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Cartel stability in an exhaustible resource model

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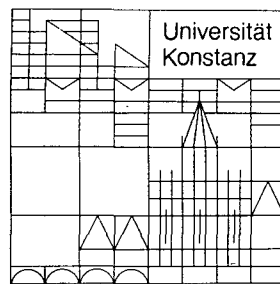
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Diskussionsbeiträge



Juristische
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Cartel Stability
in an Exhaustible Resource Model

CARTEL STABILITY IN AN EXHAUSTIBLE RESOURCE MODEL

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ABSTRACT

A simple oligopolistic common-pool exhaustible resource game is considered. By analysing punishment strategies, including optimal punishments, it is possible to determine which cartel agreements are implementable in a non-cooperative play of the game. Joint-profit maximizing allocations are sustainable for sufficiently low discounting, but in general it is shown that no folk-theorem exists for this model. In particular, for sufficiently high elasticities of demand, it is shown that optimal punishments are not sufficiently severe to enforce stationary symmetric extraction paths, thus confirming the hypothesis that sufficient market power is needed for a cartel to be stable.

INTRODUCTION

We shall look at a simple exhaustible resource model in a discrete time, infinite horizon framework. By looking at "punishments" levelled against firms which deviate from an implicitly agreed extraction path, we shall examine the extent to which different allocations can be supported as non-cooperative equilibria. We shall pay particular attention to the question of whether punishments can be severe enough to sustain implicit agreements which maximize joint profits, and to the idea (Pindyck (1979)) that cartel stability depends crucially upon market power.

This non-cooperative approach to the question of cartel stability has been used by Porter (1983) in a model with imperfect information. (See also Green and Porter (1984)). While we assume perfect observability here, the model is complicated by the existence of a state variable in the form of the resource stock; moreover we shall apply the idea of optimal punishments in this dynamic model. (Abreu, Pearce and Stacchetti (1986) have looked at general properties of the equilibrium set in the Green-Porter model.)

The model we analyse is a common property, non-renewable resource model. We follow much of the literature in assuming the following: N firms compete both in extracting the resource and in selling on the same market, extraction costs are zero, and the market demand curve has constant elasticity. Previous work has mostly concentrated on solution concepts that lead to

a unique equilibrium, a central question being whether equilibrium extraction is too high relative to joint profit maximization, whereas here we will focus on the range of equilibria which exists in a more general non-cooperative framework. Using the open-loop concept it was shown, somewhat surprisingly, that the equilibrium was efficient, which, in this context, is equivalent to joint-profit maximization (Kemp and Long (1980)). (It has been argued by Mohr (forthcoming) that this result depends critically on an exogenous finite time horizon; see also Bolle (1986), and McMillan and Sinn (1984), who criticise the reaction hypothesis of the open-loop concept). With feedback strategies the result seems to be different, namely that extraction is too fast relative to the cooperative solution. (See Eswaran and Lewis (1984), and also Reinganum and Stokey (1985), who show that shortening the period for which firms can commit leads to more rapid resource depletion). This of course corresponds better with intuition. Moving from open-loop to feedback strategies means that current actions are allowed to depend on the actual course of the game to a certain extent. Going further in this direction, and allowing actions to depend in a general way on the course of play, Cowans and Lewis (1983) show in a renewable resource model that some cooperation at least can be sustained in equilibrium - so extraction can be slowed down again. We shall take this last approach a step further and examine the precise extent to which cooperation can be sustained when actions can

be conditioned in an arbitrary way upon the previous course of play.

The particular reason why this enlargement of the strategy space leads to more chances of cooperation is that punishment strategies can be used to discourage potential deviants upsetting a particular proposed outcome; an *outcome* here means a particular sequence of play. In a sub-game perfect equilibrium these punishments are just equilibria of the sub-game which starts after the deviation has occurred and which are particularly distasteful from the point of view of the deviant. Thinking in terms of a proposed outcome, and punishments which deter players from deviating from the proposed outcome, is of course just a particular way of thinking about an equilibrium of a game. Nevertheless it follows from the work of Abreu (1988) that nothing is lost by doing this: any equilibrium of the game can be expressed in this fashion, and moreover if the *severest* punishments can be found attention can be restricted to these since any equilibrium outcome of the game can be supported by these so-called optimal punishments.

