

Broll, Udo; Wahl, Jack E.

**Working Paper**

## Missing risk sharing markets and the benefits of cross hedging in developing countries

Diskussionsbeiträge - Serie II, No. 284

**Provided in Cooperation with:**

Department of Economics, University of Konstanz

*Suggested Citation:* Broll, Udo; Wahl, Jack E. (1995) : Missing risk sharing markets and the benefits of cross hedging in developing countries, Diskussionsbeiträge - Serie II, No. 284, Universität Konstanz, Sonderforschungsbereich 178 - Internationalisierung der Wirtschaft, Konstanz

This Version is available at:

<https://hdl.handle.net/10419/101614>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

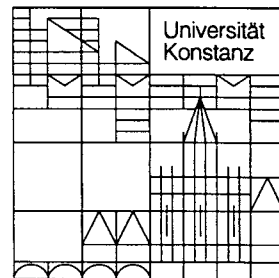
Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



**Sonderforschungsbereich 178**  
**„Internationalisierung der Wirtschaft“**

Diskussionsbeiträge

Juristische  
Fakultät

Fakultät für Wirtschafts-  
wissenschaften und Statistik

---

Udo Broll  
Jack E. Wahl

**Missing Risk Sharing Markets  
and the Benefits of Cross Hedging  
in Developing Countries**

11. DEZ. 1995 Weltwirtschaft.  
Kiel

W 113 (284) mi br sig gla

---

**Missing Risk Sharing Markets  
and the Benefits of Cross Hedging  
in Developing Countries**

646931

**Udo Broll**

**Jack E. Wahl**

**W 113 (284)**



Serie II - Nr. 284

November 1995

# Missing Risk Sharing Markets and the Benefits of Cross Hedging in Developing Countries

UDO BROLL\*

*Department of Economics,*

*University of Konstanz, D-78434 Konstanz*

AND

JACK E. WAHL

*Department of Finance,*

*University of Dortmund, D-44221 Dortmund*

October 1995

## Abstract

We study the impact of exchange rate risk on an exporting firm in a developing country when there is no forward market in the foreign currency. However there exists a forward traded asset in this country the price of which is highly correlated to the foreign currency. By indirectly hedging its foreign exchange exposure the firm can increase its economic welfare. Furthermore export production increases and promotes international trade of the developing country if the spot rate of foreign exchange has a regression relationship with the price of the forward traded asset.

*Key words: missing markets, export production, exchange rate risk, cross hedging, wealth risk*

*JEL classification: O12, O16*

---

\*We are grateful to Sugata Marjit and Itzhak Zilcha for helpful discussions.

# Missing Risk Sharing Markets and the Benefits of Cross Hedging in Developing Countries

UDO BROLL and JACK E. WAHL

We study the impact of exchange rate risk on an exporting firm in a developing country when there is no forward market in the foreign currency. However there exists a forward traded asset in this country the price of which is highly correlated to the foreign currency. By indirectly hedging its foreign exchange exposure the firm can increase its economic welfare. Furthermore export production increases and promotes international trade of the developing country if the spot rate of foreign exchange has a regression relationship with the price of the forward traded asset.

## 1. Introduction

The scenario of export price uncertainty attributable to exchange rate uncertainty is one particular case which is of interest in three specific contexts: that of multinational firms, that of price discriminating firms in international trade, and that of developing nations looking to manufacturing exports as a stimulus to economic growth, as a foundation for development of an industrial sector, and as a means for acquiring foreign currency (see, e.g., Myers (1992), Sercu (1992), Grobar (1993)). Assuming some sellers to be risk averse and none to be risk preferring, extant theory would suggest, and Grobar (1993) has provided empirical evidence of, a negative relationship between exchange rate uncertainty and developing nations' exports; or in both principle and practice risk aversion leads to cautious behavior.

Since the advent of floating exchange rates and given the great volatility of foreign exchange rates, internationally operating firms have been highly

concerned with developing ways to protect themselves from exchange rate risk. The literature reports that an international firm facing exchange rate risk can eliminate this risk altogether if it can use a currency forward market, or another forward traded asset which is perfectly correlated to the exchange rate (see Benninga, Eldor and Zilcha (1985), Kawai and Zilcha (1986), Broll and Zilcha (1992), Broll and Wahl (1995)). In the absence of such markets, the firm can reduce its income risk by engaging in a hedging activity of assets correlated to the foreign exchange rate.

In reality, not every currency is traded in a forward market (see Powell (1989), Buckley (1992), Fry (1992)). Therefore the exporting firm uses forward contracts with other underlying assets whose spot prices are highly correlated with the spot rate of the foreign currency in question. In the real world hedging must often be accomplished by using forward contracts on different deliverable instruments. Such hedging is sometimes called cross hedging (see Eaker and Grant (1987), Broll, Wahl and Zilcha (1995)) and constitutes an indirect hedging policy. Our study shows that an exporting firm is able to benefit from hedging exchange rate risk indirectly, that is to say, when no perfect hedging policy is possible.

In our study of economic implications of cross hedging, we focus on the impact of a regressibility assumption between the foreign currency spot rate and the spot price of the forward traded asset on export production, forward commitments on the asset and hedge benefits of the international firm. The regressibility assumption appears by extensive statistical research on the relation between forward and spot prices and has a long tradition in the risk management and economic literature (see Powers and Castelino (1991)). Another way of reducing foreign exposure by a cross hedge refers to some parity relationship between currencies due to nonarbitrage conditions. In the context of developing countries the regressibility approach seems to be more promising.

