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## Volatility and conditional distribution in financial markets

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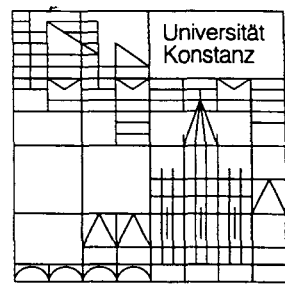
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Diskussionsbeiträge

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Klaus Abberger

**Volatility and Conditional Distribution  
in Financial Markets**

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# Volatility and conditional distribution in financial markets

Klaus Abberger

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## **Summary:**

There are various parametric models to analyse the volatility in time series of financial market data. For maximum likelihood estimation these parametric methods require the assumption of a known conditional distribution. In this paper we examine the conditional distribution of daily DAX returns with the help of nonparametric methods. We use kernel estimators for conditional quantiles resulting from a kernel estimation of conditional distributions.

# 1 Introduction

The volatility of returns is an important feature of daily financial market data (e.g. stock returns, exchange rates). Volatility varies over time. It is possible to assess volatility with conditional variances. In the statistical analysis of financial markets, ARCH and GARCH models are frequently used. Let  $\{Y_i\}$  be a stochastic process with

$$Y_i = \mu + V_i U_i \quad , \quad (1)$$

$E[U_i] = 0$ ,  $Var[U_i] = 1$  and  $\{V_i\}$  a sequence of real positive random variables. Then the ARCH(q) model [Engle (1982)] for the conditional variance is defined as

$$V_i^2 = \omega + \sum_{j=1}^q \alpha_j \varepsilon_{i-j}^2 \quad , \quad (2)$$

with  $\varepsilon_i = V_i U_i$ . Thus, squared noise in the past explains the conditional variance. The GARCH(p, q) model [Bollerslev (1986)] is

$$V_i^2 = \omega + \sum_{j=1}^q \alpha_j \varepsilon_{i-j}^2 + \sum_{j=1}^p \beta_j V_{i-j}^2. \quad (3)$$

It is similar to an ARMA(p, q) model of the squared values of the time series  $\{Y_i\}$ . Both models are estimated with the help of the maximum likelihood methods. Therefore the distribution of  $U_i$  must be assumed to be known. Usually the  $U_i$  are assumed to be independently and identically Gaussian-distributed, from which follows that the distribution of  $Y_i$  given  $V_i = v_i$  is  $\mathcal{N}(\mu, v_i^2)$ . This assumption is still in discussion: e.g. Baillie and Bollerslev (1989) think that a Gaussian distribution is unsuitable for modelling daily exchange rates. They suggest a leptocurtic conditional distribution instead and propose the use of a t-distribution. Hsieh (1989) examines further distributions. He uses a t-distribution, a normal Poisson mixture and a normal

lognormal mixture. Engle and González-Rivera (1991) favor a semiparametric GARCH model which does not require an explicit conditional distribution. Instead, the smoothness of the conditional distribution, a constant kurtosis and a constant skewness are needed. Using an iterative procedure the GARCH parameters and the conditional distribution are estimated. When these are used for different daily stock returns the authors achieve leptocurtical and skew conditional distributions.

In this paper we analyze volatility by nonparametric methods. Instead of looking at conditional variance we look at marginal quantiles. The estimation of time-dependent quantiles shows changes in variation and skewness in the time series. The nonparametrically estimated quantiles are compared with the results of a GARCH analysis using Gaussian distributions. The objective of this study is to obtain new insights in the features and changes of conditional distributions in financial market data.

The procedure is as follows: in section 2 a kernel estimator for conditional quantiles is defined and the mean squared error is examined. Section 3 gives an empirical analysis with GARCH estimations and nonparametric quantile estimations used on daily DAX returns. The results are compared and discussed. A final section gives a synopsis of the results.

## **2 Nonparametric estimation of conditional quantiles**

The characteristics of the time series of daily financial market data are analyzed with quantile regressions. The quantile regression is based on the fol-

lowing model:

Let  $\{Y, X\}$  be a bivariate random variable and  $F(y | x) = F_x(y)$  the conditional distribution of  $Y$ , given  $X = x$ . The conditional  $\alpha$ -quantile  $q_\alpha(x)$  is then given by  $q_\alpha(x) = \inf\{y \in \mathbb{R} | F_x(y) \geq \alpha\}$ ,  $0 < \alpha < 1, x \in \mathbb{R}$ . Given the observations  $\{Y_i, X_i\}_{i=1}^n$ , the model is

$$Y_i = q_\alpha(X_i) + U_i \quad , \quad (4)$$

with  $P(U_i \leq 0) = \alpha$ . In the following application of smoothing time series,  $X_i = (i - 1/2)/n$  is fixed and the conditional quantiles correspond to marginal quantiles.

In the existing literature several procedures for nonparametric estimation of conditional quantiles are discussed. Koenker, Portnoy and Ng (1992) use spline smoothing, Horváth and Yandell (1988) examine a kernel estimator, and Stute (1986) suggests a nearest-neighbour estimator.

