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Time varying covariance structures in financial markets

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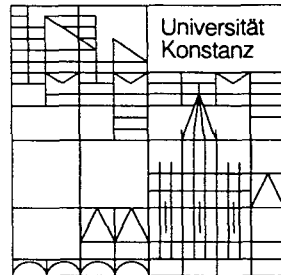
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Sonderforschungsbereich 178 „Internationalisierung der Wirtschaft“

Diskussionsbeiträge

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Time Varying Covariance Structures in Financial Markets

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Time Varying Covariance Structures in Financial Markets

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Abstract

Reliable estimates of variances and covariances are crucial for portfolio management and risk controlling. This paper investigates alternative methods to estimate time varying variance-covariance matrices: ordinary estimates and exponentially weighted moving averages in comparison to Markov switching models.

Different criteria are used to assess the quality of estimators. Bootstrapped confidence intervals for variances and correlations allow to compare the reliability of these estimates. Most importantly, we investigate the quality of these estimators directly in the context of portfolio applications and risk measurement. Therefore, we evaluate the performance of these estimation concepts on the basis of their suitability to select portfolios according to the minimum-variance criterion. This benchmark allows to infer the estimators' ability to capture diversification benefits without diluting this evaluation by any concept aiming to forecast returns. As an additional benchmark we evaluate the validity of forecasted confidence intervals.

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1 Introduction

This paper investigates various methods to estimate time varying covariances in financial markets. The focus is on the time dependence of volatilities and correlations. Reliable estimates of variances and covariances for a certain point in time are crucial for portfolio management. They are also one of the key issues in risk controlling which has gained even more importance after the new capital adequacy guidelines have been issued by the European Community.

The evidence of unstable volatilities and correlations of financial assets is widely documented in the literature. Many researchers have related changes in aggregate stock market volatility to the volatility of economic variables such as financial leverage, e.g. Black (1976), or dividends, e.g. Shiller (1981). Using monthly stock market returns from 1857 to 1987, Schwert (1989) investigates a variety of such economic variables. The seminal papers of Engle (1982) and Bollerslev (1986) have produced a large class of statistical models about changes in conditional volatility over time. A variety of studies using daily data or even intraday data support ARCH and GARCH-effects in stock market returns. For a survey see Bollerslev, Chou, and Kroner (1992).

Recently, multivariate GARCH-models are used to estimate conditional covariances between different stock markets. Among others, Hamao, Masulis, and Ng (1990), King and Wadhvani (1990), Lin, Engle, and Ito (1994), and Longin and Solnik (1995) investigate volatility spillovers and linkages between markets. For example, Longin and Solnik (1995) provide some evidence that the unconditional correlation between markets increases during highly volatile periods. On the other hand, there is no clear cut evidence that the correlation between markets has been steadily increasing due to international integration.

The focus of this paper is not the discussion of volatility spillovers or the reasons for time varying covariances between returns of different markets. We investigate some implications of these effects for risk assessment in particular portfolio applications, i.e. the selection of a minimum variance portfolio and the estimation of confidence intervals for returns.

Assume for a moment an investor faces two assets with identical expected returns, but different risks. If the correlation between these assets' returns is low, standard portfolio theory tells him that he may achieve a considerable risk reduction holding both assets instead of just the less risky one. However, to capture the maximum diversification benefit our investor has to obtain a correlation estimate. As there are various methods to estimate standard deviations and correlations, several questions immediately arise: First, how reliable are such estimates? This is simply a matter of confidence intervals (CIs). Second, for what period ahead is the estimate valid? CIs also provide an answer to this question which might be stated differently: Are there significant changes in correlations over time? Moreover, it might be interesting to know whether the quality of such estimates is changing over time, i.e. whether CIs are broader at certain points of time. These questions are not only crucial for portfolio managers. In risk controlling traders' portfolios are investigated

to measure potential losses. Besides the question to what extent the risks of traders' positions offset each other on an aggregate level the problem of how to share risk limits among associated traders is of special interest. Therefore, one has to address the costs and the sources of these imprecisions. Obviously, if volatility is underestimated, the risk of holding an asset is higher than expected. Thus, losses might be larger than expected. On the other hand, overestimating volatility reduces an investor's earnings potential. If an investor faces a shortfall constraint, then overestimation of volatility may force him to invest less in risky assets. Similar considerations are true of errors in correlation estimation.

In this paper we compare three alternative methods to estimate time varying variance-covariance matrices (VCM)¹: ordinary estimates (ORD), GARCH-type models including J.P. Morgan's methodology of exponentially weighted moving averages (EWMA), and Markov switching (MSW) models. The main difference between these estimators lies in their weighting schemes of observations.

To compare these estimators' suitability for portfolio applications, we use different concepts. First, estimators are compared on the basis of bootstrapped confidence intervals. Bootstrap results are provided for ORD, EWMA, and MSW estimates. Second, we use an economic benchmark. In order to evaluate the forecast performance of the analyzed models one normally would compare the forecast with the corresponding observation. However, there is no "natural" observation for a VCM, since VCMs have to be estimated. This would create the need for a reference estimator which is not apparent. In order to avoid this we evaluate the forecasting power of our estimators in a context closely related to portfolio applications. We investigate whether one of the above estimation concepts leads to superior portfolio selection or to improved risk measurement. Their performance is evaluated on the basis of the minimum-variance portfolios (MVP) implied by estimated VCMs. Although, in practical portfolio management MVPs do not play an important role, it is nevertheless a meaningful benchmark concept because it enables us to focus exclusively on the properties of forecasted VCMs avoiding the need to forecast returns. Calculating efficient portfolios is a task of optimization depending on expected returns and forecasted VCMs. The MVP is the only portfolio whose composition is independent of expected returns. This benchmark enables us to investigate whether one of the concepts performs significantly better than the others in terms of ex-post portfolio variance, the quality of confidence intervals for forecasted variances and trading costs.

In Section 2, we review different methods to estimate volatilities and correlations, including the MSW representation. We describe the data in section 3, present some estimation results and indicate some properties of the MSW model. Additional results of benchmark comparisons are also provided. Section 4 concludes the paper.

¹Canina and Figlewski (1993) have pointed out, that "it has become almost an article of faith in the finance profession, that the implied volatility is the market's volatility forecast, and that it is a better estimate than historical volatility". However, they provide evidence that implied volatilities and historical volatilities are nearly uncorrelated. In addition to that it is virtually impossible to obtain implied volatilities and correlations for all the markets needed in most practical applications.

2 Covariance Estimators

In this section we briefly review three different estimators: Ordinary estimators (ORD), exponentially weighted moving averages (EWMA) and Markov switching (MSW).

The fundamental differences between these concepts may be found in their weighting structures. ORD imposes equal weights on all observations in the sample, whereas EWMA and GARCH assume a decreasing information content of observations dating further back. Thus, an event's influence on the estimate erodes over time. Similarly to ORD, MSW imposes equal weights, however, not on all observations but only on those which are classified to belong to a certain state. Since states cannot be observed directly, one has to construct an inference conditional on observable variables. Note that once an event is inferred to belong to a certain state, its influence on estimates remains constant over time.

In practical applications, the main focus lies on the recent covariance estimate, i.e. taking all available information into account. Therefore, a moving window is usually implemented to obtain estimates. We employ this approach to obtain benchmark comparisons.

