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Are German stock and bond returns consistent with equilibrium asset pricing? A calibration exercise using recursive non-expected utility

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Juristische Fakultät Fakultät für Wirtschaftswissenschaften und Statistik

Bernd Meyer

Are German Stock and Bond Returns Consistent with Equilibrium Asset Pricing? A Calibration Exercise

Using Recursive Non-Expected Utility



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Using Recursive Non-Expected Utility

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Abstract

This paper reexamines the Equity Premium Puzzle for the German stock market with control for inflation and taxation. Two methods for relaxing the assumption of aggregate consumption being equal to aggregate dividends are compared: the leverage approach and the usage of a bivariate stochastic process. Markov mixtures of univariate and bivariate normal distributions for the stochastic processes of consumption and dividends are estimated directly from the data and evaluated by Monte Carlo simulations. Preferences are modeled by time-additive expected utility and, alternatively, by recursive non-expected utility.

The empirical results for the period 1960 to 1994 confirm those for the U.S. and favour the use of recursive non-expected utility which clearly distinguishes between risk preference and time preference. The leverage approach yields the first moment of the risk-free rate and the first and second moments of the risk premium on the stock market with plausible preference parameters. The bivariate approach yields smaller risk premia because the assumption of perfect correlation between consumption and dividends is relaxed; the correlation is equal to the low value historically observed, reducing the risk premia in comparison with the leverage approach.

1 Introduction

This paper investigates empirically intertemporal asset pricing models at the German stock market. The existing evidence for the U.S. (e.g., HANSEN/SINGLETON (1982, 1983), MEHRA/PRESCOTT (1985)) shows that explaining risk-free and risky returns jointly poses a problem for intertemporal asset pricing models with expected utility. MEHRA/PRESCOTT observe that the risk premia on stocks are too high and the risk-free rates are too low to be explained by plausible parameters of risk aversion and time preference in a general equilibrium model based on LUCAS (1978). This result is well-known as the "Equity Premium Puzzle".

Equilibrium asset returns in intertemporal asset pricing models can generally be characterized by an Euler equation (SARGENT (1987:93)). The Euler equation, $1 = E\{IMRS \cdot (1 + \tilde{r}_{t+1}^i)\}$, shows that the expected return on an asset *i* from date *t* to date t + 1, $E[\tilde{r}_{t+1}^i]$, depends on its correlation with the market's marginal rate of substitution between a unit of consumption at date *t*, c_t , and t + 1, c_{t+1} . This intertemporal marginal rate of substitution, IMRS, depends on the preference function *U*. It is calculated as $IMRS = \frac{\partial U_0}{\partial c_{t+1}} / \frac{\partial U_0}{\partial c_t}$, where U_0 is the discounted utility of the total future consumption. The Euler equation itself does not determine a complete general economic equilibrium. It leaves open the exact way in which the economy is imagined to fit together (SARGENT (1987:95)) and thus, describes only a partial equilibrium.

In the literature there exist two different methods for the empirical investigation of intertemporal asset pricing models.

The first method, the estimation method, is to take the observed market returns and the observed aggregate consumption and to estimate the parameters of the preference function on aggregate consumption so as to optimize the fit of the Euler equation.¹ This method is plagued by estimation problems and problems concerning the interpretation of the estimated parameters. The estimation method does not require a general equilibrium model.

The second method for the empirical investigation of intertemporal asset pricing models, the calibration method, is based on general equilibrium models (LUCAS (1978)). Consider, for example, a simple equilibrium model with a representative investor with constant relative risk aversion. Production is assumed to equal dividends and consumption in each period. The production process is i.i.d. and stationary and $\tilde{\lambda}_{t+1}$ denotes the growth rate of production from date t to date t + 1. Then the return on the market from date t to date t + 1, \tilde{r}_{t+1}^m , follows as $\tilde{r}_{t+1}^m = \Phi \tilde{\lambda}_{t+1}$, where Φ depends on the preferences of the representative investor and on the expected production growth. Now, taking the observed growth rate of consumption (= pro-

¹Within this estimation method different approaches for the estimation of the preference parameters exist. Some of them require strong distributional assumptions regarding security returns and aggregate consumption (see HANSEN/SINGLETON (1983), GIOVANNINI/WEIL (1989) and JO-RION/GIOVANNINI (1993)). BROWN/GIBBONS (1985) criticize that the estimation results may not be robust to departures from the assumed distributions and that, since the advantage of utility-based pricing theories is the lack of strong distributional assumptions regarding security returns, empirical investigations of these theories should not require distributional assumptions. The distribution-free method of estimation and inference derived by HANSEN/SINGLETON (1982) overcomes this shortcoming (for an application see EPSTEIN/ZIN (1991)). SINGLETON (1990) provides a survey of different approaches within the estimation method.

duction), this model allows us to analyse which return on the market and which risk-free rate of return the equilibrium model implies, depending on the preferences chosen. By varying the parameters of the preference function (i.e., calibrating the preference function) one tries to match the moments of the implied equilibrium rates of return with those historically observed.

Despite the disadvantage that strong assumptions have to be made in a general equilibrium model, the calibration method has several advantages. The impact of the utility specification and its parameters on the implied rates of return can easily be examined by changing one parameter, holding the others constant, whereas an analytical comparative statics approach is often impossible for more sophisticated utility specifications. Furthermore, the calibrated model allows us to analyse equilibrium expected returns conditional on different realizations of the exogenous stochastic inputs (i.e., the level of macroeconomic activity). Thus, with this kind of model, one can draw conclusions concerning the theoretical impact of the macroeconomic activity on the rates of return and on the comovement of the returns on risk-free and risky assets. These conclusions can then also be investigated empirically.

This paper follows the calibration method. Therefore we present a brief overview of the existing literature applying this method.

MEHRA/PRESCOTT (1985) tried to calibrate the parameters of a time-additive expected utility specification with constant relative risk aversion on the basis of the general equilibrium model of LUCAS (1978). The LUCAS economy is a closed economy denoted in real terms characterized by one good, a representative agent and the equality of production, dividends and consumption. MEHRA/PRESCOTT assumed a stochastic exogenously given growth rate of production modeled by a symmetric Markov switching model with two deterministic states. They found that the observed risk premia are too high and the observed risk-free rates are too low to be explained by plausible parameters of risk aversion and discount factors.

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Many attempts have been made to solve this Equity Premium Puzzle. The different attempts can be classified as follows: (1) relaxing the assumption of the equality of production, dividends and consumption, (2) modifications of the utility specification, (3) modifications concerning the exogenous stochastic processes and (4) incorporation of additional risks (idiosyncratic risks, labour risks) or constraints (e.g. liquidity constraints). These attempts will be characterized in the following.²

(1) The introduction of leverage which has already been introduced by MEHRA/PRESCOTT (1985) relaxes the equality of dividends and production. Aggregate production equals aggregate consumption in each periode, but production is paid off in two parts, a risk-free part which is the payoff on fixed claimes (interest payments, wages ...), and a risky residual, the dividends. This approach is motivated by the fact that stocks traded in reality are stocks of levered firms and, thus, the return on the stock market is the residual return to the stock holders after interest payments to debt holders. The introduction of leverage increases the risk of the stock market and therefore the implied risk premium on stock returns. Holding leverage constant, production (resp. consumption) and dividends are strongly correlated.³ CECCHETTI/LAM/MARK (1993) and HUNG (1994) relax this

²See also ABEL (1991) for a survey of research conducted until 1991.

³For a constant risk-free rate and a constant amount of debt, production and dividends have a

restriction by modeling production growth and dividend growth separately. They substitute the univariate stochastic process for production (= consumption = dividend) growth by a bivariate process for production (= consumption) growth and dividend growth with the correlation set to the historically observed value. ABEL (1994) explicitly includes labor income to break the equality between dividends and consumption. MANKIW/ZELDES (1991) do not relax the equality of production, consumption and dividends but use consumption data of stock holders rather than aggregate consumption, because they observe, that consumption of stock holders and nonstockholders differ considerably.

(2) The first step in modifying the utility specification was to allow the discount factor β in the expected utility specification to increase beyond 1. BEN-NINGA/PROTOPAPADAKIS (1990) find that theoretically implausible discount factors larger than 1 in combination with leverage can resolve the Equity Premium Puzzle. KOCHERLAKOTA (1990a, 1990b) gets similar results and shows that discount factors larger than 1 occur because the time-additive expected utility specification is not capable to distinguish between risk preference and the elasticity of intertemporal substitution. The elasticity of intertemporal substitution measures the preference concerning the smoothness of the consumption path and is restricted to equal the inverse of the parameter of risk aversion in the time-additive expected utility specification. High risk aversion is connected to low elasticity of intertemporal substitution and vice versa. For example, a risk neutral agent has an infinite elasticity of intertemporal substitution and therefore no preference for the smoothmess of his consumption path. The EPSTEIN/ZIN (1989) parametrization of the recursive non-expected KREPS/PORTEUS (1978) preferences allows to distinguish between risk aversion and the elasticity of intertemporal substitution. WEIL (1989), KAN-DEL/STAMBAUGH (1991) and HUNG (1994) use this non-time-sparable utility specification in calibration exercises. The last two articles find promising results. Utility specifications with habit formation, another kind of non-time-separable utility, are used by ABEL (1990) and CONSTANTINIDES (1990).

(3) Modifications of the exogenous stochastic processes employ Markov switching models with more than two states (KANDEL/STAMBAUGH (1990), RIETZ (1988)) and switching models with stochastic instead of deterministic states (CEC-CETTI/LAM/MARK (1990)). In switching models with deterministic states the exogenous variable is deterministic within each state, whereas in switching models with stochastic states it is stochastic within each state. For each state a univariate distribution is assumed for the variable and the switching model is estimated directly from the data. In contrast, the switching models with deterministic states are usually assumed to be symmetric, i.e. the unconditional probability of all states is equal, and the states are chosen to fit the first-order autocorrelation and the first and second moments of the observed time series. Modeling consumption growth and dividend growth separately the switching between univariate distributions is replaced by a switching between bivariate distributions (CECCHETTI/LAM/MARK (1993), HUNG (1994)). ABEL (1994) compares the Markov switching models with a model with conditionally i.i.d. stochastic shocks.

(4) For the incorporation of additional risks or constraints see MANKIW (1986),

correlation equal to 1.

DANTHINE/DONALDSON/MEHRA (1992), WEIL (1992) and LUCAS (1994).

This paper reexamines the Equity Premium Puzzle for the German stock market by applying the calibration method to quarterly German data for the period 1960 to 1994. We compare some of the modifications of the MEHRA/PRESCOTT-approach described above and generalize the existing literature in several aspects. Our approach will be characterized in the following. We are aware of no other calibration exercise for Germany.

One criticism of the MEHRA/PRESCOTT-approach on the basis of LUCAS (1978) is that the observed consumption data are strongly smoothed in comparison with stock market dividends and thus, consumption and dividends should not be assumed to be equal or to be strongly correlated.⁴ Using different time series measuring production (= consumption) and dividends allows us to relax the assumption of the equality and strong correlation of consumption and dividend whereas the leverage approach only relaxes the assumption of equality. Therefore we compare the univariate process and leverage approach with a bivariate process approach.

For the univariate and the bivariate stochastic processes we use Markov mixtures of two normal distributions estimated directly from the data. By allowing the variance and correlation to differ in the two states, these processes as well as the derived equilibrium rates of return generalize those of CECCHETTI/LAM/MARK (1993) and HUNG (1994). This generalization has the potential not only to produce conditional expected returns with different means, but also with different volatilities. This may capture the empirically observed increase in volatility of aggregate stock returns during recessions (SCHWERT (1989a, 1989b), BITTLINGMAYER (1995)). We restrict the Markov switching model to two states since it is unlikely to get good estimates for more states with the limited number of observations available. For the evaluation of the estimated switching model we generalize the Monte Carlo approach used for univariate models by CECCHETTI/LAM/MARK (1990) to the bivariate case.

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In addition, we compare time-additive expected utility and recursive non-expected utility. As we observe that consumption data are smoothed compared to dividends the utility specification should allow an independent parametrization of risk aversion and preference for the smoothness of the consumption path.

In contrast to the studies cited above, we adjust the historical risk-free rate of return and the historical risky return for taxation. The taxation of capital gains and interest or dividend payments is different for institutional and private investors and thus may have an influence on the after tax risk-free rate of return and the risk premium. Therefore we calculate benchmark returns for four different taxation scenarios.

