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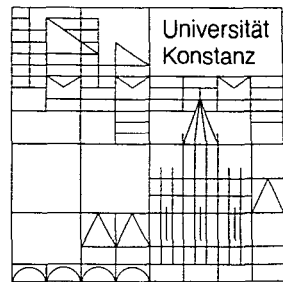
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Capital Mobility and Costs of Adjustment

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CAPITAL MOBILITY AND COSTS OF ADJUSTMENT

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Capital Mobility and Costs of Adjustment*

by Max Albert

The paper considers intersectoral capital mobility in the context of investment theory. Convex costs of adjustment explain imperfect mobility of capital between sectors. Stocks of capital are endogenous; the model essentially is a two-sector growth model with Keynesian investment functions. The paper analyzes adjustment in a small open economy with balanced and imbalanced trade and adjustment in a closed economy.

I Introduction

Models of the production sector used in trade theory traditionally entail a certain dichotomy between capital reallocation and accumulation processes. Thus it is quite common to interpret the specific-factors (SF) model as a short-run version of the Heckscher-Ohlin (HO) model (Jones & Neary 1984, p.26). According to this interpretation, adjustment of the economy to external shocks is driven by a slow process of capital reallocation in the direction of higher returns, while labor moves instantaneously between sectors to equalize wages. Each point on the adjustment path can be described by the SF model; the long-run equilibrium is described by the HO model. Capital accumulation, it seems, comes into effect only when reallocation is finished. This view is stated explicitly in the Oniki-Uzawa (OU) model in which adjustment to the long-run equilibrium is seen as a sequence of temporary HO equilibria.

From the point of view of investment theory, however, capital reallocation is only a side effect of investment and disinvestment, i.e. of capital accumulation. In many cases physical capital is sector-specific; in other cases - especially relevant for international capital movements - physical capital is immobile for all practical purposes. 'Mobility' in

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these cases is just a metaphor standing for investment and depreciation processes which involve no physical reallocation at all. Despite this fact most models concerned with international and intersectoral capital mobility consider only physical mobility. The present paper looks at the other - and, as I think, more important - extreme: the case where physical capital is completely immobile and capital reallocation is brought about by investment and depreciation alone.

Thus one may contrast the physical-mobility view with the investment-depreciation view on capital mobility. On the other hand there is the contrast between the static and dynamic perspective. This latter distinction is important if one looks at the literature concerned with imperfect or partial factor mobility. In the last years quite a few authors expressed their discontent with the extreme assumptions on factor mobility in the SF and the HO model (cf. e.g. Mussa 1982, Grossman 1983, Hill & Méndez 1983). They presented static models of partial factor mobility allowing for any degree of mobility between zero (perfect immobility) and infinity (perfect mobility). These models contain the SF and the HO model as special cases. At least in one case (Hill & Méndez 1983, p. 20) the degree of mobility is assumed to depend on the length of the adjustment period considered, connecting low degrees of mobility with the short run and high degrees of mobility with the long run. This interpretation of partial mobility matches with the interpretation of the SF model as a short-run version of the HO model. Much of the appeal of static partial-mobility models derives, I believe, from the fact that these models seem to describe something like the middle-run behavior of an economy: partial mobility accounts for the fact that adjustment is slow.

Slow adjustment, however, presupposes the existence of increasing marginal costs of adjustment speed or, with a more familiar term, convex costs of adjustment, since otherwise adjustment is instantaneous. Adjustment costs give rise to investment problems involving the optimum speed of adjustment. It is not at all clear that this adjustment process can be described adequately with the help of static models. The present paper shows how typical features of static partial-mobility models arise out of dynamic models. Explicit consideration of the dynamic model is not superfluous, however, since

factor mobility depends on expectations and the rate of interest. Both dependencies are not accounted for in the static models.

It is helpful to draw some distinctions between different kinds of capital mobility which are relevant in the present context. There are two kinds of capital mobility which we shall call *physical* and *financial* mobility, and there are two kinds of partial mobility which we shall call partial mobility in the *static* and in the *dynamic* sense (or static and dynamic PM for short).

