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## The asset market view on flexible exchange rates: A critical comment

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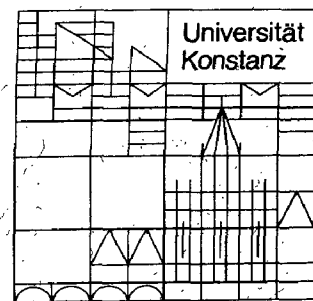
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Thusnelda Tivig

**The Asset Market View**  
**On Flexible Exchange Rates**  
A Critical Comment

# **The Asset Market View On Flexible Exchange Rates**

**- A Critical Comment -**

**Thusnelda Tivig**

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### Abstract\*

The more recent asset market literature on flexible exchange rates seems to share the opinion that it is not possible to treat the stock-flow interaction on the adjustment path to a new long-run equilibrium adequately in continuous time.<sup>1)</sup> The present paper discloses some severe inconsistencies between the verbal and formal formulation of the asset market approach (AMA) that are linked to this proposition and argues that the AMA has not reached its target yet. A possible solution is outlined.

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1) Most authors refer to Tobin and De Macedo (1980) for this statement.

## I. Introduction

The more recent literature on the determination of flexible exchange rates is mainly of the monetary type, of which the monetary approach in a narrow sense, the stock approach, the portfolio balance approach and the asset market approach are variants. The common base of these approaches can be summarized in four features:

- (i) the interpretation of the exchange rate as a relative price of two currencies which, together perhaps with other prices, provides equilibrium in the international financial markets,
- (ii) the consideration of more than one asset and of substitution possibilities between assets denominated in different currencies,
- (iii) the formulation of asset demands as stock demands, and, at least tentatively, their microeconomic foundation by portfolio theory,
- (iv) the emphasis on the role of expectations in the portfolio balance process and the determination of exchange (and interest-) rates.

With the definition of the exchange rate as the relative price of two currencies, the emphasis is shifted to the monetary sector, that is, on the supply of and demand for financial assets. As the latter are stocks, this view is called "stock approach" as opposed to the traditional analysis by means of equilibrium conditions referring to flow variables. The label "asset market approach" stresses the role of financial markets too, as compared to the customary emphasis on the real side of the economy. But whereas the monetary approach in a narrow sense concentrates on money markets only, the asset market approach explicitly takes into account asset markets, in which interest bearing, distinct assets are traded. The notion "portfolio balance approach" finally underlines the fact that when several assets are available, wealth holders have to make portfolio decisions. Despite these distinctions, the literature largely uses the four notions synonymously.

## II. A Critical Presentation Of The Asset Market Approach

The AMA was initially designed for the analysis of capital movements and interest rate changes in a system of fixed exchange rates. It is based on the idea that private wealth holders allocate their wealth between the available domestic and foreign assets, the desired wealth structure depending generally on the expected returns of the assets, the transaction volume in the good markets and the level of wealth. Changes of these factors induce revisions of the desired portfolio structure and lead to price-and/or quantity movements. In a fixed exchange rates system there may be interest movements, in a flexible exchange rates system interest- and exchange rate movements. By quantity movements there are meant stock shifts which lead alone or in interaction with the price movements to restoration of portfolio equilibrium. These stock shifts may take either the form of a direct exchange of tradable assets between private domestic and foreign wealth holders, or they can come about because of the readiness of the domestic central bank to exchange foreign assets for domestic assets at a fixed exchange rate. The latter is valid for a system of fixed or managed exchange rates only, the former can occur in either exchange rate system, provided there are at least two international tradable assets. The stock shifts imply gross capital movements which have solely a temporary stock adjustment character. They cease when the desired portfolio structure is reached. Thus short-run equilibrium requires portfolio balance and financial market equilibrium.

The private wealth holders decide not only on their portfolio structure, but also on their consumption level. If one takes into consideration the fact that price movements imply, through valuation effects, changes in the level of wealth, the consumption decision is simultaneously revised with the portfolio decision. If, to the contrary, one assumes a constant level of wealth in the short run, the saving process occurs only in time, by accumulation of wealth through new savings. Long-run equilibrium is attained when all goods and asset markets clear, portfolio balance prevails and the current account is balanced.

The starting point of the AMA was the recognition that there had been periods, after switching to a flexible exchange rate system, with large international movements of financial capital and heavy exchange rate changes within relatively short spans of time. These facts raised interest in studying portfolio balance processes, which are characterized by simultaneous price- and quantity changes. Put differently, the AMA aims at studying exchange (and interest-) rate movements in the context of international financial capital movements, where the latter result either from reshuffling of portfolios or from current account imbalances. In the following it will be argued that the AMA provides an adequate framework

for such an analysis, but that it has not been sufficiently exploited yet. The critics refer to two points

- the treatment of the simultaneity of price- and quantity movements in the short run,
- the interaction of international temporary stock shifts with sustainable flows of new savings along the adjustment path to a new long-run equilibrium.

In his seminal paper, Kouri (1976) describes the temporary portfolio balance process first as follows:

"In the stock approach it is assumed that financial markets equilibrate very fast. Transaction costs are so small that it may be assumed that financial assets are always held in desired proportions. For example, if a market participant has too many sterling assets in his portfolio, he can exchange these assets for dollar assets in a very short time. For a large segment of the international short-term money market, this is probably a reasonable simplifying assumption. Revisions of existing portfolios are much larger in size than marginal additions to asset demands from new savings." (p. 281)

It is in no way clear from the text above that the author refers to a certain exchange rate system. But finally Kouri concludes:

"Under fixed exchange rates portfolio equilibrium between domestic money and foreign assets *at a given level of wealth* is obtained through *instantaneous* capital inflows and outflows because the Central Bank supplies foreign assets at a fixed price. Under flexible exchange rates instantaneous portfolio equilibrium is obtained through changes in the *valuation* of assets - that is, through changes in the exchange rate." (p. 300) <sup>2)</sup>

Kouri's formal analysis corresponds to the distinction he made in the final part of his paper. His model is built up essentially of the equilibrium condition for the domestic money market

$$(1) \quad M = M^d(\pi Y, x, V); \quad M_{\pi Y}^d > 0, \quad M_x^d < 0, \quad M_V^d > 0,$$

and the differential equation for the wealth accumulation by new savings

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2) Note that there are no net capital movements taking place in the case of stock shifts, as the change in the private domestic stock of foreign assets is equal in size but of opposite sign to the change in the reserve holdings of the domestic central bank, so the balance on capital account is zero.

$$(2) \quad \dot{F} = Y - C(Y, V/\pi); \quad 0 < C_Y < 1, \quad C_{V/\pi} > 0,$$

where

$M$  stands for the nominal stock of the domestic money, which is not tradable internationally,

$\pi$  is the exchange rate, which by the law of one price and setting the constant foreign goods price equal to one is equal to the domestic price  $P$  of the single international tradable good,

$M^d$  is the nominal domestic demand for domestic money,

$x$  is the expected rate of change of the exchange rate,

$Y$  is the real constant production income of domestics,

$V$  is the nominal wealth of private domestics, defined as  $V = M + \pi F$ ,

$F$  is the real stock of foreign money owned by private domestics,

$C$  is the real consumption demand of the private domestics.

For ease of exposition we abstract, unlike Kouri, from government activities. The model is closed by specifying an expectation process for the determination of  $x$ . For the purpose of this part of the paper it is irrelevant what expectational form one chooses, as long as it does not pose stability problems;  $x$  could equally well be exogenous. Note that because of the identity  $P \equiv \pi$ , real variables are expressed in terms of foreign currency or good units.

