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Working Paper Factor price equalization under joint production: A general equilibrium synthesis

Diskussionsbeiträge - Serie II, No. 268

Provided in Cooperation with: Department of Economics, University of Konstanz

Suggested Citation: Albert, Max; Kohler, Wilhelm K. (1995) : Factor price equalization under joint production: A general equilibrium synthesis, Diskussionsbeiträge - Serie II, No. 268, Universität Konstanz, Sonderforschungsbereich 178 - Internationalisierung der Wirtschaft, Konstanz

This Version is available at: https://hdl.handle.net/10419/101662

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Sonderforschungsbereich 178 "Internationalisierung der Wirtschaft"

Diskussionsbeiträge



Juristische Fakultät

Fakultät für Wirtschaftswissenschaften und Statistik

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Factor Price Equalization under Joint Production: A General Equilibrium Synthesis

2 2. JUNI 1995 Weltwirtschaft W M3 (268) pr. br sig gla

Postfach 5560 D-78434 Konstanz Serie II — Nr. 268 Mai 1995

Factor Price Equalization under Joint Production:

A General Equilibrium Synthesis

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Serie II - Nr. 268

Mai 1995

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For hints and comments, we are grateful to Jürgen Meckl (Konstanz) and to an anonymous referee of an early version of the paper.

Abstract

We investigate how joint production affects the likelihood of factor price equalization (FPE) through trade. Following up on recent contributions by Samuelson (1992) and Jones (1992), we propose to take the relative size of the FPE region of endowment distributions to measure this likelihood. We show that the likelihood (1) may *increase* even if joint production leads to a *decrease* of the number of basic production processes, and (2) may *decrease* even if the number of basic production processes operated in equilibrium is *the same* as under non-joint production. Moreover, we reinterpret the formal arguments and apply them to a comparison between trade in intermediate versus trade in finished products. Our analysis throws light on the basic mechanism at work. It shows that goods trade and factor movements can be complements: Further integration of product markets can destroy FPE and create factor-price differences, thereby generating factor movements.

1 Introduction

Two recent papers by Samuelson (1992) and Jones (1992) have discussed the relevance of jointness in production for perhaps the most fundamental of all trade theory results which states that under certain conditions trade in goods alone will completely equalize factor returns across countries (FPE). Under these conditions, international factor mobility would simply be rendered unnecessary, from a welfare point of view, by international trade in goods. Under far less restrictive conditions trade can still be shown to narrow factor price gaps between countries (partial FPE), thus delivering at least part of the welfare gains that could otherwise only be achieved through factor movements. While the real-world importance of the fundamental issue has never been in doubt, the conditions under which complete FPE would emerge have been widely regarded as being too restrictive for the FPE theorem to be of any practical relevance. Less agreement exists, however, as to how certain modifications of the standard model affect the likelihood of FPE. And it is in this sense that Samuelson and Jones discuss the role of jointness in production (JP). The question is whether introducing JP into an otherwise orthodox trade model increases or decreases the likelihood of FPE. Samuelson concludes that the FPE result becomes less likely because under JP the number of profitable production processes tends to be lower. Jones argues that the latter is possible, but not inevitable. Assuming that the number of profitable processes is equal under JP and non-joint production (NJP), he arrives at the opposite conclusion if the likelihood of FPE is measured by the range of commodity price ratios that support a FPE equilibrium for given factor endowments.

On the one hand, this paper offers a general equilibrium synthesis of these two perspectives. Our aim is twofold. First, we suggest a different approach for measuring the likelihood of FPE which is based on the so-called FPE region, pioneered by Travis (1964) and rediscovered and popularized more recently by Dixit and Norman (1980). We thus place the question of FPE into a full general equilibrium setting. It turns out that the likelihood of FPE may increase or decrease, even under Jones' assumption that the number of independent production processes available under JP is the same as under NJP. Our second aim is to clarify the relationship between jointness in production and the number of processes used. In particular, we show that a reduction in the number of processes due to jointness in production may well lead to an increase in the likelihood of FPE as defined above.

On the other hand, we reinterpret the rather arcane question of comparing different technologies w.r.t. the likelihood of FPE. It can be shown that Jones' (1992) argument can be used to analyze the effects of trade in intermediate versus trade in finished products in the presence of joint production. While this serves to demonstrate that their is a real economic problem involved, it does not alter the main conclusions: The effect on the likelihood of FPE is ambiguous. However, this is now more interesting than before: As we will show, it implies that trade in goods and factor movements can be complements.

We proceed as follows. In section 2 we introduce our notation and restate the arguments of Samuelson and Jones. In section 3 we derive our general results. Section 4 considers the question of the likelihood of FPE in a situation where finished goods are traded in comparison with a situation where trade is in intermediate products. The appendices contain proofs and an example involving only two commodities, factors and countries $(2 \times 2 \times 2 \text{ case})$.

2 Joint Production and the Traditional FPE Theorem

The tradional FPE theorem, as stated by Samuelson (1953) and McKenzie (1955), exclusively looks at the production side. Consider an economy where *n* finished products $\boldsymbol{x} \in \mathbb{R}^n_+$ can be produced by *m* primary factors of production $\boldsymbol{v} \in \mathbb{R}^m_+$ with a convex NJP technology.¹ As usual, we may describe the set of feasible input-output combinations with the help of a function *g* by the condition $g(\boldsymbol{x}, \boldsymbol{v}) \leq 0$. The production equilibrium for given commodity prices $\boldsymbol{p} \in \mathbb{R}^n_+$ and given factor endowments \boldsymbol{v} maximizes the value of the total product. We thus summarize the production side of the economy by the revenue function

$$r(\boldsymbol{p}, \boldsymbol{v}) \stackrel{\text{def}}{=} \max_{\boldsymbol{x}} \left\{ \boldsymbol{p}^{\mathrm{T}} \boldsymbol{x} : g(\boldsymbol{x}, \boldsymbol{v}) \leq 0, \, \boldsymbol{x} \geq \boldsymbol{0} \right\}.$$
(1)

In the following we also make use of the dual definition of the revenue function:

$$r(\boldsymbol{p}, \boldsymbol{v}) \equiv \min_{\boldsymbol{w}} \left\{ \boldsymbol{w}^{\mathrm{T}} \boldsymbol{v} : \boldsymbol{b}(\boldsymbol{w}) \ge \boldsymbol{p}, \, \boldsymbol{w} \ge \boldsymbol{0} \right\}$$
(2)