Physical mobility amounts to the combination of two assumptions, namely that markets for second-hand capital goods exist and that these capital goods can be used in more than one sector. In the following depreciation will be of the usual exponential-decay variant (constant rate of depreciation) and there will be no other difference between old and new capital except its state of decay (no vintage capital), so that the existence of these markets just means that with perfect physical mobility of capital between sectors the rental rates of capital are equalized. PM of physical capital in the purely dynamic sense means that the process of equalization is slow because the amount of capital which has to move to bring about equalization does not move all at once. PM in the static sense means that there will never move enough capital to equalize the rental rates. It is not easy, however, to explain static PM without reference to some dynamic process. Grossman's (1983) assumption of inhomogeneity of capital goods, for example, is itself in need of an explanation. If one considers investment it becomes clear that degree and structure of inhomogeneity are endogenous. Considering this problem, one might conjecture that static PM is best explained by assumptions on preferences of capital owners. But this is not possible. Capital owners are shareholders, not owners of individual capital goods. In the presence of markets for second hand capital there is no reason why shareholders' preferences should interfere with equalization of rental rates.¹

Financial mobility of capital is perfect if all sectors face the same rate of interest on the market for financial capital, i. e. the market for

¹ Here, I think, is the most important difference between labor mobility and capital mobility: workers' preferences may lead to wage differentials, capital owners' preferences never lead to differentials in rental rates of capital.

savings. Clearly there can be static PM of financial capital if investors have a preference for one sector, e.g. because investment in that sector is less risky. Dynamic PM is rarely considered in the literature; it would amount to the assumption that fast portfolio adjustments are more costly than slow adjustments.

The present paper assumes financial mobility to be perfect in both senses. Financial mobility serves as a substitute for physical mobility which is assumed to be completely absent. As usual, markets for financial capital are assumed to distribute savings between different investment projects, i.e. between different firms representing different sectors of the economy. These firms maximize the discounted present value of the capital stock. Except when explicitly stated otherwise expectations are assumed to be static. The basic theory used is the dynamic theory of a firm operating under convex costs of adjustment. To justify the introduction of convexity we take a look at the linear case.

With linear adjustment costs the pattern of investment becomes very simple. All firms for which rental rates of capital exceed (or are equal to) marginal costs of investment are willing to raise their stock of capital by any amount. Thus the rate of interest (and therefore user costs) will go up until at least user costs will exceed rental rates for all firms but one, i.e. savings will be invested only in the sector with the highest net return to investment. When that sector's rate of return eventually has been driven down far enough, savings will be split between the two most attractive sectors, then between the three most attractive sectors, and so on. If one assumes that adjustment costs are equal between sectors and that financial capital is perfectly mobile between sectors and countries, the adjustment path will after a while coincide with that of an OU model in the factor-price-equalization region.

There are two obvious lines of further development. The theory presented so far assumes linear costs of adjustment and no preferences of shareholders between countries and sectors. Furthermore we implicitly assumed a constant propensity to save since this is the assumption governing the behavior of the OU model. Thus both sides of the capital market, firms and shareholders, are modelled in an extremely simple way, and it should be rewarding to take a closer look at the

consequences of more realistic assumptions. The present paper takes up the problem of convex costs of adjustment, thus incorporating the Keynesian investment function into trade models.

In models of the physical-mobility variety convex adjustment costs are important to account for the limited speed of reallocation. This, however, is not important in the present context. With complete physical immobility, reallocation - in the sense of equalization of rental rates of capital - is always slow as long as the supply of savings is limited. Convexity is important in other respects.

- (1) Convex costs of adjustment lead to a spreading of savings over all sectors, thus avoiding the unplausible adjustment process described above.
- (2) If user costs of capital differ between sectors - as is usually the case if adjustment costs are convex - financial mobility is an imperfect substitute for physical mobility since rental rates of capital will differ even in the long run. Therefore models describing the long-run behavior of an adjustment-cost model will show features of static partial-mobility models.

Thus convex costs of adjustment explain partial mobility in the long run and the spreading of savings over all sectors.

The idea of using investment theory to model capital mobility is not new. Fisher & Frenkel (1972) introduce convex adjustment costs to avoid the inconvenient indifference of investors in a small open economy between investing at home or abroad. Obviously they did not want to discard physical mobility of capital, since they assumed a sort of national installation of capital which involves adjustment costs depending on the national rate of investment but does not impede physical mobility. Starting from the same problem Hori & Stein (1977) look at the case of investment without adjustment costs and without physical mobility - a case which coincides with the above case of linear adjustment costs. Koch (1989) looks at a generalization of Hori & Stein's (1977) model and discusses the pattern of investment in more detail, with special reference to the OU model and other models of growth and trade. Mussa (1978) models dynamic reallocation of capital and devotes a section to the problem of capital accumulation under sector-specific costs of adjustment. The main difference to the present approach is

that he assumes adjustment to be produced by a special sector endowed with a fixed stock of sector-specific capital. Adjustment costs are equal to the value of labor used in that sector; convexity results from decreasing returns due to the fixed stock of capital. A problematic feature of the model is the impossibility of investment in the sector producing adjustment.