We use kernel estimation. With a suitable kernel function  $K(\cdot)$  and bandwidths  $h_n$ , a kernel estimator of the conditional distribution is defined as follows:

$$F_{n,x}(y) = \frac{\sum_{i=1}^n \mathbb{I}_{\{Y_i \leq y\}} K\left(\frac{x-X_i}{h_n}\right)}{\sum_{i=1}^n K\left(\frac{x-X_i}{h_n}\right)} . \quad (5)$$

For the conditional  $\alpha$ -quantile the estimator is then given by

$$q_{n,\alpha}(x) = \inf\{y \in \mathbb{R} | F_{n,x}(y) \geq \alpha\} , \quad 0 < \alpha < 1. \quad (6)$$

To learn more about the behavior of the estimator we look at the mean squared error. For independent  $\{Y_i\}$  the expectation and variance of the estimator  $F_{n,x}(y)$  are

$$E[F_{n,x}(y)] = \sum_i \frac{K\left(\frac{x-X_i}{h_n}\right)}{\left[\sum_i K\left(\frac{x-X_i}{h_n}\right)\right]} F_{X_i}(y) \quad , \quad (7)$$

$$Var[F_{n,x}(y)] = \sum_i \frac{K^2\left(\frac{x-X_i}{h_n}\right)}{\left[\sum_i K\left(\frac{x-X_i}{h_n}\right)\right]^2} [F_{X_i}(y) - F_{X_i}^2(y)]. \quad (8)$$

For further results we need assumptions of bandwidth, of kernel function and of conditional distribution. These assumptions are

$$h_n \longrightarrow 0 \quad , \quad \text{for } n \longrightarrow \infty; \quad (\text{B. 1})$$

$$n h_n \longrightarrow \infty \quad , \quad \text{for } n \longrightarrow \infty; \quad (\text{B. 2})$$

$$\text{K has compact support ;} \quad (\text{K. 1})$$

$$\text{K is symmetrical ;} \quad (\text{K. 2})$$

$$\text{K is Lipschitz-continuous ;} \quad (\text{K. 3})$$

$$\int K(u)du = 1; \quad (\text{K. 4})$$

K is bounded and there is a  $\bar{K} \in \mathbb{R}$ , with

$$K(u) \leq \bar{K} < \infty \quad , \quad \forall u \in \mathbb{R} \quad (\text{K. 5})$$

$$K(u) \geq 0, \forall u \in \mathbb{R}; \quad (\text{K. 6})$$



$F$  fixed  $y \in \mathbb{R}$  there exists

$$F_x''(y) = \frac{\delta^2 F_x(y)}{\delta x^2} \text{ in a neighbourhood of } x. \quad (\text{S. 1})$$

We assume that (K.1), (K.2) and (S.1) are fulfilled. It then from Taylor expansion  $U_i := \frac{x-X_i}{h_n}$  and  $x \in (h_n, 1 - h_n)$  it follows

$$E[F_{n,x}(y)] = F_x(y) + \frac{h_n^2}{2} \frac{\sum_i U_i^2 K(U_i)}{\sum_i K(U_i)} F_x''(y) + o(h_n^2). \quad (9)$$

Thus the bias of  $F_{n,x}(y)$  depends on the smoothness of the underlying conditional distribution function by  $F_x''(y)$ . It is now possible to give a statement about the asymptotic mean squared error.

**Theorem 1.**

Let  $\{Y_i\}$  be independent and let (B.1), (B.2), (K.1), (K.2), (K.3), (K.4) and (S.1) be fulfilled. Then it holds for  $n \rightarrow \infty$  and  $x \in (h_n, 1 - h_n)$ :

$$\begin{aligned} \text{MSE}(F_{n,x}(y)) &\approx \left[ \frac{h_n^2}{2} F_x''(y) \int u^2 K(u) du \right]^2 \\ &+ \frac{1}{nh_n} (F_x(y) - F_x^2(y)) \int K^2(u) du. \end{aligned} \quad (10)$$

**Proof:** See appendix.

□

Observe that the mean squared error depends on the second derivative of the conditional distribution and also on the difference  $(F_x(y) - F_x^2(y))$ . This means that the variance of the estimator is highest in the middle of the distributions.

From theorem 1 it also follows that the kernel estimator (5) is consistent for independent  $\{Y_i\}$  and for assumptions on the bandwidth and the kernel function. However, for time series smoothing the assumption of independence is too restrictive. Therefore we allow  $\{Y_i\}$  to be  $\alpha$ -mixing (strong mixing). Let  $A_k$  be an element of the  $\sigma$ -algebra  $\mathcal{A}_k$  generated by  $\{Y_i, i = 1, \dots, k\}$  and let  $B_{k+s}$  be an element of the  $\sigma$ -algebra  $\mathcal{B}_{k+s}$  generated by  $\{Y_i, i = k+s, \dots\}$ . Then a process  $\{Y_i\}$  is called  $\alpha$ -mixing, if for the mixing coefficient

$$\alpha_s = \sup_k \sup_{A_k, B_{k+s}} |P(A_k \cap B_{k+s}) - P(A_k)P(B_{k+s})| \quad (11)$$

