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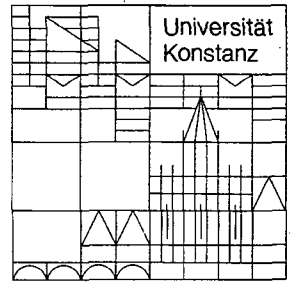
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Sonderforschungsbereich 178
„Internationalisierung der Wirtschaft“

Diskussionsbeiträge



Juristische
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Fakultät für Wirtschafts-
wissenschaften und Statistik

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**Equilibrium Term Structure Relations of
Risky Assets in Incomplete Markets**

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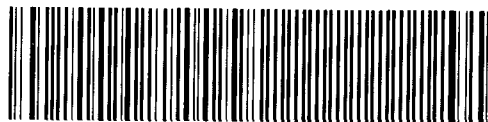
**EQUILIBRIUM TERM STRUCTURE RELATIONS
OF RISKY ASSETS IN INCOMPLETE MARKETS**

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Abstract

This paper presents a simple theoretical model of the term structure and analyzes the relations among optimal portfolio decisions, the real term structure of asset returns, and the risks and price volatilities of assets with different terms to maturity when the investor preferences are non-time-separable. It is argued that specifying utility to be a non-time-separable function of consumption allows for richer term structure relations than separable specifications. The model is capable to explain why term premiums vary and why the term structure may fail to be monotone. Our analysis also demonstrates that the planning horizon of the agents critically affects the term structure of asset returns. The competitive mechanism tends to undervalue short-term risks relative to long-term risks if the investors have short planning horizons.

1 Introduction

Equilibrium term structure relations depend critically on the specification of the intertemporal marginal rate of substitution of consumption. Yet the majority of the theoretical term structure literature uses models which either explicitly or implicitly assume additive time-separability of the investor preferences (e.g. Canova and Marrinan [1991], Cox, Ingersoll, and Ross [1985], Mankiw and Summers [1984], Shiller [1979] et. al.). An exception to this rule is the paper by Dunn and Singleton [1986] in which the assumed durability of goods induces a non-time-separable indirect utility function. To date, models based on time-separable investor preferences have not been consistent with the observed comovements of asset returns and aggregate consumption (cf. Constantinides [1990], Friend and Blume [1975]).

In this paper we explore the relations among optimal portfolio decisions, the real term structure of (risky) asset returns, and the risks and price volatilities of assets with different terms to maturity in the context of a dynamic equilibrium model in which the preferences of investors are non-time-separable. This specification provides more scope for the term structure to affect the assets' risk characteristics and price behavior. A number of empirical asset valuation puzzles and some of the observable differences among risks and average returns of assets with different terms to maturity might therefore be explainable on the basis of our approach.

The most prevalent explanation of the term structure is the expectations theory. This theory is basically only a hypothesis which holds that the return on holding a long-term asset to maturity is equal to the expected return on repeated investment in a series of short-term assets if both strategies exhibit the same (systematic) risk in terms of consumption at the maturity date. Under the expectations hypothesis, the one-period holding returns of assets with identical payoffs but different maturities are the same or differ by constant term premiums. The expectations theory also implies that the rate of return on a long-term asset can be represented as a long average of expected future short-term returns, so that the long rate should vary less than the short rate. These implications have been tested in several ways and have been found inconsistent with the data in many empirical studies presented so far (e.g. Campbell [1987], Campbell and Shiller [1987,1990], Froot [1989], Jones

and Roley [1983], Shiller [1979]). The data suggest that the term premium is time-varying and that long rates of return are too volatile to be consistent with the smoothing property implicit in the expectations hypothesis.

Our model provides an endogenous explanation for the variation in the term premium and is capable to generate fluctuations in the long rate which can be larger than those of the short rate. The volatilities of the two rates relative to each other are characterized in terms of a specific property of the risk preferences of the investors. In contrast to the expectations theory our approach does not imply that a steeply sloped yield curve should on average signal an increase in short rates of return. Another important feature of the model concerns the curvature of the term structure. According to the liquidity preference hypothesis advanced by Hicks [1946] the term premium is positive and increasing with maturity. Modigliani and Sutch [1966] disputed this conjecture by pointing out that it is not rational for individuals to prefer to lend short or to be mainly concerned with *short-term* capital losses. On theoretical grounds, term premiums can therefore as well be positive as negative. But what kind of structural model might clarify the factors which determine the slope of the term structure? Depending on the investors' attitudes towards risk, our approach is able to explain both a strictly increasing yield curve (as is claimed by the liquidity preference hypothesis) and a term structure which fails to be monotone.

In an *intertemporal* asset valuation model there is no natural way to define the riskiness of assets. An asset with one period to maturity might be relatively safe in terms of consumption next period, but very risky in terms of consumption in all following periods. Conversely, a long-term asset with, say, n periods to maturity might be safe in terms of consumption in the n -th period, but very risky in terms of consumption in the near future. In the present paper we assume that the investors have short horizons. Thus they are only concerned with the short-term risks of the assets and ignore the long-term risks in their portfolio decisions. This feature of our model leads to a 'mispricing' of short-term risks relative to long-term risks in the sense that long-term investment strategies composed of short-term assets are priced higher (per unit of expected payoff) than long-term assets although the latter involve less long-term risk. This result is in striking contrast to the implications of the expectations theory according to which the price of the long-term investment

strategy must not exceed the price of the long-term asset.

The remainder of the paper is organized as follows: In Section 2 we describe the economic environment and define equilibrium prices. Section 3 analyzes the risk characteristics of assets with different terms to maturity and relates the behavior of equilibrium prices to the investors' attitudes towards risk. In Section 4 we extend the model to allow for assets with arbitrary finite maturities. Concluding remarks are presented in Section 5, and some proofs are gathered in a separate Appendix.