We organize our paper as follows. In section 2, a model of export supply

under exchange rate risk and hedging demand is presented. In section 3 we examine the impact of cross hedging on the exporting firm's decision making. We show that the separation and the full hedging theorems break down. Nevertheless the firm benefits from accomplishing a cross hedge policy. Also the firm's output for exports increases and hence international trade of the country is promoted. In section 4 we introduce price uncertainty. We consider a situation where only a perfect hedging market for commodity forwards is available to the firm initially and then introduce indirect hedging in the foreign currency. We examine the effect of this scenario on the firm's trade and hedging decision. Section 5 introduces random initial wealth of the exporting firm and analyzes its effect on risk policy. Section 6 investigates the output effect of risky initial wealth. We show that the risk aversion function of the firm plays an important role. Section 7 offers some conclusions.

## **2. A model of export production and hedging**

As we analyze the case without a currency forward market for the exporting firm, we assume the existence of certain currency substitutes whose spot prices are highly correlated with the foreign exchange rate. Moreover, we assume that such substitutes have markets for forward contracts and that the firm has access to them. We suppose that the firm utilizes only one of the substitute assets available. The single cross hedge instrument generally is a financial asset in contrast to a cross hedge between two related foreign currencies.

Let us consider a risk-averse exporting firm in a developing country facing a random exchange rate  $\tilde{\epsilon}$ . Let the price of the commodity in the world market be  $p$  and assume that it is fixed (we relax this assumption in section 4). The firm cannot hedge its foreign exchange risk in a currency forward market, because such market does not exist. However, there is a forward market for the substitute asset, and currency and asset spot prices are correlated. The

forward market of the substitute can be utilized by the exporting firm.

When the production decision takes place the firm has access to the forward market of the substitute asset. The asset's spot price  $\tilde{g}$  is related to the foreign exchange spot rate  $\tilde{e}$  by the following equation (assumption A.1):

$$\tilde{e} = \alpha + \beta\tilde{g} + \tilde{\varepsilon}$$

and  $\beta > 0$ . Hence we propose a linear 'regression' of the foreign exchange spot rate on the spot price of the substitute asset, and we assume that the spot price  $\tilde{g}$  be conditionally independent of the mean-zero uncertainty  $\tilde{\varepsilon}$ .<sup>1</sup> The exporting firm can sell or buy forward at a given forward price  $g_f$ . The number  $\text{var}(\tilde{\varepsilon}) = \text{var}(\tilde{e} - \beta\tilde{g})$  may be interpreted as the part of exchange rate risk that cannot be hedged with the surrogate, where  $\text{var}$  denotes the variance operator.

The exporting firm chooses its optimal production  $x$  and forward contracts  $z$  in a way that maximizes its expected utility of income  $\tilde{Y}$ , where income is denominated in domestic currency. Denote by  $C(x)$  the firm's cost function compounded to the date when the foreign exchange rate materializes and assume that  $C$  is an increasing and convex function of  $x$ . Let  $U$  be the firm's von Neumann-Morgenstern utility function and assume that  $U'(Y) > 0, U''(Y) < 0$ . The firm maximizes expected utility of income  $EU(\tilde{Y})$  by choosing  $x$  and  $z$ , where  $E$  is the mathematical expectations operator. Thus the producer's problem may be written:

$$\max_{x,z} EU(I_0 + \tilde{Y}) \tag{1}$$

---

<sup>1</sup>For the notion of conditional independence see Ingersoll (1987, pp. 15-16) and Lence (1995). The random variable  $\tilde{g}$  is conditionally independent of the random variable  $\tilde{\varepsilon}$  if, and only if,  $\text{cov}(\tilde{g}, f(\tilde{\varepsilon})) = 0$  for all functions  $f(\cdot)$ . Hence  $E(\tilde{g}|\varepsilon) = E\tilde{g}$ . Note that the assumption of conditionally independence is stronger than the zero-correlation assumption of the classical linear regression model of econometrics (see, for example, Greene (1993, pp. 143-144)).



where  $I_0$  denotes compounded nonstochastic initial wealth (we relax this assumption in section 5) and export income, which is stochastic and denoted by  $\tilde{Y}$  is defined as follows:

$$\tilde{Y} = \tilde{e}px - C(x) + z(g_f - \tilde{g}).$$

The asset's price  $\tilde{g}$  is the random spot value of the asset correlated to the spot exchange rate. The joint density may be a Bayesian prior, an empirically-estimated density, or a posterior density that combines the two. Necessary and sufficient conditions for a maximum are (for notational convenience, the producer's initial wealth is henceforth suppressed):

$$EU'(\tilde{Y}^*)(\tilde{e}p - C'(x^*)) = 0, \quad (2)$$

$$EU'(\tilde{Y}^*)(g_f - \tilde{g}) = 0, \quad (3)$$

where  $U'$  denotes marginal utility and  $C'$  marginal costs and the asterisk indicates an optimum level. Since the export decision depends on the firm's attitude towards risk as well as on the distribution of the exchange rate, the separation property and the full hedge theorem does not hold with cross hedging.

In the following we use the first-order conditions (2) and (3) in order to explore the effects of hedging on the firm's allocation decisions for export production, forward commitment and economic welfare.

