r)p + \frac{[\varepsilon + E\{\tilde{\gamma}\} - (1+r)p]^2}{2\beta\sigma_\gamma} \right] \right), \quad (22)$$

if he heeds the information, while the expected utility is

$$- \exp \left( -\beta \left[ \bar{e} + (1+r)p + \frac{[E\{\tilde{\gamma}\} - (1+r)p]^2}{2\beta(\sigma_\gamma + \sigma_\varepsilon)} \right] \right), \quad (23)$$



if the information is ignored. At  $\varepsilon$ , (22) and (23) must be equal under efficient asset pricing, which implies

$$(\sigma_\gamma + \sigma_\varepsilon)[\varepsilon + E\{\tilde{\gamma}\} - (1+r)p^*(\varepsilon)]^2 = \sigma_\gamma[E\{\tilde{\gamma}\} - (1+r)p^*(\varepsilon)]^2. \quad (24)$$

Using (17) we get

$$E\{\tilde{\gamma}\} - (1+r)p^*(\varepsilon) = \beta\sigma_\gamma - \varepsilon.$$

Equation (24) then becomes

$$(\sigma_\gamma + \sigma_\varepsilon)\beta^2\sigma_\gamma = (\beta\sigma_\gamma - \varepsilon)^2.$$

Thus, if we denote the negative solution to the last equation by  $\varepsilon_1$  and the positive solution by  $\varepsilon_0$ , we get

$$\varepsilon_1 = \beta\sigma_\gamma \left[ 1 - \sqrt{1 + \frac{\sigma_\varepsilon}{\sigma_\gamma}} \right]; \quad \varepsilon_0 = \beta\sigma_\gamma \left[ 1 + \sqrt{1 + \frac{\sigma_\varepsilon}{\sigma_\gamma}} \right] \quad (25)$$

In the  $[\varepsilon_1, \varepsilon_0]$  region, agents base their investment decisions on all available information, and thus the risky asset is priced efficiently (see Figure 2). More precisely we have

**Proposition 3.2** *For any realization  $\varepsilon \in [\varepsilon_1, \varepsilon_0]$ , there exists a unique equilibrium which is of type (a) in Definition 3.1. By (21) the equilibrium price is*

$$p(\varepsilon) = \frac{\varepsilon + E\{\tilde{\gamma}\} - \beta\sigma_\gamma}{1+r}$$

and all agents will heed the signal.

**Proof:** First observe that according to (21), the efficient pricing function  $p^*(\varepsilon)$  has slope  $1/1+r$ ,

$$\frac{\partial p^*(\varepsilon)}{\partial \varepsilon} = \frac{1}{1+r}. \quad (26)$$

Step 1: By definition,  $\varepsilon_1$  and  $\varepsilon_0$  satisfy the equation

$$\begin{aligned} & \Phi(p^*(\varepsilon)) \\ & := E_{\tilde{\gamma}, \tilde{\varepsilon}} \left\{ -\exp \left( -\beta \left[ \bar{e} + p^*(\varepsilon)(1+r) [1 - \lambda_2^D(p^*(\varepsilon))] + \lambda_2^D(p^*(\varepsilon))(\tilde{\gamma} + \tilde{\varepsilon}) \right] \right) \right\} \\ & = E_{\tilde{\gamma}} \left\{ -\exp \left( -\beta [\bar{e} + \tilde{\gamma} + \varepsilon] \right) \right\} =: \Psi(p^*(\varepsilon), \varepsilon), \end{aligned} \quad (27)$$

and  $\Phi(p^*(\varepsilon)) \neq \Psi(p^*(\varepsilon), \varepsilon)$  holds  $\forall \varepsilon \in [\varepsilon_1, \varepsilon_0]$ . If we can show that the equality in (27) implies

$$\left. \frac{\partial \Phi(p^*(\varepsilon))}{\partial \varepsilon} \right|_{\varepsilon=\varepsilon_1} < \left. \frac{\partial \Psi(p^*(\varepsilon), \varepsilon)}{\partial \varepsilon} \right|_{\varepsilon=\varepsilon_1}$$