it holds:

$$\lim_{s \rightarrow \infty} \alpha_s = 0. \quad (12)$$

If  $\{Y_i\}$  is a  $\alpha$ -mixing process, then  $\{\mathbb{I}_{\{Y_i \leq y\}}\}$  is  $\alpha$ -mixing as well. For two random variables  $U$ ,  $\mathcal{A}_k$ -measurable, and  $V$ ,  $\mathcal{B}_{k+s}$ -measurable, the following inequality for the covariance holds [Doukhan (1994), p.10]:

$$|Cov(U, V)| \leq 4\alpha_s(\text{ess - sup } |U|)(\text{ess - sup } |V|), \quad (13)$$

with  $\text{ess - sup } |U| := \inf\{a \in \mathbb{R} \mid P(U > a) = 0\}$ . For the random variables  $\{\mathbb{I}_{\{Y_i \geq y\}}\}$  is  $\text{ess - sup } |\mathbb{I}_{\{Y_i \geq y\}}| = 1$ , of course.

These results allow us to look at the mean squared error of  $F_{n,x}^h$  for  $\alpha$ -mixing processes. The expectation (9) does not change. The variance (8) has to be extended by the covariance term

$$\sum_i \sum_{j \neq i} \frac{K(U_i)K(U_j)}{[\sum_i K(U_i)]^2} Cov[\mathbb{I}_{\{Y_i \leq y\}}, \mathbb{I}_{\{Y_j \leq y\}}]. \quad (14)$$

It shall be assumed that the process is covariance-stationary and that  $c(j - i) := Cov[\mathbb{I}_{\{Y_i \leq y\}}, \mathbb{I}_{\{Y_j \leq y\}}]$ . The covariance (14) is assessed by

$$\begin{aligned}
& \left| \frac{1}{[\sum_i K(U_i)]^2} \sum_i \sum_{\substack{j \\ i \neq j}} K(U_i)K(U_j) c(j-i) \right| \leq \\
& \leq \frac{2}{n^2 h_n^2} \sum_{i=1}^{n-1} \sum_{j=i+1}^n K(U_i)K(U_j) |c(j-i)| \\
& \leq \frac{2}{n^2 h_n^2} \bar{K}^2 \sum_{i=1}^{n-1} (n-i) |c(i)| \\
& \leq \frac{2}{n h_n^2} \bar{K}^2 \sum_{i=1}^{n-1} |c(i)| \\
& \leq \frac{8}{n h_n^2} \bar{K}^2 \sum_{i=1}^{n-1} \alpha_i
\end{aligned}$$

with the integral approximation and the covariance inequality (13), the conditions (K.5) and (K.6).

Thus the mixing coefficient has to fulfill the condition

$$\frac{1}{n h_n^2} \sum_{i=1}^{n-1} \alpha_i \longrightarrow 0 \quad , \quad \text{for } n \longrightarrow \infty, \quad (\text{M. 1})$$

in order for the covariance term (14) disappear and the estimator  $F_{n,x}$  to be weakly consistent.

### 3 Empirical Analysis

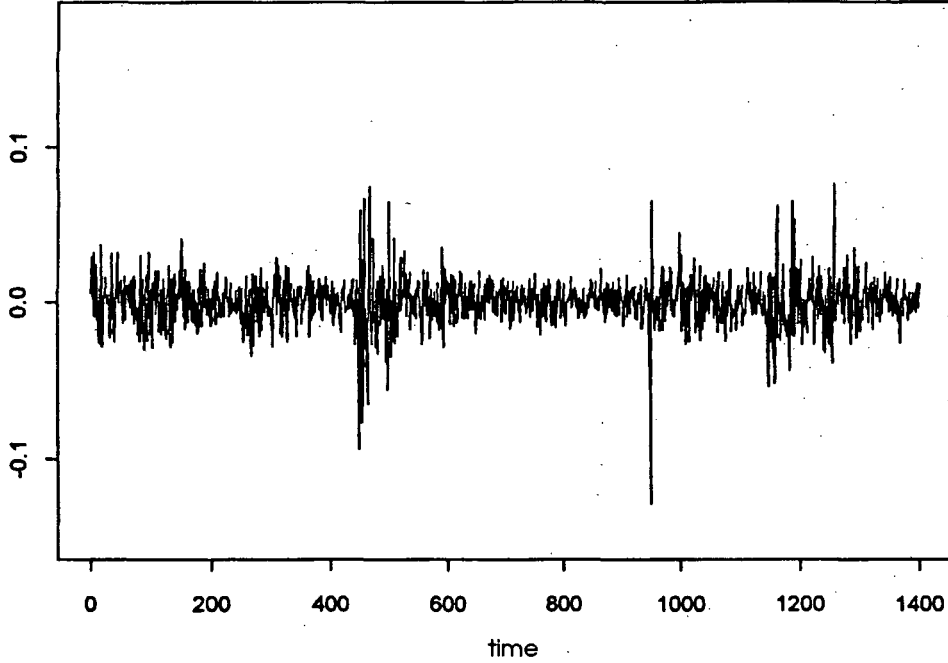
Here we apply the GARCH modelling and the kernel estimation of conditional quantiles to daily DAX returns.<sup>1</sup> Figure 1 shows the time series with 1400 observations made between 2/01/86 and 13/08/91. The returns are defined by  $Y_i = (Kurs_{[i]} - Kurs_{[i-1]})/Kurs_{[i-1]}$ .