Here $\boldsymbol{w} \in \mathbb{R}^m_+$ is the vector of factor prices and $\boldsymbol{b}(\boldsymbol{w}) \in \mathbb{R}^n_+$ is the vector of unit-cost functions for the *n* commodities. In order to avoid tedious qualifications, we assume that $\boldsymbol{b}'(\boldsymbol{w})$ is regular everywhere. The first-order conditions (FOC) derived from the minimization problem are necessary and sufficient since the technology is convex:

$$\begin{aligned} & \boldsymbol{b}(\boldsymbol{w}) \geq \boldsymbol{p}, \quad \boldsymbol{x} \geq \boldsymbol{0}, \quad (\boldsymbol{b}(\boldsymbol{w}) - \boldsymbol{p})^{\mathrm{T}} \boldsymbol{x} = \boldsymbol{0} \quad (\text{zero-profit conditions}) \\ & \mathrm{A} \boldsymbol{x} \leq \boldsymbol{v}, \quad \boldsymbol{w} \geq \boldsymbol{0}, \quad \boldsymbol{w}^{\mathrm{T}} (\mathrm{A} \boldsymbol{x} - \boldsymbol{v}) = \boldsymbol{0} \quad (\text{factor-market-clearing conditions}) \end{aligned}$$
(3)

 $A^{T} \stackrel{\text{def}}{=} \boldsymbol{b}'(\boldsymbol{w})$ denotes the matrix of input coefficients. The set

$$\mathcal{F}(\boldsymbol{p}) \stackrel{\text{def}}{=} \{ \boldsymbol{w} : \boldsymbol{b}(\boldsymbol{w}) \ge \boldsymbol{p}, \, b_j(\boldsymbol{w}) = p_j \text{ for some } j \}$$
(4)

of solutions to the zero-profit (ZP) conditions is called *factor-price frontier* (Samuelson 1962, Woodland 1977). Any $w^* \in \mathcal{F}(p)$ fixes an input coefficient matrix in the factor-market-clearing

(FMC) conditions. Moreover, it determines those production processes which can be operated without losses at \boldsymbol{w}^* . The non-negative linear combinations of the input coefficient vectors of the profitable production processes are said to form a *diversification cone* (McKenzie 1955). The dimension of the cone is the number of linearly independent input coefficient vectors forming the cone.² A solution to the FMC conditions compatible with \boldsymbol{w}^* exists if and only if the factor-endowment vector \boldsymbol{v} is in the diversification cone connected with \boldsymbol{w}^* . The traditional FPE theorem in McKenzie's (1955) version states that two countries show FPE if, for a *given* (\boldsymbol{p}) , there exists a factor-price vector $\boldsymbol{w}^* \in \mathcal{F}(\boldsymbol{p})$ such that the endowments of both countries are in the corresponding cone. Under this condition, FPE is not only *possible*, as follows from the definition of the cone, but also *inevitable*.

Although the FPE theorem is usually stated for models without JP, the argument above may be framed without explicit reference to either joint or non-joint production. This was pointed out very early on by McKenzie (1955), and more recently reemphasized by Chang, Ethier and Kemp (1980). However, more can be said if one looks at the demand side, as Samuelson (1992) does implicitly, or at goods prices, as Jones (1992) does.

If commodity prices change, the factor-price frontier shifts. Starting from a FPE equilibrium, FPE will be destroyed if the relevant cone in the FPE equilibrium has a dimension smaller than m, the number of factors of production. Therefore FPE, while certainly *possible* for any number of goods and factors, is considered *unlikely* if there are no cones of full dimension for given commodity prices.³ This is the thrust of Samuelson's (1992) argument. If there is some degree of JP, it may be unprofitable to use all the production processes available. Even if the number of goods produced is equal to the number of factors of production, the diversification cone *may* be of a dimension smaller than m since some of the goods can be produced jointly by one process.

However, as Jones (1992) argues, this is in no way *necessary*. In the extreme case, there can be *m* different production processes that, by itself, each produce all of the goods; and it may be profitable to use all of them. He proves for the 2×2 case that, under this condition, the set of commodity prices which are compatible with a FPE equilibrium for given national endowments is bigger for JP than NJP.

Three remarks on argument and counter-argument seem in order. First, both arguments are of course correct as far as they go. Second, the arguments both compare the likelihood, or structural stability, of FPE under different technologies. Obviously, the result depends on how we select these technologies. Neither Jones nor Samuelson refer to any potential real-world situation where such a comparison of different technologies might be relevant. Without such a background, it is difficult to make sense of any specific choice. We come back to this issue in section 4. Third, both arguments are based on the traditional FPE theorem which treats commodity prices as exogenous. However, in a full general equilibrium, commodity prices are endogenous. We therefore propose to discuss the comparison between JP and NJP in a full general equilibrium analysis. In doing so, we concentrate on the *modern* FPE theorem, as stated by by Dixit & Norman (1980). This not only serves to endogenize commodity prices, allowing us to readress the Jones argument from a different and, we believe, more satisfactory perspective. It also serves to shed more light on how the number of processes used in equilibrium is affected by jointness in production and how this, in turn, influences the FPE result.

3 Implications of the Modern FPE Theorem

3.1 The Likelihood of FPE

Our first point is that the modern FPE theorem suggests a specific way of measuring the likelihood of FPE which differs from the Jones criterion. Jones' way of looking at this likelihood amounts to an analysis of the robustness of FPE in the face of arbitrary price changes. However, there are no arbitrary price changes in a general equilibrium.⁴ The fundamental issue of FPE is whether, and if so to what extent, we need factor movements in addition to free trade in order to ensure efficient production in a world of *given tastes and technologies*. Thus one should endogenize commodity prices and identify the set of endowment distributions among countries which would support a free trade FPE equilibrium. The relative size of this so-called FPE region of endowment distributions is identified with the likelihood of FPE. This likelihood is a measure of how big international endowment disparities may be for them to be accommodated, in the sense of FPE, by free trade in goods alone.

