Broll, Udo; Zilcha, Itzhak

Working Paper

Exchange rate uncertainty, futures markets and the multinational firm

Diskussionsbeiträge - Serie II, No. 139

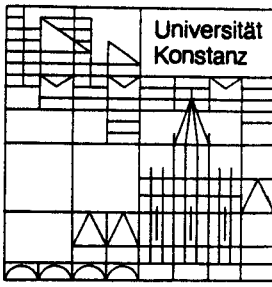
Provided in Cooperation with:
Department of Economics, University of Konstanz

Suggested Citation: Broll, Udo; Zilcha, Itzhak (1991) : Exchange rate uncertainty, futures markets and the multinational firm, Diskussionsbeiträge - Serie II, No. 139, Universität Konstanz, Sonderforschungsbereich 178 - Internationalisierung der Wirtschaft, Konstanz

This Version is available at:
https://hdl.handle.net/10419/101538

Sonderforschungsbereich 178
„Internationalisierung der Wirtschaft“

Diskussionsbeiträge



Juristische
Fakultät

Fakultät für Wirtschafts-
wissenschaften und Statistik

Udo Broll
Itzhak Zilcha

Exchange Rate Uncertainty, Futures Markets and the Multinational Firm

Serie II — Nr. 139
Mai 1991

Exchange Rate Uncertainty, Futures Markets and the Multinational Firm

Udo BROLL and Itzhak ZILCHA*

University of Konstanz, D–7750 Konstanz, FRG

University of Tel Aviv, Ramat Aviv 69978, Israel

Abstract

We consider a monopolistic, risk–averse multinational firm which sells and produces at home and abroad under exchange rate uncertainty. First we show that the stochastic exchange rate implies higher production and lower sales in the foreign country. Then we analyze the impact of currency futures markets. A separation result is obtained for a multinational firm, i.e., production and the allocation of sales are independent of the distribution of the random exchange rate and of the firm's attitude towards risk. We also examine the effect of currency futures on a multinational firm's foreign direct investments. In the absence of futures markets we obtain some comparative statics results when risk aversion increases.

*We would like to thank both our referees for their very constructive and useful suggestions and comments.

1 Introduction

The importance of multinational firms (MNFs) which own and control assets in different countries has grown rapidly over the last decade [e.g. Casson (1987); Shapiro (1989)]. MNFs have increased continuously their share of international trade and investments [see Lipsey (1989)]. About one third of the world's trade consists of trade between subsidiaries and branches within MNFs [UNCTC (1988)].

Foreign direct investments, international production and sales reflect a world in which technology and capital have become increasingly mobile. Due to the continuous changes in comparative advantages among national economies, advances in international communication, transportation and government policies, more and more firms now distribute their production plants and market service systems around the globe.

On the other hand in recent years we witness some greater fluctuations in the exchange rates of the major currencies. For international firms the need to manage foreign currency risks has increased substantially during the last decade [see Krugman (1989)]. Consequently in the developed countries a variety of hedging techniques are being offered by financial markets. However this is not true in many less developed countries where currency future markets, currency options or equivalent arrangements are either limited or nonexistent.

Recently there have been many contributions to the theory of exporting firms under stochastic currencies where the effects and the role of futures markets have been analyzed [Benninga, Eldor and Zilcha (1985), Stein (1986), Eldor and Zilcha (1987)]. One important result of these studies is that in the presence of futures markets the exporting firm will choose its output as if it were in a position of certainty. This property is known als the 'Separation property' and was first shown by Danthine (1978) for competitive firms under price uncertainty. In this paper we analyze the implications of currency

futures markets in the context of a two–country firm which is selling as well as producing at home and abroad.[1] Our objective is to investigate the effects of exchange rate uncertainty and the role of futures markets upon the international production, sales and direct investment of a risk–averse monopolistic MNF. In our model the foreign direct investments affect the demand side abroad rather than the technology in the foreign country.

