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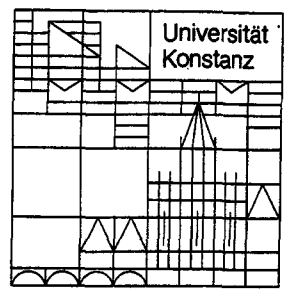
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Hans Peter Grüner

**Monetary Target Announcements,  
Reputation and Hysteresis**

# **Monetary Target Announcements, Reputation and Hysteresis\***

**Hans Peter Grüner\*\***

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## **ABSTRACT**

Two signaling games of monetary policy are considered: game one examines the effect of hysteresis on the labor market on the results of the repeated monetary policy game. Disciplinary effects of reputation disappear in presence of hysteresis. The second game compares welfare effects of monetary target announcements to those of a rigid rule under alternative assumptions on the policymaker's type space.

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## 1. Introduction

If decision makers in a central bank are interested in a high employment level, they will have, following Barro and Gordon (1983), an incentive to create surprise-inflation. If nominal wages are fixed for a certain period of time, surprise inflation lowers the real wage and employment increases if the economy is in a situation of classical unemployment. In their well-known analysis, Barro and Gordon show that such attempts to stimulate employment are defeated if the public rationally anticipates such a policy. In this case the public raises inflation expectations up to the point where the marginal loss of higher inflation equals the marginal gain from employment stimulation. The emerging situation is therefore one of inflation and unemployment. If inflation and unemployment are costly for society, then society will prefer the second-best outcome without inflation and with the given unemployment level. This, however, is only possible if the central bank can credibly precommit not to inflate after nominal contracts are made, i.e. if inflation is made more costly or if the incentives to surprise are removed. From this point of departure a vast array of literature has developed in the last ten years, dealing with the ways such precommitment can be made possible. Backus and Driffill (1985) argue that it can arise naturally from a repeated game: if there is a positive probability that the central banker is "conservative", i.e. that he is only interested in inflation, not employment, and if the public learns the type of central banker through Bayesian updating, then there is an incentive for the

"weak" policymaker (the one who is interested in the employment target) to mimic the conservative for a number of periods and to gain counterinflation-reputation. Vickers (1989), however, shows that this result can only be obtained with a rather unrealistic restriction on the two central banker's action space: if the central banker can freely choose among inflation rates, a separating equilibrium prevails, i.e. an equilibrium where the types are revealed when they choose inflation. Therefore, there is little hope that reputation and Bayesian learning have a disciplinary effect on weak policymakers. Another very simple solution would of course be to appoint someone with no interest in the employment target as the central banker. Rogoff (1985) shows that this is suboptimal if economic shocks necessitate discretionary policy. He proposes to choose an intermediate type who is less interested in employment than society. Another possibility to gain credibility instantaneously is to fix the exchange rate with a country that has already gained reputation for low inflation rates. This argument has been put forward by Mélitz (1988), Giavazzo and Pagano (1989) and Giavazzo and Giovannini (1989, Chapter 5). Giavazzi and Giovannini argue that this might be less costly in terms of employment than appointing a new (conservative) central banker, because during the process of the creation of anti-inflation reputation inflation is overestimated by the public.

The present paper extends the existing literature on the problem of the time-inconsistency of optimal plans for monetary policy into two directions:

In a first game in Section 2, the "natural" solution to the time-inconsistency problem, proposed by Backus and Driffill, is reexamined under the assumption that there is hysteresis in unemployment. Backus and Driffill assume that the incentives to create surprise inflation do not change over time. As we know from the work on hysteresis in unemployment [see for example Blanchard and Summers (1988), Grüner (1993)] this is not true for at least some European countries. Unemployment - a major incentive to create surprise inflation - is persistent. Therefore, if surprise inflation has employment effects today, it will also have employment effects tomorrow. I will examine a simplified two-period version of the Backus and Driffill model in Section 2. We will see that under the assumption of hysteresis, the type of the policymaker is revealed when he chooses policy in the first period. Thus, there is little hope for countries with hysteresis that the possibility of gaining reputation has a disciplinary effect on a weak central banker.

The rest of the paper then examines an alternative solution concept for the time-inconsistency problem: monetary target announcements. Such target announcements have been introduced in Germany, France, the US and Great Britain in the mid-seventies. Their treatment in the literature is, however, rather sparse. There exists extensive literature on the question of how a policymaker with given preferences may act in trying to convey a certain impression about these preferences [Backus and Driffill (1985), Vickers (1989)]. However, in those models, it is policy itself which serves as the signal.



In the second model of the paper, we introduce the possibility for two potential types of policymakers to make announcements about the course of their future monetary policy. The model then studies the effect of these announcements on the outcome of the monetary-policy-game between the policymaker and the public. Thus, the signal consists of an announcement about the money growth rate (or the inflation rate) and not of monetary policy itself. We will examine the effects of the institution of announcements on welfare. The only existing work on this subject is Cukierman and Liviathan (1991). Cukierman and Liviathan assume in their model that there is one policymaker who is not able to choose an inflation rate that is different from the announced rate. They call this type of policymaker "dependable". The second policymaker can cheat at no cost. This property is not known by the public. The relative interest of the potential types of policymakers in the employment and the inflation target, however, is identical and common knowledge. We here examine another interesting case: the degree of dependability is known but preferences on unemployment and inflation are unknown. We assume that deviations of policy from policy announcements are not infinitely costly for one type, but instead increasingly costly for all the potential policymakers. There are at least two reasons for this:

- 1) If the monetary aggregate is imperfectly controllable, the justification of deviations from announcements through the control error becomes less credible as the deviation of policy from the announced target becomes larger. The probability that

the policymaker is dependable in the sense of Cukierman and Liviathan (1991) decreases. This causes costs for at least two reasons: firstly, if preferences change over time, the policymaker's ability to communicate new preferences in future periods decreases<sup>1)</sup>. Secondly, the public image of the policymaker deteriorates if the public likes "dependable" or honest policymakers.

2) Rising costs of deviations of policy from self-imposed target announcements could also be imposed by legislation. This is an alternative to government imposition of financial incentives to stick to rigid rules.

The paper therefore also examines the effects of a special kind of incentive-mechanism on the outcome of the monetary policy game and adds a welfare analysis to the seminal work of Cukierman and Liviathan. This analysis is important because the deviations of monetary policy from target announcements have recurrently provoked a call for the abolition of monetary targeting. The present model offers a theoretical framework for the examination of such a policy.

I develop the basic signaling game with announcement in Section 3. Equilibria are examined in Section 4 and I derive my welfare statements in Section 5. In Section 6 the robustness of the results with respect to the assumptions on the type space of the policymaker is examined. Section 7 presents the conclusion of my results.

## 2. Reputation and Hysteresis

A natural solution to the time inconsistency problem is that a weak policymaker is interested in gaining a reputation for being tough. In a repeated monetary policy-game, Backus and Driffill (1985) show that the weak policymaker chooses a mixed strategy which will reveal his type with a certain probability. Vickers (1986), however, shows that such pooling situations disappear in a game with a larger action-space: the hard-nosed policymaker can choose a particularly low inflation rate and thus distinguishes himself from the weak one. The repeated game in this section reexamines this problem by considering hysteresis in the labour markets. We maintain Backus and Driffill's assumption that HN always chooses zero inflation. Hysteresis means that a change in unemployment has a tendency to persist over several periods; in the extreme case [Blanchard and Summers (1986)], unemployment follows a random walk if there is no unexpected policy intervention. With hysteresis, the unemployment rate becomes a state variable in the repeated game. If the policymaker can reduce unemployment today through surprise inflation, then this will reduce unemployment tomorrow. The introduction of hysteresis into the repeated game is an important extension since hysteresis is a common problem of the European countries [see Blanchard and Summers (1986) and Grüner (1993b)]. We will see that it substantially changes results: under hysteresis, it becomes uninteresting for the weak policymaker to mimic the strong policymaker.

## The Game

The game is a simple signaling game where the uninformed agent (the public) first chooses inflation expectations before the informed agent (the central bank) chooses actual inflation. The game is repeated once, time is divided in two periods, 1 and 2. Utility of the central banker is

$$Z(\pi_1, \pi_2, \pi_1^e, \pi_2^e) = -\sum_{t=1}^2 \delta^{t-1} \left[ \frac{a}{2} \pi_t^2 + \frac{b}{2} (\pi_t - \pi_t^e - u_t^B)^2 \right]$$

where  $\delta$  is a discount factor  $\pi$  and  $\pi^e$  denote actual and expected inflation. The public does not know whether the policymaker in office is a weak ( $b=\bar{b}>0$ ) or a strong one ( $b=0$ ). The second term is the loss from current unemployment,  $u_t$ . Current unemployment is lower than the reference value  $u_t^B$  if there is surprise inflation in period  $t$ . I examine the game under two different assumptions on  $u_t^B$ :

### Assumption 1

There is full hysteresis, i.e.  $u_t^B = u_1 = u_1^B + \pi^e - \pi$ .

### Assumption 2

There is no hysteresis and  $u_t^B = u_1^B$ .

$u_1^B$  is exogenously given. The initial probability for a policymaker to be hard-nosed in period  $t$  is denoted by  $q_{t-1}$ .  $q_0$  is given. The hard-nosed policymaker chooses the zero inflation rate in both periods. I assume that the public intends to minimize the sum of both period's expected expectation-error. The public

therefore tries to choose unbiased expectations. A perfect Bayesian equilibrium in pure strategies consists of

- (i) beliefs  $q^l(\pi^l)$  and
- (ii) strategies  $\pi^W_1, \pi^{HN}_1, \pi^W_2, \pi^{HN}_2, \pi^c_1, \pi^c_2$

that are consistent, i.e.