The system ((1), (2)) can be seen as an approximation by means of the moving equilibrium method of the system of differential equations

$$(1') \quad \dot{\pi} = \tau (M - M^d(\pi Y, x, V)); \quad \tau > 0, \quad \tau \rightarrow \infty,$$

$$(2) \quad \dot{F} = Y - C(Y, V/\pi).$$

Equation (1') describes the evolution of the exchange rate, which by assumption reacts instantaneously providing permanent equilibrium in the money market. In the above system,  $\pi$  is a fast and  $F$  a slow variable. The moving equilibrium method works as follows: system ((1), (2)) is approximated - usually tacitly - by the system ((1'), (2)), according to the postulated differences in the reaction speed of the endogenous variables  $\pi$  and  $F$ . The

equilibrium condition (1) serves to solve explicitly for the variable  $\pi$  as a function of  $F$  and the exogeneous variables  $x$ ,  $Y$  and  $M$

$$(3) \quad \pi = \pi(F; x, Y, M),$$

which is substituted into (2) yielding

$$(4) \quad \dot{F} = Y - C(Y, M/\pi(F; x, Y, M) + F).$$

The movement of the slow variable  $F$  is then studied under the assumption that the adjustment of the fast variable  $\pi$  to its respective short-run (temporary) equilibrium value has already occurred.

The above simplified exposition of the Kouri model reveals three points:

- a. the verbally described exchange process of financial assets is not embedded in the formal structure of the model, i.e. there are no temporary stock shifts modelled as part of the portfolio balance process;
- b. as no such stock shifts appear, one cannot identify a stock-flow interaction in the international movement of financial capital; <sup>3)</sup>
- c. short-run wealth changes by valuation effects are not captured, so the asset demands and the consumption demand do not change simultaneously.

In all asset market models there are only price- and no quantity adjustments modelled for the short run. Yet it would be wrong to conclude that this is an intrinsic feature of the AMA. The fact is that a precondition for short-run stock shifts is the existence of at least two international tradable assets. The point has been observed by several authors. So for example by Calvo and Rodriguez (1977), who write:

"The total stock of foreign currency is also given at any moment and not subject to instantaneous jumps; the only way for this economy to alter the level of its foreign currency holdings is through an excess supply or demand of traded goods..."

and add in a footnote:

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3) There is, of course, a feedback from flows to stocks, as new savings result in higher stock holdings; what is missing is the link between current account induced capital flows and those capital movements which are the outcome of reshuffling of portfolios.

"This follows from the assumption that foreign currency is the only internationally traded asset; if securities were included and allowed to be internationally traded then foreign exchange could be instantaneously acquired through sales of securities abroad." (p. 619)

Allen and Kenen (1976) argue in a similar way:

"If there is international trade in more than one asset ... the stock version produces an instantaneous rearrangement of portfolios in response to a disturbance, with a gradual accumulation in holdings of all assets when the disturbance induces savings ... When only one asset is traded internationally, by contrast, that asset can be exchanged only for a flow of goods; thus, even in the stock version of the model, any actual adjustment of portfolios is accomplished gradually as a result of saving." (p. 70)

When reading the above passages one would expect to encounter stock shifts as part of the short-run portfolio balancing process in models with two international tradable assets. Most surprisingly, this is not the case.<sup>4)</sup> The question is either simply not addressed, or, in a few papers, considered to be a methodological problem which cannot be solved in continuous time. The models are usually specified in a way which does not make the neglect of stock shifts obvious. One variant is to specify the wealth evaluated at a long-run equilibrium exchange rate as the state variable. The dynamics of the system is then described in terms of this wealth variable, which does not "jump", and not in terms of the individual wealth components.<sup>5)</sup> With this procedure, one "solves" two problems. First, the short-run exchange of assets between domestics and foreigners is not made explicit, as this exchange is neutral with respect to the level of wealth; it changes only the composition of wealth. Second, the simultaneity of wealth changes through valuation effects and through new savings is ignored. The latter problem would have to be considered in all models irrespective of the number of tradable assets, as long as they are denominated in different currencies. In Kouri's (1976) model for example, the change in nominal wealth is not  $dV/dt = \pi dF/dt$  but

$$(5) \quad \dot{V} = \dot{\pi}F + \pi\dot{F} = \dot{\pi}F + \pi B,$$

where B is the real current account balance.

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4) See Branson and Henderson (1985), De Grauwe (1983), Henderson and Rogoff (1982), Ethier (1979), Isard (1978), who all have models with two distinct international tradable assets.

5) This is the procedure that Branson and Henderson (1985) or Henderson and Rogoff (1982) follow.

The implication of neglecting  $\pi F$  in (5) is that the temporary portfolio balance process - which in the Kouri model is ensured by price adjustments only - is cut off from the saving process.

The methodological problem of modelling the stock-flow interaction adequately has been recognized by some authors in the 1975/76 literature, but has not been addressed afterwards up to the paper by Tobin and De Macedo (1980). So Frenkel and Rodriguez (1975) write in their excellent paper on the AMA for fixed exchange rates:

"... our analysis is complicated due to the fact that there are both adjustments that occur through time and adjustments that occur instantaneously in attaining a continuous portfolio balance. We have defined the capital account and the overall balance of payments in terms of changes in the holdings of securities and money ... that occur through time, therefore these definitions do not capture international transactions that occur discretely as those that are involved in the instantaneous exchange ... of cash balance for equities ... the expressions that pertain to the impact effects do not capture these initial stock adjustments" (p. 681, fn.).

Cooper (1976) put the problem as follows:

"... formulation of the problem in terms of portfolio equilibrium draws attention to the transitional dynamics, since exogenous (or policy) changes lead to transitory stock adjustments in portfolios, plus continuing debt service payment.

Unfortunately, it is very difficult to model portfolio equilibrium and stock adjustment satisfactorily without shedding much of what is of interest for stabilization policy ... A complete model cannot avoid having both stock and flow equilibria, with explicit stock adjustment responses to disequilibria. This inevitably greatly complicates the model, and the results that are crucial for assessing the stabilization impact of monetary and fiscal measures depend critically on assumptions that are made about which markets clear instantaneously and which do not, and which markets clear in the sort run by price adjustment as opposed to quantity adjustment. From the viewpoint of scientific analysis these 'dynamic' specifications, and tests of their validity, are of course desirable, however difficult they may be, for that is what economic stabilization is all about." (p. 151)

Finally, Tobin and De Macedo remark:

"The model we propose uses discrete rather than continuous time. The motivation is to include some of the effects of finite stock accumulation, which ... are not captured at all when asset markets are modeled as stock equilibrium at a point in time." (1980, p. 15).

There are at least three ways to avoid or solve the problem of modelling the stock-flow interaction in continuous time. The first is to choose a discrete time treatment, as Tobin and De Macedo do. The second is to postulate a slow portfolio adjustment process, as Karacaoglu and Ursprung (1986) do. However, they do not give a solution for an adequate treatment of

the wealth changes as the sum of valuation effects and new savings. Slow portfolio adjustment implies only that there are no instantaneous stock shifts; the exchange rate is as before a fast variable, so valuation effects occur. The third possibility is to apply the moving equilibrium method in the extended version of Witzel (1985). Witzel extended the method so as to analyze systems in which some endogeneous variables are no longer either slow or fast, but influenced in their time path by slow and fast movement components. Mathematically, a model in which such "mixed" variables appear is characterized by the usual treatment of fast or slow variables, and the formulation of differential equations for the mixed variables, in which the time derivative of other variables appears. In the simplified version of the Kouri model for example, the nominal wealth is such a mixed variable. Its time path is described by equation (5). The extended version of the moving equilibrium method states the conditions under which the initial system ((1'),(5)) may be approximated by the system ((1),(5)) and states how the impact reactions of the system ((1),(5)) have to be calculated. A detailed exposition of the method is found in Witzel (1985).