The main point of the modern theorem is easily stated. As Dixit & Norman (1980) have shown most clearly, the FPE region can be computed by starting from the so-called *intergrated* world equilibrium (IWE), which is defined by free trade and free international factor mobility. Assume that all equilibria considered are unique. Then it is clear that an equilibrium with FPE is identical to an IWE. Since the IWE is unique, the FPE equilibrium is unique, too. In the IWE, and therefore in the FPE equilibrium, we have one set of equilibrium prices (p, w) and a corresponding diversification cone. If the number of goods produced in the IWE is equal to the number of factors, it is necessary and sufficient for FPE that the factor endowments of all countries are in *this* cone. Next we introduce the so-called factor box by assuming given world endowments. The FPE region then emerges as a parallelepiped in factors in the IWE, the condition that the sum of the national output vectors satisfies IWE demand places additional restrictions on the set of FPE endowment combinations, and the FPE region is, therefore, only a part of the parallelepiped. In what follows, we largely concentrate on the even case in which m = n. The volume of the FPE region is then given by the determinant of the matrix of the sectoral factor-input vectors.⁵ The volume of the box is given by the product of the factor endowments. Let X and V be diagonal matrices with main diagonals \boldsymbol{x} and \boldsymbol{v} respectively. AX then contains the sectoral factor inputs. Thus |AX| = |A| |X| is the volume of the parallelepiped, and |V| is the volume of the box. The likelihood of FPE is therefore given by

$$\mathbf{P}_{\mathbf{FPE}} \stackrel{\text{def}}{=} \frac{|\mathbf{X}|}{|\mathbf{V}|} \cdot |\mathbf{A}| . \tag{5}$$

This expression allows us to compare the likelihood of FPE under different technologies (JP and NJP) in the even case. It is clear that the case of n < m is of no interest to us, since it implies a zero P_{FPE} in any case. The case where n > m will, however, be dealt with below. A final point to notice straight away is that P_{FPE} varies with commodity prices and that this relationship is non-monotonic in the following sense: If one varies one single price, there are only finite intervals for this price in which all of the n goods are produced. At the bounds of these intervals and outside of them P_{FPE} is zero, inside P_{FPE} is positive. Thus, starting from a diversified equilibrium with $P_{FPE} > 0$, it will almost always depend on the direction of a price change whether P_{FPE} increases or decreases.

Of course, one may argue that our measure of the likelihood of FPE depends on aggregate demand, and that it would be better to have some measure for the realistic case where aggregate demand functions are not known. Some trade theorists seem to believe that this takes us back to the traditional setting with exogenous prices. Not at all! In fact, one may take such a demand-independent measure from Travis' (1964) analysis. In the 2×2 case, the union of all possible FPE regions in the factor box is the "american football-shaped" area between the efficiency locus—the theory-of-production analogue to the contract curve—and its mirror image w.r.t. the diagonal of the box. This area is called FPE possibility set; the ratio of its size to the size of the box is a measure of the likelihood of FPE that does not depend on any specification of the demand side. Obviously, there exists a many-dimensional equivalent, even if it cannot be described that easily. We refer to this measure as the demand-independent likelihood of FPE.

3.2 Introducing Joint Production

We now consider the effects of introducing JP. Surely, introducing JP has to do much more with changing technologies than changing tastes. Let us, then, assume that JP does not affect the commodity space or preferences. Moreover, following Jones (1992) we introduce JP in a way which does not affect the basic production processes either. We thus assume that the same m basic processes are available under both JP and NJP, and they exhibit what Jones calls *independence of the input side*. In the case of JP, however, the outcome of each process is not a

single finished commodity, but some intermediate product which can be transformed into commodities over which consumers have convex preferences. The way in which this transformation may take place is specific to each process, giving rise to what Jones calls *independence of output possibilities.*⁶ Formally, we introduce the JP revenue function as

$$r^{\mathrm{JP}}(\boldsymbol{p},\boldsymbol{v}) \stackrel{\mathrm{def}}{=} \max_{\boldsymbol{x}^{j}} \left\{ \boldsymbol{p}^{\mathrm{T}}\boldsymbol{x} : g(\bar{\boldsymbol{s}},\boldsymbol{v}) \leq 0, \, \bar{s}_{j} = s_{j}(\boldsymbol{x}^{j}), \, \boldsymbol{x}^{j} \geq \boldsymbol{0}, \, \bar{s}_{j} \geq 0, \, \sum_{j=1}^{n} \boldsymbol{x}^{j} = \boldsymbol{x} \right\}.$$
(6)

Here the function g is the same as in (1). The goods vector $\boldsymbol{x} \in \mathbb{R}^n_+$, however, is replaced by the vector of intermediate products $\boldsymbol{s} \in \mathbb{R}^n_+$. Each of the functions $s_j(\boldsymbol{x}^j)$ is linearly homogenous, increasing and convex; its contour surfaces thus look like transformation surfaces. The range of commodity outputs that can be produced by one unit of s_j is given by $\{\boldsymbol{x} : s_j(\boldsymbol{x}) \leq 1\}$. The unit values $\boldsymbol{c} \in \mathbb{R}^n_+$ of the intermediate products are given by

$$c_j(\boldsymbol{p}) \stackrel{\text{def}}{=} \max_{\boldsymbol{x}} \left\{ \boldsymbol{p}^{\mathrm{T}} \boldsymbol{x} : s_j(\boldsymbol{x}) \le 1, \, \boldsymbol{x} \ge \boldsymbol{0} \right\}.$$
(7)

The derivative $c'_j(\mathbf{p}) \in {\mathbf{x}^T : s_j(\mathbf{x}) \leq 1}$ will be called the "unit output vector" of the *j*-th production process.

As ususal, equilibrium conditions can be derived more easily from the dual definition of the revenue function:

$$r^{\rm JP}(\boldsymbol{p}, \boldsymbol{v}) \equiv \min_{\boldsymbol{w}} \left\{ \boldsymbol{w}^{\rm T} \boldsymbol{v} : \boldsymbol{b}(\boldsymbol{w}) \ge \boldsymbol{c}(\boldsymbol{p}), \, \boldsymbol{w} \ge \boldsymbol{0} \right\}.$$
(8)

Obviously, we have $r^{\text{JP}}(\boldsymbol{p}, \boldsymbol{v}) \equiv r(\boldsymbol{c}(\boldsymbol{p}), \boldsymbol{v})$. The minimization problem in (8) gives rise to the following FOC, which again are necessary and sufficient:

$$\begin{aligned} & \boldsymbol{b}(\boldsymbol{w}) \geq \boldsymbol{c}(\boldsymbol{p}), \quad \boldsymbol{s} \geq \boldsymbol{0}, \quad (\boldsymbol{b}(\boldsymbol{w}) - \boldsymbol{c}(\boldsymbol{p}))^{\mathrm{T}} \boldsymbol{s} = \boldsymbol{0} \quad (\text{ZP conditions}) \\ & \mathrm{A} \boldsymbol{s} \leq \boldsymbol{v}, \qquad \boldsymbol{w} \geq \boldsymbol{0}, \qquad \boldsymbol{w}^{\mathrm{T}} (\mathrm{A} \boldsymbol{s} - \boldsymbol{v}) = \boldsymbol{0} \quad (\text{FMC conditions}) \end{aligned}$$
(9)

