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Free riding and the inefficiency of the private production of pure public goods

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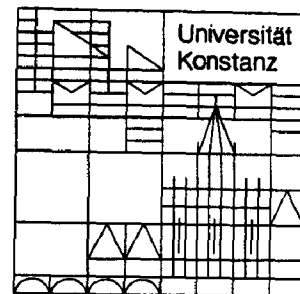
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Sonderforschungsbereich 178
„Internationalisierung der Wirtschaft“

Diskussionsbeiträge



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PRIVATE PRODUCTION OF PURE PUBLIC GOODS**

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Free Riding and the Inefficiency of the Private Production of Pure Public Goods

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Abstract

Free riding is explained in a model of voluntary production of public goods, in terms of the heterogeneity of households. To achieve this the traditional essentially Ricardian model of voluntary production of pure public goods is generalised to comprise any number of private and public goods, factors of production and households. The efficiency losses from the under-production of public goods are related to the efficiency losses from free riding and the scale of the economy. A condition for Pareto improving reallocations within the public goods sector in terms of the popularity of the various public goods is put forward.

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Free Riding and the Inefficiency of the Private Production of Pure Public Goods

Modelling the voluntary production of pure (nonexclusive and nonrivalrous) public goods as Cournot-Nash equilibria one obtains three key results: (a) the presumption that public goods are underproduced [Samuelson (1954) or Malinvaud (1972)], (b) the alternative possible interpretations of free riding [MacMillan (1979)] and (c) the invariance principle [Warr (1983), Cornes and Sandler (1986), Bergstrom, Blum and Varian (1986), Andreoni (1988) and Fries et alii (1990)].

The focus of recent attention has undoubtedly been the invariance principle especially in relation to free riding, where free riding is defined as a specialisation of contributions by households. As is well known, if there is free riding, the invariance property holds at most in a restricted sense. This may be regarded as a strength or a weakness of models of the voluntary production of pure public goods. In any case, it is agreed that free riding is an important property of these models which may be explained in terms of the heterogeneity of households and the scale of the economy.

Given the general consensus on the importance of free riding it is more than a little surprising that its normative implications especially in relation to the underproduction of public goods and to the scale of the economy have not been explored. This glaring gap is filled by the present paper. More precisely; the efficiency losses from the underproduction of public goods are formally related to the efficiency losses associated with free riding. What the role of the scale of the economy is in this relationship is also of great interest and therefore forms a key focus of the present approach.

Since free riding is defined as a specialisation in contributions by households it seems imperative to allow for specialisations in contributions in the widest possible sense. This consideration leads us to allow for any (finite) number of households, public and private goods and factors of production.

In such a general framework one can consider many important yet hitherto neglected issues: (1) Under which conditions is the allocation of resources between the various public goods inefficient? How can efficiency gains be achieved by reallocating one or more factors between the various public goods without changing the allocation of resources between the private and public goods sectors as a whole? (2) What is the precise relationship between the

efficiency losses from the underproduction of public goods and from free riding? Most importantly how is this relationship affected by changes in the scale of the economy (the number of households)? (3) In the very general model of the voluntary production of public goods to be presented below how can free riding be explained in terms of the heterogeneity of households? How is this result related to recent explanations in terms of the scale of the economy?

The structure of the paper is as follows: In part I the above mentioned generalisation of the standard Ricardian model of voluntary production of public goods is undertaken and interpreted. Part II provides a sufficient condition for the occurrence of free riding in an economy with any finite number of different households in terms of their heterogeneity. Part III focusses on the relationship between the efficiency losses due to the underproduction of public goods, the scale of the economy and specialisations in contributions as between households (i.e. free riding). Having defined a measure for the popularity of public goods a condition for a Pareto improving reallocation of factors within the public goods sector alone is derived. Finally a very general result relating the scale of the economy, free riding and the underproduction of public goods is derived and interpreted. In Part IV the center of our attention is, as in Part II, to explain the occurrence of free riding but this time in very large economies. Essentially, the well known recent results by Andreoni (1988) and Fries et alii (1990) are generalised and related to proposition I.

The main results are summarised in the conclusions.

Part I

The Model of Voluntary Production of Public Goods

(A) The Household Sector

There are H households, the preferences of household h being represented by a strictly quasi-concave and well-behaved utility function defined over n private goods and m public goods:

$$U^h(x_1^h, \dots, x_n^h, G_1, \dots, G_m) \quad h = 1, \dots, H \quad (1)$$

where x_j^h is the consumption by household h of private good j , and G_k is the total quantity of the nonrivalrous nonexcludable public good k

Each individual is endowed with a vector of factors. The endowment of household h is $\underline{v}^h \equiv (v_1^h, \dots, v_l^h)$, where v_i^h is h 's endowment of factor i . Factors can be used as inputs to produce public goods, or supplied to competitive markets to earn income for expenditure on private goods. Denote the vectors of prices of private goods and factor services respectively by $\underline{p} \equiv (p_1, \dots, p_n)$ and $\underline{w} \equiv (w_1, \dots, w_l)$. In addition, v_{ik}^h denotes the supply by household h of factor i to public good k , and $\underline{v}_{\cdot k}^h$ the vector $(v_{1k}^h, \dots, v_{lk}^h)$.