### 3. Exports and foreign exchange risk policy

Our economic setting implies that the well-known separation property (for international firms) does not hold. Hence, optimum production as well as optimum hedging demand depend upon the firm's expectations and risk behavior, i.e., production and hedging decisions cannot be separated. Suppose the forward market is unbiased (assumption A.2):  $g_f = E\tilde{g}$ . In this case we claim:

*Proposition 1:* Consider an exporting firm facing exchange rate risk and where only a cross hedging instrument is available according to regressibility assumption A.1: (a) With positive correlation between the exchange rate and the financial asset, optimal hedging implies and is implied by the short forward position  $z^* = \beta px^* > 0$ , (b) the firm underhedges export revenue, i.e.,  $\frac{\text{cov}(\tilde{\varepsilon}, \tilde{g})}{\text{var}(\tilde{\varepsilon})} z^* < px^*$ , (c) the separation property does not hold; namely, optimal export production of the firm depends upon the perceived distribution of the random variables and upon the firm's attitude towards risk.

*Proof.* (a) From condition (3) and assumption A.1 we derive

$$\text{cov}(\tilde{g}, U'[(\beta px^* - z^*)\tilde{g} + \tilde{\varepsilon}px^* + \alpha px^* + z^*g_f - C(x^*)]) = 0, \quad (4)$$

where cov is the covariance operator. Equation (4) holds at  $\beta px^* - z^* = 0$  (sufficiency). Suppose  $\beta px^* > [\leq] z^*$ . Since marginal utility is continuously decreasing and  $\tilde{g}$  is conditionally independent of  $\tilde{\varepsilon}$ , we observe in both cases a contradiction to the first-order condition (4). Hence we must have  $\beta px^* \leq [\geq] z^*$ . Given that decision problem (1) has a unique optimum it must be at  $\beta px^* = z^*$  (necessity). (b) Multiply the hedge equation  $z^* = \beta px^*$  by  $\beta' \equiv \frac{\text{cov}(\tilde{\varepsilon}, \tilde{g})}{\text{var}(\tilde{\varepsilon})}$  and obtain  $\beta' z^* = \beta' \beta px^* < px^*$  since  $\beta' \beta$  represents the determination coefficient and, therefore, is less than one as correlation is not perfect. (c) Furthermore from equations (2) and (3) we derive

$$C'(x^*) + \frac{p \text{cov}(\tilde{\varepsilon}, -U'(\tilde{Y}^*))}{EU'(\tilde{Y}^*)} = (\alpha + \beta g_f)p \quad (5)$$

which proves that the decision for export production is affected by expectations and risk behavior of the firm.  $\square$

In the unbiased case the optimal hedge position  $z^* = \beta px^*$  has two distinctive properties. First it leaves the producer's expected income unchanged. Second, it reduces all uncertainty about the exporter's future income except that uncertainty which is by its very nature unhedgeable and undiversifiable, the residual uncertainty  $\tilde{\varepsilon}$ . The optimal hedge ratio of the firm is

a fixed proportion  $\beta$  of the cash position regardless of the utility function. In other words, the optimal fraction of export revenue to be sold forward by the exporting firm is equal to the squared correlation coefficient between the foreign exchange spot rate and the spot price of the surrogate. Hence the percentage hedge ratio equals the determination coefficient, and this ratio is independent of the specific concave and increasing utility function.

The failure of the separation property is shown in equation (5) also by the effect that besides production costs the firm bears a cost of covariant risk and, therefore,  $C'(x^*) < (\alpha + \beta g_f)p$ .

### 3.1 Benefits of cross hedging exchange rate risk

Generally, the exporting firm benefits from cross hedging. We denote by  $Y_o^*$  the optimum income of the exporting firm when no hedging is available and  $x_o^*$  the corresponding optimum export production. Comparing the firm's utility level under cross hedging with no hedging markets at all, we can prove the following result:

*Proposition 2: The introduction of a cross hedging instrument increases the firm's economic welfare.*

*Proof.* We shall use the strict concavity of the utility function and the strict convexity of the cost function as follows. Since  $Y^* \neq Y_o^*$  we obtain  $E[U(\tilde{Y}^*) - U(\tilde{Y}_o^*)] > EU'(\tilde{Y}^*)(\tilde{Y}^* - \tilde{Y}_o^*) = EU'(\tilde{Y}^*)[\tilde{e}p(x^* - x_o^*) + C(x_o^*) - C(x^*) + z^*(g_f - \tilde{g})]$ . However  $C(x_o^*) - C(x^*) > C'(x^*)(x_o^* - x^*)$  whenever  $x_o^* \neq x^*$ . Hence we can write

$$EU(\tilde{Y}^*) - EU(\tilde{Y}_o^*) > E[U'(\tilde{Y}^*)(\tilde{e}p - C'(x^*))](x^* - x_o^*) + E[(g_f - \tilde{g})U'(\tilde{Y}^*)]z^* = 0,$$

due to first-order conditions (2) and (3). Thus  $EU(\tilde{Y}^*) > EU(\tilde{Y}_o^*)$ .  $\square$

Note that the above claim is independent of our regressibility assumption A.1, since the cross hedging instrument offers an opportunity to a forward

trade which cannot be disadvantageous is our context. Furthermore the analysis yields some important policy implications. Suppose individual exporters (and importers) of commodities do not face the opportunity to trade in forward foreign exchange. Then the implication of policies that bring about results equivalent to those of introducing perfect or even imperfect forward markets may be desirable to promote international trade.