2 The Equilibrium Model

We consider a stationary, infinite horizon, pure exchange economy which consists of an infinite sequence of two-period lived overlapping generations. Time is discrete, and indexed by $t = \dots - 1, 0, 1, 2, \dots$. At each trading date, there exist one perishable consumption good (numeraire) and three risky assets of different maturities.¹ These assets can be interpreted as corporate discount bonds which exhibit some positive default risk, or as life insurance policies with payoffs at the maturity date depending in part on the performance of the insurance company. To focus sharply on the impact of the term structure on the behavior of asset returns we assume that the assets differ *only* with respect to their maturity dates.

Each member of generation t is endowed with one share of the asset type π^i , $i = 0, 1, 2$. Shares of asset type π^i mature after i periods and they pay off y_{t+i} units of consumption goods at the maturity date $t + i$. y_{t+i} takes values in the interval $[\underline{y}, \bar{y}] \subset \mathbb{R}_{++}$. The probability law of the asset payoffs remains unchanged through time, i.e., y_t in an i.i.d. random variable. It is assumed that agents know the true distribution of this random variable, F , and that they have rational expectations.

Each period, trading begins after all assets which mature at that date have paid off their dividends. This implies that assets of type π^0 will not be traded since they have already paid off their dividends when the asset markets open. A price system for consumption goods and the two traded assets is given by a vector

¹We will allow for more assets in Section 4. The basic structure of the model is similar to the framework developed in Eckwert [1992,1993].

$(p^c(\cdot), p^i(\cdot); i = 1, 2)$, where each component of the vector maps the state space $[y, \bar{y}]$ into \mathbb{R}_{++} .²

A young agent decides how much to consume and the amounts of assets (with one and two periods to maturity) to acquire. In his old age he spends all asset returns on consumption. His lifetime budget constraints are then given by

$$(1) \quad c = y - p^1(y)(z^1 - 1) - p^2(y)(z^2 - 1)$$

$$(2) \quad \tilde{c} = z^1 \tilde{y} + p^1(\tilde{y})z^2,$$

where c denotes consumption and z^i is the demand for assets of type π^i . To simplify notation we have suppressed the time index; variables which refer to the agent's second period of life are distinguished by a \sim . Assets with two periods to maturity do not pay off during the lifetime of the agent. The individual is speculating in these assets because he hopes for a high resale value on the next trading date. Of course, in our stationary model, the resale value of a long term asset (type π^2) equals the price of a short term asset (type π^1) one period from now.

Individual preferences over random lifetime consumption are supposed to satisfy the expected utility hypothesis. Thus there is a function $U : \mathbb{R}_{++} \rightarrow \mathbb{R}$ evaluating sure consumption bundles such that the utility derived from deterministic current consumption, c , and stochastic future consumption, \tilde{c} , can be represented as $E\{U(c, \tilde{c})\}$. E denotes the mathematical expectation conditional on information available in the current period.

Assumption 2.1 *The intertemporal utility function U is continuously twice differentiable and strictly increasing in both arguments. Furthermore, we require that $U_{ii} < 0$ for $i = 1, 2$, $U_{21} \geq 0$, $\lim_{c \rightarrow 0} U_1 = \lim_{\tilde{c} \rightarrow 0} U_2 = \infty$, and that the measure of first period relative risk aversion, $R(c, \tilde{c}) := -U_{11}(c, \tilde{c})c/U_1(c, \tilde{c})$ exceeds or is equal to 1.*

This specification of individual preferences is strong but standard. By imposing the condition $U_{21} \geq 0$ and $R \geq 1$ we avoid ambiguities in the sign of the term pre-

²We consider only stationary equilibria, i.e., prices prevailing at date t are *time-invariant* functions of the state variable y_t . This specification is in accordance with the stationary nature of the model.

mium. Empirical studies have found that the measure of relative risk aversion for individual investors typically exceed unity (Friend and Blume [1975]). The specification $U_{21} \geq 0$ allows for some intertemporal substitution in marginal utility. As a reference situation the case of time-separable utility (i.e. $U_{21} = 0$), which is assumed in most of the efficient markets literature, is of particular interest. In our model, time-separability of the investors' preferences constitutes a special borderline case in which the traditional partial equilibrium results about term structure relations in incomplete market economies tend to be confirmed.

The necessary and sufficient first order optimality conditions are

$$(3) \quad p^1(y)E\{U_1(c, \tilde{c})\} = E\{U_2(c, \tilde{c})\tilde{y}\}$$

$$(4) \quad p^2(y)E\{U_1(c, \tilde{c})\} = E\{U_2(c, \tilde{c})p^1(\tilde{y})\},$$

where c and \tilde{c} are given by (1) and (2). Equations (3) and (4) are standard Euler equations for asset prices. The implications for term premia, the pricing of systematic risks, and the volatility of asset prices and rates of return will be discussed in the next section.

In the absence of a bequest motive (which is not modelled in this study) old agents sell all their assets and spend the proceeds on consumption, regardless of the prevailing prices. Thus, in equilibrium all assets must be held by members of the young generation. The supply (per capita of the young generation) of long term assets equals 1. The supply of short term assets is 2 since the long term assets carried over from the previous period have only one more period to run and therefore add to the current supply of short term assets. Thus, market clearing requires $z^1 = 2$, $z^2 = 1$, and the first order conditions become

$$(5) \quad p^1(y) \int U_1(c(y), \tilde{c}(\tilde{y})) dF(\tilde{y}) = \int U_2(c(y), \tilde{c}(\tilde{y})) \tilde{y} dF(\tilde{y})$$

$$(6) \quad p^2(y) \int U_1(c(y), \tilde{c}(\tilde{y})) dF(\tilde{y}) = \int U_2(c(y), \tilde{c}(\tilde{y})) p^1(\tilde{y}) dF(\tilde{y}),$$

where $c(y)$ and $\tilde{c}(\tilde{y})$ denote per capita equilibrium consumption in the first and second period of life,

$$(7) \quad c(y) = y - p^1(y), \quad \tilde{c}(\tilde{y}) = 2\tilde{y} + p^1(\tilde{y}).$$

In equilibrium, prices must be consistent with optimal individual portfolio choices and market clearing in all possible states.