The budget constraint requires that the value of expenditure on private goods cannot exceed the value of factor endowments net of the quantities supplied by the household to public goods production:

$$\underline{p} \cdot \underline{x}^h \leq \underline{w} \cdot \left(\underline{v}^h - \sum_k \underline{v}_{\cdot k}^h \right) \quad (2)$$

It remains to specify the public good technology. Let $\underline{V}_{\cdot k}$ denote the vector of total provision of factor services for production of public good k :

$\underline{V}_{\cdot k} \equiv \sum_h \underline{v}_{\cdot k}^h$ for all k . We assume that each public good is produced according to a concave production function exhibiting constant returns to scale:

$$G_k = G_k(\underline{V}_{\cdot k}), \quad \frac{\partial G_k(\cdot)}{\partial V_{lk}} > 0 \quad \forall l, k \quad (3)$$

It will be useful to distinguish between that part of the factor supply V_{lk} attributable to individual h , v_{lk}^h , and that part supplied by the rest of the community, \tilde{V}_{lk}^h . Clearly, $V_{lk} = v_{lk}^h + \tilde{V}_{lk}^h$. We shall also apply the same notation to vectors so that, for example, $\underline{V}_{\cdot k} = \underline{v}_{\cdot k}^h + \tilde{\underline{V}}_{\cdot k}^h$.

The model of utility-maximizing behaviour presented in equations (1) - (3) can helpfully be interpreted within the framework of household production theory as presented, for example, by Deaton and Muellbauer (1980, pp. 245-53). Individuals care about both private good and public good consumption levels. Since the latter depend on factor input levels, one can think of preferences, representable by appropriately defined utility functions, over private goods and the levels of factor inputs to the various public goods:

$$U^h(\cdot) = U^h[x_1^h, \dots, x_n^h, G_1(\underline{v}_{\cdot 1}^h + \tilde{\underline{V}}_{\cdot 1}^h), \dots, G_m(\underline{v}_{\cdot m}^h + \tilde{\underline{V}}_{\cdot m}^h)] \quad (4)$$

$$= \Phi^h(x_1^h, \dots, x_n^h, \underline{v}_{\cdot 1}^h + \tilde{\underline{V}}_{\cdot 1}^h, \dots, \underline{v}_{\cdot m}^h + \tilde{\underline{V}}_{\cdot m}^h). \quad (5)$$

By assumption, each individual takes as given the price vectors \underline{p} and \underline{w} and the factor supply decisions of others, $\tilde{\underline{V}}_{\cdot 1}^h, \tilde{\underline{V}}_{\cdot 2}^h, \dots, \tilde{\underline{V}}_{\cdot m}^h$. Since $U^h(\cdot)$ is strictly quasi-concave in $(x_1^h, \dots, x_n^h, G_1, \dots, G_m)$ and each $G_k(\cdot)$ is concave in all inputs, it follows that the induced utility function $\Phi^h(\cdot)$ is strictly quasi-concave in all arguments. See Rader (1978) and Milne (1981) for a discussion of such induced preferences. Cornes (1989, Chapter 6) provides a simple exposition. A self contained proof is available from the authors upon request.

The consumer's problem can now be summarized as

$$\max_{\underline{x}^h, \underline{v}_{\cdot 1}^h, \dots, \underline{v}_{\cdot m}^h} \Phi^h(\underline{x}^h, \underline{v}_{\cdot 1}^h + \tilde{\underline{V}}_{\cdot 1}^h, \dots, \underline{v}_{\cdot m}^h + \tilde{\underline{V}}_{\cdot m}^h)$$

subject to

$$\underline{p} \cdot \underline{x}^h - \underline{w} \cdot \left(\underline{v}^h - \sum_k \underline{v}_{\cdot k}^h \right) \leq 0. \quad (6)$$

$$\underline{x}^h \geq 0 \text{ and } \underline{v}_{\cdot k}^h \geq 0 \quad \forall h, k.$$

Kuhn-Tucker theory implies the following set of first-order conditions which, because of the strict quasi-concavity of $\Phi^h(\cdot)$ and linearity of the budget constraint, are sufficient as well as necessary:

$$\frac{\partial \Phi^h}{\partial x_j^h} - \lambda^h p_j \leq 0, \quad x_j^h \geq 0, \quad \left[\frac{\partial \Phi^h}{\partial x_j^h} - \lambda^h p_j \right] x_j^h = 0, \quad (7)$$

$$\frac{\partial \Phi^h}{\partial v_{ik}^h} - \lambda^h w_i \leq 0, \quad v_{ik}^h \geq 0, \quad \left[\frac{\partial \Phi^h}{\partial v_{ik}^h} - \lambda^h w_i \right] v_{ik}^h = 0, \quad (8)$$

$$\begin{aligned} \underline{p} \cdot \underline{x}^h - \underline{w} \cdot \left(\underline{v}^h - \sum_h \underline{v}_{\cdot k}^h \right) &\leq 0, \\ \lambda^h &\geq 0, \\ \left[\underline{p} \cdot \underline{x}^h - \underline{w} \cdot \left(\underline{v}^h - \sum_h \underline{v}_{\cdot k}^h \right) \right] \lambda^h &= 0. \end{aligned} \quad (9)$$

Note that the appeal to Kuhn-Tucker theory allows us to take seriously the possibility of corner solutions. As we shall see, this is of crucial importance in the context of the issues analysed in the paper. That certain properties of the public goods model; i.e., the invariance property or certain comparative static responses depend on the realisation of a corner or interior solution has been expounded by Bergstrom et alii (1986) and Cornes and Sandler (1985). However, generally speaking, it is very surprising that corner solutions have received considerable attention only relatively recently (see e.g. Andreoni 1988 or Fries et alii 1990); especially since they represent an obvious interpretation of free riding behavior. As we shall see, the latter is of great interest not only per se as a source of efficiency losses but also in the context of the underproduction of public goods and in relation to the scale of the economy. Indeed it appears there is not a single reference which has focussed on the normative implications of specialisation in contributions.

