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## A temporary equilibrium model of international migration with overlapping generations and incomplete information

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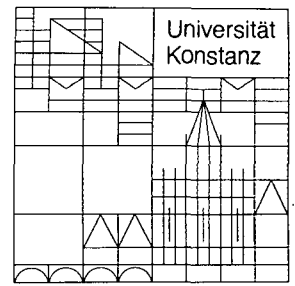
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of International Migration  
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**A TEMPORARY EQUILIBRIUM MODEL OF  
INTERNATIONAL MIGRATION WITH OVERLAPPING GENERATIONS  
AND INCOMPLETE INFORMATION\***

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The paper deals with existence and properties of temporary migration equilibria with an overlapping generations structure in a two-country-world. Individuals are living for two periods. As a young individual they are supposed to be incompletely informed about wage rates and the quality of life they will encounter in both countries during the next period. They supply their labour force in each living period in either of the two countries such that total expected utility (of the wage rate and the quality of life) over the planning horizon is maximized.

In the first part of the paper we give sufficient conditions such that there exists a temporary migration equilibrium (in a given period) that is a tuple of migration flows between the two countries and a wage profile such that the labour markets in both countries are balanced simultaneously. In the second part of the paper some interesting properties of migration equilibria are analyzed. Particularly the effects of differing distributions of the quality of life in the two countries and of differing degrees of information of the individuals on migration equilibria are investigated. Furthermore it is shown that incomplete information alone suffices to induce migration flows even between "identical" countries.

## 1. INTRODUCTION

Our purpose in the present paper is to analyze the role of incomplete information in the theory of migration. This topic has been investigated extensively by many authors (e.g. McCall/McCall (1987), Berninghaus/Seifert-Vogt (1987), (1988), Meier (1985)), within a "partial-partial" equilibrium framework. In these models potential migrants were supposed to be incompletely informed about the quality of life resp. the distribution of wages in the foreign country. Furthermore complete information could be obtained only by moving into the foreign country. Therefore the theory of migration could be integrated rather easily into more general models in the theory of optimal sequential decision making under incomplete information. As main results there could be derived propositions concerning the optimal sequencing of moving from one region into another dependent on the information accumulated about the regions so far. As one of the most striking results it could be demonstrated that a risk-neutral potential migrant would prefer to move into a region that is characterized by "more uncertainty" in the variables he is completely informed about (see e.g. Berninghaus/Seifert-Vogt (1987)).

Obviously the validity of these "partial-partial" results has to be checked within a general equilibrium framework. For one has to be careful in the interpretation of results in migration decision making that have been obtained without considering the demand for migrants in each region. In the present paper we want to attack this problem by modeling the migration decision of households within a temporary equilibrium model with an overlapping generations structure and two countries. More precisely we suppose that households are living for two periods only and that in each period there is born a new generation. Households are supposed to decide in which country to supply their labour force in each period such that total expected utility is maximized. In the first period of their lives

households are supposed to be incompletely informed about the wage rates and the quality of life they are confronted with in the next period. They build expectations about these variables that are formalized by an exogenously given subjective probability distribution.

To avoid unnecessary complications we introduce the production sector in a rather rudimentary way. More precisely we suppose that the total labour demand function in each country can be derived from the marginal productivity function of a representative firm. As a main result we are able to establish conditions that guarantee the existence of a temporary equilibrium on the labour markets of the countries in a given period.

To elaborate the role of incomplete information in the theory of migration we are able to demonstrate that even in the case of two identical countries bilateral migration will take place if households are incompletely informed about the relevant data in the foreign country. This is in contrast to well known results in the theory of International Trade. Furthermore we will analyze the impact of increasing the degree of uncertainty in the (subjective) distributions over the relevant variables on the migration flows. We can show that more uncertainty over the relevant in the foreign country might increase the incentives for migration. As mentioned before this surprising result has been derived for the first time within a partial-partial equilibrium framework.

Even though migration theory had attracted a lot of economists to do research in this field, only a few papers are concerned with migration equilibria. The models by Galor (1986) and Kemp/Kondo (1986) utilize a similar framework (overlapping generations structure). But in contrast to our model households are supposed to be completely informed. The only motive for migration between countries is a difference in the households' time preference. Whereas in Galor/Stark (1987) migration is

traced back to technological differences between the countries. Furthermore in all the papers cited above steady state migration equilibria are analyzed. In this context one could also consider the models by Djajic/ () and () respectively. The authors consider guest worker migration equilibria with special regard to age and skill structure in the home and the host country and with respect to optimal consumption decisions over a given planning horizon. Unfortunately the generation structure of the whole model has not been made very explicit. It is not clear how different generations of migrants living at the same time will interact.

In the present paper we start from a detailed description of the individual migration decision under incomplete information and derive the aggregate migration and return migration flows between two countries (section 2).

Then we give sufficient conditions for the existence of a temporary migration equilibrium in a given period. That is, we demonstrate that there exists at least one constellation of migration and remigration flows between the countries and a wage profile for both countries such that supply and demand is balanced in each country's labour market separately (section 3).

In section 4 we will analyze economic implications of our temporary migration equilibrium model under some additional reasonable assumptions about the fundamentals of the two-country economy: First we will show how the distribution of the migration costs influences the equilibrium wage profile. Second we are able to show that an increase in uncertainty of the distribution over the relevant variables of a country, measured by an increase in the mean preserving spread of the distribution will induce an increase in the equilibrium migration flow into the country. Finally we are able to demonstrate that there exists equilibrium migration even though both countries are identical in a well defined sense.

In the last part of the paper we give some suggestions for extensions.

## 2. THE MODEL

### 2.1. THE GENERAL STRUCTURE

We consider a world economy consisting of two countries  $k = i, j$  with human labour force as the only mobile production factor. It is the goal of this paper to study the flows of migration and remigration between  $i$  and  $j$  and vice versa during the course of discrete time periods.

We suppose that there exists in each country  $k$  an overlapping generation structure of the population, where each individual lives only for two periods. At the beginning of each period  $t$ , ( $t = 1, 2, \dots$ ) there is a set  $L_{k,t}$  of new-born individuals. For technical convenience we assume

(A.1) Each set  $L_{k,t}$  is a closed interval in  $\mathbb{R}$  with Lebesgue measure  $\lambda_{k,t} = \lambda(L_{k,t})$

This seems to be an appropriate approximation for describing a mass phenomenon like migration. It should be remarked that instead we could choose  $L_{k,t}$  as any atomless measure space with suitably normed masses  $\lambda(L_{i,t})$  and  $\lambda(L_{j,t})$ .

At the beginning of his life each new-born individual  $l_i \in L_{i,t}$  has the options " $i_t$  : stay at home in country  $i$  in  $t$  and  $t+1$ " and " $j_t$  : migrate to  $j$  for at least one period". In the case that  $l_i$  has chosen the option  $j_t$ , then as an old agent in period  $t+1$  he/she can choose between " $i'_{t+1}$  : remigrate to  $i$ " and " $j'_{t+1}$  : stay at  $j$  for the rest of the life".