Variances as well as covariances are evaluated around a variable's mean. However, we impose a zero mean restriction to expected returns, because this paper is mainly concerned with the evaluation of portfolio risk and does not attempt to forecast returns.² Note that this approach is not uncommon in this context.³ One might as well restrict the expected return to theoretically more meaningful values, e.g. for equity indices a long run average might be used.

2.1 Ordinary Covariance Estimator

Let r_t denote a $(m \times 1)$ vector of logarithmic returns observed at time t . The ordinary covariance estimator in absence of autocorrelation may be written as follows:

$$\begin{aligned} \text{Var}_{ORD}[r_t] &= \sum_{i=0}^{L-1} w \cdot r_{t-i} r'_{t-i} \\ \text{with } w &= \frac{1}{L} \end{aligned} \tag{1}$$

Thus, ordinary covariance estimation at date t is based on a window containing the last L observations r_{t-L+1}, \dots, r_t with equal weights w . $\text{Var}_{ORD}[r_t]$ is the $(m \times m)$ covariance matrix around the restricted zero mean. Logarithmic returns are computed in the standard way from successive daily observations.

²Note that this restriction does not imply a serious bias for estimated standard deviations. For example, differences between ordinary estimates of standard deviations computed with and without the zero mean restriction were less than 0.2% for the whole sample period (1987-95) as well as for the subsamples (1987-88 and 1994-95) used in this study.

³Cf. J.P. Morgan (1995).

2.2 Exponentially Weighted Moving Average Covariance Estimator and GARCH Models

The EWMA implemented throughout this paper is designed similarly to J.P. Morgan's RiskmetricsTM methodology. J.P. Morgan provides estimates of volatilities and correlations for a wide variety of financial time series which are updated on a daily basis. It covers currencies quoted against the US-Dollar, stock market indices, money market rates, Government bond zero rates, swap rates and commodity prices of major developed countries. Especially smaller financial institutions which cannot afford a large risk-controlling unit may use these estimates in combination with third parties' software products to monitor their portfolio risks.

J.P. Morgan uses a recursive definition of the EWMA VCM estimator

$$\text{Var}_{JPM}[r_t] = \lambda \cdot \text{Var}_{JPM}[r_{t-1}] + (1 - \lambda) \cdot r_t r'_t, \quad t = L + 1, \dots, T \quad (2)$$

using the first L observations to initialize $\text{Var}_{JPM}[r_L]$ on the basis of an ordinary estimator.

$$\text{Var}_{JPM}[r_L] := \text{Var}_{ORD}[r_L] \quad (3)$$

Thus, this estimator does not use a moving window with constant length L . Throughout this paper we use a slightly modified version of J.P. Morgan's EWMA estimator in order to obtain a fixed length moving window of 75 days. Therefore we truncate the weighted sum after lag L and correct the weights to sum up to unity:

$$\begin{aligned} \text{Var}_{EWMA}[r_t] &= \sum_{i=0}^{L-1} w_i \cdot r_{t-i} r'_{t-i} \\ \text{with } w_i &= \lambda^i \frac{1 - \lambda}{1 - \lambda^L}, \quad \lambda \in (0, 1) \end{aligned} \quad (4)$$

This yields virtually the same estimate as Var_{JPM} . This estimator assigns exponentially declining weights to observations. λ can be interpreted as a discount coefficient determining how fast the estimated variance adjusts to new observations and thus determining how fast the estimate may change over time. Low levels of λ give high weights to current observations, whereas the influence of past observations is deflated. J.P. Morgan claims that setting λ to 0.94 yields an optimal decay factor for daily VCM forecasts based on calculations of over 400 time series. Note that with $\lambda = 0.94$ weights of observations dating further back than 75 days sum up to less than 1%.

Note that the estimates of the complete covariance matrix are subject to certain assumptions concerning the time structure of observations. Due to the exponentially declining weights which are assigned to past observations, changing the order of appearance of individual observations gives different estimates of the covariance matrix. Clearly, this is not valid for ordinary covariance estimates.

It is often claimed that J.P. Morgan's EWMA estimator is a non-optimal GARCH estimator. Therefore, we have implemented a multivariate GARCH(1,1) model along the lines of Bollerslev, Engle, and Wooldridge (1988).

$$\text{Var}_{GARCH}[r_t] = AA' + B \cdot \text{Var}_{GARCH}[r_{t-1}] \cdot B' + C \cdot r_{t-1} r_{t-1}' \cdot C' \quad (5)$$

K, A and B need to be appropriately conditioned in order to ensure positive definiteness of the estimated VCM at each point of time t (cf. Bollerslev, Engle, and Wooldridge (1988)).

After having estimated this GARCH model in some subperiods of the data set described later, we can conclude that the additional computational costs of multivariate GARCH estimation does not justify the minor improvements realized in comparison to EWMA. In fact, EWMA as described above is a reasonably good approximation. Therefore we refrain from pursuing multivariate GARCH models in favor of EWMA.

2.3 Markov Switching Covariance Estimator

The MSW estimator implemented in this paper is based on the ordinary covariance estimator. Yet, MSW models are adopted because they capture structural differences between certain periods using the state-space concept. The observed series are supposed to be governed by different regimes associated with different parameter estimates. The system's unobservable evolution from one state to another is assumed to follow a Markov chain. From the data the probability to switch into a different state in the next period has to be inferred, given the state of the system at present time. Different states or regimes are characterized here by their VCM. So, depending on the estimation results one may identify "crash" states, i.e. observations "belonging" to this state are described best by high variance or by an unusual correlation with other variables. In addition, estimation often yields a "normal" state with a moderate variance and a set of quite normal correlations. As indicated before, one obtains the explicit probabilities that a future period is governed by a particular regime. More formally, the ordinary covariance estimation is extended now to a mixture of k normally distributed variables, i.e. the observed series of returns is drawn from k differently distributed random variables.

The probability that the system is in state i at time t , $\text{prob}[x_t = i]$, is defined using a scalar random variable taking on integer values $x_t \in \{1, 2, \dots, k\}$.⁴

$$\text{prob}[r_t \sim N[0, \Omega_i]] =: \text{prob}[x_t = i], \quad i = 1, \dots, k \quad (6)$$

Implicitly, we have introduced a one-dimensional discrete state space. We assume that the probability of the system taking on a certain state in the next period *solely* depends on the present state's probability distribution, i.e.

$$\text{prob}[x_t = i | x_{t-1} = j, X_{t-2}] = \text{prob}[x_t = i | x_{t-1} = j], \quad i, j = 1, \dots, k \quad (7)$$

⁴Note that we impose here a zero mean restriction as we do in the context of ORD, EWMA and GARCH.

The $(t \times 1)$ vector X_t contains the "history" of the system's states. Because of the imposed Markov structure the current state contains all information for the next state. Let the $(k \times 1)$ vector ξ_t contain the probability distribution at time t over all states the system can take on. It is obviously necessary that $\xi_{t(i)} \in [0, 1]$, and $\sum_{i=1}^k \xi_{t(i)} = 1$,⁵ so that we can define

$$\xi_t := \begin{bmatrix} \text{prob}[x_t = 1] \\ \text{prob}[x_t = 2] \\ \vdots \\ \text{prob}[x_t = k] \end{bmatrix} \quad (8)$$

Now, define a $(k \times k)$ matrix P containing the state transition probabilities, i.e. conditional probabilities the system is in state i at time t given that the system was in state j at time $t - 1$. Note, that $\text{prob}[x_t = i | x_{t-1} = j]$ is the element p_{ji} in the Markov transition matrix P :

$$p_{ji} := \text{prob}[x_t = i | x_{t-1} = j] \quad (9)$$

$$P := \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1k} \\ p_{21} & p_{22} & \cdots & p_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{kk} \end{bmatrix} \quad (10)$$

Thus, we have specified the dynamic structure of the Markov switching model.