### 3.2 Output effects of cross hedging exchange rate risk

The usage of indirect hedging devices does not necessarily increase output for exports (see Broll, Wahl and Zilcha (1995)). But this ambiguity does not occur with our regressibility assumption A.1.<sup>2</sup> We derive the following claim:

*Proposition 3: Suppose assumptions A.1 (regressibility) and A.2 (unbiasedness) hold. If initially the firm has no access to hedging devices, then introducing a cross hedging instrument will increase export production.*

*Proof.* Let  $x_o^*$  denote optimum output for exports when there is no hedging market. This output satisfies the first-order condition

$$EU'(\tilde{Y}_o^*)(\tilde{\epsilon}p - C'(x_o^*)) = 0, \quad (6)$$

where in this case income becomes  $\tilde{Y}_o^* = (E\tilde{\epsilon} + \tilde{\epsilon})px_o^* + \beta(\tilde{g} - E\tilde{g})px_o^* - C(x_o^*)$  by using the definition of income from (1) with  $z \equiv 0$  and by inserting the regression.

Now observe that  $\tilde{Y}_o^*$  coincides with optimum income  $\tilde{Y}^*$  in the case of unbiased cross hedging, if we take  $\beta = 0$ , and the optimum hedging demand is replaced according to Proposition 1. We derive  $\tilde{Y}^* = (E\tilde{\epsilon} + \tilde{\epsilon})px^* - C(x^*)$  from (1) with  $z^* = \beta px^*$  and  $g_f = E\tilde{g}$ . Therefore, let us (implicitly) differentiate output  $x_o^*$  in condition (6) with respect to  $\beta$  and analyze the locus  $\beta = 0$ .

---

<sup>2</sup>Different directions of regressibility are discussed in Briys, Crouhy and Schlesinger (1993).

We obtain:

$$\operatorname{sgn} \left. \frac{\partial x_o^*}{\partial \beta} \right|_{\beta=0} = \operatorname{sgn} EU''(\tilde{Y}_0)(\tilde{e}p - C'(x_o^*))(\tilde{g} - E\tilde{g}),$$

where  $\tilde{Y}_0 = (E\tilde{e} + \tilde{\varepsilon})px_o^* - C(x_o^*)$ . But this derivative is negative, since by sequential integration we get

$$\begin{aligned} EU''(\tilde{Y}_0)(\tilde{e}p - C'(x_o^*))(\tilde{g} - E\tilde{g}) = \\ E_g(\tilde{g} - E\tilde{g})E_\varepsilon U''(\tilde{Y}_0)(\tilde{e}p - C'(x_o^*)) < 0, \end{aligned}$$

by the conditional independence assumption and by the fact that  $E_\varepsilon U''(\tilde{Y}_0)(\tilde{e}p - C'(x_o^*))$  is monotonically decreasing in  $g$ , if  $\beta > 0$ . Hence, introducing unbiased cross hedging decreases (locally)  $x_o^*$ , which means that (locally) output is raised by cross hedging. This proves the claim.  $\square$

Proposition 2 shows that with unbiased cross hedging the impact of regression coefficient  $\beta$  on income vanishes. This holds independently of the magnitude of this coefficient, and its influence on income only occurs when there is no hedging at all. Hence  $x_o^* < x^*$ , i.e., export production is promoted by unbiased cross hedging.

Now let us consider the scenario, when cross hedging becomes perfect, and let  $x^{**}$  denote optimal output for this situation. We start with the case of unbiasedness.

*Proposition 4: If initially the firm has access to an unbiased cross hedging device, then introducing an unbiased perfect hedging instrument will increase export production.*

*Proof.* Perfect cross hedging means  $\operatorname{var}(\tilde{\varepsilon}) = 0$ , i.e., all exchange rate risk can be hedged with the surrogate. But then the corresponding optimum output  $x^{**}$  satisfies the separation property (see, for example, Benninga, Eldor and Zilcha (1985):

$$C'(x^{**}) = (\alpha + \beta g_f)p. \quad (7)$$

On the other hand, unbiased cross hedging implies from equation (2) that

$$C'(x^*) < (\alpha + \beta g_f)p, \quad (8)$$

since  $EU'(\tilde{Y}^*)(E\tilde{e}p - C'(x^*)) = \text{cov}(-U'(\tilde{Y}^*), \tilde{e}p) > 0$ . Hence, comparing (7) and (8)  $x^* < x^{**}$  must hold, if marginal costs are increasing.  $\square$

To complete the analysis of unbiasedness let us combine the results of Propositions 3 and 4, i.e.,  $x_o^* < x^*$  and  $x^* < x^{**}$ . We note immediately the following

*Corollary:* Perfect cross hedging results in higher export production compared to the case of no hedging ( $x_o^* < x^{**}$ ).

We will close the investigation of export production effect of cross hedging exchange rate risk by allowing for non-zero risk premium in the forward rate of a perfect hedging device. From equations (2) and (3) we obtain:

$$\text{cov}(\tilde{Y}^*, U'(\tilde{Y}^*)) = -EU'(\tilde{Y}^*)[(E\tilde{e}p - C'(x^*))x^* + (g_f - E\tilde{g})z^*] < 0. \quad (9)$$

The inequality sign in (9) follows from the fact that  $U'$  is a strictly decreasing function of income. Therefore, assuming unbiased cross hedging ( $g_f = E\tilde{g}$ ) and denoting the forward rate of the perfect instrument by  $\hat{g}_f$ , equation (9) and the separation condition  $C'(x^{**}) = (\alpha + \beta\hat{g}_f)p$  yield, after rearranging terms,

$$\frac{\text{cov}(\tilde{Y}^*, -U'(\tilde{Y}^*))}{x^*EU'(\tilde{Y}^*)} = C'(x^{**}) - C'(x^*) - \beta p(\hat{g}_f - E\tilde{g}) > 0, \quad (10)$$

since optimal export production and marginal utility are positive.