Definition 2.1 *A stationary rational expectations equilibrium (henceforth equilibrium) consists of continuous nonnegative price functions $(p^1(y), p^2(y))$ satisfying (5), (6) and (7) for all $y \in [\underline{y}, \bar{y}]$.*

Drees and Eckwert [1992] have shown that the existence of a solution to an equation system which is structurally equivalent to (5),(6),(7) is guaranteed if (in addition to some regularity conditions)

- the ratio between marginal utility of future and current consumption is sufficiently high,
- the measure of relative risk aversion, $\tilde{R}(c, \tilde{c}) := -U_{22}(c, \tilde{c})\tilde{c}/U_2(c, \tilde{c})$ in the *second* period of life does not exceed unity.

The first assumption is a contingent version of a condition which ensures the existence of equilibria (other than the barter equilibrium) in deterministic OLG-models. This condition requires that the intertemporal ratio of marginal utilities must exceed unity at the endowment point. The second assumption is rather unattractive because it severely restricts the investors' attitudes towards risk. Although generalization is possible we will impose the condition in order to ensure the consistency of the model.

3 The behavior of equilibrium prices and asset returns

In this section we analyze the term structure and the behavior of term premia and asset prices. According to eqs. (5),(6) and (7) both asset prices depend positively on the state variable y . We have noted earlier that the resale value of long term assets equals the future price of short term assets. Thus the properties of p^1 are of critical importance for the holding returns of assets of either maturity. As a preliminary result we prove

Lemma 3.1 *The price of short term assets is elastic, i.e. $\varepsilon[p^1, y] > 1 \forall y$.*

Proof: Differentiation of equation (5) and rearranging terms shows that the sign of $\varepsilon[p^1, y] - 1$ is identical to the sign of the expression

$$\frac{E\{U_{21}(\cdot)\tilde{y}\}}{E\{U_1(\cdot)\}}c(y) - p^1(y) \left[\frac{E\{U_{11}(\cdot)\}}{E\{U_1(\cdot)\}}c(y) + 1 \right].$$

The term in brackets is negative since $R(\cdot) > 1$ has been assumed. Recalling that U_{21} is nonnegative by Assumption 2.1, the sign of the above expression is clearly positive. ■

We will use the elasticities of the asset prices with respect to the state variable y as a measure of price volatility: e.g. the price of the long term asset will be considered more volatile than the price of the short term asset if $\varepsilon[p^2, y] > \varepsilon[p^1, y]$ holds *for all* y . The relative volatilities of the asset prices depend on the investors' attitudes towards risk as follows:

Proposition 3.1 *If the individual preferences exhibit risk complementarity (risk substitutability)³ then the price of the short term asset behaves more (less) volatile than the price of the long term asset.*

Proof: See Appendix.

Whether the prices of long-term assets should be expected to behave more volatile or less volatile than the prices of short-term assets is a heatedly debated and contentious issue in financial economics. An argument that often seems to be implicit in academic discussions claims that the yield risk is critical for the link between the price volatility and the time to maturity of assets. Treasury bills, for example, are about as safe an investment as one can get. There is no risk of default and, according to common view, the prices of these assets should be more stable the shorter the T-bills have to run. On the other hand, if asset payoffs are risky and if long rates of return can be represented as weighted averages of short rates,

³Individual preferences exhibit risk complementarity (risk substitutability) if the second period relative risk aversion $\tilde{R}(c, \tilde{c})$ is strictly increasing (decreasing) in current consumption c (cf. Sandmo [1969]).

then the smoothing implicit in the averaging implies that the prices of long-term assets should be less volatile than the prices of short-term assets.

Proposition 3.1 shows that both possibilities are consistent with rational investor behavior. In contrast to popular views, however, the link between price volatility and maturity depends on the attitudes towards risk on the market rather than on the risk characteristics of the assets. The contradictory empirical evidence on this issue (cf. Shiller [1979], Mankiw and Summers [1984]) might therefore simply reflect the fact that the investors operating on the markets for T-bills have attitudes towards risk which are different from those of investors who operate on the markets for riskier assets.

The expected one-period holding returns on short-term and long-term assets are

$$(8) \quad r_1(y) = \frac{E\{\tilde{y}\}}{p^1(y)}; \quad r_2(y) = \frac{E\{p^1(\tilde{y})\}}{p^2(y)}.$$

If we define the term premium by $\rho(y) := [r_2(y)/r_1(y)] - 1$ then ρ represents the extra return expected for holding the long-term asset rather than the short-term asset. According to common practice in economics dating at least back to Hicks [1946] the current-yield curve (i.e., the difference between current long and short yields) is specified upward sloping which implies a positive term premium. Yet there is no consensus in the theoretical literature on how term premia are related to the parameters of the economic environment and why they vary through time. We turn to these issues below.

Proposition 3.2 *The current-yield curve is upward sloping, i.e., the term premium is positive.*

Proof: See Appendix.

Under the expectations hypothesis, an upward sloping yield curve reflects the market participants' conviction that short and long yields will rise over the next trading periods. Unfortunately, several recent and older studies using various techniques and data sets have rejected this implication (Shiller, Campbell, Schoenholz

[1983], Jones and Roley [1983], Campbell and Shiller [1987, 1990]). In our equilibrium model a positive term premium is consistent both with rising and declining future yields. Since $p^1(y)$ and $p^2(y)$ are increasing functions, the holding yields of all maturities rise on average if the current state variable y is high and decline on average if the current state variable is low. Thus, in an equilibrium rational expectations context an upward-sloping term structure is *not* indicative of expectations of a rise in holding returns, but may simply reflect the fact that long-term assets are riskier in terms of consumption next period than short-term assets.

One of the major weaknesses of the expectations theory is the implied time-invariant term premium. A constant term premium can also be derived from valuation models with additively time-separable investor preferences. In our approach, intertemporal substitution in marginal utility is possible which leads to a non-trivial dependence of individual discount rates on the realized current state of the economy and hence to a time-varying term premium.

