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Capital tax competition and household mobility

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Abstract

Capital tax competition is analyzed in a model with a single private and a locally supplied public consumption good. As a benchmark case necessary conditions for efficient interregional tax structures are derived and contrasted with the outcome of beggar thy neighbor strategies. If households are immobile an intervention of a central government can be justified to raise efficiency of the Nash equilibrium. In contrast, Nash equilibria are efficient if households are imperfectly mobile because regions have incentives to ensure an efficient interregional resource allocation via granting interregional transfer payments.

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1 Introduction

Much recent public finance literature examined the role of direct taxes on highly mobile bases. Most authors conclude that a global equilibrium in a system of competing regions generally is Pareto-inefficient.\(^1\) Even though non-cooperative tax policy is rational from the viewpoint of a single region's government, isolated behavior elicits fiscal externalities on other regions and leads to a loss of welfare for the whole federation.\(^2\)

The literature distinguishes three different causes for the emergence of fiscal externalities.\(^3\) Firstly, decisions of one region negatively affect the welfare of others, and thereby their fiscal decisions, if a region's tax burden is at least partly born by non-residents, i.e. the tax is exported by taxing away the rents of immobile factors owned by foreigners. Secondly, a tax increase which influences the return of an interregionally mobile factor will drive the factor out of the taxing region, thereby causing an interregional shift of tax revenue. Third, capital flight may lead to a reduction in the after tax interest rate if the region has a certain dispose of power on the capital market.\(^4\)

The investigation submitted develops the central result that local public goods will be underprovided and identifies the sources of this inefficiency as fiscal externalities resulting from tax-base and terms-of-trade effects. We conclude that a Pigouvian subsidy of a higher level of government to the supply of local public goods can be justified to increase global efficiency.

All of these inferences cited above are made from models that ignore household mobility. However, results change dramatically if households are, even though just imperfectly, mobile. We show that the conclusions about the efficiency effects of decentralized tax autonomy are much too pessimistic if household mobility is neglected. In our model we follow up on recent contributions of Burbidge and Myers (1994), Wellisch (1995) and demonstrate that a central government's intervention can not be justified for allocative reasons if the population is mobile across regions because the Nash equilibrium is characterized by an efficient interregional allocation of resources.

We assume that capital can only be taxed according to the source principle. Actually, countries of the European Community generally adhere to the philosophy of comprehensive income taxation which implies the residence principle. On the other side, the...
source principle becomes effective when firms retain profits or countries apply the "international affiliation privilege" as it is the case for most countries of the European Community. Furthermore, it is undoubtedly true that tax administrators have strong difficulties assessing total world income of its taxpayers as long as there exists no provision for controlling income flows on a world-wide basis. Thus, the residence principle can not be carried out and therefore it is worth while investigating the effects of a source based capital tax system in an international setting with decentralized tax authorities. 

The structure of the investigation is as follows: In section 2 the model is set out. In section 3 we derive the conditions for an efficient allocation of resources before we determine the efficiency properties of the Nash equilibrium under capital and household mobility in sections 4 and 5. Finally, section 6 briefly summarizes the argumentation and gives some conclusions.

2 The Basic Model

Consider a federal economy consisting of \( t = 2 \) regions, indexed by \( i = 1, 2 \). In each region live \( N_i \) homogeneous individuals so that entire population is given by \( N = N_1 + N_2 \). The model is static. The federation as a whole has a given endowment of capital, \( K \), which is perfectly mobile and allocated across regions, \( K = K_1 + K_2 \). In addition, in each region exists a fixed (and non-tradeable) factor, labelled land, \( L_i \). Land will serve as the repository of pure profits in this model. Individuals derive utility from consumption of a private good, \( x_i \), and a local publicly-provided good, \( g_i \). The utility function of a household in region \( i \), \( U_i \), corresponds to

\[
U_i = U_i(x_i, g_i),
\]

(2.1)

with \( U_x > 0 \) and \( U_g > 0 \) where subscripts indicate partial derivatives.

We assume that \( g_i \) is produced from local public expenditure, \( z_i \), and is potentially congested by local population, or \( g_i = g_i(z_i, N_i) \). Marginal congestion costs, \( \frac{\partial z_i}{\partial N_i} \), are calculated at a fixed level of public spending. In contrast, the marginal costs of the public good, \( \frac{\partial z_i}{\partial g_i} \), give the resources required to increase the level of the public good, with the number of residents held fixed. The general specification of public costs

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6Due to the lack of coordination, every country abstains from taxing the income from capital that is invested in the 'rest of the world'. This point is made most clearly by Frenkel, Razin and Sadka (1991: 204–206): "That is, when countries ... cannot tax their residents on the income from capital that is invested in the rest of the world, then the rate-of-return arbitrage prevents each of them from taxing its residents on their income from capital invested in the other country, even though their tax authorities can cooperate on such things as tax withholding. This may explain why the EC dropped the idea of imposing a withholding tax on capital income."

7Problems associated with technological spillovers are left out of consideration.

8For example, \( U_x \equiv \frac{\partial U_i(x_i, g_i)}{\partial x_i} \).
allows to distinguish between different types of local public goods. Two of them are of special importance from a theoretical point of view. Firstly, publicly provided private goods exhibit total public costs of \( z_i = N_i g_i \), marginal costs amount to \( (\partial z_i / \partial g_i) = N_i \) and marginal congestion costs come to \( (\partial z_i / \partial N_i) = 1 \). Secondly, if \( g_i \) is purely public, then \( z_i = g_i \), \( (\partial z_i / \partial g_i) = 1 \) and \( (\partial z_i / \partial N_i) = 0 \). The private good, \( x_i \), which serves as numéraire, is produced according to a linearly-homogeneous production function \( F^i(K_i, N_i, L_i) \) in each region.\(^9\)

Each local authority has a source-based capital tax, \( t^K_i \), and a uniform head tax, \( t^N_i \), from households living within the region at its disposal. Furthermore each local government can use tax revenues to finance a non-negative interregional transfer, \( S_i \), if it would like. The region’s budget is balanced if

\[
z_i(g_i, N_i) = t^N_i N_i + t^K_i K_i - (S_i - S_j).
\]