With the help of the maximum-likelihood method a GARCH model is estimated with a Gaussian distribution as conditional distribution.

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<sup>1</sup>The data were provided by the Deutsche Finanzmarktdatenbank - DFDB, Karlsruhe.

Figure 1: Time series of daily DAX returns from 2/1/86 till 13/8/91 (n=1400)



For the kernel estimation of conditional quantiles, a bandwidth  $h_\alpha$  must be chosen. Cross-validation is a common procedure to determine a suitable bandwidth for nonparametric regression estimation and density estimation. This technique is also applied to conditional quantiles. Let  $q_{n,\alpha}^{(i)}(X_i)$  be the leave-one-out estimation for the conditional quantile.

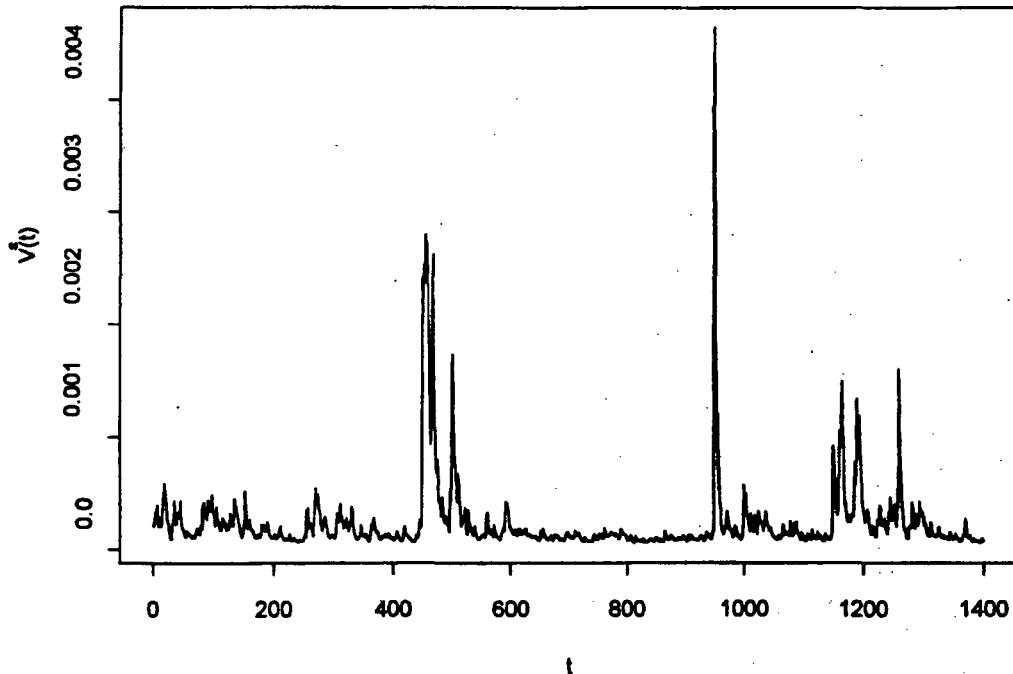
Leave-one-out means that at  $X_i$  the observation  $Y_i$  is not taken into account.

Thus the leave-one-out estimator of the conditional distribution function is

$$F_n^{(i)}(y|X_i) = \frac{\sum_{\substack{j=1 \\ j \neq i}}^n \mathbb{I}_{\{Y_j \leq y\}} K\left(\frac{X_j - X_i}{h_n}\right)}{\sum_{\substack{j=1 \\ j \neq i}}^n K\left(\frac{X_j - X_i}{h_n}\right)} \quad (15)$$

and for the leave-one-out estimator of the conditional quantile results  $q_{n\alpha}^{(i)}(X_i) =$

Figure 2: GARCH(1,2) estimation of the conditional variance for daily DAX returns



$\inf\{y \in \mathbb{R} | F_n^{(i)}(Y|X_i) \geq \alpha\}$ . Choosing a bandwidth by cross-validation means using the leave-one-out estimator in a suitable loss function and minimizing it over  $h$ . A function examined by Koenker and Bassett (1978) is chosen as the loss function:

$$\begin{aligned} \ell_\alpha(\Theta) = \frac{1}{n} \sum_i \{ & \alpha \mathbb{I}_{\{Y_i - \Theta \geq 0\}}(Y_i - \Theta) \\ & - (1 - \alpha) \mathbb{I}_{\{Y_i - \Theta < 0\}}(Y_i - \Theta) \}, \quad 0 < \alpha < 1. \end{aligned} \quad (16)$$