While Abreu's work was in the context of static repeated games, the same idea applies equally to dynamic games such as we have here. The only difference of substance is that optimal punishments will depend on the value of the state variable; so in contrast to the static case the punishment levelled at a deviant will depend on nature of his deviation, since this

affects the state variable. This complication turns out to be important.

This general approach lends itself readily to a cartel interpretation. Instead of "equilibrium", one can read "self-enforcing cartel agreement". The members of the cartel agree upon a particular outcome path, and moreover discuss how they will react in the event that one of them reneges on the implicit arrangement.¹ For the agreement to be credible, it must also be the case that the punishments themselves are credible and sufficiently severe to discourage renegeing. If optimal punishments can be identified, then all credible cartel agreements can be characterised.

While it turns out that joint-profit maximising outcomes can be sustained as equilibria for low enough discount rates, it is not the case that a folk-theorem holds in a meaningful sense: we show that no stationary symmetric extraction path below the maximum extraction rate is an equilibrium for low discount rates provided the elasticity of demand is sufficiently high.

THE MODEL

There is a single, non-renewable resource, whose stock at time t is S_t . There is an infinite horizon, and time is discrete, so

¹ The story, however, doesn't allow for the cartel members to get together later in the game and renegotiate, otherwise any punishments which are unpleasant from the punisher's point of view would be subject to renegotiation and hence incredible. See Fudenberg and Maskin (1987) for one approach to renegotiation-proofness.

$t=0,1,2,\dots$; the initial stock of the resource is S_0 . There are N identical firms, indexed by $j=1,2,\dots,N$; each can extract the resource at zero cost. Let j 's desired extraction at time t be q^j_t , and it is supposed that there is a maximum extraction rate per period, a fraction x of the current stock, so that $q^j_t \leq xS_t$. This form of rationing constraint is chosen for analytical simplicity; it corresponds however to the assumption that extraction is more difficult the smaller the remaining resource stock is (where "difficult" here simply refers to the limit on extraction). It is assumed that $x \leq 1/N$, otherwise for $x > 1/N$ it would be possible that $\sum_j q^j_t > S_t$, in which case some rationing scheme would be needed. (This latter case will be discussed below.) The stock evolves according to

$$S_{t+1} = S_t - \sum_j q^j_t .$$

All resources extracted at time t must also be sold at t (no other storage possibilities) on the same market. Hence, as is often assumed, the firms are linked from both the resource and the sales sides. It is supposed that the price at t is $p_t = (\sum_j q^j_t)^{-1/\epsilon}$, where $\epsilon > 1$ is the constant elasticity of demand. With this demand specification, marginal revenue tends to infinity as total output goes to zero. Firm j seeks to maximize discounted profits $\sum_{t=0}^{\infty} \delta^t p_t q^j_t$.

We shall look at sub-game perfect equilibria of this model. This means that after any *arbitrary* sequence of actions

by the two players, equilibrium strategies must constitute a Nash-equilibrium of the sub-game starting at this point.

First we look at the symmetric cooperative solution, which maximises joint discounted profits. This satisfies the condition that current (joint) marginal revenue equals discounted marginal revenue for each period in the future, and that $q^j_t = q^k_t$, all j, k . In fact nothing below depends on symmetry, though we shall restrict attention to it for simplicity of presentation. The optimal rule can be shown to be

$$(1) \quad q^j_t = (1 - \delta^\epsilon) S_t / N$$

provided this is less than xS_t , otherwise $q^j_t = xS_t$. Future profits discounted to period t are then

$$(1/N) S_t^{1-1/\epsilon} (1-\delta^\epsilon)^{-1/\epsilon}.$$

PUNISHMENT STRATEGIES

To see whether the cooperative outcome can be supported as an equilibrium we shall look at optimal punishments. By definition, if such punishments are not severe enough to support this outcome then cooperation cannot be supported in any other way. As everything is symmetric, we shall consider firms $2, 3, \dots, N$ punishing firm 1 , from time T onwards. The worst feasible punishment is the minimax punishment, $q^j_t = xS_t$, all $t \geq T$, $j \geq 2$, and we shall attempt to construct strategies

which support this punishment. If we succeed, we have identified optimal punishment strategies.