Inequality (10) can be used to form an upper bound to the positive risk premium  $\pi$ , as measured by the money metric  $\text{cov}(\tilde{Y}^*, -U'(\tilde{Y}^*)) / \beta p x^* EU'(\tilde{Y}^*)$ . The upper bound ensures that the firm will always increase its output for exports when a currency forward market is introduced. Our result is stated in the following claim:

*Proposition 5:* Assume that initially the exporting firm has access only to an unbiased cross hedging instrument. Then introducing a currency forward market, i.e. a perfect hedging device, will increase the firm's export production if  $\hat{g}_f \geq E\tilde{g} - \pi$ , for

$$0 \leq \pi < \frac{\text{cov}(\tilde{Y}^*, -U'(\tilde{Y}^*))}{\beta p x^* E U'(\tilde{Y}^*)}.$$

*Proof.* The claim is a direct result of (10), for if the upper bound strictly holds, then  $x^{**} > x^*$  with unbiased cross hedging ( $\beta > 0, p > 0$ ).  $\square$

Proposition 5 shows that when the risk premium of the currency forward rate is not too high, introducing a currency forward market will encourage the growth of export production, starting with an unbiased cross forward rate.

On the other hand, equation (9) reveals the possibility that starting with a biased cross hedging market ( $g_f \neq E\tilde{g}$ ) and opening up an identically biased currency forward market may decrease the firm's output for exports. Rewriting (9) using (7) implies

$$\frac{\text{cov}(\tilde{Y}^*, -U'(\tilde{Y}^*))}{x^* E U'(\tilde{Y}^*)} = C'(x^{**}) - C'(x^*) + (g_f - E\tilde{g})(z^* - \beta p x^*)/x^* > 0.$$

Hence,  $C'(x^{**}) - C'(x^*)$  may, in principle, be negative.<sup>3</sup> When this difference is negative then clearly  $x^{**} < x^*$ .

#### 4. Risk policy with exchange rate and price uncertainty

We now relax the assumption of a fixed commodity price. In the sequel this price is a risky magnitude and we assume it to be independent of the foreign exchange spot rate. Regarding the hedging policy of the exporting firm, we consider the case where a perfect commodity forward market is available to

---

<sup>3</sup>Note that with independent random variables in our scenario the product  $(g_f - E\tilde{g})(z^* - \beta p x^*)$  is always positive.

the firm. For simplicity we take export production as given and denote it by  $x_0$ . Then the firm's decision is to maximize  $EU(\tilde{Y})$  with respect to the hedge portfolio  $z$  and  $q$ , where

$$\tilde{Y} = \tilde{e}\tilde{p}x_0 + q(p_f - \tilde{p})\tilde{e} + z(g_f - \tilde{g}), \quad (11)$$

and  $p_f$  is the commodity forward price in foreign currency,  $q$  the amount of the commodity sold or purchased in the forward market. The first-order conditions for the hedging policy are

$$EU'(\tilde{Y}^*)(g_f - \tilde{g}) = 0, \quad (12)$$

$$EU'(\tilde{Y}^*)\tilde{e}(p_f - \tilde{p}) = 0. \quad (13)$$

We obtain the following result:

*Proposition 6:* Suppose that commodity price and foreign exchange rate are stochastically independent. When an unbiased commodity forward market and an unbiased cross hedging instrument are available to the exporting firm, then optimal hedging of price risk and foreign exchange risk, respectively, is given by:

$$q^* = x_0 \quad \text{and} \quad z^* = \beta p_f x_0.$$

*Proof.* (i) Let us consider first the optimal demand for commodity forward contracts. From the law of iterated expectations, if  $p_f = E\tilde{p}$ , then from equation (13) we have

$$\text{cov}_p(\tilde{p}, E[\tilde{e}U'(\tilde{Y}^*)|\tilde{p}]) = 0. \quad (14)$$

With independence of  $\tilde{e}$  and  $\tilde{p}$  differentiation of the conditional expectation yields

$$\frac{\partial}{\partial p} E[\tilde{e}U'(\tilde{Y}^*)|p] = E[\tilde{e}^2 U''(\tilde{Y}^*)|p](x_0 - q^*),$$

and, therefore, as marginal utility is monotonically decreasing equation (14) holds if, and only if,  $x_0 = q^*$ .



(ii) The optimum cross hedging follows from equation (12). If  $g_f = E\tilde{g}$ , then

$$\text{cov}_g(\tilde{g}, E[U'(\tilde{Y}^*)|\tilde{g}]) = 0. \quad (15)$$

Independence assures that under our regressibility assumption

$$\frac{\partial}{\partial g} E[U'(\tilde{Y}^*)|g] = E[U''(\tilde{Y}^*)|g](\beta p_f x_0 - z^*),$$

since  $x_0 = q^*$  from step (i). Again, since  $U'' < 0$  equation (15) holds if, and only if,  $\beta p_f x_0 = z^*$ .  $\square$

Proposition 6 reveals the interaction between perfect and imperfect hedging devices. A full hedge obtains regarding commodity price risk whereas the foreign exchange rate risk is hedged up to hedge proportion  $\beta$ . Note that the commodity forward rate determines forward export revenue which is the basis of demand for forward contracts in the surrogate. It follows that all uncertainty about the producer's income is reduced except that uncertainty which is by its very nature unhedgeable and undiversifiable, namely the noise in the spot price of the substitute asset which serves as cross hedging instrument.