It has the expectation

$$E[\ell_\alpha(\Theta)] = \alpha E[Y] - \alpha\Theta + \int_{-\infty}^{\Theta} F(y) dy,$$

which becomes minimal if  $\Theta$  corresponds to the  $\alpha$ -quantile as shown by de-

ivation:

$$\frac{\delta E[\ell_\alpha(\Theta)]}{\delta \Theta} = F(\Theta) - \alpha \stackrel{!}{=} 0,$$

$$F(\Theta) = \alpha.$$

The cross-validation method chooses a bandwidth which minimizes

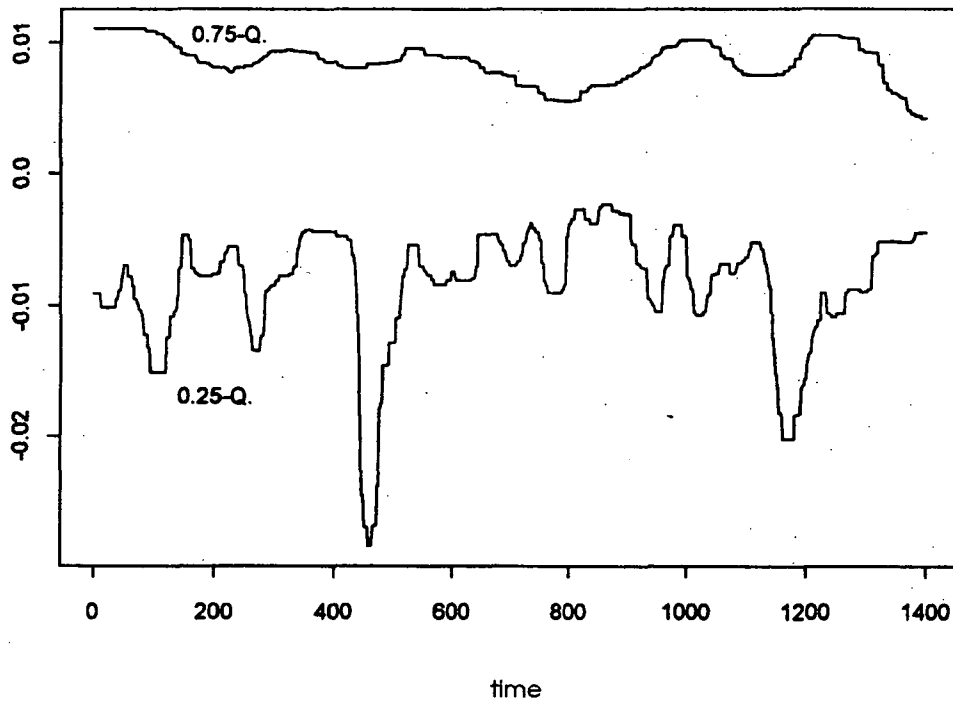
$$L_\alpha(h_\alpha) = \sum_i \left\{ \alpha \mathbb{I}_{\{Y_i - q_{n\alpha}^{(i)} \geq 0\}} (Y_i - q_{n\alpha}^{(i)}(X_i)) - (1 - \alpha) \mathbb{I}_{\{Y_i - q_{n\alpha}^{(i)} < 0\}} (Y_i - q_{n\alpha}^{(i)}(X_i)) \right\} W(i) \quad (17)$$

for a given  $\alpha$ .  $W(i) = \mathbb{I}_{\{200 < i < 1200\}}$  is an indicator function serving to avoid boundary problems. The cross-validation technique requires independent data. A block-cross-validation may be used to check the sensitivity of the resulting bandwidth for dependent data  $\{Y_i\}$ . Instead of omitting the  $i$ -th observation only, there are whole data blocks  $\{Y_i\}_{j=i-b_1}^{i+b_2}$  not taken into account in estimating the conditional distribution. However, in the present application on daily DAX returns no remarkable influences of dependences on the choice of bandwidth were observed [cf. Abberger (1994) for the detailed description of the values of  $L_\alpha(h_\alpha)$ ].

Figure 3 shows the estimations for the 0.25- and 0.75-quantiles. Whereas the 0.75-quantile fluctuates only little, the 0.25-quantile sways heavily: the conditional distribution is asymmetric.