### III. An Extended Asset Market Model

In what follows an extended Kouri model is presented. The structure of good markets is not changed. The extension refers to the asset structure of the model. I assume that the country is small in the good markets, but sufficiently large in financial markets for foreigners to find it attractive to hold domestic currency. This is in accordance with the view that when modelling an economy it may often be appropriate to give up the clear distinction between countries which are small or large in all markets and to differentiate between smallness or largeness of economies in a subset of markets.<sup>6)</sup> Real wealth of foreigners is then given by

$$(6) \quad V^* = M^* / \pi + F^* ,$$

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6) A typical example is Switzerland, which may be considered small in good markets but which surely isn't small in financial markets, in the sense that foreigners do not find it attractive to hold Swiss francs denominated assets. Even if the proportion of these assets is relatively small in the portfolios of foreigners, for Switzerland it might be crucial to consider them when analyzing the impact on the domestic economy of various disturbances. The remark is in fact not new. Tobin (1982) puts it as follows: "One step towards realism is to add foreign demands, with the same formats and properties ... as the domestic asset demand functions, to the equation for all local assets. ... This amendment can be kept within the bounds of the "small country" assumption by continuing to assume that foreign interest rates, foreign incomes and other relevant foreign variables are exogeneous." (p. 195)

where

$V^*$  is the real wealth of foreigners,

$M^*$  is the nominal stock of domestic currency held by foreigners,

$F^*$  is the real stock of foreign currency held by foreigners.

All asset stocks are assumed to be positive. Asset demands are assumed to be linear homogeneous in wealth, the proportion of wealth held in different currencies being a function of  $x$ , the expected exchange rate change

$$(7a) \quad M^d = m(x) V \quad ; \quad m_x < 0,$$

$$(7b) \quad M^{*d} = m^*(x) \pi V^* ; \quad m_x^* < 0,$$

$$(7c) \quad F^d = f(x) V \quad ; \quad f_x > 0,$$

$$(7d) \quad F^{*d} = f^*(x) \pi V^* ; \quad f_x^* > 0,$$

where

$F^d$  [ $F^{*d}$ ] is the nominal domestic [foreign] demand for foreign currency,

$M^{*d}$  is the real foreign demand for domestic currency.

The expected momentary rate of change of the exchange rate is defined to be

$$(8) \quad x = \alpha \frac{\pi^e - \pi}{\pi} ; \quad \alpha > 0,$$

where  $\pi^e$  is the expected long-run equilibrium exchange rate, the determination of which is specified below. Government expenditure is assumed to be fixed at the constant level of tax receipts. Asset supplies are exogeneous and fixed at  $M^s$  and  $F^s$ . Equilibrium in the money markets requires

$$(9) \quad M^s = M^d + M^{*d},$$

$$(10) \quad \pi F^s = F^d + F^{*d}.$$

Note that portfolio equilibrium is not implied by money market equilibrium.<sup>7)</sup> The trade balance is equal to disposable income  $Y_D$  minus private consumption  $C$

$$(11) \quad B = Y_D - C(Y_D, V/\pi); \quad 0 < C_{Y_D} < 1, \quad C_{V/\pi} > 0,$$

where  $Y_D$  is constant.

The dynamics of the system can be described by the following system of differential equations

$$(12) \quad \dot{\pi} = \tau [M^s - (M^d + M^{*d})],$$

$$(13) \quad \dot{M} = \tau (M^d - M) + (1-s) \pi B,$$

$$(14) \quad \dot{F} = -\frac{1}{\pi} \tau (M^d - M) + sB,$$

$$(15) \quad \dot{\pi}^e = a(\pi - \pi^e) + \gamma \tau [M^s - (M^d + M^{*d})],$$

$$\tau > 0, \quad \tau \rightarrow \infty, \quad 0 \leq s \leq 1, \quad 0 < a < \infty, \quad 0 \leq \gamma \leq 1.$$

Equation (12) says that exchange rate movements assure instantaneous rebalancing of the domestic money market. Equations (12) and (13) describe the changes over time of domestic holdings of domestic and foreign currency. The  $\tau$ -terms reflect the stock shifts which are part of the instantaneous portfolio balancing process, whereas the  $s$ -terms give the accumulation of  $M$  and  $F$  through current account imbalances. Thus domestic wealth holders can accumulate foreign currency either by exchanging domestic currency for foreign currency or by running a trade balance surplus. Note that by summing up equations (12) and (13) one obtains the changes in wealth through new savings. The overall change in nominal wealth is composed of valuation effects and new savings

$$(16) \quad \dot{V} = \dot{M} + \pi \dot{F} + F \dot{\pi} = \pi \dot{F} + \pi \dot{B}.$$

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<sup>7)</sup> This will usually be the case with more than one asset traded internationally.

In what follows it will be assumed that  $s = 1$  for simplicity. Equation (15) describes the formation of expectation by means of an adaptive-extrapolative process. If  $\gamma = 0$  expectations are formed adaptively. If  $\gamma = 1$  there are current value expectations held. The value of  $\pi^e$  which emerges from (15) is used to calculate  $x$ , the expected momentary rate of exchange rate change.<sup>8)</sup>

The system ((12) - (15)) is approximated by means of the extended moving equilibrium method. The fast movement components are set equal to zero

$$(17) \quad M^s - (M^d + M^{*d}) = 0 \quad ,$$

$$(18) \quad M^d - M = 0 \quad .$$

Equations (17) and (18) are used to solve explicitly for  $\pi$  and  $M$ , yielding  $\pi(F, \pi^e)$ ,  $M(F, \pi^e)$ . These functions are then substituted into (14) and (15) yielding the reduced form of the approximation system, in which only  $F$  and  $\pi^e$  are endogenous

$$(19) \quad \dot{F} = - \frac{1}{\pi(F, \pi^e)} \left( \frac{\partial M}{\partial F} \dot{F} + \frac{\partial M}{\partial \pi^e} \dot{\pi}^e \right) + B(\pi(F, \pi^e), M(F, \pi^e), F) ,$$

$$(20) \quad \dot{\pi}^e = a(\pi(F, \pi^e) - \pi^e) + \gamma \left( \frac{\partial \pi}{\partial F} \dot{F} + \frac{\partial \pi}{\partial \pi^e} \dot{\pi}^e \right) .$$

Temporary equilibrium which is characterized by portfolio balance for domestic and foreign wealth holders and equilibrium in all financial markets is thus described by equations (17) and (18),<sup>9)</sup> while long-run equilibrium further requires  $\pi^e = \pi$  and  $B = 0$ .

In order to check whether the approximation of the initial system of differential equations ((12) - (15)) by the reduced system ((19), (20)) is admissible, one has mainly to perform the

8) Note that only with  $\gamma \neq 0$  expectations are revised simultaneously with the movement of  $\pi$ . Kouri (1976) postulates that expectations are formed directly with respect to rates of change. In this case even with adaptive expectations one has instantaneous reactions in  $x$ , so the extended moving equilibrium method should be used, irrespective of how many assets are tradable internationally, as  $x$  is a mixed variable. For a detailed exposition of this point see Witzel, 1985, pp. 129 - 139.