The supply vector \boldsymbol{x} is given by the partial derivative of the revenue function w.r.t. \boldsymbol{p} ; the envelope theorem yields $\boldsymbol{x}^{\mathrm{T}} = \boldsymbol{s}^{\mathrm{T}} \boldsymbol{c}'(\boldsymbol{p})$. The matrix of unit output vectors $\boldsymbol{c}'(\boldsymbol{p})$ is intimately connected with Jones' (1992) output cone of diversification. For easier distinction we shall refer to the McKenzie cone as the *input cone of diversification*. Note that this latter cone is now determined by the unit values $c_j(\boldsymbol{p})$ instead of the price vector \boldsymbol{p} . Moreover, the dimension of the output cone is determined by the number of linearly independent unit output vectors of the profitable production processes. We assume in the following that the matrix of unit output vectors $\boldsymbol{c}'(\boldsymbol{p})$ is regular in \mathbb{R}^{n}_+ .⁷

In order to analyze the likelihood of FPE, we have to look at the IWE. For this purpose, we can interpret v in r(p, v) and $r^{JP}(p, v)$ as world factor endowments; the revenue functions thus describe world production in the IWE. In closing the model, we have to consider the world

goods-market-clearing conditions: World supply must satisfy world demand in the IWE. We denote the NJP-IWE price vector by \hat{p} . The associated factor price and output vectors are \hat{w} and \hat{x} , respectively. We now take this NJP-IWE as the benchmark against which we investigate the consequences of introducing a JP technology.

At the most general level, there is an infinite number of JP technologies that one may envisage; further insights can only be achieved by considering special cases. Perhaps the best starting point is the case of what may be called a *neutral* JP technology, by which we mean that the NJP-IWE factor price vector $\hat{\boldsymbol{w}}$ is preserved under JP. This vector determines a unique input cone of diversification and hence FPE region for a given world factor box. For ease of exposition, we assume identical homothetic preferences and normalize the utility function of our representative consumer such that it is linearly homogeneous. Demand may then be represented by a single expenditure function that is linear in utility: $e(\boldsymbol{p}) \cdot \boldsymbol{u}$. The unit expenditure function $e(\boldsymbol{p})$ is non-decreasing, linearly homogeneous and concave. Uniqueness of the IWE equilibrium is now guaranteed. Denoting the JP-IWE by a tilde, it is quite straightforward that a JP technology is neutral with respect to the NJP-IWE, if and only if it satisfies

(a)
$$\boldsymbol{c}(\tilde{\boldsymbol{p}}) = \hat{\boldsymbol{p}},$$

(b) $\hat{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{c}'(\tilde{\boldsymbol{p}}) = (\hat{\boldsymbol{w}}^{\mathrm{T}}\boldsymbol{v}/e(\tilde{\boldsymbol{p}})) \cdot e'(\tilde{\boldsymbol{p}}).$
(10)

Condition (a) states that the outcome of a unit level process has equal value under JP and NJP, given commodity prices $\tilde{\boldsymbol{p}}$ and $\hat{\boldsymbol{p}}$, respectively. Condition (b) states that the factor market clearing levels of all processes generate an overall final output vector that satisfies demand, given income $\hat{\boldsymbol{w}}^{\mathrm{T}}\boldsymbol{v}$ and commodity prices $\tilde{\boldsymbol{p}}$. Notice that $\tilde{\boldsymbol{u}} = (\hat{\boldsymbol{w}}^{\mathrm{T}}\boldsymbol{v}/e(\tilde{\boldsymbol{p}})).^{8}$

Neutrality so defined is sufficient, but not necessary for an equal P_{FPE} in the two worlds of NJP and JP, due to the non-monotonicity property of P_{FPE} mentioned in the preceding subsection. The principal insight that we gain from this borderline case is that we may produce any change in the likelihood of FPE by choosing a JP technology which is *biased the right way*. It is, of course, difficult to derive truly general results when the JP technology is biased since in general the values of all variables will differ between both IWE equilibria. But we can nevertheless use our insight to reconsider the arguments put forward by Samuelson (1992) and Jones (1992) and put them into perspective.

3.3 Reconsidering the Arguments

If every single production process can produce several goods—in the extreme case considered above: all of them—, some of the production processes may become unprofitable in the IWE. This is true also in the *even case* where (1) there are n goods and n factors of production, (2)

there are *n* basic production processes available, and (3) all *n* goods are essential. If some of the basic processes are unprofitable, the output cone and thus the input cone of diversification are of a dimension smaller than *n*; the likelihood of FPE is zero.⁹ Whether or not this is indeed the case depends on the precise interaction of demand conditions and the JP technology. A sufficient condition may be derived as follows.

We resort to the case of a linearly homogeneous utility function introduced above.¹⁰ We use the supply correspondence

$$\mathcal{X}(\boldsymbol{p}) \stackrel{\text{def}}{=} \left\{ \boldsymbol{s}^{\mathrm{T}} \boldsymbol{c}'(\boldsymbol{p}) : \, \boldsymbol{s} \ge \boldsymbol{0} \right\}$$
(11)

to define the following set of commodity price vectors:

$$\bar{\mathcal{P}} \stackrel{\text{def}}{=} \left\{ \boldsymbol{p} \in \mathbb{R}^n_+ : e'(\boldsymbol{p}) \in \mathcal{X}(\boldsymbol{p}) \right\}$$
(12)

Any $p \notin \mathcal{P}$ cannot be an equilibrium price vector. Unfortunately, even in this very simple case \mathcal{P} is not at all well-behaved. An analysis of its structure gives some indication of the problems involved in a global analysis of JP. In general, we cannot say more than that \mathcal{P} is closed and has the obvious homogeneity property.¹¹ We denote the interior of \mathcal{P} , which of course could be empty, by \mathcal{P} . We make the following assumption: The functions $-e(\mathbf{p})$ and $c_j(\mathbf{p})$ are strictly quasiconvex and continuously differentiable. With this simplification, the following holds: If and only if $\mathbf{p} \in \mathcal{P}$, all basic processes are used at positive levels.¹²

We define a further set of commodity prices as follows:

$$\mathcal{R} \stackrel{\text{def}}{=} \left\{ \boldsymbol{p} \in \mathbb{R}^n_+ : \, \boldsymbol{w}^{\mathrm{T}} = r_{\boldsymbol{v}}^{\mathrm{JP}}(\boldsymbol{p}, \boldsymbol{v}), \boldsymbol{c}(\boldsymbol{p}) = \boldsymbol{b}(\boldsymbol{w}) \right\}$$
(13)

This set contains all commodity price vectors which generate an input cone of diversification with full dimension that contains the endowment point. If c(p) is univalent, \mathcal{R} obviuously is a convex cone since its NJP analogue

$$\left\{ \boldsymbol{p} \in \mathbb{R}^n_+ : \, \boldsymbol{w}^{\mathrm{T}} = r_{\boldsymbol{v}}(\boldsymbol{p}, \boldsymbol{v}), \boldsymbol{p} = \boldsymbol{b}(\boldsymbol{w})
ight\}$$

is a convex cone. If c(p) is *not* univalent, \mathcal{R} can be a union of several convex cones that are all mapped onto the same cone by c(p).