These options can be graphically illustrated as follows:

(A.2) A young individual  $l_i \in L_{i,t}$  of period  $t$  has the following options:



Figure 1

An analogous decision structure is assumed for  $l_j \in L_{j,t}$

Using the symbols

$$m_{i,t} := \lambda(\{\text{set of young individuals } l_i \in L_{i,t} \text{ choosing } j_t \text{ in period } t\}),$$

$$r_{i,t} := \lambda(\{\text{set of old individuals } l_i \in L_{i,t} \text{ having chosen } j_{t-1} \text{ in period } t-1 \text{ and } i'_t \text{ in period } t\})$$

$$z_{i,t} := \lambda(\text{set of individuals in country } i \text{ during period } t)$$

and with an analogous notation for  $j$ , we get the following counting equations

$$(1) \quad \begin{cases} z_{i,t} = \lambda_{i,t-1} - m_{i,t-1} + m_{j,t-1} + \lambda_{i,t} - m_{i,t} + m_{j,t} + r_{i,t} - r_{j,t} \\ z_{j,t} = \lambda_{j,t-1} - m_{j,t-1} + m_{i,t-1} + \lambda_{j,t} - m_{j,t} + m_{i,t} + r_{j,t} - r_{i,t} \end{cases}$$

Here it is implicitly assumed that the sets  $L$  of different periods and countries are disjoint.

Then we may define the set

$$(2) \quad Z_t := \{z_t = (z_{i,t}, z_{j,t}) \in \mathbb{R}_+^2 : z_{i,t} + z_{j,t} = \lambda_{i,t-1} + \lambda_{j,t-1} + \lambda_{i,t} + \lambda_{j,t}\}$$

Any  $z_t \in Z_t$  describes the actual "world" population size in period  $t$ .

Among other factors which will be given exogenously and described in more detail in subsection 2.2 the wage profiles  $w_t = (w_{i,t}, w_{j,t})$  will be the most important driving force for migration and remigration flows  $m_{k,t}$  and  $r_{k,t}$  in period  $t$ . Then we can imagine a supply relation for labour (domestic and abroad) of the form

Let us take it for granted at the moment that the supply of migrants in period  $t$  is functionally dependent on the wage profiles  $w_t$  and  $w_{t-1}$ . By taking account of the fact that all agents supply the same fixed amount of labour (which is here for the sake of simplicity normalized to 1) we can identify aggregate labour supply in a country in period  $t$  with the aggregate number of people living in this country in period  $t$ . Let us denote by  $S_{Rt}(\cdot)$  the labour supply function then we have

$$(3) \quad S_t : R_+^2 \times R_+^2 \rightarrow Z_t,$$

where  $S_t(w_{t-1}, w_t) = z_t$  resulting from the migration flows  $m_{kt-1}$  in  $t-1$ , induced by  $w_{t-1}$  and the migration flows  $m_{kt}$  and remigration flows  $r_{kt}$  in  $t$ , induced by  $w_{t-1}$  and  $w_t$ .

The exact micro-foundation of this supply function will be given in 2.2.

On the other hand we suppose that there exists an aggregate demand relation for labour in each country  $k$  in each period  $t$  of the following form<sup>1)</sup>:

(A.3) The aggregate labour demand in country  $k$  at time  $t$  is implicitly given by a continuous function

$$f_{kt} : Z_{kt} = [0, \lambda_{it-1} + \lambda_{jt-1} + \lambda_{it} + \lambda_{jt}] \rightarrow R_+$$

such that  $f_{kt}(z_{kt}) = w_{kt}$  denotes the wage rate that is required on the labour market for full employment.

Now we are prepared to give our definition of a migration equilibrium.

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1) We will give no micro-foundation of this relation here. At best it could be considered e.g. to be based on the marginal productivity theory of labour.

Definition 1: For a given wage profile  $w_{t-1}$  with induced size  $m_{kt-1}$  of migration flows for period  $t-1$  and for given  $\lambda_{kt}$ , a temporary migration equilibrium (TME) consists of a quadruple  $(m_{it}^*, m_{jt}^*, r_{it}^*, r_{jt}^*)$  of migration flows and a wage profile  $w_t^*$  such that

$$(4) \quad \left[ \begin{array}{l} S_{it}(w_{t-1}, w_t^*) = r_{it}^* + m_{jt-1} - r_{jt}^* + \lambda_{it} - m_{it}^* + m_{jt}^* + \\ \qquad \qquad \qquad + \lambda_{it-1} - m_{it-1} = z_{it}^* \\ S_{jt}(w_{t-1}, w_t^*) = r_{jt}^* + m_{it-1} - r_{it}^* + \lambda_{jt} - m_{jt}^* + m_{it}^* + \\ \qquad \qquad \qquad + \lambda_{jt-1} - m_{jt-1} = z_{jt}^* \end{array} \right.$$

and

$$(5) \quad f_{kt}(z_{kt}^*) = w_{kt}^* \quad (k = i, j)$$

holds.

In the definition given above we essentially require in an equilibrium that there exist wage rates  $(w_{it}^*, w_{jt}^*)$  such that so many migrants are attracted that can be employed at the prevailing wage rates. By equ. (4) it is expressed that the migration flows  $m_{kt}^*, r_{kt}^*$  are really results of optimal decision making while equ. (5) is a version of a labour market balancing condition.

An existence proof for equilibrium states will be given further below.

## 2.2. THE MIGRATION DECISION

Let us fix an individual  $l \in L_{it}$ . Firstly we will present a model of her/his migration decision between  $i_t$  and  $j_t$  in the framework of (A.2) (part A of this subsection). In part B, we will model the remigration decision between  $i'_t$  and  $j'_t$  of an individual  $l \in L_{i't-1}$  which has migrated in  $t-1$  from  $i$  to  $j$ . From A we then get a specification of the dependence of the number  $m_{it}$  on the wage profile  $w_t$ , and from B accordingly the specification concerning the dependence of  $r_{it}$  on  $w_t$  and  $w_{t-1}$ .

This two parts puzzled together will yield the specification of the supply relation for labour by an aggregation procedure, which will be outlined in part C.