9) Equilibrium in the domestic money market is described by (17). From (17), (18) and the stock restriction  $M^s = M + M^*$  one gets  $M^{*d} = M^*$ ; this, together with the budget constraints  $V = M^d + F^d$ ,  $\pi V^* = M^{*d} + F^{*d}$  and the wealth definitions yields  $F^d = \pi F$  and  $F^{*d} = \pi F^*$ ; when further using the stock constraint  $\pi F^s = F + F^*$  one finally obtains  $\pi F^s = F^d + F^{*d}$ , which is the equilibrium condition for the foreign money market.

stability analysis of the matrix of system ((19), (20)) and of the matrix of the fast subsystem.<sup>10)</sup> It shows that both systems are locally stable if one assumes local currency preference,  $m > m^*$ .

In what follows, the effects of a once-and-for-all intervention in the foreign exchange market are analyzed. It is assumed that the central bank purchases  $\pi dF$  of foreign assets in exchange for  $dM$  of domestic currency

$$dM = - \pi dF, \quad dM > 0.$$

a) The Impact Reactions

The calculation of the impact reactions of the endogeneous variables according to the extended moving equilibrium method is left for the appendix. The results are listed below.

$$(21) \quad \frac{\partial \pi}{\partial M^S} > 0,$$

$$\frac{\partial \pi}{\partial M^S} \Big|_{\gamma=1} > \frac{\partial \pi}{\partial M^S} \Big|_{\gamma \in (0,1)} > \frac{\partial \pi}{\partial M^S} \Big|_{\gamma=0},$$

$$(22) \quad \frac{\partial (M/\pi)}{\partial M^S} = \begin{cases} \geq 0 & \text{if } \gamma \in [0,1) \text{ and } \frac{\partial M}{\partial M^S} \Big|_{dM^S = -\pi dF^S} \in [0,1),^{11)} \\ < 0 & \text{if } \gamma \in [0,1] \text{ and } \frac{\partial M}{\partial M^S} \Big|_{dM^S = -\pi dF^S} = 1, \\ < 0 & \text{if } \gamma = 1 \text{ and } \frac{\partial M}{\partial M^S} \Big|_{dM^S = -\pi dF^S} \in [0,1], \end{cases}$$

$$(23) \quad \frac{\partial F}{\partial M^S} = - \frac{1}{\pi} \frac{\partial M}{\partial M^S},$$

10) See the appendix for the calculations.

11) The partial derivation sign  $\partial (\cdot) / \partial \cdot$  is used for impact reactions, whereas  $d(\cdot) / d \cdot$  is used for long-run comparative static results.  $\partial M / \partial M^S \Big|_{dM^S = -\pi dF^S}$  denotes the extent of the disturbance.

$$(24) \quad \frac{\partial (V/\pi)}{\partial M^S} < 0,$$

$$(25) \quad \frac{\partial \pi^e}{\partial M^S} = \begin{cases} > 0 & \text{if } \gamma \in (0, 1], \\ 0 & \text{if } \gamma = 0, \end{cases}$$

$$(26) \quad \frac{\partial x}{\partial M^S} = \begin{cases} < 0 & \text{if } \gamma \in [0, 1), \\ 0 & \text{if } \gamma = 1, \end{cases}$$

$$(27) \quad \left. \frac{\partial x}{\partial M^S} \right|_{\gamma=0} < \left. \frac{\partial x}{\partial M^S} \right|_{\gamma \in (0, 1)} < \left. \frac{\partial x}{\partial M^S} \right|_{\gamma=1},$$

$$(28) \quad \frac{\partial B}{\partial M^S} > 0.$$

The foreign exchange market intervention causes an excess supply in the market for domestic money and portfolio imbalance for all wealth holders. Money market equilibrium is obtained by a depreciation of the exchange rate (21). The extent of the depreciation depends on the strength of the expectational effect  $\alpha(1-\gamma)(m_x V/\pi + m_x^* V^*)$  and of the valuation effect  $(mF + m^* F^*)$ . The stronger these effects, the weaker the exchange rate depreciation. The influence of the expectation formation process becomes intuitively clear when considering the impact reaction in  $x$ , the expected exchange rate change (26, 27). In the case of adaptive expectation ( $\gamma = 0$ ), the impact reaction in  $x$  is negative, e.g. wealth holders expect an appreciation and change their asset demands accordingly. The expectational effect thus constitutes an equilibrating tendency for the domestic money market. The stronger it is, the smaller the depreciation necessary to restore money market equilibrium. If there are current value expectations held <sup>12)</sup> ( $\gamma = 1$ ), there is no impact reaction in  $x$ . In fact, this is the only case in which overshooting occurs for sure. The valuation effect, which includes the increase in nominal domestic and foreign demand for domestic money induced by wealth changes through exchange rate fluctuations, serves to equilibrate the market for domestic money in nominal terms. However, domestic wealth holders end up

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12) In the sense of Burmeister and Turnovsky, 1976.

having a smaller real wealth immediately after the impact phase as compared to the initial equilibrium situation (24).

Portfolio imbalance induces stock shifts conform to (22), (23). As can be seen from the expressions in the appendix, it would happen by coincidence only that these stock shifts do not take place. The direction and extent of portfolio imbalance induced gross capital movements clearly depend primarily on the distribution of the disturbance among domestic and foreign wealth holders. Only in case of current value expectations the missing expectational effect causes domestics to exchange (in real terms) domestic money for foreign money, irrespective of the extent to which their portfolio composition has been changed by the disturbance.

The real wealth decline in the impact phase explains the initial trade balance surplus immediately after the impact phase, as the domestic consumption demand decreases (28).

The main differences between the above results and those by Kouri (1976) pertain on the one hand to the existence of stock shifts and, in combination with this, the occurrence of overshooting and on the other hand to the influence of the valuation effect in the impact phase (and subsequently at every moment when exchange rate jumps take place).

It seems intuitively plausible to have less or no overshooting at all when more than one channel is opened for the absorption of disturbances, as in fact is the case when allowing for stock shifts. The possibility of actual portfolio reshufflings fails only in the case of  $\gamma = 1$  to dampen the short-run exchange rate reaction, as stock shifts result in this case from equilibrium portfolio structure considerations only, which are not strong enough, if one assumes own currency preference.

Taking into account wealth changes through valuation effects helps understanding the hitherto rather obscure nature of the initial current account imbalance and the corresponding "incipient capital out- or inflows."