To check whether minimaxing is credible - that is, constitutes an equilibrium of the game - we must first look at the optimal response of firm 1 to being minimaxed. In general if firm 1's extraction at each date t is vS_t , while aggregate extraction of all other firms is zS_t , where v and z do not vary over time, then $S_{t+1} = S_t(1-z-v)$ and discounted profits to firm 1 from T onwards are

$$(2) \quad \pi(S_T, v, z) = \sum_{t=T}^{\infty} S_T^{1-1/\epsilon} (\delta(1-z-v))^{t-T} v(z+v)^{-1/\epsilon} \\ = S_T^{1-1/\epsilon} v(z+v)^{-1/\epsilon} / (1-\delta(1-z-v)^{1-1/\epsilon}).$$

So 1's optimal response to a combined extraction rate of z is to maximise this expression subject to $v \leq x$. Then if an interior solution for v exists, after some rearrangement the first order condition is

$$(3) \quad (z+v(1-1/\epsilon)) = \delta(1-z)(1-z-v)^{-1/\epsilon} (z+v(1+\epsilon/(1-z))).$$

The solution for v must be independent of the level of S_t . It may be checked that $\partial^2 \pi / \partial v^2$ is negative at any solution to (3) so there is a unique solution, and we denote it by $v^*(z, \epsilon, \delta)$; we stress that this is the solution ignoring the $v \leq x$ constraint. It also follows that if the solution has $v^* \geq x$ then the optimal response must be to set $v = x$. Thus the

optimal response to being minimaxed is to set $v = \min\{v*((N-1)x, \epsilon, \delta), x\}$.

When an interior solution exists then (3) says the marginal revenue to firm 1 from extracting one more unit of the resource must equal the discounted extra revenue which could be generated next period if this unit was not extracted: each firm j will extract x of it, so firm 1's output will rise by $(1-(N-1)x)$ and total output by 1.

The proposed punishment equilibrium strategies are to set $q^j_t = xS_t$, $j \geq 2$, and $q^1_t = vS_t$ provided these actions have been followed since the punishment was started at T . If firm j deviates at any stage from his punishment then the roles are reversed and firm j is punished from the following period onwards, until some other firm deviates from punishing firm j , in which case this firm is now punished, and so on. If firm 1 (or in general, the firm currently being punished) deviates at any stage from vS_t this is ignored, as is the simultaneous deviation by more than one player.²

Firm 1's response vS_t is by definition his best response to the other firms' strategies in any sub-game. The difficult problem is to show that each firm j 's punishment is his best response to firm 1's strategy. Suppose firm j reduces output at t ; its current profits would certainly fall and in addition it will itself be minimaxed from $t+1$. Nevertheless the state

² This may sound strange, but for the non-cooperative solution simultaneous deviations are not considered by the players, and so the consequent punishment is irrelevant.

variable S_{t+1} will be favourably affected. It is necessary to show the latter effect is no bigger than the former.