## 5. Risk policy with initial wealth uncertainty

We start with the economic setting of section 4 and relax the assumption of nonstochastic initial wealth. The exporting firm's hedging problem then becomes

$$\max_{z,q} EU(\tilde{I}_0 + \tilde{Y}), \quad (16)$$

where income  $\tilde{Y}$  is defined in equation (11) and  $\tilde{I}_0$  denotes uncertain initial wealth.<sup>4</sup> Modelling uncertainty of wealth when the production decision takes place can be motivated by the observation that firms in developing countries

---

<sup>4</sup>For a discussion of decision theoretical implications, see Kihlstrom, Romer and Williams (1981).

often face an endowment which is not certain and subject to a variety of shocks. We claim:

*Proposition 7:* *Suppose the unbiasedness assumption of Proposition 6 holds. Then, if the spot prices of the commodity and of the substitute asset are conditionally independent of the asset's price noise  $\tilde{\varepsilon}$  and of initial wealth  $\tilde{I}_0$ , then optimal risk policy of the firm remains unchanged:  $q^* = x_0$  and  $z^* = \beta p_f x_0$ .*

*Proof.* Let  $\tilde{Y}_0 = \tilde{I}_0 + \tilde{Y}$  and note the rearrangement of equation (11):

$$\tilde{Y}_0 = \tilde{I}_0 + \tilde{e}\tilde{p}(x_0 - q) + \tilde{g}(\beta p_f q - z) + \tilde{e}p_f q + k, \quad (17)$$

where  $k \equiv \alpha p_f q + g_f z$ . Unbiased markets and the optimal hedge policy of Proposition 6 imply for (16) the first-order conditions:

$$\text{cov}(\tilde{g}, U'(\tilde{I}_0 + \tilde{e}p_f q^* + k^*)) = 0, \quad (18)$$

and

$$\text{cov}(\tilde{p}, U'(\tilde{I}_0 + \tilde{e}p_f q^* + k^*)) = 0. \quad (19)$$

But (18) and (19) hold indeed by our conditional independence assumption, i.e.,  $\text{cov}(\tilde{g}, f(\tilde{I}_0, \tilde{\varepsilon})) = \text{cov}(\tilde{p}, f(\tilde{I}_0, \tilde{\varepsilon})) = 0$  for all functions  $f(\cdot)$ . Due to the strict concavity of the maximand in  $z$  and  $q$ , the first-order conditions are necessary and sufficient conditions for a maximum expected utility. Hence the unique hedge policy of Proposition 6 is still optimal.  $\square$

## 6. Exports with initial wealth uncertainty

So far we assumed in the context of volatile initial wealth that export production was given. But especially in developing countries also changes in the risk of initial wealth should be taken into account and will, in general, affect the firm's optimum output for exports.

Given the unbiasedness and conditional independence assumptions of section 5 the firm's optimal risk policy satisfies  $q^* = x_0$  and  $z^* = \beta p_f x_0$  which holds for arbitrary  $x_0 > 0$ . Hence this hedge portfolio structure is also true for optimum output  $x^* > 0$ . The exporting firm's production decision problem then becomes

$$\max_x EU(\tilde{Y}_0), \quad (20)$$

where

$$\tilde{Y}_0 = \tilde{I}_0 + (\alpha + \beta g_f + \tilde{\varepsilon}) p_f x - C(x), \quad (21)$$

using the definition of income in (17), adjusted for production costs and inserting the relationships between hedge contracting volumes and output given by optimum risk policy. This leads to the following first-order condition for optimal output:

$$EU'(\tilde{Y}_0^*)(\tilde{\varepsilon}\tilde{p} - C'(x^*)) = 0, \quad (22)$$

where as before the asterisk indicates an optimum level.

Now let us use equation (22) to compare the following scenarios: the case of random initial wealth  $\tilde{I}_0$  with the according optimum output for exports  $x^*$  and the case of  $\tilde{I}_0 = E\tilde{I}_0 \equiv \bar{I}_0$  where optimum exports be denoted by  $\bar{x}$ . We will give sufficient conditions under which  $x^*$  is less than, equal to, or greater than  $\bar{x}$ .

*Proposition 8: Assume unbiasedness for commodity and cross forward markets and independence of commodity price, foreign exchange rate and initial wealth. Then increasing risk of initial wealth will result in lower export production, if absolute risk aversion is decreasing and (nonstrictly) convex.*

*Proof.* Step 1. Define the following function:

$$\phi(\varepsilon) = \frac{E[U'(\tilde{Y}_0^*)|\varepsilon]}{U'(E[\tilde{Y}_0^*|\varepsilon])}, \quad (23)$$

where  $E[\cdot|\varepsilon]$  is the conditional expectation with respect to initial wealth  $\tilde{I}_0$  given the realization  $\varepsilon$  of random variable  $\tilde{\varepsilon}$ . Note from equation (21) that  $\tilde{Y}_0^*$

is also a function of  $\varepsilon$ , so that  $E[\cdot|\varepsilon] \neq E[\cdot]$  though  $\tilde{\varepsilon}$  and  $\tilde{I}_0$  are stochastically independent. Differentiating  $\phi(\varepsilon)$  we obtain

$$\phi'(\varepsilon) = \left( \frac{E[U''(\tilde{Y}_0^*)|\varepsilon]}{U'(E[\tilde{Y}_0^*|\varepsilon])} + R(E[\tilde{Y}_0^*|\varepsilon]) \frac{E[U'(\tilde{Y}_0^*)|\varepsilon]}{U'(E[\tilde{Y}_0^*|\varepsilon])} \right) p_f x^*, \quad (24)$$

where  $R(Y) = -U''(Y)/U'(Y)$  denotes the measure of absolute risk aversion. Rewriting (24) gives

$$\begin{aligned} \phi'(\varepsilon)U'(E[\tilde{Y}_0^*|\varepsilon])/p_f x^* & \quad (25) \\ &= R(E[\tilde{Y}_0^*|\varepsilon])E[U'(\tilde{Y}_0^*)|\varepsilon] - E[R(\tilde{Y}_0^*)U'(\tilde{Y}_0^*)|\varepsilon] \\ &= (R(E[\tilde{Y}_0^*|\varepsilon]) - E[R(\tilde{Y}_0^*)])E[U'(\tilde{Y}_0^*)|\varepsilon] - \text{cov}(R(\tilde{Y}_0^*), U'(\tilde{Y}_0^*)|\varepsilon). \end{aligned}$$