Each household is endowed, independently of its place of residence, with one unit of labor, and with \((K/N)\) units of capital. The budget of a household living in region \( i \) is balanced if labor income, \( w_i \), income from the capital endowment, \( r(K/N) \), and land rents, \( (R_i/N_i) \), equals private consumption, \( x_i \), i.e. \( x_i = w_i + r(K/N) + R_i/N_i \). Using \((w_i = F^i - t^N_i)\), \((r = F^i_K - t^K_i)\) and, from the property of the constant-returns-to-scale production function, \( R_i = F^i(K_i, N_i, L_i) - F^i_K K_i - N_i F^i_N \) yields:

\[
x_i = \frac{1}{N_i} \left[ F^i(K_i, N_i) + F^i_K M_i - N_i t^K_i K_i - N_i t^N_i \right],
\]

where \( M_i \equiv (N_i K_N - K_i) \) is the region’s net capital balance. Therefore, \( M_i < 0 \) (\( M_i > 0 \)) if region \( i \) is a net capital exporter (importer).\(^10\) For \( M_i = 0 \) no capital is traded. Owners of capital, seeking to maximize their returns, allocate their capital stock across regions so that its net rate of interest, \( r \), is equal in both regions, i.e.

\[
F^i_K - t^K_i = F^j_K - t^K_j.
\]

### 3 Efficient Allocation

The task of a central planner is to allocate resources between regions so as to maximize world utility. The Lagrangian of the problem reads:

\(^9\)The marginal product of capital is assumed to be positive, \( F^i_K > 0 \), and decreasing, \( F^i_{KK} < 0 \).

\(^10\)The capital importing country has to pay for the use of capital. This is done by trade in goods which is complementary to capital flows. The capital exporting country hence shows a trade deficit which amounts to the value of interest received.
\[
\max_{x_i, b_i, R_i, L_i} \mathcal{L} = \sum_{i=1}^{2} U_i^i(x_i, g_i) \\
+ \lambda_1 \left[ F^1(K_1, N_1, L_1) + F^2(K_2, N_2, L_2) - N_1 x_1 - N_2 x_2 \right] - z_1(g_1, N_1) - z_2(g_2, N_2) \\
+ \lambda_2 \left[ U_i^1(x_1, g_1) - U_i^2(x_2, g_2) \right] \\
+ \lambda_3 (M_1 + M_2) \\
+ \lambda_4 (N - N_1 - N_2).
\]

Equation (3.1b) describes the feasibility constraint for the numéraire good. Constraint (3.1c) reflects costless mobility of households, which rules out any interregional utility difference. Moreover, both the capital market, (3.1d), and the labor market, (3.1e), must be cleared. The first-order conditions are:

1. \[
\frac{\partial \mathcal{L}}{\partial x_i} = U_i^i - \lambda_1 N_i + \lambda_2 U_i^i = 0 \quad \forall i = 1, 2.
\]
2. \[
\frac{\partial \mathcal{L}}{\partial g_i} = U_i^i - \lambda_1 \frac{\partial z_i}{\partial g_i} + \lambda_2 U_i^i = 0 \quad \forall i = 1, 2.
\]
3. \[
\frac{\partial \mathcal{L}}{\partial K_i} = \lambda_1 F_i^i - \lambda_3 = 0 \quad \forall i = 1, 2.
\]
4. \[
\frac{\partial \mathcal{L}}{\partial N_i} = U_i^i (1 + \lambda_2) + \lambda_1 \left( F_i^i - x_i - \frac{\partial z_i}{\partial N_i} \right) = 0 \quad \forall i = 1, 2.
\]

To facilitate the interpretation we re-arrange these conditions. Inserting (3.2a) into (3.2b) yields (3.3a) and from equating (3.2c) for both regions follows (3.3b). Furthermore using (3.2a), (3.2b) in (3.2d) produces (3.3c):

\[
F_i^i = \frac{F_i^j}{K},
\]

\[
F_i^i - x_i - \frac{\partial z_i}{\partial N_i} = F_i^j - x_j - \frac{\partial z_j}{\partial N_j}.
\]

The interpretation of these conditions is familiar. Following the well-known Samuelsonian condition for an optimal provision of local public goods, (3.3a), the total of the individuals marginal valuations, \( \sum MRS \), must be taken into account when resources are shared between private and public use. Since all individuals are considered to be identical and possibilities of non-residential, external utilization are excluded by assumption, the social marginal valuation of the local public goods amounts to a single person's marginal valuation multiplied by the number of residents \( N_i \) and is equal to the marginal rate of transformation, \( MC \), which is identical one, i.e. \( \sum MRS = MC = 1 \).

In accordance with (3.3b) an efficient international capital allocation requires the equalization of the marginal products of capital. This leads to our first result:
Result 1. An efficient allocation of capital requires that source-based capital tax rates are identical across all regions, $t^K_i = t^K_j$.

Proof. From the firm's profit definition $\pi' = F'(K_i, N_i, L_i) - (r + t^K_i)K_i - w_i N_i - R_i$ it follows that the demand for capital is implicitly given by $r = F'_{K,K} - t^K_i$. From the arbitrage condition (2.4) capital's net return, $r$, must be equalized in the federation and moreover including (3.3b) into the argument completes the proof. □

Condition (3.3c) describes the efficient allocation of mobile households. According to it, the social marginal benefit, $F'_{N_i}$, of one additional resident minus the social marginal costs — the additional private consumption, $x_i$, plus marginal congestion ($\partial z_i/\partial N_i$) — must be equated across all regions. This implies our next result:

Result 2. Under a head tax, efficiency and household mobility are only compatible if the tax is set to internalize regional congestion costs.

Proof. The private budget is balanced if $x_i = w_i + r(K/N) + (R_i/N_i)$. Firm's labor demand is given implicitly by $w_i = F'_{K,K}$. Therefore, it follows for identical regions that $F'_{K,K} - z_i - t^K_i = F'_{N,N} - x_i - t^K_i$. This condition, together with (3.3c), implies $t^K_i = (\partial z_i/\partial N_i)$. □

In general, there exists no mix of head taxes and marginal congestion costs of public goods which balances the region's budget and simultaneously ensures an efficient allocation of households. Suppose, however, the special case of publicly provided private goods.

Result 3. If local public goods are publicly provided private goods, i.e. public production costs increase proportionally with the number of residents, an efficient taxation of mobile households simultaneously balances a region's public budget, (2.2).