then  $\Psi(p^*(\varepsilon), \varepsilon) > \Phi(p^*(\varepsilon))$  follows for all  $\varepsilon \in (\varepsilon_1, \varepsilon_0)$  by a simple continuity argument. In other words, given the price  $p^*(\varepsilon)$ , the expected utility of an agent who ignores the signal,  $\Phi(p^*(\varepsilon))$ , would be less than the expected utility of an agent who heeds the information,  $\Psi(p^*(\varepsilon), \varepsilon)$ , for  $\varepsilon \in (\varepsilon_1, \varepsilon_0)$ . Thus,  $p(\varepsilon) := p^*(\varepsilon)$  would be an equilibrium price and all agents would heed the signal, as is claimed in the proposition. Since  $\lambda_2^D(p^*(\varepsilon_1))$  is strictly positive by equations (20), (21) and (25), the equality  $\Phi(p^*(\varepsilon_1)) = \Psi(p^*(\varepsilon_1), \varepsilon_1)$  implies

$$\begin{aligned} \left. \frac{\partial \Phi(p^*(\varepsilon))}{\partial \varepsilon} \right|_{\varepsilon=\varepsilon_1} &= -\Phi(p^*(\varepsilon_1)) \beta p'^*(\varepsilon_1) (1+r) \left[ 1 - \lambda_2^D(p^*(\varepsilon_1)) \right] \\ &= -\Phi(p^*(\varepsilon_1)) \beta \left[ 1 - \lambda_2^D(p^*(\varepsilon_1)) \right] \\ &< -\Psi(p^*(\varepsilon_1), \varepsilon_1) \beta = \left. \frac{\partial \Psi(p^*(\varepsilon), \varepsilon)}{\partial \varepsilon} \right|_{\varepsilon=\varepsilon_1} \end{aligned} \quad (28)$$

which remained to be shown. In the second equality we used the relation (26).

Step 2 (uniqueness): In view of (12) in the proof of Proposition 3.1,  $p > p^*(\varepsilon)$  cannot be an equilibrium price as the demand for stocks in period 2 would be less than 1 for all agents.  $p < p^*(\varepsilon)$  cannot be an equilibrium price either as it would imply by Proposition 3.1 that the expected utility of agents who heed the signal is higher than  $\Psi(p^*(\varepsilon), \varepsilon)$ , and the expected utility of agents who ignore the signal is less than  $\Phi(p^*(\varepsilon))$ . ■

### 3.1.2 The Pricing Function in the Interval $(\varepsilon_2, \varepsilon_1)$

In this interval, some agents ignore the information, while others heed it. Suppose all agents ignore the information signal  $\varepsilon$ . Market clearing then requires  $\lambda_2^D(p) = 1$ , and from (20) we obtain the market clearing price  $\hat{p}(\varepsilon)$  as

$$\hat{p}(\varepsilon) = \bar{p} := \frac{1}{1+r} \left[ E\{\tilde{\gamma}\} - \beta(\sigma_\gamma + \sigma_\varepsilon) \right]. \quad (29)$$

Let  $\varepsilon$  be a realization of  $\tilde{\varepsilon}$  such that an agent is indifferent between heeding and disregarding the information  $\varepsilon$ , if the shares are valued at  $\bar{p}$ . Equating (22) and (23), using the definition of  $\bar{p}$  and solving for  $\varepsilon$ , gives two solutions

$$\varepsilon_2 = -\beta(\sigma_\gamma + \sigma_\varepsilon) \left[ 1 - \sqrt{\frac{\sigma_\gamma}{\sigma_\gamma + \sigma_\varepsilon}} \right]$$

and

$$\varepsilon_3 = -\beta(\sigma_\gamma + \sigma_\varepsilon) \left[ 1 + \sqrt{\frac{\sigma_\gamma}{\sigma_\gamma + \sigma_\varepsilon}} \right]$$