For this sake we have to make some more assumptions<sup>2)</sup>

(A.4) a) In each period  $\tau=t$  and  $t+1$  of his life an individual  $l_1 \in L_{1t}$  receives a wage  $w_\tau$  and a quality of life  $x_\tau$ ; let  $X$  denote the set of all possible qualities of life, and assume  $X$  to be a subset of  $R^n$ .

b) All individuals  $l \in L_{1t}$  have the same preferences over the set  $(R_+ \times X)^2$  of all combinations  $(w, x, w', x')$  he may experience during his life which can be represented by an additive separable utility function  $u_{11}(w, x) + u_{12}(w', x')$ , where  $u_{11}$  resp.  $u_{12}$  is the per period utility of the young resp. old agent.

c) The endowments of the qualities of life, the individual  $l_1 \in L_{1t}$  may get during his lifetime are given by functions  $g_{1t}^\alpha : L_{1t} \rightarrow X^2$ , such that  $g_{1t}^\alpha(l_1) = (x_1; x_2)$  is the vector of qualities of life,  $l$  will receive in country  $i$  resp.  $j$  as a young agent ( $\alpha=1$ ) resp. as an old agent ( $\alpha=2$ ).

d) The individual  $l_1 \in L_{1t}$  is completely informed about the actual wage profile  $w_t$  and his endowment  $g_{1t}^1(l) \in X^2$ , but only incompletely informed about the future wage profile  $w_{t+1}$  and his future endowment  $g_{1t}^2(l_1) \in X^2$ . But he will become completely informed about  $w_{t+1}$  and  $g_{1t}^2(l_1)$  at the beginning of period  $(t+1)$ .

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2) For notational convenience this assumption is formulated only for an individual from country  $i$ , but the respective assumption for a  $j$ -individual is supposed to hold.

Remarks:

1. Assumption (A.4) is an attempt to capture the idea, that there are two main factors which influence an individual migration decision, namely: income and non-pecuniary return, which could be gained in either country. The first term is measured by the per period wage and the second one measured by a vector  $x$  of quality of life indices (like price level, situation at the working place, dwelling situation etc.). The component  $x_j$  of  $g_{jt}^1(l_1)$  can be supposed to contain "costs of migration".
2. By the mapping  $g_{jt}$  we formalize the assumption, that individuals only differ with respect to their vectors of quality of life, and that these characteristics are - at least on the individual level - determined before the birth of the respective individual.
3. Finally we mention a formal convention: For the individuals  $l_j \in L_{jt}$  the mappings  $g_{jt}^a : L_{jt} \rightarrow X^2$  assign combinations  $(x_j, x_i)$ , i.e. we assume throughout that the first component of the mappings  $g_{jt}^i$  resp.  $g_{jt}^j$  gives the endowment of the  $i$ - resp.  $j$ -individuals in their respective home countries  $i$  resp.  $j$  while the second part gives the endowment in their respective countries of destination.

A. The individual migration decision

We consider an individual  $l_1 \in L_{1t}$  confronted with the options  $i_t$  resp.  $j_t$ , according to (A.2). Given the information and preference structure as described by (A.3) it seems reasonable to assume that  $l$  will form some expectation at  $t$  concerning  $w_{t+1}$ ,  $g_{i_t+1}^2(l)$ , and that this expectation may depend on his information at  $t$ .

To make this assumption precise, we consider the set  $M(R_+^2 \times X^2)$ <sup>3)</sup> of all probability measures on  $R_+^2 \times X^2$ . The assumption about the formation of the expectations may be formulated:

(A.5) There is a function  $\phi_1 : R_+^2 \times X^2 \rightarrow M(R_+^2 \times X^2)$  such that the probability which  $l_1 \in L_{1t}$  assigns to the event " $(w_{t+1}, g_{1t}^2(l_1)) \in B$ " where  $B$  is any measurable subset of  $R_+^2 \times X^2$  is given by  $\phi_1(w_t, g_{1t}^1(l_1))(B)$ , if  $w_t$  is the actual wage profile.

Thus the migration decision problem for  $l_1 \in L_{1t}$  can be represented by the following decision tree, for given  $(w_t, x_t)$  with  $x_t = g_{1t}^1(l_1)$

Figure 2

Here we denote by  $\phi'_1(w_t, x_t)$  the marginal distribution of  $(w_{t+1}, x_{t+1})$  with resp. to  $\phi_1(w_t, x_t)$ . This specific migration decision problem under incomplete information has first been modeled and analyzed in McCall (1984); there the individuals are assumed to have an infinite planning horizon, and by an application of the Gittins index method the optimal migration decisions have been determined and characterized. For further

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3) Since  $X$  is assumed to be a subset of  $R^n$  we may suppose  $R_+^2 \times X^2$  to be endowed with the suitable Borel- $\sigma$ -field, and  $M(R_+^2 \times X^2)$  is the set of Borel-measures  $\Pi$  with  $\Pi(R_+^2 \times X^2) = 1$ ; (see e.g. Parthasarathy (1967), chap. II.6).

elaboration of this approach see Berninghaus/Seifert-Vogt (1987) and McCall/McCall (1987).

Next we will characterize the optimal migration decision for an  $l_i \in L_{it}$  in our two-period framework, using a simple argument from Dynamic Programming. We define a function

$$(6) \quad V_i : R_+^2 \times X^2 \rightarrow R \text{ by}$$

$$V_i(w_{t+1}, x_{t+1}) := \max\{u_{i2}(w_{it+1}, x_{it+1}), u_{i2}(w_{jt+1}, x_{jt+1})\}$$

and

$$(7) \quad E_i : R_+^2 \times X^2 \rightarrow R \text{ by } 4)$$

$$E_i(w_t, x_t) := \int V_i d\phi_i(w_t, x_t) - \int u_{i2} d\phi'_i(w_t, x_t) \\ + u_{i1}(w_{jt}, x_{jt}) - u_{i1}(w_{it}, x_{it})$$

Then we get

Proposition 1: Let (A.4), (A.5) and additionally the following technical assumptions hold:

(T.1)  $u_{i\alpha}$  is continuous and bounded on  $R_+ \times X$  ( $\alpha = 1, 2$ ).

(T.2)  $\phi_i$  is continuous w.r. to the topology of weak convergence on  $M(R_+^2 \times X^2)$ <sup>5)</sup>

Then

a) The function  $E_i$  is continuous

b)  $l_i \in L_{it}$  will migrate to  $j$  at  $t$ , i.e. will choose the option  $j_t$ , iff

$$(8) \quad E_i(w_t, g_{it}^1(l)) > 0$$

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4) Here the first integral is taken over  $R_+^2 \times X^2$ , and the second over  $R_+ \times X$ .

5) I.e.  $(w, x) \rightarrow \int h d\phi_i(w, x)$  is continuous for all bounded and continuous  $h : R_+^2 \times X^2 \rightarrow R$ ; (compare e.g. Parthasarathy, op.cit.).

Proof: b) is an immediate consequence of the definitions (6) and (7).

a): By (T.2) the functions  $u_{12}$  and  $V_1$  are continuous and bounded. Therefore it follows from (T.3), that  $(w_t, x_t) \rightarrow$

$\int V_1 d\phi_1(w_t, x_t)$  is continuous. Since  $\phi'_1$  is continuous, we may conclude that  $(w_t, x_t) \rightarrow \int u_{12} d\phi'_1(w_t, x_t)$  is continuous

too. - Thus  $E_1$  is continuous.

Q.E.D.

## B. The individual remigration decision

Now we consider an individual  $l_1 \in L_{1,t-1}$  living (and working) in country  $j$  in period  $t-1$ . At the beginning of period  $t$ , this individual has the options  $i't$  (remigration to  $i$ ) and  $j't$  (stay in  $j$ ), according to (A.2). Let us define the function  $D_1: R_+^2 \times X^2 \rightarrow R$  by

$$(9) \quad D_1(w_t, x_t) := u_{12}(w_{it}, x_{it}) - u_{12}(w_{jt}, x_{jt})$$

Then we can easily derive the following result.