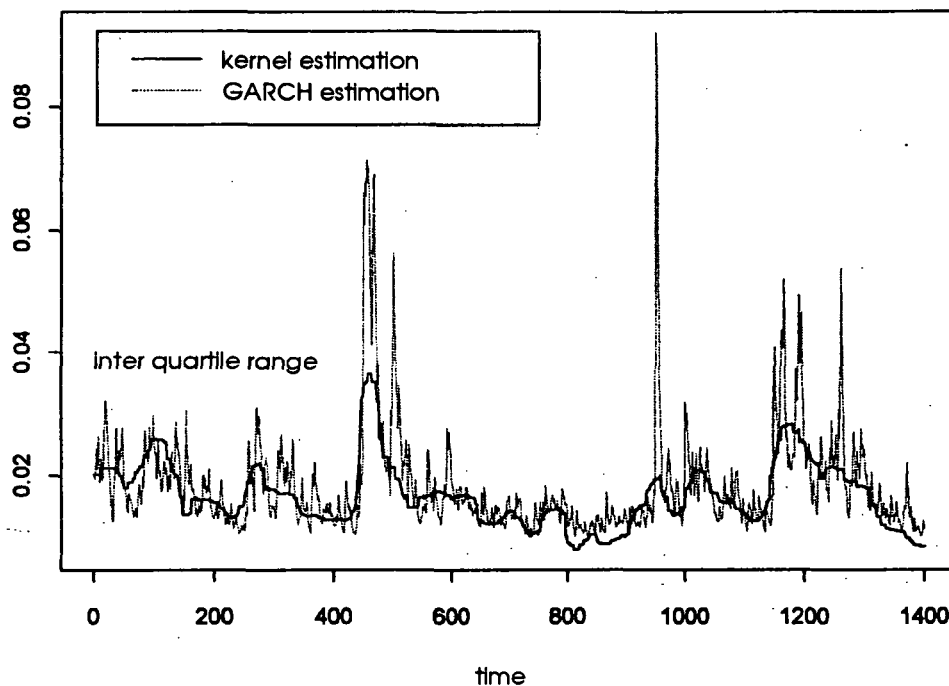
If the results of the kernel estimation are compared with those of the GARCH estimation, then quantiles must be drawn from the GARCH estimation. The Gaussian distribution being completely determined by the mean and variance makes the calculation of quantiles possible. A quantile-based variation measure is the interquartile range ( $q_{0.75} - q_{0.25}$ ). Figure 4 presents

Figure 3: Kernel estimation of 0.25- and 0.75-quantile of daily DAX returns



interquartile ranges resulting from kernel and GARCH estimations. The deviation in area 450-550, 950-1000 and 1100-1300 is striking. Instead of the interquartile range let us look at both quantiles separately - as in figure 5 - then we can distinguish two reasons for the deviation in the areas described above: the asymmetrical distribution found by kernel estimation faces a symmetrical distribution by GARCH estimation with assumption of a Gaussian distribution. In the areas of 450-550 and 1100-1300 both estimations indicate low values for the 0.25-quantiles. Because of the implicit assumption of symmetry, the GARCH estimation also indicates a high value of the 0.75-quantile whereas the kernel estimation remains flat. A further reason for the difference in both estimations is the GARCH estimation's sensitivity to outliers. This becomes evident in the area of 950-1000.

Figure 4: Interquartile ranges of GARCH(1,2) estimations and kernel estimations of daily DAX returns



Another question concerns the kurtosis of the conditional distributions. Are the conditional distributions of daily DAX returns heavy-tailed? To answer this question one has to examine the tails of the conditional distribution, facilitated by various quantiles shown in the figures 6, 7 and 8. Nevertheless there are no signs of leptokurtic conditional distributions. In the figures the skewness eclipses the kurtosis.