For signals  $\varepsilon$  in the interval  $(\varepsilon_2, \varepsilon_1)$  (see Figure 2), there does not exist a pure-strategy Nash equilibrium as both  $\Psi(p^*(\varepsilon), \varepsilon) < \Phi(p^*(\varepsilon))$  and  $\Psi(\bar{p}, \varepsilon) > \Phi(\bar{p})$  hold simultaneously. If all other agents behave rationally, then ignoring the signal is an optimal response, while it would be optimal for an agent to heed the information if all other agents ignore it. But there exist mixed-strategy Nash equilibria where one group of agents heeds the signal  $\varepsilon$ , while the other group ignores the information (equilibria of type (c) in Definition 3.1). Investors self-select endogenously into the two groups so that there is a unique market-clearing price  $p(\varepsilon')$  for  $\varepsilon' \in (\varepsilon_2, \varepsilon_1)$  at which they are indifferent between ignoring the information and heeding it (see point *A* in Figure 1 where  $\Psi(p(\varepsilon'), \varepsilon') = \Phi(p(\varepsilon'))$  holds). For investors who observe  $\varepsilon'$ , shares look relatively cheap at the price  $p(\varepsilon')$  (since  $p(\varepsilon') < p^*(\varepsilon')$ ), hence they hold more than one share. Investors who ignore the information  $\varepsilon'$  hold less than one share. The equilibrium fraction of investors who ignore the information is thus endogenously determined to achieve market clearing.

These mixed-strategy equilibria are stable. To see this, denote the fraction of agents who ignore  $\varepsilon'$  in equilibrium by  $\mu(\varepsilon')$ . Suppose more than  $\mu(\varepsilon')$  agents ignore  $\varepsilon'$ . Do observing agents now have an incentive to ignore  $\varepsilon'$ , so that we would move further away from the equilibrium fraction  $\mu(\varepsilon')$ ? Or do ignoring agents now have an incentive to heed  $\varepsilon'$ ?

If more than  $\mu(\varepsilon')$  investors ignore  $\varepsilon'$ , the market clearing price will be at  $p' < p(\varepsilon')$ . At  $p'$  (see points *B* and *C* in Figure 1), agents are better off observing  $\varepsilon'$ . Thus the out-of-equilibrium dynamics lead back to the equilibrium fraction  $\mu(\varepsilon')$ ; and the mixed-strategy equilibrium is thus stable.<sup>6</sup>

<sup>6</sup>According to Figure 1, there is another asset price  $p''(\varepsilon')$  such that  $\Psi(p''(\varepsilon'), \varepsilon') = \Phi(p''(\varepsilon'))$ .

For any  $\varepsilon \in (\varepsilon_2, \varepsilon_1)$ , the equilibrium price is such that agents are indifferent between heeding  $\varepsilon$  and ignoring it; i.e.  $p(\varepsilon)$  is such that

$$(\sigma_\gamma + \sigma_\varepsilon)[\varepsilon + E\{\tilde{\gamma}\} - (1+r)p(\varepsilon)]^2 = \sigma_\gamma[E\{\tilde{\gamma}\} - (1+r)p(\varepsilon)]^2 \quad (30)$$

holds. Differentiation of (30) yields

$$\frac{\partial p(\varepsilon)}{\partial \varepsilon} = \frac{1}{1+r} \left[ 1 - \frac{[E\{\tilde{\gamma}\} - (1+r)p(\varepsilon)]\sigma_\gamma}{[\varepsilon + E\{\tilde{\gamma}\} - (1+r)p(\varepsilon)](\sigma_\gamma + \sigma_\varepsilon)} \right]^{-1}. \quad (31)$$

Combining (30) and (31), we get

$$\frac{\partial p(\varepsilon)}{\partial \varepsilon} = \frac{1}{1+r} \left[ 1 - \sqrt{\frac{\sigma_\gamma}{\sigma_\gamma + \sigma_\varepsilon}} \right]^{-1} > \frac{1}{1+r}. \quad (32)$$

Thus the equilibrium pricing function is linear between  $\varepsilon_2$  and  $\varepsilon_1$ , and steeper than the pricing function for  $\varepsilon \in [\varepsilon_1, \varepsilon_0]$  (see Figure 2).

Why is the equilibrium pricing function in this region steeper than the efficient pricing function without cognitive dissonance? Agents who heed the information find shares less risky than the agents who ignore the signal. As a consequence, they hold more shares in their portfolios, i.e.  $\lambda_2^H > 1$ . Thus rational agents buy shares, and quasi-rational agents – those who ignore the information – sell shares, which implies that an increase in the stock price will hurt rational agents (whose share purchases become more expensive) and benefit quasi-rational agents (who collect larger revenues from their sale of shares). Now fix some  $\varepsilon \in [\varepsilon_2, \varepsilon_1]$ . At the price  $p^*(\varepsilon)$  agents would prefer to ignore the information. Thus a lower price than  $p^*(\varepsilon)$  is needed to equate the welfare of rational and quasi-rational investors. Critical for this argument is the observation that rational agents hold more shares in their portfolios than quasi-rational agents. Quasi-rational investors face a more uncertain environment and thus prefer relatively safe asset portfolios.