Suppose that a horizontally integrated, price discriminating MNF produces and sells the same differentiated commodity in two countries. We allow the production technology used in each country to differ. Our model includes the case where the MNF has a firm–specific advantage that enables it to operate at home and in the foreign country as a monopolist [see Helpman and Krugman (1985), Horstmann and Markusen (1989)]. Each country's currency is freely exchanged via some *random* exchange rate. Purchasing power parity does not hold. The firm chooses production and sales in both countries to maximize expected profits denominated in the home currency [see Calderon–Rossell (1985)]. In our model it is assumed that production levels, allocation of sales and direct investments, must be determined *before* the exchange rate is known.

In *section 2* the model is described. In *section 3* we study the effects of uncertainty in the exchange rate. We show that the stochastic exchange rate implies higher production in the foreign country and lower sales abroad. In *section 4* we analyze the impact of currency futures markets. A *separation result* [see Danthine (1978)] is obtained for this two–country firm, i.e., when a currency futures market is available *production* and the *allocation of sales* are independent of the distribution of the random exchange rate and of the firm's attitude towards risk. In *section 5* we study the effect of currency futures on this firm's foreign direct investment. In *section 6* we derive the effect of increases in the firm's risk aversion in the absence of futures markets.

[1]This aspect was also developed in a different framework by Itagaki (1981).

2 Multinational Firm

Consider a multinational firm (MNF) producing the same different commodity in the home country and abroad.[2] The production functions in the two countries may differ, this is reflected by the cost functions: $C(x)$ is the cost function in the domestic country and $C^*(x^*)$ is the cost function abroad (denominated in foreign exchange), where x, x^* are the outputs. We assume that the cost functions satisfy, as assumed often in economic models, $C'(x) > 0$, $C''(x) \geq 0, C^{*'}(x^*) > 0$ and $C^{*''}(x^*) \geq 0$. Thus we assume nondecreasing marginal costs.

The MNF sells its product in the domestic market and in the foreign market, where in both segmented markets it has a monopoly power. We denote the total revenue functions by $R(y)$ in the home market and by $R^*(y^*)$ in the foreign market. We assume that both $R(y)$ and $R^*(y^*)$ are concave functions.

The MNF chooses the levels of production, sales and trade between the two markets *before* the exchange rate is known. To simplify our analysis we assume that the cost of transportation in *intra-firm trade*, i.e., trade between the parent and foreign subsidiary of the MNF, is negligible. We also assume that the MNF is risk–averse. The only source of uncertainty in our economy is the exchange rate $\tilde{e}$, which has a known distribution function. Thus the multinational firm is assumed to maximize the managers' expected utility of profits denominated in the home currency subject to technological and markets constraints. Given the von Neumann–Morgenstern utility $U(\cdot)$, where $U' > 0, U'' < 0$, the firm chooses its product levels (x, x^*) and sales (y, y^*) in both countries. So the decision problem can be written as (under the market constraint $x + x^* = y + y^*$):

$$max_{(x,x^*,y)} EU(\tilde{\Pi}) \tag{1}$$

[2]We denote by * the *foreign* quantities.

$$\text{s.t.} \quad \tilde{\Pi} = R(y) + \tilde{e}R^*(x + x^* - y) - C(x) - \tilde{e}C^*(x^*). \tag{2}$$

We shall not consider here the extreme case where it is optimal to produce in one country only, i.e., $x = 0$ or $x^* = 0$.[3] Since the cost functions are convex and the total revenue functions are concave, Π is concave in x, x^* and y. Thus, necessary and sufficient conditions for a maximum in problem (1), assuming that x, x^* and y are positive at the optimum are

$$E[R'(y) - \tilde{e}R^{*'}(y^*)]U'(\tilde{\Pi}) = 0 \tag{3}$$

$$E[\tilde{e}R^{*'}(y^*) - C'(x)]U'(\tilde{\Pi}) = 0 \tag{4}$$

$$E[\tilde{e}R^{*'}(y^*) - \tilde{e}C^{*'}(x^*)]U'(\tilde{\Pi}) = 0. \tag{5}$$

The expectation is with respect to the given distribution function of $\tilde{e}$. In the following sections we shall use these first order conditions in order to explore the effects of uncertainty in the exchange rate on the MNF's global decisions, intra–firm trade transactions and futures contracting.

