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## Analyzing firm mortality using life table techniques

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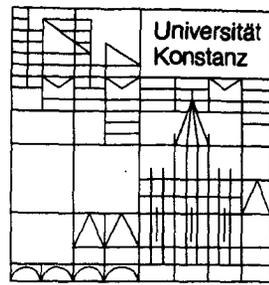
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Diskussionsbeiträge



Juristische  
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**Using Life Table Techniques**

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Serie II - Nr. 232

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# ANALYZING FIRM MORTALITY USING LIFE TABLE TECHNIQUES

Peter Pflaumer

**ABSTRACT:** The mortality of German joint stock companies is investigated using life table techniques. An IDB-distribution or Hjorth-distribution and an exponential distribution are fitted to the hazard rates, whereby the population is divided into small and large firms. Survivor functions and life expectancy functions are presented. The life expectancy of all firms is circa 40 years. In the absence of data, it is recommended that the life expectancy of a firm be estimated as the reciprocal value of the proportion of firms that have disappeared from the market.

**KEY WORDS:** force of mortality, IDB-distribution, exponential distribution, value of a firm

## I. INTRODUCTION

The value of a firm is obtained with the present value method as the sum of all future cash flows, which are discounted with the required minimum rate of return on new investments. If one assumes an average cash flow of  $c$  and a life expectancy of  $n$ , then the firm value  $v$  results as:

$$v = c \frac{1}{q^n} \frac{q^n - 1}{q - 1}$$

with

$n$  the life expectancy

$q = 1 + \frac{p}{100}$  the discount factor

$p$  the discount rate.

In addition to the cash flow to be estimated, which lies in the future and cannot be accurately predicted at the time of evaluation, the life expectancy  $n$  must also be determined. If one assumes an infinite life expectancy, then the firm value  $v$  is given through

$$v = \frac{c}{q - 1}$$

This model is only then adequate if the firm has a long life expectancy. The difference between finite and infinite present value can generally be disregarded if  $n$  is longer than 50 years, as the example calculations show.

In the following, period life-table analysis is used to estimate the life expectancy of firms in Germany. The analysis is based on data on joint stock-holding firms in West Germany between 1980 and 1986. This time period was chosen because it includes the course of an entire business cycle. Thus the mortality that is influenced by the economic situation is neither under- nor overestimated. The source of the data is the "Handbuch der deutschen Aktiengesellschaften" (Handbook of German Stock Corporations), which is published quarterly by the Hoppenstedt-Verlag, Darmstadt, Germany. The collection and analysis of the data was done as part of a master's thesis that I assigned to Beate Herrchen (cf. Herrchen, 1990). The average number of firms in the period of investigation was more than 2100. 382 firms disappeared from the market, while 361 firms were newly established. Firm mortality can happen for various reasons. A corporation may go bankrupt, be suspended from trading or delisted, in which case its shareholders are likely to experience unfavorable results (unfavorable mortality, see Queen and Roll, 1987). Alternatively, a firm may merge with or be acquired by another firm, in which case the results for shareholders may be quite favorable (favorable mortality, see Queen and Roll, 1987). Unfortunately, the data could not be used to distinguish between favorable and unfavorable mortality. In demography, one would understand unfavorable mortality to be mortality in the literal sense and favorable mortality to be migration.

The average age of the corporations in this period of investigation was 48 years. The oldest corporation was 176 years old. Due to the foundation of corporations after the Second World War, the age-structure is bimodal. Peaks can be observed in the age classes 0-8 and 56-64 (cf. Herrchen, 1990).

## II. LIFE TABLE ANALYSIS

A life table is a statistical model for studying the mortality which is generated from age-specific mortality rates. It calculates the life expectancy by taking a hypothetical population and following it "through life", assuming that at all times people die according to the age-specific rates at year  $x$ .

The function  $l(x)$  represents the numbers of survivors up to the exact age  $x$  for an original number of births  $l(0) = 1000$  or  $l(0) = 1$ .

The probability of dying during the next  $n$  years for a person who has reached age  $x$  is

$${}_nq_x = 1 - \frac{l(x+n)}{l(x)}$$

which is estimated from the age-specific death rate that is defined as the ratio of the number of deaths in a group during a given period to the average number of persons in that group (see, e.g., Keyfitz, 1977).