Given these two sets, we can formulate our result. If $\mathcal{R} \cap \mathcal{P} = \emptyset$, then the JP-IWE will have a FPE region of less than full dimension and, hence a zero probability of FPE. The equilibrium production structure will involve less than n profitable processes. Note that $\mathcal{R} \cap \bar{\mathcal{P}} \neq \emptyset$, since the existence of an equilibrium is guaranteed.

If the intersection $\mathcal{R} \cap \mathcal{P}$ is non-empty, an equilibrium with a positive P_{FPE} may or may not exist. Any $p \in \mathcal{R}$ determines some level of intermediate production s and some level of income $\boldsymbol{w}^{\mathrm{T}}\boldsymbol{v}$. Any $\boldsymbol{p} \in \cap \bar{\mathcal{P}}$ determines another value $\bar{\boldsymbol{s}}$ of intermediate production satisfying unit demands, i.e. fulfilling $e'(\boldsymbol{p}) = \bar{\boldsymbol{s}}^{\mathrm{T}}\boldsymbol{c}'(\boldsymbol{p})$. If, for $\boldsymbol{p} \in \mathcal{R} \cap \mathcal{P}$, \boldsymbol{s} and $\bar{\boldsymbol{s}}$ are collinear, \boldsymbol{p} is an equilibrium price vector: With $u = \boldsymbol{w}^{\mathrm{T}}\boldsymbol{v}/e(\boldsymbol{p})$, goods markets clear since $e'(\boldsymbol{p}) \cdot \boldsymbol{u} = \boldsymbol{s}^{\mathrm{T}}\boldsymbol{c}'(\boldsymbol{p})$.

Our condition $\mathcal{R} \cap \mathcal{P} \neq \emptyset$ is only necessary for the existence of an JP-IWE with full dimensionality of the FPE region. In other words, $\mathcal{R} \cap \mathcal{P} = \emptyset$ is sufficient for the case Samuelson had in mind. The NJP analogue of $\overline{\mathcal{P}}$ is \mathbb{R}^n_+ , so that this condition can never arise, whereas under JP $\overline{\mathcal{P}}$ is a potentially small subset of \mathbb{R}^n_+ . So there are demand systems which lead to FPE in the NJP case but which are incompatible with FPE under JP. We can also identify some of the factors determining the size of \mathcal{P} . The more similar the output intensities of the basic processes, and the higher the elasticities of substitution in demand, and the smaller the elasticity of substitution in s_j , the smaller is \mathcal{P} . \mathcal{R} will be the smaller, the higher the elasticities of primary factor substitution in the basic processes.

The fact that introducing JP may endogenously reduce the number of profitable processes clearly weakens Jones' (1992) point, which rests on the arbitrary assumption that all processes can be operated at positive levels in equilibrium. It is, therefore, tempting to argue that it is only the Samuelson argument which survives in a general equilibrium context and to conclude, therefore, that FPE is less likely under JP than under NJP. However, this is true only in the even case. If the number of processes exceeds the number of factors, then a switch from NJP to JP can increase the likelihood of FPE because some production processes become unprofitable. This results from the fact that the FPE region increases if production processes that are in the interior of the input cone of diversification become obsolete. Thus the Samuelson argument, which draws on the number of profitable production processes, does not generally hold in a full general equilibrium setting.

Jones' argument is based on a 2×2 "magnification effect". If there are only two basic production processes and two primary factors, the fact that the unit-value functions c(p) are increasing, linearly homogenous, and concave means that an interval of relative goods prices translates into some *smaller* interval of relative unit values. Thus any given interval of relative unit-values compatible with FPE translates into a *bigger* interval of relative goods prices. This provides an example of a case where FPE may be said to become more likely under a JP technology, *if commodity prices are treated as exogenous as in the traditional FPE theorem.*

In a higher-dimensional framework, the magnification effect is not guaranteed to work. But it is nevertheless *possible* that the set \mathcal{R} is bigger than its NJP analogue, especially if the unitvalue functions are not univalent. However, if prices are endogenized, as in the modern FPE theorem, this is only one side of the story. The other side is described by the set $\bar{\mathcal{P}}$, which is smaller than its NJP analogue \mathbb{R}^n_+ . In our view, Jones and Samuelson just look at different parts of the same story. It is difficult to understand why they do not put these parts together. The whole story has been given in the last subsection. If we judge the likelihood of FPE by P_{FPE} as defined in (4) above, a switch from NJP to JP may just as well make FPE more likely, even if the number of processes remains to be equal to the number of goods. We show this by means of a $2 \times 2 \times 2$ example in appendix B, but we must emphasize that our point is perfectly general. If the JP technology is not neutral in the sense of preserving the NJP-IWE factor prices, the size of the FPE region may increase or decrease, depending on the precise nature of the JP technology. The example in appendix B nicely serves to highlight the main forces that drive the exact results.

Of course, one can argue that the question of the likelihood of FPE should be answered without recourse to demand because aggregate demand functions are in fact unknown. However, this does not bring us back into the world of the traditional theorem, as noted above. Trivially, the demand-independent likelihood of FPE is the same for JP and NJP if, following Jones (1992), one makes the assumption that the basic production processes are the same under JP and NJP. This follows from the fact that the efficiency loci in the factor box are the same in this case. Of course, this assumption is completely arbitrary in the context of the Samuelson-Jones discussion. However, as the next section shows, there is a reinterpretation justifying it.

4 Trade in Intermediate Versus Trade in Finished Products

The discussion in Samuelson (1992) and Jones (1992) is very far from any real problem. Whether FPE is more or less likely with technology 1 as compared with technology 2 is relevant only if there is a choice between these technologies or if, at least, they coexist. This seems to be rather unlikely for a NJP and a JP technology sharing the same (in the case of JP: basic) production processes. So why bother, then? Because it is not *necessary* to look at the whole question as a comparison of two different technologies. It is possible to reinterpret it as a comparison of two different trading regimes: trade in intermediate products only (IP trade) versus trade in finished products only (FP trade).