- (i)  $q^l(\pi^l)$  is the conditional probability for a HN if  $\pi^l$  is chosen and
- (ii) the strategies form a subgame perfect Nash equilibrium.

Chances for a pooling equilibrium to prevail are best if utility is not discounted. For simplicity I therefore assume:

### **Assumption 3**

$\delta=1$ .

Let us now examine the existence of pooling equilibria under the different assumptions about the unemployment process. Under hysteresis, repeating the Barro Gordon game has no disciplinary effect on the weak policymakers actions; we obtain:

### **Proposition 1**

Under Assumptions 1 and 3, the game has no pooling equilibria.

### **Proof 1**

I first deduce the necessary and sufficient conditions for the existence of a pooling equilibrium.

The only candidate for a pooling equilibrium is  $\pi^{\bar{W}1} = \pi^{HN1} = \pi^{e1} = 0$ . The best enforcement is that  $\mu(\pi_1) = 0$  for all inflation rates different from zero. I consider the case of zero inflation and  $u_0$  unemployment in both periods as a case of reference. Differences in utility of the weak type of policymaker are calculated in comparison to that situation and denoted by  $dz$ .

In equilibrium, at the beginning of period 2, the public still has the initial beliefs because there is pooling in period 1. Equilibrium inflation therefore satisfies:

$$\pi_2^e = (1 - q_0) \frac{\bar{b}}{a + \bar{b}} (\pi_2^e + u_1^B) \Leftrightarrow \pi_2^e = \frac{(1 - q_0) \bar{b}}{a - q_0 \bar{b}} u_1^B.$$

and the surprise  $W$  creates is

$$e_2 = \pi_2^W - \pi_2^e = \frac{q_0 \bar{b}}{a - q_0 \bar{b}} u_1^B.$$

Therefore, one obtains for the utility gain

$$dz^{pooling} = -\frac{a}{2} \left[ \frac{\bar{b}}{a + q_0 \bar{b}} u_1^B \right]^2 + \frac{\bar{b}}{2} [2e_2 u_1^B - e_2^2]$$

Now suppose that  $W$  does not choose his equilibrium strategy but inflates at  $e_2$  in period 1. Consequently, he is discovered and does not surprise in period 2. In period 1 expected inflation is at the equilibrium value 0. The gain is two times the gain from surprise inflation minus the costs of the inflation from both periods.

$$dz^{\pi_1=e_2} = -\frac{a}{2}e_2^2 + 2\frac{\bar{b}}{2}[2e_2u_1^B - e_2^2] - \frac{a}{2}\left(\frac{\bar{b}}{a}(u_1^B - e_2)\right)^2$$

A separating equilibrium exists only if this gain is smaller than the equilibrium gain, i.e.:

$$\begin{aligned} & -\frac{a}{2}e_2^2 + 2\frac{\bar{b}}{2}[2e_2u_1^B - e_2^2] - \frac{a}{2}\left(\frac{\bar{b}}{a}(u_1^B - e_2)\right)^2 \\ & < \\ & -\frac{a}{2q_0^2}e_2^2 + \frac{\bar{b}}{2}(2e_2u_1^B - e_2^2) \end{aligned}$$

This is

$$\begin{aligned} & \frac{b^2}{a}e_2u + \frac{\bar{b}}{2}[2e_2u_1^B - e_2^2] \\ & < \frac{a}{2}\left(1 - \frac{1}{q_0^2} + \frac{1}{2}(\bar{b}/a)^2\right)e_2^2 + \frac{a}{2}(\bar{b}/a)^2[u_1^B]^2 \end{aligned}$$

or

$$\begin{aligned} & \frac{\bar{b}^2}{a}e_2u_1^B + \bar{b}e_2u_1^B \\ & < \frac{a}{2}\left(1 - \frac{1}{q_0^2}\right)e_2^2 + \left(\frac{b}{2} + \frac{b^2}{2a}\right)e_2^2 + \frac{\bar{b}^2}{2a}u_1^{B^2} \end{aligned}$$

Substituting the equilibrium-surprise for  $e_2$  and division by  $(u_1^B)^2$  yields

$$\begin{aligned} & \left(\frac{\bar{b}^2}{a} + \bar{b}\right) \frac{q_0\bar{b}}{a+q_0\bar{b}} \\ & < \\ & \frac{a}{2}\left(1 - \frac{1}{q_0^2}\right)\left(\frac{q_0\bar{b}}{a+q_0\bar{b}}\right)^2 + \frac{1}{2}\left(\frac{b^2}{a} + b\right)\left(\frac{q_0\bar{b}}{a+q_0\bar{b}}\right)^2 + \frac{b^2}{2a} \end{aligned}$$

Multiplication with  $(a+q_0\bar{b})^2$  gives

$$\begin{aligned} & \left(\frac{\bar{b}^2}{a} + b\right)q_0\bar{b}(a+q_0\bar{b}) \\ & < \\ & \frac{a}{2}\left(1 - \frac{1}{q_0^2}\right)q_0\bar{b} + \frac{1}{2}\left(\frac{\bar{b}}{a} + \bar{b}\right) + \frac{\bar{b}^2}{2a}(a^2 + 2a\bar{b}q_0 + \bar{b}^2q_0^2) \end{aligned}$$

Collecting terms one finally obtains

$$\frac{B^4 q_0^2}{a} + B^3 q_0^2 + B^3 q_0 + a q_0 B^2$$

$$<$$

$$\frac{a}{2} q_0^2 B^2 + B^3 q_0 + \frac{1}{a} B^4 q_0^2 + q_0^2 B^3 / 2$$

The left hand side is, however, larger than the right hand side, because  $q_0$  is not larger than 1. Therefore, inflating at  $e_2$  in period 1 increases the utility of W and a pooling equilibrium does not exist. ■

With hysteresis the game has no pooling equilibrium. The opposite is true if we assume the absence of hysteresis:

### Proposition 2

Under Assumptions 2 and 3 there is a pooling equilibrium if  $q$  exceeds a certain threshold  $q^*$ .

### Proof 2

In the pooling equilibrium both types must inflate at the rate of zero in period 1.  $\mu(0)=q_0$ . In period 2 W inflates  $\pi^W_2 = \bar{b}/(a+\bar{b})(\pi^e_2+u_2^B)$  with given expectations and with  $u_2^B=u_1^B$ . Unbiased expectations fulfill:  $\pi^e_2=(1-q)\bar{b}/(a+\bar{b})(\pi^e_2+u_1^B)$ . The surprise is  $e=q\bar{b}/(a+q\bar{b})u_1^B =:x u_1^B$ . The utility gain is  $-a/2 (e/q_0)^2 + \bar{b}/2 (2eu_1^B - e^2) =$

$(-a/2(x/q_0)^2 + \bar{b}/2(2x-x^2))(u_1^B)^2 u_0^2$ . If W inflates in period 1,  $q_1 = 0$ . Therefore, the expected and actual rates of inflation in



period 2 are  $b/a u_1$ . Expected inflation in period 1 is zero and inflation is  $\bar{B}/(a+\bar{B})u_0 =: x_1 u_0$ . The utility gain is  $(-a/2 x_1^2 + \bar{B}/2(2x_1 - x_1^2) - \bar{B}^2/(2a))(u_1^B)^2$ .

The necessary condition for the existence of the equilibrium is:

$$(-a/2(x/q_0)^2 + \bar{B}/2(2x - x^2))(u_1^B)^2 > (-a/2 x_1^2 + \bar{B}/2(2x_1 - x_1^2) - \bar{B}^2/(2a))(u_1^B)^2.$$

Substituting  $x$  and  $x_1$  results in

$$\begin{aligned} & -a \frac{\bar{B}^2}{2(a+q_0\bar{B})^2} + q_0 \frac{\bar{B}^2}{a+q_0\bar{B}} - q_0^2 \frac{\bar{B}^3}{2(a+q_0\bar{B})} > \\ & -a \frac{\bar{B}^2}{2(a+\bar{B})^2} + \frac{\bar{B}^2}{a+\bar{B}} - \frac{\bar{B}^3}{2(a+\bar{B})} - \frac{\bar{B}^2}{2a} \end{aligned}$$

The left-hand side exceeds the right-hand side for  $q_0 = 1$ . Continuity of the LHS in  $q$  amounts to the proposition.

■

This result is quite intuitive: if the conservative type is highly probable and if there is no hysteresis then gains from inflating in period one or period two bring the same employment gains for the weak policymaker. Cheating in period one is, however, costly if  $q$  is high, because then expected inflation in period two is much higher than under pooling.

Whether there are separating equilibria in the game with hysteresis remains to be verified. I will show here that there are parameters  $a, b$  and  $q$  for such that a unique separating equilibrium exists<sup>3)</sup>.

In the separating equilibrium, there is a certain temptation for  $W$  to choose zero inflation in period one because then the public takes him for a HN in period 2. However, the price he has to pay

for that course of action is high, since in a separating equilibrium initial inflation expectations exceed zero and gaining reputation is costly in terms of employment. We have:

### Proposition 3

Let  $u_1^B > 0$ . Under Assumptions 1 and 3 there are parameters  $a$ ,  $b$  and  $q_0$  in a neighbourhood of  $(b/a, q_0) = (1, 1/2)$  such that the game has exactly one perfect Bayesian equilibrium in pure strategies; this equilibrium is a separating equilibrium.