The state inference is adopted from Hamilton (1994, Chap. 22) and Kim (1994)⁶. It has to be calculated iteratively. First, set the starting value for the state inference equal to the model's parameter ρ , i.e. the $(k \times 1)$ vector of probabilities that the system is governed by a particular regime in period one given no information from observations. Then, the system is propagated forward including new observations. This is achieved in a second and third step: the system is iterated between the incorporation of the most recent observation into the optimal inference and the state's one step ahead forecast. This is done until state inferences are constructed for every observation. Finally, starting at $t = T$ going backwards smoothed inferences are constructed, i.e. the information set available at $t = T$ is used to improve the state inferences for $t < T$. Let R_τ denote the $(\tau \times 1)$ vector of return observations up to time τ . Then, the $(k \times 1)$ vector $\hat{\xi}_{t|\tau(i)} := \widehat{\text{prob}}[x_t = i | X_\tau, R_\tau]$ contains the estimated conditional probabilities that the system is in a certain state at

⁵For a vector c the (i) Index indicates the i th element ($c_{(i)}$).

⁶With further references in Hamilton (1994, Chap. 22).

time t , given the information set available at time τ .⁷

$$\begin{aligned}
\text{Starting value} \quad \hat{\xi}_{1|0} &:= \rho \\
\text{Optimal inference} \quad \hat{\xi}_{t|t} &= \frac{\hat{\xi}_{t|t-1} \odot \eta_t}{\imath' (\hat{\xi}_{t|t-1} \odot \eta_t)} \\
\text{Forecast} \quad \hat{\xi}_{t+1|t} &= P \cdot \hat{\xi}_{t|t} \\
\text{Smoothed inference} \quad \hat{\xi}_{t|T} &= \hat{\xi}_{t|t} \odot \left(P' (\hat{\xi}_{t+1|T} \oslash \hat{\xi}_{t+1|t}) \right)
\end{aligned} \tag{11}$$

Here \imath is a $(k \times 1)$ vector of ones. The $(k \times 1)$ vector η_t contains the densities of observed returns conditional on the regime.

$$\eta_t := \begin{bmatrix} \frac{1}{\sqrt{(2\pi)^l |\Omega_1|}} \exp \left[-\frac{1}{2} r_t' \Omega_1^{-1} r_t \right] \\ \frac{1}{\sqrt{(2\pi)^l |\Omega_2|}} \exp \left[-\frac{1}{2} r_t' \Omega_2^{-1} r_t \right] \\ \vdots \\ \frac{1}{\sqrt{(2\pi)^l |\Omega_k|}} \exp \left[-\frac{1}{2} r_t' \Omega_k^{-1} r_t \right] \end{bmatrix} \tag{12}$$

$t = 1, \dots, T$

and thus the log likelihood can be expressed as

$$L[\theta] = \sum_{t=1}^T \log \imath' (\hat{\xi}_{t|t-1} \odot \eta_t) \tag{13}$$

The vector θ contains the non-redundant elements of the system's parameters Ω_i, ρ, P for $i = 1, \dots, k$, which are estimated by maximum likelihood. Following Hamilton (1990) we use the EM algorithm introduced by Dempster, Laird, and Rubin (1977) for the maximization. The EM algorithm is used because it converges quickly from relatively poor initial starting values to a maximum of the likelihood function.⁸

2.4 Quality of Point Estimates

To provide an illustration of the quality of point estimates we compute bootstrapped CIs.⁹ We do not use asymptotic statistics to determine CIs for the following reasons.

⁷The symbol " \odot " denotes the Hadamard product, " \oslash " respectively element by element division.

⁸In practical optimization we used a large number of randomly drawn starting values to get a fairly good confidence to reach a global optimum. Optimality was determined on the ground of the euclidean norm ($< 10^{-8}$) of parameter changes between successive optimization steps within the EM algorithm. See Hamilton (1996).

⁹Cf. Efron and Tibshirani (1986, Sec. 2) or Jeong and Maddala (1993) and Vinod (1993) for an overview.

First, there are no asymptotic standard deviations of ORD correlation estimates nor of EWMA correlation estimates. Considering the properties (see Sec.3) of the series used in this study, we cannot use the results for joint normally distributed variables or for other special cases which allow direct estimation of parameter's standard deviations (see e.g. Johnson and Kotz (1970, Chap. 32)). Second, the restrictions imposed on the MSW model give rise to strong doubts about the validity of asymptotic results for the model's parameter estimates. In the course of estimation we sometimes observed large deviations between asymptotic and bootstrapped standard deviations.¹⁰ In particular the constraints on the Markov transition matrix have to be considered in this context.

Basically, the bootstrap allows us to estimate the density functions of the estimated parameters and estimated VCMs. Having inferred the VCMs' densities, we compute the desired measures, i.e. CIs and standard deviations. We compute bootstrapped CIs by the "*percentile method*" (Efron and Tibshirani (1986))¹¹, i.e. using the empirical parameter distribution the 10% CI borders are given by 5% and 95% percentiles. We have to recur to the moving block bootstrap technique since we cannot always reject the hypothesis of autocorrelation in the observed return series.¹²

3 Empirical Results

3.1 Data

Our sample contains four series of daily logarithmic returns during a nine year period (i.e. 2287 trading days) reaching from January 1987 to December 1995: A stock market index for Germany and for the US, the DEM/USD exchange rate and a German long term interest rate. German stock market returns were computed from daily closing prices of the "Deutscher Aktienindex" (DAX). For the US stock market we use the closing prices of the Dow Jones Industrial Average (DJI) denominated in USD. Exchange rate returns stem from the official DEM/USD fixing in Frankfurt. The interest rate series (LTR) is computed from yields to maturity of German Government Bonds with a maturity ranging between $9\frac{1}{2}$ and $10\frac{1}{2}$ years, which was kindly provided by the Deutsche Bundesbank.

As we want to evaluate the above estimators from a portfolio management perspective, we concentrate on the information set available to a German portfolio manager after floor trading in Germany has closed at 13:30h Frankfurt time. At this point of time he knows this day's USD fixing, the closing prices of the German stock and bond markets, and yesterday's closing of the US stock market. Therefore we lag observations of the DJI once.

¹⁰Hamilton (1996) and Hansen (1992) report similar results.

¹¹So far, we have not considered any further adjustments, as described for instance in Efron and Tibshirani (1986, Sec. 7).

¹²Cf. e.g. Kuensch (1989).