Proof. An efficiently set head tax must, by (3.3c), satisfy $t^K_i = (\partial z_i/\partial N_i)$. This, by $z_i = N_i g_i$, simultaneously satisfies the region's public budget constraint. □

4 Nash Equilibria of Capital Tax Competition

There exists a large literature on fiscal competition and most of it is devoted to capital tax competition. Most authors conclude that a Nash equilibrium in a system of competing regions generally generates an inefficiently low supply of public goods. In order to concentrate on the problem we exclude household mobility in this section. Hence, population sizes, $N_i$, are set exogenously and reflect the initial labor endowments of each region.\(^{12}\)

We assume each government behaves locally efficient, i.e. it maximizes the utility of the representative individual. In choosing the capital tax rate, $t^K_i$, the head tax rate, $t^K_i$,\(^{12}\)
and the interregional transfer payment, $S_i$, every region acts under the Nash assumption that the rates of the other jurisdiction, $t^{K}_j, t^{N}_j, S_j$, are not a function of one's own choices. Its problem reads:

$$\max_{t^N, t^K, S} U_i(x_i, g_i),$$  \hspace{1cm} (4.1)$$

where $x_i$ must be substituted by (2.3) and $g_i$ is given by (2.2). Some manipulation of the first-order conditions yields:

\[
\frac{\partial U^i}{\partial t^K_i} = G_i = 0, \quad (4.2a) \\
\frac{\partial U^i}{\partial t^N_i} = -\frac{U^i}{N_i} M_i + \frac{K_i}{N_i} G_i + U_K \frac{\partial K_i}{\partial t^K_i} = 0, \quad (4.2b) \\
\frac{\partial U^i}{\partial S_i} = -U^i \frac{\partial g_i}{\partial z_i} \leq 0, \quad S_i \geq 0, \quad \text{and} \quad S_i \frac{\partial U^i}{\partial S_i} = 0. \quad (4.2c)
\]

where $G_i$ is a re-arrangement of the Samuelson condition (3.3a):

\[
G_i = -U^i + N_i \frac{\partial g_i}{\partial z_i}, \quad (4.3a) \\
U^i_K = \frac{U^i}{N_i} \left( F_{KK} M_i + t^K_i \left( 1 + \frac{G_i}{U^i_K} \right) \right). \quad (4.3b)
\]

Equation (4.2c) immediately implies that $S_i = 0$, i.e. a region will never make a unilateral transfer payment to the other.

Rationally acting governments take capital market responses, $(\partial K_i/\partial t^K_i)$, into account when determining their tax rates. The reactions of capital flows are calculated from the following three-equation system which determines three endogenous variables, $K_i, K_j, r$, given six parameters, $t^K_i, t^K_j, N_i, N_j, \tau_i, \tau_j$:

\[
G^i(K_i, r; t^K_i, N_i, \tau_i) = F^i_K(K_i, N_i) - (1 - \tau_i) t^K_i - r = 0, \quad (4.4a) \\
G^j(K_j, r; t^K_j, N_j, \tau_j) = F^j_K(K_j, N_j) - (1 - \tau_j) t^K_j - r = 0, \quad (4.4b) \\
K = K_i + (l - 1) K_j . \quad (4.4c)
\]

Our basic analytical objective is to rewrite conditions (4.2a) and (4.2b) in order to compare them with the efficiency conditions (3.3a) and (3.3b). Condition (4.2a) immediately shows:

**Result 4.** If regional governments have unlimited recourse to a non-distortionary head tax, then the provision of local public goods will be efficient.

\[^{13}\text{The comparative static effects} (\partial K_i/\partial t^K_i) \text{ and} (\partial r/\partial t^K_i) \text{ are derived in appendix A.}\]
Proof. If $t_i^N > 0$, condition (4.2a) must be fulfilled with strict equality. But $G_i = 0$ implies that the efficiency condition (3.3a) is also fulfilled.

Inserting the capital market reactions $(\partial K_i/\partial t^K_i)$ into the first-order condition (4.2b) yields:

$$\frac{U^1_{ij} \partial g_i}{U^1_{ik} \partial z_i} = \frac{1 + M_i}{K_i} \left( 1 - \frac{F_{KK}^i}{F_{KK}^i + F_{KK}^j} \right) \frac{1}{1 - \varepsilon_i},$$

(4.5)

where $\varepsilon_i$ is the elasticity of capital demand of region $i$ with respect to the tax rate, defined as $\varepsilon_i \equiv -(t^K_i/K_i)(\partial K_i/\partial t^K_i) > 0$. Wildasin (1989) identifies this term as a basic source of inefficiency and moreover elaborates the concept of a fiscal externality: Each region strategically lowers its capital tax, thereby trying to attract capital in order to raise the return accruing to fixed factors. The taxing region does not take account of the negative tax-base effects in other regions.¹⁴ A second source of inefficiency is depicted by the terms-of-trade effect $M_i/K_i \left( 1 - F_{KK}^i/(F_{KK}^i + F_{KK}^j) \right)$, which leads to inefficiency once one allows for asymmetric jurisdictions. In general, both externalities drive a wedge between the local, or private, and the social marginal costs of public funds and consequently prevent the non-cooperative Nash equilibrium from being efficient.

Tax-Base Effects

To separate between the two causes of inefficiency, the tax-base and the terms-of-trade effects, we firstly exclude the latter and consider the case of identical regions. Hence, tax competition can not result in international capital re-allocations, i.e. $M_i = 0$. However, the typical region takes as given the other’s tax rates because of the Nash assumption. Consequently regional governments become aware of high capital inflows due to an isolated tax decrease.

Result 5. Efficiency increases when the number of competing regions is reduced.

Proof. According to equation (A-3) every region perceives that $(\partial K_i/\partial t^K_i) = (1 - (1/l)) (1/F_{KK}^i)$. In contrast to the case of a small region, i.e. $\lim_{l \to \infty}(\partial K_i/\partial t^K_i) = 1/F_{KK}^i < 0$, regions have no incentives to act non-cooperatively if they are joining together since $\lim_{l \to 1}(\partial K_i/\partial t^K_i) = 0$.

Essentially, this result confirms the intuition stemming from the Ramsey rule of optimal taxation theory. The higher the perceived elasticity of capital, the lower the optimal capital tax rate set by the typical government, i.e. heightened rivalry for a mobile factor reduces the tax burden which the factor has to bear. This intuition leads to our next result:

¹⁴In a seminal article Flatters, Henderson and Mieszkowski (1974) demonstrate that free mobility of households may cause fiscal externalities if pure local public goods are financed by a head tax on residents since every additional immigrant lowers the per capita costs of the public good but raises the price of public goods in the emigrating region. However, mobile households ignore the effects on the regional public budgets when they decide on their migration decisions.
**Result 6.** Whenever head taxes are unconstrained and region $i$ is small, the optimal capital tax rate is set to zero.