The results for Germany confirm the findings for the U.S. and favour the usage of recursive non-expected utility which clearly distinguishes between risk preference and time preference. Using non-expected recursive utility and the leverage approach the first moment of the risk-free rate and the first and second moments of the risk-premium historically observed can be obtained by the equilibrium model with plausible preference parameters. The bivariate approach yields smaller risk

⁴The equality of production, dividends and consumption is a critical assumption especially for countries like Germany, which are small and have a relatively large volume of exports and imports of goods and services. I thank Richard C. Stapleton for this comment.

premia because the observed correlation of aggregate consumption and stock market dividend is small, reducing the risk premia for given preference parameters in comparision with the leverage approach which assumes a strong correlation between consumption and dividends.

The paper is organized as follows. In section 2, we present an illustrative equilibrium model with i.i.d. production growth and specify the different preference functions. Section 3 presents the equilibrium model with a Markov switching process of production growth and derives closed form solutions for the expected risk-free rate and the expected risk premium on the market portfolio. The German consumption data and the estimation of the Markov switching model for consumption growth are described in section 4. Section 5 explains the historical German returns for different taxation scenarios. We present and discuss the calibration results in section 6. In section 7, we introduce leverage and present calibration results for a levered market portfolio. The bivariate approach with separate time series of consumption and dividend growth is analysed in section 8. A brief conclusion ends the paper.

2 Model with i.i.d. production growth

2.1 The economy

In this subsection we calculate the equilibrium market return and the equilibrium risk-free rate for the simplest equilibrium model. It is based on the following assumptions:

- 1. The economy is a one good, closed economy. In each period aggregate production \tilde{m}_t equals aggregate dividends \tilde{d}_t and aggregate consumption \tilde{c}_t .
- 2. The production process and hence the dividend and consumption process is exogenously given. The stochastic process of the gross growth rate of production $\tilde{\lambda}_{t+1} = \tilde{m}_{t+1}/m_t$ is stationary and i.i.d..
- 3. The preferences of the representative investor are given and exhibit constant relative risk aversion (CRRA).

From CRRA-preferences and the i.i.d. stationary process of production growth follows that the ex dividend market value of the productive sector at date t, M_t , equals $\Theta \cdot m_t$, where Θ depends on the parameter of the preference function and the expected future production growth.⁵ Hence the net return on the market portfolio follows as

$$\tilde{r}_{t+1}^{m} = \frac{\tilde{m}_{t+1} + \tilde{M}_{t+1}}{M_t} - 1 = \frac{\tilde{\lambda}_{t+1} \cdot m_t + \Theta \cdot \tilde{\lambda}_{t+1} \cdot m_t}{\Theta \cdot m_t} - 1 = \frac{1 + \Theta}{\Theta} \tilde{\lambda}_{t+1} - 1.$$
(1)

Hence the growth rate of consumption together with Θ determines the market return.

⁵Since aggregate production is paid off as dividend in this model, Θ can be interpreted as a price-dividend-ratio. Note, that the price-dividend-ratio is constant in this simple model.

2.2 Time-additive expected utility

We now solve for Θ and derive the equilibrium risk-free rate of return for the standard time-additive (time-separable) expected utility

$$U_0 = \sum_{t=0}^{\infty} \beta^t E(U(\tilde{c}_t)) \qquad 0 < \beta \le 1$$
(2)

where U(c) is the constant relative risk aversion utility function

$$U(c) = \frac{c^{1-\alpha}-1}{1-\alpha} \quad \text{for} \quad 0 \le \alpha < \infty \quad , \quad \alpha \ne 1 \quad \text{and}$$
$$U(c) = \log(c) \quad \text{for} \quad \alpha = 1.$$

 α is the parameter of relative risk aversion defined as -cU''(c)/U'(c) and β is the discount factor measuring the preference of early or late consumption. The discount factor should be lower than 1.

The representative investor maximizes the discounted utility of his consumption path, $(c_0, \tilde{c_1}, \ldots, \tilde{c_t}, \ldots, \tilde{c_{\infty}})$, s.t. the intertemporal budget constraint $\tilde{w}_{t+1} = (w_t - c_t)(1 + \tilde{r}_{t+1}^i)$ where \tilde{r}_{t+1}^i is the random net return on the investor's assets and w_t the investor's wealth at date t. The first-order condition for this optimization problem is the Euler equation

$$1 = E\left\{\beta \frac{U'(\tilde{c}_{t+1})}{U'(c_t)} \left(1 + \tilde{r}_{t+1}^i\right)\right\} = E\left\{\beta \left(\frac{\tilde{c}_{t+1}}{c_t}\right)^{-\alpha} \left(1 + \tilde{r}_{t+1}^i\right)\right\}.$$
 (3)

Using equation (1) this Euler equation yields in the case of the market portfolio

$$1 = E\left\{\beta \cdot \tilde{\lambda}_{t+1}^{-\alpha} \cdot \tilde{\lambda}_{t+1} \frac{1+\Theta}{\Theta}\right\}$$
$$\Leftrightarrow \frac{1+\Theta}{\Theta} = \beta^{-1} \left(E\left[\tilde{\lambda}_{t+1}^{1-\alpha}\right]\right)^{-1}$$

and thus

$$\tilde{r}_{t+1}^{m} = \beta^{-1} \left(E\left[\tilde{\lambda}_{t+1}^{1-\alpha} \right] \right)^{-1} \tilde{\lambda}_{t+1} - 1.$$
(4)

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The risk-free rate of return, r_{t+1}^{f} , can be obtained from the Euler equation (3) as

$$1 = E\left\{\beta \cdot \tilde{\lambda}_{t+1}^{-\alpha} \cdot (1+r_{t+1}^{f})\right\}$$

$$\Leftrightarrow r_{t+1}^{f} = \beta^{-1}\left(E\left[\tilde{\lambda}_{t+1}^{-\alpha}\right]\right)^{-1} - 1.$$
(5)

2.3 Recursive non-expected utility

Alternatively to the time-additive expected utility we use the EPSTEIN/ZIN (1989) parametrization of the recursive non-expected KREPS/PORTEUS (1978) preferences

$$V_t(c_t, \tilde{V}_{t+1}) = \left[(1-\beta)c_t^{1-\rho} + \beta \left(E_t \left[\tilde{V}_{t+1}^{1-\alpha} \right] \right)^{\frac{1-\rho}{1-\alpha}} \right]^{\frac{1}{1-\rho}}$$
(6)
for $0 < \beta \le 1, 0 < \rho < \infty, \rho \ne 1, 0 \le \alpha < \infty, \alpha \ne 1.$

This utility specification⁶ can be thought of as two separate functions, an aggregator function and a certainty equivalent function. The aggregator function $W[c_t, \mu_t(\tilde{V}_{t+1})] = [(1-\beta)c_t^{1-\rho} + \beta\mu_t(\tilde{V}_{t+1})^{1-\rho}]^{\frac{1}{1-\rho}}$ combines the consumption at date t, c_t , and the date t certainty equivalent $\mu_t(.)$ of the time (t+1) utility of the random wealth at date $(t+1), \tilde{V}_{t+1}$. Thus, this aggregator function fully captures the time preference, consisting of the discount factor β and the elasticity of intertemporal substitution $\sigma = 1/\rho$.⁷ The discount factor measures the preference of early or late consumption and the elasticity of intertemporal substitution measures the preference concerning the smoothness of the consumption path. The certainty equivalent function $\mu_t(\tilde{V}_{t+1}) = (E_t[\tilde{V}_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}}$ captures the risk preference. The parameter α is again the parameter of relative risk aversion and $E_t[.]$ denotes the expectation operator given the information available at date t. In contrast to the time-additive expected utility specification this recursive non-expected utility representation clearly distinguishes time and risk preference because the elasticity of intertemporal substitution is not restricted to equal the inverse of the parameter of risk aversion.⁸ As has been shown by EPSTEIN/ZIN (1989) and WEIL (1989) the Euler equation for this utility specification is

$$1 = E\left\{ \left[\beta \left(\frac{\tilde{c}_{t+1}}{c_t} \right)^{-\rho} \right]^{\frac{1-\alpha}{1-\rho}} \left(1 + \tilde{r}_{t+1}^m \right)^{\frac{1-\alpha}{1-\rho}-1} \left(1 + \tilde{r}_{t+1}^i \right) \right\}.$$
 (7)

Notice that the intertemporal marginal rate of substitution in this case depends not only on the random growth rate of consumption, \tilde{c}_{t+1}/c_t , but also on the random return on the market portfolio, \tilde{r}_{t+1}^m .⁹

If $\alpha = \rho$ we get the Euler equation for the time-additive expected utility (equation (3)). Thus the well known time-additive expected utility representation in equation

⁷The elasticity of intertemporal substitution is defined as $\sigma = (\partial \frac{c_{t+1}}{c_t} / \partial IMRS)(IMRS/\frac{c_{t+1}}{c_t})$, where *IMRS* is the intertemporal marginal rate of substitution between a deterministic unit of consumption at date t and date t + 1.

⁸By choosing the relation between the preference over the smoothness of the consumption path and the risk preference it is possible to model investors with specific preferences about the resolution of uncertainty in temporal lotteries. In the presented parametrization of the recursive KREPS/PORTEUS preference the investor prefers early (late) resolution of uncertainty if $\alpha > (<)\rho$. For $\alpha = \rho$ (risk aversion equals the inverse of the elasticity of intertemporal substitution) the investor is indifferent about the resolution of uncertainty. In this case the non-expected utility equals the time-additive expected utility.

⁹From the Euler equation for time-additive expected utility (3) the classical consumption-based CAPM, $E_t[\tilde{r}_{t+1}^i - r_{t+1}^f] = (E_t[(\tilde{c}_{t+1}/c_t)^{-\alpha}])^{-1}cov_t[-(\tilde{c}_{t+1}/c_t)^{-\alpha}; \tilde{r}_{t+1}^i]$, can be derived. In this C-CAPM systematic risk of an asset is measured by the covariance of its return with consumption growth (LUCAS (1978), BREEDEN (1979)), whereas in the market-portfolio-based CAPM systematic risk is measured by the covariance with the return on the market. From the Euler equation for the recursive non-expected utility (7) a two-factor asset pricing model can be derived, where the systematic risk is measured by the covariance with consumption growth and the covariance with the return on the market (GIOVANNINI/WEIL (1989:9f), EPSTEIN (1992:32f)). Thus this generalization of the utility representation leads to a combination of the consumption-based and the market-portfolio-based CAPM.

⁶For $\rho = 1$ the recursive utility function is $V_t(c_t, \tilde{V}_{t+1}) = c_t^{1-\beta} (E_t[\tilde{V}_{t+1}^{1-\alpha}])^{\frac{\beta}{1-\alpha}}$. Using L'Hôpital it can be shown that this is the limit of equation (6) for $\rho \to 1$. For $\alpha = 1$ the utility function equals $V_t(c_t, \tilde{V}_{t+1}) = [(1-\beta)c_t^{1-\rho} + \beta(\exp(E(\ln(\tilde{V}_{t+1}))))^{1-\rho}]^{\frac{1}{1-\rho}}$ and for $\alpha = 1$ and $\rho = 1$ $V_t(c_t, \tilde{V}_{t+1}) = c_t^{1-\beta}(\exp(E(\ln(\tilde{V}_{t+1}))))^{\beta}$.

(2) is a special case of the more general recursive non-expected utility representation (equation (6)).

Using equation (1) the Euler equation (7) yields in the case of the market portfolio

$$1 = E\left\{ \left[\beta \cdot \tilde{\lambda}_{t+1}^{-\rho} \right]^{\frac{1-\alpha}{1-\rho}} \left(\tilde{\lambda}_{t+1} \frac{1+\Theta}{\Theta} \right)^{\frac{1-\alpha}{1-\rho}} \right\}$$
$$\Leftrightarrow \left(\frac{1+\Theta}{\Theta} \right)^{\frac{1-\alpha}{1-\rho}} = \beta^{-\frac{1-\alpha}{1-\rho}} \left(E\left[\tilde{\lambda}_{t+1}^{1-\alpha} \right] \right)^{-1}$$
$$\Leftrightarrow \frac{1+\Theta}{\Theta} = \beta^{-1} \left(E\left[\tilde{\lambda}_{t+1}^{1-\alpha} \right] \right)^{-\frac{1-\rho}{1-\alpha}}$$

and thus

$$\tilde{r}_{t+1}^{m} = \beta^{-1} \left(E\left[\tilde{\lambda}_{t+1}^{1-\alpha} \right] \right)^{-\frac{1-\rho}{1-\alpha}} \tilde{\lambda}_{t+1} - 1.$$
(8)

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The risk-free rate of return, r_{t+1}^{f} , can be obtained from the Euler equation (7) and equation (8) as follows

$$1 = E\left\{\left[\beta \cdot \tilde{\lambda}_{t+1}^{-\rho}\right]^{\frac{1-\alpha}{1-\rho}} \left(\beta^{-1} \left(E\left[\tilde{\lambda}_{t+1}^{1-\alpha}\right]\right)^{-\frac{1-\rho}{1-\alpha}} \cdot \tilde{\lambda}_{t+1}\right)^{\frac{1-\alpha}{1-\rho}-1} \left(1+r_{t+1}^{f}\right)\right\}$$

$$\Leftrightarrow \frac{1}{1+r_{t+1}^{f}} = E\left\{\beta \cdot \tilde{\lambda}_{t+1}^{-\alpha} \left(E\left[\tilde{\lambda}_{t+1}^{1-\alpha}\right]\right)^{\frac{1-\rho}{1-\alpha}-1}\right\}$$

$$\Leftrightarrow r_{t+1}^{f} = \beta^{-1} \left(E\left[\tilde{\lambda}_{t+1}^{-\alpha}\right]\right)^{-1} \left(E\left[\tilde{\lambda}_{t+1}^{1-\alpha}\right]\right)^{1-\frac{1-\rho}{1-\alpha}} - 1.$$
(9)

Note, that for $\alpha = \rho$ the equations (8) and (9) equal their time-additive expected utility counterparts (4) and (5).