#### b) The Long-Run Reactions

After the adjustments have taken place, the domestic economy shows up - as in Kouri (1976) - with a higher exchange rate and the same level and composition of real wealth as in

the initial equilibrium state.<sup>13)</sup> It is obvious that the admission of temporary stock shifts will not change long-run comparative static results. What it does change are the impact reactions and the possible adjustment paths.

c) An Adjustment Path

A possible adjustment path is illustrated by means of a phase-diagram in the  $(\pi^e, F)$ -plane, drawn by reference to the reduced approximation system ((19),(20)) under assumption of  $s = 1$ , e.g. the case where trade deficits are financed by foreign assets only. Unfortunately, it is very tedious to determine the slopes of the  $\dot{\pi}^e = 0$  - and the  $\dot{F} = 0$  -curves, without further assumptions.<sup>14)</sup> Take the  $\gamma = 0$  case for exemplification (Fig. 1). The  $\dot{\pi}^e = 0$  -curve is definitely negatively sloped. The  $\dot{F} = 0$ -curve, however, could have a negative or positive slope, depending on the sign of the expression

$$(i) \quad m_X m^* f^* - m_X m f.$$

If one assumes (i)  $> 0$ , the sign of  $\partial \dot{F} / \partial F$  is still open. I assume  $\partial \dot{F} / \partial F$  to be negative, as the algebraic stability analysis revealed the system ((19), (20)) to be locally stable. We then have

$$\left. \frac{dF}{d\pi^e} \right|_{\dot{F}=0} = - \frac{\partial \dot{F} / \partial \pi^e}{\partial \dot{F} / \partial F} = - \frac{+}{-} > 0,$$

$$\left. \frac{dF}{d\pi^e} \right|_{\dot{\pi}^e=0} = - \frac{\partial \dot{\pi}^e / \partial \pi^e}{\partial \dot{\pi}^e / \partial F} = - \frac{-}{-} < 0.$$

For reference, the  $B = 0$  - curve is drawn too. It has a positive slope and is less steep than the  $\dot{F} = 0$  -curve. Note that the horizontal axis shows values of  $\pi^e$ , the expected long-run equilibrium exchange rate. With  $\gamma \neq 1$  one cannot exactly deduce the path of the exchange rate from the adjustment path of  $\pi^e$ . Only for time  $t = 0$  (initial values) and  $t = E$  (long-run equilibrium) the values of  $\pi$  and  $\pi^e$  coincide.

<sup>13)</sup> It might be interesting to note that results are totally different if one allows for two internationally tradable interest bearing distinct assets and explicitly considers interest receipts in the current account. See Tivig, 1986.

<sup>14)</sup> The experience with the application of the extended moving equilibrium method shows that it is generally easier to perform the stability analysis algebraically than graphically.

The foreign exchange market intervention causes a decline in  $F$ , the real stock of foreign assets held by domestics. The  $\pi^e$ -value doesn't change in the impact phase. The "starting point"  $(F_{0+}, \pi_{0+}^e)$  for the approximation system lies in the  $B > 0$  -region, as indicated by equation (28). In  $(F_{0+}, \pi_{0+}^e)$  there is money market equilibrium and portfolio balance. The inflow of foreign assets according to the trade balance surplus changes the composition and level of domestic private wealth, causing new portfolio reshufflings (stock shifts) and exchange rate jumps, as asset demands change with the increase in wealth through new savings. The increase in wealth also causes a rise in the consumption demand, so the trade balance surplus diminishes, changing subsequently to a deficit. The movement of the system is oscillating convergent. There will be some periods when the exchange rate appreciates, at least immediately after the impact phase and then again when the system has passed through the point of intersection of the trajectory with the  $\dot{\pi}^e = 0$  -locus.

In the new equilibrium position  $(\pi_E, F_E)$ , the exchange rate is higher, but the real stocks of assets held by domestic wealth owners are the same as in the initial equilibrium situation.

#### IV. Concluding Remarks

The AMA to flexible exchange rates took the existence of large international financial capital movements which result from reshuffling of portfolios, that is, which are not directly linked to current account imbalances, as a point of departure for analyzing temporary exchange rate fluctuations. However, in order for stock shifts to take place, at least two international tradable assets must exist. The dynamic analysis of models which explicitly incorporate stock shifts raises some problems when formulating the model in continuous time. Similar problems are posed by the inclusion of wealth changes through valuation effects. The inclusion of these effects helps - besides of closer approaching reality - to understand the concept of "incipient capital out- or inflows" which is rather obscure in the literature. The present paper showed that these problems can be solved by applying the extended moving equilibrium method (Witzel, 1985).

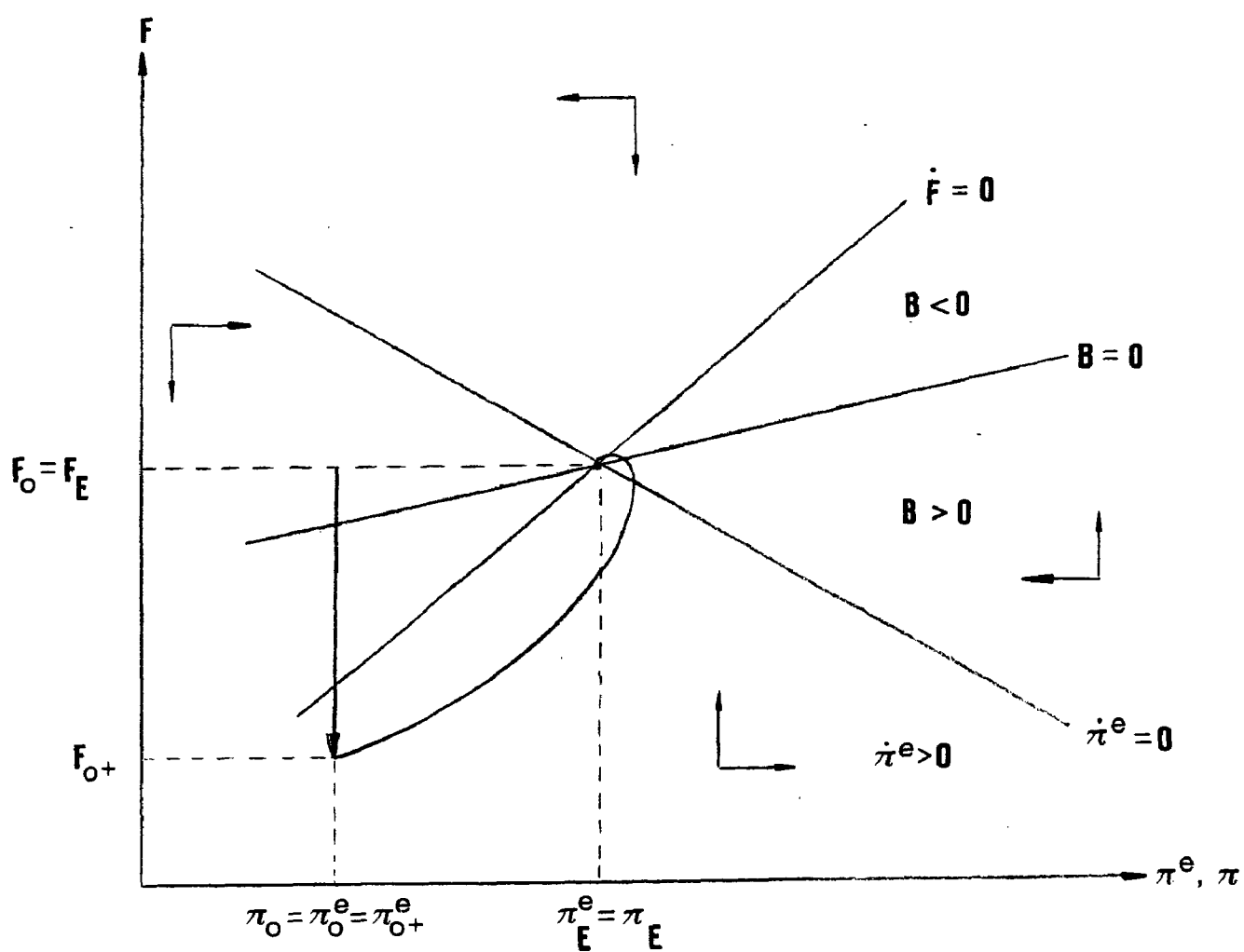


Fig.1

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# APPENDIX

Linearising the system (12)-(15) for  $s = 1$  around the equilibrium point (E) yields