*Proof.* Whenever $t_i^N$ is unconstrained (4.2a) holds with strict equality. Using $G_i = 0$, further taking into account that $\lim_{tN \to \infty} (\partial K_t / \partial t^K) = 0$ and solving (4.5) for $t_i^K$ completes the proof. □

Result 6 suggests that small regions offer a 'tax holiday' for internationally mobile capital. This result corresponds to OATES, SCHWAB (1988:339) and to FRENKEL, RAZIN, SADKA (1991:206) who conclude that the outcome of a non-cooperative tax competition equilibrium in source-based capital taxes is PARETO inferior to a second-best equilibrium in residence-based taxes. Therefore, a region's optimal second-best tax policy is characterized by choosing endogenously a zero source-based and a non-zero residence-based capital tax rate.

Figure 1 depicts the situation of a typical region. It extends the analysis in ZODROW and MIESZKOWSKI (1986). Regional production of the local public good $g^h$ is measured by the length of the horizontal axis. The height of the vertical axis measures the total amount of the rival private good, $x^h$. Curve $a'a'$ denotes the production possibility locus of the federation which has a slope of minus one since production efficiency is supported by the assumption of identical regions. The federation as a whole has an exogenously given endowment of capital, hence a tax increase throughout the federation yields extra public revenue of $(\partial z_t / \partial t^K) = K_t$. The curves $b'b'$ and $e'e'$ show production possibility loci as being perceived by a typical government. Due to the tax-base effect an isolated increase of the tax rate will only yield additional tax revenue of $(\partial z_t / \partial t^K) = K_t + (\partial K_t / \partial t^K)K_t$. The costs of public funds are perceived to be higher and this explains the steeper gradient of both curves.

15The source-based capital tax considered here could be interpreted as a tax on domestic investment income. Taxation under the residence principle is applied to income from all savings. However, in the static model a residence tax is lump sum and therefore excluded from the analysis presented.
From standard microeconomic theory, an equilibrium situation is characterized by the production possibility locus being a tangent on an indifference curve. Point a depicts an efficient situation in which tax rates are set cooperatively and interregional investment decisions are undistorted. No trade in capital takes place. Point b portrays a single region's incentives for non-cooperative policy setting. A single region commits itself to an inefficiently low capital tax rate, expecting that a tax reduction causes interregional capital re-allocations and hence leads to higher private and public income. Playing Nash strategies, every government acts as if it could neglect the negative effect of a decrease in its own tax on the tax bases of neighboring regions. However, the symmetric Nash equilibrium must yield low tax rates on capital in the whole federation. Hence, as shown in point e, lower levels of public expenditure prevail in all regions. But this can not be a tangent on an indifference curve and therefore the federation as a whole would be better off if tax rates of all regions were to be re-adjusted at a higher level.

Inevitably, as no other limitless and undistorting tax is to a region's disposal, local public goods are in undersupply even if production efficiency is ensured.

Terms-of-Trade Effects
In order to concentrate on fiscal externalities associated with terms-of-trade effects we assume each region has a non-distorting head tax at its disposal. Therefore, capital tax rates are set solely in order to alter the international distribution of income.

Result 7. If regions are not identical, equilibrium tax rates differ between countries: The capital-exporting region effectively subsidizes capital whereas the capital-importing region endogenously sets a positive rate. However, trade in capital, \( M_i \neq 0 \), will occur in a tax-distorted Nash equilibrium.\(^{16}\)

Proof. We assume (4.2a) is fulfilled with strict equality. For \( t^K_i > 0 \) condition (4.2b) also holds true with strict equality, i.e.

\[
-M_i + \frac{\partial K_i}{\partial t^K_i} (F^K_{iK} M_i + t^K_i) = 0. \tag{4.6}
\]

First the prove for the tax rates: Inserting the full expression of \( (\partial K_i/\partial t^K_i) \) for \( l = 2 \) into (4.6) gives \( t^K_i = M_i F^K_{iK} \). Therefore, a capital importing region, \( M_i < 0 \), chooses a positive tax rate and a capital exporting region, \( M_i > 0 \), subsidizes capital.

Subsequent the proof that a regime which features no trade in capital, \( M_i = 0 \), could not be optimal

\(^{16}\)This result tallies with JANEBA (1995) who restricts the analysis to positive tax rates and concludes that the tax rate of the capital exporting country is driven down to zero if it stimulates international capital flows. The capital importing country sets a positive tax rate to improve its terms-of-trade.

The statement that trade occurs in the Nash equilibrium of non-identical regions does not involve a tautology. If both country's rational acting governments set taxes 'too' high this must result in an standstill of capital movements as is shown by BOND and SAMUELSON (1989) who concludes that the capital exporter will always raise its capital tax rate above the rate set by the capital importing country. On the other hand, the capital importing country adopts the tax rate of the capital exporter. The capital exporting region hence eliminates capital flows in the Nash equilibrium. However, the result of BOND and SAMUELSON is supported by the assumption that the capital exporting country can fully discriminate against foreign capital investment by choosing a different tax rate on foreign interest income and credits the taxes paid abroad.
for either region: Inserting $M_t = 0$ into (4.6) yields $(\partial U^i/\partial t^K_i) = t^K_i (\partial K_i/\partial t^K_i) < 0$. When no capital is traded, every region reduces its tax rate to allow for welfare increasing international trade in capital.

The illustrative figure 2 tallies with the theoretical results. Consider the two large regions which differ in initial endowments such that in a cooperative situation which involves free trade, i.e. ($t^K_i = t^K_j$), region $i$ is capital-importing, $M_i < 0$, whereas region $j$ is capital-exporting, ($M_j = -M_i > 0$). By setting tax rates, both regions form NASH expectations with respect to the rate of the rival’s capital tax. Reaction functions of region $i$ and $j$, $R_i$ and $R_j$, are illustrated with the NASH equilibrium occurring at the intersection of both curves in point $a$.

In order to show the existence of a NASH equilibrium in tax rates notice that due to result 7 an equilibrium point can not be situated in regime $M_t = 0$, hence we rule it out. Now consider a point in regime $M_t > 0$. The optimal tax rate for region $j$ must be positive since any small positive tax improves region $j$’s terms-of-trade as well as its tax revenue. Region $i$’s best reply is to set a negative tax rate. This must lead to a reversal of capital flows. Therefore, a point in $M_t > 0$ can not characterize an equilibrium situation.

The last case is regime $M_t < 0$. The capital importing region, $i$, sets a positive tax rate in order to cut back its excess capital demand, thereby reducing its interest payments and increasing regional tax revenues. The capital exporter, $j$, hence is confronted with bad terms-of-trade due to the lower return of capital. Region $j$’s best response is a negative tax rate because any higher rate will increase the capital outflows into region $i$. For a capital exporting country, the negative terms-of-trade effect of a tax increase always dominates the positive public revenue effect. The point of intersection of the two best-reply curves, $a$, denotes the equilibrium situation in which a very small tax rate, ($t^K > 0$), does not lead to a reversal of capital flows.\(^{17}\)

From the above argument it follows that the NASH equilibrium must yield trade in capital and differences in initial endowments reflect the direction of capital movements from region $i$ to $j$. But at least one of both regions must be worse off at the NASH

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\(^{17}\) The argumentation given here is to some extend similar to the well understood parallel of tariff wars. For an overview over this subject see, for example, MCMILLAN (1982).
equilibrium than at the free trade equilibrium, \((t_f^K = t_f^F)\), in which production efficiency is ensured.

