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**Working Paper**

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Diskussionsbeiträge - Serie II, No. 52

**Provided in Cooperation with:**

Department of Economics, University of Konstanz

*Suggested Citation:* König, Peter; Möller, Joachim (1988) : Deviations from uncovered interest parity: A Kalman filter approach to the Mark/Dollar rate and the Swiss Franc/Dollar rate, Diskussionsbeiträge - Serie II, No. 52, Universität Konstanz, Sonderforschungsbereich 178 - Internationalisierung der Wirtschaft, Konstanz

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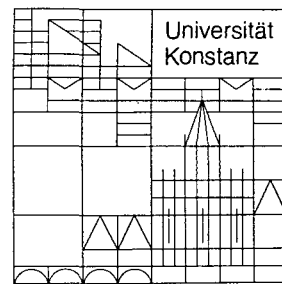
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# Sonderforschungsbereich 178 „Internationalisierung der Wirtschaft“

Diskussionsbeiträge



Juristische  
Fakultät

Fakultät für Wirtschafts-  
wissenschaften und Statistik

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Peter König  
Joachim Möller

**Deviations from  
Uncovered Interest Parity**  
A Kalman Filter Approach  
to the Mark/Dollar Rate  
and the Swiss Franc/Dollar Rate

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# DEVIATIONS FROM UNCOVERED INTEREST PARITY

## A Kalman Filter Approach to the Mark/Dollar Rate and the Swiss Franc/Dollar Rate

Peter König and Joachim Möller \*

Serie II – Nr. 52

April 1988

Ag 2764/88  
Weltwirtschaft  
Kiel

### Abstract

The present paper analyzes deviations from Uncovered Interest Parity by applying a Kalman filter procedure based on optimal initial information to the estimation of this arbitrage equilibrium for the Mark/Dollar rate and the Swiss Franc/Dollar rate. The results demonstrate that the hypothesis of a time-varying deviation from Uncovered Interest Parity in these equations cannot be rejected. Furthermore, these models are shown to out-perform the random walk in forecasting. An examination of the stochastic disturbances indicates that the shocks responsible for the deviations cover a wide range of economic and political issues.

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\* We are grateful to the members of the research seminar of the SFB 178 'Internationalisierung der Wirtschaft' for suggestions and helpful comments. The research for this paper was supported by the Deutsche Forschungsgemeinschaft.

## 1. Introduction

The failure of some representative structural economic models of the portfolio balance and of the monetary approach to explain and forecast exchange rate behaviour [Meese and Rogoff (1983a), (1983b) and (1985)] has often been attributed to the fast reaction of exchange rates to "news". The random walk hypothesis which was used as a reference model in these studies is based exclusively on the instantaneous incorporation of stochastic "news" into the current spot rate and thereby neglects any other economic structure. Maybe for this very reason the random walk approach proved more successful in the empirical estimation and forecasting of exchange rate behaviour. One direction pursued in recent theoretical and empirical research to overcome this rather disenchanting performance of the structural economic models has been the elaboration of approaches linking exchange rate determination exclusively to interest rate and forward rate arbitrage. The theoretical foundations of these approaches, notably the Uncovered Interest Parity condition, had already played key-roles in other structural economic approaches to exchange rate determination. Nevertheless, the avoidance of a commitment to additional specific macroeconomic structures, which were often too clear-cut as in the monetary approach or too extended as in many large-scale models to shed light on the empirically relevant determinants of flexible exchange rates, is apt to yield a more accurate conception of one of the main adjustment processes in international macroeconomics. Consequently, the (empirical) research focussing on a single structural relationship has resulted in an accentuation of several other problems associated with the pure arbitrage models. These problems originated mainly from the empirical observation of deviations from the Uncovered Interest Parity condition, i.e. "spreads" between the prices on those markets, among which the arbitrage process was supposed to work.

Referring to the corresponding literature, various theoretical and empirical approaches explaining the appearance of those spreads in the arbitrage equation will be discussed in section 2. According to these arguments, the deviations from Uncovered Interest Parity can be modeled as biases in the (estimated) expectation formation process [Frankel and Froot (1986), (1987a) and (1987b)], "news" about the structures of the underlying economies [Dornbusch (1980), Frenkel (1983)], time-varying risk premia in an international portfolio balance framework [Frankel (1979), (1982a), (1982b), (1987)], and time-varying risk premia in an intertemporal asset pricing context [Lucas (1978) and (1982), Hansen and Hodrick (1983), Hodrick and Srivastava (1984), (1987)].

In section 3, a small model of the determination of the current spot rate in an international interest rate arbitrage equilibrium will be presented, which combines elements from various branches of the literature discussed in section 2. With regard to exchange rate expectations, several hypotheses about the expectation formation processes will be considered, as described by Frankel and Froot (1987a), or De Jong (1987). The spread itself will be introduced as a state variable driven by stochastic shocks. This is basically in line with the "news" literature as well as with a representation of it as a (stochastically) time-varying risk premium. The main feature of this approach is the identification of two different stochastic processes in a model of exchange rate determination: the disturbances in the exchange rate formation process, which are assumed to be "white noise", and a random walk

process describing the development of the spread as a latent variable, which reflects accumulated past information about structural characteristics of the economies. Loosely speaking, the shocks affecting the exchange rate are divided into those which are forgotten after the current period, and those which are kept in memory. This latent variable approach avoids the explicit need for taking the pick of a variety of data series to explain the spread, as is necessary in the "news" models or in the portfolio balance approaches. In this respect, it is equivalent to the instrumental variable approach used by Hodrick and Srivastava (1984). In contrast to their method, however, the identification of the two separate stochastic processes pays regard to the arguments brought forward in the "news" literature [Dornbusch (1980) p.21, Frenkel (1983) p.19].

The implementation of this theoretical structure is based on the Kalman algorithm. Following an approach suggested by Watson and Engle (1983), a Maximum-Likelihood method of estimating the optimal partition of the stochastic processes will be outlined in section 4. Using this optimal initial information, the spread will be filtered out in a state space model as a time-varying intercept in the exchange rate equation.

In section 5 empirical results will be presented for the Mark/Dollar and the Swiss Franc/Dollar rate for the time period between 1974,1 and 1987,2. All estimations were carried through for a set of alternative assumptions about expectation formation. The results of the Kalman procedure are contrasted with the corresponding OLS results. Moreover, the models were re-estimated for 1974,1 to 1984,4, and out-of-sample forecasts were run for 1985,1 to 1987,2, again with OLS and also the random walk hypothesis as reference models.

Then in section 6, an attempt is made to shed some light on the determinants of the spread variable by comparing some of its major changes as estimated in section 5 with events, which supposedly had significant and large impact on the information set of market participants, changed their behaviour towards risk or, alternatively, could be regarded as important "news" about the structure of the underlying economies. The changes in the spread variable are shown to reflect several economic and political developments, ranging from the Afghanistan crisis in 1979/80 over the Reagan election in 1980/81 to the Plaza Agreement in 1985, or the announcement of exchange rate stabilization programs like like Carter's in 1978.