While all of the above papers discuss problems and propose solutions which are relevant for the present paper, none of them refers explicitly to the dynamic theory of the firm. The present paper argues that this theory is the natural starting point for modelling capital reallocation.

After stating the general assumptions in section II and summarizing the necessary elements of the dynamic theory of the firm in section III, the paper turns to two-sector models in section IV. The properties of the simplest two-sector model are analyzed: the model of a small open economy which is confronted with constant prices on goods markets and with a constant rate of interest on the world market for financial capital. The latter assumption means that the balanced-trade condition is absent and that financial capital is perfectly mobile internationally. Section V looks at the more complicated case of a small open economy without access to international capital markets; this means that balanced trade is enforced or that savings have to equal investment. Section VI eventually is concerned with a model which can be interpreted in two ways, as a model of a closed economy or as a model of trade and capital mobility between two specialized countries. Section VII offers some concluding remarks.

All models are shown to be globally stable at unique equilibria (in sections V and VI this requires simplifications). The stability proof for section IV's model is so simple that only the line of argument is indicated in the text; stability proofs for the other models are given in some detail in two appendices.

II Assumptions and Notation

There are two goods, an investment good x_i and a consumer good x_c . The respective production functions are linearly homogeneous functions using two factors of production, labor L_i and capital K_i , as inputs:

$$(1) \quad (a) \quad x_I = f_I(K_I, L_I)$$

$$(b) \quad x_C = f_C(K_C, L_C)$$

Labor endowments are assumed to be fixed throughout the paper:

$$(2) \quad L_I + L_C = L$$

All markets clear instantaneously. The goods prices p_i equal marginal costs; the price of the investment good serves as *numeraire* ($p_I \equiv 1$). The relative price of the consumer good is denoted by p . Capital rents r_i and wages w_i equal the respective marginal value products (in terms of the investment good). The rate of interest i is exogenous in the models of section III and IV and endogenous in the other sections. All investors have static expectations, i.e. they expect the current values of all prices to remain unchanged. Only in section IV other forms of expectations are briefly discussed. Firms maximize the present value of the capital stock. For simplicity an infinite time horizon is assumed. Labor is perfectly mobile, i.e. workers move instantaneously such that there never is a wage differential between sectors ($w_I = w_C = w$).

III The Dynamic Theory of the Firm

To free the arguments of the following sections from expositions which are more or less standard in the dynamic theory of the firm a short account of the required material is given here.²

The problem of the firm is to maximize the present value of the returns to capital minus the costs of adjustment, i.e.

$$(3) \quad \max_{L, I} \left\{ \int_0^{\infty} (p \cdot x - w \cdot L - p_I \cdot C(I)) \cdot e^{-i \cdot t} dt \right\}$$
$$\text{s.t.} \quad \dot{K} = I - \delta \cdot K, \quad I \geq 0$$
$$x = f(K, L)$$

with I denoting gross investment and δ the rate of depreciation. C is an increasing and strictly convex function with $C(0) = 0$, $DC(0) = 0$. It denotes the costs of adjustment which are assumed to depend on gross investment; the assumption of convexity leads to increasing marginal costs of adjustment speed. $C(I)$ denotes the quantity of investment goods required to produce gross investment I . The price of the

² See Söderström (1976) for a survey.

investment good is denoted by p_I for greater clarity; indices pertaining to the good produced are dropped for simplicity.

The solution of the above problem is given by

$$(4) \quad r = p_I \cdot (i + \delta) \cdot DC(I)$$

which can be solved for I . The left side of (4) gives the marginal return to investment, the right side gives the marginal cost of investment or the user costs of capital. The firm finances investment by selling shares on perfect capital markets. The solution (4) can be substituted into the dynamic constraint; this yields the flexible-accelerator form of the differential equation:

$$(5) \quad \dot{K} = \delta \cdot (K^* - K)$$

Here $K^* = I/\delta$ is the desired long-run capital stock under static expectations. The equations (3) to (5) in the following represent the behavior of a sector.

Some remarks on the interpretation of the C function seem to be in order. In growth models like the OU model it is usually assumed that the rate of growth of the national capital stock is proportional to the input of investment goods. In the context of the present approach this amounts to the assumption of perfect physical mobility together with the assumption that C is constant and equal to unity. The primary motive for choosing the above formalization of adjustment costs has been its closeness to the assumptions of standard growth models.