A Pigouvian Subsidy
Due to the existence of tax-base and terms-of-trade effects the Nash equilibrium has the property of being simultaneously individually rational and collectively irrational. Neither region can, by its own actions, make itself better off than at the Nash equilibrium. However, if some degree of cooperation can somehow be achieved, both countries can be made better off. An approach which immediately suggests itself in order to achieve a gain in efficiency lies in an intervention of a higher level of government. The idea of raising a Pigouvian subsidy in order to internalize fiscal externalities provoked by mobile households goes back to Boskin (1973). Wildasin (1989) takes up the idea and applies it to symmetric capital tax competition.

Result 8. If fiscal externalities are present, an intervention of a higher authority in the regional tax autonomy improves global efficiency. If regions are identical the higher level authority grants a Pigouvian subsidy.

Proof. Inserting \((1 - \tau_i)t_i^K\) for \(t_i^K\) everywhere in equation (2.3), and maximizing the utility of a representative household according to the problem (4.1) yields:

\[
\frac{\partial U_i}{\partial t_i^K} = -U_i^r(1 - \tau_i)\frac{K}{N} + U_i^g\frac{\partial g_i}{\partial z_i}K_i + \frac{U_i^g}{N_i}F_i F_i K M_i(1 - \tau_i)\frac{\partial K_i}{\partial t_i^K} + U_i^g\frac{\partial g_i}{\partial z_i}K_i(1 - \tau_i)\frac{\partial K_i}{\partial t_i^K} = 0. \tag{4.7}
\]

If the intervention should guarantee efficiency \(\tau_i\) must be set according to:

\[
\tau_i = -\frac{1}{\left(\frac{N_iK_i}{K_i\frac{F_i}{F_i + F_i + F_i}}\right) + \varepsilon_i} + 1. \tag{4.8}
\]

Due to the counteracting terms-of-trade and tax-base effects the sign of \(\tau_i\) is generally indeterminate. Only if regions are identical, i.e. when terms-of-trade effects are excluded, is \(\tau_i\) unambiguously positive. Re-arranging (4.8) yields:

\[
\tau_i = \frac{\varepsilon_i}{1 + \varepsilon_i} > 0. \tag{4.9}
\]

By means of granting a proportional Pigouvian subsidy according to (4.9), a central government can fill the gap between the locally perceived marginal costs and the social marginal costs of public funds. Every region's government hence has an incentive to increase capital taxes because it can raise public revenue of \((t_i^K K_i)\) by imposing an effective tax rate of \((1 - \tau_i)t_i^K\). All subsidies are financed by a contribution, \(S_i\), which each region \(i\) is obliged to pay to the central government, i.e. \(\sum_i S_i = \sum_i \pi_i t_i^K(K/N)\).  

11
5 Tax Competition and Household Mobility

Most of the literature on capital tax competition has assumed a perfectly immobile population. In this section we refer to some recent contributions who investigate the effects of incomplete household mobility on the outcome of a Nash equilibrium using an approach which goes back to DePalma (1988) and Mansoorian and Myers (1993).

Assume that there are \( N \) households differing from each other only in their attachment to home. Each household, \( n \), receives additional utility if he resides in the preferred region:

\[
V(n) = \begin{cases} 
U^i(x_i, z_i) + a(N - n) & \text{if } n \text{ resides in } i, \\
U^j(x_j, z_j) + aN & \text{if } n \text{ resides in } j.
\end{cases}
\] (5.1)

According to (5.1) utility is additively separable with respect to the utility derived out of the consumption of private and public goods and the utility derived from the psychological component. Each individual differs from the others only in his attachment parameter \( n \), which measures the utility derived from residing in region \( j \). Hence, \( (N-n) \) is the utility resulting from being in region \( i \). The parameter \( a \) measures the degree of household mobility. For \( a = 0 \), regional attachment does not enter the utility function and households are perfectly mobile between regions. If \( a > 0 \) then households are imperfectly mobile. The migration equilibrium is characterized by the marginal household, \( N_i \), being on the same utility level in both regions:

\[
U^i(x_i, z_i) + a(N - N_i) = U^j(x_j, z_j) + aN_i.
\] (5.2)

In choosing the capital tax rate, \( t^K_i \), the head tax rate, \( t^H_i \), and the interregional transfer payment, \( S_i \), the government in each region \( i \) maximizes utility of its residents, taking into account the public and private resource constraints. The problem is to maximize

\[
\max_{t^H_i, t^K_i, S_i} U^i(x_i, g_i),
\] (5.3)

where \( x_i \) must be substituted by (2.3) and \( g_i \) by (2.2). Taking into account all migration responses, some manipulation of the first-order conditions yields:

---

18In a well-known article Gordon (1983) derives first-order conditions for regional behavior in a general model but does not offer an internalization instrument, \( S_i \), to governments. The approach, that costless mobility of households gives incentives for regional governments to strive for efficiency goes back to Boadway (1982) and Myers (1990).


20Therefore, \( N_i \) denotes not only the marginal individual but is also the size of the population in region \( i \), see Mansoorian and Myers (1993).
Rationally acting governments must take migration into account when they decide on their tax policies. Migration responses of households and capital can be derived from the following four-equations system:

\[ U^i \frac{\partial U^i}{\partial t^i} = G_i + U^i_N \frac{\partial N_i}{\partial t^i} + U^i_K \frac{\partial K_i}{\partial t^i} = 0, \quad (5.4a) \]

\[ \frac{\partial U^i}{\partial t^i} = -\frac{U^i}{N_i} M_i + K_i \frac{G_i}{N_i} + U^i_N \frac{\partial N_i}{\partial t^i} + U^i_K \frac{\partial K_i}{\partial t^i} \leq 0, \quad t_i^K \geq 0, \quad \text{and} \quad t_i^K \frac{\partial U^i}{\partial t^i} = 0, \quad (5.4b) \]

\[ \frac{\partial U^i}{\partial S_i} = -U^i \frac{\partial g_i}{\partial z_i} + U^i_N \frac{\partial N_i}{\partial S_i} + U^i_K \frac{\partial K_i}{\partial S_i} \leq 0, \quad S_i \geq 0, \quad \text{and} \quad S_i \frac{\partial U^i}{\partial S_i} = 0, \quad (5.4c) \]

where \( M_i \equiv (N_i/N - K_i) \), \( G_i, U^i_K \) as defined in (4.3a) and (4.3b), respectively, and besides

\[ U^i_N = \frac{U^i}{N_i} \left[ (F^i_N - z_i) + F^i_{KN} M_i + K^i \left( F^i_K - t_i^K \right) + \frac{G_i}{U^i} + N_i \frac{U^i}{U^i} \frac{\partial g_i}{\partial N_i} \right]. \quad (5.5a) \]

Our basic analytical goal is to insert the factor market reactions into equations (5.4) and re-write the latter in such a way that we are able to compare them with the efficiency conditions (3.3).