Section 7 contains concluding remarks.

## 2. Theoretical Explanations for Deviations from Uncovered Interest Parity

Three main issues have been addressed in the course of the theoretical examination of possible deviations from the Uncovered Interest Parity condition.

First, one of the reasons for the existence of spreads might be the problems associated with the explicit consideration and measurement of expectation formation behaviour as discussed by Frankel and Froot (1986), (1987a) and (1987b). The lack of reliable data on this issue requires the broadly accepted imposition of pre-set assumptions about this process. Though this problem tends to be reduced if different sets of models are allowed to represent the expectation formation, it must be emphasized that an incorrect specification will lead to biases in the estimation of the arbitrage equation. This systematic error might explain spreads between the current forward premium or interest differential and the expected depreciation of the spot rate given by the economic model.

In a second branch of the literature, the spreads between forward rate or interest differential and the expected spot rate have been attributed to "news" which alter the expected future spot rate, and "news" concerning the structures of the underlying economies, respectively [Dornbusch (1980), Frenkel (1983)]. In principle, these structural "news" may contain unexpected changes of inflationary expectations, unexpected changes of interest rates, or - with respect to a most recent shift of focus - news about trade balance figures. Besides the obvious difficulty of evaluating which data series are likely to reflect these unexpected structural changes adequately, there remains a second problem involved with the treatment of past "news" in those models: since the spread between the expected future spot rate and the current forward rate or interest differential is derived as being proportional to the difference between last period's expected and the realized values of the data series reflecting the structural change, the spread must be equal to zero in the case of a correct prediction of the future values. Thus, all past structural changes are assumed to be incorporated in the current forward rate or interest differential. For unbiased expectations about these structural changes, this assumption is equivalent to the spread being random in its level - not its change.

The third conception of modeling spreads between forward rate and expected spot rate revolves around the assumption of time-varying risk premia in international arbitrage. It has been pursued along two main lines of research: one approach analyzes risk premia in the foreign exchange markets in an international portfolio balance framework [Frankel (1979), (1982a), (1982b), (1987)]; according to the other approach, time-varying risk premia are derived in an intertemporal asset pricing context [Lucas (1978), (1982), Hansen and Hodrick (1983)].

In the portfolio balance approach, the different default risks of bonds issued by different borrowers induce the spread between the rates of return as measured by expected exchange rate changes and the interest rate differential. These risk premia are derived as being proportional to the degree of risk aversion of investors, the variances and the covariances of the exchange rates, and the difference between the actual "speculative" and the "minimum variance" portfolio. The empirical testing of this theory was conducted under the assumptions of constant degrees of

exchange rate variability, constant degrees of risk aversion and a constant "minimum variance" portfolio. The risk premium then becomes a (linear) function of the relative supply of "outside" assets, with several assumptions about the structure of these asset supplies being tested simultaneously with the exchange rate equations [Frankel (1982a), (1982b)]. Clearly, the necessity of having to pick and choose a data series representing the spread turns out to be a restrictive and rather arbitrary feature of the analysis. In contrast to the pure "news" models, however, the spread is modeled as a state variable. Past developments of the determinants of the spread are reflected in its current level and not assumed to be incorporated in the current forward rate or interest differential.

In the intertemporal asset pricing context investors maximize discounted expected utility defined on consumption. In equilibrium, the conditional covariance of the  $n$ -period return on foreign exchange speculation and the marginal rate of substitution of money between periods  $t$  and  $t + n$  is required to be proportional to the risk premium on forward foreign exchange or international interest rate speculation, respectively. Thus, the risk premium is supposed to vary according to changes in the information sets of the market participants. In the empirical assessment of this theory, the currently occurring forecast error in the arbitrage equation was used as an instrumental variable representation of the risk premium, resulting in the spread between expected exchange rate appreciation and the forward premium or interest differential being a linear function of last period's spread [Hodrick and Srivastava (1984)]. This approach avoids the difficulties associated with selecting a data series modeling the spread, furthermore the "news" are allowed to affect a state variable, i.e. last period's spread. Yet, in taking last period's spread as an instrument, it is not possible to distinguish between "news" that change the risk premium and "news" that change the current forward rate (or measurement errors in the dependent variable), the difference having been emphasized by Dornbusch (1980). This superimposing of the stochastic structures may amount to an error-in-variables problem, resulting in biased estimates.

In the following section, a spread in an international interest rate arbitrage equilibrium is allowed for without an exclusive commitment to any of the theoretical aspects as described above. The only theoretical structure imposed on the spread will be the assumption that its changes are purely random, which reflects some of the elements of the "news" approach as of the model with time-varying risk premia in the intertemporal asset pricing context.



### 3. Exchange Rate Determination and Expectation Formation

The spread in an arbitrage equilibrium for international interest speculation is given via the modified Uncovered Interest Parity condition (in logarithmic terms) as

$$sp_t = -(E_t e_{t+1} - e_t) + E_t i_{t+1} - E_t i_{t+1}^* \quad (1)$$

where  $i$  and  $i^*$  are the logs of one plus the interest rates on perfectly substitutable home and foreign bonds,  $e$  is the log of the home currency price of one unit foreign currency and  $E$  is the expectations operator.

This spread in the arbitrage equilibrium can alternatively be regarded as a bias in exchange rate expectations, as a representation of structural "news", or as an expected risk premium on foreign bonds in international interest speculation. Assuming that all past information relevant for the determination of the spread is incorporated in the current spread, then, with future "news" about its determinants being stochastic, the spread will follow a random walk

$$sp_t = sp_{t-1} + w_t \quad (2)$$

with  $w_t$  being the "news" affecting the spread and

$$E_t w_{t+1} = 0 \text{ and } \text{cov}(w_t, w_{t+1}) = 0.$$

Assuming that interest rates are fixed over one period in advance and solving for the current spot rate as the endogenous variable yields the well-familiar (modified) arbitrage equation

$$e_t = E_t e_{t+1} + sp_t - i_t + i_t^* \quad (3)$$

The following models are to be considered alternatively for the exchange rate expectation formation process [see Frankel and Froot (1987a), de Jong (1987)]:

Extrapolative expectations (model A) with

$$E_t e_{t+1} = g e_{t-1} + (1 - g) e_t + z_{t+1}. \quad (4)$$

yield bandwagon effects for  $-1 < g < 0$  and a distributed lag process for  $0 < g < 1$ . For  $z_t$  as the "noise" in the expectation proxies [see Frankel and Froot (1987a) p.148, or, Frenkel (1983) p.8] it is assumed that  $E_t z_{t+1} = 0$  and  $\text{Cov}(z_{t+i}, z_{t+j}) = 0$  for  $i \neq j$ .