Proposition 2: Let (A.5), and (T.1) hold, then

- a) The function  $D_1$  is continuous
- b) An individual  $l_1 \in L_{1,t-1}$ , which has migrated to  $j$  in  $t-1$ , will remigrate to  $i$  at  $t$ , i.e. will choose the option  $i't$ , iff

$$(10) \quad D_1(w_t, g_{1,t-1}^2) > 0$$

Proof: Obvious by inspection of figure 2 and the definition of  $D_1$ .

Q.E.D.



C. The aggregate migration and remigration decisions

For given wage profiles  $w_{t-1}$  and  $w_t$  the aggregate of the migration and remigration flows is depending on the form how the quality of life endowments are distributed among the individuals. This leads us to the distributions of the mappings

$$g_{k\tau} = (g_{k\tau}^1, g_{k\tau}^2) : L_{k\tau} \rightarrow X^2 \times X^2 =: X^4, \quad k = i, j, \quad \tau = t-1, t.$$

These are given in the usual way by<sup>6)</sup>

$$(11) \quad \mu_{k\tau} : B(X^4) \rightarrow [0, \lambda_{k\tau}], \quad \mu_{k\tau}(B) := \lambda(\{l \in L_{k\tau} : g_{k\tau}(l) \in B\})$$

By  $\mu_{k\tau}^\alpha$  we denote the marginal distribution of  $\mu_{k\tau}$  induced by  $g_{k\tau}^\alpha$  ( $\alpha = 1, 2$ ).

Using these distributions we may define

Definition 2:<sup>7)</sup> a) The migration function from i to j in period  $\tau$  is given by  $M_{i\tau} : R_+^2 \rightarrow [0, \lambda_{i\tau}]$  as follows

$$(12) \quad M_{i\tau}(w_\tau) := \mu_{i\tau}^1(\{x \in X^2 : E_i(w_\tau, x) > 0\}); \quad \tau = t-1, t$$

b) The remigration function from j to i in period t is given as  $R_{it} : R_+^2 \times R_+^2 \rightarrow [0, \lambda_{it-1}]$  with

$$(13) \quad R_{it}(w_{t-1}, w_t) := \mu_{it-1}(\{x, x' \in X^4 : E_i(w_{t-1}, x) > 0 \text{ and } D_i(w_t, x') > 0\})$$

Since  $E_k$  resp.  $D_k$  are continuous and  $g_{k\tau}$   $\lambda$ -measurable, these functions are well defined.  $M_{i\tau}(w_\tau)$  resp.  $R_{it}(w_{t-1}, w_t)$  is a measure of the size of the set of migrants from i to j resp. of remigrants from j to i in period  $\tau$  resp. t. This yields now the announced specification of the labour supply function.

Proposition 3: a) Let (A.1), ..., (A.5) and (T.1), ..., (T.3) hold. Then the labour supply function  $S_t : R_+^2 \times R_+^2 \rightarrow Z_t$  is given by

6) Here we suppose that the technical assumption "(T.3)  $g_{k\tau}$  is  $\lambda$ -measurable" holds.

7) For country j the functions  $M_{j\tau}$  and  $R_{j\tau}$  are defined analogously.

$$(14) \left\{ \begin{array}{l} S_{1t}(w_{t-1}, w_t) = \lambda_{1t-1} - M_{1t-1}(w_{t-1}) + M_{1t-1}(w_{t-1}) \\ \quad + \lambda_{1t} - M_{1t}(w_t) + M_{1t}(w_t) + R_1(w_{t-1}, w_t) - R_1(w_{t-1}, w_t) \\ S_{jt}(w_{t-1}, w_t) = \lambda_{jt-1} - M_{jt-1}(w_{t-1}) + M_{jt-1}(w_{t-1}) \\ \quad + \lambda_{jt} - M_{jt}(w_t) + M_{jt}(w_t) + R_j(w_{t-1}, w_t) - R_j(w_{t-1}, w_t) \end{array} \right.$$

b) If additionally we make the technical assumptions:

(T.4) The distribution  $\mu_{k\tau}$  of  $g_{k\tau}$  on  $X^4$  is absolutely continuous w.r. to the Lebesgue measure on  $B(X^4)$ <sup>8)</sup>

(T.5) For each  $w$  the sets  $\{x \in X^2 : E_k(w, x) = 0\}$  and  $\{x \in X^2 : D_k(w, x) = 0\}$  have empty interiors in  $X^2$ ,<sup>9)</sup> then  $S_{1t}(w_{t-1}, \cdot)$  and  $S_{jt}(w_{t-1}, \cdot)$  are continuous.

Proof: a) Obvious.

b) We must show b1)  $M_{k\tau}$  is continuous, b)  $R_{k\tau}$  is continuous; (in the following we omit the index k.)

b1) Let  $(w_n)$  be a sequence in  $R_+^2$ , converging to some  $w$ , and define  $B_n := \{x \in X^2 : E(w_n, x) > 0\}$  resp.  $B := \{x \in X^2 : E(w, x) > 0\}$ .

Then it must be shown:  $\mu_\tau(B_n) \rightarrow \mu_\tau(B)$  resp.<sup>10)</sup>  $\int_{B_n} 1 \, d\mu_\tau \rightarrow \int_B 1 \, d\mu_\tau$ .

We will apply the Dominated Convergence theorem to the sequence  $(1_{B_n})$ . It must be shown that  $(1_{B_n})$  converges to  $1_B$  for  $\mu_\tau$ -almost  $x \in X^2$ . Because of the continuity of  $E(\cdot, x)$  we know:  $E(w_n, x) > 0 \forall n \Rightarrow E(w, x) \geq 0$  and  $E(w_n, x) \leq 0 \forall n \Rightarrow E(w, x) \leq 0$ . Therefore we get as a consequence of (T.4) and (T.5) the  $\mu_\tau$ -almost-everywhere convergence of  $(1_{B_n})$  to  $1_B$ ; and thus the continuity of  $M_\tau$ .

b2) Let  $\{w_n\}$  be a sequence converging to some  $w$ . We define the sets  $A_n := \{(x, x') \in X^4 : D(w_n, x) > 0\}$  and  $B := \{(x, x') \in X^4 : E(w_{t-1}, x) > 0\}$ . Then we can write  $R_t(w_{t-1}, w_n) =$

8) For any subset  $B \in B(X^4)$  with Lebesgue measure zero we have  $\mu_{k\tau}(B) = 0$ .

9) This is e.g. fulfilled, if the equations  $E_k(\underline{w}, x_t) = 0$  resp.  $D_k(\underline{w}, x_{t-1}) = 0$  admit only countably many solutions  $x_t$  resp.  $x_{t-1}$  for any  $\underline{w}$ .