Inspection of (6) makes it clear that the individual's behaviour can be described using a restricted indirect utility function:

$$I(\underline{p}, \underline{w}, \underline{\tilde{v}}_{\cdot 1}^h, \dots, \underline{\tilde{v}}_{\cdot m}^h, \underline{w} \cdot \underline{v}^h) \equiv \max_{\underline{x}^h, \underline{v}_{\cdot 1}^h, \dots, \underline{v}_{\cdot m}^h} \{ \Phi^h(\cdot) \mid \underline{p} \cdot \underline{x}^h + \underline{w} \cdot \left(\sum_k \underline{v}_k^h \right) \leq \underline{w} \cdot \underline{v}^h \} \quad (10)$$

For further discussion of such functions in the context of quantity - constrained behaviour, see Cornes (1989). By applying the envelope property to (6) or, equivalently, by appeal to a 'restricted' version of Roy's Identity, the existence of the following restricted demand functions for private goods and supply functions of factors can be inferred:

$$x_j^h = x_j^h(\underline{p}, \underline{w}, \underline{\tilde{V}}_{.1}^h, \dots, \underline{\tilde{V}}_{.m}^h, M^h) \quad \forall h, j \quad (11)$$

$$v_{ik}^h = v_{ik}^h(\underline{p}, \underline{w}, \underline{\tilde{V}}_{.1}^h, \dots, \underline{\tilde{V}}_{.m}^h, M^h) \quad \forall h, i, k \quad (12)$$

Notice that equations (11) and (12) contain a single income term, $M^h \equiv \underline{w} \cdot \underline{v}^h$. The absence of additional constraints for the individual factor endowments V_1^h, \dots, V_l^h can be interpreted in either of two ways. Either there exist factor markets, enabling h to acquire additional increments of factors if desired and to supply more of a particular factor than his initial endowment, or, in the absence of such markets, the equilibrium may nevertheless be an interior one for h at which none of the constraints $\sum_k v_{ik}^h \leq V_i^h$ are binding.

The functions in expressions (11) and (12) imply that, once $\underline{p}, \underline{w}$ and the patterns of factor supplies of others are given, the equilibrium response by household h is uniquely determined. This follows straightforwardly from competitive behaviour and the strict quasi-concavity of the induced utility function $\Phi^h(\cdot)$ and the linearity of the constraints. It should be emphasized that this does not by any means guarantee a unique allocation within the household sector. In the first place, even for given values of the price vectors \underline{p} and \underline{w} and total endowments, there may be multiple equilibria in the sense that, for every household, there exist other vectors $(\underline{\tilde{V}}_{.1}^{h*}, \dots, \underline{\tilde{V}}_{.m}^{h*})$ that lead to alternative values $x_j^{h*}(\cdot)$ and $v_{ik}^{h*}(\cdot)$. In the simple textbook model, this corresponds to multiple intersections of reaction curves. In the second place, once we bring in supply in a closed multi-household economy there is the familiar problem that there may in general be many vectors $(\underline{p}, \underline{w})$ that equilibrate the system.

(B) THE PRODUCTION SECTOR

We assume that the production functions for private goods are strictly quasi-concave and homogeneous of degree one. Private goods are produced by profit-maximizing firms which behave competitively, treating output and input prices parametrically and the supply of factors to the private good sector as perfectly inelastic. Since the total endowment of household h is \underline{v}^h , and the supply of inputs by that household to the public good sector is $\sum_k \underline{v}_k^h$, total factor supplies to the private good sector are given by

$$\sum_h (\underline{v}^h - \sum_k \underline{v}_k^h) \quad (13)$$

Equilibrium of the private good production sector can be characterized by the private sector revenue function:

$$R(\underline{p}, \sum_h (\underline{v}^h - \sum_k \underline{v}_k^h)) \equiv \max_{\underline{y}} \{ \underline{p} \cdot \underline{y} \mid [\underline{y}, \sum_h (\underline{v}^h - \sum_k \underline{v}_k^h)] \text{ feasible} \} \quad (13a)$$

where \underline{y} represents the non-negative vector of outputs of private goods. The existence of such a revenue function, together with its properties, is discussed by Dixit and Norman (1980) and also by Woodland (1982).

As long as there are no more private goods produced than there are factors supplied to the private goods sector, and the Jacobian determinant of cost functions associated with the private goods is nonvanishing, we avoid the possibility of ruled surfaces in the private goods production possibility frontier, and to any set of values of \underline{p} and $\sum_h (\underline{v}^h - \sum_k \underline{v}_k^h)$ there corresponds a unique revenue-maximizing private goods output vector, \underline{y} .

\underline{y} is generated by partial differentiation of $R(\cdot)$ with respect to \underline{p} :

$$\underline{y} = \underline{R}_p(\cdot). \quad (14)$$

In a similar way, partial differentiation of $R(\cdot)$ with respect to input quantities generates the vector of competitive factor prices:

$$\underline{w} = \underline{R}_v(\cdot). \quad (15)$$

This completes the setting out of the model. As explained before there are no firms which choose cost minimising techniques for the production of public goods. Production decisions concerning public goods are entirely in the hands of households. If we separated the production decisions concerning public goods from households we would have to assume that public goods firms are guided by some prices or that they have a complete knowledge of the preferences of households.