Period		Mean	Std. Dev.	Skewness	Kurtosis	Percentile	
						5%	95%
DAX	1987-88	-0.02	1.60	-0.85	10.20	-2.55	1.96
	1994-95	-0.00	0.95	-0.30	3.89	-1.52	1.49
DJI	1987-88	0.02	1.82	-5.63	83.77	-2.20	2.24
	1994-95	0.06	0.64	-0.23	5.03	-1.00	1.02
USD	1987-88	-0.02	0.68	-0.06	4.69	-1.16	1.10
	1994-95	-0.04	0.72	-0.55	8.16	-1.18	0.99
LTR	1987-88	0.02	0.66	-1.05	14.27	-0.90	1.02
	1994-95	0.02	0.80	0.73	6.53	-1.20	1.46

Table 1: Descriptive statistics of return series (log differences)

We use two 2-year subsamples, 1987-88 and 1994-1995, to evaluate estimators. Descriptive statistics for these subsamples are given in table 1. Standard tests of normality based on skewness and kurtosis were performed also.¹³ The hypothesis of normality was clearly rejected on all series for all subsamples as well as for the whole sample at a 5% confidence level.

To gather some information about a possible time structure in the series used in this study we performed *Breusch-Godfrey* tests (BG) assuming an AR(20) model.¹⁴ Table 2 shows that the hypothesis of no autocorrelation in the first period is rejected at a 5% confidence level for DAX and DJI, whereas there is no evidence for autocorrelation in the second subperiod. The hypothesis of no autocorrelation can neither be rejected for USD and LTR in the first and second subperiods.

Period		Observations	R^2	BG	p-value
DAX	1987-88	500	0.08	39.65	0.01
	1994-95	501	0.05	25.38	0.19
DJI	1987-88	500	0.08	39.33	0.01
	1994-95	501	0.06	28.51	0.10
USD	1987-88	500	0.05	26.40	0.15
	1994-95	501	0.03	14.98	0.78
LTR	1987-88	500	0.06	27.27	0.13
	1994-95	501	0.06	27.76	0.12

Table 2: Breusch-Godfrey test, H_0 : no autocorrelation.

3.2 Properties of the MSW Model

Before we can proceed to assess the estimators' properties in an economic context, we need to consider the specification of the MSW model, i.e. the number of states used and other distributional assumptions like autocorrelation and ARCH effects.

¹³See for example D'Agostino, Balanger, and D'Agostino Jr. (1990).

¹⁴See for example Godfrey (1978). Tests on the MA(20) structure lead to comparable results.

Since the VCM estimate at date t should be based exclusively on the data available up to that point of time, we use a moving window for the economic applications. Here we take a closer look at the first and the last window in order to evaluate the MSW structure, i.e. the subperiods 1987-88 and 1994-95.

The ordinary VCM estimator could be seen as a MSW estimator with only one state. So, before examining the MSW estimates we show one set of ordinary estimates for each subperiod, 1987-88 and 1994-95, which is calculated in exactly the same way as for MSW, i.e. using one window of 500 observations. These ordinary estimates are given in table 3 for the 1987-88 period and in table 4 for 1994-95. Point estimates ($\hat{\theta}$) are provided as well as bootstrap results, namely CIs, means and standard deviations derived from the estimated distribution of the parameter estimates. The small differences between the point estimates and the bootstrap means suggest the absence of a serious bias.

Parameter	Variable	$\hat{\theta}$	CI	Mean	Std. Dev.
Std. dev.	DAX	1.60	(1.40;1.79)	1.59	0.12
	DJI	1.82	(1.26;2.45)	1.76	0.37
	USD	0.68	(0.62;0.73)	0.67	0.03
	LTR	0.66	(0.57;0.76)	0.66	0.06
Correlation	DAX,DJI	0.26	(0.13;0.40)	0.27	0.08
	DAX,USD	0.50	(0.42;0.56)	0.49	0.04
	DAX,LTR	-0.04	(-0.17;0.08)	-0.05	0.08
	DJI,USD	0.11	(0.02;0.25)	0.13	0.07
	DJI,LTR	0.11	(-0.25;0.37)	0.05	0.20
	USD,LTR	0.16	(0.04;0.26)	0.16	0.07

Table 3: Ordinary standard deviations and correlations estimated on a 500 days sample (1987-88)

Parameter	Variable	$\hat{\theta}$	CI	Mean	Std. Dev.
Std. dev.	DAX	0.95	(0.89;1.01)	0.95	0.04
	DJI	0.64	(0.59;0.70)	0.64	0.03
	USD	0.72	(0.65;0.80)	0.73	0.05
	LTR	0.80	(0.73;0.88)	0.80	0.05
Correlation	DAX,DJI	0.43	(0.36;0.49)	0.43	0.04
	DAX,USD	0.44	(0.38;0.50)	0.44	0.04
	DAX,LTR	-0.54	(-0.59;-0.48)	-0.53	0.03
	DJI,USD	0.18	(0.10;0.26)	0.17	0.05
	DJI,LTR	-0.31	(-0.38;-0.24)	-0.31	0.04
	USD,LTR	-0.18	(-0.25;-0.08)	-0.16	0.05

Table 4: Ordinary standard deviations and correlations estimated on a 500 days sample (1994-95)

The MSW estimation results shown below are based on a two state model ($k = 2$) using the specification indicated in section 2.3 restricting means to zero. Results for a three state model are omitted because there was little evidence found for a third state. See for example the results of the score tests presented below (cf. table 9).

Table 5 (7) reports estimation and bootstrap results of the first (second) subperiod for a two state Markov switching model. These MSW results have to be compared with the results for the ordinary estimator given in tables 3 and 4, respectively. Tables 6 and 8 show the results for the estimated Markov transition matrix (P).

Parameter	Variable	State	$\hat{\theta}$	CI	Mean	Std. Dev.
Std. dev.	DAX	1	1.10	(0.87;1.16)	1.04	0.09
		2	4.05	(1.98;4.69)	3.47	0.88
	DJI	1	1.05	(0.79;1.14)	1.00	0.11
		2	5.10	(1.67;7.72)	4.40	2.02
	USD	1	0.61	(0.46;0.65)	0.56	0.06
		2	1.19	(0.87;1.35)	1.10	0.15
	LTR	1	0.52	(0.41;0.56)	0.49	0.05
		2	1.46	(0.75;1.94)	1.33	0.38
Correlation	DAX,DJI	1	0.42	(0.34;0.48)	0.42	0.04
		2	0.17	(-0.00;0.44)	0.21	0.13
	DAX,USD	1	0.41	(0.34;0.52)	0.43	0.05
		2	0.68	(0.47;0.74)	0.61	0.09
	DAX,LTR	1	-0.19	(-0.38;-0.08)	-0.22	0.09
		2	0.12	(-0.11;0.29)	0.07	0.12
	DJI,USD	1	0.20	(0.10;0.29)	0.20	0.06
		2	0.04	(-0.09;0.27)	0.07	0.11
	DJI,LTR	1	-0.19	(-0.29;-0.09)	-0.19	0.06
		2	0.35	(-0.26;0.66)	0.21	0.32
	USD,LTR	1	0.18	(-0.07;0.28)	0.12	0.11
		2	0.11	(-0.19;0.43)	0.15	0.19

Table 5: MSW standard deviations and correlations 1987-88

$p[s_{t+1} s_t]$		$s_{t+1} = e_1$	$s_{t+1} = e_2$
$s_t = e_1$	$\hat{\theta}$	0.93	0.07
	CI	(0.82;0.92)	(0.08;0.18)
	Mean	0.88	0.12
	Std. dev.	0.03	0.03
$s_t = e_2$	$\hat{\theta}$	0.21	0.79
	CI	(0.19;0.42)	(0.58;0.80)
	Mean	0.31	0.69
	Std. Dev.	0.07	0.07

Table 6: MSW transition probabilities 1987-88

For all four return series in both subperiods state 1 is characterized by a lower standard deviation (cf. table 5, $\hat{\theta}$) and the quality of parameter estimates is different for the two states with respect to CIs as well as parameters' standard deviations. The parameters' standard deviations for state 2 are relatively high. Obviously, in the first subperiod there is also a substantial difference between the correlation estimates of state 1 and state 2. The estimates of the second subperiod suggest that the difference between both states is

found in their volatilities but there is no substantial difference in correlations. Table 6 and 8 suggest a clear cut Markov structure for both subperiods.