Under the proposed punishment strategies firm 1's profits, discounted to T, are

$$(4) \quad S_T^{1-1/\epsilon} v((N-1)x+v)^{-1/\epsilon} / (1-\delta(1-(N-1)x-v)^{1-1/\epsilon})$$

while firm j gets

$$(5) \quad S_T^{1-1/\epsilon} x((N-1)x+v)^{-1/\epsilon} / (1-\delta(1-(N-1)x-v)^{1-1/\epsilon}).$$

The discounted profits of j if he deviates at T by playing bS_T are

$$(6) \quad S_T^{1-1/\epsilon} (b((N-2)x+v+b))^{-1/\epsilon} + \\ \delta(1-((N-2)x+v+b))^{1-1/\epsilon} v((N-1)x+v)^{-1/\epsilon} / (1-\delta(1-(N-1)x-v)^{1-1/\epsilon}).$$

Minimaxing is optimal provided (6) is no bigger than (5), for all $b \leq x$. (Because everything is stationary, it is only necessary to check this in one period.) there are N firms competing both in extracting the resource and in selling on the same market. there are N firms competing both in extracting the resource and in selling on the same market.

In general it is very difficult to compare the expressions in (5) and (6), but we can deal with the case when the response to being minimaxed is up against the extraction constraint. A

first step towards finding a condition for this is to set $z = (N-1)v$ in (3):

$$(7) \quad \delta^\epsilon = (N\epsilon - 1)^\epsilon (1 - Nv) / (N\epsilon(1 - (N-1)v) - 1)^\epsilon.$$

Denote by $f(v; \epsilon)$ the right hand side of (7). The solution, v^c , to this equation, $\delta^\epsilon = f(v^c; \epsilon)$, will prove to be important below, and it corresponds to a stage-game Nash-equilibrium in a static repeated-game. Thus if all other players are playing $q^j t = v^c$ each period then it is in j 's interests to do likewise, and this is therefore a closed-loop equilibrium of the game whenever $v^c \leq x$. It is straightforward to show that at $v = 0$, $f(0; \epsilon) = 1$ and $\partial f / \partial v(0; \epsilon) > 0$; it has a single turning point, and at $v = 1/N$, $f = 0$. Consequently there is a unique solution to (7) which is bounded away from zero, as δ varies, by the positive solution for v to $f(v; \epsilon) = 1$; denote this bound by \underline{v} . See Figure 1. Using this function f it is now possible to give a condition on the parameters under which the optimal minimax response is against the extraction constraint (i.e. $v^* \geq x$).

Lemma: $v^*((N-1)x, \epsilon, \delta) \geq x$ if and only if $\delta^\epsilon \leq f(x; \epsilon)$.

Proof: See Appendix.

Note that $\delta^\epsilon \leq f(x; \epsilon)$ is also equivalent to $v^c \geq x$, so the Lemma also establishes that $v^* \geq x$ if and only if $v^c \geq x$. Hence

the mutual playing of $v^c S_t$ is an equilibrium when $\delta^\epsilon > f(x; \epsilon)$; in general the mutual playing of $\min\{v^c, x\}$ is an equilibrium.

We can now prove

Proposition 1: For parameter values such that $\delta^\epsilon \leq f(x; \epsilon)$, the optimal punishment is the minimax punishment.

Proof: The strategies are those specified above. Since the player being punished is also playing, by the Lemma, $q^j_t = x S_t$, then the punisher's strategy is also a best response to this strategy. This is true in all sub-games. Q.E.D.

When $\delta^\epsilon \leq f(x; \epsilon)$ it is now possible to answer the question of whether cooperation can be supported in equilibrium. It is only necessary to compare the discounted profits obtained by cooperation with the potential gains from deviation assuming that optimal punishments are imposed thereafter.

Proposition 2: For parameter values for x and ϵ such that $\delta^\epsilon \leq f(x; \epsilon)$ for all δ , cooperation can be sustained for δ near enough to 1.

Proof: Consider a deviation by one of the players at time t . The potential initial gains to deviation are bounded above, since $\epsilon > 1$, by $S_t^{1-1/\epsilon}$. The punishment utility, discounted to $t+1$, is $S_{t+1}^{1-1/\epsilon} x (2x)^{-1/\epsilon} / (1 - \delta(1-2x)^{1-1/\epsilon})$, which is bounded above in δ . So the total profits from any deviation strategy

are bounded above. For δ near 1, however, profits from cooperation will exceed this bound, so deviation cannot be profitable for such δ . Q.E.D.