Adjustment costs formalized in this way are of the kind envisaged by Rothschild (1971, p. 609). At the level of the individual firm the stock of capital can be increased only by adding indivisible units of new capital. Adding one further unit requires time and resources; for the sake of simplicity an homogeneous investment good stands for all the resources required. It is possible to substitute time by resource inputs; the idea is roughly that total costs of, say, building a factory are higher if the factory is built in six months instead of a year. As usual, discontinuities due to indivisibilities will be smoothed out more or less by aggregation; the result can be approximated by a continuous convex function relating gross investment to necessary resource inputs. Rothschild argues in favor of concave costs of adjustment at a sufficiently low speed of adjustment: at very low speed it may become

cheaper to do some things more quickly. It is not clear *a priori*, however, whether the resulting additional discontinuity at firm level necessarily makes for any difference at the sectoral level. Therefore the present analysis is restricted to the simpler case of strict convexity.

Furthermore it is not clear whether adjustment costs should depend on gross or net investment.³ Both versions are acceptable. If maintenance of the existing stock of capital is more like repairing a machine once and again, then there is no argument for convexity of maintenance costs. If on the other hand maintenance is more like building a new factory instead of an old one, maintenance costs have not to be distinguished from costs resulting from net investment. The latter view which is adopted here seems to be more robust since it is still a good approximation if maintenance is a mixture of repair and replacement.

In the following we will make use of a static maximization problem which directly yields the steady-state condition $K=K^*$. To formulate the problem we look at steady-state capital costs. These consist of two components: replacement costs and opportunity costs of holding capital.

Replacement costs are given by $p_I \cdot C(\delta \cdot K)$. To compute opportunity costs one has to look at replacement costs per unit of newly installed capital, $p_I \cdot C(\delta \cdot K) / (\delta \cdot K)$. Investors financing the installation of one unit of new capital incur opportunity costs of $i \cdot p_I \cdot C(\delta \cdot K) / (\delta \cdot K)$. Since old and new capital are perfect substitutes, opportunity costs per unit must be equal; opportunity costs of holding the total stock are then given by $i \cdot p_I \cdot C(\delta \cdot K) / \delta$. We define $V(K) := p_I \cdot C(\delta \cdot K) / \delta$; $(i + \delta) \cdot V(K)$ then denotes the sum of replacement costs and opportunity costs. The static problem of choosing the steady-state size of the firm is consequently

$$(6) \max_{L, K} \{ p \cdot x - w \cdot L - (i + \delta) \cdot V(K) : x = f(K, L) \} .$$

The first-order conditions of (6) are identical to the steady-state conditions of the differential equation derived from the original dynamic problem. The computations above throw some light on the optimality

³ Sargent (1987, pp. 132) assumes that only net investment gives rise to convex costs of adjustment.

conditions derived by Jorgenson (1967). Expressed in our terminology he assumed C to be linear with $DC \equiv 1$; this yields $V(K) \equiv p_I \cdot K$.

IV A Small Open Economy: Imbalanced Trade

We now turn to the model of a small open economy facing constant goods prices and a constant rate of interest. We only look at the production side. The relevant dynamic system is given by

$$(7)(a) \quad \dot{K}_I = \delta_I \cdot (K_I^* - K_I)$$

$$(b) \quad \dot{K}_C = \delta_C \cdot (K_C^* - K_C)$$

and the condition of equal wages in both sectors.

We first analyze a static problem which corresponds to last section's static problem of the firm. Since allocation of labor poses no interesting problems, we define with $p_I \equiv 1$, $p := p_C$

$$(8) \quad y(p, L, K_I, K_C) := \max_{L_I, L_C} \{x_I + p \cdot x_C : L_I + L_C = L, x_j = f_j(K_j, L_j)\} .$$

(8) is the national product function of the SF model; it is concave and linearly homogeneous in factor quantities and therefore strictly concave in capital stocks. The two-sector problem, then, is given by

$$(9) \quad \max_{K_I, K_C} \{y(p, L, K_I, K_C) - (i + \delta_I) \cdot V_I(K_I) - (i + \delta_C) \cdot V_C(K_C)\} .$$

It is easy to see that any steady state of (7) satisfies the first-order conditions of (9). Since the maximand of (9) is strictly concave in capital stocks, there can be only one solution to the first-order conditions and therefore to the steady-state conditions. Global stability can be proved by using the maximand of (9) as a Liapunov function. Since the maximand of (9) has a unique maximum at the steady state, one only has to prove that it increases monotonically under the dynamic system. To verify this a glance at the time derivative suffices.

A closer look at the time derivative of the Liapunov function reveals that there is a class of very simple investment rules which lead to globally stable adjustment. Consider the term $(i + \delta) \cdot DC(\delta \cdot K_i)$. The term gives the user costs of capital if net investment is zero. Global stability is ensured if net investment (1) is positive if the rental rate of capital exceeds the user costs connected with zero net investment, (2) is negative if these user costs exceed the rental rate, and (3) is zero in case of equality. At least in a neighborhood of the steady state the

rational-expectations investment rule seems to be a member of this class of investment rules. In any case the stability analysis is not restricted to the case of static expectations; any form of expectations compatible with the rules indicated above is included.