This alternative interpretation applies immediately to Jones' small-open-economy argument. If there is only trade in intermediate products s, the unit-value functions c(p) do not enter the picture. Given the magnification effect, FPE is more likely under FP trade, because the range of finished-product prices p supporting FPE is greater than the range of intermediate-product prices c supporting FPE.¹³

Again it is more satisfying to consider this question in a full general equilibrium setting. Note first that, if c(p) is univalent, equalization of intermediate-product prices implies equalization

of finished-product prices if *all* basic processes are profitable. Thus there is nothing to choose between IP trade and FP trade: FP trade makes IP trade superfluous anyway; the univalence assumption guarantees that the reverse also holds.

If we give up the univalence assumption, we may have FPE without equalization of finishedgoods prices in the case of IP trade, but only if additionally preferences differ between countries. Of course, an FPE equilibrium where finished-goods prices and therefore *all* prices are equalized is possible, too. The FPE regions corresponding to these equilibria can be computed from different IWEs. The first kind (FPE without equalization of p) can be computed from a not completely integrated world equilibrium where factors are mobile and where intermediate goods are traded, but finished goods are not. The second kind (full price equalization) can be computed from a fully integrated world equilibrium. Factor prices of these IWEs and therefore the likelihoods of FPE under both trading regimes may differ.

Again it depends on which goods are favored by a change from IP trade to FP trade whether the likelihood of FPE increases or decreases. The interesting point is that nothing excludes the possibility that FPE reached by trade in intermediate products only is *destroyed* by full integration of product markets. This is quite likely, in fact, if trade in intermediate products alone leaves room for big differences in finished-product prices. Thus trade in goods and factor movements can be complements in such a model: Integrating more product markets can create factor-price differences and therefore generate factor movements.

5 Conclusion

Does the factor-proportions model understate or overstate the likelihood of FPE by trade if production is assumed non-joint, as in most of the literature? The answer is difficult for two reasons. First, there is a high degree of arbitrariness if jointness in production is introduced into an otherwise orthodox trade model. Samuelson (1992) argues that introducing jointness carries a potential for reducing the number of basic production processes, thus making FPE less likely. However, if one assumes from the outset that there exist fewer basic production processes than primary factors in the JP case, the result that FPE is unlikely in no way depends on jointness or non-jointness of production. The issue only arises if enough basic production processes are *available*.

The second difficulty relates to the exact meaning of the likelihood of FPE. Is this notion adequately captured by the size of the range of commodity price ratios that can support a free trade equilibrium with complete FPE for a given inter- country distribution of world factor endowments? Jones (1992) shows that under this criterion one can say that FPE is more likely under joint production than under non-joint production, provided the nature and number of the basic production processes is the same in both worlds. However, there is another criterion that one might want to employ which focuses more on alternative inter-country factor endowment distributions, such as might be realized with varying degrees of factor mobility. This criterion leads one to measure the likelihood of FPE by the *amount of endowment disparities* that free trade is able to accommodate, in the sense that it leads to FPE. Under the second criterion, one considers the possibility of FPE in a full general equilibrium context where *all* prices *and* the number of profitable production processes are endogenous. We have argued that in this context neither the Samuelson nor the Jones argument survives. It is then very well possible, though not inevitable, that joint production *lowers* the likelihood of FPE if the number of profitable processes does *not* decrease, and *increases* the likelihood of FPE if the number of profitable processes does decrease. Whether or not this is the case depends on how the JP technology interacts with demand. Moreover, as we have demonstrated by means of a 2×2 example, the likelihood of FPE may *decrease* under JP even with a given (and sufficiently high) number of profitable production processes.

However, not only is there no clear-cut answer to the question discussed by Samuelson and Jones. The question itself is highly questionable. What sense does it make to compare two arbitrarily chosen technologies w.r.t. the likelihood of FPE or anything else? Of course the interactions between demand and technology are more complicated in the case of JP than in the case of NJP. This is relevant if one computes the likelihood of FPE, but it does not at all demonstrate that a *comparison of technologies* is of any interest. However, it is possible to reinterpret the question as a special case of a more general question: How does it affect the likelihood of FPE if one changes from trade in intermediate to trade in finished products? Our analysis throws light on the basic mechanism at work. It moreover shows that integration of product markets can destroy FPE and generate factor-price differences that lead to factor movements.

Appendix A Existence of Non–Trivial Neutral JP Technologies

This section proves that a non-trivial neutral JP technology always exists, and that it is not unique.

We start from a given NJP-IWE; equilibrium values are denoted by "hats". Remember that we have assumed identical homothetic preferences and that we have normalized the utility function such that it is linearly homogeneous. Moreover, by assumption we have an equal number of goods and factors. The NJP-IWE satisfies

$$e(\hat{\boldsymbol{p}}) \cdot \hat{\boldsymbol{u}} = \hat{\boldsymbol{w}}^{\mathrm{T}} \boldsymbol{v}$$

$$\boldsymbol{b}(\hat{\boldsymbol{w}}) = \hat{\boldsymbol{p}}$$

$$\hat{\boldsymbol{u}} \cdot e'(\hat{\boldsymbol{p}}) = \boldsymbol{v}^{\mathrm{T}} (\mathrm{A}^{\mathrm{T}})^{-1}.$$
(14)

Here again, $A^{T} = b'(\hat{w})$, and the final equation states factor market equilibrium. Next we consider an JP-IWE which we denote by a tilde. Since neutrality requires that factor prices are equal to those in the NJP-IWE we may substitute $b(\hat{w})$ by $b(\hat{w}) = \hat{p}$. Moreover, we assume for simplicity that the matrix of input coefficients $A^{T} = b'(\hat{w})$ is of full rank; this requires that the activity levels of the basic production processes are equal in both cases, i. e. $\tilde{\boldsymbol{s}} = \hat{\boldsymbol{x}}$ in the JP-IWE. By substituting accordingly, the system for the JP-IWE becomes:

$$e(\tilde{\boldsymbol{p}}) \cdot \tilde{\boldsymbol{u}} = \hat{\boldsymbol{w}}^{\mathrm{T}} \boldsymbol{v}$$

$$C^{\mathrm{T}}(\tilde{\boldsymbol{p}}) \tilde{\boldsymbol{p}} = \hat{\boldsymbol{p}}$$

$$\tilde{\boldsymbol{u}} \cdot e'(\tilde{\boldsymbol{p}}) = \hat{\boldsymbol{x}}^{\mathrm{T}} C^{\mathrm{T}}(\tilde{\boldsymbol{p}}).$$
(15)