The life expectancy at age  $x$  is the average number of years to be lived by those surviving to age  $x$

$$e(x) = \frac{\int_x^{\infty} l(a) da}{l(x)}.$$

Theoretical statements on mortality are often expressed with a measure which is called force of mortality or hazard rate or failure rate at age  $x$ :

$$\mu(x) = \frac{-1}{l(x)} \frac{dl(x)}{dx}.$$

The force of mortality is estimated from the probability of dying, using a Taylor series expansion up to the 2nd degree, which yields the following approximation (see Clairin and Condé, 1986):

$$\mu(x) \approx \frac{1}{n} \left( {}_nq_x + \frac{({}_nq_x)^2}{2} \right).$$

Solving the differential equation that defines  $\mu$  in the case that  $\mu(x) = \mu$  gives the exponential distribution

$$l(x) = e^{-\mu x}$$

with its life expectancy

$$e(x) = \frac{1}{\mu}.$$

The life expectancy at age  $x$  is constant and independent of  $x$ .

Exponential distributions and Weibull distributions, which are not described here, can be used to model constant, decreasing and increasing hazard rates. However, the hazard rate often has a bathtub-shaped course. At first the mortality is high, then it is constant for a certain time and in the end it increases again.

The IDB-distribution (Increasing, Decreasing, Bathtub-shaped) introduced by Hjorth (1980) is a distribution that also allows the modelling of bathtub-shaped failure rates, in addition to constant, decreasing and increasing failure rates.

The hazard rate of the IDB(a,b,c)-distribution is

$$\mu(x) = ax + \frac{c}{1+bx}$$

From this, then one calculates the survivor function

$$l(x) = \frac{e^{-ax^2/2}}{(1+bx)^{c/b}}$$

Special cases of the IDB-distribution are:

- $c = 0$ : a special Weibull distribution
- $a = b = 0$ : the exponential distribution (a Weibull distribution)
- $a = 0$ : decreasing failure rate
- $a \geq c/b$ : increasing failure rate
- $0 < a < c/b$ : bathtub curve

The life expectancy can only be computed numerically:

$$e(0) = \int_0^{\infty} \frac{e^{-ax^2/2}}{(1+bx)^{c/b}} dx$$

A measure for the description of a mortality table is the so-called entropy  $H$ , which is defined as follows:

$$H = - \frac{\int_0^{\infty} (\ln l(x)) l(x) dx}{\int_0^{\infty} l(x) dx}$$

$H$  measures the degree of convexity of  $l(x)$ . If the mortality is slight, i.e. the overwhelming number of deaths only occur after a certain limit is surpassed (e.g. in the case of human populations), then  $H$  is approximately zero. If this limit approaches the younger ages, then  $H$  increases. In the case of  $H=1$ , the mortality is distributed equally among all ages. If an exponential distribution is present, then  $H=1$ . Keyfitz (1977) cites  $H$ . Deevey, who gives a number of curves for animal species that resemble the curves of Fig. 5, with fruit flies near  $H=0$ , oysters below  $H=1$ , and hydra near  $H=0.5$ . In the latter case  $l(x)$  is a straight line.

### III. EMPIRICAL RESULTS

The hazard rates were calculated from the mortality rates. The total was divided into two subpopulations. The one subpopulation contains "small" firms (balance-sheet total under 75 million DM), whereas the other subpopulation contains large firms (balance-sheet total 75 million DM and over). This division yielded a proportion of 54% for the small firms and 46% for the large firms. The hazard rates for the total and for the two subpopulations can be seen in Fig.1 to 3. Due to the relatively small number of observations, the empirical hazard rates show an erratic course. Using a nonlinear method of least square estimation, the following IDB-distributions were fitted to the observations:

	a	b	c
all firms:	0.000164	0.0872	0.0543
	(0.0000388)	(0.0411)	(0.0198)
average hazard rate			
$\mu = 0.023$			
minimum of the hazard function $x_{\min} = 50.2$ years			
small firms:	0.000393	0.0759	0.069
	(0.0000864)	(0.0575)	(0.0198)
average hazard rate			
$\mu = 0.0409$			
minimum of the hazard function $x_{\min} = 34.9$ years			
large firms:	0.000052	0.0735	0.0187
	(0.000024)	(0.0568)	(0.0054)
average hazard rate			
$\mu = 0.0081$			
minimum of the hazard function $x_{\min} = 56.3$ years			

The standard deviations are located under the estimators in parentheses. The mortality is the highest in the first three age groups. The mortality risk sinks in the subsequent age classes and reaches minimal values for the corporations between 30 and 60 years old. After this, the mortality risk rises slightly. This risk is significantly higher for small businesses than for large ones.