Long-run comparative-static results are easily derived; we concentrate on the effects of price changes and state the argument in an informal way.

Assume that there is no investment. Then the results are those of the SF model: a rise in the relative price of a good reduces the capital intensity in its production because labor endowments are partially shifted to this sector. This transforms the change in prices into an overproportional rise in the rental rate of this sector's capital and into an underproportional rise in the wage rate. In the other sector the capital intensity is increased; this leads to an overproportional reduction in the rental rate of capital. (Cf. Dixit & Norman 1980, ch. 2.)

Now let investment come into the picture: an increase (decrease) in the rental rate of capital will lead to an increase (decrease) in the capital stock. Whether investment will dampen or sharpen the effects of price changes depends on the rates of substitution (see Meckl 1989).

V A Small Open Economy: Balanced Trade

We now assume that there is a constant marginal propensity to save s and that there is no mobility of financial capital between the small open economy we consider and the rest of the world. The dynamic system is extended by an equation which requires savings to equal investment, i.e.

$$(10) \quad s \cdot y(p, L, K_I, K_C) = C_I(I_I) + C_C(I_C).$$

It is difficult to analyze the system in general. To demonstrate the difficulties and to motivate the introduction of a convenient simplification we look at another static problem of production in a two-sector economy.

In the following we treat $V_j(K_j)$ in analogy to a factor of production and assume that the sum of the V_j is fixed. This means that there is a concave trade-off between the two types of capital. The model therefore is similar to static PM models like that of Grossman (1983) or Mussa

(1982).⁴ The first-order conditions of the following maximization problem describe the production side of our present model's steady state. The problem is to maximize steady-state consumption under the restriction of a given V :

$$(11) \max_{K_I, K_c} \{y(p, L, K_I, K_c) - \delta_I \cdot V_I(K_I) - \delta_c \cdot V_c(K_c) : V_I(K_I) + V_c(K_c) = V\}$$

The problem yields a set of V allocations fulfilling the first-order conditions. If the allocation of V is in this set and if V is such that savings equal investment the economy is in the steady state and the rate of interest is equal to the shadow price of V . It is not important whether the point in question is actually a maximum of (11). It is easy to see that uniqueness of the solution to the first-order conditions is not guaranteed: The maximand is strictly concave and has a single maximum. If V is so high that this maximum is exceeded the relevant parts of the level lines of the maximand are concave. Since the restriction also is concave, there can be any number of points of tangency.

If the propensity to save is high enough, this possibility of multiple solutions becomes relevant and it seems to be impossible to say something about the number of steady states. This becomes obvious if one looks at the extreme case of $s = 1$. The maximand of (13) is equal to consumption. If $s = 1$, this part is 0. The zero level line of consumption in the K_I, K_c plane has the form of a transformation curve; the restriction has the same form. Multiple solutions are possible, and each solution corresponds to a steady state. Less extreme cases can show the same feature.

What causes trouble in the above problem is the possibility of differing rates of depreciation. Differing rates of depreciation lead to a valuation of V differing between sectors; therefore it is not possible to banish the V_j from the maximand. In order to get rid of the problem, we introduce the simplifying assumption of identical rates of depreciation, $\delta_I = \delta_c = \delta$. The maximization problem then reduces to

$$(12) g(p, L, V) := \max_{K_I, K_c} \{y(p, L, K_I, K_c) : V_I(K_I) + V_c(K_c) = V\}$$

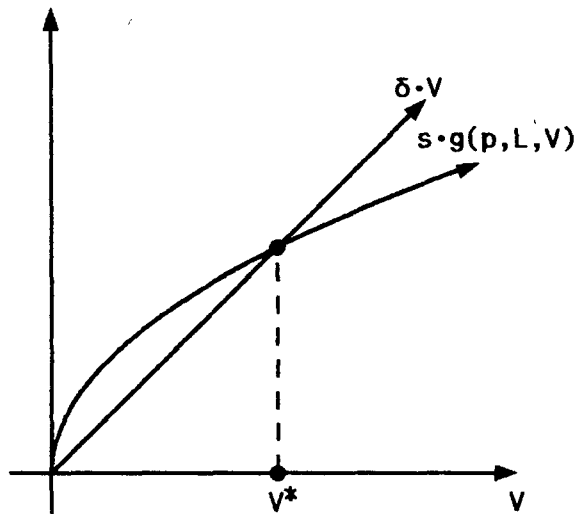
⁴ See Meckl (1989) for a thorough discussion of this point.

and the shadow price of V must be equal to $(i+\delta)$ in the steady state. The condition (10) now can be written as $s \cdot g = \delta \cdot V$. The solution of (12) exists and is unique, and since g shows decreasing returns in V there is exactly one steady state (see fig. 1).