### 3.1.3 The Pricing Function in the Interval $[\varepsilon_3, \varepsilon_2]$

Any signal  $\varepsilon \in [\varepsilon_3, \varepsilon_2]$  will be ignored by all agents, since by construction of this interval we have  $\Phi(\bar{p}) \geq \Psi(\bar{p}, \varepsilon)$ . By an argument symmetric to the one used in

But  $p''(\varepsilon') > p(\varepsilon') > \bar{p}$ , thus the per-capita asset demand at the price  $p''(\varepsilon')$  is less than 1 for both groups of agents, contradicting market clearing.

Section 3.1.1,  $\bar{p}$  (defined in (29)) is the unique equilibrium price for any realization of  $\varepsilon \in [\varepsilon_3, \varepsilon_2]$ . At  $\varepsilon_3$  agents are again indifferent between ignoring the information and heeding it. The efficient pricing function  $p^*(\varepsilon)$  intersects the equilibrium price  $p^*(\hat{\varepsilon}) = \bar{p}$  at  $\hat{\varepsilon} := -\beta\sigma_\varepsilon$  (see Figure 2). For  $\varepsilon > \hat{\varepsilon}$ , the risky asset is undervalued and for  $\varepsilon < \hat{\varepsilon}$  the risky asset is overvalued (relative to the pricing process in an efficient market).

### 3.1.4 The Pricing Function in the Intervals $(-\infty, \varepsilon_3)$ and $(\varepsilon_0, \infty)$

The situation in these regions is similar to the case analyzed in Section 3.1.2. Again a pure-strategy Nash equilibrium does not exist as we have  $\Psi(p^*(\varepsilon), \varepsilon) < \Phi(p^*(\varepsilon))$  and  $\Psi(\bar{p}, \varepsilon) > \Phi(\bar{p})$  for all  $\varepsilon \in (-\infty, \varepsilon_3) \cup (\varepsilon_0, \infty)$ . The unique equilibrium price which supports a mixed-strategy Nash-equilibrium will equate the welfare of rational and quasi-rational agents. Applying the same procedure as in Section 3.1.2 to the present case yields

$$\frac{\partial p(\varepsilon)}{\partial \varepsilon} = \frac{1}{1+r} \left[ 1 + \sqrt{\frac{\sigma_\gamma}{\sigma_\gamma + \sigma_\varepsilon}} \right]^{-1} < \frac{1}{1+r} \quad (33)$$

for  $\varepsilon \in (-\infty, \varepsilon_3) \cup (\varepsilon_0, \infty)$ .

Relative to the efficient price  $p^*(\varepsilon)$ , shares are overvalued in the region  $(-\infty, \varepsilon_3)$ . The risky asset holdings of the agents who heed the information are negative; these rational agents take larger short positions, the smaller is the observed  $\varepsilon$ - realization. Also note that  $p^*(\varepsilon_3)$  is smaller than  $\bar{p}$ , otherwise rational agents would hold more than 1 share at  $(\varepsilon_3, \bar{p})$  rather than a negative amount.

For realizations of  $\varepsilon < \varepsilon_3$ , some agents will switch to heeding the information because the asset price distortions are large enough so that the extra profit gained by rational agents compensates them for their psychological conflict in such a way that their expected utility at date 2 (despite taking the bad information into account) is equal to the expected utility of the naive agents who ignore  $\varepsilon$ . Ex post (at date 3) the agents who heed the information are (on average) better off than the naive agents (abstracting from their disutility caused by cognitive dissonance). In other words, investors face a trade-off: By ignoring  $\varepsilon$ , agents are avoiding the disutility of

cognitive conflict, but they forego the bigger (average) wealth at date 3 from acting ‘rational’.

The asset pricing process in the region  $(\varepsilon_0, \infty)$  lends itself to an analogous interpretation. Here shares are undervalued relative to the efficient price  $p^*(\varepsilon)$  and the quasi-rational investors take short positions in risky assets. Since these agents do not acknowledge the positive information signal, from their perspective the shares appear to be expensive, and so they try to make some arbitrage profits by selling them short. In this region, acknowledging the strongly positive signal  $\varepsilon > \varepsilon_0$  causes cognitive dissonance because the signal suggests that the previously held portfolio contained too few shares.