Empirical evidence suggests that term premiums fluctuate significantly through time but not necessarily in such a way as to equalize on average the holding yields of assets (e.g. Mankiw [1986]). According to traditional wisdom based on the equalization of expected holding return profiles, the rate of increase of the long rate should be greater the greater is the spread between the long rate and the short rate. The economic intuition behind this conjecture is simple: If the spread between the long rate and the short rate is positive, the long asset has a higher current yield which ought to be offset by an expected capital loss in the future in order to narrow the gap between the intertemporal return profiles of the two assets. A capital loss is, of course, equivalent to an increase in long rates.

Unfortunately, the data do not seem to be consistent with this implication. In fact, empirical studies which try to relate the slope of the yield curve to the behavior of long rates tend to generate a relationship in the direction opposite to the one predicted by the traditional theory, i.e., when long rates are high relative to short rates they tend to move down in the subsequent period (e.g. Mankiw [1986], Shiller [1979]).

Thus the data suggest that a high (low) term premium at time t is likely to induce a decrease (increase) in the long rate at time $t + 1$. In our model, the term premium

$$\rho(y) = \frac{p^1(y)}{p^2(y)} \frac{E\{p^1(\tilde{y})\}}{E\{\tilde{y}\}} - 1$$

is a strictly monotone increasing function of the state variable y in case of risk complementarity and a monotone decreasing function in case if risk substitutability (cf. the proof of Proposition 3.1). The relationship between the behavior of long rates and the term premium generated by our model is therefore broadly consistent with the data if the investor preferences exhibit risk substitutability. In this case a low value of y implies both a high term premium and a high long rate. Since the long rate is an i.i.d. random variable it is likely to decrease next period if it assumes a high value today. Thus a high term premium today would be indicative of a lower long rate tomorrow, which is exactly the statistical relationship suggested by the data.

Under the assumption of additively time-separable investor preferences, i.e. $U_{21} = 0$, our model generates a constant term premium. The approach developed in this paper therefore includes the expectations theory as a special case and offers a new opportunity to relate the poor empirical performance of the expectations theory to an overly restrictive (but ubiquitously used) specification of the attitudes towards risk in the economy.

It is instructive to consider the following (hypothetical) investment strategy: Buy one share of the short-term asset and reinvest the return again into short-term assets. The expected rate of return (minus 1), r_s^2 , of this 2-period strategy s is

$$(9) \quad r_s^2(y) = \frac{1}{p^1(y)} E \left\{ \tilde{y} \frac{y'}{p^1(\tilde{y})} \right\}.$$

If we denote the expected 2-period holding return on long-term assets by r_2^2 then

$$(10) \quad \frac{r_2^2(y)}{r_s^2(y)} = \frac{r_2^2(y)}{\frac{1}{p^1(y)} E \left\{ \tilde{y} \frac{y'}{p^1(\tilde{y})} \right\}} = \frac{p^1(y)}{p^2(y)} \frac{1}{E\{\tilde{y}/p^1(\tilde{y})\}} > 1$$

holds for all y .⁴ We may summarize this result in

⁴Inequality (10) is proved in the Appendix.

Proposition 3.3 *The expected 2-period holding return on long-term assets is higher than the expected rate of return on an investment strategy which invests into short-term assets for 2 consecutive periods.*

The dominance in expected return of long-term assets relative to long-term investment strategies composed of short-term assets is puzzling at first sight because it seems to contradict the Efficient Markets Hypothesis. The rate of return after two periods of a long-term asset is $\tilde{y}/p^2(y)$. If one buys $[1/p^2(y)]/E\{\tilde{y}/p^1(\tilde{y})\}$ short-term assets and reinvests the payoff into short-term assets on the next trading date, then the payoff after two periods of this strategy, s , is

$$\frac{\tilde{y}}{p^2(y)} \left(\frac{y'}{p^1(y')} \middle/ E \left\{ \frac{\tilde{y}}{p^1(\tilde{y})} \right\} \right).$$

Since y' and \tilde{y} are stochastically independent the long-term asset dominates the investment strategy s in the sense of Second Order Stochastic Dominance and thus exhibits less long-term risk: Any (hypothetical) individual at time t who has continuous risk averse preferences for time $t + 2$ consumption would prefer the long-term asset to the investment strategy s (cf. Huang and Litzenberger [1988], pp. 45). Despite this fact, in our model the long-term asset pays a higher long-term risk premium than the investment strategy s according to Proposition 3.3. This ‘mispricing’ of long-term risks occurs because the agents who have access to the market for long-term assets at date t have no preferences for consumption at the time when the asset matures. They buy the long-term assets only for the sake of their resale value. Since the long rate of return on long-term assets does not enter the agents’ choice problems it should not come as a surprise that this rate is not a correct measure of the consumption risk associated with the assets’ payoff pattern.

Although they involve less long-term risk (in the fairly objective sense of Second Order Stochastic Dominance) long-term assets pay a higher long-term risk premium than long-term portfolio strategies which are composed of short-term investment activities. Thus, if we accept the view that an *efficient* capital market assigns larger risk premia to higher risks, then in our model the above portfolio strategy would be overpriced compared to assets with long maturities. This implies that the short-term risks (the risks of the short-term assets contained in the portfolio)

are underpriced relative to the long-term risks, i.e., the short rates of return are 'excessively volatile' in the sense that they are riskier than would be justified by the market valuation of short-term assets.

The seeming inconsistency of the equilibrium price system with the Efficient Markets Hypothesis can be dissolved by taking two observations into consideration. First, the agents are willing to hold short-term assets without 'adequate' compensation for the implied risk because they have a strong liquidity preference in this model. Since the individuals have short horizons they are only interested in liquid portfolios which pay off during their lifetimes. The 'underpricing' of short-term risks constitutes an endogenous liquidity premium imposed by the market on short-term investments.