3 The Effect of Uncertainty

Uncertainty in the exchange rate has real effects on production, sales and intra–firm trade structure. In our study we suppose that the net foreign revenue is not negative $R^*(y^*) - C^*(x^*) \geq 0$. This is a crucial assumption and implies that the MNF is making a non negative profit abroad.[4] To explore the impact of uncertainty let us use (3)–(5). Since $E\tilde{e}U'(\tilde{\Pi}) > 0$ we obtain from (5) that,

$$R^{*'}(y^*) = C^{*'}(x^*). \tag{6}$$

[3]The model where $x^* \equiv 0$ has been treated by Eldor and Zilcha (1987). Production takes place at the home country only while the product is sold at the domestic and foreign markets.

[4]For a study which allows for positive as well as negative profits under certain conditions of international taxation see Itagaki (1981), and Cushman (1988).

Also, from (3) and (4) we obtain that

$$R'(y) = C'(x). \tag{7}$$

Using (4) and (5) we derive also,

$$E[C'(x) - \tilde{e}C^{*'}(x^*)]U'(\tilde{\Pi}) = 0. \tag{8}$$

Since $\tilde{\Pi}$ increases as $\tilde{e}$ attains higher realizations (note that at the optimum $R^*(y^*) - C^*(x^*) \geq 0$) and since U' is a decreasing function, by our assumption that $U'' < 0$, we have $Cov(\tilde{e}, U'(\tilde{\Pi})) < 0$. Thus (8) yields (with $E\tilde{e} \equiv \bar{e}$),

$$\bar{e}C^{*'}(x^*) - C'(x) > 0. \tag{9}$$

Denote by x_c^* and x_c the optimal output when $\bar{e}$ is the exchange rate, i.e., the *certainty-equivalent* case. Then from the optimality conditions for the certainty case, we obtain

$$\bar{e}C^{*'}(x_c^*) = C'(x_c). \tag{10}$$

Now we can state the following relations regarding the optimal levels of production used under uncertainty and under its certainty equivalent.

Lemma: $x^* > x_c^*$ and $x < x_c$.

Proof: Assume that $x \geq x_c$, then by assumptions about the cost functions $C'(x) \geq C'(x_c)$. This implies, using (7), that $R'(y) \geq R'(y_c)$, i.e., $y \leq y_c$. Since $R'(y_c) = \bar{e}R^{*'}(y_c^*)$ and from (3) we derive also,

$$R'(y) - \bar{e}R^{*'}(y^*) < 0. \tag{11}$$

We obtain that $R^{*'}(y^*) > R^{*'}(y_c^*)$ hence $y^* < y_c^*$. This implies that $y + y^* < y_c + y_c^*$, i.e., total sales in the certainty equivalent case is higher. However, since by our assumption $x \geq x_c$ and $x + x^* = y + y^*$, we must have $x^* < x_c^*$. But from (6), (9) and (10) we have: $C^{*'}(x^*) = R^{*'}(y^*) > R^{*'}(y_c^*) = C^{*'}(x_c^*)$. Which implies that $x^* > x_c^*$. This contradiction proves that $x < x_c$. Assume now, to

the contrary of our claim, that $x^* \leq x_c^*$. Then $R^{*'}(y^*) = C^{*'}(x^*) \leq C^{*'}(x_c^*) = R^{*'}(y_c^*)$. Hence, since $R^{*'}$ is a decreasing function, $y^* \geq y_c^*$. Also, by our earlier result $x < x_c$ hence by a similar argument it implies that $R'(y) < R'(y_c)$, i.e., $y > y_c$. But $y_c^* + y_c = x_c^* + x_c$ thus we obtain a contradiction. This shows that we must have $x^* > x_c^*$.