### Proof 3

I still consider the case with zero inflation and  $u_0$  unemployment in both periods as a case of reference. Differences in utility of the weak policymaker are calculated in comparison to this reference-situation and denoted by  $dz$ . Inflation surprises are denoted by  $e$ . The weak policymaker can choose to surprise the public either in period one or in period 2. Supposing there is a separating equilibrium and equilibrium expectations in period 1 are  $\pi_1^e$ , then the gain realized by  $W$  by playing  $e$  in period 1 is:

$$\begin{aligned} \Delta z^{\pi_1 = \pi_1^e + e} = & -\frac{a}{2} (e + \pi_1^e)^2 + \frac{\bar{b}}{2} [2eu_1^B - e^2] \\ & - \frac{a\delta}{2} \left( \frac{\bar{b}}{a} (u_1^B - e) \right)^2 + \frac{\bar{b}\delta}{2} (2u_1^B e - e^2) \end{aligned} \quad (1)$$

This is maximized by

$$\frac{\partial \Delta z^{\pi_1 = \pi_1^e + e}}{\partial e} = -a(e + \pi_1^e) - \frac{\delta \bar{b}^2}{2a} (2e - u_1^B) + (1 + \delta) \bar{b} (u_1^B - e) = 0$$

or

$$e((1 + \delta) \bar{b} + \frac{\bar{b}^2}{a} \delta + a) = \left( \frac{\bar{b}^2}{2a} \delta + (1 + \delta) \bar{b} \right) u_1^B - a\pi_1^e$$

Expectations are unbiased iff

$$e = \frac{q_0}{1-q_0} \pi_1^e$$

Thus the surprise in equilibrium is

$$\begin{aligned} e &= \frac{\frac{b^2}{2a} \delta + (1+\delta) b}{\frac{b^2}{a} \delta + (1+\delta) b + \frac{a}{q_0}} u_1^B \\ &= \frac{\frac{b}{a} \delta + (1+\delta)}{\frac{b}{2a} \delta + (1+\delta) + \frac{a}{b} \frac{1}{q_0}} u_1^B =: x u_1^B < u_1^B. \end{aligned}$$

Substituting this into (6) amounts to the gain from separation.

Now consider the case where W inflates at rate 0 in period 1. Doing so, he incurs a loss through unfavorable expectations because the public will expect  $(1-q_1)/q_1 x u_1^B$ . Unemployment at the end of the period is therefore  $u_2^B = [1+(1-q_1)/q_1 x] u_1^B$ .  $q_1=1$ , therefore  $\pi_2^e=0$  and inflation will be  $b/(a+b)u_1$ . It follows for the complete gain from mimicking HN in period 1 that

$$\begin{aligned} \Delta Z^{\pi_1=\pi_2} &= \left[ -\frac{\delta a}{2} \left( \frac{b}{a+b} \left( 1 + \frac{1-q_x}{q} \right) \right)^2 \right. \\ &\quad \left. - \frac{b}{2} \left( \left( \frac{1-q_x}{q} \right)^2 + 2 \frac{1-q_x}{q} \right) \right] u_1^{B^2} \\ &\quad + \frac{b\delta}{2} (u_1^{B^2} - (u_2^B - (\pi_2 - 0))^2) \end{aligned}$$

The necessary and sufficient condition for the existence of a separating equilibrium is that the expression in (1) exceeds the gain from mimicking HN. We assume  $\delta=1$ . Substituting  $[1+(1-q_1)/q_1 x] u_1^B$  for  $u_2^B$  we obtain the condition:

$$\Delta Z^{W, sep. eq.} > \Delta Z^{\pi_1 = e_2} \Leftrightarrow -\frac{a}{2} \frac{1}{q^2} x^2 + \bar{b}(2x - x^2) - \frac{\bar{b}^2}{2a} (1 - 2x + x^2) \\
> \\
\left[ -\frac{a\bar{b}^2}{2(a+\bar{b})^2} \left(1 + \frac{1-q}{q} x\right)^2 \right. \\
\left. - \frac{\bar{b}}{2} \left(\frac{1-q}{q} x\right)^2 - \bar{b} \frac{1-q}{q} x \right] u_1^{B^2} \\
+ \left[ -\frac{\bar{b}}{2} \left(\frac{1-q}{q}\right)^2 x^2 - \frac{\bar{b}a}{a+\bar{b}} \frac{1-q}{q} x + \frac{\bar{b}^2}{a+\bar{b}} x - \frac{\bar{b}^3}{2(a+\bar{b})^2} \right] u_t^{1^2}.$$

We have  $a, \bar{b} > 0$ . Therefore,  $\bar{b}^2/a x > \bar{b}^2/(a+\bar{b}) x$ , which is the only positive term on the right hand side. On the right-hand side there remains therefore a negative term and on the left-hand side we are left with  $(-a/2 \cdot 1/q^2 + \bar{b} - \bar{b}^2/(2a))x^2 + 2\bar{b}x$ . Dividing this by  $b$  results in that this term being negative only if  $(-a/2\bar{b} \cdot 1/q^2 + 1 - \bar{b}/(2a))x^2 + 2x < 0$ . As  $x < 1$ , this demands at least that  $a/2\bar{b} \cdot 1/q^2 + \bar{b}/(2a) > 3$ . This is not true for  $\bar{b}=a$  and  $q=0.5$ .

■

Propositions 1, 2 and 3 support - like the results of Vickers (1986) - the existence of separating rather than pooling equilibria in repeated games of monetary policy. Vickers obtains this result because he allows the conservative central banker to choose a low inflation rate. I maintain Backus and Driffill's assumption that the conservative central banker always chooses zero inflation. The reason why I obtain no pooling equilibria is different: persistent gains through surprise inflation always create an incentive for a weak policymaker to surprise as soon as possible. It follows that in a country with hysteresis, a weak central banker will never choose a low inflation rate to mimic a hard-nosed policymaker and that - in such a situation -

different mechanisms are necessary to solve the time-inconsistency problem. Section 3 analyses the possibility that monetary target announcements can serve as such a mechanism in the context of a one-period game.

### 3. The one-period signaling game with a target announcement and uncertainty on $b$

This section presents my basic one-period signaling game with an endogenous monetary target announcement. The central bank possesses private information on its relative interest in different conflicting targets. I assume the public to be uncertain about the relative dislike of the policymaker for deviations from a known socially desired inflation rate,  $\bar{\pi}$ , and for deviations from full employment. We disregard uncertainty about an economic shock, about the central bank's time preference, or about  $\bar{\pi}$ .

#### Preferences

The policymaker's preferences are represented through the utility function:

$$Z = -\frac{a}{2} (\pi - \bar{\pi})^2 - \frac{b}{2} (\pi - \pi^e - u)^2 - \gamma \frac{c}{2} (\pi - \pi^a)^2. \quad (1)$$

where  $\pi$  is inflation,  $\pi^e$  expected inflation and  $\pi^a$  announced inflation. The first term represents the loss which stems from an inflation rate different from the desired level  $\bar{\pi}$ .  $\gamma$  is one, if the central bank chooses to make an announcement, and zero otherwise. I call the inflation rate  $\bar{\pi}$  the full-employment inflation rate (FEIR) because it is the inflation rate one obtains if there is no incentive to create surprise inflation, i.e. if there is no unemployment,  $u$ , in the economy<sup>2)</sup>. The second term is the loss from unemployment. Unemployment can be below the

threshold level  $u$  if there is surprise inflation.  $u$  can be thought of as the natural rate or, alternatively under hysteresis, as lagged unemployment. We assume that  $b_i$  takes a low value  $\underline{b}$  with the probability  $q$  ("hard-nosed"- policymaker, HN) and a high value  $\bar{b}$  with the probability  $1-q$  ("weak" policymaker, W). I finally assume that there are increasing costs of deviations from the target announcement and these costs are known. There are at least two justifications for this assumption:

- 1) One can assume that the policymaker loses more personal reputation the larger his deviation from the announcement. This is due to the existence of a small control error that serves to justify deviations. The justification becomes less credible as the deviation increases. The modelization of the game as a one-period game with rising costs of deviations can be seen as a shortcut under this assumption.
- 2) Another reason for rising costs is a salary contract for the central banker that punishes deviations from the announcement.

This is why I assume that both types face the same costs of deviations from the announcement. A control error is not modeled explicitly here because it would not change the results and only complicates the notation.

I assume that the public always chooses inflation expectations so that they equal the expected value of inflation given the beliefs about the type of central banker. Formally one can write the choice of the public as  $\pi^e = \operatorname{argmin}_x (x - E(\pi(\pi^1, x)))^2$ . In the

game the public always reaches its preferred outcome i.e. unbiased expectations.

In our game the central banker unilaterally selects the inflation rate  $\pi$ . It should be noted that all the results in the rest of the paper continue to hold if we introduce a demand side through a quantity equation and let the central banker choose the money growth rate. The choice of  $\pi$  as the strategic variable is only made for the reasons of simplicity.