Parameter	Variable	State	$\hat{\theta}$	CI	Mean	Std. Dev.
Std. dev.	DAX	1	0.74	(0.68;0.89)	0.77	0.07
		2	1.20	(1.00;1.33)	1.17	0.10
	DJI	1	0.55	(0.47;0.66)	0.55	0.06
		2	0.77	(0.54;0.92)	0.77	0.11
	USD	1	0.49	(0.42;0.56)	0.49	0.06
		2	0.97	(0.79;1.19)	0.99	0.14
	LTR	1	0.51	(0.45;0.66)	0.54	0.08
		2	1.10	(0.91;1.27)	1.08	0.14
Correlation	DAX,DJI	1	0.43	(0.30;0.54)	0.43	0.07
		2	0.43	(0.32;0.53)	0.43	0.06
	DAX,USD	1	0.40	(0.25;0.52)	0.39	0.08
		2	0.46	(0.36;0.57)	0.47	0.07
	DAX,LTR	1	-0.54	(-0.64;-0.44)	-0.55	0.06
		2	-0.54	(-0.62;-0.44)	-0.54	0.06
	DJI,USD	1	0.13	(-0.00;0.30)	0.14	0.09
		2	0.21	(0.08;0.32)	0.20	0.07
	DJI,LTR	1	-0.32	(-0.45;-0.18)	-0.32	0.08
		2	-0.32	(-0.46;-0.19)	-0.33	0.08
	USD,LTR	1	-0.22	(-0.38;-0.07)	-0.22	0.10
		2	-0.15	(-0.28;-0.00)	-0.14	0.09

Table 7: MSW standard deviations and correlations 1994-95

$p[s_{t+1} s_t]$		$s_{t+1} = e_1$	$s_{t+1} = e_2$
$s_t = e_1$	$\hat{\theta}$	0.84	0.16
	CI	(0.79;0.86)	(0.13;0.20)
	Mean	0.82	0.18
	Std. dev.	0.02	0.02
$s_t = e_2$	$\hat{\theta}$	0.18	0.82
	CI	(0.13;0.34)	(0.66;0.87)
	Mean	0.23	0.77
	Std. dev.	0.06	0.06

Table 8: MSW transition probabilities 1994-95

Since MSW models need quite a few specification assumptions, we tested for the most common misspecifications. Therefore, we implemented a set of score tests according to White (1987) along the lines of Hamilton (1996) for autocorrelation, ARCH effects and a misspecified Markov structure. Scores are defined as derivatives of each observation's likelihood contribution with respect to the elements of the parameter vector θ . Intuitively, the scores are used as a substitute for residuals. One can show that these scores should be approximately white noise. Lagged scores should contain no information about present scores. So, tests can be performed on particular scores' autocorrelations (cf. table 9).

Here we have used the derivatives with respect to the elements of μ (observations means) to test for misspecification through autocorrelation, derivatives with respect to standard deviations ($\sigma_{(.)}$) to test for misspecification through ARCH effects not picked up by the model and finally various parameter combinations to test for misspecified Markov dynamics, caused for example by a wrong number of states or the assumption of a homogeneous Markov chain.

Table 9 reports results of these score tests for both subperiods. We face serious problems in the first subperiod. Here, autocorrelation and ARCH effects seem to be present. Specification tests for the Markov structure raise some doubt about the two state model. However, the tests indicate no such misspecifications in the second subperiod. One reason might be that the impact of the crash in October 1987 and the period of market disturbances afterwards induce the need to correct for autocorrelation or to introduce a third state picking up the particular ARCH effects around the crash. We refrained from doing so, since the second subperiod does not show similar effects. Nevertheless, in practical applications one should pay attention to the specification problems which arise using MSW models.

Test on	Parameters involved	Period 1987-88		Period 1994-95	
		Statistic	p-value	Statistic	p-value
Autocorrelation	$\mu(1)$	19.03	0.00	1.90	0.75
	$\mu(2)$	4.09	0.39	3.49	0.48
	$\mu(3)$	5.97	0.20	4.40	0.35
	$\mu(4)$	11.80	0.02	3.66	0.45
ARCH	$\sigma_{(11)}$	12.23	0.02	3.50	0.50
	$\sigma_{(22)}$	9.91	0.04	3.74	0.44
	$\sigma_{(33)}$	29.97	0.00	4.08	0.40
	$\sigma_{(44)}$	15.59	0.00	4.82	0.31
Markov specification 1	$\mu(1)$ and lag of p_{11}, p_{22}	5.97	0.20	0.55	0.97
	$\mu(2)$ and lag of p_{11}, p_{22}	8.83	0.07	3.49	0.48
	$\mu(3)$ and lag of p_{11}, p_{22}	5.97	0.20	4.40	0.35
	$\mu(4)$ and lag of p_{11}, p_{22}	11.13	0.03	3.66	0.45
Markov specification 2	$\mu(1), p_{11}, p_{22}$	43.58	0.00	8.44	0.39
	$\mu(2), p_{11}, p_{22}$	38.44	0.00	9.93	0.27
	$\mu(3), p_{11}, p_{22}$	45.66	0.00	11.61	0.17
	$\mu(4), p_{11}, p_{22}$	38.16	0.00	12.90	0.12
Markov specification 3	p_{11}, p_{22}	35.67	0.00	7.72	0.10

Table 9: Score tests for dynamic specification

Finally, we tested whether the imposed zero mean restrictions cause a significant loss in fit measured in terms of the likelihood. This is also based on scores.¹⁵ Here we use the fact that the average score of an unrestricted parameter should be zero. The tests reported in table 10 show that the average scores of the restricted means are significantly different

¹⁵Cf. Hamilton (1996).

from zero, allowing the conclusion that we experience a decrease in the likelihood induced by this restriction.

Period	Statistic	p-value
1987-88	61.81	0.00
1994-95	32.06	0.00

Table 10: Tests on significant decrease in likelihood induced by restrictions

3.3 Comparison of Estimation Results

After having discussed the specification of the MSW estimator, i.e. two states, a zero restricted mean and a joint normal distribution of returns, we turn to a discussion of differences between ORD, EWMA, and MSW. Therefore plots of estimated correlations between DAX and DJI (figure 1) as well as DAX standard deviations (figure 2) are shown below for the first subperiod.

Both ORD and EWMA correlations were estimated and bootstrapped using a moving time window of 75 days. For simplicity, MSW, is estimated and bootstrapped using the entire subsample 1987-88. So, different information sets are used here.

Trying to interpret the point estimates while ignoring their quality is strictly misleading. A more meaningful comparison can be achieved on the basis of CIs.¹⁶ We use a moving block bootstrap to compute CIs since we could not reject the hypothesis of no autocorrelation structure in all subperiods. In figures 1 and 2 correlation estimates and estimated standard deviations are plotted against time (YYMMDD). Solid lines depict point estimates. Dotted lines show CI's.