Theorem 1. Uncertainty in the exchange rate results in: (a) higher production abraod and lower production in the home country (i.e., $x^* > x_c^*$ and $x < x_c$); (b) lower sales in the foreign country and higher sales in the home country (i.e., $y^* < y_c^*$ and $y > y_c$).

The *proof* is a direct result of the *Lemma* and (6) and (7).

It has been shown in Theorem 1 that the MNF repatriates profits under exchange rate risk by repatriating the physical output, thus reducing its net–exposure to this risk. Or, in other words, the MNF *internalizes* the missing hedging markets by increasing foreign production and lowering foreign sales thus increasing costs and decreasing revenues denominated in foreign currency. This is a further example where MNFs can internalize some market imperfections [see Helpman and Krugman (1985)]. In our model the internalization of the MNF encourages foreign production and intra–firm trade when there are no external financial markets for hedging transactions.

4 Futures Markets

Consider now the behavior of the MNF in the presence of foreign currency futures markets.[5] In this case when the firm makes its decision about the production levels and allocation of the sales in the two markets, it chooses its optimal contract for currency sale in the futures market. Suppose the forward exchange rate e_f is given, then z is the amount of its futures currency

[5]There is no basis risk, which implies that there is no distinction between futures and forward markets.

commitment. The MNF maximizes now

$$max_{(x,x^*,y,z)} EU(\tilde{\Pi}) \tag{12}$$
$$\text{s.t.} \quad \tilde{\Pi} = R(y) + \tilde{e}R^*(x + x^* - y) - C(x) - \tilde{e}C^*(x^*) + z(e_f - \tilde{e}).$$

The necessary and sufficient conditions for an optimum are, basically, conditions (3), (4) and (5) for this case and an additional obtained by differentiating w.r.t. z, i.e.,

$$E(e_f - \tilde{e})U'(\tilde{\Pi}) = 0. \tag{13}$$

From (3) and (13) we obtain

$$R'(y) = e_f R^{*'}(y^*). \tag{14}$$

Like in the classical case of a price–discriminating firm, the MNF allocates its output in a way that equates marginal revenues in both markets taking the futures rate as the exchange rate. Condition (14) implies that the MNF is hedging perfectly against the foreign *marginal* revenue risk. This hedging behavior is valid even when the MNF finds it not optimal to hedge against the *total* net revenue risk.

From (4), (5) and (14) we also derive that,

$$C'(x) = e_f C^{*'}(x^*). \tag{15}$$

Thus the optimization in the presence of currency futures markets results in equating the marginal cost in both countries, where e_f is the contemplated exchange rate. Now we prove a *separation* theorem for this model.

Theorem 2. (Separation). When currency futures markets are available the MNF's optimal decisions about production and allocation of sales are given by (14) and (15). Hence, they are independent of its attitude towards risk and the distribution function of the random exchange rate.

Thus, Theorem 2 shows that when a futures market is available the MNF behaves just as in the full certainty case where e_f plays the role of the "certain" exchange rate: The allocation of production and the intra–firm trade to

allocate sales are determined by the usual rules, equating marginal revenues and marginal costs.[6]

The separation theorem was first proved by Danthine (1978) for a competitive firm [see also Kawai and Zilcha (1986)]. Its extension to our model demonstrates that the MNF's decisions are independent of the utility function or the probability distribution of the exchange rate changes. This means that two MNFs with identical revenue functions and technologies but with different attitudes towards risk and different probability beliefs will choose the same levels of outputs and sales.