It is even possible to insert (10) into (12) as further restriction. The modified problem then directly yields the steady state of the system.

Global stability of the adjustment process is proved in appendix I. It is shown there that the steady state lies on the boundary of a compact global attractor, namely the region of all K_I, K_C which yield at most the steady state V . Because we operate in two dimensions, a unique steady state at the boundary of a compact global attractor excludes closed orbits. Since there is no higher dimensional counterpart of this theorem, the proof does not generalize to models with more than two sectors.

Fig. 1: Uniqueness of the Steady State



Comparative-static results are ambiguous (cf. Meckl 1989). One problem is posed by the line of optimum V allocations which has downward sloping parts if the curvature of the C functions is small, i.e. if the model is close to an HO model. But even if one assumes that the line is upward sloping in a neighborhood of the initial equilibrium, this leaves the question of how the line is shifted by shocks.

VI Intersectoral Capital Mobility in a Closed Economy

In this section the last section's static model of production is simplified and placed in the context of a closed economy. The so far missing equations are given by market-clearing conditions for goods. We analyze the resulting system with the help of maximization problems which are analogous to the problems of the last section.

We define

$$(13) \quad u(L, K_I, K_C) := \max_{L_I, L_C} \{x_I^s \cdot x_C^{1-s} : L_I + L_C = L, x_j = f_j(L_j, K_j), j=I, C\}$$

to get the problem of labor allocation out of the way. The Cobb-Douglas utility function used as maximand just yields the demand functions corresponding to the rate of savings s . (13) is the analogue of (8) above and describes the SF equilibrium in a closed economy. The problem yielding the optimum allocation of V is given by

$$(14) \quad \max_{K_I, K_C} \{u(L, K_I, K_C) : V_I(K_I) + V_C(K_C) = V\}$$

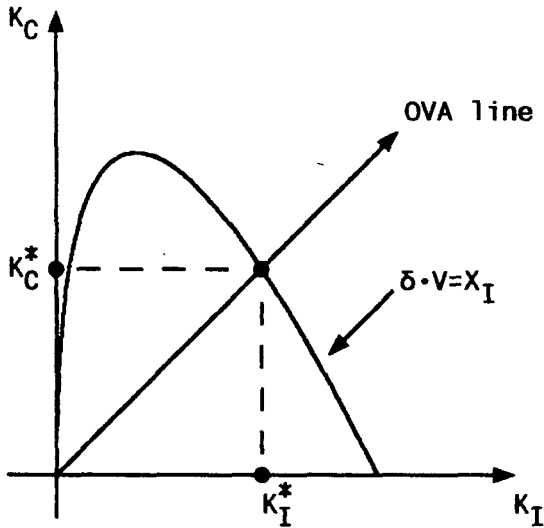
under the assumption of equal rates of depreciation. (14) is the analogue of (12) above; its solution is unique. It is now a familiar property of this kind of maximization problem that every steady state fulfills its first-order conditions, i.e. that the allocation of the steady state V is optimum. The line of optimum V allocations in the K_I, K_C plane is, as before, not necessarily positively sloped everywhere.

The missing equation to determine the steady state is the market-clearing condition for the investment good which under our assumptions can be written as $x_I = \delta \cdot V$. This condition yields K_C as an implicit function of K_I which is strictly concave. The graph of this function starts at the origin of the K_I, K_C plane and ends at (or is no longer relevant beyond) a maximum K_I with a corresponding K_C of zero. All intersection points of this curve with the optimum- V -allocation line are steady states of the system. At this level of generality nothing can be said about the number of steady states. Therefore we look at a very special case where the system can easily be analyzed.

We assume that the production functions are Cobb-Douglas; as a consequence $u(L, K_I, K_C)$ is homothetic in the K_j . If additionally $V_I(K_I) + V_C(K_C)$ is homothetic - i.e. if the C functions are identical between sectors and are of the form $C(x) = x^{1+z}$, $z > 0$ -, the optimum- V -

allocation line is linear. This ensures uniqueness as can be seen from fig. 2.

Fig. 2: Uniqueness of the Steady State



Global stability is guaranteed. The proof proceeds in two steps. At first one shows that the ratio of capital stocks converges to its long-run value. This means that the optimum-V-allocation line is a global attractor. Global stability then follows if the system is stable on that line. The latter is trivial to establish; the behavior of the capital ratio is discussed in appendix II.