Three points are worthwhile noting. First, a significant change in the level of correlation between DAX and DJI can hardly be observed during the 1987-88 period (figure 1), i.e. we are not able to reject the hypothesis of a stationary correlation for this period. CIs for EWMA correlations are typically broader than corresponding CIs of ORD and MSW estimates. Second, the width of CIs changes over time, i.e. the point estimator's quality fluctuates during the observation period. CIs broaden substantially after the October 1987 crash, e.g. in October 1987 CIs of ORD estimates double. None of these two statements could have been based on point estimates only. Third, a strong persistence of shocks can be noted in both, ORD and EWMA correlation estimates. Since the impact of the 1987 stock market crash slowly decreases in the EWMA estimates, the correlation slowly approaches the original level as the weight laid on the shock diminishes.

¹⁶These intervals are based on a 10% confidence level.

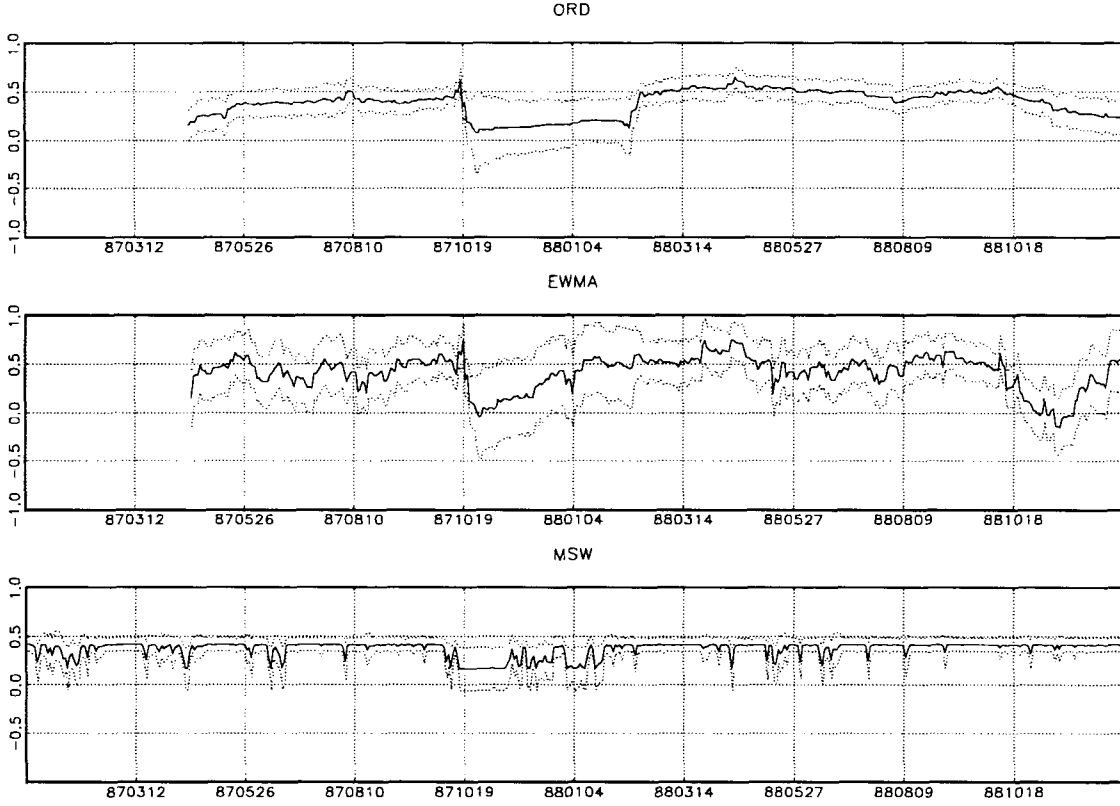


Figure 1: ORD, EWMA and MSW correlation DAX/DJI, 1987-88

This event's influence on the ordinary estimates remains more or less constant as long as the shock is within the 75-days window. As soon as this observation is excluded the estimates jump back to their pre-crash levels. This interpretation is confined to the point estimators. Looking at the CIs we cannot reach the same conclusion since they are just broader after the shock. In-sample MSW point estimates are not as heavily influenced by the crash as in the other models. Moreover, CIs suggest that there is no change in the correlation during this period.

The difference between the three estimators is much more pronounced considering estimated standard deviations (figure 2). The crash is incorporated quicker by EWMA and MSW estimates in comparison to ORD. After the shock, MSW estimates reach the base level faster than EWMA or ORD. In the MSW model the crash information is also used to estimate variances in minor market disruptions before and after the crash.

Again, these results were obtained using different information sets for the estimators. Thus, only general properties can be compared but no information is to be gained about their forecast performance. This is investigated in the following section.

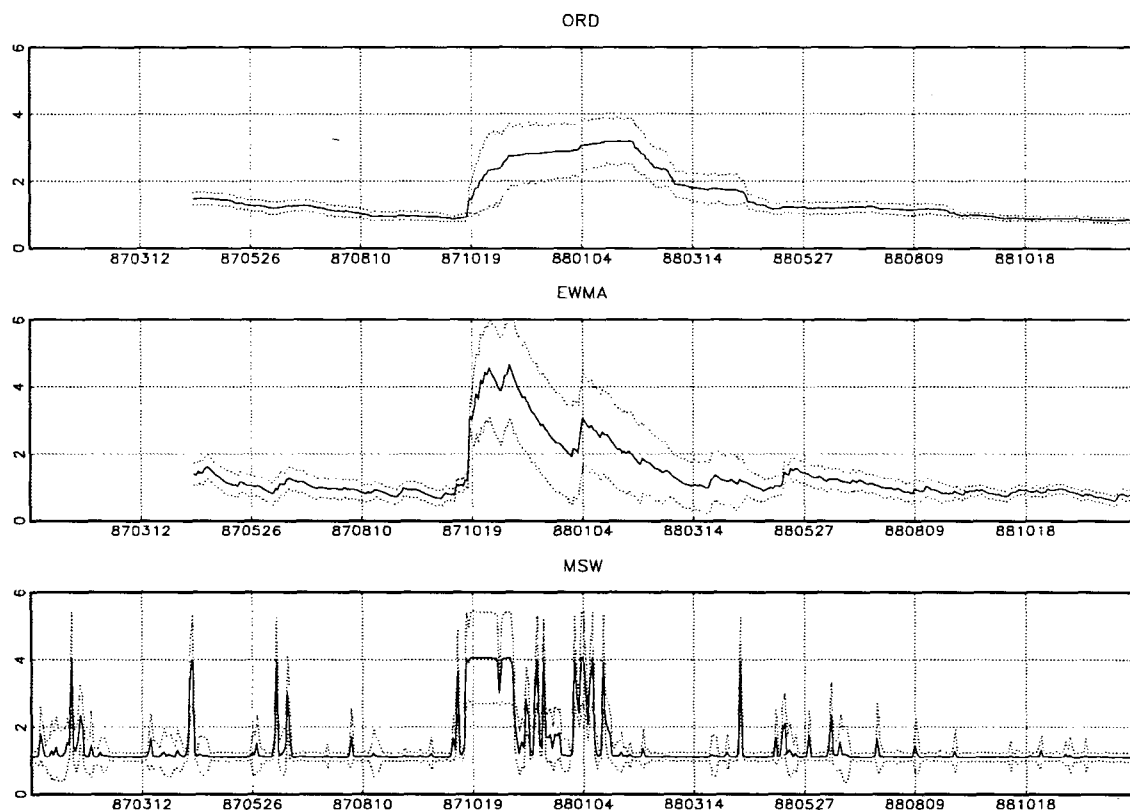


Figure 2: ORD, EWMA and MSW standard deviation DAX, 1987-88

3.4 Benchmark Results

Comparing the three estimators on a purely theoretical basis, the MSW estimator shows some desirable properties: first, it has a "longer memory" and second, the Markov transition matrix explicitly maps the probability to change into a highly volatile state. However, it might be questionable whether these properties lead to a superior performance of the MSW estimator in practical portfolio applications. Here we compare estimators in the context of a MVP. This seems to be a natural benchmark to measure the quality of different covariance forecasts. Since the selection of such portfolios is solely based on covariance estimates, the evaluation of estimators' performance is not diluted by any concept aiming to forecast returns.¹⁷