However, the *hedging behavior* of the MNF will depend on the attitude towards risk *and* the distribution of $\tilde{e}$. If the risk premium is positive, i.e., $e_f < \bar{e}$, then the futures sale of currency is less than the net foreign revenue. With unbiasedness, i.e., $e_f = \bar{e}$, the firm hedges such that its profits are totally independent of the realization of the exchange rate. If the risk premium is negative (i.e., $e_f > \bar{e}$) the MNF sells forward more than its net revenues abroad, i.e., it speculates. This is summarized in (where $\hat{z}$ is the optimal futures sales of currency)

Corollary. Unbiasedness case: If $e_f = \bar{e}$ then $\hat{z} = R^*(y^*) - C^*(x^*)$, hence $\tilde{\Pi}$ is independent of $\tilde{e}$. This is the full–hedging case. *Normal backwardation*: If $e_f < \bar{e}$ then the firm *hedges*, i.e., $\hat{z} < R^*(y^*) - C^*(x^*)$. *Contango*: If $e_f > \bar{e}$ then the firm *overhedges* ,i.e., $\hat{z} > R^*(y^*) - C^*(x^*)$.

Proof: Let us rewrite $\tilde{\Pi}$ as follows $\tilde{\Pi} = R(y) - C(x) + \hat{z}e_f + \tilde{e}[R^*(y^*) - C^*(x^*) - \hat{z}]$. Using (13), we obtain that

$$(e_f - \bar{e})EU'(\tilde{\Pi}) = Cov(\tilde{e}, U'(\tilde{\Pi})). \tag{16}$$

If the LHS of (16) is zero, hence the covariance of $\tilde{e}$ with the strictly decreasing function of $\tilde{\Pi}$, $U'(\tilde{\Pi})$, is zero then $\tilde{\Pi}$ must be independent of $\tilde{e}$, i.e.,

[6]Note that when the forward price is unbiased, i.e., $e_f = \bar{e}$, the values of x, x^*, y and y^* are the same as in the certainty equivalent case.

$\hat{z} = R^*(y^*) - C^*(x^*)$. Similarly if the LHS of (16) is positive, $\tilde{\Pi}$ must be a decreasing function of $\tilde{e}$ (since the covariance is positive in this case), i.e., $\hat{z} > R^*(y^*) - C^*(x^*)$ (and similarly for normal backwardation).[7]

5 Foreign Investments and Futures Markets

Let us expand our model to include the possibility that the MNF can invest in the foreign market in a way that affects the demand for its product. In reality we observe many such cases, for example Japanese car producers invest in the United States and the EC in advertising, dealership, and other service facilities that increase the attractiveness of their cars. This type of foreign direct investment does not include investment in improving the technology of production in the foreign country.

To include such foreign investments in our two-country firm model let us assume that the *total revenue* in the foreign market depends not only upon the sales y^* but also upon such foreign direct investments S^*, i.e., $R^* = R^*(y^*, S^*)$. We assume that R^* satisfies the following conditions: (a) $R^*(y^*, S^*)$ is increasing in S^*, i.e., $R^*_{S^*} = \partial R^*/\partial S^* > 0$, but at a decreasing rate, namely $R^*_{S^*S^*} = \partial^2 R^*/\partial S^{*2} \leq 0$; (b) the marginal revenue $R^{*\prime}(y^*, S^*)$ is non-decreasing in S^*, i.e., $R^{*\prime}_{S^*} = \partial^2 R^*/\partial y^* \partial S^* \geq 0$.

Condition (a) is economically clear. Condition (b) requires that as the firm invests more the marginal revenue can either increase or remain unchanged. This includes the case where the addition to total revenue function is an increasing function of S^* (i.e., independent of the level of sales). Now let us rewrite the firm's optimization problem with foreign investment, assuming that futures currency markets exist.

$$max_{(x,x^*,y,S^*,z)} EU(\tilde{\Pi}) \qquad \text{s.t.} \tag{17}$$

[7]The expressions normal backwardation and contango are used to describe downward and upward biasedness of the expected exchange rate.

$$\tilde{\Pi} = R(y) + \tilde{e}R^*(x + x^* - y; S^*) - C(x) - \tilde{e}C^*(x^*) - \tilde{e}S^* + z(e_f - \tilde{e}).$$