3 Model with a two-state Markov switching process of production growth

3.1 The economy

In the illustrative example in the last section an i.i.d. process (e.g. a single normal distribution) was used for the growth rate of production. However, empirical data are often autocorrelated or skewed and exhibit excess kurtosis or nonstationarities like volatility clusters or boom and depression states. Therefore we will now introduce a Markov switching process for the growth rate of production. The process is again stationary but the distribution of the growth rate of production is conditional on the state of the Markov switching process. We assume that the growth rate of production is normally distributed in each state and hence call this process a Markov mixture of normal distributions.

A Markov mixture of normal distributions has several advantages. First, it allows us to model skewness, excess kurtosis and first-order autocorrelation in the distribution of the growth rate. Second, nonstationarities can be modeled and third, conditional expectations can be modeled since at date t the investors may have different expectations about the future growth depending on the state at date t. We analyse an economy which can switch between two growth states. Therefore we assume the distribution of $\ln(\tilde{\lambda}_t)$ to be an ergodic Markov mixture of two normal distributions $N(\mu_{c1}, \sigma_{c1}^2)$ and $N(\mu_{c2}, \sigma_{c2}^2)$. For this switching model with two states we get a 2 × 2 transition matrix with the transition probabilities $p_{ij} = Prob(\ln(\tilde{\lambda}_{t+1}) \sim N(\mu_{cj}, \sigma_{cj}^2) | \ln(\tilde{\lambda}_t) \sim N(\mu_{ci}, \sigma_{ci}^2))$ for $i, j \in \{1, 2\}$.

In contrast to the i.i.d. production growth the ex dividend market value of the productive sector at date t, M_t , now equals $\Theta_i \cdot m_t$, $i \in \{1, 2\}$, where Θ_i is statedependent. Hence the net return on the market portfolio in a period from state i to state j can be expressed as

$$\tilde{r}_{ij}^m = \frac{\tilde{\lambda}_j m_t + \tilde{M}_{j,t+1}}{M_{i,t}} - 1 = \frac{\tilde{\lambda}_j m_t + \Theta_j \tilde{\lambda}_j m_t}{\Theta_i m_t} - 1 = \tilde{\lambda}_j \frac{1 + \Theta_j}{\Theta_i} - 1 \quad \text{for} \quad i, j \in \{1, 2\}.(10)$$

3.2 Equilibrium returns: expectation and variance

Since the time-additive expected utility specification is the special case of the nonexpected recursive utility specification where $\alpha = \rho$, it is sufficient to derive the equilibrium returns and the equilibrium risk premia for the non-expected utility specification. This derivation follows that of KANDEL/STAMBAUGH (1991) and HUNG (1994) but is generalized to allow a two state Markov mixture of normal distributions with state-dependent variances for the exogenous production (= consumption) process.

3.2.1 Return on the market portfolio

It has been noted that the marginal rate of substitution depends on the return on the market portfolio. This is the reason why we have to determine the expected equilibrium return on the market first.

Substituting equation (10) together with the transition probabilities p_{ii} and p_{ij} and $\tilde{c}_{t+1}/c_t = \tilde{\lambda}_{t+1}$ the Euler equation (7) for an economy in state *i* now yields in the case of the market portfolio

$$1 = \sum_{j=1}^{2} p_{ij} E\left\{ \left[\beta \tilde{\lambda}_{j}^{-\rho}\right]^{\frac{1-\alpha}{1-\rho}} \left(\tilde{\lambda}_{j} \frac{1+\Theta_{j}}{\Theta_{i}}\right)^{\frac{1-\alpha}{1-\rho}} \right\}$$

$$\Leftrightarrow \Theta_{i}^{\frac{1-\alpha}{1-\rho}} = \sum_{j=1}^{2} p_{ij} \beta^{\frac{1-\alpha}{1-\rho}} E\left[\tilde{\lambda}_{j}^{1-\alpha}\right] (1+\Theta_{j})^{\frac{1-\alpha}{1-\rho}} \quad \text{for} \quad i \in \{1,2\}.$$
(11)

Given the distribution of $\tilde{\lambda}_i$, $E[\tilde{\lambda}_i^{1-\alpha}]$ equals $\exp[(1-\alpha)\mu_{ci}+0.5(1-\alpha)^2\sigma_{ci}^2]$. Equation (11) is a system of two non-linear equations with the unknowns Θ_1 and Θ_2 which in general cannot be solved analytically.¹⁰ For the time-additive expected utility specification, where $\alpha = \rho$, equation (11) becomes a system of two linear equations

$$\Theta_i = \sum_{j=1}^2 p_{ij} \beta E[\tilde{\lambda}_j^{1-\alpha}](1+\Theta_j) \quad \text{for} \quad i \in \{1,2\}$$
(12)

¹⁰For the calibration exercise this system is solved numerically using the SAS procedure "model".

which can be solved analytically. Solving this system yields

$$\Theta_{i} = \frac{q_{ii}(1-q_{jj}) + q_{ij}(1+q_{ji})}{1-q_{ii} - q_{jj} + q_{ii}q_{jj} - q_{ij}q_{ji}} \quad \text{for} \quad i, j \in \{1, 2\} \quad \text{and} \quad i \neq j$$
(13)

where $q_{ii} = p_{ii}\beta E[\tilde{\lambda}_i^{1-\alpha}]$ and $q_{ij} = p_{ij}\beta E[\tilde{\lambda}_j^{1-\alpha}]$.

Substituting the calculated Θ_1 and Θ_2 in equation (10) the expected return on the market portfolio from state *i* to state *j* equals $E[\tilde{r}_{ij}^m] = E[\tilde{\lambda}_j](1+\Theta_j)/\Theta_i - 1$, where $E[\tilde{\lambda}_j] = \exp(\mu_{cj} + 0.5\sigma_{cj}^2)$. The unconditional expected return on the market portfolio follows as

$$E[\tilde{r}^m] = \sum_{i=1}^2 \sum_{j=1}^2 p_i p_{ij} E[\tilde{r}^m_{ij}].$$
 (14)

6

The variance of the return on the market portfolio from state *i* to state *j* is calculated from equation (10) and the distribution of $\tilde{\lambda}_j$ as follows: $Var[\tilde{r}_{ij}^m] = Var[\tilde{\lambda}_j(1 + \Theta_j)/\Theta_i - 1] = ((1 + \Theta_j)/\Theta_i)^2 \exp[2\mu_{cj} + \sigma_{cj}^2](\exp[\sigma_{cj}^2] - 1)$. The unconditional variance of the return on the market portfolio then is

$$Var[\tilde{r}^{m}] = \sum_{i=1}^{2} \sum_{j=1}^{2} p_{i} p_{ij} (Var[\tilde{r}_{ij}^{m}] + (E[\tilde{r}_{ij}^{m}] - E[\tilde{r}^{m}])^{2}).$$
(15)

3.2.2 Risk-free rate of return

Knowing the expected return on the market portfolio from state i to state j, the equilibrium risk-free rate can be calculated. Using the transition probabilities p_{ii} and p_{ij} together with equation (10), the Euler equation (7) can be rewritten for the risk-free rate and an economy in state *i* as

$$\frac{1}{1+r_i^f} = \sum_{j=1}^2 p_{ij} E\left\{ \left[\beta \tilde{\lambda}_j^{-\rho}\right]^{\frac{1-\alpha}{1-\rho}} \left(\tilde{\lambda}_j \frac{1+\Theta_j}{\Theta_i}\right)^{\frac{1-\alpha}{1-\rho}-1} \right\}$$

$$\Leftrightarrow r_i^f = \left(\sum_{j=1}^2 p_{ij} \beta^{\frac{1-\alpha}{1-\rho}} E[\tilde{\lambda}_j^{-\alpha}] \left(\frac{1+\Theta_j}{\Theta_i}\right)^{\frac{1-\alpha}{1-\rho}-1} \right)^{-1} - 1 \quad \text{for} \quad i \in \{1,2\}.$$
(16)

Given the distribution of $\tilde{\lambda}_i$, $E[\tilde{\lambda}_i^{-\alpha}]$ equals $\exp[-\alpha\mu_{ci} + 0.5\alpha^2\sigma_{ci}^2]$. For the time-additive expected utility, where $\alpha = \rho$, equation (16) becomes

$$r_{i}^{f} = \left(\sum_{j=1}^{2} p_{ij} \beta E[\tilde{\lambda}_{j}^{-\alpha}]\right)^{-1} - 1 \quad \text{for} \quad i \in \{1, 2\}.$$
(17)

Using the unconditional stationary probabilities for state 1 and 2, p_1 and p_2 , the unconditional expected risk-free rate follows from equation (16) or (17) as

$$E[\tilde{r}^f] = p_1 r_1^f + p_2 r_2^f.$$
(18)

The unconditional variance of the risk-free rate of return is calculated as

$$Var[\tilde{r}^{f}] = \sum_{i=1}^{2} p_{i}(r_{i}^{f} - E[\tilde{r}^{f}])^{2}.$$
(19)

3.2.3 Risk premium on the market portfolio

The unconditional expected risk premium on the market portfolio is

$$E[\tilde{p}^m] = E[\tilde{r}^m] - E[\tilde{r}^f]$$
(20)

and the unconditional variance of the risk premium on the stock market is

$$Var[\tilde{p}^{m}] = \sum_{i=1}^{2} \sum_{j=1}^{2} p_{i} p_{ij} (Var[\tilde{r}_{ij}^{m} - r_{i}^{f}] + (E[\tilde{r}_{ij}^{m} - r_{i}^{f}] - E[\tilde{p}^{m}])^{2}).$$
(21)

4 The observed growth rate of consumption

4.1 Description of data

The model requires consumption data for each quarter. Since consumption data always measure consumption expenditures instead of consumption from which utility is gained in the same period, only the usage of those expenditures which are totally converted to utility in the same period is theoretically justifiable.¹¹ Therefore the time series of consumption expenditures on non-durables and services is usually used in this context. Unfortunately no such time series is available for Germany on a quarterly basis.

The employed data on aggregate consumption expenditures are from two different sources. Seasonally adjusted quarterly aggregate consumption expenditure from 1960 to December 1994 on food (C1), clothing and shoes (C2), housing and energy (C3) and other goods and services (C4) have been provided by the Deutsches Institut für Wirtschaftsforschung. More detailed yearly data are available from the Statistisches Bundesamt for the same period. An explicit time series of expenditures on non-durables and services is available since 1980 on a yearly basis. Using the yearly data from the Statistisches Bundesamt it is possible to calculate for example the yearly percentage of C3 spent for energy or the yearly percentage of C4 representing medical expenditures. Quarterly percentage data are estimated from these time series of yearly data in the following way: A continuous curve is fitted to the data by connecting successive straight line segments. Interval midpoints are used as the break points and ordinates are chosen so that the average of the four estimated quarterly data of a year equals the yearly input data.

The following consumption time series is used as non-durables and services: food plus 20 % of clothing and shoes plus energy plus medical expenditures plus information transfer plus personal equipment plus education, entertainment and leisure. The time series of this combination comes closest to that of the yearly non-durables and services time series which is published by the Statistisches Bundesamt since 1980. In order to express the consumption expenditure in per capita terms, we divided this time series by the quarterly average population. Real annualized quarterly per capita growth rates of consumption are gained from $4(\ln(c_{t+1}) - \ln(c_t) - \ln(1 + \pi))$, where c_t is the per capita consumption in quarter t and π is the rate of inflation.

¹¹For a discussion of this argument see Epstein/Zin (1991:272f)

Table 1: Descriptive statistics of annualized quarterly logarithmic real growth rates of per capita consumption expenditures in the period April 1960 to December 1994 (139 observations).