Figure 5: Estimation of 0.25- and 0.75-quantiles of daily DAX returns

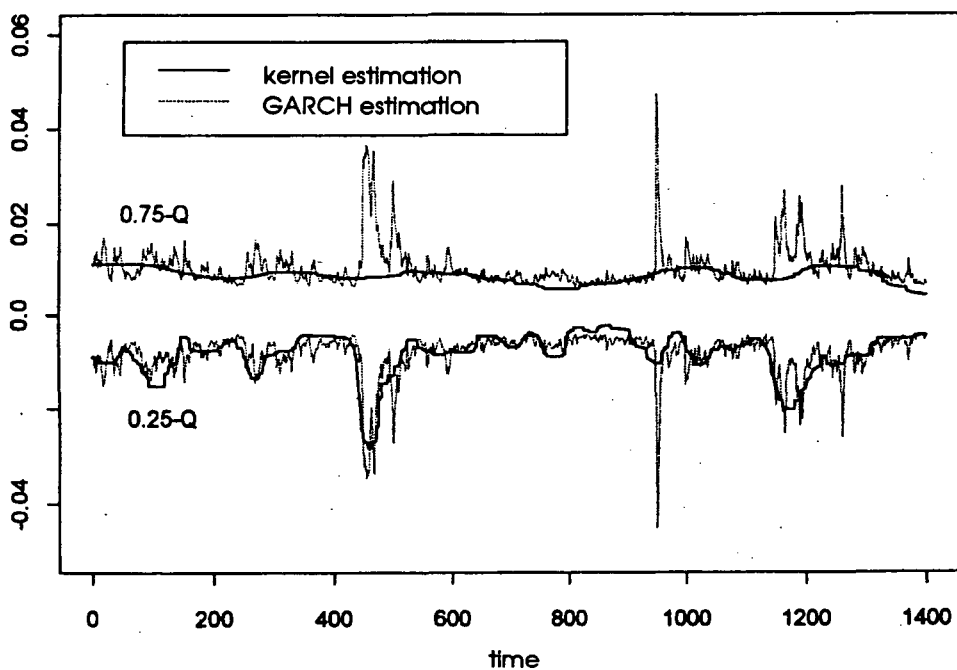


Figure 6: Estimation of 0.10- and 0.90-quantiles of daily DAX returns

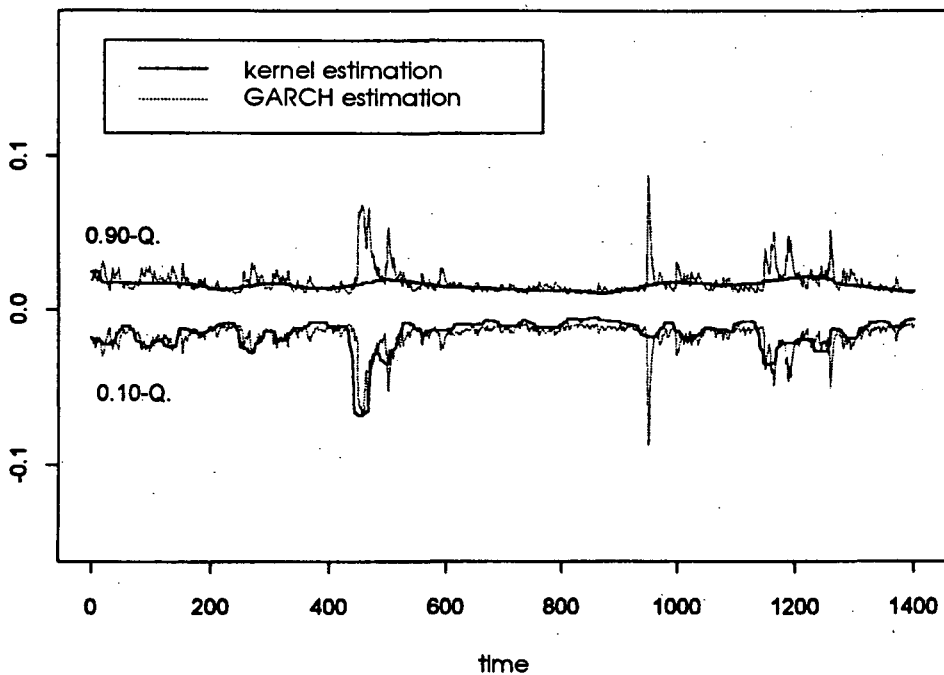


Figure 7: Estimation of 0.05- and 0.95-quantiles of daily DAX returns

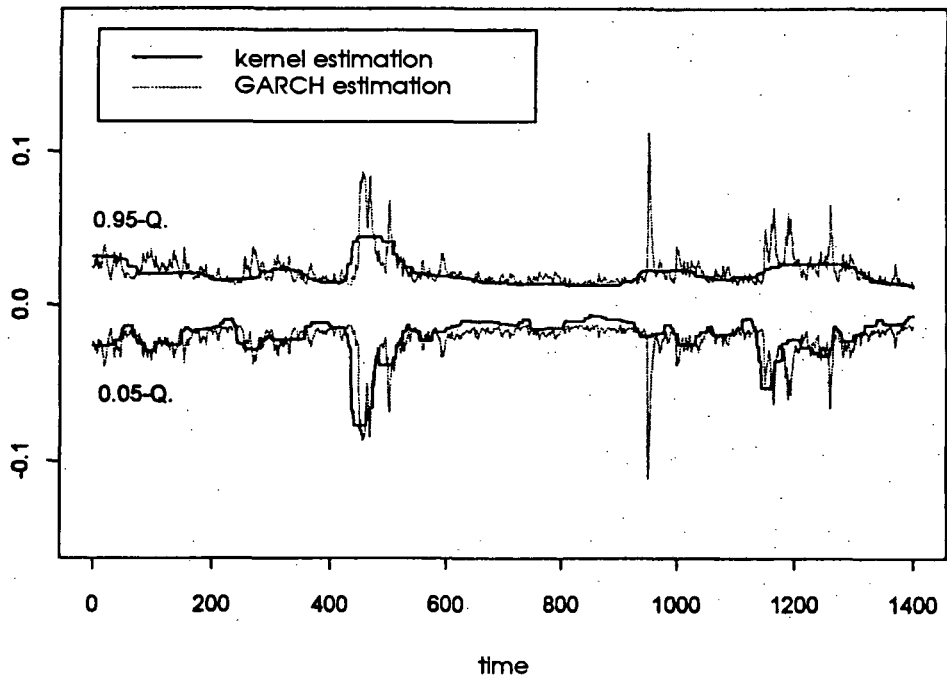
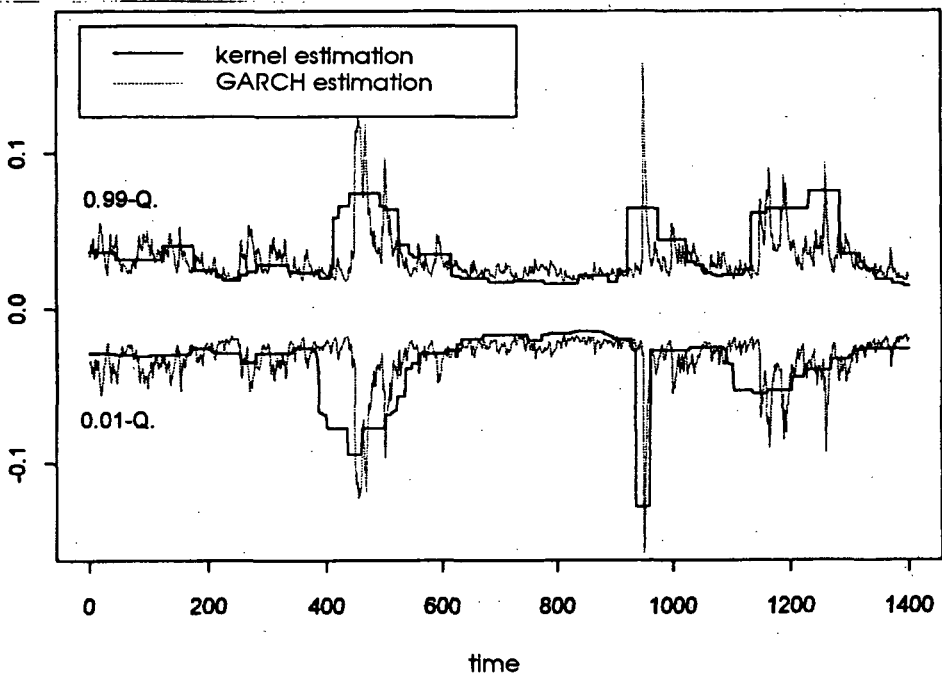