As  $|\varepsilon|$  gets larger in the  $(-\infty, \varepsilon_3) \cup (\varepsilon_0, \infty)$  region, more agents will heed the information. Yet the share of rational agents is bounded above by  $1/2$ ; i.e. most of the agents choose to ignore the information irrespective of how strongly the released information deviates from their ex ante predictions and regardless of the price distortions thus created. The share  $\alpha(\varepsilon)$  of rational traders is determined by the equation

$$\alpha(\varepsilon)\lambda_2^H(\varepsilon, p(\varepsilon)) + [1 - \alpha(\varepsilon)]\lambda_2^D(p(\varepsilon)) = 1. \quad (34)$$

Using (19), (20) and (33) in (34),  $\alpha := \lim_{|\varepsilon| \rightarrow \infty} \alpha(\varepsilon)$  can be calculated as

$$\alpha = \left[ 1 + \frac{1}{\sqrt{1-I}} \right]^{-1} \in \left( 0, \frac{1}{2} \right), \quad (35)$$

where  $I := \sigma_\varepsilon / (\sigma_\varepsilon + \sigma_\gamma)$  measures the informativeness of the signal  $\varepsilon$ . The less informative the signal is, the less attention will it receive, i.e., the fewer agents will return eventually (as  $|\varepsilon| \rightarrow \infty$ ) to a rational assessment of their portfolio position.

It is interesting that quasi-rational agents do not eventually (for  $|\varepsilon| \rightarrow \infty$ ) disappear from the market despite the fact that they are taken advantage of by the rational investors. This is so because an agent has an incentive to behave rationally in the face of unfavorable information only if he can exploit the distorted price structure at the expense of quasi-rational agents. Thus the risky asset cannot be valued efficiently since an efficient price would eliminate any incentive to hold rational beliefs in the face of extremely bad news.

### 3.2 Changing the Informativeness of the Signal

The risky asset is efficiently priced, if the information released at date 2 is sufficiently close to its ex ante expected value. The asset is undervalued (and the associated risk overvalued) if the realization of the signal is between  $\hat{\varepsilon}$  and  $\varepsilon_1$  or if it is larger than  $\varepsilon_0$ ; and the asset is overvalued for signals less than  $\hat{\varepsilon}$ . Taking the efficient pricing function  $p^*(\varepsilon)$  as a reference point and measuring the (local) price volatility by the state derivative of the price function, we find excessive price volatility between  $\varepsilon_2$  and  $\varepsilon_1$ , while for signals less than  $\varepsilon_2$  or larger than  $\varepsilon_0$  asset prices are excessively stable.

The general shape of the equilibrium pricing function (with cognitive dissonance) depends critically on the informativeness of the signal  $\varepsilon$ . Here the informativeness,  $I$ , can be defined as  $I := \sigma_\varepsilon/\sigma$ . It is the portion of the overall uncertainty,  $\sigma := \sigma_\varepsilon + \sigma_\gamma$ , that is resolved at date 2. By holding  $\sigma_\varepsilon$  fixed and varying  $\sigma_\gamma$ , we can control for the informativeness of the signal. However, since the efficient pricing function  $p^*(\varepsilon)$  serves as a reference point in our analysis, it is expedient to perturb the distribution of  $\tilde{\gamma}$  in a way that leaves the position of  $p^*(\varepsilon)$  unchanged. In view of (21) this implies

$$\beta d\sigma_\gamma = dE\{\tilde{\gamma}\}.$$

We will model an increase in the informativeness of the signal by a decrease in  $\sigma_\gamma$  according to the above perturbation, i.e.,  $dI = (\partial I/\partial \sigma_\gamma)d\sigma_\gamma$ . Notice that this perturbation also leaves unchanged the values of  $\bar{p}$  and  $\hat{\varepsilon}$ . From (25) we derive

$$-\frac{\partial \varepsilon_2}{\partial I} = \frac{\partial \varepsilon_1}{\partial I} = -\beta \frac{\sigma}{I} \left[ 1 - \frac{1 + (1 - I)^{-1}}{2(1 - I)^{-1/2}} \right] > 0 \quad (36)$$

$$-\frac{\partial \varepsilon_3}{\partial I} = \frac{\partial \varepsilon_0}{\partial I} = -\beta \frac{\sigma}{I} \left[ 1 + \frac{1 + (1 - I)^{-1}}{2(1 - I)^{-1/2}} \right] < 0. \quad (37)$$