Time series	Mean	Std^a	Skewness	Kurtosis	T-Statistic ^b
Total consumption	.0284	.0343	2083	.1861	9.7732***
Non-durables and services	.0231	.0337	1899	.3636	$8.1007^{***}_{\circ\circ\circ}$

^aStandard deviation.

^{b***} (**, *)[$\circ \circ \circ (\circ \circ, \circ)$]: The hypotheses of zero mean can be rejected at a 1 % (5 %, 10 %) significance level on the basis of the parametric standard T-statistic [on the basis of the nonparametric Wilcoxon-Sign-Rank statistic].

Table 1 exhibits descriptive statistics of this time series. For comparison summary statistics for the growth rate of total consumption are reported, too. The 2.31% average growth rate of consumption expenditures on non-durables and services nearly equals the 2.39% mean observed for the period 1959 to 1978 by MEHRA/PRESCOTT (1985:147) for the U.S.. The total consumption has grown stronger than the consumption of non-durables and services. This seems plausible since with growing wealth the population can spend a larger amount of wealth on durables and luxury goods.

4.2 Estimation of the Markov switching model

Besides the possibility to model nonstationarities and conditional expectations (see section 3.1) the Markov mixture of normal distributions allows us to model excess kurtosis, skewness and first-order autocorrelation. A mixture of two normal distributions with different variances leads to leptokurtic distributions whereas a mixture of two normal distributions with different means and different stationary probabilities leads to skewed distributions. Often negative skewness and leptokurtosis of high frequency financial time series data are reasons for the hypothesis of a single normal distribution to be rejected. For lower frequency data the assumption of a single normal distribution often cannot be rejected. This is also true for the time series of consumption growth calculated above. Tests were made using the Anderson-Darling-Statistic¹² A^2 and the Shapiro-Wilk-Statistic W ($A^2 = .1871$, W = .9928) and the hypothesis of normal distributed consumption growth cannot be rejected. Moreover the time series exhibits only low negative skewness, low excess kurtosis (see table 1) and low first-order autocorrelation (-.02081). Hence, if a Markov switching model does explain the structure of the data significantly better than a single normal distribution, then nonstationarities in the time series (e.g. volatility clusters) must be the reason rather than distributional aspects like skewness, kurtosis or first-order autocorrelation.

¹²The Anderson-Darling-Statistic is based on the vertical distance between the empirical and the theoretical cumulative density function, like the Kolmogorov-Statistic. However D'AGOSTINO/STEPHENS (1986:404) find that the Anderson-Darling-Statistic is more powerful.

Table 2: ML-estimates of the Markov mixtures of two normal distributions for the annualized quarterly real growth rate of per capita consumption expenditures on non-durables and services.

State $i =$	1		2	
p_{ii}	.00000	(.11765) ^a	.73291	(.40496)
$p_i{}^b$.21079		.78921	
μ_{ci}	00122	(.03241)	.02967	(.00529)
σ_{ci}^2	.00127	(.00069)	.00089	(.00016)
LR ^c	-	1.80	91	

^aStandard deviations in parentheses.

^bThe unconditional stationary probabilities of the states are calculated as $p_1 = (1 - p_{22})/(2 - p_{11} - p_{22})$ and $p_2 = 1 - p_1$.

^cLikelihood ratio statistic of H₀: single normal distribution.

The Markov mixture of two normal distributions for consumption growth are estimated using the expectation-maximization (EM) algorithm (HAMILTON (1989, 1990, 1993, 1996)). The difficulty in estimating the model is that the state variable is not observable. BAUM/PETRIE/SOULES/WEISS (1970) showed that, under certain regularity conditions, the EM-algorithm increases the likelihood function monotonically and that it converges to the maximum-likelihood estimates.¹³

The maximum likelihood (ML) estimates are shown in table 2. Standard deviations are given in parentheses. The estimates identify an up-state with low volatility of consumption growth (state 2) and a down-state with high volatility (state 1). Thus our generalization to allow the variance to differ in the states leads to an estimation result consistent with the empirical findings of increased volatilities during recessions, reflecting macroeconomic uncertainty.

The down state is characterized by an average consumption decrease of .1% and the conditional probability of staying in this state is nearly equal to zero. Hence, the down state identified can be interpreted as a short and strong crash state. The first and third graph of figure 8 in appendix A confirm these results. The first graph exhibits the time series of consumption growth. The third graph shows the time series of the smoothed conditional probabilities for being in state 1, the crash state, of the estimated switching model. It can be seen that the model switches fast between the states and that often the current regime cannot be identified exactly as state 1 or state 2.

Until now the discussion has shown that the ML-estimated model does not separate the regimes exactly. To evaluate the estimated Markov mixture we will test, if the Markov mixture describes the time series significantly better than a single normal distribution. As can be seen from figure 1 the density function of the mixture nearly equals that of a single normal distribution and thus does not describe the data significantly better. This result can also be obtained from the insignificant likelihood

¹³For the process we have also calculated a Markov switching model with two deterministic states as applied by MEHRA/PRESCOTT (1985). See appendix B.1.

Figure 1: The empirical density function of the annualized quarterly growth rate of per capita consumption and the density functions of the estimated Markov mixture of two univariate normal distributions in comparision with the density function of a single normal distribution are shown.



ratio statistic (LR=1.8091) which compares the loglikelihood of the Markov mixture with that of the single normal distribution.¹⁴

5 The historical tax adjusted returns

The calibration exercises try to match the first and second moments of the implied equilibrium returns with those returns historically observed. Therefore, this chapter describes the historically observed tax adjusted German risk-free rate and risk premium on the stock market for four different taxation scenarios. Scenario 1 is the zero tax scenario, τ^0 . The scenarios 2 and 3 refer to private investors with a marginal tax rate of 36%, τ_{Priv}^{36} , and private investors with the highest marginal tax rate, τ_{Priv}^{max} . Scenario 4 refers to an institutional investor, $\tau_{Corp}^{.15}$.

Table 3 reports summary statistics for the real annualized quarterly risk-free rate and risk premium in the period January 1960 to December 1994 for the four taxation scenarios. It can be seen that the average risk premia are positive for all scenarios. For the zero tax scenario for example an average annualized quarterly risk-free return of 3.05% and an average risk premium of 3.3% is observed. Increasing the tax rate of the private investor reduces the risk-free rate and increases the risk premium, so that the investor with the highest marginal tax rate receives a negative real risk-free

¹⁴Since the Markov switching model is not identified under the null of no changes in regime the likelihood ratio statistic does not have a standard chi-squared distribution (HAMILTON (1990:61)). Therefore we test the switching model against a single normal distribution using a Monte Carlo approach with 1000 realizations, following CECCHETTI/LAM/MARK (1990:404).

¹⁵For a detailed description of the data and the calculation of the historical tax adjusted returns see appendix C.

Table 3: Descriptive statistics for the calculated historical tax adjusted risk-free rates and risk premia for the four different taxation scenarios in the period January 1960 to December 1994.

Tin	ne series	Mean	Std^a	Skewness	Kurtosis	T-Statistic ^b
	Scenario	$ 1 (\tau^0): $	Annuali	zed ^c quarter	rly real dat	a
r_{τ}^{f}	Risk-free rate	.0305	.0208	1391	1653	17.3301***
$p^s_{ au}$	Risk premium	.0330	.3756	1339	2.3251	1.0405
	Scenario	$2 (au_{Priv}^{36})$: Annua	alized quarte	erly real da	ta
r_{τ}^{f}	Risk-free rate	.0079	.0162	4011	.2698	5.7608***
$p^s_{ au}$	Risk premium	.0424	.3741	1435	2.3357	1.3402_{\circ}
_	Scenario	$\overline{3} (\tau_{Priv}^{max})$: Annua	alized quarte	erly real da	ta
r_{τ}^{f}	Risk-free rate	0038	.0157	3713	.3166	-2.8583***
$p^s_{ au}$	Risk premium	.0472	.3736	1483	2.3353	1.4940_{\circ}
	Scenario	$4 (\tau_{Corp})$: Annua	alized quarte	erly real da	ta
$r_{ au}^{f}$	Risk-free rate	0027	.0157	3611	.2418	-2.0689°**
$p^s_{ au}$	Risk premium	.0144	.1779	0829	2.3932	.9562

^aStandard deviation.

^{b***} (**, *)[$\circ \circ \circ (\circ \circ, \circ)$]: The hypotheses of zero mean can be rejected at a 1 % (5 %, 10 %) significance level on the basis of the parametric standard T-statistic [on the basis of the nonparametric Wilcoxon-Sign-Rank statistic].

^cThe quarterly data are multiplied by 4 before the calculation of the descriptive statistics.

rate of .38% and a risk premium of 4.72%.¹⁶ Comparing the results shown in table 3 with those shown in table 1 we find the well known result that the time series of consumption growth is much smoother than the time series of the risk premia on the stock market.

6 Calibration results

We now analyze the expected equilibrium returns implied by the model, dependent on the parameters of the preference function. We compare the results using time-additive expected utility (section 6.1) with those using non-expected recursive utility (section 6.2). For the stochastic exogenous input we use the mixture of univariate normal distributions as described in section 4.2.¹⁷ We use graphs to illustrate the implied risk-free rates and risk premia for different combinations of the preference parameters. First, we analyze the implied returns using time-additive expected utility.

¹⁶Resulting from the German tax system the highest risk premia are earned by private investors with the highest marginal tax rate. In contrast to the standard assumption of constant or decreasing relative risk aversion this favours increasing relative risk aversion. Hence, an existing constant or decreasing relative risk aversion in combination with the tax system could be a reason for the low attractiveness of the German stock market to most Germans.

¹⁷For a comparison of the calibration results applying switching models with stochastic states described in this section with those applying switching models with deterministic states see appendix B.1.

Figure 2: Using TIME-ADDITIVE EXPECTED UTILITY: set of admissible average risk premia and risk-free rates using the ML-estimated univariate model. Discount factor $\beta \leq 1$ in the left graph and $\beta \leq 1.02$ in the right graph. Risk aversion $\alpha \geq 0$.



6.1 Using time-additive expected utility

We calculate the implied returns for all combinations of discount factor β , $\beta \in \{.9, .901, \ldots, 1\}$, and risk aversion α , $\alpha \in \{0, .1, \ldots, 10\}$. The left graph in figure 2 illustrates the implied risk-free rate / risk premium - combinations for risk-free rates $\leq 5\%$. We find that no risk premium larger than .28 % can be obtained with the specified equilibrium model for risk-free rates $\leq 5\%$. Even an increase in the parameter of risk aversion, α , does not help to resolve this puzzle. The highest risk premium for risk-free rates $\leq 5\%$ is obtained using $\alpha = 2.2$ and $\beta = 1$. An increase in α does not only increase the risk premium but also increases the risk-free rate. Decreasing β leads to higher risk-free rates at the same level of risk premia. Thus, the Equity Premium Puzzle does not only describe the fact that high risk premia can only be obtained with implausibly high parameters of risk aversion but also that those implausibly high parameters of risk aversion but also that those implausibly high parameters of risk aversion.¹⁸

The results show that for the time-additive expected utility specification an increase in risk aversion does not only increase the risk premium but also increases the riskfree rate. This effect results from the fact that in this preference function risk aversion α and the elasticity of intertemporal substitution σ are not separately parametrized (see section 2.3). Since σ is restricted to be the inverse of α , an increase in α decreases σ . The low elasticity of intertemporal substitution σ leads to a higher risk-free rate and, hence, the increase in risk aversion α increases the risk-free rate and the risk premium simultaneously. KOCHERLAKOTA (1990a, 1990b) argues that the implicit relation between risk aversion α and elasticity of intertemporal substitution σ in the time-additive expected utility makes it economically plausible to allow the discount factor β to increase beyond 1. If, for example, an investor with constant time preference, i.e. constant elasticity of intertemporal substitution σ and constant discount factor β , increases his risk aversion, this increase in α leads implicitly to

¹⁸For a discussion of the plausible level of risk aversion see, for example, MEHRA/PRESCOTT (1985:154).

Figure 3: Using NON-EXPECTED RECURSIVE UTILITY: set of admissible average risk premia and risk-free rates using the ML-estimated univariate model. Discount factor $\beta = .99$ in the left graph and $\beta = .97$ in the right graph. Risk aversion $.5 \le \alpha \le 10$ and elasticity of intertemporal substitution $.45 \le \sigma \le 1.85$. In addition, the right graph shows the returns for $\sigma = 3, 6$ and 9.



a decrease in σ and hence an increase in the risk-free rate. But, since the investor has a constant time preference the risk-free rate should stay almost constant. To keep the risk-free rate constant we may increase the discount factor β beyond 1. Thus, for high parameters of risk aversion even discount factors larger than 1 may be plausible as compensation for the implicitly chosen low elasticity of intertemporal substitution.