The model has a number of advantages compared with standard models:

(1) It allows for the lack of perfect substitutability of household contributions. Households may be endowed with different factors which may be substitutes or complements in the production of the various public and private goods.

(2) It can be solved under a great variety of assumptions with regard to the tradability or non-tradability of private goods and factors between households and/or internationally. Also there is considerable flexibility in the interpretation of the Cournot-Nash equilibrium. We may, for example, allow for household cooperation between some households concerning some or all factors, some or all public goods.

(3) In particular by making the small country assumption and assuming that all reallocations of factors take place on the same diversification cone of private goods we can isolate the externality from the price effects. This considerably facilitates the interpretation of the results.

(4) It can endogenise the production patterns of private goods and changes therein which are, for example, due to changes in the economy endowments of factors. This is important if changes in the scale of the economy and the heterogeneity of households are considered simultaneously. Changes in the economy endowments generally change the vector of factor prices and therefore the outputs of public goods.

Part II

Free Riding and the Distribution of Factor Endowments between Households

A very fundamental feature of private goods models (and the small country assumption) is that all private goods are produced (for given goods prices) if and only if the factor endowment vector of the economy lies within the diversification cone (i.e.: the cone spanned by the input coefficient vectors of the various goods, see e.g. Woodland (1982, p. 72)). This holds whatever the distribution of factor endowments between households. Surprisingly, it can be shown that this fundamental property does not carry over to public goods even if all the factors of production are taken to be private. The distribution of factor endowments between households acquires a crucial significance in the context of public goods production. As we shall see, it is also of extreme importance in explaining free riding in terms of the heterogeneity of households in an economy with any finite number of households.

The main result of part II is stated as proposition I:

Proposition I: Assume that expression (6) correctly describes the optimisation problem of each household. Denote the sets of factor endowments of households by $\Omega^1, \dots, \Omega^H$ and the sets of factors required by the various public goods (at the given prices of private goods) by: S^1, \dots, S^m . Define the set theoretic intersection of the sets of factor endowments of households, $\Omega = \Omega^1 \cap \Omega^2 \cap \Omega^3 \dots \cap \Omega^H$.

If $\Omega \cap S^1, \Omega \cap S^2, \Omega \cap S^3, \dots, \Omega \cap S^m$ are all empty but public goods are produced then there is free riding.

Proof: We first prove that if a public good k is produced from the contributions of two or more households the set theoretic intersection of factors contributed by two or more households to the public good k cannot be empty.

From equation (8):

$$\left(\frac{\partial \Phi^h}{\partial v_{ik}^h} \right) = \lambda^h w_i \quad (16)$$

if household h contributes factor i to the production of public good k . This may be rearranged in a revealing way. Recall from expression (5) that:

$$\frac{\partial \Phi^h}{\partial v_{ik}^h} = \left(\frac{\partial U^h}{\partial G_k} \right) \left(\frac{\partial G_k}{\partial v_{ik}^h} \right) \quad (17)$$

Substituting expression (17) into expression (16) and dividing through by λ^h we obtain:

$$\frac{1}{\lambda^h} \left(\frac{\partial U^h}{\partial G_k} \right) \left(\frac{\partial G_k}{\partial v_{ik}^h} \right) = w_i \quad (18)$$

The term is the virtual demand price of household h for public good k . Let us denote this demand price by ζ_k^h .

Expression (18) then becomes:

$$\zeta_k^h \left(\frac{\partial G_k}{\partial v_{ik}^h} \right) = w_i \quad (19)$$

From the assumption of perfect substitutability of contributions of all households, we know that the term in the round brackets is independent of h . It follows that if any two households contribute at least one and the same factor to public good k the virtual demand price of public good k is equalised between these two households.

Assume now that household 1 supplies only factor 1 to public good k and household 2 supplies only factor 2 to public good k . This implies that:

$$\zeta_k^1 \left(\frac{\partial G_k}{\partial v_{1k}^1} \right) = w_1 > \zeta_k^2 \left(\frac{\partial G_k}{\partial v_{1k}^2} \right) \quad (20)$$

$$\text{and} \quad \zeta_k^2 \left(\frac{\partial G_k}{\partial v_{2k}^2} \right) = w_2 > \zeta_k^1 \left(\frac{\partial G_k}{\partial v_{2k}^1} \right) \quad (21)$$

Since by assumption:

$$\frac{\partial G_k}{\partial v_{1k}^1} = \frac{\partial G_k}{\partial v_{1k}^2} > 0$$

$$\text{and} \quad \frac{\partial G_k}{\partial v_{2k}^2} = \frac{\partial G_k}{\partial v_{2k}^1} > 0$$

we find that expressions (20) and (21) imply a contradiction. More generally, we have proven that the set theoretic intersection of factors contributed by two or more households to a public good k cannot be empty.

We now prove that all households which contribute to a public good k contribute exactly the same set of factors.

Assume that household 1 contributes to public good k factors 1 and 2 but household 2 contributes only factor 1. In this case:

$$\zeta_k^1\left(\frac{\partial G_k}{\partial v_{1k}^1}\right) = w_1 = \zeta_k^2\left(\frac{\partial G_k}{\partial v_{1k}^2}\right) \quad (22)$$

$$\zeta_k^1\left(\frac{\partial G_k}{\partial v_{2k}^1}\right) = w_2 > \zeta_k^2\left(\frac{\partial G_k}{\partial v_{2k}^2}\right) \quad (23)$$

Clearly, expressions (22) and (23) imply again a contradiction. We have therefore established that if any two households contribute to a public good k they contribute the same set of factors. Free riding occurs whenever at least one household does not contribute at least some or all of his factor endowments to all public goods, i.e.: free riding occurs if: $v_{ik}^h = 0$ for at least one h, i and k .