The conditional covariance in equation (25) is positive since the functions involved are both monotonically decreasing for all  $\varepsilon$ . Hence

$$\phi'(\varepsilon)U'(E[\tilde{Y}_0^*|\varepsilon])/p_f x^* < (R(E[\tilde{Y}_0^*|\varepsilon]) - E[R(\tilde{Y}_0^*)])E[U'(\tilde{Y}_0^*)|\varepsilon]. \quad (26)$$

Assuming nonconcave absolute risk aversion, Jensen's inequality and positive marginal utility imply that the right-hand side of (26) is less or equal to zero. With  $p_f > 0$  and  $x^* > 0$  we therefore have  $\phi'(\varepsilon) < 0$ .

Step 2. By the independence assumption  $ep$  is an increasing function of  $\varepsilon$ . Therefore, let  $\varepsilon = \bar{\varepsilon}$  be the level of noise when  $ep = C'(x^*)$ . Then, since  $\phi(\varepsilon)$  is decreasing

$$\phi(\varepsilon) < \phi(\bar{\varepsilon}), \quad \text{if } \varepsilon > \bar{\varepsilon}. \quad (27)$$

It follows that we also have  $ep > C'(x^*)$ , so that substituting from the definition of  $\phi(\varepsilon)$ , we obtain, after multiplication with  $U'(E[\tilde{Y}_0^*|\varepsilon])(ep - C'(x^*)) > 0$ ,

$$E[U'(\tilde{Y}_0^*)|\varepsilon](ep - C'(x^*)) < \phi(\bar{\varepsilon})U'(E[\tilde{Y}_0^*|\varepsilon])(ep - C'(x^*)). \quad (28)$$

Now consider the case when  $\varepsilon < \bar{\varepsilon}$  or,  $ep < C'(x^*)$ . Then inequality (27) is reversed, but multiplication by  $U'(E[\tilde{Y}_0^*|\varepsilon])(ep - C'(x^*)) < 0$  reverses it once more, so that (28) holds for all values of  $\varepsilon$ . Taking expectations in (28) gives

$$EU'(\tilde{Y}_0^*)(\tilde{ep} - C'(x^*)) < \phi(\bar{\varepsilon})EU'(E[\tilde{Y}_0^*|\tilde{\varepsilon}](\tilde{ep} - C'(x^*)). \quad (29)$$

But optimum output  $\bar{x}$ , i.e. when  $\tilde{I}_0 = \bar{I}_0$ , satisfies

$$EU'(\bar{I}_0 + (\alpha + \beta g_f + \tilde{\varepsilon})p_f \bar{x} - C(\bar{x}))(\tilde{e}\tilde{p} - C'(\bar{x})) = 0. \quad (30)$$

Hence the right-hand side of (29) is zero for export production  $x = \bar{x}$  and the left-hand side, accordingly, becomes negative at optimum  $\bar{x}$ . Since expected utility is strictly concave in  $x$ , we therefore conclude from equations (29) and (30) that  $x^* < \bar{x}$ . This proves the claim.  $\square$

Note that Proposition 8 holds for the widely used linear risk tolerance family of utility functions  $R(Y)^{-1} = A + BY > 0$ :<sup>5</sup>

$$U(Y) = \begin{cases} \frac{1}{B-1}(A + BY)^{1-1/B}, & \text{for } 0 < B \neq 1 < \infty, \\ \ln(A + Y), & \text{for } B = 1, \end{cases}$$

if  $B$  is restricted to be positive and finite, implying decreasing absolute risk aversion. Note that this class of utility functions has (nonstrictly) convex absolute risk aversion, in general.

*Corollary:* (i) *If absolute risk aversion is increasing and (nonstrictly) concave, then increasing risk in initial wealth increases the level of the firm's export production.* (ii) *If absolute risk aversion is constant or if the utility function is quadratic, then an increase in volatility of  $\tilde{I}_0$  does not affect the production level of the firm.*

*Proof.* (i) The claim follows from the proof of Proposition 8 since the sign of  $\phi'(\varepsilon)$  is reversed. (ii) If  $R(Y)$  is a constant, then  $\phi'(\varepsilon) = 0$ . Hence, again according to the proof of Proposition 8, export production cannot change when initial wealth becomes more volatile. But also if  $U'(Y)$  is linear in  $Y$ , as it is with quadratic utility, i.e.  $U'(Y) = a + bY > 0, b < 0$ , the first-order

---

<sup>5</sup>Another notion for this class of utility functions is hyperbolic absolute risk aversion (HARA) preferences (see, e.g., Pratt, Raiffa and Schlaifer 1995, p. 806).

condition (22) becomes

$$\frac{E(\tilde{\varepsilon}\tilde{p} - C'(x^*))}{x^*} = \frac{-b}{a + bE\tilde{Y}_0^*} \text{var}(\tilde{\varepsilon})p_f^2.$$

Hence  $x^*$  does not depend upon higher moments of the random variable  $\tilde{I}_0$  then  $E\tilde{I}_0$ , because  $E\tilde{Y}_0^* = \bar{I}_0 + (\alpha + \beta g_f)p_f x^* - C(x^*)$  from equation (21). This completes the proof.  $\square$

In the light of the above mentioned linear risk tolerance class of utility functions, part (i) of the Corollary cannot obtain. On the other hand, part (ii) holds if for  $A > 0$  we have  $B = 0$  or  $B = -1$ . But remember that we only derived sufficient conditions for signing the effect of risky initial wealth on export production. This can be emphasized by considering again quadratic utility which implies increasing but strictly convex absolute risk aversion for positive marginal utility.