According to (36) and (37), the undervaluation regime  $(\hat{\varepsilon}, \varepsilon_1) \cup (\varepsilon_0, \infty)$  becomes smaller, and thus overpricing of risk less likely, the less informative the signal is. At the same time, the pricing function gets steeper in the interval between  $\varepsilon_2$  and  $\varepsilon_1$  so that the asset price becomes more volatile. In the limit, when  $I$  goes to zero,

$\varepsilon_0$  tends to  $\infty$ , and both  $\varepsilon_2$  and  $\varepsilon_1$  approach  $\hat{\varepsilon}/2$ :

$$\begin{aligned} \varepsilon_2|_{I \rightarrow 0} = \varepsilon_1|_{I \rightarrow 0} &= \lim_{\sigma_\gamma \rightarrow \infty} \beta \frac{1 - \left(1 + \frac{\sigma_\varepsilon}{\sigma_\gamma}\right)^{1/2}}{1/\sigma_\gamma} \\ &= \lim_{\sigma_\gamma \rightarrow \infty} -\frac{1}{2}\beta\sigma_\varepsilon \left(1 + \frac{\sigma_\varepsilon}{\sigma_\gamma}\right)^{-1/2} = -\frac{1}{2}\beta\sigma_\varepsilon = \frac{1}{2}\hat{\varepsilon}, \end{aligned}$$

where the third equality follows from an application of l'Hospital's rule.

With a large amount of unresolved uncertainty (i.e. a nearly irrelevant signal), the equilibrium price function will be nearly vertical in the neighborhood of  $\hat{\varepsilon}/2$ ; a situation which looks very much like a stock market crash. A small deterioration of the released information results in a huge reduction in the asset price. This price drop occurs although the information signal eliminates only a very small fraction of the overall uncertainty about the asset payoffs and, in this sense, is almost uninformative. Note that a crash can only occur in a small part of the undervaluation region that borders on the efficient pricing region. Thus a crash does not reflect a correction of overpriced assets. Rather the risky asset will be undervalued after the crash while it was priced efficiently just before the sudden price drop.

Turning to the sensitivity of the  $\varepsilon_3$  threshold, we find that the amount of negative news that induces some investors to return to an accurate assessment becomes larger (i.e.,  $\varepsilon_3$  moves to the left), the less informative the signal is (cf. eq. (37)). In addition, if the information content of the signal is smaller, the equilibrium pricing function becomes flatter for realizations less than  $\varepsilon_3$ . Thus, while the size of the overvaluation regime does not depend on the informativeness of the signal, the extent to which the risky asset is overpriced on average in the overvaluation region strictly decreases as the signal becomes more informative. Finally, it is immediate from the derived pricing formulas that  $I \rightarrow 1$  implies  $p(\varepsilon) \rightarrow p^*(\varepsilon) \forall \varepsilon$  from which we conclude that the distortion of the pricing process is more severe the less information is provided by the signal about the future asset payoffs.

Our analysis suggests that the price system may be significantly distorted if the market receives news that contain little information about asset fundamentals. Almost irrelevant news can lead to sudden declines in asset prices and to large devi-



ations of the market price from its fundamental value. In the rational expectations literature irrelevant news which do not contain any information about economic fundamentals are called ‘sunspots’. The choice-theoretic concept of cognitive dissonance therefore suggests a mechanism for the genesis of agents’ beliefs in the influence of sunspots on the price system and the allocation in an economy.

#### 4 Concluding Remarks

This paper contains a first attempt to apply the notion of cognitive dissonance to the valuation of assets. We think that this new approach to asset pricing has potential for generating new insights into the functioning of financial markets. Our results can briefly be summarized using the graphic representation in Figure 2: Viewed from a rational standpoint, the individual portfolio decisions (of some or all agents) are suboptimal in the region  $\mathbb{R} \setminus [\varepsilon_1, \varepsilon_0]$  as they are based on only part of the available information. The more significant (i.e. the more negative or the more positive) the ignored information is, the more distorted are the portfolios. As a consequence, the risky asset will be mispriced relative to the safe asset. If the extent to which the assets are mispriced becomes large enough, rational agents can exploit the distorted price structure and realize arbitrage profits. In the region to the left of  $\varepsilon_3$  the rational agents exploit the quasi-rational traders: they sell overpriced assets to them. The same is true in the region between  $\varepsilon_2$  and  $\varepsilon_1$  and between  $\varepsilon_0$  and  $\infty$ . Here the rational traders buy underpriced assets from the quasi-rational agents.