To check that Proposition 2 is not empty we need only show that there are values of x and ϵ such that $f(x;\epsilon) \geq 1$, in which case the condition $\delta^\epsilon \leq f(x;\epsilon)$ holds for all δ between 0 and 1. From the discussion on the properties of f , it follows that for any given ϵ this is true for an interval of values for x containing 0; e.g. if $\epsilon=2$, $N=2$, then the interval is $[0,.325]$.

In fact there are three possible regimes, depending on the discount factor. If the discount factor is sufficiently low then the cooperative output levels may be up against the extraction constraint. The condition for this, from (1), is

$$(8) \quad \delta^\epsilon \leq 1-Nx.$$

Notice that the right hand side of (8) is smaller than f , so (8) implies $\delta^\epsilon \leq f(x;\epsilon)$, and hence by the lemma, the minimax response is also against the extraction constraint. For higher values of δ the cooperative output level first falls below the constraint, and then the minimax response also falls below x , unless $f(x;\epsilon) \geq 1$, in which case it remains at x .

While we are only able to show that minimax punishments are credible (that is, equilibrium) punishments when $\delta^\epsilon \leq f(x;\epsilon)$,³ the result of Proposition 2 can be generalised to the

³ I have not been able, though, using numerical solutions, to find any parameter values for which minimaxing isn't credible.

case $\delta^\epsilon > f(x; \epsilon)$ by using less severe punishments. In particular, consider the equilibrium when all players play $q_t^t = v^c S_t$ each period. (Recall that this is an equilibrium when $\delta^\epsilon \leq f(x; \epsilon)$.) Since $v^c > \underline{v} > 0$, the utilities in the equilibrium are bounded above. So using this equilibrium as a punishment consequent upon any deviation from cooperation, the rest of the proof of Proposition 2 can be repeated, and we have:

Proposition 3: Given any values for x and ϵ , cooperation can be sustained for δ near enough to 1.

In view of this proposition, it is reasonable to ask whether an even stronger version - a folk-theorem - can be proved; that is, can all individually rational outcomes be sustained as equilibria for low enough discount rates? The answer depends partly upon how we define the concepts involved. First, however, a general remark about this game. The declining resource stock itself has an effect very similar to a positive discount rate, in that the game is scaled down whenever the stock declines. So it is to be expected that the standard arguments used in proving folk-theorems will not apply here. As a counter example to a folk-theorem we shall show that any stationary symmetric extraction path below the maximum

extraction rate cannot be sustained for low discount rates provided the elasticity of demand is high enough.^{4, 5}

Proposition 4: Consider the outcome path with $q_j^t = cS_t$, all j, t , where $0 < c < x$. Then there exists an ϵ^ , and $\delta^* < 1$, such that for $\epsilon > \epsilon^*$ and $1 > \delta > \delta^*$ this outcome path cannot be sustained in equilibrium.*

Proof: Future profits along the path, discounted to t , are

$$(9) \quad S_t^{1-1/\epsilon} c (Nc)^{-1/\epsilon} / (1-\delta(1-Nc)^{1-1/\epsilon}).$$

Suppose one of the players deviates at t , by extracting xS_t instead of cS_t , and suppose moreover that he is minimaxed thereafter. He gets

$$(10) \quad S_t^{1-1/\epsilon} (x((N-1)c+x)^{-1/\epsilon} + \delta(1-(N-1)c-x)^{1-1/\epsilon} v((N-1)x+v)^{-1/\epsilon} / (1-\delta(1-(N-1)x-v)^{1-1/\epsilon}).$$

This must be a lower bound on the best deviation payoff since xS_t may not be the best deviation and the minimax punishment is at least as severe as the optimal punishment. Letting $\epsilon \rightarrow \infty$ and $\delta \rightarrow 1$, the expression in (9) tends to

⁴ Computer simulations suggest that at lower elasticities it may be possible to support all stationary symmetric paths as equilibria.