Second, at time t a long-term asset and a long-term investment strategy composed of short-term assets involve the same systematic long-term risk (their long-term returns are perfectly correlated with respect to changes in y_{t+2}). However, the long term asset involves more systematic *short-term risk* than the investment strategy since $p^1(y_{t+1})$ is more volatile than y_{t+1} according to Lemma 3.1. Thus the higher short-term risk accounts for the higher long-term risk premium. If in this model an investor would exist (of negligible size) who is interested only in consumption two periods from now, then he would invest in long-term assets. The market would reward him for the high short-term risk of these assets although the investor does not have to bear it. If there were many such investors then they would drive up the price for long-term assets, thereby eliminating the 'mispricing' of short-term risks relative to long-term risks.

Thus far our analysis has shown that a speculative asset⁵ exhibits more systematic risk than a non-speculative asset, but it does not necessarily possess a more volatile price. We have also seen that some anomalies in the behavior of the term premium which result from overly restrictive implicit assumptions in the expectations theory can be resolved by allowing for intertemporal substitution in marginal

⁵We call an asset 'speculative' if its maturity date lies beyond the planning horizon of the agents who are currently alive. A speculative asset is traded in the hope of a high resale value rather than in order to cash the contractual asset payoff.

utility. Finally, the planning horizons of the investors have been shown to be of critical importance for the valuation of short-term risks and long-term risks on a competitive asset market. Below we extend our analysis to include assets of arbitrary (but finite) maturities and investigate how the relationship between the risks and the price volatilities depends on the time to maturity of the assets.

4 Time to maturity, risk, and price volatility

In this section we introduce a finite number of additional assets π^i with $i = 3, 4, \dots, n$ periods to maturity. All these assets differ only with respect to their maturity dates: an asset which matures at time t pays off y_t units of the consumption good, and the distribution of the random variables y_t is the same as in the previous section. This generalization allows us to analyze the relationship between the time to maturity of assets on the one hand and the risk-(price) volatility link on the other hand. Obviously, the riskiness of an asset with k periods to maturity (henceforth k -period asset) depends on the price volatility of a $(k-1)$ -period asset because at the next market date a k -period asset will be traded at the price of a $(k-1)$ -period asset. Thus there exists an intertemporal link (delayed by one period) between price volatility and short-term risk premia. Given the investors' attitudes towards risk, this link depends on the time to maturity of an asset as will be shown below.

To ease the presentation and in order to avoid changes in the notation introduced in Section 3, we assume that the assets of type π^i , $i \geq 3$ are in zero net supply and that the endowment process of the agents is the same as before. Equilibrium consumption remains unchanged then and is given by (7) while the asset prices are determined by the first order conditions for utility maximization

$$(11) \quad p^1(y) \int U_1(c(y), \tilde{c}(\tilde{y})) dF(\tilde{y}) = \int U_2(c(y), \tilde{c}(\tilde{y})) \tilde{y} dF(\tilde{y})$$

$$(12) \quad p^k(\tilde{y}) \int U_1(c(y), \tilde{c}(\tilde{y})) dF(\tilde{y}) = \int U_2(c(y), \tilde{c}(\tilde{y})) p^{k-1}(\tilde{y}) dF(\tilde{y}), \quad (k = 2, \dots, n),$$

where p^k denotes the share price of the k -period asset.

The next proposition establishes a relationship between the price volatility and

the time to maturity of an asset. We stick to our earlier definition of volatility according to which the price of a k -period asset behaves more volatile than the price of an l -period asset if $\varepsilon[p^k, y] > \varepsilon[p^l, y]$ holds for all y . This condition may be expressed more conveniently by using the self-explanatory notation $p^k \succ_{vol} p^l$.

Proposition 4.4 *Suppose the agents' preferences exhibit risk substitutability. Then the price volatility of an asset is strictly increasing in the time to maturity of the asset, i.e., $p^1 \not\succ_{vol} p^2 \not\succ_{vol} \dots \not\succ_{vol} p^n$. If the agents' preferences are characterized by risk complementarity, then the size of the price fluctuations is non-monotone in the time to maturity in the sense that $p^1 \succ_{vol} p^2 \not\succ_{vol} p^3 \succ_{vol} p^4 \not\succ_{vol} p^5 \dots$ holds.*

Proof: See Appendix.

An asset with a long time to maturity must be traded a large number of times until the final holder of the asset can cash its dividend payoff at the maturity date. In line with common usage in financial economics, such an asset might be considered more speculative than an asset which pays off its dividends after a short period of time. In terms of this terminology Proposition 4.4 says that in risk substitutionary economies a more speculative asset possesses a more volatile price. In risk complementary economies, however, a highly speculative asset can, but need not, exhibit large percentage price fluctuations relative to other less speculative assets. The relative amount of an asset's price volatility increases and decreases alternately as the time to maturity becomes shorter.

Since in our model the individual planning horizons extend just one period into the future, the agents are only concerned about the *short-term* risks of the assets, i.e., the risks implied by the stochastic behavior of the short-term (one period) rates of return. In this economy it seems sensible, therefore, to measure the riskiness of an asset by its short-term risk premium.⁶ The short-term rate of return (minus 1)

⁶The short-term risk premium and the short-term rate of return differ only by a constant, namely the risk-free rate. For our purposes the risk premium and the rate of return are equivalent concepts.

of asset π^k , $k \geq 2$, is given by

$$(13) \quad r_k(y) := \frac{E\{p^{k-1}(\tilde{y})\}}{p^k(y)}.$$

Asset π^k , $k \geq 3$, is said to be riskier (less risky) than asset π^{k-1} iff it pays a higher (lower) risk premium in all states, i.e. iff

$$(14) \quad \frac{r_k(y)}{r_{k-1}(y)} = \frac{E\{p^{k-1}(\tilde{y})\}}{E\{p^{k-2}(\tilde{y})\}} \frac{E\{U_2(c(y), \tilde{c}(\tilde{y}))p^{k-2}(\tilde{y})\}}{E\{U_2(c(y), \tilde{c}(\tilde{y}))p^{k-1}(\tilde{y})\}} \begin{matrix} (<) \\ > \end{matrix} 1$$

holds for all y . An argument symmetric to the proof of Proposition 3.2 reveals that the inequality in (12) is satisfied if $p^{k-1}(\tilde{y})/p^{k-2}(\tilde{y})$ is strictly monotone increasing (decreasing) or, equivalently, if $p^{k-1} \succ_{\text{vol}} p^{k-2}$ ($p^{k-1} \prec_{\text{vol}} p^{k-2}$). Thus, we get as a corollary to Proposition 4.4