(A)

Regressive expectations (model B) with regard to long-run equilibrium considerations [see also Dornbusch (1976)] can be modeled as

$$E_t e_{t+1} = h \bar{e}_t + (1 - h) e_t + z_{t+1} \quad (5)$$

with  $0 < h < 1$ . As for the long-run equilibrium rate, Purchasing Power Parity will be assumed, i.e.

$$\bar{e}_t = e_0 + (p_t - p_0) - (p_t^* - p_0^*), \quad (6)$$

with  $p$  and  $p^*$  the logs of home and foreign consumer price levels. In the empirical estimations, the term  $(e_0 - p_0 + p_0^*)$  will be included as a shift in the intercept (see section 5).

A combination of regressive and extrapolative expectations (model C) yields an expectation process of the form

$$E_t e_{t+1} = h \bar{e}_t + g e_{t-1} + (1 - g - h) e_t + z_{t+1} \quad (7)$$

with  $0 < h < 1$  and  $0 < g < 1$ .

Substituting eq.(7) into eq.(4) and solving again for the current spot rate  $e_t$  as the endogenous variable leads to

$$e_t = \frac{1}{g+h} s p_t - \frac{1}{g+h} (i_t - i_t^*) + \frac{h}{g+h} \bar{e}_t + \frac{g}{g+h} e_{t-1} + \frac{1}{g+h} z_{t+1}. \quad (8)$$

For  $g = 0$  eq.(8) yields the case of pure regressive and for  $h = 0$  the case of pure extrapolative expectations. Obviously, eq.(8) is over-identified, which results in the restriction that the sum of the coefficients of the long-run equilibrium rate and of the lagged dependent is equal to one.

Eq.(8) is the reduced form exchange rate equation to be tested in section 5. Simultaneously, the spread is allowed to vary over time according to eq. (2) via a time-varying intercept in the state equation of the Kalman filter, which will be presented in the following section.

#### 4. A Kalman Filter Method Based on Optimal Initial Information

For estimation purposes a state-space model of the following form will be used:

$$y_t = X_t \beta_t + u_t \quad (\text{measurement equation}) \quad (9)$$

$$\beta_t = \beta_{t-1} + v_t, \quad (\text{state equation}) \quad (10)$$

with  $y_t$  the dependent variable,  $X_t$  the  $(1 \times k)$ -vector of explanatory variables and  $\beta_t$  the vector of coefficients in time period  $t$  ( $t = 1, \dots, T$ ). The stochastic disturbances  $u_t$  and  $v_t$  are Normally and independently distributed:

$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} \sim NID \left[ 0, \begin{pmatrix} \sigma_u^2 & 0 \\ 0 & Q \end{pmatrix} \right], \quad (11)$$

with  $Q = \text{cov}(v'v)$  and  $\sigma_u^2 = \text{var}(u)$ , where  $v$  is a  $(T \times k)$ -matrix with  $v_t$  as row  $t$ , and  $u$  a  $(T \times 1)$ -vector with typical element  $u_t$ . If the prior information (i.e.  $\sigma_u^2$  and  $Q$ ) is known, the time path of the estimated coefficients can be calculated using the Kalman smoothing algorithm [see for example Anderson and Moore (1979)]. But in an econometric context this important initial information is unknown and has to be extracted from the data. In the corresponding literature [see Engle and Watson (1981), Watson and Engle (1983), Chow (1984)], different Maximum-Likelihood estimation methods are discussed [Fisher-Scoring-algorithm, EM-algorithm, see Dempster, Laird, Rubin (1977) for the latter].

Current experience shows that in some cases the EM-algorithm requires an extremely high number of iterations before reaching convergence. Furthermore, severe problems arise when diagonal elements of  $Q$  tend to zero. Therefore, the (numerically rather costly but also more informative) Scoring method seems more appropriate. This method will be briefly outlined following Watson and Engle (1983).

Given the innovations

$$\eta_t = y_t - X_t \beta_{t-1} \quad (12)$$

and their covariance matrix

$$H_t = X_t(P_{t-1} + Q)X_t' + \sigma_u^2 \quad \text{with} \quad P_{t-1} = \text{var}(\beta_{t-1}) \quad (13)$$

the kernel of the likelihood function is given by

$$\ln L_t = \text{const.} - 0.5 \sum_{t=1}^T (\ln |H_t| + \eta_t' H_t \eta_t). \quad (14)$$

A vector of unknown parameters  $\theta$  can be defined as containing  $\sigma_u^2$  and the elements of  $Q$ . The first derivatives of the likelihood function with respect to the  $i$ -th element of  $\theta$  are recursively calculated following Engle and Watson (1981):

$$\frac{\partial \ln L}{\partial \theta_i} = \sum_{t=1}^T \frac{\partial \ln L_t}{\partial \theta_i} \quad (15)$$

where

$$\frac{\partial \ln L_t}{\partial \theta_i} = -0.5 \text{ spur} \left( H_t^{-1} \frac{\partial H_t}{\partial \theta_i} \right) - \left( \frac{\partial \eta_t}{\partial \theta_i} \right)' H_t^{-1} \eta_t - 0.5 \eta_t' H_t^{-1} \frac{\partial H_t}{\partial \theta_i} H_t^{-1} \eta_t. \quad (16)$$

A typical element of the information matrix  $J = -E[\partial^2 L / \partial \theta \partial \theta']$  is obtained by the following equation:

$$[J]_{ij} = \sum_{t=1}^T \left\{ 0.5 \text{ spur} \left( H_t^{-1} \frac{\partial H_t}{\partial \theta_i} H_t^{-1} \frac{\partial H_t}{\partial \theta_j} \right) + E \left[ \left( \frac{\partial \eta_t}{\partial \theta_i} \right)' H_t^{-1} \frac{\partial \eta_t}{\partial \theta_j} \right] \right\}. \quad (17)$$

After the  $n$ -th iteration, the new vector  $\theta^{n+1}$  is given as

$$\theta^{n+1} = \theta^n + c J^{-1} \lambda,$$

where  $\lambda$  is the vector of first derivatives of the likelihood function with respect to  $\theta$  and  $c$  is a suitable chosen constant [see Berndt et. al. (1974)]. During the optimization it must be checked that the covariance matrices are not negative definite. In some cases the corresponding rows and columns of the covariance matrices have to be restricted to zero.

The iteration process is stopped when the Lagrange-multiplier-test

$$\mu = \lambda' J^{-1} \lambda / T$$

becomes sufficiently small (say,  $10^{-4}$ ). This will be the case when the slope of the likelihood function tends to zero.

In practice, the covariance matrix  $Q$  will be restricted to a diagonal matrix. If the  $i$ -th diagonal element  $q_{ii}$  on a priori reasons or as a result of the optimization procedure is set to zero, the corresponding element of the parameter vector does not vary over time. The hypothesis  $q_{ii} = 0$  can be tested using a likelihood ratio test.

In the case of the theoretical model described in section 3, the focus is on the hypothesis  $q_{11} = 0$  which has to be rejected if the spread variable can be adequately modeled as a random walk. It should be mentioned that for the estimations described in the next section the hypothesis  $q_{ii} = 0$  could not be rejected at the 5% level for all  $i > 1$ .