The right graph in figure 2 presents the implied returns for discount factor $\beta \leq 1.02$. The increase in β from 1 to 1.02 raises the obtainable risk premia by about 50% and, hence, risk premia up to .42% can be obtained for risk-free rates lower of equal to 5%. Thus, for a given risk-free rate the risk premium can be raised by simultaneously increasing the coefficient of risk aversion α and the discount factor β . For example with $\alpha = 10$ and $\beta = 1.1$ the model yields a risk premium of about 1% for a risk-free rate of 5%.

6.2 Using non-expected recursive utility

We now analyse the implied returns using non-expected recursive utility. Therefore, we calculate the implied returns for different combinations of the parameters of the preference function. The discount factor is firstly assumed to be constant and lower than 1, $\beta = .99$; the risk aversion is chosen as $\alpha \in \{.5, 2, 3, ..., 10\}$ and the elasticity of intertemporal substitution is $\sigma \in \{.45, .55, ..., 1.85\}$. The left graph in figure 3 illustrates the implied risk-free rates and risk premia for risk-free rates $\leq 5\%$. We observe that for risk-free rates $\leq 5\%$ risk premia up to 1.3% can be obtained with risk aversion $\alpha \leq 10$ and discount factor $\beta = .99$.

The graph shows that for constant elasticity of intertemporal substitution an increase in the risk aversion leads to an increase in the risk premium and also affects the risk-free rate. However, this effect is much smaller than for time-additive expected utility and decreases the risk-free rate. A small decrease in the risk-free rate for an increase in risk aversion is plausible, because higher risk aversion may lead to an adjustment in the investor's portfolio to include more risk-free assets and less risky assets, resulting in a simultaneously increasing risk premium and a decreasing risk-free rate.¹⁹

A change in the elasticity of intertemporal substitution clearly influences the risk-free rate and has nearly no effect on the risk premium. A high elasticity of intertemporal substitution produces a lower risk-free rate and vice versa. High elasticity of intertemporal substitution means that β -adjusted consumption at different dates are nearly perfect substitutes to each other so that the investors are indifferent between a smooth or a volatile consumption path whereas investors with low elasticity of intertemporal substitution prefer a smoother consumption path. Endowed with a relatively smooth consumption path, investors with a low elasticity of intertemporal substitution are only willing to unsmooth their consumption by giving up one unit of consumption at date t + 1. Hence, a low elasticity of intertemporal substitution requires higher risk-free rates.

The theoretically satisfying result is that the parameter of risk aversion determines the risk premium whereas the elasticity of intertemporal substitution and the discount factor determine the risk-free rate. However, as in the case of time-additive expected utility, with risk aversion $\alpha \leq 10\%$ only risk premia up to about 1% can be obtained for risk-free rates $\leq 5\%$.

A decrease in β changes the results as expected. The right graph in figure 3 illustrates the same situation as the left graph but for $\beta = .97$ instead of $\beta = .99$. We find that the risk-free rate decreases with the decrease in β and the risk premium stays almost constant.

8

6.3 Summarizing the results

Non-expected recursive specifies the risk preference and the time preference separately. The risk premium is determined by the risk aversion whereas the risk-free rate is determined by the discount factor and the elasticity of intertemporal substitution. This result is theoretically more satisfying than compensating the wrong but implicitly chosen elasticity of intertemporal substitution through discount factors beyond 1 in the time-additive expected utility framework. Although non-expected recusive utility allows to decrease the risk-free rate, holding the discount factor constant and lower than 1, the historically observed risk-premia can still only be obtained with implausibly high parameters of risk aversion.

7 Analysis of a levered market portfolio

7.1 Stock market equals levered market portfolio

Until now the model assumed that the market portfolio pays off the total production as dividends. In reality stockholders receive only the residual after payments on fixed

¹⁹KIMBALL/WEIL (1992) find in a two period setting that higher risk aversion tends to increase the strength of the precautionary saving motive using preferences which allow risk preference and intertemporal substitution to be varied independently.

claims as wages or interest payments on debt have been deducted. Hence, the payoff to the stockholders in reality is more risky than in the model discussed until now. This may be a reason why the model does not yield risk premia comparable to those observed historically.

Now the firms are assumed to be financed by debt and equity. The portfolio of equity, i.e. the stock market, is equal to the levered market portfolio. It is assumed that at every date t a constant proportion L, 0 < L < 1, of the market portfolio with ex dividend value M_t is financed with one-period debt²⁰ so that the dividend on equity equals the production (= consumption) minus interest payments on debt²¹. Since the real risk-free rate does not fluctuate very much, dividend and production (= consumption) are strongly correlated.

7.2 Return and risk premium on the levered market portfolio

For the exogenous production modeled by a two-state Markov switching model, the expected return on the stock market in a period from state i to state j follows as

$$E[\tilde{r}_{ij}^s] = E[\tilde{r}_{ij}^m] + (E[\tilde{r}_{ij}^m] - r_i^f) \frac{L}{1 - L} \quad \text{for} \quad i, j \in \{1, 2\} \quad \text{and} \quad 0 \le L < 1$$
(22)

where L is the proportion of the market portfolio financed with debt, $E[\tilde{r}_{ij}^m]$ is the expected return on the market portfolio in a period from state *i* to state *j* as calculated in section 3.2.1 and r_i^f is the risk-free rate of return. The unconditional expected return on the stock market then follows as

$$E[\tilde{r}^s] = \sum_{i=1}^2 \sum_{j=1}^2 p_i p_{ij} E[\tilde{r}^s_{ij}].$$
(23)

The variance of the return on the stock market from state *i* to state *j* is calculated from this equation as $Var[\tilde{r}_{ij}^s] = (1-L)^{-2} Var[\tilde{r}_{ij}^m]$. The unconditional variance of the return on the stock market equals

$$Var[\tilde{r}^{s}] = \sum_{i=1}^{2} \sum_{j=1}^{2} p_{i} p_{ij} (Var[\tilde{r}^{s}_{ij}] + (E[\tilde{r}^{s}_{ij}] - E[\tilde{r}^{s}])^{2}).$$
(24)

The unconditional expected risk premium on the stock market is

$$E[\tilde{p}^s] = E[\tilde{r}^s] - E[\tilde{r}^f]$$
(25)

and the unconditional variance of the risk premium on the stock market is

$$Var[\tilde{p}^{s}] = \sum_{i=1}^{2} \sum_{j=1}^{2} p_{i} p_{ij} (Var[\tilde{r}_{ij}^{s} - r_{i}^{f}] + (E[\tilde{r}_{ij}^{s} - r_{i}^{f}] - E[\tilde{p}^{s}])^{2}).$$
(26)

 $^{^{20}}$ It is assumed that bankruptcy does not occur. Thus interest payments are equal to the risk-free rate.

²¹ More general: consumption equals dividends plus payments on fixed claims (e.g. interest payments, wages ...).

Figure 4: Using TIME-ADDITIVE EXPECTED UTILITY and LEVERAGE: set of admissible average risk premia and risk-free rates using the ML-estimates of the univariate model. Three levels of leverage are compared using $\beta = 1$ in the left graph and $\beta = 1.1$ in the right graph. The historical returns for the four taxation scenarios are marked in the right graph.



Table 4: Using TIME-ADDITIVE EXPECTED UTILITY and LEVERAGE: First and second moments of the expected equilibrium risk-free rate and risk premium for specific parameter combinations of the preference function and the ML-estimated univariate model.

Process	Model parameter ^a		Risk-free rate		Risk premium		
	α	β	L	$E[\tilde{r}^f]$	$Std[\tilde{r}^{f}]$	$E[ilde{p}^s]$	$Std[ilde{p}^s]$
Historical taxation scenario 1^b				.0305	.0208	.0330	.3756
Univariate model	5.3	1.080	.8	.0305	.0205	.0350	.2011

 ${}^a\alpha$ is the parameter of constant relative risk aversion, β is the discount factor and L is the level of leverage.

^bThe taxation scenario 1 is the zero-tax scenario.

7.3 Calibration results with leverage

We start using time-additive expected utility. The left graph of figure 4 exhibits the implied returns for discount factor $\beta = 1$ and leverage L equal to 0, .4 or .8. Since the highest risk premia are obtained for discount factor $\beta = 1$, only those implied returns are presented. As can be seen, an increase in leverage leads to higher risk premia, holding risk aversion α and discount factor β constant. However, with leverage $L \leq .8$ and $\beta \leq 1$ no risk premium larger than 1.4% can be obtained for risk-free rates $\leq 5\%$ and thus, introducing leverage is not sufficient to explain the historical risk premia.

In addition to leverage, we again allow the discount factor β to increase beyond 1. The right graph of figure 4 presents the implied returns for discount factor $\beta = 1.1$ and leverage L equal to 0, .4 or .8. The empirically observed combinations of risk-free rates and risk premia for the four taxation scenarios are marked in this graph. It can be seen that scenario 1 and scenario 4 exhibit lower risk preFigure 5: Using NON-EXPECTED RECURSIVE UTILITY and LEVERAGE: set of admissible average risk premia and risk-free rates using the ML-estimated univariate model with leverage L = .8. $\beta = .99$ in the left graph and $\beta = .97$ in the right graph. The historical returns for the four taxation scenarios are marked.



mia than those obtained with $\beta = 1.1$ and L = .8. This result parallels that of BENNINGA/PROTOPAPADAKIS (1990). They find that with leverage and discount factors larger than 1 the MEHRA/PRESCOTT (1985) Equity Premium Puzzle can be resolved.

Table 4 presents the first and second moments of the implied risk-free rate and risk premium for a specific parameter combination of the preference function. Comparing line 1 with line 2 of this table shows that the univariate model discussed until now produces the average risk-free rate and risk premium of taxation scenario 1 with risk aversion $\alpha = 5.3$, discount factor $\beta = 1.08$ and leverage L = .8. The standard deviation of the risk-free rate is nearly equal to the standard deviation observed historically, whereas the standard deviation of the risk premium is smaller than the value observed historically.

In the next step we use the non-expected recursive utility specification. The left graph of figure 5 exhibits the implied returns for leverage L = .8 and shows the known result that leverage increases the risk premia. The first moments of the returns of the historical scenario 1 can be obtained with risk aversion $\alpha = 5.8$, discount factor $\beta = .99$, elasticity of intertemporal substitution $\sigma = .88$ and leverage L = .8 (see second line in table 5). In contrast to the time-additive expected utility the implied standard deviation of the risk-free rate is too small to match the value observed historically.

A decrease in β changes the results as expected. The right graph in figure 5 exhibits the same situation as the left graph but for $\beta = .97$ instead of $\beta = .99$. We find that the risk-free rate decreases with the decrease in β and the risk premium stays more or less constant. The third and fourth row of table 5 show that with decreasing discount factor, $\beta = .98$ and $\beta = .97$, we have to increase the elasticity of intertemporal substitution to produce the first moments of the historically observed returns of scenario 1, whereas the parameter of risk aversion is nearly unchanged. Table 5: Using NON-EXPECTED RECURSIVE UTILITY and LEVERAGE: First and second moments of the expected equilibrium risk-free rate and risk premium for specific parameter combinations of the preference function and the ML-estimated univariate model.

Process	Model parameter ^a			Risk-f	Risk-free rate		Risk premium	
	α	eta	σ	L	$E[\tilde{r}^f]$	$Std[\tilde{r}^f]$	$E[\tilde{p}^s]$	$Std[ilde{p}^s]$
Historical scenario 1^b					.0305	.0208	.0330	.3756
Univariate model	5.8	.99	.88	.8	.0306	.0046	.0333	.1735
Univariate model	5.85	.98	1.55	.8	.0306	.0027	.0331	.1711
Univariate model	5.95	.97	7.45	.8	.0304	.0008	.0331	.1689

 $^{a}\alpha$ is the parameter of constant relative risk aversion, β is the discount factor, σ is the elasticity of intertemporal substitution and L is the level of leverage.

^bThe taxation scenario 1 is the zero-tax scenario.

7.4 Summarizing the results with leverage

It is shown that the first and second moment of the historically observed risk-free rate and the first moment of the risk premium on the stock market (scenario 1) are consistent with the univariate model with leverage (L = .8) and time-additive expected utility with risk aversion α near 5 and discount factor β near 1.1.

6

Using the univariate model with leverage (L = .8) and non-expected recursive utility the first moments of the historically observed risk-free rate and the risk premium are consistent with risk aversion $\alpha = 5.8$, discount factor $\beta = .99$ and elasticity of intertemporal substitution σ near .9. The implied standard deviation of the risk-free rate is smaller than in the case of expected utility and cannot explain the standard deviation observed historically.