Corollary 4.1 *Suppose the agents' preferences exhibit risk substitutability. Then the systematic risk of an asset is strictly increasing in the time to maturity of the asset, i.e., $\pi^1 \prec \pi^2 \prec \dots \prec \pi^n$ holds in obvious notation. If the agents' preferences are characterized by risk complementarity, then the systematic risk of an asset depends non-monotonically on the time to maturity in the sense that $\pi^1 \prec \pi^2 \succ \pi^3 \prec \pi^4 \succ \pi^5 \dots$ holds.*

A comparison of Proposition 4.4 with the above corollary reveals that in risk substitutionary economies the price volatility ordering and the risk ordering across assets with different maturities coincide. Both the riskiness and the price volatility of an asset decrease as the maturity date approaches. In risk complementary economies the time to maturity has opposite effects on the relative riskiness and the relative price volatility of assets: An asset which matures after k periods is riskier (less risky) than an asset which matures either after $(k+1)$ or after $(k-1)$ periods if and only if it has a less (more) volatile price. Thus, as time passes by and the maturity date of an asset comes nearer, the transition of the economy from some period t to $t+1$ either simultaneously stabilizes the price and increases the systematic risk of a given asset or results in a combined decline in the riskiness of the asset and a destabilization of the equilibrium asset price.

The liquidity preference hypothesis implies that the expected return on assets increases monotonically with remaining time to maturity. The data provide only partial support for the liquidity preference hypothesis. E.g. Fama [1984] concludes from evidence on term premiums for T-bills that in certain time intervals expected returns are not monotonic. Without an explicit theory of the term premium, however, Fama's characterization of the data has limited value. Our model not only explains the term premiums of assets but it is also capable to generate non-monotonic expected returns. In view of Corollary 4.1, Fama's finding concerning the non-monotonic behavior of expected returns suggests that the attitudes towards risk of investors operating in the markets for T-bills are characterized by risk complementarity.

5 Conclusion

This paper has presented a simple theoretical model of the term structure in terms of the ultimate objectives of economic agents and the stochastic properties of forcing variables. We have argued that specifying utility to be a non-time-separable function of consumption allows for richer term structure relations than separable specifications. The model is capable to explain why term premiums vary and why the term structure may fail to be monotone. Our analysis also shows that the link between the one-period holding return and the price volatility on the one hand and the time to maturity of an asset on the other hand depends critically on the investors' risk preferences. The assumption of time-separable utility implicit in the expectations theory and in most term structure models developed in the last twenty years severs this link and might therefore account for the fact that these models have not held up well in empirical tests of the returns on assets with different terms to maturity.

While our study leaves unanswered the question how we should measure (or compare) the riskiness of assets with different maturities and how term premiums are related to some measure of risk, it clearly demonstrates that the planning horizon of the agents has an important impact on how a competitive market values long-term risks relative to short-term risks and hence on the term structure of asset

returns: Short-term risks tend to be undervalued compared to long-term risks if the investors have short planning horizons. This observation may explain why the empirical evidence on risk premiums provided by Fama [1984], Fama and Bliss [1987], Stambough [1988], Starz [1982], and others have not been found to be consistent with the implications of the consumption-based asset pricing model developed by Lucas [1978] and Breeden [1979] which uses the assumption of an infinitely-lived representative investor.

Appendix

In this appendix we prove Proposition 3.1, Proposition 3.2, inequality (10) from the main text, and Proposition 4.4.

Proof of Proposition 3.1: Since both asset prices are monotone increasing in the state variable y , the short term asset exhibits more (less) price volatility than the long term asset if the price ratio $p^1(y)/p^2(y)$ is strictly monotone increasing (decreasing) in y . For notational convenience let f denote the marginal utility of future equilibrium consumption, $f(y, \tilde{y}) := U_2(c(y), \tilde{c}(\tilde{y}))$. We assess the derivative with respect to y of the price ratio $p^1(y)/p^2(y) = \int f(y, \tilde{y})\tilde{y} dF(\tilde{y}) / \int f(y, \tilde{y})p^1(\tilde{y}) dF(\tilde{y})$ (cf. (5) and (6)). Define $g(y) := \alpha y$, $y \in [\underline{y}, \bar{y}]$, and choose $\alpha > 0$ such that

$$(A1) \quad \int f(y, \tilde{y})p^1(\tilde{y}) dF(\tilde{y}) = \int f(y, \tilde{y})g(\tilde{y}) dF(\tilde{y}).$$

Clearly,

$$(A2) \quad \frac{\partial}{\partial y} \left(\frac{\int f(y, \tilde{y})\tilde{y} dF(\tilde{y})}{\int f(y, \tilde{y})g(\tilde{y}) dF(\tilde{y})} \right) = 0$$

and the result in Lemma 3.1 implies $[p^1(y) = g(y) \Rightarrow p^{2'}(y) > g'(y)]$, from which we conclude ⁷

$$(A3) \quad \left\{ y \mid p^1(y) > g(y) \right\} > \left\{ y \mid p^1(y) < g(y) \right\}.$$

⁷ $A > B$ means $x > z$ for all $x \in A$ and $z \in B$.

Lemma A1 *If $f_1(y, \tilde{y})/f(y, \tilde{y})$ is strictly increasing in \tilde{y} then*

$$(A4) \quad \int f(y+\varepsilon, \tilde{y}) p^1(\tilde{y}) dF(\tilde{y}) > \int f(y+\varepsilon, \tilde{y}) g(\tilde{y}) dF(\tilde{y}), \quad \varepsilon > 0.$$

The inequality in (A4) are reversed if $f_1(y, \tilde{y})/f(y, \tilde{y})$ is a strictly decreasing function of \tilde{y} .