### Equilibrium without announcement

Before I turn to the equilibrium of the signaling game, I will briefly review the equilibrium of the game without an announcement. In the absence of an announcement, the optimal policy of the hard-nosed policymaker is  $\pi^{HN} = 1/(a+b)$   $[a\bar{\pi} + b(\pi^e + u)]$ , whereas the weak policymaker chooses  $\pi^W = 1/(a+\bar{b})$   $[a\bar{\pi} + \bar{b}(\pi^e + u)]$ . Thus, expectations are unbiased iff:

$\pi^e = q[1/(a+b) [a\bar{\pi} + b(\pi^e + u)]] + (1-q)[1/(a+\bar{b}) [a\bar{\pi} + \bar{b}(\pi^e + u)]]$ . So

Equilibrium expectations are then:

$$\pi_t^e = \bar{\pi} + \frac{\frac{qb}{a+b} + \frac{(1-q)\bar{b}}{a+\bar{b}}}{1 - \frac{qb}{a+b} - \frac{(1-q)\bar{b}}{a+\bar{b}}} u_{t-1} \quad (3)$$

In Section 5 I will compare the outcome of the game with an announcement to this reference situation. In Section 4 I calculate the equilibria of the game with an announcement.



#### 4. Perfect Bayesian Equilibria in Pure Strategies

A perfect Bayesian equilibrium in pure strategies of our game is given by

- (i) beliefs about the probability of a HN-policymaker, contingent on the announcement,  $\mu(\pi^a)$
- (ii) a signal for each type of policymaker
- (iii) inflation expectations  $\pi^e$

so that

(i) given the beliefs and expectations, each policymaker chooses the optimal announcement and

(ii) given the signals, the beliefs are formed according to Bayes's law whenever possible.

(iii) Given beliefs and strategies of the central bankers, expected inflation is the public's best response. As I assume that the public wishes to minimize the difference between expected inflation  $\pi^e$  and the expected value of inflation, this means that both values are identical.

Considering only pure-strategies equilibria it is natural to restrict beliefs  $\mu(\pi^a)$  about the type to the values 0, 1, and  $q$ . I first specify conditions for the existence of a separating equilibrium in which both types make an announcement. We will see

that under certain conditions there is an infinite number of separating equilibria and I will therefore introduce a new refinement method. I then examine pooling equilibria where both types choose the same announcements. Finally I will examine cases where one or both policymakers choose not to announce.

We need

### Lemma 1

If there is a separating equilibrium and if W announces, then he announces  $\bar{\pi}$  and  $\mu(\bar{\pi})=0$ .

### Proof of Lemma 1

Suppose  $\mu(x)=0$  for  $x \in R$ . Then expectations in stage two of the game are:  $\pi^e = 1/(a+c) (a\bar{\pi} + bu + c\pi^{AW})$ . Substitution of  $\pi^e$  into (1), maximisation by choice of  $\pi$ , substitution of  $\pi$  and  $\pi^e$  into (1), and maximization by choice of the announcement results in  $\pi^{AW} = \bar{\pi}$ . However, in a separating equilibrium  $\mu(\bar{\pi})$  is at least zero. Therefore, it is better for W to announce  $\bar{\pi}$  than to announce any other rate  $x$  with  $\mu(x)=0$ .  $\bar{\pi}$  is therefore the announcement that W chooses in a separating equilibrium and  $\mu(\bar{\pi})$  must equal zero. ■

Suppose now that a separating equilibrium exists. We define the difference between the equilibrium signals,  $\delta$ , as  $\delta := \pi^{AW} - \pi^{AHN}$ .  $\delta$  has to satisfy two conditions in order for a separating equilibrium to exist:

- (i)  $\delta$  has to be sufficiently large to avoid that W is better off announcing  $\pi^{a,HN}$  instead of  $\bar{\pi}$ ,
- (ii)  $\delta$  has to be sufficiently small to avoid that HN is better off announcing the FEIR.

We have

**Proposition 4 Separating Equilibrium**

There is an  $s \in \mathbb{R}$  so that if  $(a+c)/\bar{b} > s$ , the game has an infinity of separating equilibria. Given  $a$ ,  $c$ ,  $\bar{b}$ , and  $b$ , there is a nonempty interval  $[\delta_1, \delta_2]$  with  $\delta_1 > \delta_2 > 0$ , so that for all  $\delta \in [\delta_1, \delta_2]$  the following strategies and beliefs form an equilibrium:

- beliefs are  $\mu(\pi^a) = 0$  for  $\pi^a > \bar{\pi} - \delta$  and 1 otherwise
- W announces  $\bar{\pi}$  and HN announces  $\bar{\pi} - \delta$

There are no inflation surprises in equilibrium.

**Proof 4**

We have to consider and compare four values of the utility functions:

- (i) Utility of W in equilibrium
- (ii) Utility of W if he announces  $\pi^{a,HN}$  with  $\mu(\pi^{a,HN}) = 1$
- (iii) Utility of HN in equilibrium
- (iv) Utility of HN if he announces  $\pi^{a,W}$  with  $\mu(\pi^{a,W}) = 0$

ad (i) Expectations satisfy:

$$\pi^e = \pi^W = \bar{\pi} + \frac{b}{a+b+c} (\pi^e + u) \Leftrightarrow \pi^e = \bar{\pi} + \frac{b}{a+c} u.$$

Substituting expectations  $\pi^W$  and  $\pi^e$  into the utility function of W yields

$$Z^{W, sep. eq} = -\frac{b^2}{2(a+c)} u^2 - \frac{b}{2} u^2.$$

ad (ii) Expectations fulfill

$$\begin{aligned} \pi^e &= \frac{1}{a+b+c} [a\bar{\pi} + b(\pi^e + u) + c\bar{\pi} - \delta] \Leftrightarrow \\ \pi^e &= \bar{\pi} + \frac{b}{a+c} u - \frac{c}{a+c} \delta. \end{aligned}$$

W responds with

$$\begin{aligned} \pi^W &= \frac{1}{a+b+c} [a\bar{\pi} + b(\bar{\pi} + \frac{a+b+c}{a+c} - \frac{c}{a+c} \delta) + c\bar{\pi} - \delta] = \\ &\bar{\pi} + \frac{b}{a+c} F^W u - \frac{c}{a+c} \delta. \end{aligned}$$

where

$$F^W = \frac{a+b+c}{a+b+c} < 1.$$

Substituting  $\pi^W$ ,  $\pi^e$ , and  $\pi^a$  into the utility function of W yields:

$$\begin{aligned} Z_{\pi-\delta}^W &= [-\frac{b^2}{2(a+c)} F^{W^2} - \frac{ac^2 + ca^2}{(a+c)^2} \delta^2 \\ &\quad - \frac{b}{2} [\frac{bF^W - b + a + c}{a+c}]^2] u_0^2 \end{aligned}$$

The necessary condition for the existence of a separating equilibrium is then:

$$\frac{1}{2} \left[ \frac{\bar{b}^2}{(a+c)} + \bar{b} - \frac{(\bar{b}F^W)^2}{a+c} - \bar{b} \left[ \frac{\bar{b}F^W - \bar{b} + a + c}{a+c} \right]^2 \right] u_0^2 \leq \frac{ac^2 + ca^2}{a+c} \delta^2$$

If this condition is not fulfilled, W prefers to announce  $\pi^{a,HN}$ .

ad (iii) We have  $\pi^{a,HN} = \bar{\pi} - \delta$ ,  $\mu(\bar{\pi} - \delta) = 1$  and

$$\pi^e = \pi^{HN,eq} = \bar{\pi} + \frac{b}{a+c} u - \frac{c}{a+c} \delta.$$

Substituting  $\pi^{a,HN}$ ,  $\pi^{HN,eq}$  and  $\pi^e$  into the utility function of HN yields:

$$Z^{HN,sep.eq} = -\frac{b^2}{2(a+c)} u^2 - \frac{ac^2 + ca^2}{(a+c)^2} \delta^2 - \frac{b}{2} u^2.$$

ad (iv)  $\pi^{a,HN} = \bar{\pi}$ ,  $\mu(\bar{\pi}) = 0$  and

$$\pi^e = \pi^W = \bar{\pi} + \frac{\bar{b}}{a+c} u.$$

Therefore HN inflates at

$$\pi^{HN} = \frac{1}{a+b+c} \left[ a\bar{\pi} + b \left( \bar{\pi} + \frac{a+\bar{b}+c}{a+c} u \right) + c\bar{\pi} \right] = \bar{\pi} + \frac{b}{a+c} F^H u$$

with

$$F^H = \frac{a+\bar{b}+c}{a+b+c} > 1.$$

Utility is

$$Z^{HN} = -\frac{b^2}{2(a+c)} F^{H^2} u^2 - \frac{b}{2} \left[ \frac{bF^H - \bar{b} + a + c}{a+c} \right]^2 u^2.$$

Utility from (iii) must at least equal utility from (iv). The sufficient condition for the existence of a separating

equilibrium is:

$$\begin{aligned} & \frac{Z_{\pi}^{HN} \leq Z^{HN, eq} \wedge Z_{\pi-\delta}^W \leq Z^{W, eq}}{[-\frac{b^2}{2(a+c)} - \frac{b}{2} + \frac{(bF^{HN})^2}{2(a+c)} + \frac{b}{2} [\frac{bF^{HN} - \bar{b} + a + c}{a+c}]^2] u_0^2 \geq \frac{ac^2 + ca^2}{a+c} \delta^2} \\ & \end{aligned}$$

If this condition is not satisfied, HN will announce  $\bar{\pi}$ .