It should be noted that the simplifying assumptions introduced above are so strong that even with differing rates of depreciation the steady state is unique and globally stable. The proof is not very gratifying, however, so we leave it at that. The simplifications essentially turn the model into an one-sector model of capital accumulation. This results mainly from combining Cobb-Douglas production functions with a constant propensity to save.

We discuss the effect of a rise in the marginal propensity to save in order to demonstrate that the intuitively plausible conjectures hold. If savings go up, labor and capital are drawn to the sector producing investment goods. The rate of interest goes down; the social product rises, and so do wages. The rise in the wage rate shows that labor in the investment-goods sector goes up by a lower percentage than capital, i.e. the capital intensity goes up. The effect on the capital stock in the consumer-goods sector depends on whether the marginal

propensity to save has already been high or not: beyond a certain point more capital in the investment-goods sector can be maintained only by reducing capital in the other sector. Necessarily the same goes for consumption: there is a point beyond which a higher propensity to save leads to less consumption.

VI Conclusion

The model introduced in the present paper has some attractive features. It is clear that even a small degree of convexity in adjustment costs - elasticity of the C functions just greater than one - preserves section V's model from the well-known stability problems which turn up in models without costs of adjustment if production of investment goods is relatively capital intensive. This attractive feature of the model is connected with a further point which deserves to be recalled: the demand functions for investment goods lead to investment in both sectors even if rental rates of capital are not equal. This is an important point. A model of capital mobility is not acceptable if it implies that, as an immediate consequence of shocks, investment will be concentrated on the favored sector alone. In a context of international capital mobility this would imply that a shock to the world economy would shift world investment to one sector of one country. In the present model this feature is absent; thus the model seems to be fit for the task of analyzing the dynamics of imbalanced trade. For this task which requires a two-country model numerical studies seem to be the method of choice, at least until the dynamic interactions are better understood.

Of course one can interpret the model of a closed two-sector economy as a model of trade in goods and equities between specialized countries, as it is done by Hori & Stein (1977). As mentioned above, the present model avoids the discontinuous changes in the adjustment process which result from the fact that without costs of adjustment the demand for investment goods in a country is either zero or bounded only by the supply of savings. Thus the model has a limited usefulness for trade-theoretical purposes even in its present form; the interesting problem for further research along these lines, however, is the generalization to trade and mobility of financial capital between diversified countries.

Appendix I

The following proof shows that the dynamic version of section V's model is globally stable at a unique equilibrium.

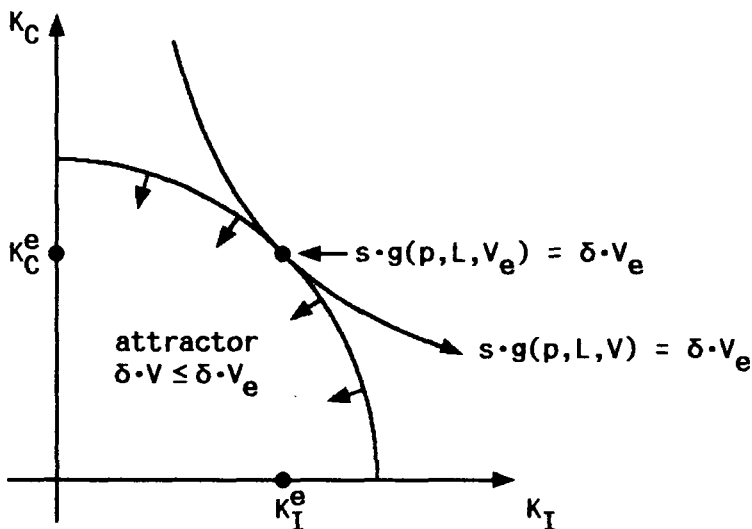
The proof makes use of the fact that $V(K_I)+V(K_C)$ - a term we will for simplicity write as $V(K)$ - is a convex function of the vector $K:=(K_I,K_C)^T$. For all convex functions V the following result holds

$$DV(K) \cdot (K^* - K) < 0 \quad \text{if} \quad V(K) \geq V(K^*) \quad \text{and} \quad K \neq K^*$$

where K^* is the vector of the desired K_j . The result follows directly from the definition of convexity. We use the result to prove that $V(K)$ falls under the dynamic system as long as savings are smaller than or equal to $\delta \cdot V(K)$ and the system is not in the steady state.

Savings in temporary equilibrium are equal to $\delta \cdot V(K^*)$. The condition of savings being smaller than or equal to $\delta \cdot V(K)$ then reduces to $V(K) \geq V(K^*)$. The condition means that savings are smaller than or equal to replacement costs for the current stocks of capital.