For the sake of clarity notice that the effects triggered by the head tax are similar to those caused by the capital tax since both taxes lead to reallocations of mobile factors, i.e. \( \frac{\partial N_i}{\partial t^i_N} \neq 0 \). In the following, two alternative sets of regional tax instruments will therefore be considered. The first is that there exists an exogenous upper bound on the tax on capital, \( t^K = 0 \), whereas in the second both mobile factors can be optimally taxed.

In the case of an exogenous upper bound on the taxation of capital the associated first-order condition (5.4b) is no longer binding and thus redundant. In order to identify the conditions under which decentralized tax policies lead to an efficient outcome we insert the migration responses \( \left( \frac{\partial N_i}{\partial t^i_N} \right) \) and \( \left( \frac{\partial K_i}{\partial t^i_N} \right) \) into the first-order condition (5.4a). This yields:\(^{21}\)

\[ G_i A_2 = 0. \quad (5.7) \]

\(^{21}\)Derivations of this and the following equations are collected in appendix B.
Following condition (5.7), local public goods are provided efficiently, i.e. \( G_i = 0 \), if the determinant \( |A_2| \) is unequal to zero. We make use of (5.4c) to test for the size of \( |A_2| \).

Inserting \( (\partial N_i/\partial S_i) \) and \( (\partial K_i/\partial S_i) \) into the first-order condition (5.4c) results in:

\[
\frac{1}{U^j} \frac{\partial U^j}{\partial z_i} |A_2| = \frac{1}{U^j} \frac{\partial U^j}{\partial z_i} |A_i| \leq 0. \tag{5.8}
\]

Equation (5.7) together with (5.8) implies our next result on the efficiency properties of a Nash equilibrium in taxes:

**Result 9.** If governments are able to choose \( t_i^N \) according to (5.7) and \( S_i \) according to (5.8) then the Samuelson-condition is fulfilled, i.e. local public goods are provided efficiently, and households are allocated efficiently across regions.

**Proof.** Stability requires \( |A| = |A_1| + |A_2| < 0 \) and hence \( |A_2| < -|A_1| \). Additionally taking into account (5.8) proves that \( |A_2| < 0 \). Therefore, condition (5.7) implies \( G_i = 0 \) and (5.8) ensures an efficient allocation of households.

The interpretation of result 9 is straightforward. Although the head tax distorts the migration decisions of mobile households — and therefore causes fiscal externalities comparable to those arising from source–based capital taxes — the tax revenues yielded both are sufficient for an efficient provision of local public goods and for an interregional transfer. If a region performs a transfer it serves to neutralize the fiscal externalities caused by the distortive head tax and hence the transfer guarantees an efficient interregional allocation of mobile households in the Nash equilibrium.

The Samuelson condition enables us to re-write condition (5.8):

\[
\frac{N^j}{U^j} |A_2| - \frac{N_i}{U^j} |A_i| \leq 0, \tag{5.9}
\]

or, equivalently,

\[
\left| (F_j^N - x_j) - (F_i^N - x_i) + N_j \frac{\partial U^j}{\partial N_j} \frac{\partial N_j}{\partial \lambda_i} - N_i \frac{\partial U^j}{\partial N_i} \frac{\partial N_i}{\partial \lambda_i} - 2a \frac{N_i}{U^j} (t_j^N - t_i^N) \right| \leq 0. \tag{5.10}
\]

Before we discuss the efficiency properties of the capital tax, it is useful to identify the cases under which (5.9), or equivalently (5.10), is fulfilled with strict equality.

**Result 10.** Condition (5.9) holds with strict equality if households are perfectly mobile, \( (a = 0) \). If households are imperfectly mobile at least one region does not perform the transfer, i.e. condition (5.9) holds with strict inequality.
Proof. To prove the first part of result 10 insert \((a = 0)\) into (5.10). Then equation (5.10) holds simultaneously and with strict equality for both regions. Hence, both regions grant transfers.

Next, we prove by contradiction that at least one region does not pay a transfer if households are imperfectly mobile, \((0 < a < \infty)\). Assume that region \(i\) would pay a transfer. Then region \(i\)'s first-order condition becomes \(G_i^f (t_i^f - t_i^f) + G_i^f ((F_i^f - x_i) - (F_i^f - x_i)) + N_i (U_i^f / U_i^f) (\partial g_i / \partial N_i) - N_i (U_i^f / U_i^f) (\partial g_i / \partial N_i) = -2aG_i^f (N_i / U_i^f) \neq 2aG_i^f (N_i / U_i^f)\), what completes the prove. Evidently, if both regions are identical, i.e. \((t_i^f - t_i^f) = 0\) and \((F_i^f - x_i) = (F_i^f - x_i)\) + \(N_i (U_i^f / U_i^f) (\partial g_i / \partial N_i) - N_i (U_i^f / U_i^f) (\partial g_i / \partial N_i) = 0\), no region performs a transfer.

In order to interpret the result we follow the intuition given in WELLISCH (1995: 253). Due to the migration equilibrium, perfect household mobility ensures that governments maximize the utility of households in the whole federation. Hence, regional governments agree over the interregional transfer. If households are imperfectly mobile, both regions disagree over the desired population size and at least one region does not provide a transfer, i.e. behaves non-cooperatively. According to MANSOORIAN and MYERS (1993) the better endowed with immobile land a region is, the more likely it makes a transfer.