Reformulation of the necessary and the sufficient condition yields:

$$\begin{aligned} & \frac{Z_{\pi}^{HN} \leq Z^{HN, eq} \wedge Z_{\pi-\delta}^W \leq Z^{W, eq}}{\frac{1}{2} [-\frac{\bar{b}^2}{(a+c)} - \bar{b} + \frac{(\bar{b}F^W)^2}{a+c} + \bar{b} [\frac{\bar{b}F^W - \underline{b} + a + c}{a+c}]^2] u_0^2 \geq} \\ & [\frac{b^2}{2(a+c)} + \frac{b}{2} - \frac{(bF^H)^2}{a+c} - \frac{b}{2} [\frac{bF^H - \bar{b} + a + c}{a+c}]^2] u_0^2 \end{aligned}$$

Multiplication of this condition by  $2(a+c)^2$  leads us to

$$\begin{aligned} & \frac{Z_{\pi}^{HN} \leq Z^{HN, eq} \wedge Z_{\pi-\delta}^W \leq Z^{W, eq}}{(a+c) ((\bar{b}F^W)^2 + (bF^H)^2) + \bar{b} [\bar{b}F^W - \underline{b} + a + c]^2 + b [bF^H - \bar{b} + a + c]^2 \geq} \\ & (a+c) (b^2 + \bar{b}^2) + (a+c)^2 (\underline{b} + \bar{b}). \end{aligned}$$

Following from  $\lim_{(a+c)/\bar{b} \rightarrow \infty} F^W = \lim_{(a+c)/\bar{b} \rightarrow \infty} F^H = 1$ , the limit of the left-hand side of this expression for  $(a+c)$  tending towards infinity is:

$$\begin{aligned} & (a+c) ((\bar{b})^2 + (\underline{b})^2) + \bar{b} [\bar{b} - \underline{b} + a + c]^2 + \underline{b} [\underline{b} - \bar{b} + a + c]^2 \\ & = (a+c) (b^2 + \bar{b}^2) + (a+c)^2 (\underline{b} + \bar{b}) + 2(\bar{b} - \underline{b})(a+c)(\bar{b} + \underline{b}). \end{aligned}$$

This is more than the expression on the right hand side. Since both sides of the inequality are continuous in  $a$  and  $c$ , it follows directly that there exists a positive value for  $(a+c)/\bar{b}$  so that the left-hand side is larger than the right hand side. Therefore, there is a value of  $(a+c)/\bar{b}$ ,  $s$ , so that for all  $(a+c)/\bar{b} > s$  the necessary and sufficient condition for the existence of a separating equilibrium is fulfilled. The values  $\delta_1$  and  $\delta_2$  are the supremum (and infimum) for  $\delta$  for which the two

above conditions hold.

Consistent beliefs for the case of no announcement remain to be found. The above equilibrium extends to an equilibrium of the whole game if  $\mu(\text{NO})=0$ . First consider W. In equilibrium he inflates at  $\bar{\pi}+b/(a+c)u$  and gets  $-b^2/2(a+c)u^2 - b/2u^2$ . If he decides not to announce, he inflates at  $\bar{\pi}+\bar{b}/a u$  and gets  $-\bar{b}^2/2a u^2 - \bar{b}/2u^2$ . This is less. Now consider HN. We have seen that he prefers announcing  $\pi^{a,\text{HN}}$  to  $\bar{\pi}$  if beliefs are  $\mu(\bar{\pi})=0$ . In this case expectations are  $\pi^e = \bar{\pi} + \bar{b}/(a+c) u$ . Optimal inflation is  $\bar{\pi}+b/a (a+\bar{b}+c)/(a+b+c) u$ . This is less than if HN does not announce: then expectations are (under  $\mu=0$ )  $\pi^e = \bar{\pi} + \bar{b}/a u$  and inflation is  $\bar{\pi}+b/a (a+\bar{b})/(a+b) u$ . The negative surprise  $\bar{b}/a u - b/a (a+\bar{b})/(a+b) u$  exceeds the surprise  $\bar{b}/(a+c) u - b/a (a+\bar{b}+c)/(a+b+c) u$  and HN inflates more. Utility is smaller and thus HN prefers his equilibrium strategy.

■

We found that there is either no or an infinity of separating equilibria. For a refinement, I define a criterion that is somewhat stronger than the criterion of dominated messages [Kreps (1990,p.436)]. In this game, Kreps' criterion of dominated messages would demand that a player does not choose a signal that - in combination with any expectation of the public - always makes him worse off than if he chooses some other signal with any value of expected inflation. My criterion builds on a similar

intuitive argument, but I restrict the public to choose expectations that are consistent with beliefs about the probability distribution of types. I define:

**Definition 1 x-dominated signal**

Suppose there are two signals  $\pi^1$  and  $\pi^2$ . Denote  $E_1$  and  $E_2$  the set of expectations that are induced by all possible beliefs and by the signals respectively. For a player  $i$ ,  $\pi^1$  is x-dominated by  $\pi^2$  iff the maximum obtainable under  $\pi^1$  with expectations from  $E_1$  is smaller than the minimum obtainable under  $\pi^2$  with expectations from  $E_2$ .

**Definition 2 x-proof equilibrium**

An equilibrium fails the test of x-domination iff there is a signal that is x-dominated for exactly one player and if the equilibrium cannot be sustained with beliefs that assign zero probability to this player if the signal occurs. An equilibrium is called x-proof if it does not fail the test of x domination.

Define  $B$  as the set of possible beliefs. This refinement demands that a player will not choose a signal  $s$  if the expectations consistent with beliefs  $b$  are worse for all  $b$  from  $B$  than expectations for another signal consistent with all beliefs  $b$  in  $B$ . There may, however, be signals where this is true for all players and one cannot assign zero probability to all of them. This is why the criterion requires that the test is not failed for exactly one player.



**Proposition 5** x-proof separating equilibria

All equilibria where  $d$  is taken from  $(d_1, d_2]$  are not x-proof.

**Proof 5**

We know that for all the  $\pi^a$  from  $[\bar{\pi}-d_2, \bar{\pi}-d_1)$   $W$  is better off by announcing the FEIR even if  $\mu(\pi^a)=1$  (this follows directly from our definition of  $[d_1, d_2]$ ). These signals are not x-dominated for HN. Therefore, in an x-proof-equilibrium the public has to assign zero probability for  $W$  to these signals. In this case, if  $\mu(\bar{\pi}-d_1)=1$ , HN chooses to announce  $\pi^a=\bar{\pi}-d_1$ . If  $\mu(\bar{\pi}-d_1)$  is smaller than one, then a optimal response for HN does not exist. ■

It follows directly that the game has exactly one x-proof separating equilibrium.

I now turn to the pooling equilibria of the game. In a pooling equilibrium we have  $\pi^{a,W}=\pi^{a,HN}$  and  $\mu(\pi^{a,HN})=q$ . Many such equilibria with different announcements exist, but a refinement is possible. We have:

**Proposition 6**

- (i) The signaling game has a pooling equilibrium.
- (ii) There is exactly one pooling equilibrium of the game, where out-of-equilibrium beliefs equal initial beliefs (criterion of unchanged out-of-equilibrium beliefs). In this equilibrium

$\pi^{a,W} = \pi^{a,HN} = \bar{\pi}$  and expectations are:

$$\pi_t^e = \bar{\pi} + \frac{\frac{qb}{a+b+c} + \frac{(1-q)\bar{b}}{a+\bar{b}+c}}{1 - \frac{qb}{a+b+c} - \frac{(1-q)\bar{b}}{a+\bar{b}+c}} u_{t-1} \quad (5)$$

**Proof 6**

(i) It is useful to examine the case where beliefs are  $\mu(\pi^a) = q$  for  $\pi^a \in R$  with  $q \in [0,1]$  and to calculate the best announcement for both types. For every announcement, expectations satisfy  $\pi^e = q \pi^{HN}(\pi^e, \pi^a) + (1-q) \pi^W(\pi^e, \pi^a)$  with obvious notations. This is:

$$\pi^e = \frac{q}{a+b+c} [a\bar{\pi} + b(\pi^e + u) + c\pi^a] + \frac{1-q}{a+\bar{b}+c} [a\bar{\pi} + \bar{b}(\pi^e + u) + c\pi^a]$$

Expectations are then:

$$\pi^e = \frac{1}{(a+c)^2 + (a+c)(q\bar{b} + (1-q)b) + (b\bar{b} + (a+c)(qb + (1-q)\bar{b})u)} [a\bar{\pi} + c\pi^a]$$

or

$$\pi^e = \frac{1}{a+c} [a\bar{\pi} + c\pi^a] + Xu.$$

We now consider, without restricting generality, the HN-policymaker. His optimal action, given  $\pi^e$ , and  $\pi^a$ , is:

$$\pi^H = \frac{1}{a+b+c} [a\bar{\pi} + c\pi^a + b(\frac{a\bar{\pi} + c\pi^a}{a+c} + (1+X)u)]$$

We obtain the optimal announcement by substitution of inflation and expectations into (1)

$$\begin{aligned} z^H = & -\frac{a}{2} \left( \frac{c}{a+c} (\pi^a - \bar{\pi}) + \frac{b}{a+b+c} (1+X)u \right)^2 \\ & - \frac{b}{2} \left( \frac{b(1+X) - X(a+b+c)}{a+b+c} u \right)^2 \\ & - \frac{c}{2} \left( \frac{a}{a+c} (\bar{\pi} - \pi^a) + \frac{b}{a+b+c} (1+X)u \right)^2 \end{aligned}$$

and derivation with respect to  $\pi^a$ . The optimal announcement is  $\pi^{a,HN} = \bar{\pi}$ . Exactly the same calcul applies to W if we exchange  $b$  for

$\bar{b}$ . Therefore, if beliefs are  $\mu(\pi^a)=q$  for  $\pi^a \in R$ , both policymakers announce  $\bar{\pi}$ . The given beliefs thus prove to be correct.