For both preference specifications the implied standard deviation of the risk premium is about 50% of that observed historically.

8 Separating consumption and dividends

8.1 The bivariate model

The models analyzed until now assumed the stock market to be equal to the market portfolio resp. the levered market portfolio, where the market portfolio contains the whole productive sector and pays off the total production of the economy. The stock market in reality, however, does not equal the whole productive sector of the economy. Thus, the assumption of equal or strongly correlated aggregate production (= consumption) and stock market dividends should be relaxed. Another argument in favour of this demand is that the observed consumption data are heavily smoothed in comparison to stock market dividends. Consumption smoothing can result from stock-keeping of the consumption good or from export and import.

Therefore we now assume the productive sector and thus the market portfolio to consist of two separate firms. Together they produce \tilde{m}_t which equals \tilde{c}_t . Only one

of these firms is traded at the stock market. It has the ex dividend market value S_t at date t and its exogenous stochastic production (= dividend) \tilde{s}_t exhibits a random gross growth rate $\tilde{\gamma}_{t+1} = \tilde{s}_{t+1}/s_t$. The date t + 1 net return of this firm is $\tilde{r}_{t+1}^s = (\tilde{s}_{t+1} + \tilde{S}_{t+1})/S_t - 1$. \tilde{s}_t and \tilde{m}_t are separate stochastic variables and, thus, aggregate production (= consumption) and stock market dividend need not be equal or strongly correlated. We assume that the joint distribution of $\ln(\tilde{\lambda}_t)$, the growth rate of aggregate consumption, and $\ln(\tilde{\gamma}_t)$, the growth rate of stock market dividends, is a Markov mixture of two bivariate normal distributions $N(\mu_{c1}, \mu_{s1}, \sigma_{c1}^2, \sigma_{s1}^2, \sigma_{cs1})$ and $N(\mu_{c2}, \mu_{s2}, \sigma_{c2}^2, \sigma_{s2}^2, \sigma_{cs2})$. Again we get a 2×2 transition matrix. This approach will be referred to as the bivariate model.

Parallel to the argument in section 2.1 and 3.1 the ex dividend market value of the firm traded at the stock market at date t, S_t , equals $\Psi_i \cdot s_t$, $i \in \{1, 2\}$, where s_t is the current level of dividend on the stock market and Ψ_i is state- and preference-dependent.²² Hence the net return on the stock market in a period from state i to state j follows as

$$\tilde{r}_{ij}^{s} = \frac{\tilde{\gamma}_{j}s_{t} + \tilde{S}_{j,t+1}}{S_{i,t}} - 1 = \frac{\tilde{\gamma}_{j}s_{t} + \Psi_{j}\tilde{\gamma}_{j}s_{t}}{\Psi_{i}s_{t}} - 1 = \tilde{\gamma}_{j}\frac{1 + \Psi_{j}}{\Psi_{i}} - 1 \quad \text{for} \quad i, j \in \{1, 2\}.$$
(27)

For the market portfolio, which consists of both firms, the firm traded at the stock market and the other firm, equation (10) still holds.

8.2 Return and risk premium on the stock market

The derivation of the expected return on the stock market is quite similar to that of the expected return on the market portfolio. Again we start with the Euler equation (7):

$$1 = E\left\{ \left[\beta \left(\frac{\tilde{c}_{t+1}}{c_t} \right)^{-\rho} \right]^{\frac{1-\alpha}{1-\rho}} (1 + \tilde{r}_{t+1}^m)^{\frac{1-\alpha}{1-\rho}-1} (1 + \tilde{r}_{t+1}^s) \right\}.$$
 (28)

Using (10) and (27) together with the transition probabilities p_{ii} and p_{ij} and $\tilde{c}_{t+1}/c_t = \tilde{\lambda}_{t+1}$ the Euler equation (28) yields a system of two linear equations with the unknowns Ψ_1 and Ψ_2

$$1 = \sum_{j=1}^{2} p_{ij} E\left\{ \left[\beta \tilde{\lambda}_{j}^{-\rho}\right]^{\frac{1-\alpha}{1-\rho}} \left(\tilde{\lambda}_{j} \frac{1+\Theta_{j}}{\Theta_{i}}\right)^{\frac{1-\alpha}{1-\rho}-1} \tilde{\gamma}_{j} \frac{1+\Psi_{j}}{\Psi_{i}} \right\}$$

$$\Leftrightarrow \Psi_{i} = \sum_{j=1}^{2} p_{ij} \beta^{\frac{1-\alpha}{1-\rho}} E[\tilde{\lambda}_{j}^{-\alpha} \tilde{\gamma}_{j}] \left(\frac{1+\Theta_{j}}{\Theta_{i}}\right)^{\frac{1-\alpha}{1-\rho}-1} (1+\Psi_{j}) \quad \text{for} \quad i \in \{1,2\} \quad (29)$$

 $^{^{22}\}Psi_i$ can be interpreted as a state-dependent price-dividend or price-earnings ratio. By multiplying the current dividend (resp. earnings) with this ratio we obtain the current price of an asset. In the following we derive equilibrium price-dividend (resp. price-earnings) ratios. As intuitively expected these equilibrium ratios depend on the preference function, the risk-free rate and the expected future growth of consumption and dividend (earnings) (see equations (30), (13) and (17)). In future research we will empirically investigate these equilibrium price-dividend (resp. price-earnings) ratios.

which can be solved analytically. Solving this system yields

$$\Psi_{i} = \frac{q_{ii}(1-q_{jj}) + q_{ij}(1+q_{ji})}{1-q_{ii} - q_{jj} + q_{ii}q_{jj} - q_{ij}q_{ji}} \quad \text{for} \quad i, j \in \{1, 2\} \quad \text{and} \quad i \neq j$$
(30)

where $q_{ij} = p_{ij}\beta^{\frac{1-\alpha}{1-\rho}} E[\tilde{\lambda}_j^{-\alpha}\tilde{\gamma}_j]((1+\Theta_j)/\Theta_i)^{\frac{1-\alpha}{1-\rho}-1}$ for $i, j \in \{1, 2\}$. Substituting W_i and W_j in equation (27) the expected return

Substituting Ψ_1 and Ψ_2 in equation (27), the expected return on the stock market from state *i* to state *j* equals $E[\tilde{\tau}_{ij}^s] = E[\tilde{\gamma}_j](1+\Psi_j)/\Psi_i - 1$, where $E[\tilde{\gamma}_j] = \exp(\mu_{sj} + 0.5\sigma_{sj}^2)$ and $E[\tilde{\lambda}_j^{-\alpha}\tilde{\gamma}_j] = \exp(-\alpha\mu_{cj} + \mu_{sj} + 0.5\alpha^2\sigma_{cj}^2 + 0.5\sigma_{sj}^2 - \alpha\sigma_{scj})$. The unconditional expected return on the stock market then equals

$$E[\tilde{r}^s] = \sum_{i=1}^2 \sum_{j=1}^2 p_i p_{ij} E[\tilde{r}^s_{ij}].$$
(31)

The variance of the return on the stock market from state *i* to state *j* is calculated from equation (27) and the distribution of $\tilde{\gamma}_j$ as follows: $Var[\tilde{r}_{ij}^s] = Var[\tilde{\gamma}_j(1 + \Psi_j)/\Psi_i - 1] = ((1 + \Psi_j)/\Psi_i)^2 \exp[2\mu_{sj} + \sigma_{sj}^2](\exp[\sigma_{sj}^2] - 1)$. The unconditional variance of the return on the stock market can then be calculated using equation (24).

The unconditional expected risk premium on the stock market and the unconditional variance of the risk premium follow from equations (25) and (26).

8.3 Growth rate of corporate profits

For the bivariate model, where the growth rate of consumption and the growth rate of dividends are separately modeled, we have to calculate the quarterly time series of the growth rate of per capita dividends of the corporations traded at the stock market. Since no such time series exists for the German stock market from 1960 to 1994 we approximate this time series. All data used in this context have been provided by the Deutsches Institut für Wirtschaftsforschung and the Statistisches Bundesamt.

The stock market dividend in the equilibrium model is the payoff to the owners of the firm traded at the stock market. Again one could argue that in reality this firm is financed with debt and equity and, thus, the owners of equity only receive the residual payment after payments on fixed claims as wages and interests. Instead of using total production of the firm traded at the stock market in combination with a leverage approach, we use original data for the residual payment the owners of equity receive. This could be the historically observed aggregate dividend payment of the stock market, but since dividend payments in reality are usually smoothed in comparison with corporate profits because changes in dividends are interpreted as signals of good or bad news we will use data on corporate profits instead of dividend data.

We assume that the growth rate of the profits of the corporations traded at the stock market equals that of all corporations (i.e. unincorporated enterprises are excluded). This growth rate is calculated as follows: On a yearly basis the ratio of the aggregate profits of all corporations to the aggregate national property and entrepreneurial income is calculated. Quarterly ratios are estimated from this time series of yearly ratios by fitting a continuous curve to the data by connecting successive straight line segments. Interval midpoints are used as the break points and ordinates are chosen so

Table 6: Descriptive statistics of annualized quarterly logarithmic real growth rates of per capita national income in the period April 1960 to December 1994 (139 observations).

Time series	Mean	$\operatorname{Std}^{\overline{a}}$	Skewness	Kurtosis	T-Statistic ^b
Property and entrepr. income	.0188	.1201	2497	.4973	1.8468*
Corporate profits	.0391	.1408	4737	1.6094	$3.2699_{\circ\circ\circ}^{***}$

^aStandard deviation.

that the average of the four estimated quarterly data of a year equals the yearly input data. By multiplying these quarterly ratios with the seasonally adjusted quarterly aggregate national property and entrepreneurial income we obtain an approximation of the aggregate quarterly profits of all corporations. We need this approximation from yearly data to quarterly data because on a quarterly basis only the aggregate time series of national porperty and entrepreneurial income is available. In order to express the corporate profits in per capita terms, we divide the time series obtained by the quarterly average population. Real annualized quarterly per capita growth rates of corporate profits are gained from $4(\ln(s_{t+1}) - \ln(s_t) - \ln(1 + \pi))$, where s_t are the per capita corporate profits in quarter t and π is the rate of inflation.

This appoximated time series of growth rates of corporate profits leads to biased results. Since we assume the corporate profits to be a part of the national property and entrepreneurial income we assume both to be equally volatile although income from property is less volatile than entrepreneurial income.²³ Thus the approximation procedure underestimates the volatility of the growth rate of corporate profits.

Table 6 exhibits descriptive statistics of this time-series. For comparison summary statistics for the growth rate of the total national property and entrepreneurial income are reported, too. The Anderson-Darling-Statistic A^2 and the Shapiro-Wilk-Statistic W for the growth rate of corporate profits are $A^2 = .4787$ and W = .9825 and, thus, the hypothesis of normally distributed growth rates cannot be rejected. The time series exhibits only low negative skewness, (see table 6) and low first-order autocorrelation (.23502). These results are quite similar to those for the growth rate of consumption.

We discussed the argument, that consumption and stock market dividend should not be assumed to be equal or perfectly correlated because consumption growth is smoothed. Comparing the time series in table 6 with those in table 1 confirms this argument; the standard deviation of the growth of corporate profits is more than 4 times larger than the standard deviation of the growth of expenditures on non-

^{b***} (**, *)[$\circ \circ \circ (\circ \circ, \circ)$]: The hypotheses of zero mean can be rejected at a 1 % (5 %, 10 %) significance level on the basis of the parametric standard T-statistic [on the basis of the nonparametric Wilcoxon-Sign-Rank statistic].

 $^{^{23}}$ In research not reported here we find, for example, that on a yearly basis the standard deviation of the nominal growth rate of corporate profits is 1.57 times larger than the standard deviation of the nominal growth rate of the national property and entrepreneurial income, whereas it is only 1.17 (=.1408/.1201) times larger for the approximated quarterly growth rate (table 6).

Table 7: ML-estimates of the Markov mixture of two bivariate normal distributions for the joint distribution of the annualized quarterly real growth rate of per capita consumption and the annualized quarterly real growth rate of per capita corporate profits.

State $i =$	1		2	
p_{ii}	.92254	(.03811)	.87021	(.06480)
$p_i{}^a$.62624		.37376	
μ_{ci}	.01767	(.00439)	.03233	(.00487)
μ_{si}	.00416	(.01729)	.09754	(.02069)
σ_{ci}^2	.00144	(.00025)	.00046	(.00014)
σ_{si}^2 .	.02206	(.00380)	.01023	(.00246)
σ_{csi}	.00052	(.00063)	.00120	(.00044)
LR^{b}		22.1	.835	

^aThe unconditional stationary probabilities of the states are calculated as $p_1 = (1 - p_{22})/(2 - p_{11} - p_{22})$ and $p_2 = 1 - p_1$.

 b Likelihood ratio statistic of H₀: single normal distribution.

durables and services although the approximated growth of corporate profits may still be biased to lower volatility. The correlation of both time series is .2324.