$$(12') \quad \dot{\pi}|_E = \tau(\pi\Phi_{\pi} + M\Phi_M + F\Phi_F + \pi^e\Phi_e),$$

$$(13') \quad \dot{M}|_E = \tau(\pi\Psi_{\pi} + M\Psi_M + F\Psi_F + \pi^e\Psi_e),$$

$$(14') \quad \dot{F}|_E = -\frac{1}{\pi} \tau(\pi\Psi_{\pi} + M\Psi_M + F\Psi_F + \pi^e\Psi_e) + (\pi B_{\pi} + MB_M + FB_F + \pi^e B_e),$$

$$(15') \quad \dot{\pi^e}|_E = a(\pi\Omega_{\pi} + M\Omega_M + F\Omega_F + \pi^e\Omega_e) + \gamma\tau(\pi\Phi_{\pi} + M\Phi_M + F\Phi_F + \pi^e\Phi_e),$$

with the endogeneous variables in deviation form from their equilibrium values and

$$\Phi = M^S - [m(x)V + m^*(x)\pi V^*],$$

$$\Psi = m(x)V - M$$

$$\Omega = \pi - \pi^e,$$

with the partial derivatives

$$\Phi_{\pi} = \alpha(m_x \frac{V}{\pi} + m^*_x V^*) - (mF + m^*F^*) < 0,$$

$$\Phi_M = m^* - m < 0, \quad 1)$$

$$\Phi_F = \pi(m^* - m) < 0,$$

$$\Phi_e = -\alpha(m_x \frac{V}{\pi} + m^*_x V^*) > 0,$$

$$\Psi_{\pi} = -\alpha m_x \frac{V}{\pi} + mF > 0, \quad B_{\pi} = C_{V/\pi} \frac{M}{\pi^2} > 0$$

$$\Psi_M = -f < 0, \quad B_M = -\frac{1}{\pi} C_{V/\pi} < 0$$

---

1) By assumption of own currency preference  $m > m^*$ .

$$\Psi_F = \pi m > 0, \quad B_F = -C_{V/\pi} < 0$$

$$\Psi_e = \alpha m_x \frac{V}{\pi} < 0, \quad B_e = 0,$$

$$\Omega_\pi = 1, \quad \Omega_M = 0, \quad \Omega_F = 0, \quad \Omega_e = -1.$$

### Stability

The linearized reduced approximation system is obtained by setting the  $\tau$ -terms in (12') and (13') equal to zero, solving for  $\pi$  and  $M$  in terms of  $F$  and  $\pi^e$  and substituting the results into (14') and (15'). The matrix of the resulting system is denoted by  $\underline{S}$ . In order to determine  $\underline{S}$  more easily the following notation is introduced

$$\underline{A} = \begin{pmatrix} \Phi_\pi & \Phi_M \\ \Psi_\pi & \Psi_M \end{pmatrix}, \quad \underline{B} = \begin{pmatrix} \Phi_F & \Phi_e \\ \Psi_F & \Psi_e \end{pmatrix}, \quad \underline{C} = \begin{pmatrix} B_\pi & B_M \\ a\Omega_\pi & a\Omega_M \end{pmatrix}, \quad 1)$$

$$\underline{D} = \begin{pmatrix} B_F & B_e \\ a\Omega_F & a\Omega_e \end{pmatrix}, \quad \underline{G} = \begin{pmatrix} 0 & -\frac{1}{\pi} \\ \gamma & 0 \end{pmatrix}.$$

Then

$$\underline{S} = (\underline{I} + \underline{G}\underline{A}^{-1}\underline{B})^{-1}(\underline{D} - \underline{C}\underline{A}^{-1}\underline{B}),$$

where  $\underline{I}$  is the unit matrix. Calculation of  $\underline{S}$  yields

$$(\underline{I} + \underline{G}\underline{A}^{-1}\underline{B})^{-1} = \begin{bmatrix} \frac{\Phi_M(\Psi_\pi + \gamma\Psi_e) - \Psi_M(\Phi_\pi + \gamma\Phi_e)}{\Phi_\pi + \gamma\Phi_e} & \frac{\Psi_\pi\Phi_e - \Psi_e\Phi_\pi}{\pi(\Phi_\pi + \gamma\Phi_e)} \\ -\frac{\gamma\pi\Phi_M}{\Phi_\pi + \gamma\Phi_e} & \frac{\Phi_\pi}{\Phi_\pi + \gamma\Phi_e} \end{bmatrix}$$

1) This notation has been chosen in order to facilitate a comparison with the presentation of the extended moving equilibrium method by Witzel (1985). Underlined letters denote matrices or vectors.

with

$$\text{Det } (\underline{I} + \underline{G}\underline{A}^{-1}\underline{B}) = - \frac{\Phi_{\pi} + \gamma\Phi_e}{\text{Det } \underline{A}}$$

where  $\text{Det } \underline{A} = -\alpha(m_x f^* \frac{V}{\pi} + m_x^* f V^*) + f f^* \frac{M^S}{\pi} > 0$ , so  $\text{Det } (\underline{I} + \underline{G}\underline{A}^{-1}\underline{B}) > 0$ .

Further,

$$(\underline{D} - \underline{C}\underline{A}^{-1}\underline{B}) = \begin{bmatrix} \frac{\pi(B_{\pi}\Phi_M - B_M\Phi_{\pi})}{\text{Det } \underline{A}} & \frac{B_{\pi}(\Psi_e\Phi_M - \Psi_M\Phi_e) + B_M(\Psi_{\pi}\Phi_e - \Psi_e\Phi_{\pi})}{\text{Det } \underline{A}} \\ \frac{a\pi\Phi_M}{\text{Det } \underline{A}} & \frac{a[\Phi_M(\Psi_{\pi} + \Psi_e) - \Psi_M(\Phi_{\pi} + \Phi_e)]}{\text{Det } \underline{A}} \end{bmatrix}$$

with

$$\text{Det } (\underline{D} - \underline{C}\underline{A}^{-1}\underline{B}) = \frac{a\pi[B_M(\Phi_{\pi} + \Phi_e) - B_{\pi}\Phi_M]}{\text{Det } \underline{A}} > 0.$$

It follows that

$$\text{Det } \underline{S} > 0$$

and

$$\text{trace } \underline{S} = \pi B_M - \frac{a(\Phi_{\pi} + \Phi_e) + \pi B_{\pi}\Phi_M}{\Phi_{\pi} + \gamma\Phi_e} < 0,$$

thus the Routh-Hurwitz stability conditions for  $\underline{S}$  are fulfilled. In order to secure that the solutions of the initial system converge with  $\tau \rightarrow \infty$  towards the solutions of the approximation system one has to check the stability of the matrix  $(\underline{A} + \underline{B}\underline{G})$  of the "fast" subsystem. It can be easily verified that

$$\text{Det } (\underline{A} + \underline{B}\underline{G}) = - (\Phi_{\pi} + \gamma\Phi_e) > 0,$$

$$\text{trace } (\underline{A} + \underline{B}\underline{G}) = \Phi_{\pi} + \gamma\Phi_e - 1 < 0.$$