Figure 8: Estimation of 0.01- and 0.99-quantiles of daily DAX returns



## 4 Conclusion

In the statistical analysis of financial market data, the determination of volatility by ARCH and GARCH models plays an important role. In this context the determination of the form of the conditional distribution is difficult. Therefore in this paper we use nonparametric methods, which do not require a knowledge of the functional form of the conditional distribution. It is shown that the kernel estimation of the conditional distribution is weakly consistent, assuming different characteristics of the kernel, the bandwidth and the smoothness of the conditional distribution. This consistency also holds for strong mixing processes. The kernel estimation of different conditional quantiles of daily DAX returns shows asymmetries in the conditional distributions. These asymmetries are not detected by the GARCH model because of the implicit assumption of symmetry in the use of the Gaussian distribution. We also looked for tailedness in the conditional distributions, but we could not find any signs of a generally leptocurtic conditional distribution.

## 5 Appendix

**Lemma 1: Integral approximation of the sum over the kernel function.**

With (K.1) Lipschitz-continuity (K.3) and the mean value theorem of integration, it follows:

$$\begin{aligned}
 & \left| \sum_{i=1}^n K(U_i) - nh_n \int K(u) du \right| \\
 & \leq \sum_{i \in J} \left| \left( \frac{1}{nh_n} \right) K(U_i) - \int_{U_i}^{U_{i-1}} K(z) dz \right| \\
 & = \sum_{i \in J} \left| \left( \frac{1}{nh_n} \right) K(U_i) - (U_{i-1} - U_i) K(\zeta_i) \right| \\
 & = \frac{1}{nh_n} \sum_{i \in J} |K(U_i) - K(\zeta_i)| \\
 & = \frac{1}{nh_n} \sum_{i \in J} L |U_i - \zeta_i| \leq \frac{1}{nh_n} \sum_{i \in J} O\left(\frac{1}{nh_n}\right) = O\left(\frac{1}{n^2 h_n^2}\right) \sum_{i \in J} 1 = O\left(\frac{1}{nh_n}\right).
 \end{aligned}$$

Let  $J$  be the index set of observations with a weight larger than zero. It holds that  $|J| = O(nh)$ , where  $|J|$  denotes cardinality. In the same way it is possible to approximate  $\sum_i K^2(U_i)$  and  $\sum_i U_i^2 K(U_i)$  by integrals.

**Proof of theorem 1.**

The bias results from equation (9) and the integral approximation in lemma 1.

Taylor expansion yields the variance (8)

$$\begin{aligned}
 F(y | x - h_n U_i) &= F(y | x) - h_n U_i F'(y | x) + h_n^2 U_i^2 F''(y | x) + o(h_n^2), \\
 F^2(y | x - h_n U_i) &= F^2(y | x) - 2h_n U_i F(y | x) F'(y | x) + h_n^2 U_i^2 F'(y | x) \\
 &\quad + h_n^2 U_i^2 F(y | x) F''(y | x) + o(h_n^2),
 \end{aligned}$$

Let condition (K.2) hold and let be  $A := 1 / \left[ \sum_i K(U_i) \right]^2$ . Then

$$\text{Var}[F_{n,x}(y)] = A [F(y | x) - F^2(y | x)] \sum_i K^2(U_i)$$

$$\begin{aligned}
& + Ah_n^2 [F''(y | x) - 2F'(y | x) - 2F(y | x)F''(y | x)] \sum_i U_i^2 K^2(U_i) \\
& + A o(h_n^2) \sum_i K^2(U_i).
\end{aligned}$$

With integral approximation, it holds

$$\text{Var} [F_{n,x}(y)] \approx \frac{1}{nh} [F(y | x) - F^2(y | x)] \cdot \int K^2(u) du.$$

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