To summarize, our discussion shows that riskiness of initial wealth, which may be a meaningful assumption in the context of developing countries, does not necessarily imply that export production of firms decreases and, hence, does not necessarily interfere with the promotion of international trade. An essential factor of influence is risk attitude of exporting firms or, more precisely, the kind of absolute risk aversion function the firms possess.

## 7. Concluding remarks

We have presented a model of a risk-averse exporting firm in a developing country under exchange rate risk when there are no perfect hedging devices available. However there exist related assets the prices of which are correlated to the foreign currency. This situation calls for hedging a quantity of the currency spot market with forward contracts delivering other assets. We consider the market for forward contracts in stochastically related assets and we investigate the hedging policy of the exporting firm that enters that forward market. For simplicity we suppose that the firm utilizes only one of the

substitute assets available. The single cross hedge instrument is some asset in contrast to a cross hedge between two related foreign currencies using a parity relation. This is different from the literature.

We show that with cross hedging the well-known separation theorem (for international firms) does not hold. Although we assume that the forward market is unbiased, we demonstrate that the firm underhedges in the case of cross hedging. Furthermore, the firm's export production, when using cross hedging, is lower than its production when perfect hedging is available. On the other hand, the firm's output for exports increases if cross hedging becomes possible though it means an imperfect hedge. This has implications for policy and international trade: Exporting firms benefit when cross hedging devices are offered by governments, for instance, and it is not really important that the hedging instrument may be not perfect. Furthermore the output effect is such that cross hedging opportunities promote international trade.

We then consider the implications of commodity price risk and uncertain initial wealth for the hedging policy of the exporting firm. If an unbiased commodity forward market is available, the firm fully hedges its output in this market. The cross hedging of the firm is affected by the commodity forward rate which determines the forward export revenue to be hedged. The optimum fraction of forward export revenue sold in the cross hedging market is given by the determination coefficient between the foreign exchange spot rate and the spot price of the substitute asset. This holds for all concave utility functions. Furthermore there are conditions under which these results still hold when initial wealth becomes risky. Finally, the interference of risky initial wealth with export production is shown to depend upon the shape of the firm's risk aversion function.

## References

- Benninga, S., Eldor, R. and I. Zilcha, 1985, Optimal international hedging in commodity and currency forward markets, *Journal of International Money and Finance* 4, 537-552.
- Briys, E., Crouhy, M. and H. Schlesinger, 1993, Optimal hedging in a futures market with background noise and basis risk, *European Economic Review* 37, 949-960.
- Broll, U. and J.E. Wahl, 1995, Imperfect hedging and export production, *Southern Economic Journal* (forthcoming).
- Broll, U., Wahl, J.E. and I. Zilcha, 1995, Indirect hedging of exchange rate risk, *Journal of International Money and Finance* 14, 667-678.
- Broll, U. and I. Zilcha, 1992, Exchange rate uncertainty, futures markets and the multinational firm, *European Economic Review* 36, 815-826.
- Buckley, A., 1992, *Multinational finance*, New York: Prentice Hall.
- Eaker, M.R. and D.M. Grant, 1987, Cross hedging foreign currency risk, *Journal of International Money and Finance* 6, 85-105.
- Fry, M., 1992, *Money, interest, and banking in economic development*, Baltimore: Johns Hopkins University Press.
- Greene, W.H., 1993, *Econometric analysis*, 2nd ed., New York: Macmillan.
- Grobar, L.M., 1993, The effect of real exchange rate uncertainty on LDC manufactured exports, *Journal of Development Economics* 41, 367-376.
- Ingersoll, Jr., J.E., 1987, *Theory of financial decision making*, Totowa: Rowman & Littlefield.
- Kawai, M. and I. Zilcha, 1986, International trade with forward-futures markets under exchange rate and price uncertainty, *Journal of International Economics* 20, 83-98.



- Kihlstrom, R.E., Romer, D. and S. Williams, 1981, Risk aversion with random initial wealth, *Econometrica* 49, 911-920.
- Lence, S.H., 1995, On the optimal hedge under unbiased futures prices, *Economics Letters* 47, 385-388.
- Myers, R.J., 1992, Incomplete markets and commodity linked finance in developing countries, *The World Bank Research Observer* 7, 79-94.
- Powell, A., 1989, The management of risk in developing country finance, *Oxford Review of Economic Policy* 5, 69-87.
- Powers, M.J. and M.C. Castelino, 1991, *Inside the financial futures markets*, New York: Wiley & Sons.
- Pratt, J.W., Raiffa, H. and R. Schlaifer, 1995, *Introduction to statistical decision theory*, Cambridge: The MIT Press.
- Sercu, P., 1992, Exchange risk, exposure, and the option to trade, *Journal of International Money and Finance* 11, 579-593.

*Udo Broll, University of Konstanz, Department of Economics, D-78434 Konstanz (Germany),*

*Jack E. Wahl, University of Dortmund, Department of Finance, D-44221 Dortmund (Germany).*

---