(ii) Other pooling equilibria can lie in a neighborhood of  $\bar{\pi}$  if  $\mu$  is zero in a sufficiently large neighborhood of the pooling announcement (otherwise both types announce  $\bar{\pi}$ ). This, however, is excluded if out-of-equilibrium beliefs must equal prior beliefs.

■

The refinement applied in (ii) intuitively means that - if a signal has been chosen that has the probability of zero for both types - then there is no new information in this announcement. There is then no reason to change the initial beliefs [see, for a similar idea, Fudenberg and Tirole (1991, p.332)]. It can also easily be seen that the separating equilibrium from proposition 4 is not robust with respect to the criterion of unchanged out-of-equilibrium beliefs. (The weak policymaker would deviate if beliefs were  $q$  around  $\pi^a=\bar{\pi}$ .) In contrast to the separating equilibrium the pooling one exists for all positive values of  $a, b, c$  and  $q$ . It is the most obvious way to play the game. For the pooling equilibrium, expectations are:

$$\pi_t^e = \bar{\pi} + \frac{\frac{qb}{a+b+c} + \frac{(1-q)\bar{b}}{a+\bar{b}+c}}{1 - \frac{qb}{a+b+c} - \frac{(1-q)\bar{b}}{a+\bar{b}+c}} u_{t-1} \quad (5)$$

We obtain a new interesting long-run inflation-unemployment relationship from this equilibrium: current inflation rises with past unemployment. One obtains two different "long-run Phillips

curves" which are positively sloped; each curve refers to one type of central banker. If central bankers frequently change over time, a counter-clockwise movement in the unemployment-inflation space results.

It remains to be verified whether there are equilibria when one or two central bankers do not announce.

### Proposition 7

- (i) There is no equilibrium where W does not announce and HN announces.
- (ii) Given  $a$  and  $\underline{b}$ ,  $\bar{b}$ , there is a  $c^* > 0$ , so that for all  $c > c^*$  there is no equilibrium when HN does not announce.

### Proof 7

- (i) If W announces the FEIR, he gets at least

$$-\frac{a+c}{2} \left( \frac{\bar{b}}{a+c} u \right)^2 - \frac{\bar{b}}{2} u^2.$$

If HN announces and we have a separating equilibrium, then beliefs must be zero for the case of no announcement. Utility of W is then:

$$-\frac{a}{2} \left( \frac{\bar{b}}{a} u \right)^2 - \frac{\bar{b}}{2} u^2.$$

- (ii) It is sufficient to consider that with high values for  $c$ ,

utility of HN tends towards the utility in the case where it is known that he is HN and he announces the FEIR:

$$\lim_{c \rightarrow \infty} [\min[z^{HN, \text{announces}}] - (-\frac{a+c}{2} (\frac{\bar{b}}{a+c}u)^2 - \frac{\bar{b}}{2}u^2)] = 0.$$

The second term is larger than that for HN if his type is known and he separates without announcement:

$$-\frac{a}{2} (\frac{b}{a}u)^2 - \frac{b}{2}u^2.$$

This exceeds the utility of HN if both policymakers do not announce. ■

An interesting point is that there is a tendency for both types to announce the FEIR if they know that they will be discovered. The reason for this is that because deviating is painful and gains from cheating are impossible anyway, one can bind himself more strongly to the desired rate by announcing it. We can already observe here that the announcement is rather a tool to communicate the FEIR than to reveal  $b$ . To observe this more clearly, we will have to add another dimension to the type space. This will be done in section 5. It may seem paradoxical that a central banker chooses to make an announcement knowing that he will never stick to it and knowing that deviations are costly. But it is exactly the fact that deviations are costly (and the public knows it) that makes the announcement an attractive strategy.

## 5. Credibility and Welfare

Two measures of credibility are useful if we consider the equilibria from Section 4:

1) Following the terminology introduced by Cukierman and Meltzer (1986), one can call the inverse of the distance of expectations from the announcement "average credibility of the announcement".

2) One can also distinguish revealing and non-revealing announcements. A revealing announcement is not fully credible in the sense of average credibility, but it carries all the relevant information.

In the separating equilibrium, the announcement is revealing. Average credibility depends on the type of policymaker in office.

Consider the pooling equilibrium from Section 3. From (3) we see that average credibility rises with rising probability for a conservative banker, with rising costs of cheating and inflation and with falling unemployment.

The model offers a new way to test empirically for whether a new institutional arrangement has a positive effect on the average credibility of a central bank's target announcements, i.e. for whether the parameters  $a$ ,  $b$ ,  $c$  and  $q$  are affected. In this case, a regression of expectations on the temptation-variables (unemployment, government debt) should show a structural change. (If, for example, the EMS really changes costs of inflation, then both types' parameter  $a$ , rises,  $\bar{\pi}$  falls, and the influence of  $u$

on  $\pi^e$  falls. The nomination of a new central banker might increase  $q$ , etc..) Another possibility is to directly analyze the influence of the unemployment rate and of government debt on the deviation of the money growth rate from the target announcement.

## **Welfare**

A natural formulation for a social welfare function is

$$W = E \left( -\frac{a}{2} (\pi_t - \bar{\pi})^2 - \frac{b^s}{2} (\pi_t - \pi_t^e - u_{t-1})^2 \right) \quad (5)$$

where  $b^s \in [\underline{b}, \bar{b}]$ . We have

## **Proposition 8 Welfare**

- (i) Welfare in the pooling equilibrium is higher than without an announcement.
- (ii) Take  $a$ ,  $\bar{b}$ ,  $\underline{b}$ , and  $u$  as given. Suppose that for all  $c > c^* > 0$  we have a separating equilibrium.  $c^{**} \geq c^*$  so that for all  $c > c^{**}$ , expected welfare in this equilibrium exceeds expected welfare in the equilibrium of the game with no announcement.

## **Proof 8**

- (i) We must compare the pooling equilibrium from Section 3 with the situation without an announcement: without an announcement, expectations satisfy  $\pi^e = q \pi^{HN}(\pi^e) + (1-q) \pi^W(\pi^e)$  so that expected inflation is higher than in the pooling equilibrium with an announcement (equation (4)):

$$\pi_t^e = \bar{\pi} + \frac{\frac{qb}{a+b} + \frac{(1-q)b}{a+b}}{1 - \frac{qb}{a+b} - \frac{(1-q)b}{a+b}} u_{t-1} \quad (3)$$

Therefore, the first-loss term in the welfare function is smaller in the presence of an announcement. It remains to be considered that  $E((-b^2/2) (\pi - \pi^e - u)^2) = E((-b^2/2) ((\pi - \pi^e)^2 - 2(\pi - \pi^e)u + u^2)) = E((-b^2/2) (\pi - \pi^e)^2) - b/2 u^2$ . The loss is smaller with an announcement, as can easily be seen from (3) and (4). Therefore, the pooling equilibrium from Section 4 is welfare-improving for every positive  $c$  compared to the no-announcement case considered in the end of Section 3.

(ii) Expected welfare without an announcement consists of two expected losses: the expected loss from deviations from  $\bar{\pi}$  and the expected loss of deviations from  $\pi^e + u$ . With an announcement, in the separating equilibrium, the second loss term is simply  $-b^2/2u^2$ . In the absence of an announcement, inflation expectations equal the expected value of inflation, too, but the expected loss is larger because of the quadratic loss function. As  $c$  tends towards infinity, the first expected-loss term in the announcement equilibrium tends towards zero. This loss is smaller than the corresponding loss without an announcement. The proposition then follows from the continuity of the welfare function.

■

The institution of a monetary announcement, therefore, turns out



to be welfare-improving for both equilibria from Section 3, provided that  $c$  is sufficiently high. Note also that in this game, without shocks an increase in  $c$  unambiguously increases welfare. The reason for this is that the higher the  $c$ , the larger the possibility for both types of policymaker to commit to the second-best outcome with no inflation surprises and the inflation rate  $\bar{\pi}$ . It is probable that introducing shocks like in the famous Rogoff (1985) paper would lead to the optimality of some positive value for deviation costs. One can conclude from this analysis that monetary targeting is welfare-improving, although in the model both types of policymakers do not stick to their announcement. Monetary targets are used, if the costs of deviating from them are sufficiently high. Abandoning monetary targeting would reduce welfare. The fact that such deviations from target announcements do actually occur is no good reason for abandoning targets. Note, finally, that the pooling equilibrium is fully equivalent to the outcome under a rule that fixes costs  $c(\pi - \bar{\pi})^2$  for deviations from the inflation target. It, however, does not necessitate the explicit specification of  $\bar{\pi}$  and leaves the independence of the central bank intact.

## 5. The signaling game with uncertainty on $\bar{\pi}$

We have seen in Section 3 that the signal is equal to the desired inflation rate  $\bar{\pi}$ . The announcement could therefore be a tool to communicate the FEIR. This section studies the game from section

3 under a different assumption about the type-space:  $b$  is a known positive parameter, but  $\bar{\pi}$  is unknown and distributed uniformly on  $[\pi^L, \pi^U]^3$ .  $\bar{\pi}$  can be thought of as a convex combination of different conflicting targets like for example a foreign inflation rate, the seignorage-maximising inflation rate and the zero-inflation rate. Different individuals will assign different weight to each of these objectives. We therefore assume in this section that the FEIR of the policymaker and not  $b$  is uncertain. We again restrict the analysis to pure strategy equilibria and to the analysis of linear expectations of the form

$$\pi^e = \pi^a + \delta \tag{6}$$

This is a somewhat restrictive assumption and I cannot exclude that there do exist equilibria with nonlinear or even discontinuous expectation functions<sup>4)</sup>. However, with our assumption on the uniform distribution of central bankers on the linear type space there is no good reason why two announcements should be treated differently in the expectation formation process, and this supports the above linear expectations.