Here, we have exploited linear homogeneity of c(p) to replace $c(\tilde{p})$ by $c'(\tilde{p})\tilde{p} = C^{T}(\tilde{p})\tilde{p}$. We eliminate \tilde{u} and normalize prices such that $\hat{w}^{T}v = 1$. This yields

$$C^{\mathrm{T}}(\tilde{\boldsymbol{p}})\tilde{\boldsymbol{p}} = \hat{\boldsymbol{p}}$$

$$e'(\tilde{\boldsymbol{p}})/e(\tilde{\boldsymbol{p}}) = \hat{\boldsymbol{x}}^{\mathrm{T}}C^{\mathrm{T}}(\tilde{\boldsymbol{p}}).$$
 (16)

It should be noticed that we do not require unit level processes to have identical values for NJP and JP at a common price level, say \hat{p} . Instead, they must have equal values at their corresponding equilibrium prices \tilde{p} and \hat{p} , respectively. In other words, a neutral JP technology may well exhibit an equilibrium price vector which is different from that of the NJP-IWE. The second part of (16) incorporates market clearing for both goods and factors, and it establishes the precise way in which the output intensities of the JP technology must be related to demand characteristics. This can be made more transparent by introducing diagonal matrices \tilde{P} and \hat{P} whose diagonals are the price vectors \tilde{p} and \hat{p} , respectively. We may then transform (16) into

$$M^{T} \boldsymbol{\tau} = \boldsymbol{\tau}$$

$$M\boldsymbol{\theta}(\hat{\boldsymbol{p}}) = \boldsymbol{\theta}(\tilde{\boldsymbol{p}}),$$
(17)

where θ is the vector of relative shares in consumption, $M \stackrel{\text{def}}{=} \tilde{P}^{-1}C\hat{P}$ is a matrix of relative value shares on the output side of the JP technology, and τ is a vector of 1's. The matrix M is the only relevant feature of the output side of the JP technology, and its value shares interpretation follows from the fact that, by construction, a unit level process *i* has a value of \hat{p}_i . Let us assume for the moment that the utility function of the representative consumer is of the Cobb-Douglas type. Then θ is a vector of constants, and a neutral JP technology has to satisfy the following conditions:

$$M^{\mathrm{T}}\boldsymbol{\tau} = \boldsymbol{\tau}$$

$$M\boldsymbol{\theta} = \boldsymbol{\theta}.$$
(18)

The first condition is fulfilled by the definition of the matrix as a matrix of value shares. The second condition may be fulfilled in infinitely many ways. Let T and I denote the square matrix with n identical column vectors $\boldsymbol{\theta}$ and the identity matrix, respectively. Then all the matrices

$$\mathbf{M} = \alpha \cdot \mathbf{T} + (1 - \alpha) \cdot \mathbf{I}$$

fulfill the conditions (18). Of course, there are several matrices doing the same job, each of which may be thought of as being generated as a value shares matrix for a given price vector \tilde{p} by an appropriate JP technology. For Cobb-Douglas preferences, a neutral JP technology therefore exists, is non-unique, and allows \tilde{p} to be different from \hat{p} . For more general preferences the same argument establishes the existence and non-uniqueness of a netural JP technology which gives rise to the same commodity prices as in the NJP-IWE.

Appendix B A $2 \times 2 \times 2$ Example

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We consider a $2 \times 2 \times 2$ model where the NJP and JP technologies are described by the 2×2 versions of (2) and (8), respectively. We assume that in the JP-IWE as in the NJP-IWE both basic production processes are used. Indicating NJP-IWE values with hats again, we introduce the following definition: The JP technology is *biased in favor of the first basic production process* if and only if $c_1(\tilde{p}_1)/c_2(\tilde{p}_2) > \hat{p}_1/\hat{p}_2$, where \tilde{p}_j is the JP-IWE price of commodity j. Given a factor-intensity condition, we can translate biasedness in favor of a production process into biasedness in favor of a factor by the Stolper-Samuelson theorem. Note that this definition of biasedness is complementary to the above definition of neutrality in the sense that nonbiasedness implies neutrality as defined in (10).

If we assume that the bias is *marginal*, we can derive the change in the likelihood of FPE by a simple comparative-static analysis of the impact of a price change on the production side of the IWE. One may note that our analysis is essentially equivalent to that of a demand shift in a conventional NJP model. If one makes use of the factor-market clearing conditions, expression (5) can be written as

$$\mathbf{P}_{\rm FPE} = \frac{a_{11}a_{22} + a_{12}a_{21} - a_{22}a_{21}\frac{v_1}{v_2} - a_{11}a_{12}\frac{v_2}{v_1}}{a_{11}a_{22} - a_{21}a_{12}},\tag{19}$$

where $a_{ij} \stackrel{\text{def}}{=} \partial b_j / \partial w_i$ is the input coefficient of factor *i* in sector *j*. With $k_j \stackrel{\text{def}}{=} \frac{a_{1j}}{a_{2j}}$ and $k \stackrel{\text{def}}{=} \frac{v_1}{v_2}$, (19) transforms into

$$P_{\rm FPE} = \frac{k_1 + k_2 - k - k_1 k_2 / k}{k_1 - k_2} \,. \tag{20}$$

Since the intensities k_j depend on the relative factor price $\omega \stackrel{\text{def}}{=} w_1/w_2$, we discuss the likelihood of FPE as a function $P_{\text{FPE}} = \Psi(\omega)$. We compute the elasticity of Ψ with respect to ω by taking into account that the elasticity of k_j with respect to ω is the elasticity of substitution in sector j, in the following denoted by $\sigma_j < 0$. The elasticity η of Ψ is given by

$$\eta = \frac{k_1 \cdot \sigma_1}{k_1 - k_2} \cdot \frac{k - k_2}{k_1 - k} - \frac{k_2 \cdot \sigma_2}{k_1 - k_2} \cdot \frac{k_1 - k}{k - k_2}.$$
(21)

Equation (21) makes it easy to state the condition for an increase in the likelihood of FPE in response to a change in ω , as brought about by the introduction of JP. Let us denote the favored and the non-favored process by f and n, respectively. The changes of ω in response to changes in c_j are well known from the Stolper-Samuelson theorem: ω goes up if and only if $k_f > k_n$. Defining $\lambda \stackrel{\text{def}}{=} \frac{k_f - k}{k - k_n}$, we use the Stolper-Samuelson theorem and (21) to arrive at the following condition:

$$d\Psi > 0 \iff \lambda^2 \cdot \frac{k_n}{k_f} > \frac{\sigma_f}{\sigma_n}$$
(22)