The first order conditions are (3)–(5), (13) and

$$E[\tilde{e}R^*_{S^*}(y^*_f, S^*_f) - \tilde{e}]U'(\tilde{\Pi}) = 0, \tag{18}$$

where S^*_f and y^*_f denote the optimum foreign investment and foreign sales in the presence of futures markets. Since $E\tilde{e}U'(\tilde{\Pi}) > 0$, we obtain from (18),

$$R^*_{S^*}(y^*_f, S^*_f) = 1. \tag{19}$$

Now let us prove that introducing currency futures market results in higher foreign direct investment if the risk premium is not too high. Denote by S^* the optimal foreign investments, under exchange rate uncertainty, without currency futures market.

Theorem 3. The introduction of a currency futures market which is unbiased results in a higher foreign investment of the multinational firm, i.e., $S^*_f > S^*$.

Proof: Let us note first that the optimal foreign sales and foreign investment change in the same direction, as we vary $\bar{e}$ or the variance of $\tilde{e}$. This follows directly from (19) which holds always, since $\partial R^*_{S*}/\partial y^* > 0$ and $R^*_{S^*S^*} < 0$. Consider the case where the futures market is unbiased. As we have shown earlier in the presence of futures markets sales y^* (and other decision variables) are independent of the distribution of $\tilde{e}$. Hence if $e_f = \bar{e}$ then $y^*_f = y^*_c$, where y^*_c and S^*_c are the *certainty equivalent* foreign sales and foreign investment. By Theorem 1, $y^*_c > y^*$ therefore we obtain from $R^*_{S^*}(y^*_f, S^*_f) = R^*_{S^*}(y^*_c, S^*_c) = R^*_{S^*}(y^*, S^*) = 1$ that $R^*_{S^*}(y^*_f, S^*_f) < R^*_{S^*}(y^*_f, S^*)$ which implies that $S^*_f > S^*$. Thus whenever $e_f = \bar{e}$ we obtain that $S^*_f > S^*$. Due to the continuity of all the functions this inequality holds for $e_f \geq \bar{e} - \bar{\theta}$, for some positive $\bar{\theta}$ which depends on the various parameters of the model.

Let us consider now the effects of changes in the futures exchange rate e_f on the foreign direct investment S^*_f. Such variations may result from changes

in the expected value of $\tilde{e}$, i.e., $\bar{e}$ or perhaps as a consequence of changes in the variance of $\tilde{e}$, which will be reflected in the futures price.

Theorem 4. In the presence of currency futures market, an increase in the futures price e_f will result in higher foreign investment S_f^* and higher sales abroad y_f^*.

Proof: The proof is based on (14), (15) and (19) which hold in the presence of a futures market. Assume that the contrary to our claim holds, i.e., S_f^* declines as a result of higher e_f. This implies by (19) that y_f^* declines as well when e_f increases since

$$R_{S^*}^{*'} \frac{\partial y_f^*}{\partial e_f} + R_{S^*S^*}^{*} \frac{\partial S_f^*}{\partial e_f} = 0. \tag{20}$$

Thus, using the strict concavity of $R^*(y^*, S^*)$ in S^*,

$$sgn \frac{\partial y_f^*}{\partial e_f} = sgn \frac{\partial S_f^*}{\partial e_f}. \tag{21}$$

Therefore, by our assumption about $y_f^*, e_f R^{*'}(y_f^*)$ increases as e_f increases hence, by (14), the domestic sales y decreases and thus $y + y_f^*$ decreases. Now let us apply (6) and (7). The decline of y_f^* and y will cause x^* and x to increase (since the marginal cost functions are increasing). Thus $x + x^*$ increase. But $x + x^* = y + y_f^*$, which is a contradiction. Therefore we conclude that $\partial y_f^*/\partial e_f \geq 0$. By (21), this implies that $\partial S_f^*/\partial e_f \geq 0$ which proves the theorem.