### Impact Reactions

$\underline{V}$  be the vector whose elements indicate the effect of the foreign exchange market operation on the fast movement-components

$$\underline{V} = \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \end{pmatrix}, \text{ with } \underline{v}_1 = \frac{\partial \Phi}{\partial M^S} \Big|_{dM^S} = -\pi dF^S = 1,$$

$$\underline{v}_2 = \frac{\partial \Psi}{\partial M^S} \Big|_{dM^S} = -\pi dF^S = - \frac{\partial M}{\partial M^S} \Big|_{dM^S} = -\pi dF^S,$$

$$\text{where } \frac{\partial M}{\partial M^S} \Big|_{dM^S} = -\pi dF^S \in [0,1].$$

The impact reactions are calculated from the system below

$$\begin{bmatrix} \frac{\partial \pi}{\partial M^S} \\ \frac{\partial M}{\partial M^S} \end{bmatrix} = -(\underline{A} + \underline{B}\underline{G})^{-1} \underline{V}, \quad \begin{bmatrix} \frac{\partial F}{\partial M^S} \\ \frac{\partial \pi^e}{\partial M^S} \end{bmatrix} = \underline{G} \begin{bmatrix} \frac{\partial \pi}{\partial M^S} \\ \frac{\partial M}{\partial M^S} \end{bmatrix}.$$

One obtains

$$\frac{\partial \pi}{\partial M^S} = \frac{1}{\text{Det}(\underline{A} + \underline{B}\underline{G})} > 0,$$

and

$$\frac{\partial \pi}{\partial M^S} \Big|_{\gamma=1} > \frac{\partial \pi}{\partial M^S} \Big|_{\gamma \in (0,1)} > \frac{\partial \pi}{\partial M^S} \Big|_{\gamma=0},$$

$$\frac{\partial M}{\partial M^S} = - \frac{\alpha(1-\gamma)m_x \frac{V}{\pi} - mF}{\text{Det}(\underline{A} + \underline{B}\underline{G})} - \frac{\partial M}{\partial M^S} \Big|_{dM^S} = -\pi dF^S$$

$$= \begin{cases} \geq 0 & \text{if } \frac{\partial M}{\partial M^S} \Big|_{dM^S = -\pi dF^S} \in (0,1), \\ < 0 & \text{if } \frac{\partial M}{\partial M^S} \Big|_{dM^S = -\pi dF^S} = 1, \\ > 0 & \text{if } \frac{\partial M}{\partial M^S} \Big|_{dM^S = -\pi dF^S} = 0. \end{cases}$$

However,

$$\frac{\partial M}{\partial M^S} + \frac{\partial M}{\partial M^S} \Big|_{dM^S = -\pi dF^S} > 0.$$

In real terms one gets

$$\frac{\partial (M/\pi)}{\partial M^S} = - \frac{\alpha(1-\gamma)m_x \frac{V}{\pi} + m \frac{M}{\pi}}{\text{Det } (A+BG)} - \frac{\partial (M/\pi)}{\partial M^S} \Big|_{dM^S = -\pi dF^S}$$

$$= \begin{cases} \geq 0 & \text{if } \gamma \in [0,1) \text{ and } \frac{\partial M}{\partial M^S} \Big|_{dM^S = -\pi dF^S} \in [0,1), \\ < 0 & \text{if } \gamma \in [0,1] \text{ and } \frac{\partial M}{\partial M^S} \Big|_{dM^S = -\pi dF^S} = 1, \\ < 0 & \text{if } \gamma = 1 \text{ and } \frac{\partial M}{\partial M^S} \Big|_{dM^S = -\pi dF^S} \in [0,1], \end{cases}$$

so

$$\frac{\partial (M/\pi)}{\partial M^S} + \frac{\partial (M/\pi)}{\partial M^S} \Big|_{dM^S = -\pi dF^S} = \begin{cases} \geq 0 & \text{if } \gamma \in [0,1), \\ < 0 & \text{if } \gamma = 1, \end{cases}$$

$$\frac{\partial F}{\partial M^S} = - \frac{1}{\pi} \frac{\partial M}{\partial M^S}$$

$$= \begin{cases} > 0 \text{ if } \frac{\partial M}{\partial M^S} \Big|_{dM^S = -\pi dF^S} \in (0,1), \\ > 0 \text{ if } \frac{\partial M}{\partial M^S} \Big|_{dM^S = -\pi dF^S} = 1, \\ < 0 \text{ if } \frac{\partial M}{\partial M^S} \Big|_{dM^S = -\pi dF^S} = 0, \end{cases}$$

and

$$\frac{\partial F}{\partial M^S} + \frac{\partial F}{\partial M^S} \Big|_{dM^S = -\pi dF^S} < 0,$$

$$\frac{\partial (V/\pi)}{\partial M^S} = - \frac{M/\pi^2}{\text{Det}(\underline{A+BG})} < 0,$$

$$\frac{\partial \pi^e}{\partial M^S} = \gamma \frac{\partial \pi}{\partial M^S} = \begin{cases} > 0 \text{ if } \gamma \in (0,1], \\ 0 \text{ if } \gamma = 0, \end{cases}$$

$$\frac{\partial x}{\partial M^S} = - \frac{\alpha}{\pi} (1-\gamma) \frac{\partial \pi}{\partial M^S} = \begin{cases} < 0 \text{ if } \gamma \in [0,1), \\ 0 \text{ if } \gamma = 1, \end{cases}$$

and

$$\frac{\partial x}{\partial M^S} \Big|_{\gamma=0} < \frac{\partial x}{\partial M^S} \Big|_{\gamma \in (0,1)} < \frac{\partial x}{\partial M^S} \Big|_{\gamma=1},$$

$$\frac{\partial B}{\partial M^S} = - C_{V/\pi} \frac{\partial (V/\pi)}{\partial M^S} > 0.$$

### Long Run Comparative Statics

Solving the system

$$\phi_{\pi} \frac{d\pi}{dM^S} + \phi_M \frac{dM}{dM^S} + \phi_F \frac{dF}{dM^S} + \phi_e \frac{d\pi^e}{dM^S} = -1$$

$$\psi_{\pi} \frac{d\pi}{dM^S} + \psi_M \frac{dM}{dM^S} + \psi_F \frac{dF}{dM^S} + \psi_e \frac{d\pi^e}{dM^S} = \frac{\partial M}{\partial M^S} \Big|_{dM^S} = -\pi dF^S$$

$$B_{\pi} \frac{d\pi}{dM^S} + B_M \frac{dM}{dM^S} + B_F \frac{dF}{dM^S} + B_e \frac{d\pi^e}{dM^S} = 0$$

$$a_{\Omega_{\pi}} \frac{d\pi}{dM^S} + a_{\Omega_M} \frac{dM}{dM^S} + a_{\Omega_F} \frac{dF}{dM^S} + a_{\Omega_e} \frac{d\pi^e}{dM^S} = 0$$

in the original variables  $\pi$ ,  $M$ ,  $F$  and  $\pi^e$  one obtains the long run reactions

$$\frac{d\pi}{dM} = \frac{a_{C_{V/\pi}}}{\Delta} > 0,$$

where  $\Delta$  is the matrix of the above system

$$\Delta = a_{C_{V/\pi}} f^* \frac{M^S}{\pi} > 0;$$

$$\frac{dM}{dM^S} = \frac{M}{f^* M^S} > 0, \quad \frac{d(M/\pi)}{dM^S} = 0,$$

$$\frac{d(\pi F)}{dM^S} = \frac{\pi F}{f^* M^S} > 0, \quad \frac{dF}{dM^S} = 0,$$

$$\frac{dV}{dM^S} = \frac{V}{f^* M^S} > 0, \quad \frac{d(V/\pi)}{dM^S} = 0,$$

$$\frac{d\pi^e}{dM^S} = \frac{d\pi}{dM^S} \Rightarrow \frac{dx}{dM^S} = 0, \quad \frac{dB}{dM^S} = 0.$$