10)  $1_B$  is the indicator function of  $B$ .

$$= \mu_{t-1}(B \cap A_n) = \int_{A_n} 1_B \cdot 1_{A_n} d\mu_{t-1}, \text{ and for showing that}$$

$R_t(w_{t-1}, w_n)$  converges to  $R_t(w_{t-1}, w)$  it suffices to establish that  $(1_{A_n})$  converges to  $(1_A)$   $\mu_{t-1}$  - almost everywhere. - But this can be shown similarly as in b1) by making use of (T.5).

Q.E.D.

### 3. THE EXISTENCE OF A TME

In this part of the paper we will show that the assumptions made in 2. for our model of international migration plus one additional technical assumption about the demand relations  $f_k$  for labour according to (A.3) will suffice to establish the existence of a TME. This assumption on the demand relation will be (not surprisingly):

(T.6) The inverse demand for labour functions  $f_{kt}$  are continuous.

Now let us define a mapping:

$$(15) \quad H_t : R_+^2 \times Z_t \rightarrow Z_t \\ H_t(w_{t-1}, z_{1t}, z_{2t}) := S_t(w_{t-1}, (f_{1t}(z_{1t}), f_{2t}(z_{2t})))$$

If  $(z_{1t}^*, z_{2t}^*)$  is a fixed point of  $H_t(w_{t-1}, \cdot)$  then  $(M_{1t}(w_t^*), M_{2t}(w_t^*), R_{1t}(w_{t-1}, w_t^*), R_{2t}(w_{t-1}, w_t^*))$  with  $w_t^* = (f_{1t}(z_{1t}^*), f_{2t}(z_{2t}^*))$  and  $M_{kt}$  resp.  $R_{kt}$  given by (12) and (13) is a TME for given  $w_{t-1}$ . Thus we are led to

Theorem 1: If in the model of international migration as specified by (A.1), ..., (A.5) the technical assumptions (T.1), ..., (T.6) are fulfilled, then there exists a TME for each given  $w_{t-1}$ .

Proof: Since  $Z_t$  is convex and compact, and  $H_t(w_{t-1}, \cdot)$  is a continuous mapping from  $Z_t$  into  $Z_t$ , there exists a fixed point, and this generates a TME, as outlined above.

Q.E.D.

### Remark

For later purposes, not pursued in this paper, we should have the following more striking results:

1. Uniqueness of the TME for given  $w_{t-1}$
2. Continuity of the mapping  $(w_{t-1}, \mu_{1t}, \mu_{jt}, \phi_1, \phi_j) \rightarrow w_t^*$ , which assigns to different wage profiles of period  $t-1$  and different distributions of the quality of life endowments and different expectation functions the corresponding TME  $w_t^*$  of period  $t$ ; (here  $\mu_{kt}$  is the vector  $(\mu_{kt-1}, \mu_{kt}^{(1)})$ ).

For 1. of course we would need some monotonicity assumptions on the utility functions  $u_{ka}$  and the wage functions  $f_{kt}$ . Given this, we claim that 2. can be proven without additional assumptions. - Since we need not these results 1. and 2. in this paper, we will not outline these themes here.

## 4. ECONOMIC IMPLICATIONS

### 4.1. INTRODUCTORY REMARKS

In this section we will analyze properties of a TME which seem to be interesting from an economic point of view. We will deal with the following questions:

1. Consider countries  $i$  and  $j$  which are 'identical in the aggregate' except for the distributions  $\mu_i^1$ ; resp.  $\mu_j^1$ ; of the

young period endowments of the quality of life of the individuals  $l \in L_{it}$  resp.  $l \in L_{jt}$ . What can be said about the relation between  $w_{it}^*$  and  $w_{jt}^*$  in a TME, if the quality of life is 'more favorable for migration' for the  $i$ -individuals than for the  $j$ -individuals? (This problem will be analyzed in 4.2).

2. Again consider two countries  $i$  and  $j$  which are 'identical in the aggregate' but now except for the expectations  $\varphi_i$  resp.  $\varphi_j$ . How are the TME wages  $w_{it}^*$  and  $w_{jt}^*$  related, if the  $j$ -individuals seem to be less well informed than the  $i$ -individuals? (The analysis of this problem will be carried out in 4.3).

3. In the last subsection we analyze the concept of 'identity' of countries in more detail. Especially there will be shown, that in a TME the migration flows will be zero only in the case of static point expectations, i.e. where  $\varphi_i(w, x) = \delta_{(w, x)^{11}}$ , despite 'identity of the countries on the individual level'.

Now we will first define the concepts of identity we will use in the following subsections. Primarily let us consider the following relations.

$$(16) \quad w_{it-1} = w_{jt-1}$$

$$(17) \quad f_{it-1} = f_{jt-1} = f_{it} = f_{jt} =: f$$

$$(18) \quad \lambda_{it-1} = \lambda_{jt-1} = \lambda_{it} = \lambda_{jt} =: \lambda_0$$

$$(19) \quad u_{i\alpha} = u_{j\alpha} \quad (\alpha = 1, 2)$$

$$(20)^{12)} \quad a) \quad \varphi_{it-1} = \varphi_{jt-1} \quad b) \quad \varphi_{it} = \varphi_{jt}$$

---

11)  $\delta_z$  denotes the singular distribution with all its mass concentrated on  $z$ .

12) We admit here time dependent expectation functions. It can easily be checked, that the results of sections 2 and 3 could be carried out for this more general case too.

$$(21) \quad a) \quad \mu_{it-1} = \mu_{jt-1} \quad b) \quad \mu_{it} = \mu_{jt}$$

(22) The marginal distributions  $\mu_{it}^1$  resp.  $\mu_{jt}^1$  of  $\mu_{it}$  resp.  $\mu_{jt}$  and  $\mu_{it-1}^2$  resp.  $\mu_{jt-1}^2$  of  $\mu_{it-1}$  resp.  $\mu_{jt-1}$  have their mass concentrated on the diagonal  $\Delta = \{(x, x') \in X^2 : x = x'\}$  in  $X^2$

Definition 3: We will say:

- a) 'The countries  $i$  and  $j$  are identical in the aggregate in period  $t$ ', if the relations (16), ..., (21) hold.
- b) 'The countries  $i$  and  $j$  are identical on the individual level in period  $t$ ', if the relations (16), ..., (22) hold.

We need these concepts, introduced above, for the comparative static analysis we want to make in the following subsections. The idea of our procedure can roughly be described as follows: We consider two countries that are identical with respect to all characteristics but one. Then we isolate the effect of a variation of this particular characteristic on the equilibrium values of wages and migration flows by assuming that the two countries differ just with respect to this particular characteristic. Throughout this section 4 we assume that (A.1), ..., (A.5), (T.1), ..., (T.6) hold and furthermore will make two technical assumptions concerning the labour demand functions and the utility functions.

(T.7) The inverse demand functions for labour  $f_{kt}$  are strictly decreasing.