Fig. 4 shows the survival functions of all firms. The empirical survival function is compared to theoretical survival functions which are based partially on the exponent distribution

$$l(x) = e^{-0.023x}$$

and partially on the IDB-distribution

$$l(x) = \frac{e^{-0.000164x^2/2}}{(1 + 0.0872x)^{\frac{0.0543}{0.0872}}}$$

In the lower age groups the exponential distribution underestimates the mortality. The fit of the IDB-distribution is rather good, although the mortality is overestimated in the middle age groups. The survival functions of the two sub-populations have a comparable course. The entropies of the IDB-distribution are  $H=1.12$  for all,  $H=1.095$  for small and  $H=0.86$  for large firms. The mortalities are thus not concentrated in certain age groups. The entropies do not differ that much from that of the exponential distribution, which is always  $H=1$ .

For comparison, the survivor function of the mortality table of the female population of Germany 1980/82 is given in Fig. 5. Its entropy of  $H=0.12$  is significantly smaller. In human populations, the mortality is concentrated in higher age groups. In the last 100 years, the entropy of the survivor functions of the female population in Germany has decreased from  $H=0.57$  to  $H=0.12$  because of the decrease in infant mortality.

Fig. 6 shows the IDB-survivor functions for small, large and all firms. The median age is 21 years for all firms. The small firms are at 14 years significantly younger than the large ones, whose median age is 85 years.

Fig. 7 shows the development of the life expectancy at age  $x$  based on an IDB-distribution. The life expectancy of a newly established firm is about 40 years for all firms, 24 years for small ones and 108 for the large ones. The life expectancy at age  $x$  first rises for all groups and then slowly sinks after having reached a maximum at the age of 20 to 30 years. Taking the exponential distribution as a basis, the life expectancies of newly established firms are calculated to be 43.5 years, 24.4 years and 123.5 years respectively.