## 5. Empirical Results

All data used in the estimation were taken from the International Monetary Fund's International Financial Statistics. The model was estimated with quarterly end of period data for the Mark/Dollar rate and the Swiss Franc/Dollar rate (lines ae) from 1974,1 to 1987,2. As for the interest rates, government bond yields (lines 61) were chosen. For the long-run equilibrium rates, the relative consumer price indices (lines 64) were inserted.

Tables 1 and 2 show empirical results for the estimation of eq. (8) and eq. (2) for the Mark/Dollar rate and the Swiss Franc/Dollar rate for the three different hypotheses about the expectation formation process. The OLS regressions of eq. (8) with a constant intercept are presented as reference models.

As can be seen from both tables, the models with regressive expectations (B) are clearly inferior to the two other specifications. Bad autocorrelation statistics, significantly 'wrong' signs of coefficients with regard to the theoretical model, as well as a zero-variance of the measurement equation as a result of the Maximum Likelihood estimation of the priors in the state space model suggest severe specification errors. For model A the coefficient of the lagged endogeneous is significantly different from one when estimated with the state space model (which is not in accordance with the theoretical derivations), while for model C all results are in line with the theory.

By comparing the values of the respective likelihood functions, the assumption of a constant intercept must clearly be rejected for the Mark/Dollar rate in all specifications. For the Swiss Franc/Dollar rate the direct comparison of OLS and Kalman yields ambiguous results: the likelihood ratio test is significant at a high level only for model B, it is close to the 5% significance level for model C, whereas it is not significant even at the 10% level for model A. A clear discrimination between model A and C under the hypothesis of a constant intercept is difficult: the likelihood ratio test between the two specifications is not significant at the 5% level but slightly exceeds the critical value at the 10% level. So it seems appropriate to test a joint hypothesis. Comparing the likelihood value of the least restrictive model, i.e. the Kalman approach with combined expectations (C), with the OLS model for extrapolative expectations (A), gives a likelihood ratio of 6.534, which exceeds the

TABLE 1  
Estimation Results for the Mark/Dollar-Rate (1974,1 – 1987,2)

	Model A		Model B		Model C	
	OLS (1)	Kalman (2)	OLS (3)	Kalman (4)	OLS (5)	Kalman (6)
intercept	0.011	t.v.	0.844	t.v.	-0.027	t.v.
s.e.	0.045	—	0.043	—	-0.046	—
$q_{11} \times 100$	—	0.109	—	0.288	—	0.103
$i_t - i_t^*$	-3.636	-13.876	1.956#	-11.666	-7.353	-14.996
s.e.	1.502	3.405	0.307	3.811	2.209	4.387
$p_t - p_t^*$	—	—	-0.040#	0.868	0.196	0.783
s.e.	—	—	0.257	0.701	0.088	0.477
$e_{t-1}$	0.960	0.601#	—	—	0.975	0.583
s.e.	0.051	0.105	—	—	0.049	0.104
DW	1.590	1.734	0.148	1.182	1.604	1.876
Durbin's h	1.625	1.536	—	—	1.560	0.706
$\sigma_u^2 \times 100$	0.320	0.085	2.590	0.000	0.300	0.081
ln likelih.	79.854	85.231	23.529	81.335	82.409	86.646
L-R-Test	10.754		115.612		8.474	

Notes:

- A : extrapolative expectations, B : regressive expectations, C : combined hypothesis
- $q_{11}$  : first diagonal element of the covariance matrix of the state equation
- $\sigma_u^2$  : variance of the measurement equation
- L-R-Test : Likelihood-Ratio-Test, testing the hypothesis  $q_{11} = 0$
- t.v.: the corresponding coefficient is time-varying
- # : not consistent with theoretical model

critical value of the  $\chi^2$  distribution with 2 degrees of freedom at the 5% level. Thus, also for the Swiss Franc/Dollar rate, neither should the price differential be excluded from the set of explanatory variables, nor should the intercept be restricted to be constant. To check the restrictions on the estimated parameters imposed by the theoretical model, the models were re-estimated correspondingly and additional likelihood-ratio tests were carried through. The results of these tests are in line with the interpretation of the estimated coefficients as discussed above: the parameter restrictions had to be rejected for the Kalman version of model A and the OLS version of model B, whereas they could not be rejected for the other specifications.

TABLE 2

Estimation Results for the Swiss Franc/Dollar-Rate (1974,1 – 1987,2)

	Model A		Model B		Model C	
	OLS (1)	Kalman (2)	OLS (3)	Kalman (4)	OLS (5)	Kalman (6)
intercept	0.066	t.v.	0.714	t.v.	-0.076	t.v.
s.e.	0.053	—	0.073	—	-0.053	—
$q_{11} \times 100$	—	0.329	—	0.373	—	0.246
$i_t - i_t^*$	-4.435	-15.109	2.309#	-15.265	-6.222	-17.042
s.e.	1.593	4.095	4.920	4.196	1.911	4.076
$p_t - p_t^*$	—	—	0.614	1.532	0.139	1.303
s.e.	—	—	0.215	0.749	0.085	0.650
$e_{t-1}$	1.002	0.329#	—	—	0.973	0.375
s.e.	0.053	0.125	—	—	0.055	0.119
DW	1.918	1.882	0.190	1.561	1.888	1.754
Durbin's h	0.327	1.097	—	—	0.450	1.863
$\sigma_u^2 \times 100$	0.410	0.019	2.790	0.000	0.390	0.052
ln likelih.	73.505	74.760	21.517	74.336	74.909	76.772
L-R-Test	2.510		105.638		3.726	

Notes: See Table 1

It should be added that in all cases but for the regressive expectation model with constant intercept, the interest differential is shown to have a large significant impact on the corresponding spot rate, which is clearly in contrast to the pure random walk hypothesis. The influence of equilibrium considerations on exchange rate expectations has played a small role in the determination of the current spot rate, which is in accordance with the results presented by Frankel and Froot (1987a). The expectation formation process seems to exhibit strong elements of static expectations, especially when it is allowed for "news" to change the intercept. Nevertheless, the test statistics in general do not allow the exclusion of the price differential and especially of the lagged endogeneous variable from the analysis.