From expressions (16) to (23) any two contributing households contribute the same set of factors. From the condition of proposition I there does not exist a set of factors which is common to all households and used in the production of any one public good. It follows that if at least one public good is produced not all households are contributing to it.

Q.E.D.

Proposition I states only a sufficient condition for the occurrence of free riding as a result of the heterogeneity of factor endowments of households. However it holds in a very general model and it is independent of the preferences of households provided that preferences are such that at least one public good is produced.

It has no equivalents in the received literature because the latter does not allow (a) for the production of public goods, (b) the possibility of heterogeneity of household endowments (and factors of production) and (c) the possibility that the production of different public goods may require different factors. A striking implication of proposition I (which highlights the distinctive feature of production models of public goods as compared with private

goods) is that for certain distributions of economy factor endowments among households public goods cannot be produced (in a Cournot-Nash equilibrium without household rationing). This holds whatever the strength of preferences of households for public goods. This is, of course, in sharp contrast to Arrow-Debreu or Walrasian general equilibrium models with only private goods.

For the relationship between proposition I and the recent results derived by Andreoni (1988) and Fries et alii (1990) see part IV.

Part III

The Underproduction of Public Goods, Free Riding and the Scale of the Economy

In this part our main aim is twofold: (1) a fundamental relationship between the efficiency losses due to the underproduction of public goods, free riding and the scale of the economy is derived and interpreted. (2) a condition for the Pareto improving reallocation of factors within the public good sector alone in terms of the popularity of the various public goods is put forward. None of these results have any equivalents in the received literature. It is indeed one of the paradoxes of the literature on the voluntary provision of public goods that even though recently there have been some important contributions to explain the occurrence of free riding in large economies (see Andreoni 1988 and Fries et alii 1990) the literature is silent on the normative implications of free riding and the precise normative relationship between free riding, the underproduction of public goods and the scale of the economy. The same applies to the analysis of the normative implications of reallocations of resources (factors) within the public goods sector alone, i.e.: between various public goods.

Since these are obviously important issues which are right at the heart of the concept of the voluntary production of pure public goods it can only be conjectured that their neglect is due to the continued usage of standard models (e.g.: with only one public good).

As before we shall make extensive use of duality concepts to derive our main result [expression (29)].

The main tool of analysis on the demand side is the following restricted expenditure function of household h defined over the whole expenditure of household h :

$$E^h(\underline{p}, \underline{w}, \underline{\tilde{V}}_{.1}^h, \dots, \underline{\tilde{V}}_{.m}^h; u^h) \equiv \min_{\underline{x}^h, \underline{v}_k^h} \{ \underline{p} \cdot \underline{x}^h + \underline{w} \cdot (\sum_k \underline{v}_k^h) \mid \Phi^h(\cdot) \geq u^h \} \quad (23a)$$

where $\Phi^h(\cdot)$ is the reduced form direct utility function, see expression (5).

On the supply side we make use of the revenue or national product function of the private sector, expression (13a).

We are now in a position to define the following trade expenditure function (for details of interpretation see Lloyd and Schweinberger 1988):

$$\bar{B} = \sum_h E^h(\underline{p}, \underline{w}, \underline{\tilde{V}}_{.1}^h, \dots, \underline{\tilde{V}}_{.m}^h; u^h) - R(\cdot) - \underline{w} \cdot \sum_h (\sum_k \underline{v}_k^h) \quad (24)$$

To derive our main result it is assumed that a small rationing of contributions to the production of public goods occurs in the Cournot-Nash equilibrium. It is important to understand that the rationing takes the following form: all households which contribute a factor i to the production of a public good k are coerced to contribute a (infinitesimally) bigger amount of factor i to the public good k .

From the assumption of perfect substitutability of contributions and the household equilibrium conditions:

$$-\frac{\partial E^h(\cdot)}{\partial \tilde{V}_{ik}^h} \leq w_i \quad \forall h, k \text{ and } i \quad (25)$$

Totally differentiating expression (24), keeping all the u^h fixed, we have:

$$d\bar{B} = (\sum_h E_{\underline{p}}^h - \underline{y}) d\underline{p} + (\sum_h E_{\underline{w}}^h - \sum_h \sum_k \underline{v}_{.k}^h) d\underline{w} - \underline{p} d\underline{y} + \sum_h \sum_i \sum_k (\frac{\partial E^h(\cdot)}{\partial \tilde{V}_{ik}^h}) d\tilde{V}_{ik}^h \quad (26)$$

Expression (26) follows because the supplies of all primary factors to the whole economy are fixed, i.e.:

$$d(\sum_h \sum_k \underline{v}_{.k}^h)w = -d(\sum_h \underline{v}^h - \sum_k \underline{v}_{.k}^h)w$$

where the right hand side stands for the inner product of the derivatives of $R(\cdot)$ with respect to the supplies of factors to the private sector times the changes in these factor supplies. From the envelope properties of the function $\bar{B}(\cdot)$ and since the partial derivatives are evaluated in the equilibrium and $\text{pdy} = 0$, expression (26) can be simplified to:

$$d\bar{B} = \sum_h \sum_i \sum_k \frac{\partial E^h(\cdot)}{\partial \tilde{V}_{ik}^h} d \tilde{V}_{ik}^h \quad (27)$$

If and only if expression (27) is negative there is a Pareto improvement as a result of the assumed small rationing of contributing households to contribute more of each factor to each public good (to which they contribute in the Cournot-Nash equilibrium).