**Proposition 9** Existence of an equilibrium with linear expectations

There is exactly one equilibrium with linear expectations. In equilibrium  $\delta = b/c$  u.

**Proof 9**

We first deduce the optimal strategy for the central bank, given linear expectations. We then proof that there is one and only

one mark-up  $\delta$  for which expectations of this form prove to be correct. Given expectations from (6), the utility of a  $(b, \bar{\pi})$ -banker is:

$$z = -\frac{a}{2} (\pi - \bar{\pi})^2 - \frac{b}{2} (\pi - \pi^a - \delta - u)^2 - \frac{c}{2} (\pi - \pi^a)^2.$$

Therefore, given the announcement  $\pi^a$  utility is maximised iff

$$\pi = \frac{1}{(a+b+c)} [a\bar{\pi} + (b+c)\pi^a + b(\delta+u)].$$

Utility contingent on  $\pi^a$  therefore is given by:

$$\begin{aligned} z = & \frac{1}{(a+b+c)^2} \left( -\frac{a}{2} ((b+c)(-\bar{\pi} + \pi^a) + b(\delta+u))^2 \right. \\ & - \frac{b}{2} (a\bar{\pi} - a\pi^a - (a+c)(\delta+u))^2 \\ & \left. - \frac{c}{2} (a\bar{\pi} - a\pi^a + b(\delta+u))^2 \right). \end{aligned}$$

The first order condition for a maximum of this expression is:

$$\pi^a = \bar{\pi} - \frac{bc + b^2 + ba}{(b+c)^2 + ba + ca} [\delta + u].$$

this is

$$(7) \quad \pi^a = \bar{\pi} - \frac{b}{b+c} [\delta + u].$$

One therefore obtains that, given  $\pi^a$ , expectations have to fulfill the following condition to be unbiased

$$\pi^e = \pi^a + \delta = \frac{1}{a+b+c} [a(\pi^a + b/(b+c)(\delta+u)) + b(\pi^a + \delta + u) + c\pi^a].$$

Solving this equation for  $\delta$  yields  $\delta = b/c u$ .

■

We are now ready to make our welfare statements. I assume that society cares about deviations of inflation from its desired rate  $\pi^s$  and about unemployment. The relative importance of the employment target is  $b^s/a$ . For  $\pi^s$  we assume  $\pi^s = (\pi^u - \pi^L)/2$ . One

interpretation is that society's preferences are distributed over the set B and that the central banker is drawn randomly among the members of society. Society's preferences are the preferences of the median voter. The welfare for a particular inflation rate  $\pi$  is:

$$w = -\frac{a}{2} (\pi - \pi^s)^2 - \frac{b^s}{2} (\pi - \pi^e - u)^2.$$

Taking uncertainty into account we have to consider the expected value of  $w$ , i.e.:

$$W = E(w).$$

### **Proposition 10**

Assume that an announcement is compulsory for each type of central banker. Welfare increases with  $c$ .

### **Proof 10**

There is no uncertainty on inflation for all  $c > 0$  because the announcement is fully revealing. But the interval of actions of the potential policymakers approaches the type space for higher  $c$ . Welfare rises because society dislikes deviations of inflation from the mean of the type space. ■

It is thus optimal to make  $c$  infinitely large in the absence of economic shocks. If  $c$  tends to infinity, the announcement and the action of a  $(b, \bar{\pi})$ -type tend to  $\bar{\pi}$ , while  $\delta$  tends to zero. It remains to be verified whether the announcement is an institution which is beneficial to society at all. There are two effects if we compare the announcement- to the no-announcement- case:

firstly, announcements reduce inflation uncertainty. But secondly, they give the opportunity to deviate from the socially desired rate at low costs because bankers are then able to manipulate expectations. We compare our equilibrium under infinit costs  $c$  to the situation where announcements are impossible.

### Proposition 11

(i) There is a unique value for  $b^s/a$ ,  $(b^s/a)^*$ , so that a mechanism, which imposes infinit costs on the central bank for deviations from announcements, raises welfare if and only if  $(b^s/a) > (b^s/a)^*$

(ii)  $(b^s/a)^*$  increases with  $\pi^u - \pi^L$  and decreases with  $u$ .

### Proof 11

Expectations without announcement must satisfy

$$E^N(\pi) = \int_{\pi^L}^{\pi^U} \left[ \frac{1}{\pi^U - \pi^L} \right] \frac{1}{a+b} (a\bar{\pi} + b(\pi^e + u)) d\bar{\pi}.$$

which can be written as

$$E^N(\pi) = \int_{\pi^L}^{\pi^U} \left[ \frac{1}{\pi^U - \pi^L} \right] \left( \bar{\pi} + \frac{b}{a} u \right) d\bar{\pi} = \pi^s + \frac{b}{a} u.$$

Expected welfare without announcement is

$$E(w)^N = \int_{\pi^L}^{\pi^U} \left[ \frac{1}{\pi^U - \pi^L} \right] \left[ -\frac{a}{2} \left( \frac{1}{a+b} (a\bar{\pi} + b(\pi^e + u)) - \pi^s \right)^2 - \frac{b^s}{2} \left( \frac{1}{a+b} (a\bar{\pi} + b(\pi^e + u)) - \pi^e - u \right)^2 \right] d\bar{\pi}.$$

Using  $\pi^e = \pi^s + b/a u$ , this is

or

$$\begin{aligned}
E(w)^n &= \int_{\pi^L}^{\pi^U} \left[ \frac{1}{\pi^U - \pi^L} \right] \left[ -\frac{a}{2} \left( \frac{a}{a+b} (\bar{\pi} - \pi^s) + \frac{b}{a} u \right)^2 \right. \\
&\quad \left. - \frac{b^s}{2} \left( \frac{a}{a+b} (\bar{\pi} - \pi^s) + \left( \frac{b}{a} + 1 \right) u \right)^2 \right] d\bar{\pi} . \\
E(w)^n &= - \int_{\pi^L}^{\pi^U} \left[ \frac{1}{\pi^U - \pi^L} \right] \frac{a^2 + b^s a}{2(a+b)^2} (\bar{\pi} - \pi^s)^2 d\bar{\pi} - \\
&\quad - \frac{a}{2} \left( \frac{b}{a} \right)^2 u^2 - \frac{b^s}{2} \left( \left( \frac{b}{a} + 1 \right) \right)^2 u^2 .
\end{aligned}$$

c is infinite, so all the central bankers choose to announce  $\bar{\pi}$  and inflate  $\bar{\pi}$ . This follows from taking limits of  $\delta$  and (7). Therefore, expected welfare is simply:

$$\begin{aligned}
E(w)^a &= - \int_{\pi^L}^{\pi^U} \left[ \frac{1}{\pi^U - \pi^L} \right] \frac{a}{2} (\bar{\pi} - \pi^s)^2 d\bar{\pi} \\
&\quad - \frac{b^s}{2} u^2 .
\end{aligned}$$

The announcement therefore increases welfare iff

$$\begin{aligned}
\left( -\frac{a^2 + b^s a}{2(a+b)^2} + \frac{a}{2} \right) \int_{\pi^L}^{\pi^U} \left[ \frac{1}{\pi^U - \pi^L} \right] (\bar{\pi} - \pi^s)^2 d\bar{\pi} < \\
\frac{a}{2} \left( \frac{b}{a} \right)^2 u^2 + \frac{b^s}{2} \left( \frac{b}{a} + 1 \right)^2 u^2 - \frac{b^s}{2} u^2 .
\end{aligned}$$

or iff

$$\begin{aligned}
\left( -\frac{a^2 + b^s a}{2(a+b)^2} + \frac{a}{2} \right) \int_{\pi^L}^{\pi^U} \left[ \frac{1}{\pi^U - \pi^L} \right] (\bar{\pi} - \pi^s)^2 d\bar{\pi} < \\
\frac{a}{2} \left( \frac{b}{a} \right)^2 u^2 + \frac{b^s}{2} \left( \left( \frac{b}{a} \right)^2 + 2 \frac{b}{a} \right) u^2 .
\end{aligned}$$

The left hand side is decreasing with  $b^s/a$  and the right hand side is increasing. (i) and (ii) follow immediatly.

■

#### Announcent vs. Rule

Proposition 11 says that, if  $b^s/a$  is sufficiently large, an announcement is a beneficial institution because it reduces

inflation-uncertainty that comes from the uncertainty about the central banker's objective. Therefore it would be beneficial to impose a contract on the central banker, that punishes deviations from the self-imposed target announcement. One can argue that a contract that severely punishes the deviation from society's desired inflation rate would perform better. Imagine a punishment of the form  $d(\pi - \hat{\pi})^2$ . With such a contract the utility function of the central banker is:

$$z_t = -\frac{a}{2} (\pi - \bar{\pi})^2 - \frac{b}{2} (\pi - \pi^e - u_{t-1})^2 - \frac{d}{2} (\pi - \hat{\pi})^2. \quad (8)$$

It would then be optimal to choose  $\hat{\pi} = \pi^s$  and  $d$  infinitely large. Society then obtains  $-b^s/2 u^2$ . Such a rule, however, is only helpful if it can not be changed easily. Otherwise the well known time-inconsistency problem arises, because  $\hat{\pi}$  could be adjusted after nominal contracts are written. This is why the rule has the disadvantage of inflexibility. This is a problem, if society's objective  $\pi^s$  can change over time. In this section, I compare the optimal rule and the optimal contract with announcement under the following three assumptions:

**Assumption 3** Given the rule  $\hat{\pi}$ , society's mean FEIR,  $\pi^s$ , is distributed uniformly on  $[\hat{\pi} - X, \hat{\pi} + X]$  with  $X$  nonnegative.