This is easily interpreted. If $\sigma_f > \sigma_n$, intensities become closer as a consequence of the switch from NJP to JP. The closer the factor intensities, the smaller is the input diversification cone. However, there is a second effect which is captured by λ . The closer one of the intensities is to the factor endowment ratio, the more important is the change in this intensity; with λ very big, it tends to dominate the effect of a change in the width of the diversification cone. The case where one intensity is almost equal to the factor endowments ratio is obviously the case where the output of the production process in question is relatively high. We can thus express the general principles emerging from this by the following statements. If both processes are equally important in terms of resource use ($\lambda \approx 1$), FPE is more likely under JP than NJP if and only if the favored process's elasticity of factor substitution is relatively high in the sense of $\sigma_f/\sigma_n \gg k_n/k_f$. If the favored process's elasticity of substitution is relatively small, FPE is more likely under JP than NJP if and only if the process is important enough in terms of resource use $(\lambda \gg 1)$. It should be noted that this proposition does not depend on the factor intensity ranking since any change in this ranking also changes the direction in which ω moves upon introducing JP.

Notice that the switch to JP endogenously changes λ , hence the conditions just derived are not sufficient for signing the effect of a *non-marginal* bias on the likelihood of FPE. But this observation only reinforces our main point that the effect on the likelihood of FPE of introducing JP is, in general, ambiguous.

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Notes

1. If not stated otherwise, all vectors are column vectors. A prime denotes the first derivative. The derivative of a function with a vector argument is a row vector. Vector-valued functions are defined as column vectors of the component functions. The scalar product is treated as a special case of matrix multiplication. The superscript T denotes transposition.

2. We include the cases where there is only *one* profitable production process. This allows for a simple general statement.

3. It would be more accurate to speak about the *robustness* or *structural stability* instead of the *likelihood* of FPE. The real question is what it takes to destroy (or accomplish) FPE. No *chance mechanism* is presupposed if one discusses the likelihood of FPE under this perspective.

4. Of course, there could be arbitrary shifts in tastes. But this is not our concern. Since Heckscher's (1919) path-breaking analysis, interest has been centered on the influence of *factor* endowments on trade equilibria. In our view, the FPE literature, with the exception of Uzawa (1959) and Travis (1964), had lost sight of this central problem, until Dixit & Norman (1980) again pressed the general equilibrium perspective. However, some trade theorists still claim that the Samuelson-McKenzie version of the FPE theorem is in no need of demand-side supplements; we have encountered this view in connection with the present paper. Readers may judge for themselves. In our view, the FPE story, especially the univalence discussion (cf. Albert 1994), demonstrates that a glance at the demand side once and again saves a lot of irrelevant argument. Moreover, as we noted above, Samuelson (1992) implicitly does look at the demand side. Why not do it more systematically?

5. The volume of a parallelepiped is equal to the determinant of the matrix formed by the vectors defining its edges [(Lang 1970: 202-211)].

6. Technically, the independence of output possibilities from the input combination is a certain separability property. At this stage of the argument, it does not matter whether some intermediate product physically exists or not.

7. As became clear in the univalence debate, it is neither necessary not sufficient for global univalence that the Jacobian matrix is regular everywhere in the domain of a function from a subset of \mathbb{R}^n to \mathbb{R}^n if n > 1. We therefore discuss univalence of the vector-valued function c(p), when relevant, independently from the regularity of c'. c(p) is univalent if c' is regular in \mathbb{R}^n_+ and some further conditions hold on the boundary of \mathbb{R}^n_+ (Mas-Colell 1979).

8. A more detailed treatment of this concept of neutrality is contained in appendix A where we also show that a non-trivial neutral JP technology always exists and is non-unique.

9. If c'(p) is singular, a case explicitly excluded above, the *dimension* of the output cone of diversification is no longer crucial. Even if it is possible to satisfy demand without using all processes from the output side, it might be necessary to use all of them in order to clear factor markets.

10. A generalization to arbitrary demand functions is possible; the increased complexity, hoewever, is not justified by the additional insight.

11. Closedness follows from continuity of the functions involved. It seems that $\bar{\mathcal{P}}$ is simply connected. This is, however, not essential and is certainly not guaranteed for generalizations to arbitrary preferences. Any further conjectures concerning stronger general properties of $\bar{\mathcal{P}}$ can be refuted by considering the three-dimensional case. One reduces dimensions by defining the functions $\bar{e}(\mathbf{p}) \stackrel{\text{def}}{=} e(g(\mathbf{p}))$ and $\bar{c}_j(\mathbf{p}) \stackrel{\text{def}}{=} c_j(g(\mathbf{p}))$. It is straightforward to show that $e'(\mathbf{p})$ is in the cone of the gradients $c'_j(\mathbf{p})$ if and only if there is a nonnegative linear combination of the gradients of $\bar{c}_j(\mathbf{p})$ with not all coefficients zero that adds up to the zero vector. Because of homogeneity of degree zero one can restrict considerations to the unit simplex. Since the contour sets of the functions $\bar{c}_j(\mathbf{p})$ on the unit simplex are convex, counterexamples to several conjectures may be constructed by means of two-dimensional drawings. It can immediately be shown, e.g., that $\bar{\mathcal{P}}$ is *not* convex in general.

12. Note that s is an implicit function of p under our assumptions. If all basic processes are used at positive levels, small price changes do not alter this fact because of continuity; therefore p is in the interior of $\overline{\mathcal{P}}$. The other direction is slightly more complicated. Assume e(p) = 1 for all prices considered. If $s_1 = 0$ at p, there is a small change dp in prices such that at p + dpwe have $s_1 \neq 0$: The existence of an extended region of prices characterised by $s_1 = 0$ implies the existence of linear segments of c(p) and e(p), which is excluded by the assumptions on curvature. However, if $s_1 > 0$ at p + dp, then $s_1 < 0$ at p - dp. Thus p is a boundary point of $\overline{\mathcal{P}}$ if any process is not used at p.

13. If there is separability of cost functions without any physical intermediate product that could be traded, there is still a trading arrangement that yields the same results. One has to assume that firms must choose whether to export *all* their products or *none* of them. Under such an arrangement, prices of finished products may differ between exported and non-exported goods, but the value of c_j must be the same for both. However, this arrangement is quite artificial.

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