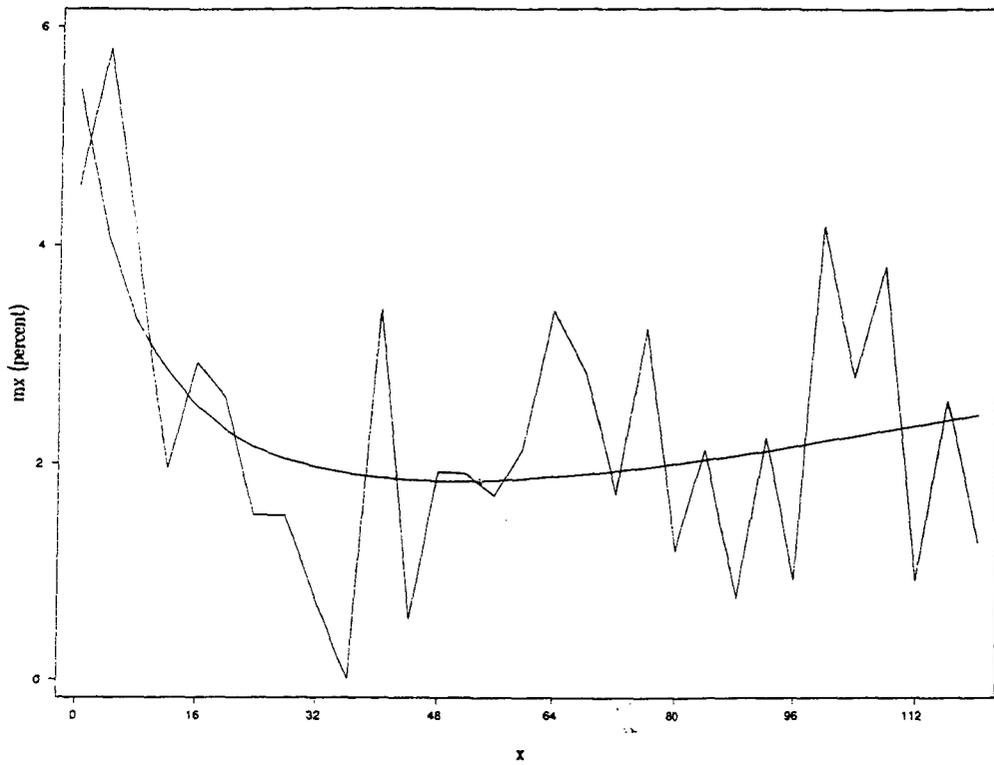
#### IV. CONCLUSION

The estimation of a firm's life expectancy based on a life table requires collecting the appearance and disappearance data of very many firms. Although the analysis carried out here yielded the result that the force-of-mortality function of the firms showed a bath-tub-shaped course, only the exponential distribution can be used to estimate the life expectancy of a firm

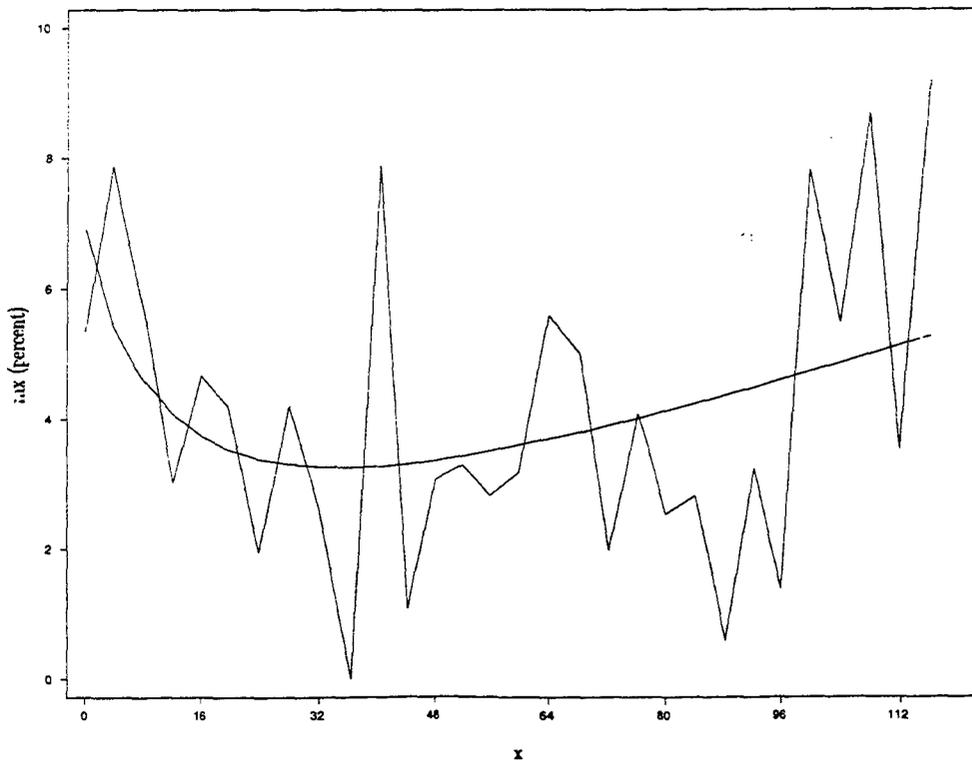
for practical purposes, especially when available data is not present. The life expectancy then results as a reciprocal value of the percentage of firms that disappeared from the market in a given period. If this is smaller than 2%, it can be assumed that the life expectancy is at least 50 years. In this case the equation of the firm evaluation can be simplified by assuming an infinite life expectancy. When calculating the percentage of firms that have disappeared from the market, one should do it for each branch separately and include several years in order to compensate for economic fluctuations.