In our model, investors have rational expectations, but once they have made their investments their (utility-maximizing) beliefs change. This appears to be consistent with psychological research. Since the endogenous beliefs of the investors can lead to persistent over- and undervaluation of stocks, our theory could possibly explain the ‘equity premium puzzle’; i.e., the fact that over long time horizons the return on equity appears to be ‘too large’ relative to the return on safe government securities. Cognitive dissonance could also possibly generate more asset price volatility which is closer to the actually observed volatility on financial markets. It could thus explain parts of the ‘excess volatility puzzle’.

In further research, we hope to provide a theory of stock market crashes. This goal has been elusive for the traditional theory. According to our view, the mechanism would - loosely speaking - go like this: Once a critical mass of adverse information (which initially has been ignored by investors because of cognitive dissonance) has accumulated, investors suddenly realize their biased view and return to a more accurate assessment. This could trigger a sudden drop in stock prices.

And finally, this new view of investor behavior could also have important implications for the effective regulation of financial markets.

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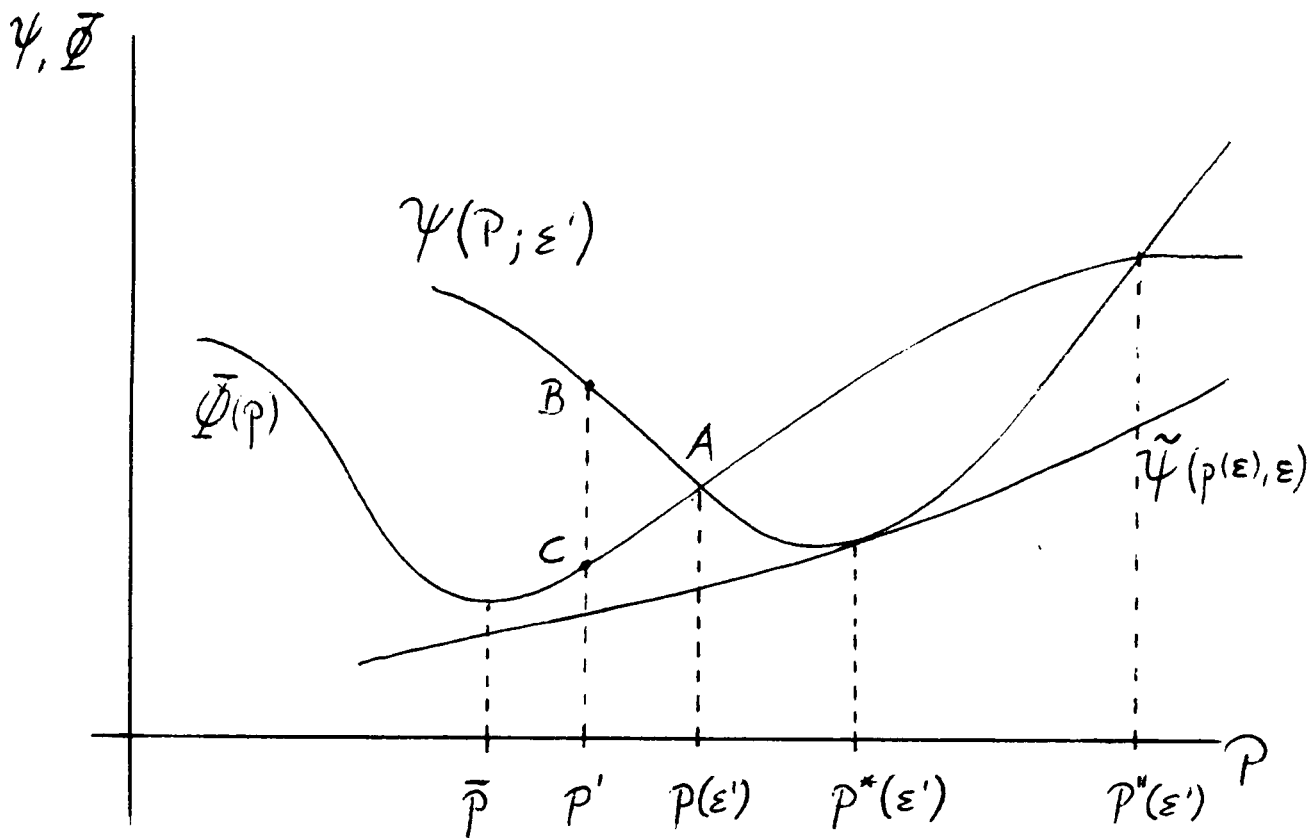