(T.8) The utility functions  $u_{k\alpha}$  are strictly increasing in each of their arguments  $w$  and  $x$ .

These rather mild assumptions will turn out to be crucial for the intended analysis.

#### 4.2. THE IMPACT OF DIFFERENT QUALITY OF LIFE DISTRIBUTION FOR THE YOUNG AGENTS ON THE TME WAGE PROFILE

In this subsection we will consider a situation where the two countries  $i$  and  $j$  are identical in the aggregate except for (21)b). With this exception we will make an attempt to capture the idea that the quality of life is 'more favorable for migration' of the  $i$ -individuals than of the  $j$ -individuals. Loosely speaking this will be the case if the first component (= the home country component) of the functions  $g_i^1$  tend to take lower values in  $X$  (with respect to the usual vector ordering in  $R^n$ ) than the corresponding parts of  $g_j^1$ , and if the second component (= the guest country component) of  $g_i^1$  are taking tendentially higher values than the corresponding components of  $g_j^1$ . To make things not too complicated we will assume now

(T.9)  $X \subset R$ , i.e. the quality of life in each country is measurable by only one real number.

(T.10) The qualities of life for the young individuals in period  $t$  in the two countries are "independently" distributed, i.e. there are measures  $\nu_{ki}$  and  $\nu_{kj}$  on  $B(X)$ , such that for any  $B, B' \in B(X)$  the relation  $\mu_{kt}^1(B \times B') = \nu_{ki}(B)\nu_{kj}(B')$  holds

In the case of (T.9) we denote the distribution functions of  $\nu_{ki}$  resp.  $\nu_{kj}$  by  $F_{ki}$  resp.  $F_{kj}$ .

Then we can define

Definition 4: The quality of life distribution for the young individuals of period  $t$  is more favorable for migration of the individuals in  $i$  than of those in  $j$ , iff  $F_{jj}$  dominates  $F_{ii}$  and  $F_{ij}$  dominates  $F_{ji}$  w.r. to the first-order dominance, i.e. iff  $F_{jj}(x) \leq F_{ii}(x)$  for all  $x \in X$ , with strict inequality for those  $x$  where  $F_{jj}(x) < \lambda_0 = \lambda(L_{kt})$  and  $F_{ij}(x) \leq F_{ji}(x)$

for all  $x \in X$ , with strict inequality for those  $x$  where  $F_{ij}(x) < \lambda_0$ .

Now we can give an answer to the first question in 4.1:

Theorem 2: Let in the model of international migration as specified by the assumptions (A.1), ..., (A.5) the technical assumptions (T.1), ..., (T.10) be fulfilled, and let the countries  $i$  and  $j$  be identical in the aggregate with the only exception that the quality of life distribution for the young individuals of period  $t$  is more favorable for migration of the individuals in  $i$  than of those in  $j$ . Furthermore let the following monotonicity assumption

$$(Mon) \quad \left\{ \begin{array}{l} E_k(w, w', x, x') \text{ is strictly increasing in } w' \text{ and } x' \\ \text{and strictly decreasing in } w \text{ and } x, \quad (k = i, j)^{13} \end{array} \right.$$

be fulfilled. Then for any TME, given  $w_{t-1}$ , we have for the corresponding wage profile  $w_t^*$

$$(23) \quad w_{jt}^* < w_{it}^*$$

Obviously the assumption (Mon) imposes some restrictions concerning the individuals' expectation functions. As an example where (Mon) holds consider the following specification for  $i$ , and similarly for  $j$ :

$$(24) \quad \varphi_i \text{ does not depend on } x$$

$$(25) \quad \text{There are functions } \varphi_{ik}: R_+ \rightarrow M(R_+) \quad (k = i, j) \text{ and a measure } \Pi_i \in M(X^2), \text{ such that}$$

$$\varphi_i(w, w')(B \times B' \times C) = \varphi_{ii}(w)(B) \varphi_{ij}(w')(B') \Pi_i(C)$$

$$\text{for all } B, B' \in B(R_+), C \in B(X^2)$$

---

13) For  $E_k(\cdot)$  we use the convention that the first component of  $(w, w')$  denotes the wage in the home country  $k$  and the second one denotes the wage in the foreign country (the second convention has already been made for  $(x, x')$ ).



- (26)  $\varphi_{ik}$  is "increasing" in the following sense:  
 $w > w'$  implies:  $\varphi_{ik}(w)([0,y]) \leq \varphi_{ik}(w')([0,y])$  for all  $y \in \mathbb{R}$  with strict inequality for these  $y$  where  $\varphi_{ik}(w)([0,y]) < 1$ ; ( $k=i,j$ ); (in other words: if  $\varphi_{ik}(w)$  is better than  $\varphi_{ij}(w')$  in the sense of first-degree stochastic dominance.

The proof of Theorem 2 is prepared by some lemmata:

Lemma 1: Given (T.1), ..., (T.8) and (16), ..., (21), then

$$(27) M_{it-1}(w_{t-1}) = M_{jt-1}(w_{t-1}), \text{ and}$$

$$(28) R_{it}(w_{t-1}, w, w') = R_{jt}(w_{t-1}, w', w) \quad \forall w, w' \in \mathbb{R}_+$$

Lemma 2: Given (T.1), ..., (T.6) and (T.8), then:

$R_{it}(w_{t-1}, w_i, w_j)$  is increasing in  $w_i$  and decreasing in  $w_j$ , for any  $w_{t-1}$ .

Lemma 3: Given (T.1), ..., (T.8) and (16), ..., (18), then for any

TME  $w_t^*$  we have:

If  $w_{it}^* = w_{jt}^*$  (rsp.  $w_{it}^* < w_{jt}^*$ ), then:

$M_{it}(w_t^*) =$  (rsp.  $<$ )  $M_{jt}(w_t^*)$  and similarly

$R_{it}(w_{t-1}, w_t^*) =$  (rsp.  $\leq$ )  $R_{jt}(w_{t-1}, w_t^*)$ .

(The proofs of these lemmata follow easily from the definitions. They are available by request from the authors.)

Proof of the Theorem 2:

a) First we assume that for the TME  $w^* := w_t^*$  we have the equality of wages:  $w_i^* = w_j^*$ . Because of (Mon) there is a strictly increasing function  $e: X \rightarrow X$ , depending on  $w^*$ , such that  $E_i(w^*, x, e(x)) = 0$  for all  $x \in X$ , and  $E_i(w^*, x, x') > 0$  if and only if  $x' > e(x)$ . We define  $A := \{(x, x') \in X^2 : x' > e(x)\}$ , and will show that

$$(29) \quad \mu_{it}(A) > \mu_{jt}(A).$$

$$\begin{aligned} \text{For we can write } \mu_{jt}(A) &= \int v_{jj}(\{x' \in X: x > e(x')\}) dv_{ji}(x) = \\ &= \int F_{jj}(e^{-1}(x)) dF_{ji}(x) < \int F_{ii}(e^{-1}(x)) dF_{ji}(x) \\ &< \int F_{ii}(e^{-1}(x)) dF_{ij}(x) = \int v_{ii}(\{x' \in X: x > e(x')\}) dv_{ij}(x) = \mu_{it}(A); \end{aligned}$$

where the integrals are taken over  $X$  and the two inequalities follow from the dominance relations in Def. 4.