For the purpose of interpretation it is useful to rewrite expression (27) as follows:

$$d\bar{B} = - \sum_h \sum_i \sum_k w_i d \tilde{V}_{ik}^h + \sum_h \sum_i \sum_k \left(\frac{\partial E^h(\cdot)}{\partial \tilde{V}_{ik}^h} + w_i \right) d \tilde{V}_{ik}^h \quad (28)$$

From the definition of $d \tilde{V}_{ik}^h$ this becomes:

$$d\bar{B} = -(H - I) \sum_h \sum_i \sum_k w_i dv_{ik}^h + \Delta \quad (29)$$

$$\text{where: } \Delta = \sum_h \sum_i \sum_k \left(\frac{\partial E^h(\cdot)}{\partial \tilde{V}_{ik}^h} + w_i \right) d \tilde{V}_{ik}^h \geq 0$$

because of expression (25) and the assumption that $dv_{ik}^h > 0 \forall h, k$ and i .

To make the economic meaning of Δ transparent note that from the envelope theorem applied to expression (23a), it follows that:

$$-\frac{\partial E^h(\cdot)}{\partial \tilde{V}_{ik}^h} = \frac{1}{\lambda^h} \frac{\partial \Phi^h}{\partial \tilde{V}_{ik}^h} = \frac{1}{\lambda^h} \frac{\partial \Phi^h}{\partial V_{ik}^h} \quad (30)$$

because of the assumed perfect substitutability of contributions. Expression (30) should be compared with expression (8).

Expression (29) is a fundamental result which relates the welfare losses from the underproduction of public goods to the welfare losses from free riding and to the scale of the economy.

The relationship between the welfare losses from the underproduction of public goods and the existence of free riding is formalised in the following proposition II.

Proposition II The welfare losses from the underproduction of public goods are greater in an economy without free riding than in an economy with free riding.

Proof: Assume that there is no free riding. In this case all households contribute all factors to all public goods and therefore expression (25) holds as an equality. It follows that

$$\Delta = 0 \text{ and } d\bar{B} = -(H - I) \sum_h \sum_i \sum_k w_i dv_{ik}^h$$

The aggregate welfare gain from a small reallocation of factors from the production of private goods to the public sector reaches a maximum. If there is free riding then expression (25) holds as a strict inequality for at least some h, i and k . In this case $\Delta > 0$ and the welfare gain from the same small reallocation of factors from private to public goods is reduced

Q.E.D.

Proposition II is important because [see proposition I and the recent contributions by Andreoni (*op.cit.*) and Fries et alii (*op.cit.*)] there are good reasons to believe that free riding is an important property of models of the voluntary production of public goods. Its rationale is clear: if one or more households do not contribute one or more factors to public goods in the Cournot- Nash equilibrium, they value these factors in the production of public goods less than the prices of these factors (which are determined in private markets). These noncontributing households therefore gain less from the small increase in the contributions of contributing households.

As explained already in the introduction, one of our main aims in this paper is to put forward a rule for Pareto improving reallocations of factors within the public goods sector alone. From expression (29) it is clear that if there is no free riding and therefore Δ equal to zero there is no scope for such Pareto improving reallocations. However if there is free riding the allocation of resources within the public goods sector is inefficient. This is a source of inefficiency which apparently has been completely overlooked. It follows because the household ignores in his decision to contribute or not to contribute the sum of the utility changes of other households. The sum of the marginal utilities of noncontributing households may be greater for a public good to which a given household decides not to contribute than the corresponding sum of marginal utilities for another public good to which the given household decides to contribute. Clearly we have here a source of inefficiency in Cournot-Nash equilibria which is quite distinct from the inefficiency caused by the underproduction of public goods in general. At the bottom of it are what may be called popularity reversals as between public goods and households. The marginal valuation of household H of public good k may be greater than the marginal valuation of public good k-1 but all the other households taken together may value public good k-1 more than public good k.

More generally, whenever the social demand shadow price of a factor i : $\sum_h \frac{\partial E^h(\cdot)}{\partial V_{ik}}$ differs between any two public goods (i.e. not all households contribute to the two public goods) it is clear that small Pareto improving reallocations of the factors contributed by a household to the said two public goods exist. This follows directly from expression (29). This result is formalised as proposition III.

Proposition III: Assume that there exists at least one household which contributes at least one factor i to at least two public goods (e and f) and that not all households contribute to the public goods e and f. Assume further that the social demand shadow prices of the public goods e and f are not equal. Then there exist small Pareto improving reallocations of the contributions of at least one household. The reallocation has to be such that contributions to more popular public goods increase and contributions to less popular goods decrease. Public good e is more popular than public good f if:

$$\sum_h \frac{\partial E^h(\cdot)}{\partial \tilde{V}_{ie}^h} < \sum_h \frac{\partial E^h(\cdot)}{\partial \tilde{V}_{if}^h}$$

Proposition III follows directly from expression (29).

Q.E.D.