In table 3 the autocorrelation statistics up to order 8 and a sign test for the randomness of the innovations  $\eta_t$  are presented for the specifications with time-varying intercepts. The autocorrelation coefficients  $\rho_i$  do not exceed a confidence interval of twice the standard deviation in either case but for  $\rho_1$  in model B for the Mark/Dollar rate. According to the sign test, the hypothesis of randomness of the innovations

TABLE 3  
Test Statistics for Innovations

	DM/Dollar Rate			Sfr./Dollar Rate		
	Model A	Model B	Model C	Model A	Model B	Model C
<i>autocorrelation coefficients</i>						
$\hat{\rho}_{-1}$ (0.149)	0.110	0.329	0.123	0.033	0.257	0.128
$\hat{\rho}_{-2}$ (0.151)	0.032	0.144	0.022	0.072	0.100	0.077
$\hat{\rho}_{-3}$ (0.152)	0.196	0.266	0.173	0.038	0.066	0.028
$\hat{\rho}_{-4}$ (0.154)	0.071	0.246	0.011	0.189	0.159	0.058
$\hat{\rho}_{-5}$ (0.156)	0.191	0.241	0.120	0.005	-0.078	-0.155
$\hat{\rho}_{-6}$ (0.158)	0.057	0.063	0.012	-0.118	-0.188	-0.196
$\hat{\rho}_{-7}$ (0.160)	0.060	0.015	0.048	0.102	-0.070	0.005
$\hat{\rho}_{-8}$ (0.162)	0.135	0.045	0.118	0.083	0.057	0.062
<i>sign test on randomness</i>						
positive signs	20	20	23	21	25	27
iterations	25	19	23	23	21	21
lower conf.lim.	15	15	17	16	19	21
upper conf.lim.	26	26	30	27	32	34

Notes:

- Taking account for the erratic fluctuation of the filter process at the beginning of the sample, the first 8 observations were excluded for the autocorrelation statistics of the innovations
- $\hat{\rho}_{-i}$  : estimated autocorrelation of order  $i$ , standard error in parenthesis
- For the sign test the lower and upper bounds of the 95% confidence region refer to the number of iterations

cannot be rejected.

In order to check the prediction abilities of the models presented here, a set of out-of-sample forecasts was run. Taking 1974,1 to 1984,4 as a base period, 10 one-step predictions were calculated with the model parameters (including the prior statistics of the Kalman algorithm), re-estimated after each step. The specifications of model A, B, and C with constant intercept and the random walk hypothesis were chosen as reference models. The results are presented in *table 4* for the Mark/Dollar rate and in *table 5* for the Swiss Franc/Dollar rate. Reported are the bias, the autocorrelation statistics, the root mean square errors (RMSE), Theil's inequality coefficient (TIC), and its decomposition into characteristic sources [see for example Pindyck and Rubinfeld (1981)].

Again it can be seen that model B performs worst with rather high autocorrelation coefficients, RMSE, and TIC, especially when estimated with OLS. But the random walk also exhibits relatively strong first order autocorrelation of the forecast errors, larger RMSE and a higher TIC when compared with models A and C. The latter



TABLE 4  
Out-of-Sample Forecasts for the Mark/Dollar Rate (1985,1 – 1987,2)

	Model A		Model B		Model C		Random
	OLS (1)	Kalman (2)	OLS (3)	Kalman (4)	OLS (5)	Kalman (6)	Walk (7)
mean	0.888	0.861	0.956	0.879	0.862	0.854	0.886
bias	0.056	0.029	0.054	0.047	0.030	0.022	0.054
$\hat{\rho}$	0.655	0.153	0.938	0.539	0.478	-0.005	0.663
RMSE	0.069	0.051	0.233	0.061	0.058	0.046	0.068
TIC	0.039	0.029	0.129	0.035	0.033	0.027	0.039
bias prop.	0.674	0.328	0.284	0.576	0.273	0.222	0.636
var. prop.	0.026	0.006	0.337	0.001	0.280	0.015	0.005
cov. prop.	0.300	0.666	0.379	0.423	0.448	0.764	0.359

Notes:

- $\hat{\rho}$ : Estimated autocorrelation coefficient of forecast errors
- RMSE : root mean square error
- TIC : Theil's inequality measure and decomposition into characteristic sources

TABLE 5  
Out-of-Sample Forecasts for the Swiss Franc/Dollar Rate (1985,1 – 1987,2)

	Model A		Model B		Model C		Random
	OLS (1)	Kalman (2)	OLS (3)	Kalman (4)	OLS (5)	Kalman (6)	Walk (7)
mean	0.677	0.677	0.820	0.687	0.643	0.672	0.700
bias	0.031	0.031	0.173	0.040	-0.004	0.025	0.053
$\hat{\rho}$	0.275	0.015	0.943	0.353	-0.015	-0.092	0.514
RMSE	0.061	0.057	0.247	0.060	0.056	0.051	0.073
TIC	0.044	0.041	0.165	0.043	0.041	0.037	0.052
bias prop.	0.258	0.291	0.489	0.453	0.006	0.239	0.525
var. prop.	0.088	0.003	0.211	0.012	0.263	0.001	0.002
cov. prop.	0.654	0.706	0.300	0.535	0.731	0.760	0.474

Notes: See Table 3.

two specifications yield ambiguous results for the Swiss Franc/Dollar rate: with respect to Theil's inequality coefficient and the RMSE the Kalman version of model C performs best, whereas for the autocorrelation statistics and the bias the evidence is not clear. As for the Mark/Dollar rate, however, the Kalman version of model C again is clearly superior to all other specifications including the random walk.

As a resume of this forecasting simulations, which are important benchmarks in evaluating the estimation performance of the various models considered, it can be established that allowing for a time-varying spread in the arbitrage equation improves forecasting abilities markedly. Though a discrimination between the model with pure extrapolative expectations and the model with the combined expectations hypothesis is not unambiguous for the Swiss Franc/Dollar rate, for the Mark/Dollar rate the latter model performed best when the estimation technique permitted the spread to change stochastically. The random walk hypothesis, however, was shown to be inferior in the forecasting experiment.

## 6. The Development of the Spread Variable

*Figure 1* shows the development of the log of the Mark/Dollar rate as the endogenous variable versus the exogenous time-varying intercept as calculated by the Kalman smoothing algorithm. This variable, which is a linear transformation of the deviation from Uncovered Interest Parity (spread), refers to the model with combined exchange rate expectations (model C). This specification was taken as representative because of its better over-all performance as discussed in the previous section. The difference between this time series and the log of the exchange rate is proportional to the estimated influence of interest differentials and of deterministic exchange rate expectations. *Figure 2* shows the corresponding scenario for the Swiss Franc/Dollar rate. As can be seen from both figures, the time-varying intercepts show high correlation with the exchange rates. This indicates that the factors determining the spreads exert considerable impact on the level of the actual exchange rates.

The following subperiods in the development of the spread variables should be emphasized, however, since they are likely to give insights into the interaction of the variables in exchange rate determination: In the second quarter of 1976, the Swiss National Bank in contrast to the German Bundesbank, took strong measures to stop the decline of the Dollar by introducing a set of restrictions on international capital movements. These measures comprehended restrictions on foreign currency deposits for Swiss private banks as well as a negative interest rate for deposits by foreign privates, which was accomplished via a commission fee. Though these steps succeeded in stopping the decline of the spread and of the Dollar versus the Swiss Franc for approximately two quarters, the subsequent fall of spread and Dollar was all the more steeper as compared with the Mark/Dollar development, the German Bundesbank having refrained from such measures.