One might be tempted to evaluate the forecasting power of the estimators comparing "ex-post observed" covariance matrices with forecasted ones. However, these covariances are

¹⁷The assets' weights in the MVP do not depend on expected returns. We have already addressed the potential bias induced in the covariance estimate by restricting expected returns to zero.

not observable but have to be estimated. Here we face substantial problems in specifying the "correct" ex-post estimator. Two straightforward ways to proceed seem to be obvious. First, a benchmark for the variance (σ_*^2) can be computed depending on the forecast horizon. This can be achieved on the basis of one observation ($\sigma_*^2 \approx r_{t+\tau}^2$) or using all observations within the forecast horizon ($\sigma_*^2 \approx \frac{1}{\tau} \sum_{i=1}^{\tau} r_{t+i}^2$). Considering, however, that these estimators rely only on few observations they incorporate a substantial measurement error. Second, we might consider a benchmark estimator based on an "appropriate" information set. As examples might serve the Newey-West estimator based on a window ranging from $t + \tau - \frac{L}{2}$ to $t + \tau + \frac{L}{2}$, a symmetric EWMA estimator on the same window, or an ORD estimator $t + \tau - L$ to $t + \tau$. Arguments in favor of and against any of these estimators can be easily found e.g. on the basis of consistency or variability. As results are highly dependent on the estimator chosen and specifications appear to be quite arbitrary we turn to a different method.

Instead, we use the MVP benchmark to facilitate the comparison of the VCM estimators. The MVP gives us a well defined context to measure the estimators forecast performance, i.e. if the estimator under consideration is appropriate the implied MVP should have the properties suggested by theory. Since we do not know the variance baseline this benchmark can only yield a relative quality measure. Thus, we can conclude that the estimator producing the lowest standard deviation of portfolio returns is better than the others. Autocorrelation and heteroskedasticity of the MVP returns does not cause any problems for the benchmark as long as the variance estimator used is consistent and the relative performance of the estimators to be evaluated is stationary, i.e. the order is invariant with respect to size and offset of the sample used (cf. table 12 below to verify these properties).

To construct the MVP benchmark, we have to assume a particular situation an investor faces. In this paper we assume that the investor minimizes the risk of nominal DEM returns. Furthermore, we restrict the investor's decision space to three assets: DAX, DJI and a generic German long term bond (LTB). In a second step, we test the robustness of results dropping the long term bond. Hence, benchmark results are reported for a three asset portfolio consisting of DAX, DJI, and LTB as well as for a two asset portfolio of DAX and DJI.

Under these assumptions, an investment in the DJI incorporates a currency risk in addition to the index risk. Since we have calculated returns in terms of changes in log prices the return on the DJI for a German investor is simply given by $r_{DJI(DEM),t} = r_{DJI(USD),t} + r_{USD(DEM),t}$. Thus, an investor buying the DJI has an equally weighted portfolio consisting of the DJI expressed in USD ($DJI(USD)$) and the USD expressed in DEM ($USD(DEM)$). Therefore, we can use standard portfolio theory to evaluate the risk and return of such a portfolio.

Another issue the hypothetical investor would have to face is the investment in an interest rate, which can be achieved by buying a bond of comparable maturity. One way to

approximate the risk of this bond is to apply standard duration methods.¹⁸ Using the MVP benchmark concept, we proceed as follows: First, we select asset weights (x_t) in the MVP given the covariance forecast at a certain day t according to $x_t = \frac{1}{i' \Omega_t^{-1} i} \cdot \Omega_t^{-1} i$ where Ω_t is the (3×3) VCM of asset's returns denominated in DEM. Next, we evaluate the ex-post returns of this portfolio, i.e. the returns during the succeeding period $(t+1)$ to $(t+\tau)$. Afterwards, we compute portfolio adjustments necessary to match the optimal weights for the next date $(t+\tau)$. Moving on τ days and repeating the procedure we finally get a series of ex-post returns for each covariance estimator. Here, we analyze one day ($\tau = 1$) VCM forecast periods as well as five day ($\tau = 5$) forecasts. However, only results for a one day forecast period are reported because both do not substantially differ. Finally, the portfolio's performance is evaluated on the basis of its ex-post standard deviation. We also report percentiles of realized returns and portfolio adjustments serving as a proxy for transaction costs. As we need 500 observations for the first MSW covariance forecast we can construct portfolios for the remaining 1787 days, i.e. roughly seven years (1989-95). The first and last window's MSW estimates have already been discussed in Sec. 3.2.

In table 11 results are displayed for equally weighted portfolios (EQU) as well as for the MVPs computed on the basis of ORD, EWMA and MSW covariance estimates (entries are based on daily returns, denominated as percentage points). Descriptive statistics of portfolio returns suggest that there is no substantial difference between MVPs on the basis of ORD, EWMA and MSW. Nevertheless, EQU portfolios show a higher standard deviation as well as a higher kurtosis compared to ORD, EWMA or MSW MVPs for the three asset portfolio. Whereas the two asset portfolio produces quite different results.

	3 assets (DAX, DJI, LTB)				2 assets (DAX, DJI)			
	EQU	ORD	EWMA	MSW	EQU	ORD	EWMA	MSW
Std. dev.	0.653	0.312	0.320	0.317	0.922	0.924	0.933	0.926
Percentile 10%	-0.716	-0.307	-0.300	-0.311	-1.002	-0.963	-0.978	-1.027
Percentile 90%	0.790	0.369	0.384	0.377	1.089	1.062	1.086	1.100
Mean	0.034	0.031	0.033	0.029	0.035	0.021	0.017	0.036
Skewness	-0.874	-0.423	-0.259	-0.627	-1.242	-1.620	-1.645	-1.436
Kurtosis	13.573	7.618	9.358	7.248	20.497	23.933	21.124	23.784

Table 11: One day portfolio returns: descriptive statistics

To ensure that the order of MVPs' standard deviations is stable, annual results are reported in table 12.

¹⁸To facilitate the analysis we assume a bond of ten years time to maturity paying an annual coupon of 6%.