Figure 1

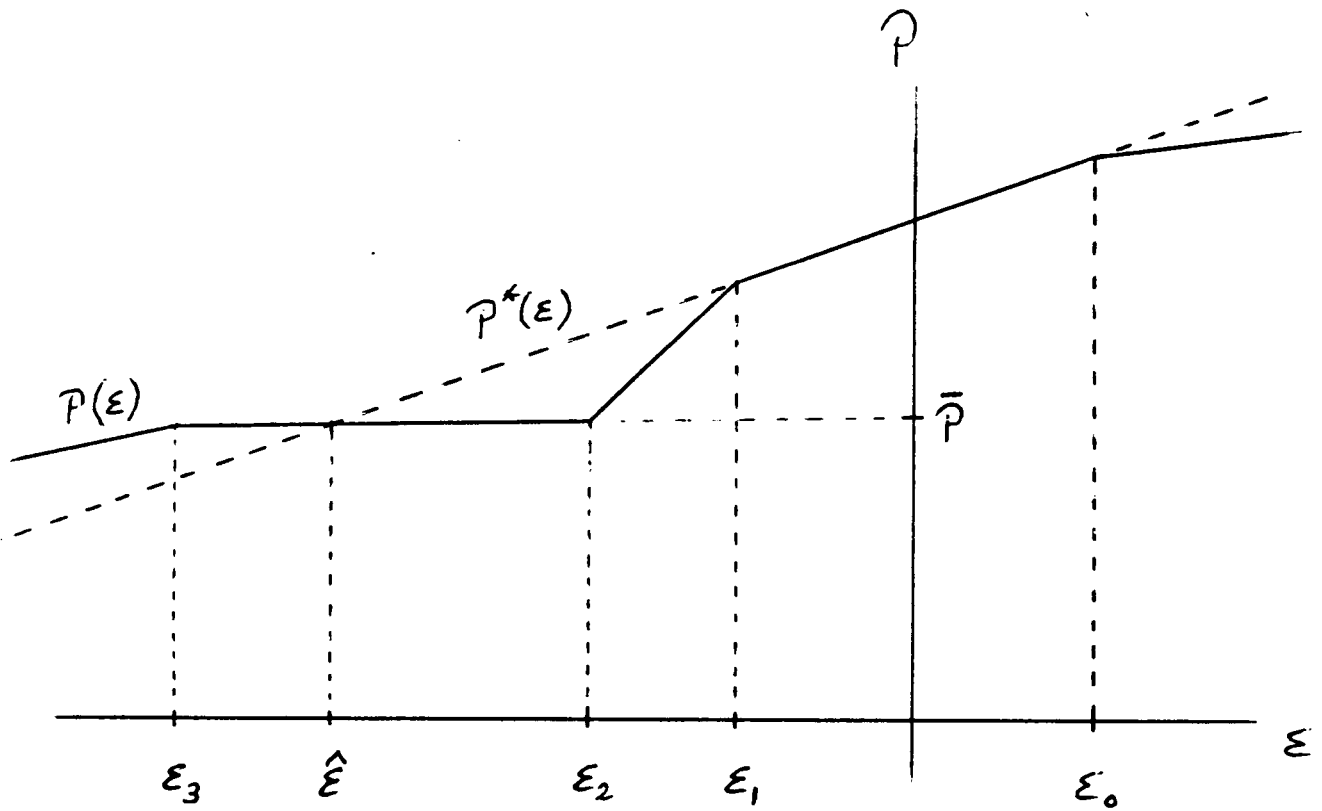


Figure 2