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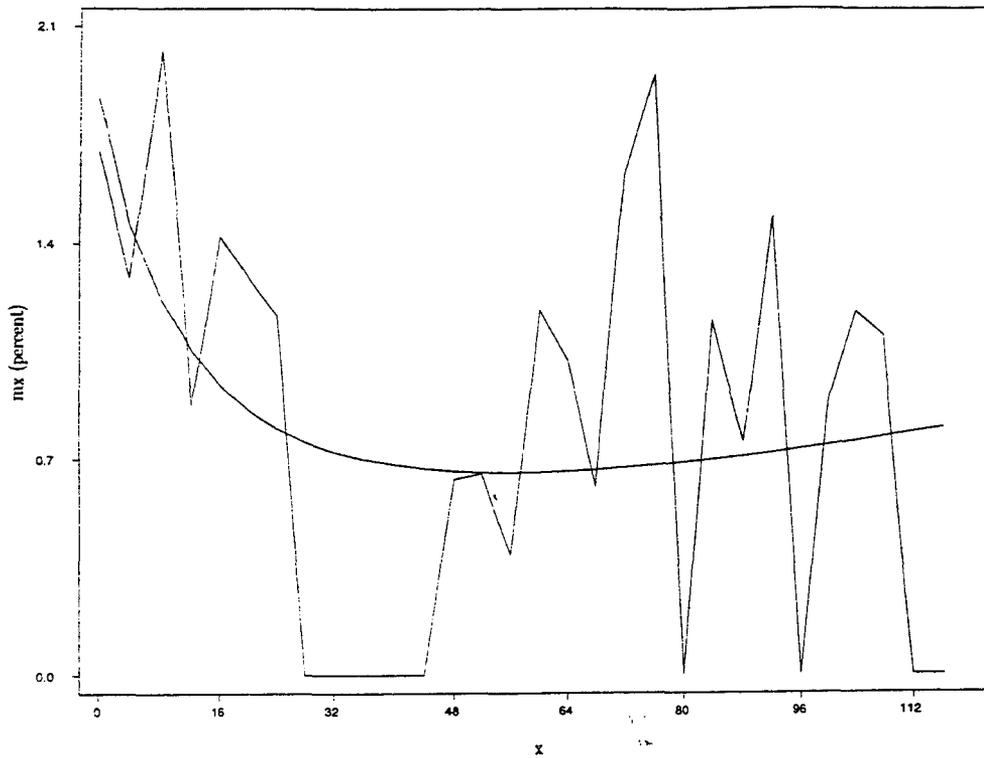
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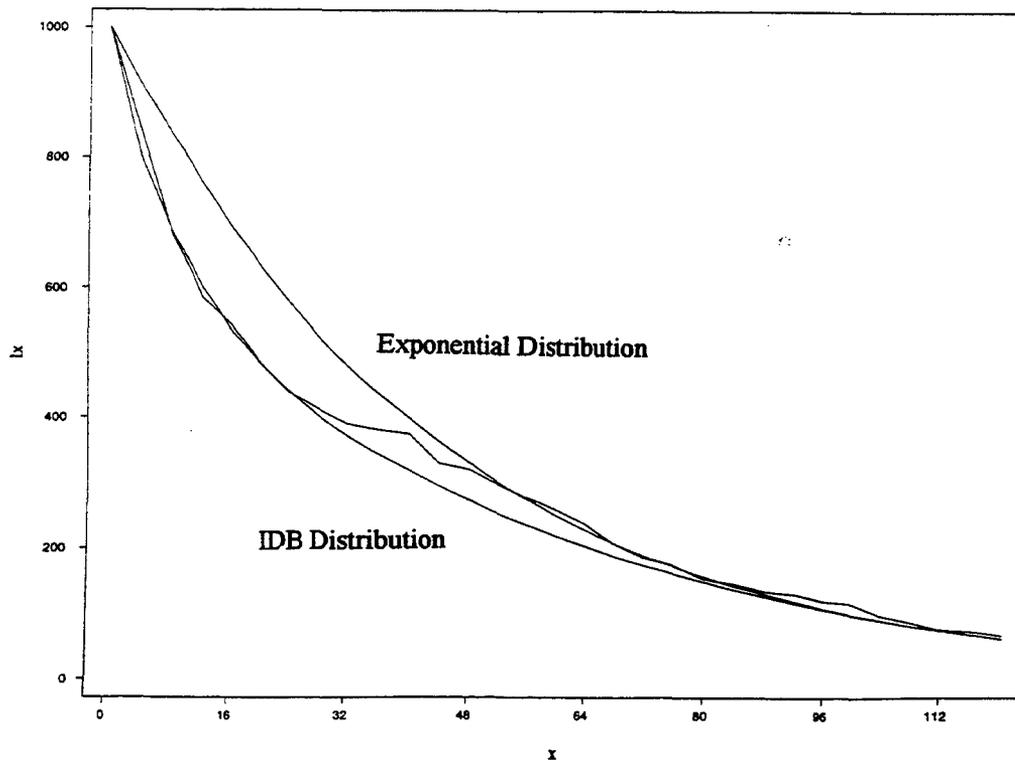
**Fig. 1 : Empirical and Theoretical Hazard Rates of All Firms**