8.4 Estimation of the bivariate Markov switching model

Similar to the Markov mixture of two univariate normal distributions for consumption growth (section 4.2), the Markov mixture of two bivariate normal distributions for the joint consumption/corporate profits growth process is estimated using the expectation-maximization (EM) algorithm.²⁴

The ML-estimates for the bivariate model are shown in table 7. Standard deviations are given in parentheses. The estimates of the bivariate model as well as those of the univariate model (see table 2) identify an up-state with low volatility of consumption growth and growth of corporate profits (state 2) and a down-state with high volatility (state 1). In the bivariate model the variance of consumption growth and the variance of the growth of corporate profits is more than two times larger in the down-state than in the up-state and the covariance of both growth rates is higher in the up-state. Thus, parallel to the estimated univariate model, our generalization to allow the variance and covariance to differ in the states leads to an estimation result consistent with the empirical findings of increased volatilities during recessions, reflecting macroeconomic uncertainty.

Comparing growth of consumption with growth of corporate profits confirms the argument, that consumption growth is smoothed. The average growth of corporate profits switches between .42% and 9.75% whereas the average consumption growth switches between 1.77% and 3.23%. Moreover the volatility of the growth of cor-

²⁴For the bivariate process we have also calculated a Markov switching models with two deterministic states as applied by KANDEL/STAMBAUGH (1990, 1991). See appendix B.1.

porate profits in both states is more than 15 times larger than the volatility of consumption growth.

Let us now compare the estimates of the bivariate model with those obtained for the univariate model (section 4.2). The down state in the univariate model is characterized by an average consumption decrease of .1% whereas the down state identified by the bivariate model exhibits a low increase in consumption of 1.77%. Thus, the univariate model identifies a stronger down state which occurs with lower stationary probability (21%) as compared with the down state of the bivariate model. The conditional probability of staying in one of the states is much smaller in the univariate model than in the bivariate model. Hence, the states identified by the bivariate model can be characterized as longer lasting boom or depression regimes in contrast to short and strong crash- and up-states identified by the univariate model. The graphs in figure 8 in appendix A confirm these results. The upper two graphs exhibit the time series of consumption growth and growth of corporate profits. The lower two graphs show the time series of the smoothed conditional probabilities for being in state 1, the crash, resp. the recession state, of the univariate and the bivariate model. In contrast to the univariate model, which switches fast between the states and that often cannot exactly identify the current regime as state 1 or state 2, the bivariate model produces long up- and down-regimes and the current regime can be identified nearly always exactly as state 1 or 2. The vertical lines in figure 8 separate the recession and boom phases identified by the bivariate model. Comparing the growth rates in the upper two graphs accross these regimes illustrates whe result obtained above. During the boom phases the growth rates are nearly always positive and less volatile than during the recession phases.

The discussion has shown that the ML-estimated bivariate model identifies two states which can be distinguished clearly, whereas the ML-estimated univariate model does not separate the regimes exactly. To evaluate the estimated bivariate model we will test, if the Markov mixtures describe the time series significantly better than a single bivariate normal distribution. The upper part of figure 6 compares the density functions of a single bivariate normal distribution and of the estimated mixture of two bivariate normal distributions. The density function of the mixture is skewed and exhibits excess kurtosis. Both density functions are not equal. The result that the mixture of bivariate normal distributions fits the data significantly better than a single bivariate normal distribution can also be drawn from the large likelihood ratio statistic (LR=22.185). To test the bivariate switching model against the single bivariate distribution we generalize the Monte Carlo approach used for univariate models by CECCHETTI/LAM/MARK (1990:404).²⁵ Only 12 of the 1000 simulated likelihood ratios exceed the likelihood ratio (LR=22.1835) of the estimated model. Thus the hypothesis of a single bivariate normal distribution against the hypothesis of the Markov mixture can be rejected at a 2% level of significance.

²⁵One thousand realizations from samples of 139 growth rates of consumption and 139 growth rates of corporate profits were generated under the null of a bivariate normal distribution with means, variances and correlation set to the values computed from the observed data. To generate the random numbers of the bivariate normal distribution with the prespecified correlation the algorithm described in BOSWELL/GORE/PATIL/TAILLIE (1993:708f) was used. For each realization the Markov switching model was estimated and the likelihood ratio statistic was computed.

Figure 6: The upper part exhibits the density function of a single bivariate normal distribution fitted to the joint consumption/corporate profits growth in comparison with the density function of the estimated Markov mixture of two bivariate normal distributions. In the lower part the estimated density functions of the two states of the Markov mixture are shown.



Figure 7: Set of admissible average risk premia and risk-free rates using the MLestimated BIVARIATE MODEL. TIME-ADDITIVE EXPECTED UTILITY is used in the left graph, NON-EXPECTED RECURSIVE UTILITY is used in the right graph.



Table 8: Using TIME-ADDITIVE EXPECTED UTILITY in the ML-estimated BI-VARIATE MODEL: First and second moments of the expected equilibrium risk-free rate and risk premium for specific parameter combinations of the preference function.

Process	Model parameter ^a		Risk-free rate		Risk premium	
	α	eta	$E[\tilde{r}^f]$	$Std[\tilde{r}^f]$	$E[ilde{p}^s]$	$Std[ilde{p}^s]$
Historical taxation scenario 1^b			.0305	.0208	.0330	.3756
Bivariate model	5.3	1.080	.0314	.0364	.0047	.1377
Modified bivariate model ^c	5.3	1.080	.0364	.0562	.0249	.1460
Modified bivariate model	7.3	1.117	.0307	.0531	.0331	.1408

 ${}^{a}\alpha$ is the parameter of constant relative risk aversion and β is the discount factor.

^bThe taxation scenario 1 is the zero-tax scenario.

^cThe covariances of the ML-estimates are modified to produce perfect correlation between the growth rate of consumption and the growth rate of corporate profits.

8.5 Calibration results using the bivariate model

We start analysing the bivariate model using time-additive expected utility.²⁶ The left graph in figur 7 exhibits the implied returns with risk-free rates $\leq 5\%$ for risk aversion $\alpha > 0$ and discount factor $\beta < 1$. It is surprising that the implied risk premia are much smaller than for the univariate model with leverage (left graph of figure 4). They are nearly equal to those of the univariate model without leverage (left graph of figure 2). Line 2 in table 8 shows that risk aversion $\alpha = 5.3$ and discount factor $\beta = 1.080$, which produce the first moments of the historically observed returns in the univariate model with leverage (L = .8), only imply a risk premium of .47% in the bivariate model. However, this result is not that surprising. Remember that in contrast to the univariate model with leverage the bivariate model does not assume a strong correlation of the growth rate of consumption and the growth rate of corporate profits. The implied strong correlation between consumption and dividends in the univariate models maximizes the risk premium for given variances of consumption and dividends. An asset with high payoffs in states of high consumption and low payoffs in states of low consumption is more risky than one with payoffs which are lower correlated with consumption. Thus, the bivariate model produces lower risk premia than the univariate model with leverage because the correlation between the growth rate of consumption and the growth rate of corporate profits is equal to the historically observed low correlation of .2324. It is shown in the last two lines of table 8, that this correlation is the main reason for the obtained small risk premia. We modified the covariances in both states of the ML-estimates to produce a perfect correlation between the growth rate of consumption and the growth rate of corporate profits. For these modified ML-estimates nearly the same risk aversion and discount factor as in the univariate model with leverage produce the first moments of the historically observed returns (scenario 1).

²⁶For a comparison of the calibration results applying switching models with stochastic states described in this section with those applying switching models with deterministic states see appendix B.2.

Table 9: Using NON-EXPECTED RECURSIVE UTILITY in the ML-estimated BIVARIATE MODEL: First and second moments of the expected equilibrium risk-free rate and risk premium for specific parameter combinations of the preference function.

Process	Model parameter ^a		Risk-free rate		Risk premium		
	α	eta	σ	$E[\tilde{r}^f]$	$Std[\tilde{r}^{f}]$	$E[ilde{p}^s]$	$Std[ilde{p}^s]$
Historical scenario 1^b				.0305	.0208	.0330	.3756
Bivariate model	5.8	.99	.88	.0303	.0086	.0150	.1738
Modified bivariate model ^c	5.8	.99	.88	.0303	.0086	.0417	.1942
Modified bivariate model	4.7	.99	.92	.0306	.0079	.0333	.1965

 $^{a}\alpha$ is the parameter of constant relative risk aversion, β is the discount factor and σ is the elasticity of intertemporal substitution.

^bThe taxation scenario 1 is the zero-tax scenario.

^cThe covariances of the ML-estimates are modified to produce perfect correlation between the growth rate of consumption and the growth rate of corporate profits.

We now analyse the bivariate model using non-expected recursive utility. The right graph in figure 7 exhibits the implied returns with risk-free rates $\leq 5\%$ for the discount factor $\beta = .99$, risk aversion α , $.5 \leq \alpha \leq 10$ and elasticity of intertemporal substitution σ , $.45 \leq \sigma \leq 1.85$. The bivariate model again produces lower implied risk premia (see also the second line of table 9) than the univariate model with leverage. However, in comparison with the bivariate model using the expected utility specification, the bivariate model produces much higher risk premia using the non-expected utility. The lower implied risk premia using the bivariate model as compared with the univariate model can again be contributed to the low correlation of the growth rate of consumption and the growth rate of corporate profits. Modifying the covariances of both states of the ML-estimates to produce perfect correlation between both growth rates again leads to implied returns comparable to those of the univariate model with leverage (see last two lines of table 9).

8

8.6 Summarizing the results using the bivariate model

The univariate model with leverage assumes a strong correlation between consumption and dividends. The bivariate model relaxes this assumption, and the low correlation between the growth rate of consumption and the growth rate of corporate profits (i.e. dividends) leads to low risk premia in comparison with the univariate model with leverage. However, this result could be caused by problems with the data which may be biased towards a lower correlation between the growth rate of consumption and the growth rate of corporate profits.²⁷

²⁷If we assume, for example, a correlation of .6 and modify the covariances on the ML-estimates to produce this correlation, we obtain a risk premium of more than 1.5% with $\alpha = 5.3$ and $\beta = 1.08$ using time-additive expected utility and the ML-estimated bivariate model (table 8).

9 Discussion of the results and conclusion

We have reexamined the Equity Premium Puzzle for the German stock market and our empirical results for the time period 1960 to 1994 confirm those for the U.S.. The results favour the usage of non-expected recursive utility which clearly distinguishes between risk preference and time preference. Using non-expected recursive utility the univariate model with leverage produces the first moment of the quarterly average risk-free rate and the first and second moments of the risk premium on the stock market with plausible parameters of risk aversion, discount factors and elasticity of intertemporal substitution. However, the implied second moment of the risk-free rate is too low. It can better be explained using expected utility.

The evaluation of the estimated Markov switching models shows that the mixture of two bivariate normal distributions fits the historically observed consumption and dividend data significantly better than a single bivariate normal distribution. Long recession and boom states are identified. A similar result could not be obtained for the mixture of univariate normal distributions for the consumption data.

The mixture of two bivariate normal distributions relaxes the assumption of strong correlation between consumption and dividends. A lower correlation between consumption and dividends decreases the risk premium rather than solving the Equity Premium Puzzle. Thus, the univariate model with leverage produces better results than the bivariate model. However, as in the univariate case in the bivariate model more reliable results are obtained with non-expected recursive utility, too. Besides the possibility that our data may be biased towards low correlation between consumption growth and growth of corporate profits, another explanation for the low historical correlation may be that in reality only a small fraction of the people in the economy own stocks and that the consumption of stockholders may differ considerably from the consumption of non-stockholders. Hence, aggregate consumption may not be the right measure for consumption of the stockholders.

A puzzling result which remains is, that the scenarios with taxation can only be explained, if at all, with an implausibly high elasticity of intertemporal substitution.

A Time series plots

Figure 8: The upper two graphs exhibit the time series of annualized quarterly real growth rates of per capita consumption expenditures on nondurables and services and the annualized quarterly real growth rates of per capita corporate profits. The lower two graphs show the conditional smoothed probabilities of state 1, the crash, resp. the recession state, of the univariate Markov switching model for consumption growth and of the bivariate Markov switching model for consumption growth and growth of corporate profits, both estimated using the EM-algorithm.