Proof: Suppose $f_1(y, \tilde{y})/f(y, \tilde{y})$ is strictly increasing in \tilde{y} (the remaining case can be treated analogously). Then $f_1(y, \tilde{y}) = \gamma(\tilde{y})f(y, \tilde{y})$ with $\gamma'(\tilde{y}) > 0$ and

$$\int f_1(y, \tilde{y}) [p^1(\tilde{y}) - g(\tilde{y})] dF(\tilde{y}) = \int \gamma(\tilde{y}) f(y, \tilde{y}) [p^1(\tilde{y}) - g(\tilde{y})] dF(\tilde{y}) > 0,$$

where the inequalities follow from (A1), (A3) and $\gamma'(\tilde{y}) > 0$. ■

Using the result in Lemma A1 and equations (A1),(A2) we now conclude (for the case where $f_1(y, \tilde{y})/f(y, \tilde{y})$ is strictly increasing in \tilde{y})

$$\frac{\int f(y+\varepsilon, \tilde{y}) \tilde{y} dF(\tilde{y})}{\int f(y+\varepsilon, \tilde{y}) p^1(\tilde{y}) dF(\tilde{y})} < \frac{\int f(y+\varepsilon, \tilde{y}) \tilde{y} dF(\tilde{y})}{\int f(y+\varepsilon, \tilde{y}) g(\tilde{y}) dF(\tilde{y})} = \frac{\int f(y, \tilde{y}) \tilde{y} dF(\tilde{y})}{\int f(y, \tilde{y}) p^1(\tilde{y}) dF(\tilde{y})}; \quad \varepsilon > 0$$

or, solving (5) and (6) for p^1/p^2 and using the above assessment,

$$(A5) \quad \frac{\partial}{\partial y} \left(\frac{p^1(y)}{p^2(y)} \right) = \frac{\partial}{\partial y} \left(\frac{\int f(y, \tilde{y}) \tilde{y} dF(\tilde{y})}{\int f(y, \tilde{y}) p^1(\tilde{y}) dF(\tilde{y})} \right) < 0.$$

It is straightforward to verify that the inequality in (A5) will be reversed in case $f_1(y, \tilde{y})/f(y, \tilde{y})$ is strictly decreasing as a function of \tilde{y} . The following lemma therefore completes the proof of Proposition 3.1:

Lemma A2 *$f_1(y, \tilde{y})/f(y, \tilde{y})$ is strictly increasing (decreasing) in \tilde{y} if the relative degree of risk aversion, $\tilde{R}(c, \tilde{c})$, is strictly decreasing (increasing) in c or, formally,*

$$\text{sign} \left[\frac{\partial}{\partial \tilde{y}} \left(\frac{f_1(y, \tilde{y})}{f(y, \tilde{y})} \right) \right] = -\text{sign} \left[\frac{\partial}{\partial x} \tilde{R}(x, \tilde{c}(\tilde{y})) \Big|_{x=c(y)} \right].$$

Proof: Noting that future consumption, $\tilde{c}(\tilde{y})$, is strictly increasing in \tilde{y} the assertion follows from

$$\begin{aligned}
 \frac{\partial}{\partial \tilde{y}} \left(\frac{f_1(y, \tilde{y})}{f(y, \tilde{y})} \right) &= \frac{\partial}{\partial \tilde{y}} \left[\frac{\partial}{\partial y} \left(\ln f(y, \tilde{y}) \right) \right] = \frac{\partial}{\partial y} \left[\frac{f_2(y, \tilde{y})}{f(y, \tilde{y})} \right] \\
 &= \frac{\partial}{\partial y} \left[\frac{U_{22}(c(y), \tilde{c}(\tilde{y})) \tilde{c}'(\tilde{y})}{U_2(c(y), \tilde{c}(\tilde{y}))} \right] = \frac{\partial}{\partial y} \left[-\tilde{R}(c(y), \tilde{c}(\tilde{y})) \frac{\tilde{c}'(\tilde{y})}{\tilde{c}(\tilde{y})} \right] \\
 &= - \left(\frac{\partial}{\partial \tilde{y}} \ln \tilde{c}(\tilde{y}) \right) c'(y) \frac{\partial}{\partial x} \tilde{R}(x, \tilde{c}(\tilde{y})) \Big|_{x=c(y)}
 \end{aligned}$$

■

Proof of Proposition 3.2: The term premium is positive if

$$(A6) \quad \frac{r_2(y)}{r_1(y)} = \frac{E\{p^1(\tilde{y})\} p^1(y)}{E\{\tilde{y}\} p^2(y)} = \frac{E\{p^1(\tilde{y})\}}{E\{\tilde{y}\}} \frac{E\{f(y, \tilde{y}) \tilde{y}\}}{E\{f(y, \tilde{y}) p^1(\tilde{y})\}}$$

exceeds unity. Choose $\beta > 0$ such that $E\{\beta \tilde{y}\} = E\{p^1(\tilde{y})\}$ and define $h(\tilde{y}) := \beta \tilde{y}$.

(A6) can then be reformulated as

$$(A7) \quad \int f(y, \tilde{y}) h(\tilde{y}) dF(\tilde{y}) > \int f(y, \tilde{y}) p^1(\tilde{y}) dF(\tilde{y}).$$

Since $p^1(\tilde{y})$ and $h(\tilde{y})$ have identical expected values and since $p^1(\tilde{y})/h(\tilde{y})$ is strictly monotone increasing by Lemma 3.1, there exists a unique $y^* \in [\underline{y}, \bar{y}]$ with $p^1(y^*) = h(y^*)$ and $p^1(\tilde{y}) \stackrel{(>)}{<} h(\tilde{y})$ for $\tilde{y} \stackrel{(>)}{<} y^*$ (see Figure 1).