Proposition III defines the popularity of public goods in terms of their respective social demand shadow prices. If there is free riding the social demand shadow prices are, except by coincidence, not equalised between public goods. Therefore proposition III is rather general (it hinges only on the connectedness of the economy in the sense that at least one household contributes one and the same factor to more than one public good). It is however not readily applicable because social demand shadow prices are not directly observable. Another approach is to define popularity in terms of the number of actual contributors. If connectedness is satisfied it can easily be shown that Pareto improvements are achievable if the contribution of a household to a public good e (to which more than half the households contribute) increase and (at the same time) contributions to a public good f (to which less than half the households contribute) decrease.

To conclude Part III the scale of the economy H is related to the under-production of public goods and free riding as reflected in Δ [see expression (29)].

To understand the key issue the following thought experiment is helpful. Assume that in a more populous economy the same value of factors is reallocated from the production of private goods to the production of public goods as in a less popular economy.

Clearly in the more populous economy the additional households may substitute public goods for private goods. Keeping their utility levels unchanged their consumption of private goods may be decreased as factors are reallocated from the production of private goods to the production of public goods. This decrease in their consumption of private goods can be brought about through lump sum taxation. The proceeds of these lump sum taxes may then be transferred to the remaining households. In the less populous economy there is clearly no scope for the achievement of such additional welfare gains. This is reflected in expression (29) in the coefficient (H-1). What

is of great interest in the comparison of more and less populous economies is that subject to the assumptions stated in e.g. Fries et alii (1990) the more populous economy will be characterised by more free riding. This entails that as H rises Δ in general rises too. We therefore have established a relationship between the scale of the economy, the underproduction of public goods and free riding.

Proposition IV: Public goods are more underproduced the more populous the economy. If an increase in the scale of the economy is associated with an increase in free riding [see e.g. Fries et alii (1990)] the increase in the underproduction of public goods as a result of an increase in the scale of the economy is less than in proportion to the increase in $(H-I)$, where H is the number of households.

Proposition IV again follows directly from expression (29).

Q.E.D.

It should be remembered that the extent of free riding is not only dependent upon the scale of the economy but also the degree of heterogeneity of households (see e.g. proposition I).

Propositions II, III and IV to the best of our knowledge have no equivalents in the received literature. As mentioned before it is extremely surprising that the normative implications of free riding especially in relation to the well known underproduction of public goods have not been explored. Maybe this is due to the fact that the issue of free riding has been mainly discussed if public goods are produced by the government and financed by taxation. If the latter is the case it is obvious that the issue of free riding assumes quite a different meaning and dimension.

Two observations now seem appropriate to put the results of propositions II to IV into perspective.

Firstly, it has been assumed throughout the analysis that lump sum taxes and transfers are feasible between all households. This assumption could be relaxed and the results contained in propositions II to IV restated. Without lump sum transfers at most $H-I$ households may lose from an increase in the production of public goods because the price changes may have immiserisation effects. The latter may dominate the welfare increasing externality effects. What is of considerable interest in this context is that as the scale of the economy becomes larger externality effects are bound to dominate price

effects because the former depend on the number of households. Price effects can, of course, be ruled out by the small country assumption.

Secondly, all our results are relevant to the invariance property. As is well known, the invariance property cannot hold globally if there is free riding.

Finally the reader's attention is drawn to the fact that expression (29) contains also a useful measure of the extent of the public good nature of the goods $k=1, \dots, m$. Assume that for each household h , public good k and factor i , $\frac{\partial E^h(\cdot)}{\partial V_{ik}} \approx 0$. It then follows at once that $d\bar{B} \approx 0$. Then the Cournot-Nash equilibrium of the voluntary production of public goods is approximately (almost) Pareto efficient.

Part IV

Free Riding in Very Large Economies

Andreoni (1988) and Fries et alii (1990) recently proved important theorems which yield new insights into the occurrence of free riding as the scale of the economy tends to infinity. Subject to standard assumptions and that there is only one public good which is normal in demand they prove that whatever the degree of heterogeneity of households in preferences and endowments, only one household type will contribute to the public good as the scale of the economy tends to infinity. Free riding will be rampant in very large economies.

Since the occurrence of free riding is of crucial relevance to us because we are concerned with the normative analysis of free riding per se and in relation to the underproduction of public goods (in a Cournot-Nash equilibrium) a generalisation of the said theorem is now sketched and later on related to proposition I.

Assume H different household types. Each type has N members. There are any finite number of private goods and factors of production but there is only one public good. The degree of heterogeneity of households is such

that for some N (scale of the economy) all households contribute all factors to the one public good.

The initial equilibrium is characterised by the following set of equations:

$$E^1 \left[\underline{p}, \underline{w}, N \sum_h \underline{v}^h - \underline{v}^1, u^1 \right] = \underline{w} \bar{v}^1 \quad (31)$$

$$E^2 \left[\underline{p}, \underline{w}, N \sum_h \underline{v}^h - \underline{v}^2, u^2 \right] = \underline{w} \bar{v}^2 \quad (32)$$

$$E^H \left[\underline{p}, \underline{w}, N \sum_h \underline{v}^h - \underline{v}^H, u^H \right] = \underline{w} \bar{v}^H \quad (33)$$

$$E_{\underline{w}}^1 \left[\underline{p}, \underline{w}, N \sum_h \underline{v}^h - \underline{v}^1, u^1 \right] = \underline{v}^1 \quad (34)$$

$$E_{\underline{w}}^2 \left[\underline{p}, \underline{w}, N \sum_h \underline{v}^h - \underline{v}^2, u^2 \right] = \underline{v}^2 \quad (35)$$

$$E_{\underline{w}}^H \left[\underline{p}, \underline{w}, N \sum_h \underline{v}^h - \underline{v}^H, u^H \right] = \underline{v}^H \quad (36)$$

where the set of equations (31) to (33) state the expenditure income equalities for the H households and the equations (34) to (36) the Hicksian contribution functions for the factors to the public good. They are derived from the expenditure (minimum cost) functions of each household [see expression (23a)]. The equations (31) to (36) determine for given \underline{p} and \underline{w} the utility level, u^1, \dots, u^H and the vectors of factor contributions $\underline{v}^1, \dots, \underline{v}^H$. It is assumed that all households are endowed with all factors and that their preferences are such that they contribute all factors to the public good.