As in Theorem 4 we see that if the currency futures price is positively correlated to $\bar{e}$ then an increase in $\bar{e}$ results in higher foreign investment S_f^*. In general, if we know how a mean-preserving spread in the distribution of $\tilde{e}$ affects the currency futures price e_f we can use Theorem 4 to analyze the impact of such an increase in the $Var(\tilde{e})$ on the foreign investment S_f^*. For example, if $\partial e_f/\partial \sigma^2 < 0$ (where σ^2 is the variance of $\tilde{e}$ for a fixed $\bar{e}$) then we conclude that $\partial S_f^*/\partial \sigma^2 < 0$.

For example when the market is expecting an appreciation of the foreign currency the MNF increases the amount of foreign direct investment and increases foreign sales. Clearly this result is related to the specific type of foreign direct investment. In our model direct investments are a decision variable which directly improve the market condition for foreign sales. Direct investments with other characteristics, for example affecting the foreign technology of production, would imply different effects with respect to the interaction between expected changes in the exchange rate and foreign investment activities.

6 The Effect of Increasing Risk Aversion

In this section we compare the decisions about production, sales and trade of two MNFs, which differ only in their attitude towards risk. We consider the case where no foreign investment takes place (i.e., $S^* = 0$) and where futures markets do not exist; since otherwise the firms' behavior is independent of the utility function (from the *separation* theorem). Now we claim:

Theorem 5. Consider a multinational firm when currency futures markets are not available. Increasing risk aversion results in: (a) higher sales in the domestic market and lower sales in the foreign market; (b) higher production in the foreign country and lower production in the domestic country.

Thus if risk aversion increases (and if futures markets are absent) more intra–firm trade is induced.

Proof: Let us use (7) to define the function: $x = \alpha(y)$. Since $R'(y)$ is decreasing and $C'(x)$ increasing, it is easy to verify that $\alpha' \leq 0$. Now when we consider the optimization problem (1) we can maximize the expected utility of profits with respect to (x^*, y) replacing x by $\alpha(y)$. In this case equation (3) reduces to,

$$E[R'(y) - \tilde{e}(1 - \alpha')R^{*'}(y^*) - \alpha' C'(\alpha(y))]U'(\tilde{\Pi}) = 0. \qquad (22)$$

Let one MNF, which is either risk averse or risk neutral, have a utility function U while the other's utility is $V(\Pi) \equiv f[U(\Pi)]$, with f satisfying $f' > 0, f'' < 0$; i.e., V is a concave transformation of U. The optimality conditions for the firm V (denoted by $(^-)$) are:

$$E\{[R'(\bar{y}) - \tilde{e}(1-\alpha')R^{*\prime}(\bar{y}^*) - \alpha' C'(\alpha(\bar{y}))]f'(U(\bar{\Pi}))U'(\bar{\Pi})\} = 0 \quad (23)$$

$$E\{[\tilde{e}R^{*\prime}(\bar{y}^*) - \tilde{e}C^{*\prime}(\bar{x}^*)]f'(U(\bar{\Pi}))U'(\bar{\Pi})\} = 0. \quad (24)$$

Now we shall use the fact that f' and U' are decreasing while $\bar{\Pi}$ is an increasing function of $\tilde{e}$, hence

$$Cov\{f'(U(\bar{\Pi})); [R'(\bar{y}) - \tilde{e}(1-\alpha')R^{*\prime}(\bar{y}^*) - \alpha' C'(\alpha(\bar{y}))]U'(\bar{\Pi})\} > 0.$$

Thus by (23) we derive that

$$E\{[R'(\bar{y}) - \tilde{e}(1-\alpha')R^{*\prime}(\bar{y}^*) - \alpha' C'(\alpha(\bar{y}))]U'(\bar{\Pi})\} < 0. \quad (25)$$

Now consider maximization of $EV(\bar{\Pi})$ with respect to y and x^*. Since the maximand is a concave function of $(\bar{y}, \bar{x}^*)$ then comparing the optimal levels of V (determined by (23) and (24)) with the optimal levels of (y, x^*) when we maximize $EU(\tilde{\Pi})$, we come to the following conclusion: Comparing (25) and (5) with (22) and (5), we find that

$$\bar{y} > y \quad \text{and} \quad \bar{x}^* > x^* \quad . \quad (26)$$

Using (6) and (7) we derive from (26) that $x > \bar{x}$ and $\bar{y}^* < y^*$, which completes the proof.