A comparison of the impact reaction and the long-run reaction of the exchange rate yields

$$\begin{aligned} \frac{d\pi}{dM^S} - \frac{\partial \pi}{\partial M^S} &= \frac{\alpha(1-\gamma) (m_X V + m_X^* \pi V^*) + (m - m^*) M}{f^* M^S [\alpha(1-\gamma) (m_X \frac{V}{\pi} + m_X^* V^*) - (mF + m^* F^*)]} \\ &= \begin{cases} < 0 & \text{if } \gamma = 1, \\ \geq 0 & \text{if } \gamma \neq 1. \end{cases} \end{aligned}$$

### Phase-diagrams

In order to draw phase-diagrams in the  $(\pi^e, F)$ -plane of the reduced approximation system (19)-(20) one has to calculate the slopes of the  $F=0$ - and  $\pi^e=0$ -curves. For this, the system (19)-(20) is rewritten as

$$\dot{F} = \frac{B(1 - \gamma \frac{\partial \pi}{\partial \pi^e}) - \frac{a(\pi - \pi^e)}{\pi} \frac{\partial M}{\partial \pi^e}}{\Delta'},$$

$$\dot{\pi^e} = \frac{a(\pi - \pi^e) (1 + \frac{1}{\pi} \frac{\partial M}{\partial F}) + \gamma B \frac{\partial \pi}{\partial F}}{\Delta'},$$

where

$$\Delta' = (1 + \frac{1}{\pi} \frac{\partial M}{\partial F}) (1 - \gamma \frac{\partial \pi}{\partial \pi^e}) + \gamma \frac{1}{\pi} \frac{\partial M}{\partial \pi^e} \frac{\partial \pi}{\partial F}$$

and  $\frac{\partial \pi}{\partial F}$ ,  $\frac{\partial \pi}{\partial \pi^e}$ ,  $\frac{\partial M}{\partial F}$ ,  $\frac{\partial M}{\partial \pi^e}$  are the elements of the matrix

$$-\underline{A}^{-1} \underline{B} = \begin{bmatrix} \frac{\pi \Phi_M}{\text{Det } \underline{A}} & \frac{\Psi_e \Phi_M - \Psi_M \Phi_e}{\text{Det } \underline{A}} \\ \frac{\Psi_\pi \Phi_F - \Psi_F \Phi_\pi}{\text{Det } \underline{A}} & \frac{\Psi_\pi \Phi_e - \Psi_e \Phi_\pi}{\text{Det } \underline{A}} \end{bmatrix}$$

which describes the dependence of the variables  $\pi$  and  $M$  from the variables  $F$  and  $\pi^e$  as given by the equilibrium conditions (17) and (18). As usual only the linear part of the equilibrium conditions is considered. One finds

$$\frac{\partial \pi}{\partial F} < 0, \quad \frac{\partial \pi}{\partial \pi^e} > 0, \quad \frac{\partial M}{\partial F} > 0 \text{ and}$$

$$\frac{\partial M}{\partial \pi^e} \gtrless 0 \quad \text{if} \quad m_x m^* f^* - m_x^* m f \gtrless 0.$$

Then

$$\left. \frac{dF}{d\pi^e} \right|_{\dot{F}=0} = - \frac{\dot{\partial F} / \partial \pi^e}{\dot{\partial F} / \partial F} = - \frac{(1-\gamma \frac{\partial \pi}{\partial \pi^e}) \frac{\partial B}{\partial \pi^e} + \frac{a}{\pi} (1 - \frac{\partial \pi}{\partial \pi^e}) \frac{\partial M}{\partial \pi^e}}{(1-\gamma \frac{\partial \pi}{\partial \pi^e}) \frac{\partial B}{\partial F} - \frac{a}{\pi} \frac{\partial M}{\partial \pi^e} \frac{\partial \pi}{\partial F}},$$

and

$$\left. \frac{dF}{d\pi^e} \right|_{\pi^e=0} = - \frac{\dot{\partial \pi^e} / \partial \pi^e}{\dot{\partial \pi^e} / \partial F} = - \frac{\gamma \frac{\partial \pi}{\partial F} \frac{\partial B}{\partial \pi^e} - a(1 - \frac{\partial \pi}{\partial \pi^e}) (1 + \frac{1}{\pi} \frac{\partial M}{\partial F})}{\gamma \frac{\partial \pi}{\partial F} \frac{\partial B}{\partial F} + a(1 + \frac{1}{\pi} \frac{\partial M}{\partial F}) \frac{\partial \pi}{\partial F}}.$$

With the assumption

$$(i) \quad m_x^{m^*f^*} - m_x^{mf} > 0, \text{ that is } \frac{\partial M}{\partial \pi^e} > 0$$

and noting that

$$\frac{\partial B}{\partial \pi^e} = \frac{C_{V/\pi}}{\pi} \left( \frac{M}{\pi} \frac{\partial \pi}{\partial \pi^e} - \frac{\partial M}{\partial \pi^e} \right) = - \frac{\alpha C_{V/\pi}}{\pi^3 \text{Det } \underline{A}} m_x^{f^*M^S V} > 0,$$

$$\frac{\partial B}{\partial F} = \frac{C_{V/\pi}}{\pi} \left( \frac{M}{\pi} \frac{\partial \pi}{\partial F} - \frac{\partial M}{\partial F} - \pi \right) = \frac{C_{V/\pi}}{\pi \text{Det } \underline{A}} [\alpha (m_x^{V+m_x^* \pi V^*}) - f^* M^S] < 0$$

one obtains

$$\left. \frac{dF}{d\pi^e} \right|_{\dot{F}=0} = - \frac{+}{\pm}, \quad \left. \frac{dF}{d\pi^e} \right|_{\pi^e=0} = - \frac{-}{\pm}.$$

I make the assumption  $\dot{\partial F} / \partial F < 0$ , as the system was found to be stable when analyzing it algebraically. So the  $\dot{F}=0$ -curve has a positive slope. In order to determine the sign of  $\dot{\partial \pi^e} / \partial F$ , further assumptions are necessary. If

$$(ii) \quad af > \gamma C_{V/\pi} f^*,$$

the  $\dot{\pi}^e=0$ -curve is negatively sloped. Note that condition (ii) is always satisfied with  $\gamma = 0$ , that is in the adaptive-expectation case. With assumptions (i) and (ii), the singular point is a stable focus, so the movement is oscillating convergent.

The  $B=0$ -curve is positively sloped and flatter than the  $\dot{F}=0$ -curve.

Note that in the  $\gamma = 1$  case, equations (12) and (15) coincide. The reduced approximation system shrinks to the differential equation

$$(19') \quad \dot{F} = \frac{B(\pi(F), M(F), F)}{1 + \frac{1}{\pi(F)} \frac{\partial M}{\partial F}}$$

which is locally stable as  $\partial \dot{F} / \partial F = - \frac{C_V / \pi^{f^* M^S}}{\pi(mF + m^* F^*)} < 0$ , when taking into account the linear part only.