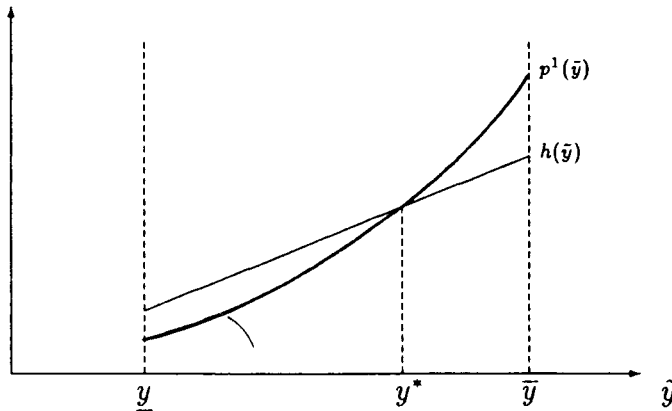


Figure 1

We then have

$$\int_{\underline{y}}^{y^*} p^1(\tilde{y}) - h(\tilde{y}) dF(\tilde{y}) = \int_{y^*}^{\bar{y}} h(\tilde{y}) - p^1(\tilde{y}) dF(\tilde{y}),$$

from which we conclude

$$\begin{aligned} \int_{\underline{y}}^{y^*} f(y, \tilde{y}) [h(\tilde{y}) - p^1(\tilde{y})] dF(\tilde{y}) &> \int_{\underline{y}}^{y^*} f(y, y^*) [h(\tilde{y}) - p^1(\tilde{y})] dF(\tilde{y}) \\ &= \int_{y^*}^{\bar{y}} f(y, y^*) [p^1(\tilde{y}) - h(\tilde{y})] dF(\tilde{y}) > \int_{y^*}^{\bar{y}} f(y, \tilde{y}) [p^1(\tilde{y}) - h(\tilde{y})] dF(\tilde{y}) \end{aligned}$$

since $f(y, \tilde{y})$ is strictly decreasing in its second argument. This shows the validity of (A7) and completes the proof. \blacksquare

Proof of inequality (10): The inequality is an immediate consequence of the following lemma.

Lemma A3 *Let $h, g : [y, \bar{y}] \rightarrow \mathbb{R}_+$ be integrable functions and assume that g is strictly monotone increasing. Then*

$$E \left\{ \frac{h(\tilde{y})}{g(\tilde{y})} \right\} \begin{matrix} (<) \\ > \end{matrix} \frac{E\{h(\tilde{y})\}}{E\{g(\tilde{y})\}}$$

if $h(\tilde{y})/g(\tilde{y})$ is strictly monotone decreasing (increasing).

Proof: The assertion follows from the identity

$$E\{h(\tilde{y})\} = E \left\{ \frac{h(\tilde{y})}{g(\tilde{y})} \right\} E\{g(\tilde{y})\} + \text{cov} \left[\frac{h}{g}, g \right]. \quad \blacksquare$$

Lemma 3.1 and $\tilde{R}(c, \tilde{c}) \leq 1$ imply that $U_2(c, \tilde{c})p^1(\tilde{y}) = U_2(c, \tilde{c})\tilde{c}\frac{p^1(\tilde{y})}{2\tilde{y}+p^1(\tilde{y})}$ is strictly increasing in \tilde{y} . Setting $h = U_2\tilde{y}$ and $g = \widehat{U_2}p^1$ in Lemma A3 yields the assessment

$$\frac{p^1(y)}{p^2(y)} = \frac{E\{U_2(c, \tilde{c})\tilde{y}\}}{E\{U_2(c, \tilde{c})p^1(\tilde{y})\}} > E \left\{ \frac{\tilde{y}}{p^1(\tilde{y})} \right\},$$

which has been claimed in inequality (10). \blacksquare

Proof of Proposition 4.3: The proof is by induction over k . From eqs. (10) we derive

$$(A8) \quad \frac{p^k(y)}{p^{k-1}(y)} = \frac{\int U_2(c(y), \tilde{c}(\tilde{y})) p^{k-1}(\tilde{y}) dF(\tilde{y})}{\int U_2(c(y), \tilde{c}(\tilde{y})) p^{k-2}(\tilde{y}) dF(\tilde{y})}, \quad k = 3, 4, \dots, n.$$

We know already that $p^2 \succsim_{\text{vol}} p^1$ in case of risk substitutability and $p^1 \succsim_{\text{vol}} p^2$ in case of risk complementarity (cf. Proposition 3.1). If $p^{k-1} \succsim_{\text{vol}} p^{k-2}$ then $p^{k-1}(\tilde{y})/p^{k-2}(\tilde{y})$ is a strictly increasing function of \tilde{y} . Thus, by (A8) and an argument symmetric to the proof of Proposition 3.1, $p^k(y)/p^{k-1}(y)$ is strictly increasing (decreasing) in y if the agents' preferences exhibit risk substitutability (risk complementarity). Similarly, if $p^{k-1} \precsim_{\text{vol}} p^{k-2}$ then $p^k(y)/p^{k-1}(y)$ is strictly decreasing in case of risk substitutability and strictly increasing in case of risk complementarity. Thus we have

$$\begin{aligned} \boxed{\text{risk substitutability}} &\implies \begin{cases} p^k \succsim_{\text{vol}} p^{k-1} & \text{if } p^{k-1} \succsim_{\text{vol}} p^{k-2} \\ p^k \precsim_{\text{vol}} p^{k-1} & \text{if } p^{k-1} \precsim_{\text{vol}} p^{k-2} \end{cases} \\ \boxed{\text{risk complementarity}} &\implies \begin{cases} p^k \precsim_{\text{vol}} p^{k-1} & \text{if } p^{k-1} \succsim_{\text{vol}} p^{k-2} \\ p^k \succsim_{\text{vol}} p^{k-1} & \text{if } p^{k-1} \precsim_{\text{vol}} p^{k-2} \end{cases} \end{aligned}$$

for $k = 3, \dots, n$. The proof is complete. ■

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