- Since we have just by definition  $M_{it}(w^*) = \mu_{it}^1(A)$  and  $M_{jt}(w^*) = \mu_{jt}^1(A)$ , from (29) there follows a contradiction to the assumption  $w_i^* = w_j^*$  (because of Lemma 3).

b) Next let us assume for the TME  $w^* = w_t^*$  the relation  $w_i^* < w_j^*$ . Then by (Mon) we get for all  $x \in X^2$ :  $E_i(w_i^*, w_j^*, x) > E_i(w_j^*, w_j^*, x) = E_j(w_j^*, w_j^*, x) > E_j(w_j^*, w_i^*, x)$ . From this we can easily conclude:  $M_{it}(w_i^*, w_j^*) \geq M_{it}(w_j^*, w_j^*) > M_{jt}(w_j^*, w_j^*) \geq M_{jt}(w_j^*, w_i^*)$ , where the inner inequality follows from (29). Thus the assumption  $w_i^* < w_j^*$  leads again to a contradiction to the result of Lemma 3. Taking the steps a) and b) together yields the desired result.

Q.E.D.

The economic interpretation of this result is straightforward. If the incentives for young agents in  $i$  to migrate would be higher than the incentives for the young agents in  $j$  we would expect the wage rate in country  $i$  going up relative to the wage rate in county  $j$  as labour force has become more scarce in  $i$ . But the increasing wage rate in  $i$  on the other hand attracts more remigrants which has a dampening effect on the wage rate. As the result shows there still remains in equilibrium a positive net effect on  $w_i$  caused by the young migrants.

An interesting interpretation of the result would be obtained if we regard  $x_{jt}$  as migration costs for young agents in  $i$ . In this case we could infer directly from theorem 2 that a "more favourable" distribution of migration costs in country  $i$  induces a wage increase.

#### 4.3. THE IMPACT OF DIFFERENT EXPECTATIONS ABOUT THE QUALITY OF LIFE OR THE WAGE RATE IN THE GUEST COUNTRY ON THE TIME WAGE PROFILE

The purpose of this section is to make precise our statement in the introduction that expectation functions with more uncertainty (= larger mean preserving spread) induce a larger migration flow. It has been argued elsewhere (see Berninghaus/Seifert-Vogt (1987)) that more uncertainty in the expectation functions sometimes can be interpreted as "less information". In the decision theoretical literature on migration it has been shown (e.g. McCall/McCall (1984)) that less informed individuals are more willing to migrate. We will establish rigorous conditions below that imply the validity of this result in a temporary equilibrium framework (Theorem 3). Now let us at first consider the case where the countries  $i$  and  $j$  are again identical in the aggregate, but now with an exception concerning the expectations in period  $t$  for period  $t+1$ , i.e. we suppose now (16), ..., (20a), (21), and instead of (20b) we assume:

$$(30) \quad \varphi_{i,t}(w, x) = \delta_{(w, x)}$$

i.e. the  $i$ -individuals expect the same wage profile and qualities of life in  $t+1$  as in the previous period.

$$(31a) \quad \varphi_{j,t}(w, x, x') = \delta_{(w, x)} \times \Psi(x'), \text{ or}$$

$$(31b) \quad \varphi_{j,t}(w, w', x) = \delta_{(w, x)} \times \Pi(w')$$

where  $\Psi(x')$  resp.  $\Pi(w')$  are mean preserving spreads of  $\delta_{x'}$  resp.  $\delta_{w'}$ ; i.e. the  $j$ -individuals are only uncertain either about the quality of life or about the wage rate in  $t+1$  in the guest country.

The specifications (30a) presupposes the following specialization of (T.9).

(T.9)' X is a (bounded) interval in R

Furthermore we must make an assumption concerning the risk behavior of the j-individuals. The following will turn out to be sufficient

- (T.11)<sup>14)</sup> a)  $u_{j2}$  is convex with respect to x  
b)  $u_{j2}$  is convex with respect to w

Then we may assert the following Theorem, which gives us an answer to the second question posed in 4.1.

Theorem 3: Let in the model of international migration as specified by (A.1), ..., (A.5) the technical assumptions (T.1), ..., (T.9') and the specifications (16), ..., (19), (21), (21') and (30) be fulfilled.

a) If we assume (T.11)a) and (31a), then in any TME  $w_t^*$  for given  $w_{t-1}$  the relation

$$(32) \quad w_{it}^* \leq w_{jt}^*$$

must hold.

b) If we assume (T.11)b) and (31b), then (32) is the only possible relation for a TME  $w_t^*$  too.

Proof of a):

i) Let us first prove

$$(33) \quad M_{it}(w, w) \leq M_{jt}(w, w) \text{ for any } w \in R_+$$

The functions  $E_i(w, w, x)$  and  $E_j(w, w, x)$  differ only in the terms

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14) In Seifert-Vogt (1988) the implications of different assumptions about the risk behavior of migrants in the framework of the migration decision problem as outlined in Figure 2 are analyzed.

$$V_i(w, w, x) \text{ resp. } I = \int V_j(w, w, x, \cdot) d\Psi(x'), \text{ for any } x = (x, x') \in X^2.$$

Since  $V_j(w, w, x, \cdot)$  is convex and  $\Psi(x')$  is a mean preserving spread of  $\delta_x$ , we get:  $V_i(w, w, x, x') < I$ . And this yields

$$(34) \quad E_i(w, w, x, x') < E_j(w, w, x, x')$$

Now (33) is an obvious consequence of (34).

ii) Next we will establish:

$$(35) \quad M_{it}(w', w) \leq M_{it}(w, w) \leq M_{it}(w, w') \text{ for } w < w'$$

But this follows, since  $E_i(w, w', x)$  is increasing in  $w'$  and decreasing in  $w$ .

iii) Since the same holds for  $E_j$  and therefore for  $M_{jt}$  we reach the conclusion, by (33) and (35):

$$(36) \quad M_{it}(w', w) \leq M_{jt}(w, w') \text{ for } w < w'$$

iv) We may apply Lemma 1 and Lemma 3, and obtain that in the case where for the TME  $w_t$  the relation  $w_{it} > w_{jt}$  would hold, we would have  $M_{it}(w_t) > M_{jt}(w_t)$ . But this contradicts (33) resp. (36); therefore only (32) is possible.

The proof of part b) runs completely analogous.

Q.E.D.