Equation (5.9) proves helpful concerning the derivation of the last results. We enlarge the set of available regional tax instruments and re-introduce capital taxation into the model, i.e. \(t_i^f > 0\). Hence, the associated first-order condition (5.4b) is not redundant any longer and \((\partial U_i / \partial t_i^f) \leq 0\) must hold with strict equality. From inserting the migration responses \((\partial N_i / \partial t_i^f)\) and \((\partial K_i / \partial t_i^f)\) into (5.4b) follows that:

\[
K_i G_i^f \left[ \frac{N_i U_i^f}{U_i^f} |A_2| - \frac{N_i U_i^f}{U_i^f} |A_1| \right] - \frac{N_i U_i^f}{U_i^f} K_i \left[ 1 - \frac{N_i U_i^f}{U_i^f} |A_1| \right] + \left[ t_i^f \left( 1 + \frac{G_i}{U_i^f} \right) - t_i^f \left( 1 + \frac{G_i}{U_i^f} \right) \right] \frac{N_i U_i^f}{U_i^f} |A_2| = 0. \tag{5.11}
\]

This equation will now be interpreted for two special cases. To highlight the implications of household mobility on regional tax policies we consider both perfect and imperfect mobility. Firstly turn to the case of perfect mobility. Condition (5.11) implies:

**Result 11.** If households are perfectly mobile and a region's policy instruments, \(t_i^f, t_i^f, S_i\), are chosen locally optimal, decentralized fiscal decision making, i.e. NASH behavior of regional governments, achieves global efficiency.

**Proof.** If households are perfectly mobile, condition (5.9) holds with strict equality. Using \(G_i = 0\) from the first–order condition (5.7) in condition (5.11) produces the result that capital tax rates are set efficiently, \(t_i^f = t_i^f\).

Result 11 enlarges our previous result 9 by the statement that capital taxes are also set efficiently by governments in the Nash equilibrium if households are perfectly mobile. The rationale behind is driven by an interpretation of the equal utility migration equilibrium which proves to be a powerful device for coordinating non-cooperative fiscal policies.\(^\text{22}\) The core of the argument is that, driven by their own interest, all governments maximize world welfare because they have to take into account that households

\(^{\text{22}}\)Recently, this has been analyzed in MYERS (1990).
must attain the same utility level everywhere in the whole federation. Hence, they agree on an interregional income distribution for which to achieve the transfer constitutes the first-best instrument. According to result 10 no government is constrained in the use of the transfer and this excludes head or capital taxes being used strategically. Non-cooperative tax policies would not only lead to a change in the interregional distribution of income but additionally generate distortions and hence are only second-best efficient.

To set an example for a combination of taxes which yields an efficient allocation but does not require an international transfer payment, suppose regions are identical and local public goods are publicly provided private goods \( (z_i = N_i g_i) \). In this case, the condition for an efficient population allocation and the public revenue requirements, \( G_i = 0 \), could simultaneously be fulfilled by regional governments.\(^{23}\) The intuition is reflected by our next result:

**Result 12.** If households are costlessly mobile, regions are identical and have both access to a head tax and to a source-based tax on capital then the Nash equilibrium yields efficiency without the necessity for an interregional transfer.

Proof. Inserting the first-order condition (5.7) into (5.11) yields for identical regions \( |A_2| = (N_i/U_i^2) / (N_j/U_j^2)|A_1| \neq 0 \). Therefore, \( G_i = 0 \) and \( t_i^K = t_j^K \).

Our last result involves imperfect household mobility:

**Result 13.** If households are imperfectly mobile, regional governments have access to a head tax and moreover to a source-based capital tax the Nash equilibrium between regions results in \( t_j^K = f_{KK}^j M_j \).

However, the region which voluntarily makes an interregional transfer ensures an efficient allocation adopting the tax rate of its rival, i.e. \( t_i^K = t_j^K \).

Proof. If households are imperfectly mobile at least one region does not make a transfer. Hence, condition (5.9) holds with strict inequality for that region. Inserting (5.9) and \( G_i = 0 \) into (5.11) yields \( t_j^K = f_{KK}^j M_j \), which is positive (negative) for a capital importing (exporting) country and zero in the case of identical countries. Next, we prove the second part of result 13. A region which voluntarily pays a transfer sets its tax rate efficiently, \( t_i^K = t_j^K \), since then equation (5.9) holds with strict equality.

This result is related to Burbidge and Myers (1994) who state that an interregional transfer always ensures efficiency. Even though one region performs a non-cooperative tax policy, which is second-best in order to generate interregional income redistributions, the other region uses the first-best instrument and sets tax rates efficiently. The region which grants a transfer cannot avoid that the other region uses tax rates strategically to influence the interregional distribution of income, neither by granting a transfer nor by using the second-best instrument in the form of taxes.\(^{24}\)

\(^{23}\)See result 3.

\(^{24}\)Contrary to Wellisch (1995) decentralized fiscal policies lead to an efficient allocation of resources since we excluded interregional spillovers of the public goods.
6 Summary

We have investigated various reasons for inefficient behavior of regional governments and proposed some remedies for welfare losses caused by non-cooperative tax policy. Firstly, a single country is aware of the fact that a tax increase induces capital flight to other countries. By raising the costs of capital above its after-tax return, the capital tax encourages private firms to reduce the capital intensity of production. Since the world’s capital stock is fixed, the induced capital outflow implies capital imports of identical magnitude elsewhere in the federation. Due to the fact that a region neglects the consequence of its own fiscal decisions on the other regions utility level, the tax base effect represents a positive fiscal externality, in that any tax increase creates an external benefit for the other regions but lowers the utility of only its inhabitants. Secondly, regions which are in the position to manipulate interregional prices create an additional source of inefficiency because they strategically influence the interregional distribution of income via terms-of-trade effects to their own advantage.

A means to hold off inefficiencies of tax competition is the harmonization of tax rates or the intervention of a supra-regional level of government which grants Proovian subsidies and aims at internalizing fiscal externalities. However, in our simple model none of the fiscal externalities lasts if consumers are mobile across regions. Now that regions are granting transfer payments of their own accord they have incentives to adopt the tax rate of its rival.