The second subperiod which is characteristic for the model is marked by Carter's stabilization program in the last quarter of 1978. Among other measures, it was an-

nounced that facilities in Swiss Francs, German Marks and Japanese Yen totalling 30 billion US Dollar had been arranged in order to stop a further decline of the Dollar. The spreads reacted instantaneously to the "news". The Mark/Dollar rate and the Swiss Franc/Dollar rate remained at approximately the same level for another year, however, which induced Hodrick and Srivastava to put the Carter intervention at November 1979 [(Hodrick and Srivastava (1984), p.19)]. According to the model presented here, these deviations are due to the fact that the intervention announcement was not accompanied by a corresponding interest rate policy or exchange rate expectation process in the first place.

The same argument applies in reversed order for the most recent decline of the Dollar. Though the "news" associated with the Plaza Agreement in the third quarter of 1985 made the spreads decline, the announcement in itself cannot be fully responsible for the strong decline of the exchange rates. The latter seems to have been accomplished by an even stronger reaction especially of the Mark/Dollar rate to interest rate differentials and pure exchange rate expectations with regard to equilibrium considerations.

The stochastic shocks to the spread variable ("news") are shown in *figure 3* for the Mark/Dollar rate, and in *figure 4* for the Swiss Franc/Dollar rate. *Table 6* contains the means, variances and a  $\chi^2$  test on Normal distribution for the disturbances in the state equation  $v_t$ . The results indicate that the assumption of a zero mean and of Normal distribution cannot be rejected. Furthermore, a Breusch-Pagan test and a Goldfeld-Quandt test on heteroskedasticity were performed. According to the test statistics, the hypothesis of heteroscedastic residuals of the state equation can be rejected in all cases. The autocorrelation statistics show that the autocorrelation coefficient of order one exceeds twice the standard error in the case of the Mark/Dollar rate. The Swiss Franc/Dollar results exhibit no autocorrelation. According to the non-parametric sign test randomness of the residuals of the state equation cannot be rejected for both exchange rates. It can be concluded that the residuals of the state equation satisfy the statistical requirements imposed on them as described in section 3 and 4, with the exception of the slight autocorrelation of order one in the case of the Mark/Dollar rate.

The five largest shocks to each of the spread variables are listed in *table 7*. Three shocks rank high for the Mark/Dollar spread as well as for the Swiss Franc/Dollar spread. The largest positive shock for both variables was induced by the Afghanistan crisis in the first quarter of 1980. Large negative shocks were caused by the Plaza Agreement in the third quarter of 1985, and by negative "news" about the prospects of the US economy in the third and fourth quarter of 1977. The latter included a strong criticism of Carter's energy program and its expected consequences for business development, created inflationary expectations and a baisse on Wall Street. They were accompanied by a Secretary of Treasury Blumenthal "talking the Dollar down", resulting in the largest shock to the Mark/Dollar spread and two lower ranking shocks to the Swiss Franc/Dollar spread.

Certain shocks turned out to have an exclusive effect on each of the exchange rates considered: the inauguration of the first Reagan administration in the first quarter of 1981, as well as a sort of a reversion of the enthusiasm associated with it in the

TABLE 6

Distribution and Test Statistics for Residuals of State Equation (Model C)

	DM/Dollar Rate	Sfr./Dollar Rate
<i>estimated distribution parameters</i>		
mean	0.001	0.001
standard deviation	0.023	0.043
t-stat. for mean=0	0.166	0.121
$\chi^2$ test on Normal distribution	8.399 [ $\sim \chi^2(5)$ ]	6.378 [ $\sim \chi^2(9)$ ]
<i>tests on heteroskedasticity</i>		
Breusch-Pagan test	42.781 [ $\sim \chi^2(3)$ ]	58.212 [ $\sim \chi^2(3)$ ]
Goldfeld-Quandt test	1.226 [ $\sim F(19, 19)$ ]	1.869 [ $\sim F(19, 19)$ ]
<i>autocorrelation coefficients</i>		
$\hat{\rho}_{-1}$ (0.140)	0.349	0.170
$\hat{\rho}_{-2}$ (0.141)	0.145	0.107
$\hat{\rho}_{-3}$ (0.143)	0.159	-0.011
$\hat{\rho}_{-4}$ (0.144)	0.126	0.121
$\hat{\rho}_{-5}$ (0.146)	0.159	-0.006
$\hat{\rho}_{-6}$ (0.147)	0.096	-0.168
$\hat{\rho}_{-7}$ (0.149)	0.124	0.057
$\hat{\rho}_{-8}$ (0.151)	0.148	0.064
<i>sign test on randomness</i>		
number positive signs	23	26
number iterations	20	23
lower confid. limit (95%)	17	20
upper confid. limit (95%)	30	33

## Notes:

- The  $\chi^2$  test was based on the hypothesis of a zero mean, and a standard deviation as estimated above; the tests was carried through with 8 classes for the Mark/Dollar rate and for 12 classes for the Swiss Franc/Dollar rate to provide for a minimum of 5 observations in each class of the theoretically expected distribution
- For the Breusch-Pagan test the lagged dependent, the square of the lagged dependent, the exponent of the lagged dependent and a constant were used as regressors
- For the Goldfeld-Quandt test a subperiod of 13 quarters was omitted in the middle of the sample
- $\hat{\rho}_{-i}$  : estimated autocorrelation of order i, standard error in parenthesis
- For the sign test the lower and upper bounds of the 95% confidence region refer to the number of iterations

TABLE 7  
Major Changes of the Spread Variables (Model C)

DM/Dollar Rate		Sfr./Dollar Rate	
1977,4 (-0.373)	Critique of Carter's Energy Program	1978,3 (-0.675)	Announced Introduction of European Currency System
1985,3 (-0.367)	Plaza Agreement	1980,1 ( 0.640)	Afghanistan Crisis
1980,1 ( 0.280)	Afghanistan Crisis	1985,3 (-0.505)	Plaza Agreement
1981,1 ( 0.260)	Inauguration of Reagan Administration	1977,4 (-0.499)	Critique of Carter's Energy Program
1981,3 (-0.253)	Disillusion over Reagan Administration	1977,3 (-0.475)	Critique of Carter's Energy Program

Note: Extent of shock in parenthesis ( $\times 10^2$ )

third quarter of the same year, were of considerable impact to the Mark/Dollar rate, whereas this political shift was of minor impact to the Swiss Franc. On the other hand, the announcement of the establishment of the European Currency System worked strongly in favour of the Swiss Franc, possibly via an even effect on Dollar and German Mark, since the spread concerning the ratio of these currencies was not affected.

In summary, the shocks changing the spread variable in both exchange rate equations by and large satisfy the statistical requirements imposed by the estimation procedure as well as the postulations given by economic reason. With reference to the various theoretical explanations for the deviations from Uncovered Interest Parity, there seem to be large conformities with their representation as a time-varying risk premium changed by "news" in the information set of the market participants. This resemblance might be indicated by the "risky" nature of the shocks to the state variable, which was emphasized for instance by the reaction of the deviation from Uncovered Interest Parity during the Afghanistan crisis working in favour of the US Dollar according to the "save heaven" argument. In case of a constant degree of risk aversion, however, the hypothesis that those time-varying deviations from Uncovered Interest Parity might be equivalent to biases in the pure exchange rate expectation formation process should not be set aside. The variety of issues driving this latent variable suggests, however, that it is improbable to match these developments with pure economic data series, thus making the use of more sophisti-

cated estimation procedures than ordinary least squares a necessary condition when applied to the analysis of exchange rate behaviour.