⁵ This game does not satisfy either of the jointly sufficient conditions, given by Lockwood(1986), for a folk-theorem to hold in a dynamic game with no discounting.

$$(11) \quad S_t^{1-1/\epsilon}(1/N),$$

while that in (10) tends to

$$(12) \quad S_t^{1-1/\epsilon}(x + (1-(N-1)c-x)v/((N-1)x+v)).$$

Since v by definition maximizes the second term in (12), we can set $v = x$ to get a lower bound of this expression. As $c < x$, we have $Nx - (N-1)c - x > 0$, and so

$$x + (1-(N-1)c-x)x/(Nx) > 1/N.$$

Thus in the limit, the deviation utility is larger. Q.E.D.

It is argued by Pindyck (1979) that cartel stability is higher the lower the elasticity of demand, the reason being that the potential gains from cartel formation relative to the competitive outcome are then greater. While we get essentially the same result, namely that by choosing a high enough elasticity almost any stationary symmetric path fails to be supportable as an equilibrium, the explanation is somewhat different, partly because the comparison here is with punishment utilities rather than the competitive outcome, and partly because we explicitly consider the one-period gains from deviation. At higher elasticities the benefits from deviating increase because marginal revenue is higher. More importantly,

the cost of being punished falls as higher extraction rates are actually desirable when ϵ is high.

While in outcome space no general folk-theorem holds, in dynamic games the distinction between outcomes and normalised discounted payoffs is more important than in static repeated-games (see Lockwood(1986)). If payoffs were normalised so that utility from cooperation remains constant, then it is simple to prove a folk-theorem in payoff space. To normalise, using the expression below (1), discounted payoffs must be multiplied through by $(1-\delta^\epsilon)^{1/\epsilon}$. Maximum normalised joint profits are then $S_0^{1-1/\epsilon}$. Since in the limit the normalised minimax utilities are zero, the folk-theorem presented below captures the idea of feasible individually rational payoffs.

Proposition 5. Any normalised payoff vector $(\pi^1, \pi^2, \dots, \pi^N)$, satisfying $\pi_j > 0$ all j , and $\sum_j \pi^j \leq S_0^{1-1/\epsilon}$, can be sustained as an equilibrium for δ near enough to 1.

Proof: Consider a solution in $(0,1]$ for C to the following equation:

$$S_0^{1-1/\epsilon} (1-\delta^\epsilon)^{1/\epsilon} C^{1-1/\epsilon} / (1-\delta(1-C)^{1-1/\epsilon}) = \sum_j \pi^j.$$

The left hand side is total normalised profits when the total output rate is stationary at C (see (2)). It attains its maximum value of $S_0^{1-1/\epsilon}$ when $C = (1-\delta^\epsilon)$, from (1). Also, for fixed C , it tends to 0 as δ tends to 1. By continuity, therefore, there exists a solution (depending upon δ) when δ is in a neighbourhood of 1. Denote this solution $C(\delta)$ and consider

the outcome path which has $q^j_t = (\pi^j / \sum_i \pi^i) C(\delta) S_t$. For such δ near 1 this path delivers the required payoff vector. The punishment strategies $q^j_t = \min\{v^c, x\} S_t$ are again sufficiently severe in the limit to support this outcome path, because the non-normalised payoffs from the path become arbitrarily large in the limit, while punishment utilities tend to zero.

Q.E.D.

The explanation of how this folk-theorem result can obtain despite Proposition 4 is simple. The actions which were constructed to give rise to the payoff vector in question vary with the discount factor, and in fact the extraction rates tend to zero as δ tends to 1, as of course the cooperative extraction rates also do. So in a sense Proposition 5 only concerns itself with outcomes which are very close to cooperation, which we already know are sustainable in equilibrium. On the other hand it is not clear whether any other normalisation could be profitably used. Overall, Proposition 4 would seem to be the more meaningful result.