Table 10: The left hand side shows the M/P-estimates of the Markov mixtures of two univariate normal distributions for the annualized quarterly real growth rate of per capita consumption expenditures on non-durables and services. The hand right side gives the M/P-estimates of the Markov mixture of two bivariate normal distributions for the joint distribution of the growth rate of consumption and the annualized quarterly real growth rate of per capita corporate profits.

	Univaria	te model	Bivariat	e model
i =	1	2	1	2
p_{ii}	48960	.48960	.48960	.48960
$p_i{}^a$.50000	.50000	.50000	.50000
μ_{ci}	01054	.05684	01054	.05684
μ_{si}			10175	.17985
σ_{ci}^2	.00000	.00000	.00000	.00000
σ_{si}^2			.00000	.00000
σ_{csi}			.00000	.00000

^aThe unconditional stationary probabilities of the states are calculated as $p_1 = (1 - p_{22})/(2 - p_{11} - p_{22})$ and $p_2 = 1 - p_1$.

B Switching models with deterministic states

In this appendix we estimate Markov switching models with two deterministic states as applied by MEHRA/PRESCOTT (1985) and KANDEL/STAMBAUGH (1990, 1991). For comparison with the results obtained with the ML-estimated Markov switching models we present some calibration results using the Markov switching models with deterministic states. In appendix B.1 we analyse the univariate model and in appendix B.2 we analyse the bivariate model.

B.1 The univariate model with deterministic states

For the univariate model with deterministic states the two growth states are chosen as $\mu_{c1} = \mu_c - \sigma_c$ and $\mu_{c2} = \mu_c + \sigma_c$, where μ_c and σ_c are the mean and the standard deviation of consumption growth. The conditional standard deviation of consumption growth is set to zero in both states. The transition probabilities are chosen as $p_{11} = p_{22} = .5 + .5\delta$ to explain the first-order autocorrelation δ of consumption growth, because the first-order autocorrelation of the state variable of a Markov switching model can be calculated as $p_{11} + p_{22} - 1$. Choosing these parameters leads to a switching model which totally explains the mean, the variance and the firstorder autocorrelation of consumption growth, but since $p_1 = p_2$ and $\sigma_1 = \sigma_2$ no excess kurtosis or skewness can be explained.

The left part of table 10 presents the estimated switching model with deteministic states. Since this estimation method was used by MEHRA/PRESCOTT (1985) these estimates will be called M/P-estimates.

The left graph in figure 9 illustrates the implied risk-free rate / risk premium - combinations for risk-free rates lower or equal to 5 % using the M/P-estimates of

Figure 9: Set of admissible average risk premia and risk-free rates for the M/Pestimated UNIVARIATE MODEL without leverage. Results are shown using TIME-ADDITIVE EXPECTED UTILITY in the left graph and NON-EXPECTED RE-CURSIVE UTILITY in the right graph.



Figure 10: Set of admissible average risk premia and risk-free rates for the M/Pestimated BIVARIATE MODEL. Results are shown using TIME-ADDITIVE EX-PECTED UTILITY in the left graph and NON-EXPECTED RECURSIVE UTIL-ITY in the right graph.



the univariate model and time-additive expected utility. Comparing this graph with its ML-estimated counterpart in figure 2 yields the result that in the univariate case with time-additive expected utility the estimation method has no considerable influence on the first moments of the implied rates of return. This result also holds for non-expected recursive utility, which can be seen by comparing the right graph of figure 9 with its ML-estimated counterpart in figure 3.

B.2 The bivariate model with deterministic states

For the bivariate model with deterministic states the consumption growth and the growth of corporate profits are assumed to be perfectly correlated; in addition to the univariate model with deterministic states the conditional growth rates of corporate profits are chosen as $\mu_{s1} = \mu_s - \sigma_s$ and $\mu_{s2} = \mu_s + \sigma_s$ where μ_s and σ_s are the mean and the standard deviation of the growth of corporate profits. The conditional standard

deviation of the growth rate is set to zero in both states. The right part of table 10 presents the M/P-estimates of the bivariate switching models with deterministic states.

Comparing the implied rates of return using the M/P-estimated bivariate model with those using the ML-estimates does not yield clear results. The left graph in figure 10 exhibits the implied rates of return using the M/P-estimated bivariate model and time-additive expected utility. It can be seen, that risk premia up to 1.1% can be obtained for risk-free rates $\leq 5\%$ with discount factor $\beta \leq 1$. Thus, much higher risk premia are implied by the M/P-estimated bivariate model using expected utility than by the ML-estimated bivariate model (left graph of figure 7). However, these different results can be explained by the implied perfect correlation between consumption and dividends in the M/P-estimated bivariate model. The perfect positive correlation maximizes the risk premium for given variances of consumption and dividends and hence increases the risk premia in comparision to those implied by the ML-estimated bivariate model. This result also holds for the non-expected recursive utility specification (see right graph in figure 10 and its ML-emstimated counterpart in figure 7).

Further analyses have shown, that the second moment of the implied risk-free rate is much smaller using the M/P-estimates instead of the ML-estimates. This result is independent of the preference specification and the model (univariate or bivariate) applied.

6

C Calculation of the tax adjusted historical returns

In this appendix we describe the data and the calculation of the historically observed tax adjusted German risk-free rate and risk premium on the stock market for the four different taxation scenarios. The results are described in section 5 and are used in the calibration exercises in section 6, 7.3 and 8.5.

C.1 Description of data

The quarterly rate of inflation is calculated from monthly data of the German consumer price index for private households. The calculated time series of inflation rates shows a strong seasonality, resulting from the commodity basket underlying the consumer price index. To adjust for this seasonality a seasonally adjusted time series is calculated using the SAS-adaptation of the U. S. Bureau of the Census X-11 Seasonal Adjustment Program. Descriptive statistics for the time series are displayed in the first row of table 11.

The risk-free rate of return is taken to be the interest rate on three-month interbank deposits, i.e. the three-month money market rate, in Germany. Quarterly averages are gained from weekly means in the period January 1960 to February 1967 and from daily means in the period March 1967 to December 1994. Descriptive statistics are shown in the second row of table 11. Both time series, the consumer price index and the three-month money market rates, are provided by the Deutsche Bundesbank. The risky return on the stock market is measured by the Deutsche Aktien-Forschungsindex DAFOX (German Stock Price Research Index) which was constructed especially for research needs (GÖPPL/SCHÜTZ (1993)). Several subindices are available. The DAFOX_{Perfo} is a capital-weighted performance index including all German stocks which are traded in the Amtlicher Markt at the Frankfurt Stock Exchange. It is adjusted for dividends and proceeds from the sale of rights. German investors receive dividends with a tax credit which is deductible from personal income tax since 1977. The size of this tax credit has changed over time (compare last column of table 12). The tax credit is not included in the calculation of the $DAFOX_{Perfo}$, thus assuming implicitly a personal tax rate since 1977 which equals the corporate tax rate on dividends. The $DAFOX_{Price}$ contains the same stocks as the DAFOX_{Per fo} but reflects only the capital gains and the proceeds from the sale of rights; in contrast to the DAFOX_{Perfo} dividends are not included. For both indices annualized quarterly returns are calculated (descriptive statistics are shown in row three and four of table 11) from daily data which are provided by the Deutsche Finanzdatenbank (DFDB).

C.2 Calculation of the tax adjusted returns for different taxation scenarios

From the observed rates of return on the aggregate stock market and on risk-free interbank deposits described in chapter C.1 real risk-free rates and real stock returns are derived for four different taxation scenarios. Scenario 1 is the zero tax scenario,

Time s	eries	Mean	Std^a	Skewness	Kurtosis	T-Statistic ^b
	Original annualiz	zed ^c qua	rterly n	ominal data	$(N^{d}=140)$	
π	Rate of inflation	.0325	.0196	.4312	.1624	19.5896***
r_n^f	Interbank deposits	.0632	.0256	.9066	.2129	29.1883***
T _{Per} fo	DAFOX _{Perfo}	.0893	.3744	1408	2.5144	$2.8220_{\circ\circ\circ}^{***}$
r_{Price}	DAFOX _{Price}	.0591	.3752	1655	2.2851	$1.8639^{*}_{\circ\circ}$
r^d	Est. dividend return	.0370	.0302	.8425	.2550	$14.4930_{\circ\circ\circ}^{***}$

Table 11: Descriptive statistics for different time series of annualized quarterly data in the time period January 1960 to December 1994.

^aStandard deviation.

^{b***} (**, *)[$\circ \circ \circ (\circ \circ, \circ)$]: The hypotheses of zero mean can be rejected at a 1 % (5 %, 10 %) significance level on the basis of the parametric standard T-statistic [on the basis of the nonparametric Wilcoxon-Sign-Rank statistic].

 $^c\mathrm{The}$ quarterly data are multiplied by 4 before the calculation of the descriptive statistics. $^d\mathrm{Number}$ of observations .

 τ^0 . The scenarios 2 and 3 refer to private investors. In these cases capital gains are not taxed if the stock is held for at least six months, whereas dividend and interest payments are taxed at the personal progressive tax rate. Scenario 2 [3] refers to a private investor with a marginal tax rate of 36%, τ_{Priv}^{36} [with the highest marginal tax rate, τ_{Priv}^{max}]. The highest marginal tax rate for private investors has changed over time (table 12). Scenario 4 refers to an institutional investor, for whom capital gains as well as dividend and interest payments are subject to the corporate tax rate, $= \tau_{Corp}$. The corporate tax rate is shown in table 12.

We assume that the total risk-free return comes from interest payments. Thus, in each period the net real risk-free rate, r_{τ}^{f} , for the different tax rates τ can be calculated as $r_{\tau}^{f} = (1 - \tau)(1 + r_{n}^{f})/(1 + \pi) - 1$, where π is the rate of inflation and r_{n}^{f} is the nominal three-month money market rate.

To calculate the risk premia for the taxation scenarios the dividend payments of stocks and their capital gains have to be separated. Unfortunately no dividend return time series with higher than yearly frequency is available for Germany. We will use the different returns on the $DAFOX_{Perfo}$ and the $DAFOX_{Price}$ to estimate the dividend return for each quarter. The estimation procedure described below leads to good estimates of dividend returns and can also be applied for time series with higher than quarterly frequency.

The calculation of the DAFOX_{Perfo} assumes, that the dividend payment is directly reinvested in the dividend paying stock. If we assume that the dividend payments are continuously paid over the quarter and reinvesting in the dividend paying stock equals reinvesting in the whole index portfolio then the before tax dividend return, r^d , can be estimated as $r^d = (r_{Perfo} - r_{Price})/((1 - .5 \cdot r_{Perfo})(1 - \tau_{credit}))$ where r_{Perfo} and r_{Price} are the net nominal returns on the DAFOX_{Perfo} and the DAFOX_{Price} and τ_{credit} is the tax credit the investors receive on dividends. The fifth row of table 11 exhibits descriptive statistics for the estimated quarterly dividend returns. The 3.70% mean annualized quarterly dividend return including the tax credit is nearly

Time period	$ au_{Priv}^{maxa}$	τ_{Corp}^{b}	τ_{Credit}^{c}
1960-1967	53.00 %	51.00 %	0.00~%
1968-1974	54.59~%	52.53~%	0.00~%
1975-1977	57.68~%	52.53~%	0.00~%
1977-1978	57.68~%	57.68~%	37.08~%
1978-1989	56.00~%	56.00~%	36.00~%
1990-1993	53.00~%	50.00~%	36.00~%
1994	53.00 %	$45.00 \ \%$	30.00 %

Table 12: Private and corporate tax rates and tax credit on dividend payments.

^aHighest private marginal tax rate including supplementary tax but not including church tax. ^bCorporate tax rate including supplementary tax.

^cTax credit on dividend payments including supplementary tax.

equal to the 3.74% mean of the yearly dividend returns including tax credit published by the Statistisches Bundesamt.²⁸ Thus the employed estimation of dividend returns can be used as a good approximation.

The after tax net real returns on the stock market, r_{τ}^{s} , are calculated for private investors (scenarios 2 and 3) as $r_{\tau}^{s} = (r_{Price} + r^{d}(1 - \tau_{Priv}) + 1)/(1 + \pi) - 1$ where π is the rate of inflation²⁹. The returns to institutional investors (scenario 4) are given by $r_{\tau}^{s} = ((r_{Price} + r^{d})(1 - \tau_{Corp}) + 1)/(1 + \pi) - 1$.

²⁸Employing the estimation procedure to monthly returns on the DAFOX_{Perfo} and the DAFOX_{Price} and analysing the seasonal structure of the dividend payments yields the same plausible results found by other authors for the German stock market (BAY (1990:76), MORAWIETZ (1994:123))

²⁹Adding r_{Price} and after tax r^d assumes that the after tax dividend payments are reinvested at the end of each quarter.

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