## 7. Summary and Conclusion

The paper analyzed deviations from the Uncovered Interest Parity condition by applying a Kalman filter procedure based on optimal initial information to the estimation of this arbitrage equilibrium for the Mark/Dollar rate and the Swiss Franc/Dollar rate. Several economic models were allowed to represent the expectation formation process, where a combination of regressive and extrapolative expectations yielded the best results when implemented into the reduced form exchange rate equations.

Allowing for a time-varying intercept induced not only an improvement of the ex-post estimation results especially for the Mark/Dollar rate, but also yielded superiority with regard to forecasting abilities compared with the reference models, which included the random walk hypothesis.

Since there are several competing approaches in the literature to explain the nature of these deviations from Uncovered Interest Parity, an examination of the statistics of the stochastic processes determining the development of these deviations was submitted, supplemented by a comparison with historical events. This rather stylized analysis suggests, however, that the stochastic disturbances driving these deviations from the pure arbitrage equilibrium cover a wide range of economic and political developments. Characteristic for these "news" as they were modeled here is that they are not forgotten, but are stored in a state variable reflecting accumulated information about the structure and the institutional setting of the underlying economies. The restrictive features of the analysis do not yet allow for an assignment of these empirically filtered "news" to a specific theoretical model. The basic results suggest, however, that it is improbable to construct a set of economic data series which adequately captures all factors determining the deviations from Uncovered Interest Parity. The use of advanced statistical procedures then can be regarded not only as a compensation for theoretical deficiencies, but also as a necessary condition for the empirical estimation of exchange rate behaviour.