**Assumption 4** Given society's mean  $\pi^s$ , the FEIR of the central banker in office is distributed uniformly on  $[\pi^s - Y, \pi^s + Y]$  with  $Y$  nonnegative.

**Assumption 5** The rule can not be made contingent on  $\pi^s$ .

Proposition 12 compares expected welfare in two different games: the game from section 4.1, with announcement and the optimal value for  $c$  (i.e.  $c$  is infinite), and the same game with a rule where  $d$  is chosen optimally. In the first stage of both games the policymakers are determined according to assumption 3 and 4.

**Proposition 12** Assume assumption 3,4 and 5. Given the divergence of the objectives of the potential policymakers ( $Y$ ) there always is a minimum value  $X^* > 0$  for  $X$  so that for all  $X > X^*$  society prefers the best contract with announcement to the best contract with a rule.

**Proof 12**

I call  $W^A(Y)$  the maximal value of expected welfare obtainable in the announcement-game. I call  $W^R(X,Y)$  the maximal value of expected welfare in the game with a rule.  $W^A$  is obtained under  $c = \infty$  and is independent of  $X$  because  $\hat{\pi}$  and  $\pi^s$  do not play a role in any of the decisions taken in the game. We have  $W^A(0) = W^R(0,Y) = b^s/2u^2$  for all nonnegative values of  $Y$ . We also need that

$$\frac{\partial W^R(X,Y)}{\partial X} < 0 \quad \forall X, Y \geq 0.$$

This can be seen as follows: Take any value for  $d$ , given  $X$  and  $Y$ . If  $X$  increases welfare decreases. So the optimal value for  $d$  under  $X$  and  $Y$  does outperform the optimal value for  $d$  under any higher  $X$ . Therefore  $W^R$  decreases with  $X$ .  $W^R$  is not bounded below. From this and  $W^A(0) = W^R(0,Y)$  follows the proposition. ■

The result is intuitive: the rigid rule is more costly, the more



volatile the inflation-objective of society is. The announcement is therefore preferable to the rule if society's objective is sufficiently volatile. We have seen that monetary target announcements contain some information. They work, even though it does not always look like that. The public can use the announcements to extract information about the central banker's full-employment inflation rate. We see that results are sensitive to our assumption on the type space. The assumption that  $\bar{\pi}$  is private information leads to a refinement of the welfare results from section 3.

## 6. Conclusion

In our first game with hysteresis on the labour market, we have seen that the results from Backus and Driffill (1985) fail to persist if there is hysteresis: in this case, the weak policymaker always reveals himself when he chooses inflation. This confirms the result by Vickers (1986), who examines a game where the conservative policymaker is not restricted to choose a certain inflation rate and where he can distinguish himself by the choice of a very low inflation rate. The reason is different in my model: persistent gains from playing tough in repeated prisoners'-dilemma situations abolish the disciplinary effects of reputation.

We have examined an alternative mechanism that shall help to solve the time inconsistency problem: monetary target announcements. In view of recurring deviations of monetary policy from target announcements one might be tempted to see announcements as totally useless for predicting money growth and therefore for establishing a low inflation rate. The signaling games in sections 3 and 6 of this paper shows that this need not be correct. Using a model that incorporates increasing costs of deviations from target announcements I show that target announcements can be welfare improving institution although policy deviates from the announcements. The results are obtained from a model without shocks that would require an intervention of the central bank. If such shocks were present some positive value for the marginal cost of deviations should prove to be

optimal.

It is interesting to see that announcements are used deliberately by both types in spite of their costs: they are painful for the central banker because deviations are costly. But - for the same reason - they moderate expectations. We have seen that for both central bankers the beneficial effect of expectation-moderation is dominant whatever the costs of deviations are.

Social (or financial) pressure on the central banker to keep his promises is beneficial with respect to expected welfare. One may object that the easiest way to reduce inflation uncertainty is to create incentives to stick to a given rule. The price that has to be paid for such a mechanism, however, is a loss of flexibility if rules cannot be changed. As soon as one allows that the rule may be changed by government and the government's preferences are uncertain, one obtains a result similar to the no-announcement case from section 4 with high inflation-uncertainty. The above analysis offers an alternative to giving direct incentives for a central banker to stick to a certain inflation rate. Incentives to stick to target announcements reduce inflation uncertainty without putting central bank independence in danger. Or to put it differently: central bank independence necessarily produces uncertainty that can be reduced through incentives to stick to a target announcement. We have seen that the announcement outperforms the rule if societies inflation-objective is sufficiently variable and if the central banker's inflation-objective is linked to society's objective.

We have also seen that an announcement is beneficial if society is sufficiently interested in the output target. But we have seen that the revelation of information is not always beneficial to society because partial revelation gives the banker the possibility to pursue at low cost a policy which is far away from the socially desired outcome.

Our second normative result is that the central banker's deviation-cost parameter  $c$  should be as large as possible. In this case, however, the central banker will not react to shocks that occur during the period. In the presence of shocks that can be offset by monetary fine tuning, an intermediate value for  $c$  will be optimal (see Rogoff (1985) for the elaboration of this argument in a model without signaling). One way to avoid this trade-off between flexibility and the possibility to commit might be target ranges for announced policy. They should provide some space to react, even for the central banker, who is bound strongly to his announcement.

It follows from the above analysis, that a strong link between society and central banker through target announcements is desirable. One can argue that the strength of such a link is related to the number of persons that decide on the course of monetary policy. The bad publicity that results from a missed target might be stronger for one single person that is solely responsible for monetary policy than for the members of a large and anonymous central bank council. This is because there exists the possibility that a member of a council did not vote in favor

of a certain decision if the decisions are taken anonymously. Note that this result changes radically if society does not punish a banker for dishonest behavior but for carrying out a conservative policy. In this case even a conservative central banker, who is strongly interested in his public image, is under pressure to create surprise inflation if expected inflation is low. The public anticipates this and one obtains the discretionary outcome as in the Barro-Gordon (1983)-paper. The consequence would be that societies which are not concerned about monetary stability are better off with a central banker who is independent of the public. However, announcements lose precision in this case. Central banks can therefore work better, the more concerned about stability a society is. Apart from such rather speculative psychological arguments, our analysis shows that financial incentives to stick to an announced policy unambiguously have positive welfare effects.

#### **Notes:**

1) Another way to model this would be to introduce directly one dependable type, an updating function, changing preferences and a control error. Our short-cut is meant to prevent untractable calculations in an already complicated signaling game.

2) There is a number of determinants of the full-employment inflation rate: A first rate desired by a policymaker might be the zero-inflation rate because deviations induce "shoe-leather-costs" and because inflation uncertainty rises with inflation.

(This correlation with uncertainty has to be exogenous to the model. It can also be the consequence of a game: A correlation arises from this model because higher unemployment will lead to a higher expected value of inflation and more variance. See also Ball (1992) for a model where this correlation is explained with a monetary policy game). For an country which participates in a fixed exchange rate system the inflation rate of another participating country will be another target. A third target is obtained if we consider the government's interest in seignorage revenues. For the loss from deviation from a number of  $k$  targets,  $\bar{\pi}_i$  ( $i=1..k$ ), we may write in general

$$-\frac{1}{2} \sum_{i=1}^k \eta_i (\pi - \bar{\pi}_i)^2, \text{ with } \eta_i > 0, i=1, \dots, k,$$

and we can additionally introduce the employment target and the announcement:

$$-\frac{1}{2} \sum_{i=1}^{k+2} \eta_i (\pi - \bar{\pi}_i)^2 \text{ with } \bar{\pi}_{k+1} = \pi^e + u_{t-1}, \bar{\pi}_{k+2} = \pi^a.$$

Maximization of this expression yields the same as the maximization of

$$-[\frac{1}{2} \sum_{i=1}^k \eta_i] [\pi - \sum_{i=1}^k \frac{\eta_i}{\sum_{i=1}^k \eta_i} \bar{\pi}_i]^2 - \frac{1}{2} \eta_{k+1} (\pi^e - u_{t-1})^2 - \frac{1}{2} \eta_{k+2} (\pi - \pi^a)^2$$

or by defining

$$a := \sum_{i=1}^k \eta_i, \quad b := \eta_{k+1}, \quad c := \eta_{k+2}, \quad \bar{\pi} = \sum_{i=1}^k \frac{\eta_i}{\sum_{i=1}^k \eta_i} \bar{\pi}_i,$$

as the result of the maximization of (1). We therefore obtain our above utility function. Our assumption that both types have the same full-employment inflation target  $\bar{\pi}$  therefore means that they attach the same relative weight to the first  $k$  targets and that they differ in the relative valuations of the employment

target. Entering a fixed exchange rate-system changes the values of  $a$  and of the FEIR itself.

3) Grüner (1993a) examines the game under the assumption that the type space is two-dimensional.

4) It is easily seen that there exists no equilibrium with  $\pi^c = k\pi^a + \delta$  where  $k < 1$  or  $k > 1$ .

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