**Fig. 2 : Empirical and Theoretical Hazard Rates of Small Firms**



**Fig. 3 : Empirical and Theoretical Hazard Rates of Large Firms**



**Fig. 4 : Survival Functions of All Firms**

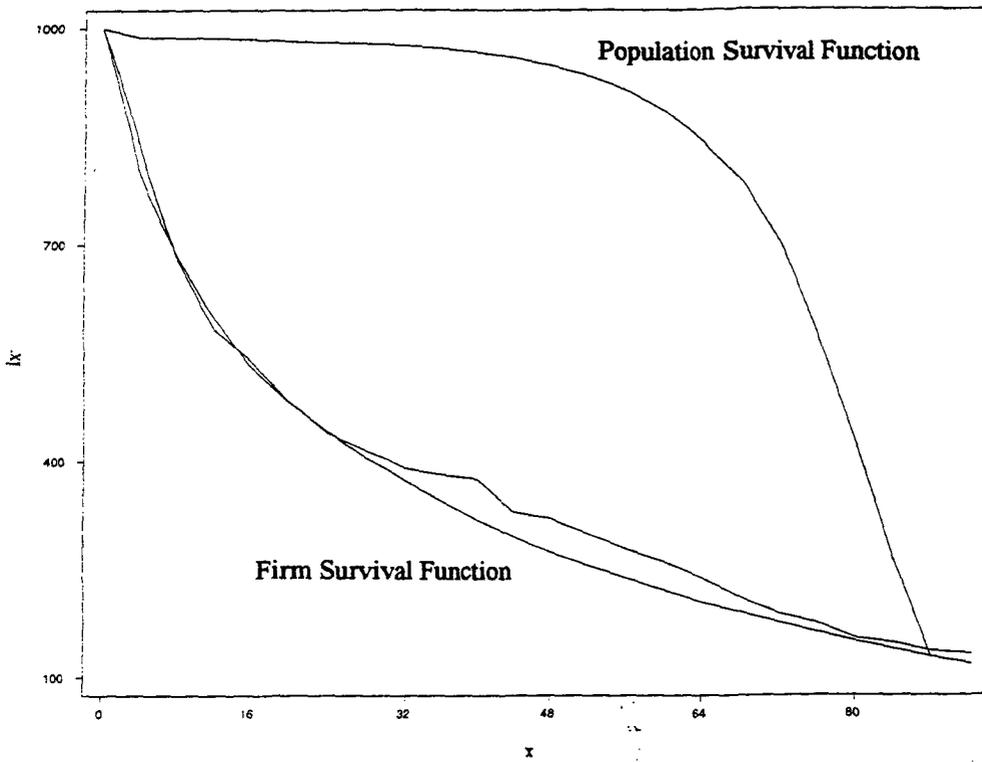


Fig. 5 : Population Survival Function and Firm Survival Function