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FIGURE 1: LOG OF THE MARK/DOLLAR RATE AND CORRESPONDING INTERCEPT

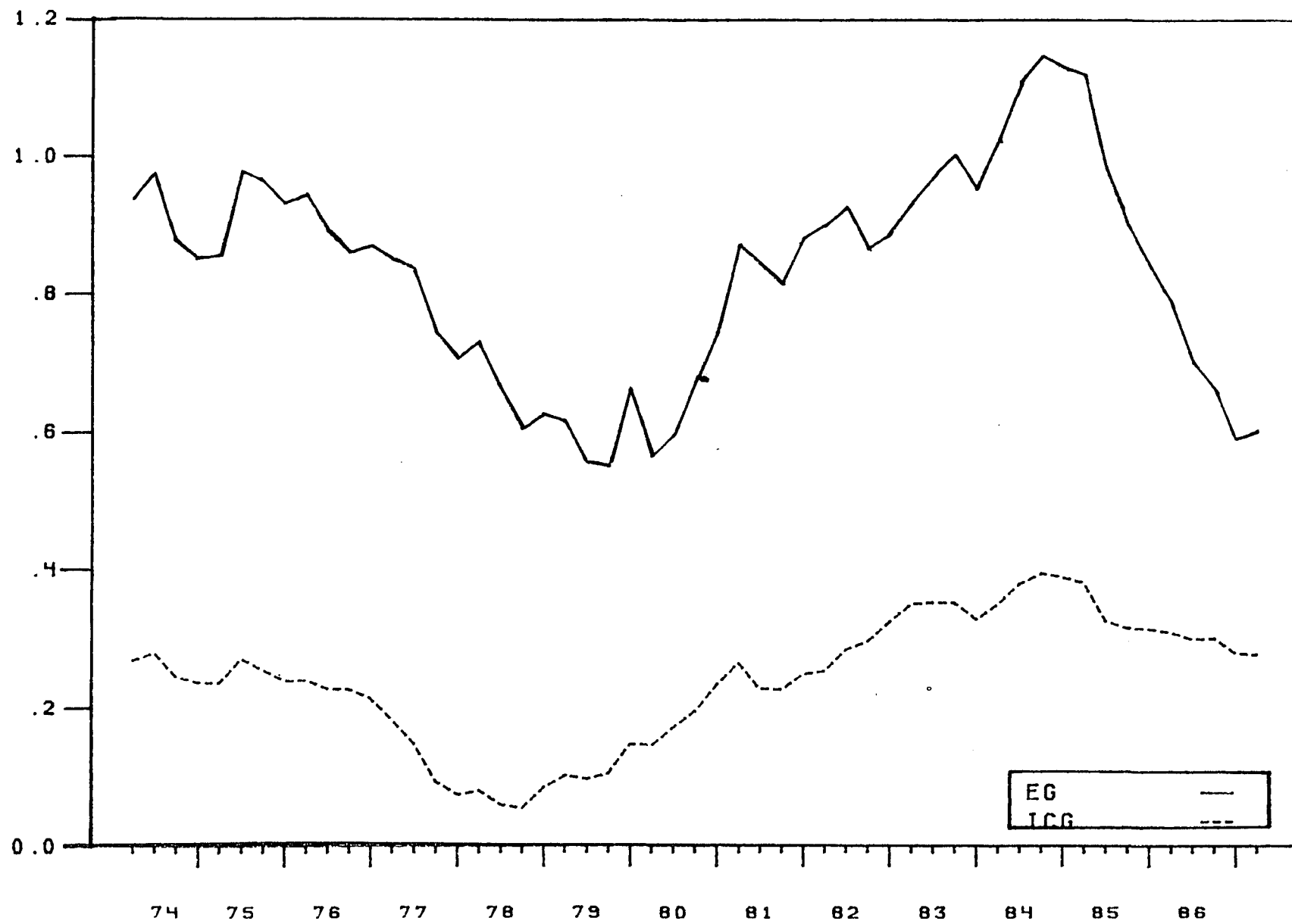


FIGURE 2: LOG OF THE SWISS FRANC/DOLLAR RATE AND CORRESPONDING INTERCEPT

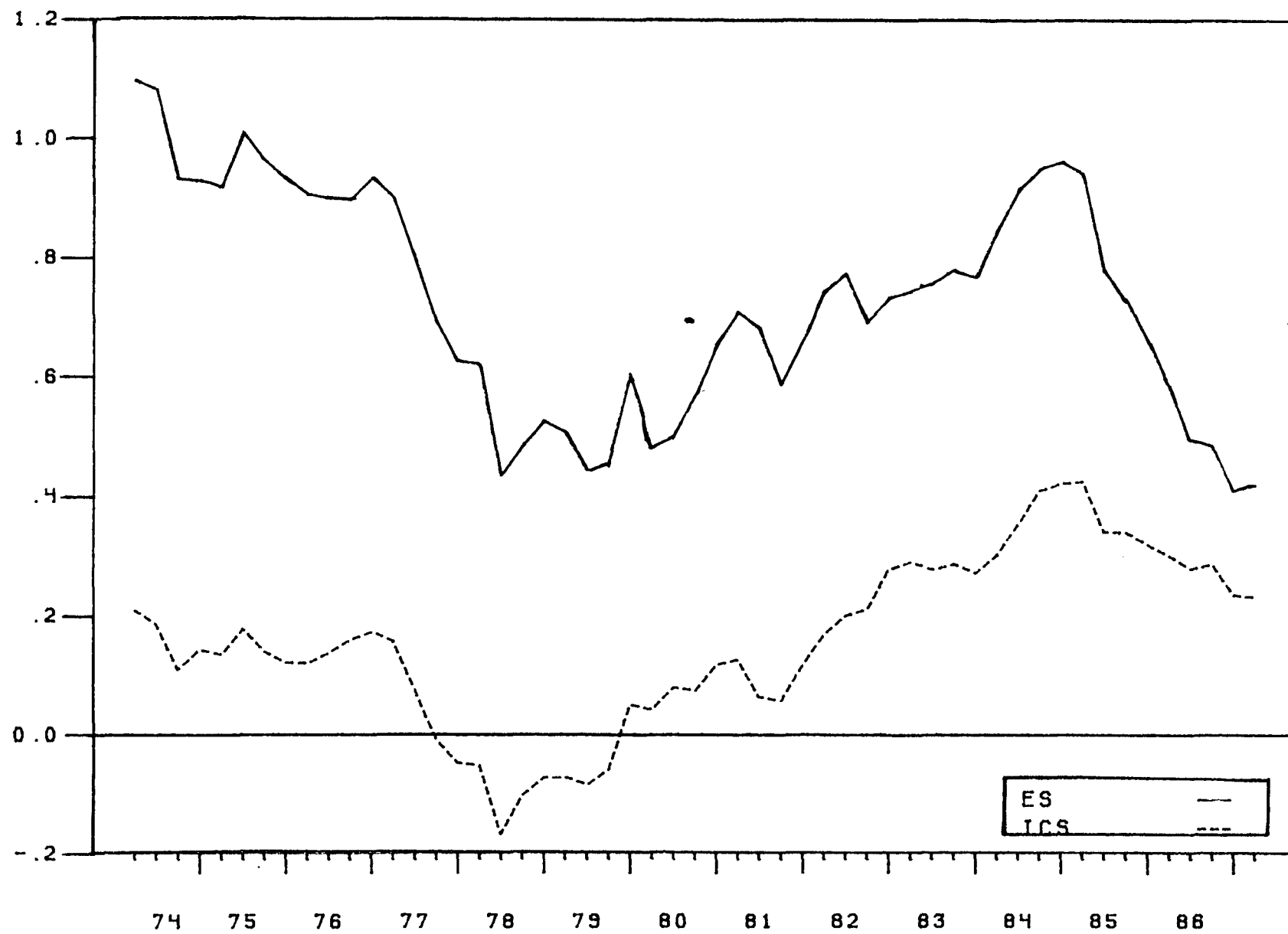


FIGURE 3: CHANGES OF THE SPREAD FOR THE MARK/DOLLAR RATE

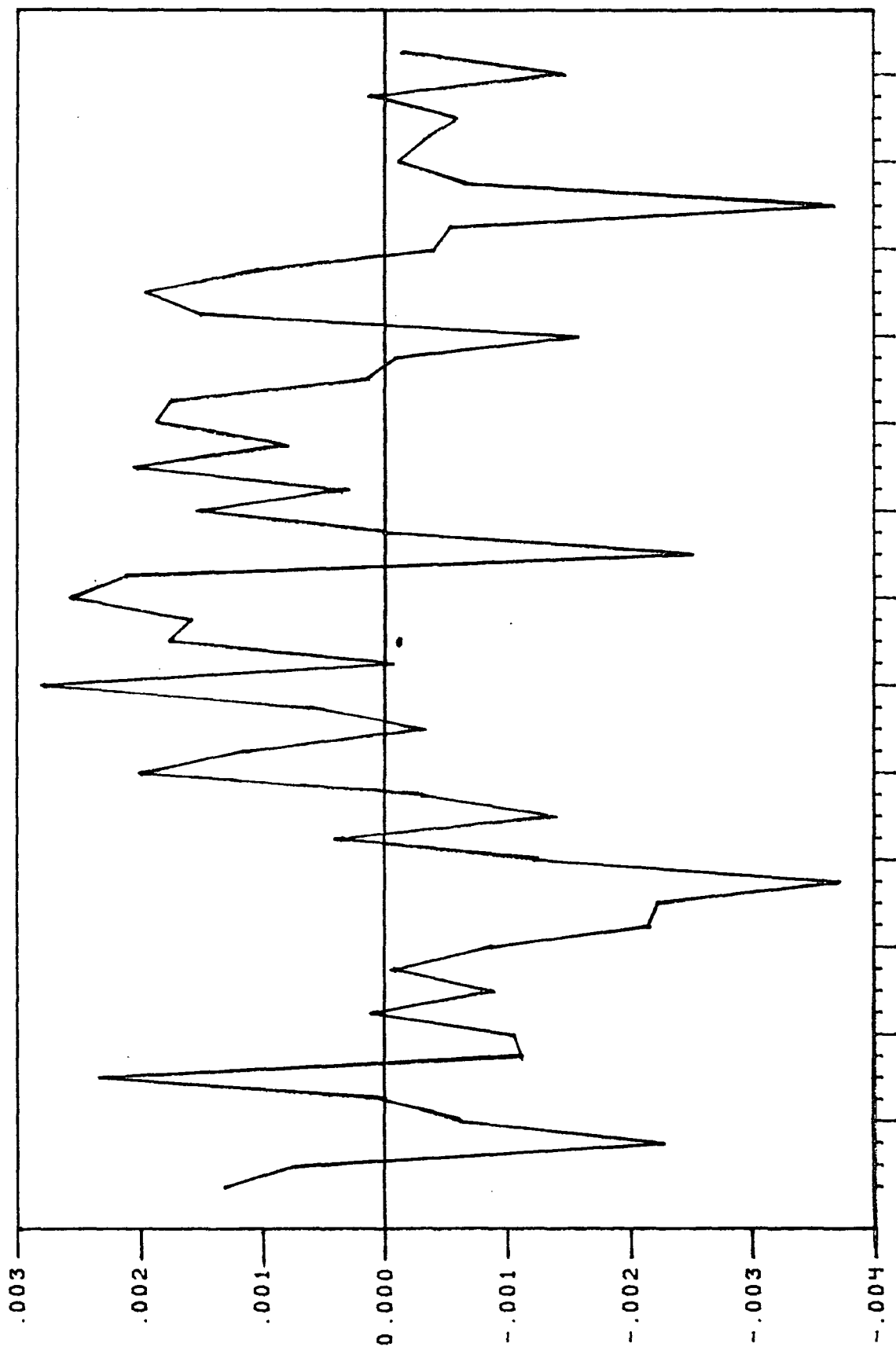


FIGURE 4: CHANGES OF THE SPREAD FOR THE SWISS FRANC/DOLLAR RATE