From the above proof we can easily derive the following Corollary

Corollary 1: Let  $\mu_{it}^1 = \mu_{jt}^1$  be strictly isotonic (i.w. if  $B \neq B'$  for  $B, B' \in B(X^2)$ , then  $\mu_{it}^1(B) < \mu_{it}^1(B')$ ). Then we obtain instead of (32) the relation

$$(32') \quad w_{it}^* < w_{jt}^*$$

Finally we can summarize the arguments of the proof of Theorem 3 and the previous Corollary to obtain

Corollary 2: If there is uncertainty in both countries but if the uncertainty in  $i$  is smaller than in  $j$ , in the sense that the expectation in  $j$  has a larger mean preserving spread than the expectation in  $i$ , and if  $\mu_t^i$  is strictly isotonic, then in any TME  $w_t^*$  the relation (32') must hold.

Corollary 2 contains a precise version of the statement that "more uncertainty" in the expectation functions of the agents in a country generates a larger migration stream off from this country. The "partial-partial" equilibrium result obviously can be shown to be true even in an equilibrium framework. As increasing streams of young migrants raise the wage rate of the country we expect (32') to be true. But a same reasoning as in the previous subsection shows that the validity of this result does not follow from a straightforward translation of the decision theoretical result to an equilibrium framework. For a rising wage rate in one country attracts a larger number of return migrants that might offset the wage increase.

#### 4.4. THE MIGRATION FLOWS IN A TME FOR IDENTICAL COUNTRIES

In the present section our differentiation of the "identical country" assumption (see (16)-(22)) in "identity of the aggregate" and in "identity on the individual level" proves to be useful. Indeed the identity concept between countries is not trivial if the individuals in each country are subject to random shocks in their characteristics.

The results in our theorems 4 and 5 below make precise our statement, given in the introduction, that even in case of "identical countries" there might be (symmetrical) migration flows from one country into the other one. As it is implied by

Theorem 5 we need some further restrictions to show that the migration flows are really non-empty. One can easily prove

Theorem 4: If the assumptions (A.1), ..., (A.5) together with the technical assumptions (T.1), ..., (T.8) hold and if the two countries i and j are identical in the aggregate in period t, then for the TME  $w_t^*$  the equality of wages  $w_{it}^* = w_{jt}^*$  and therefore the equality of migration flows  $M_{it}(w_t^*) = M_{jt}(w_t^*)$  and remigration flows  $R_{it}(w_{t-1}, w_t^*) = R_{jt}(w_{t-1}, w_t^*)$  must hold.

Proof: Follows by Lemma 1 and 3.

From theorem 4 we can obviously only infer that the migration and remigration streams from i to j and from j to i are identical but we cannot guarantee that there takes place non-trivial migration and remigration at all. To investigate this problem further we will prove a result in the following theorem from which conditions can be derived that guarantee non-trivial migration even for "identical" countries. For this sake we utilize the strongest possible identity assumption between countries, that is "identity between i and j on the individual level". To get non-trivial results about  $M_{it}(w_t)$  and  $M_{jt}(w_t)$  in this case, we must drop (T.4), because with (T.4) we would always have  $M_{it}(w_t) = M_{jt}(w_t)$  for all  $w_t$ , since the Diagonal  $\Delta(x^2)$  has Lebesgue measure zero. But if we cancel (T.4) the existence of a TME can not be guaranteed any longer. Therefore the following discussion is based on the assumption

(T.12) For  $w_{t-1}$  there exists a TME  $w_t^*$  in period t

Then we can show that even 'identity in the individual level' alone is not sufficient to guarantee that no migration will take place in period t; more precisely:

Theorem 5: Let in the model (A.1), ..., (A.5) with (T.7), (T.8), (T.9'), (T.11) and (T.12) the countries i and j be identical on the individual level in period t. Then in addition to the conclusion of Theorem 4 the following holds

a) If  $\varphi_i(w, x, x') = \varphi_j(w, x, x') = \delta_{(w, x)} \times \Psi(x')$  for all  $(w, x, x')$  then in the TMW  $w_t^*$  the migration flows  $M_{it}(w_t^*)$  and  $M_{jt}(w_t^*)$  are both zero if and only if the set  $N := \{x \in X: \Psi(x) \text{ is a MPS of } \delta_x\}^{15}$  has measure zero with respect to the marginal distribution of  $v := \mu_i^1 = \mu_j^1$  for the second factor.

b) If  $\varphi_i(w_{it}, w_{jt}, x) = \delta_{(w_{it}, x)} \times \Pi(w_{jt})$  and  $\varphi_j(w_{jt}, w_{it}, x) = \delta_{(w_{jt}, x)} \times \Pi(w_{it})$  for the TME  $w_t^* = (w_{it}^*, w_{jt}^*)$ , then the migration flows  $M_{it}(w_t^*)$  and  $M_{jt}(w_t^*)$  are both zero if and only if  $\Pi(w)$  is no MPS of  $\delta_w^*$  for  $w^* = w_{it}^* = w_{jt}^*$ .

Proof: a)  $M_{it}(w_t^*) = 0$  iff  $v(\{(x, x) \in \Delta(X^2): E_i(w, w, x, x) > 0\}) = 0$ . Since on the other hand  $x \in N$  iff  $E_i(w, w, x, x) > 0$ , we have established the first assertion.

b) can be proven similar.

Q.E.D.

Concerning the remigrating TME flows we remark finally: Even if the migrants of period  $t-1$  had no correct foresight, then there will be no remigratory flows in the TME  $w_t^*$  if the countries are identical on the individual level.

## 5. CONCLUDING REMARKS

It was the aim of this paper to present a temporary equilibrium model of international migration in which incomplete information of the migrants is the focal point. Building on a theory of

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15) MPS is the usual abbreviation for Mean Preserving Spread.



migration decisions under incomplete information by McCall (1984), we have constructed such a TME-model and given a series of conditions which guarantee its consistency in the sense of existence of a TME; (in sect. 2 and 3). Finally in section 4 first modest steps have been undertaken, to exploit the model for answering some questions of economic interest.

For future research within the framework of this TME model the analysis of a dynamic process of temporary migration equilibria under different assumptions about the economy is planned, especially concerning

- a) the expectation functions,
- b) the distributions of qualities of life.

With respect to a) we intend to endogenize the expectation functions  $\Psi_i(\cdot)$  and ask whether there exist learning processes of the individuals over the generations such that the individual expectations will coincide with the actual distributions of equilibrium wage rates and the individuals' quality of life. - Concerning b) a central point of interest will be the study of some kind of "Markov dependence" of the sequence of distributions  $\dots, \mu_{k,t-1}, \mu_{k,t}, \dots$  such that e.g.  $\mu_{k,t}^2$  depends on the size of migrants  $M_{k,t}$ . Such a specification would be intended to capture the idea, that the quality of life e.g. for migrants from  $i$  in  $j$  would be better (to some extent), if there are many other individuals from  $i$  in  $j$ .

We are convinced that the model as presented in this paper will enable us to study the mentioned points in a rigorous manner. Finally it should be remarked, that in principle the 'partial framework' of our equilibrium approach can be generalized, if we specify the production possibilities in both countries, and regard the migrants as consumers of (a part of) the production. But it is not yet clear to us whether this would not lead us too

far away from our central theme, namely the analysis of the role of incomplete information on international migratory movements.

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