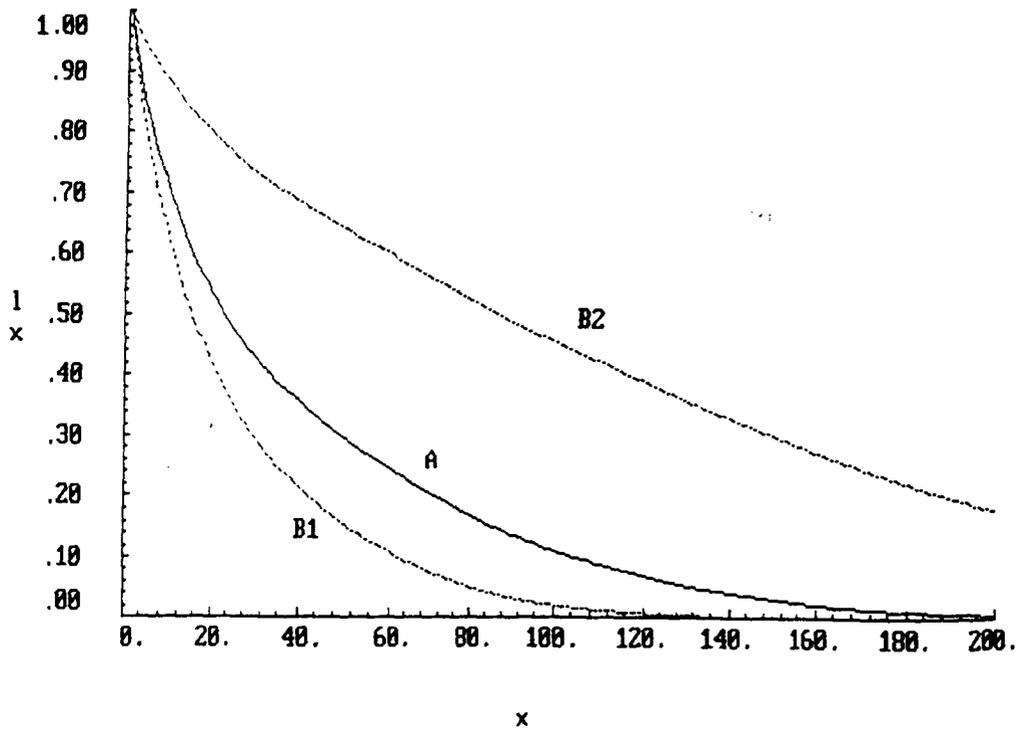


Fig. 6 : IDB-Survival Function of all Firms (A), Small Firms (B1) and Large Firms (B2)

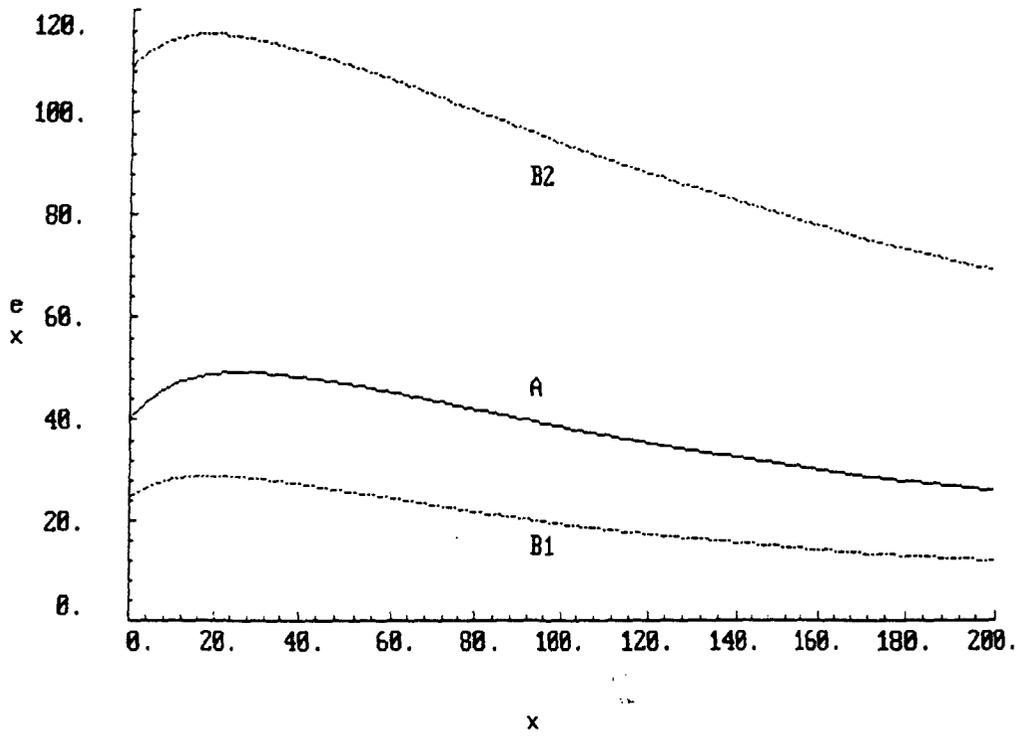


Fig. 7 : Expectation of Life at Age  $x$  of All Firms (A), Small Firms (B1) and Large Firms (B2)