CONCLUDING COMMENTS

We have looked at the limits to cooperation in a non-cooperative model of resource extraction. While the most cooperative outcomes can be sustained as equilibria for low enough discount rates, other outcomes cannot be sustained if the elasticity of demand in the market is sufficiently high. It would be interesting to see whether similar results obtain when the demand function is not iso-elastic: in particular, if the

demand curve intercepts the price axis then the type of argument used to show that cooperative outcomes can be sustained would break down.

APPENDIX

Proof of Lemma: Clearly when $\delta^\epsilon = f(x; \epsilon)$ then (3) holds with $v=x$ and $z=(N-1)x$, and so $v^*=x$, and vice versa. Suppose the parameters are such that $\delta^\epsilon > f(x, \epsilon)$. Consider increasing x from its current value to some higher value, x' . The inequality $\delta^\epsilon > f(x', \epsilon)$ remains unchanged, see Figure 1, and whatever the inequality was between v^* and x , this too remains unchanged since otherwise by continuity there would be an $x'' > x$ such that $v^*(\delta, \epsilon, (N-1)x'') = x''$ and hence such that $\delta^\epsilon = f(x'', \epsilon)$. If $x' = 1/N$ then from (3) $v^*(\delta, \epsilon, (N-1)x') < 1/N = x'$ (otherwise the RHS of (3) is unbounded). Hence also $v^*(\delta, \epsilon, (N-1)x) < x$, thus establishing necessity. The reverse implication, that $\delta^\epsilon < f(x, \epsilon)$ implies $v^* > x$, is likewise established by considering x tending to zero, thus establishing sufficiency. Q.E.D.

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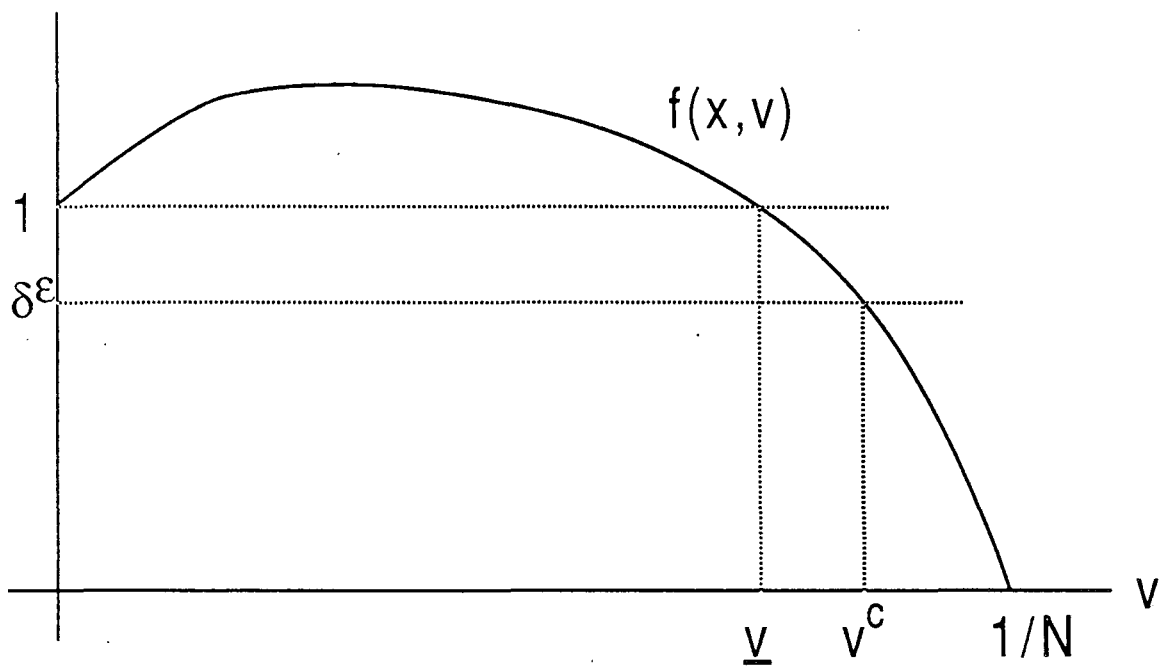
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