7 Concluding Remarks

Futures markets are frequently used by international commercial firms to insure against price and exchange rate risks. This paper has provided a theoretical framework for analyzing the effects of futures markets in foreign exchange

on the allocation of production and sales of a two–country monopolistic firm. The cost functions, and hence technology of production, in the two countries may differ.

Our model assumes that the decisions of the firm regarding production and allocation of sales between the two markets are made *before* the realization of the random exchange rate. Such an approach has been taken in a model of an *exporting* firm by Katz, Paroush and Kahana (1982). A different approach can be found in the literature where all production decisions are prior to the realization of the exchange rate but decisions about sales in the two markets are made *after* the exchange rate is observed [Eldor and Zilcha (1987)].

Note that there is some "symmetry" between the home and the foreign countries in the profit function (2). Thus our assumption that the MNF maximizes expected profits denominated in the home currency may be replaced by an assumption that $\tilde{\Pi}$ is denominated in the foreign currency, without having much impact on the results. In some cases we should reverse the inequalities (e.g., in Theorem 1 and Theorem 5). However, the analysis can be conducted similarly.

The paper has extended to multinational firms the results in the literature demonstrating the importance of establishing currency futures markets for exporting firms which do not produce in both countries. Particulary the extension of the well–known 'Separation property' to the case of a multinational firm with some monopolistic power is very interesting.

References

Benninga, S.R., Eldor, R. and I. Zilcha, 1985, Optimal international hedging and output policies in commodity and currency forward markets, Journal of International Money and Finance 4, 537–552.

Calderon–Rossell, J. R., 1985, Towards the theory of foreign direct investment, Oxford Economic Papers 37, 282-291.

Casson, M., 1987, The firm and the market (Oxford University Press, Oxford).

Cushman, D.O., 1988, Exchange–rate uncertainty and foreign direct investment in the united states, Weltwirtschaftliches Archiv 124, 322–336.

Danthine, J.P., 1978, Information, Futures Markets and Stabilizing Speculation, Journal of Economic Theory 17, 79–98.

Eldor, R. and I. Zilcha, 1987, Discriminating monopoly, forward markets and international trade, International Economic Review 28, 459–469.

Helpman, E. and P. R. Krugman, 1985, Market Structure and Foreign Trade (MIT Press, Cambridge, London).

Horstmann, I. and J. R. Markusen, 1989, Firm–specific assets and the gains from direct foreign investment, Economica 56, 41–48.

Itagaki, T., 1981, The theory of the multinational firm under exchange rate uncertainty, Canadian Journal of Economics 14, 277–297.

Katz, E., Paroush, J. and N. Kahana, 1982, Price uncertainty and the price discriminating firm in international trade, International Economic Review 23, 389–400.

Kawai, M. and I. Zilcha, 1986, International trade with forward–futures markets under exchange rate price–uncertainty, Journal of International Economics 20, 83–98.

Krugman, P. R., 1989, Exchange–Rate Instability (MIT Press, Cambridge).

Leland, H.E., 1972, Theory of the firm facing uncertain demand, American Economic Review 62, 278–291.

Lipsey, R.E., 1989, The internationalization of production, National Bureau of Economic Research, Working Paper No. 2923.

Shapiro, A. C., 1989, Multinational Financial Management (Allyn and Bacon, Boston, London).

Stein, J. L., 1986, The Economics of Futures Markets (Basil Blackwell, Oxford).

UNCTC, 1988, (United Nations Centre of Transnational Corporations), Transnational Corporations in World Development, Trends and Prospects (United Nations, New York).