Future work will aim to generalize the analysis towards endogenous labor and capital supply and should also involve a more sophisticated tax system. This enables an efficiency analysis of a general tax system in a framework which takes account of the high interregional mobility of factors.
A Appendix to Section 4

Using region i's Nash assumption and total differentiation of (4.4) yields the following system:

\[
\begin{pmatrix}
F_{KK}^i & 0 & -1 \\
0 & F_{KK}^j & -1 \\
1 & (l-1) & 0 \\
\end{pmatrix}
\begin{pmatrix}
dK_i \\
dK_j \\
\frac{dr}{dt} \\
\end{pmatrix}
= \begin{pmatrix}
dt_i^f \\
dt_j^f \\
0 \\
\end{pmatrix}.
\]  

(A-1)

The determinant \( |A| \) of the Jacobian on the left hand side of (A-1) is negative:

\[
|A| = F_{KK}^j + (l-1)F_{KK}^i < 0.
\]  

(A-2)

Applying Cramer's rule on (A-2) yields:

\[
\frac{\partial K_i}{\partial t_i^f} = \frac{1}{|A|} \begin{vmatrix}
1 & 0 & -1 \\
0 & F_{KK}^j & -1 \\
0 & (l-1) & 0 \\
\end{vmatrix} = \frac{(l-1)}{F_{KK}^j + (l-1)F_{KK}^i} < 0,
\]  

(A-3a)

\[
\frac{\partial r}{\partial t_i^f} = \frac{1}{|A|} \begin{vmatrix}
F_{KK}^i & 0 & 1 \\
0 & F_{KK}^j & 0 \\
1 & (l-1) & 0 \\
\end{vmatrix} = -\frac{F_{KK}^j}{F_{KK}^j + (l-1)F_{KK}^i} < 0.
\]  

(A-3b)

B Appendix to Section 5

The migration equilibrium (A-1) is defined by a system of four equations in four endogenous variables, \( K_i, N_i, K_j, N_j \), with six exogenous variables, \( t_i^K, t_j^K, t_i^N, t_j^N, S_i, S_j \). Inserting (5.6c) and (5.6d) everywhere in (5.6a), (5.6b), and total differentiation yields:

\[
\begin{pmatrix}
G_N^i & G_K^i \\
H_N^i & H_K^i \\
\end{pmatrix}
\begin{pmatrix}
dN_i \\
dK_i \\
\end{pmatrix}
= -\begin{pmatrix}
G_{tK}^i & G_{tN}^i & G_S^i \\
H_{tK}^i & H_{tN}^i & H_S^i \\
\end{pmatrix}
\begin{pmatrix}
dt_i^f \\
dt_i^N \\
\frac{ds_i}{dt} \\
\end{pmatrix}.
\]  

(B-1)

where

\[
G_N^i = F_{KN}^i + F_{KN}^j, \quad G_K^i = F_{KK}^j + F_{KK}^i, \\
H_N^i = U_N^i + U_N^j - 2a, \quad H_K^i = U_K^i + U_K^j, \\
G_{tK}^i = -1, \quad G_{tN}^i = 0, \quad G_S^i = 0, \\
H_{tK}^i = \frac{K_i}{N_i} G_i - \frac{U_i}{N_i} M_i, \quad H_{tN}^i = G_i, \quad H_S^i = -U_{g_i} \frac{dg_i}{dz_i} - U_{g_j} \frac{dg_j}{dz_j}.
\]
Let $|A| \equiv |A_1| + |A_2|$ denote the matrix on the left hand side of (B-1):

$$
|A| = \begin{vmatrix}
G^i_N & G^i_K \\
U^i_N & U^i_K
\end{vmatrix} + \begin{vmatrix}
G^i_N & G^i_K \\
U^j_N & U^j_K
\end{vmatrix}.
$$

Assuming $|A| < 0$, which ensures local stability of the migration equilibrium (5.6), we can use Cramer’s rule in order to derive the migration responses:

$$
\frac{\partial N_i}{\partial t^K} = \frac{1}{|A|} \begin{vmatrix}
G^i_K \\
-H^i_K
\end{vmatrix},
$$

(B-2a)

$$
\frac{\partial N_i}{\partial t^N} = \frac{1}{|A|} \begin{vmatrix}
G^i_K \\
-G_i
\end{vmatrix},
$$

(B-2b)

$$
\frac{\partial N_i}{\partial S_i} = \frac{1}{|A|} \begin{vmatrix}
-G^i_K \\
-H^i_K
\end{vmatrix},
$$

(B-2c)

$$
\frac{\partial K_i}{\partial t^K} = \frac{1}{|A|} \begin{vmatrix}
G^i_N \\
-H^i_N
\end{vmatrix},
$$

(B-2d)

$$
\frac{\partial K_i}{\partial t^N} = \frac{1}{|A|} \begin{vmatrix}
G^i_N \\
-G_i
\end{vmatrix},
$$

(B-2e)

$$
\frac{\partial K_i}{\partial S_i} = \frac{1}{|A|} \begin{vmatrix}
-G^i_N \\
-H^i_N
\end{vmatrix},
$$

(B-2f)

Inserting (B-2c) and (B-2f) into (5.4c) and multiplying the whole expression by $|A|$ yields:

$$
\begin{vmatrix}
G^i_N; \\
U^i g^j_g (U^j_N - 2a) - U^j g^j_j g^j_j \\
U^i g^j_g U^j_K - U^j g^j_j g^j_j K
\end{vmatrix} \leq 0.
$$

(B-3)

Multiplying (B-3) by $1/(U^j g^j_g (\partial g^j / \partial z_N) U^j (\partial g^j / \partial z_N))$ gives (5.8). For $G_i = 0$ we can re-arrange (B-3):

$$
\begin{vmatrix}
G^i_N; \\
U^i N_i (U^i_N - 2a) - U^i g^j_g N_j \\
U^j g^j_j g^j_j K
\end{vmatrix} \leq 0.
$$

(B-4)

Using the full expressions for $U^i_N$, $U^i_K$ and $U^j_K$, multiplying (B-4) by $(N_i N_j)/(U^i N_i)$ and adding the by $M_i$ multiplied first row results of equation (5.10) in the main text.

Inserting (B-2a) and (B-2d) into (5.4b) and multiplying by $|A|$ gives:

$$
\begin{vmatrix}
-\frac{U^i_N}{N_i} M_i + \frac{K_i}{N_i} G_i \\
Ni N_j (U^i_N - 2a) - U^i g^j_g N_j \\
U^j g^j_j g^j_j K
\end{vmatrix} + \begin{vmatrix}
\frac{Ni N_j}{U^i g^i g^i} U^i_N - U^i g^i g^i \\
U^j g^j_j g^j_j K
\end{vmatrix} = 0.
$$

(B-5)
Using the full expressions for $U_N^i, U_N^j, U_K^i$ and $U_K^j$ leads to:

$$\frac{K_I}{U_z} G_i \frac{N^i_j}{U_z} |A_2| + \frac{N_j}{U_z^i} (F_N^i - x_i) + N_i \frac{U_i^j}{U_z^i} \frac{\partial g_{ij}}{\partial N_i} - (F_N^j - x_j) - (t^K - t^j) + t^K \frac{G_i}{U_z^i} - t^j \frac{G_j}{U_z^j};$$

$$U_N - 2a;$$

$$U_K^i;$$

Further re-arranging and dividing the expression by $(N_i/U_z^i)|A_1|$ gives the first-order condition (5.11).
References