	EQU	ORD	EWMA	MSW	EQU	ORD	EWMA	MSW
1989	0.74	0.27	0.29	0.25	b, c, d	a	a,d	a,c
1990	0.77	0.39	0.38	0.43	b, c, d	a	a	a
1991	0.61	0.22	0.23	0.24	b, c, d	a	a	a
1992	0.60	0.20	0.21	0.20	b, c, d	a	a	a
1993	0.48	0.23	0.24	0.23	b, c, d	a	a	a
1994	0.71	0.47	0.49	0.47	b, c, d	a	a	a
1995	0.61	0.29	0.30	0.29	b, c, d	a	a	a

Table 12: Annual one day MVP forecast performance (3 assets)

- a Std. dev. significantly different from EQU on a 5 % level.
- b Std. dev. significantly different from ORD on a 5 % level.
- c Std. dev. significantly different from EWMA on a 5 % level.
- d Std. dev. significantly different from MSW on a 5 % level.

Table 13 displays tests on equality of standard deviations of portfolio returns for the whole sample period. For the 3 asset portfolios standard deviations of all three MVPs are significantly different from the EQU portfolio. Whereas there is no significant difference between ORD, EWMA and MSW portfolios. In the 2 asset case none of the portfolios differs significantly from the other portfolios with respect to the standard deviation.

		3 assets		2 assets	
H_0		F(1786,1786)	p-value	F(1786,1786)	p-value
$Std[r_{EQU}] = Std[r_{ORD}]$		4.38	0.00	1.00	0.92
$Std[r_{EQU}] = Std[r_{EWMA}]$		4.17	0.00	1.02	0.62
$Std[r_{EQU}] = Std[r_{MSW}]$		4.24	0.00	1.01	0.83
$Std[r_{ORD}] = Std[r_{EWMA}]$		1.05	0.29	1.02	0.70
$Std[r_{MSW}] = Std[r_{EWMA}]$		1.02	0.73	1.01	0.78
$Std[r_{ORD}] = Std[r_{MSW}]$		1.03	0.48	1.00	0.92

Table 13: Test on equal variances of portfolio returns

The main difference between ORD, EWMA and MSW is found in portfolio adjustments necessary to maintain the desired MVP property. Note that the EQU portfolios need to be adjusted as well, since the effects of unequal returns in different assets have to be offset. Comparing cumulated adjustments reveals a turnover of the EQU portfolio of about 6 times during the 7 years under consideration. MSW turnover is $2\frac{1}{2}$ times as large, ORD roughly 3 times and EWMA about 13 times (cf. table 14). In the case of the 2 asset portfolio adjustments do also differ substantially.

		EQU	ORD	EWMA	MSW
3 assets (DAX, DJI, LTB)					
Cum. pf. adj.	DAX	2.05	6.65	27.04	3.82
Cum. pf. adj.	DJI	2.35	5.81	22.24	6.11
Cum. pf. adj.	LTB	1.77	7.45	29.47	5.18
Cum. pf. adj.	total	6.16	19.01	78.74	15.10
2 assets (DAX, DJI)					
Cum. pf. adj.	total	6.16	32.40	131.69	43.57

Table 14: Cumulated portfolio adjustments (daily horizon)

Despite the large portfolio adjustments suggested by EWMA this estimator does not outperform ORD or MSW in terms of the standard deviation of the MVP. One reason might be that the EWMA estimator reacts too sensible. This point might be illustrated by the standard deviation of the estimated correlation coefficient in the 2 asset case, i.e. the correlation between DAX and DJI which is 0.31 for EWMA, 0.27 for ORD and 0.13 for MSW.

	3 assets			2 assets		
	ORD	EWMA	MSW	ORD	EWMA	MSW
Mean	0.557	2.204	0.423	0.907	3.687	1.220
Std.dev.	0.713	3.123	0.599	1.528	6.257	1.827
10%-percentile	0.074	0.153	0.034	0.061	0.148	0.065
50%-percentile	0.327	1.164	0.213	0.449	1.709	0.508
90%-percentile	1.264	5.316	1.037	2.010	8.876	3.192

Table 15: Descriptive statistics of individual portfolio adjustments (given in percentage points for a daily horizon)

Note that entries in table 15 are given in percentage points in contrast to table 14. The high sensitivity of the EWMA estimator is reflected by the large variability of individual portfolio adjustments documented in table 15. However, additional research is needed to clarify this point.

Furthermore, an important point is whether individual covariance estimates provide reliable CIs for the returns of the selected portfolios. Therefore, we perform a test once again based on the realized portfolio returns. We consider a binomially distributed random variable which takes on the value 0 if the observed return lies within the 90% CI based on the corresponding VCM forecast and 1 otherwise. As a matter of construction, one would expect this random variable's parameter p to be equal to 0.10 if the VCM forecasts were of some use in this context. As a matter of fact, none of the estimators performs well under this test. Table 16 reports results for the entire sample as well as for the sample split into halves. Even though in the three asset case the MSW estimates appear to be valid for the whole sample period, the estimator clearly misperforms on each of the sample's halves. The overall performance is just caused by an overestimation of CIs in the first half of the

sample and an underestimation in the second half, thus pretending an overall acceptable performance. For the two asset portfolio estimated CIs do not perform very well either.

		3 assets (DAX, DJI, LTB)		2 assets (DAX, DJI)	
		Observations outside 90% CI	p-value	Observations outside 90% CI	p-value
ORD	total	11.86%	0.01	10.41%	0.56
	1st half	11.87%	0.06	8.73%	0.21
	2nd half	11.86%	0.06	12.09%	0.04
EWMA	total	14.77%	0.00	12.20%	0.00
	1st half	15.23%	0.00	12.43%	0.02
	2nd half	14.33%	0.00	11.97%	0.05
MSW	total	9.90%	0.89	6.60%	0.00
	1st half	6.27%	0.00	3.70%	0.00
	2nd half	13.55%	0.00	9.51%	0.62

Table 10: Test on validity of CIs based on one day forecasts of VCMs

The benchmark results above were analyzed on a five day horizon as well, i.e. VCMs were forecasted on a five day time span, the performance of portfolios was evaluated for this horizon and adjustments were made every five days. Results are omitted here because there is no substantial difference for these five day horizon portfolios.

So far, MSW does not outperform ORD and EWMA in terms of standard deviations of realized returns, whereas it has advantages over these considering transaction costs.

4 Conclusion and further Research

In this paper we addressed the issue of estimating time varying covariances needed for portfolio management and risk measurement. With respect to these economic applications, we investigated the suitability of different estimation concepts: We compared ordinary estimators, exponentially weighted moving averages and Markov switching models. EWMA estimators serve as a reasonably good approximation of the widely used GARCH estimators. Markov switching models were considered a promising alternative to ARCH type estimators because they explicitly model different states of a decision space. Thus, they allow to quantify the risk to change into an adverse state. Another advantage of Markov switching models is that the influence of particular observations does not erode over time as in GARCH models.

The considered applications, namely portfolio selection as well as evaluation of a portfolio's risk, incorporate the anticipation of future risk structures. Therefore, the centerpiece of the models' performance is its forecasting power. In contrast to other forecasting applications involving observable targets, VCM forecasts cannot be evaluated by comparing them to observed values. Thus, we introduce here the concept of an observable economic

benchmark, i.e. the minimum variance portfolio (MVP) selected on the basis of forecasted VCMs.

Using this economic benchmark, we cannot claim that the two state Markov switching model implemented here performs better than the other models in terms of the realized returns' standard deviation of the implied MVPs. Nevertheless, results for the quality of confidence intervals as well as for trading costs suggest that further research might be fruitful. E.g., it might be promising to investigate alternative Markov switching structures in comparison to different ARCH type models. However, ordinary VCM estimates did show a surprisingly good performance involving only moderate computational costs. .

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