The key assumption which has to be made and which is derived from more primitive assumptions in Andreoni (op.cit.) and Fries et alii (op.cit.) is that as the scale of the economy, i.e.: N , tends to infinity, the output of the (one) public good tends to a finite amount. Subject to this assumption it can be proven that only one household type will contribute to the public good as $N \rightarrow \infty$.

If the output of the public good tends to a finite amount as $N \rightarrow \infty$ it

follows at once that $\underline{v}^1, \underline{v}^2, \dots, \underline{v}^H \rightarrow 0$ as $N \rightarrow \infty$. Therefore:

$$\begin{aligned} \lim_{N \rightarrow \infty} : E^1 \left[\underline{p}, \underline{w}, N \sum_h \underline{v}^h - \underline{v}^1, u^1 \right] &= E^1 \left[\underline{p}, \underline{w}, N \sum_h v^h, u^1 \right] \\ \lim_{N \rightarrow \infty} : E_{\underline{w}}^1 \left[\underline{p}, \underline{w}, N \sum_h \underline{v}^h - \underline{v}^1, u^1 \right] &= E_{\underline{w}}^1 \left[\underline{p}, \underline{w}, N \sum_h v^h, u^1 \right] \end{aligned}$$

also as

$$N \rightarrow \infty : E_w^1 \left[\underline{p}, \underline{w}, N \sum_h v^h, u^1 \right] = 0$$

Analogous results hold for all the other households.

Now assume that as $N \rightarrow \infty$ the set of equations (31) to (36) continue to hold as interior solutions and therefore equations. This leads to a contradiction. In the limit there are only the following variables: u^1, \dots, u^H and $N \sum_h \underline{v}^h$. Hence not all the equations (31) to (36) can be satisfied. From part II and the proof of proposition I it is known that in a Cournot-Nash equilibrium without rationing two or more contributing households contribute the same set of factors. It follows that if the public good requires as inputs all the factors of production that only one household makes contributions to the public good as $N \rightarrow \infty$.

Though this result is of considerable interest, because it isolates the effects of scale on the specialisation pattern of contributions it has two limitations. Firstly in a world with many different public goods (subject to the same assumptions) it can readily be shown that as $N \rightarrow \infty$ there remain as many contributing households as there are public goods. Secondly, changes in the scale of the economy may be associated with simultaneous changes in the heterogeneity of households (i.e.: the household structure of the economy).

More general conditions under which an increase in the number of households is associated with an increase in free riding are readily derivable from an extension of proposition I, taking also into account changes in the prices of private goods and factors. If the number of households is increased, clearly the set Ω cannot become larger. The sets S^1, \dots, S^m may change because a change in the number of households entails a change in the equilibrium product and factor prices. If changes in the latter are small and the sets of factors required in the production of the various public goods do not change (or change very little) it follows at once that an increase in the number of households cannot reduce the extent of free riding.

Conclusions

Though free riding is one of the hallmarks of models of voluntary production of pure public goods its normative implications especially in relation to the underproduction of public goods and the scale of the economy have not been explored. To fill this amazing gap in the received literature is the key purpose of the present paper.

The principal result is formalised in expression (29) which relates the efficiency losses from the underproduction of public goods to the existence (or lack) of free riding and the scale of the economy. The recent literature sheds new light on the relationship between free riding and the scale of the economy [see Andreoni (1988, op.cit.) and Fries et alii (1990, op.cit.)]. Expression (29) and propositions II to IV (which are based on it) may therefore be regarded as the natural normative complements of the recent positive results on free riding not only in relation to the scale of the economy but also in relation to the underproduction of public goods.

The welfare losses from the underproduction of public goods for a given number of households as well as the comparative welfare losses in more or less populous economies are crucially affected by the existence or absence of free riding (i.e.: specialisation in contributions to public goods as between households). These important issues can only be analysed in a model with many public goods, private goods and factors of production. Making use of basic duality theory it is straightforward to construct and to interpret a very general model of the voluntary production of public goods (see part I).

If more than one public good is allowed for it can be shown that the received literature overlooked a potentially important source of inefficiency: there may be significant welfare gains from a reallocation of factors within the public goods sector alone if there is free riding. This is the subject matter of proposition III.

The main focus is on the normative implications of free riding; however a novel proposition relating the occurrence of free riding to the degree of heterogeneity of households is also put forward (see proposition I). This proposition is related to results contained in the recent literature on free riding in very large economies (see part IV).

Finally it must be emphasized that there are many possible applications of the general model of the voluntary production of public goods (put forward in this paper) which have not been touched upon. The real world may be

characterised by many production or consumption distortions of the private sector such as, for example, involuntary unemployment of labour or imperfect competition. The production model with many private goods and factors opens the door to these applications. Something similar holds with respect to the incorporation of produced public intermediate goods.

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