Fig. 3: Global Attractor of the Dynamic System



We now look at the time derivative of $V(K)$. By using the flexible-accelerator form of the differential equations (see equations (9)) we get

$$(15) \quad \dot{V}(K) = DV(K) \cdot \delta \cdot (K^* - K) .$$

By convexity of V this is smaller than zero as long as savings are smaller than or equal to $\delta \cdot V(K)$ and the system is not in the steady state.

We consider the curve in the K plane where $V(K)$ is equal to its steady-state value V_e . Except for the steady state itself all points on this curve have the property that $\delta \cdot V(K)$ exceeds savings. This follows from the fact that at the steady state savings are equal to $\delta \cdot V(K)$ in connection with the fact that the steady state is the point on the curve where savings take on their maximum value. From the above considerations it follows that for all points of the curve but the steady state V locally falls under the dynamic system. Furthermore the same goes for all $V > V_e$. The set $V \leq V_e$ then is a global attractor for the dynamic system and $(V - V_e)^2$ is a kind of Liapunov function (see fig. 3).

So far we have proved the following: there is a (compact) global attractor with the unique steady state lying on its boundary. We did not use the fact that K is of dimension two; so far the proof generalizes to any number of sectors. The last step of the proof, however, is only valid in the plane: a unique steady state on the boundary of a two-dimensional compact global attractor is globally stable (1) because a closed orbit in the attractor would require the existence of a second steady state enclosed by the orbit while (2) a closed orbit around the steady state is impossible because no orbit can leave the attractor. QED

Appendix II

The following proof shows that the dynamic version of section VI's model is globally stable at a unique equilibrium. As stated in the text it suffices to show that the system converges to the optimum-V-allocation line since behavior on that line is trivially stable.

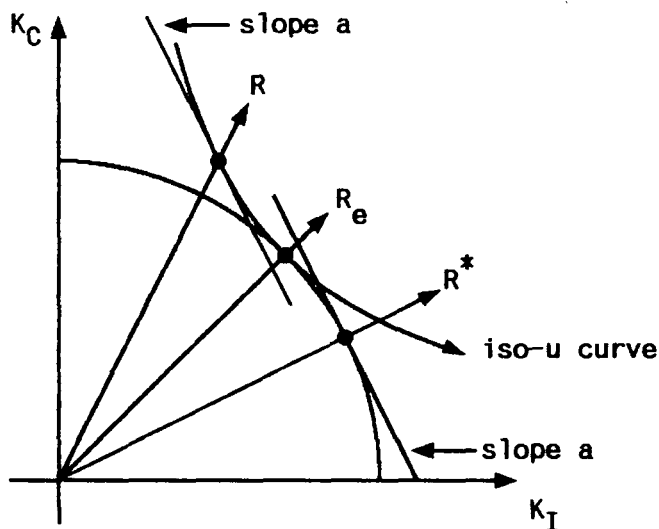
The proof is very simple. From the dynamic theory of the firm we know that

$$(16) \quad r_I/r_C = DC_I(I_I)/DC_C(I_C), \quad I_j = \delta \cdot K_j^* \quad .$$

Because the left side of (16) just is the slope of an iso-V curve, and because we assumed the V function to be homothetic, (16) yields a ratio of the I_j and therefore of the desired capital stocks depending on r_I/r_C .

We look at fig. 4 to see what is implied for the capital ratio $R:=K_I/K_C$, the steady-state capital ratio R_e (which is equal to the slope of the optimum-V-allocation line) and the ratio of the desired capital stocks R^* .

Fig. 4: Actual, Desired and Long-Run Ratio of Capital Stocks



With $R > R_e$, the slope of the iso-utility curves (see equation (13)) at the points where R prevails is steeper than at the line where R_e prevails. Let us denote that slope by a . Since the line where R_e prevails is defined by the points of tangency between the iso-V- and the iso-u curves, the line where the iso-V curves have the slope a is always on

the other side of R_e than R . This line - where the iso-V curves have slope a - is the line with slope R^* . This follows from the fact that by the definition of u the slope a is equal to r_1/r_c , and from (16) above. By analogous arguments for the case $R < R^*$ we get the result that R^* is always between R and R_e - except for the case where all three ratios coincide and the system is on the optimum-V-allocation line. We use this result to show that the distance $H := (R - R_e)^2$ is a kind of Liapunov function which proves that the line where R_e prevails is a global attractor.

By simple computations the time derivative of H under the dynamic system can be written as

$$\dot{H} = 2 \cdot \delta \cdot (K_c^*/K_c) \cdot (R - R_e) \cdot (R^* - R)$$

if one uses the flexible-accelerator form of the differential equations (see equations (7)). This is of course negative *iff* $R \neq R^*$. QED

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