

Albert, Max; Meckl, Jürgen

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## Expectations and adjustment dynamics in a two-sector model of a small open economy

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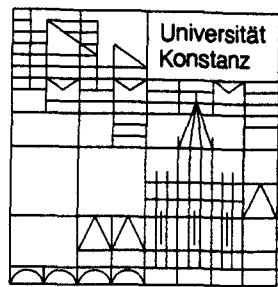
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# **Sonderforschungsbereich 178** **„Internationalisierung der Wirtschaft“**

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Max Albert  
Jürgen Meckl

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in a Two-Sector Model  
of a Small Open Economy**

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Postfach 5560  
D-7750 Konstanz

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**EXPECTATIONS AND ADJUSTMENT DYNAMICS IN A  
TWO-SECTOR MODEL OF A SMALL OPEN ECONOMY**

**Max Albert**

**Jürgen Meckl**

**Serie II - Nr. 132**

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# Expectations and Adjustment Dynamics in a Two-Sector Model of a Small Open Economy

Max Albert and Jürgen Meckl  
University of Konstanz/SFB 178\*

*This paper generalizes the analysis in Albert (1989) and Meckl (1990) of adjustment in a dynamic specific-factors model with endogenous capital stocks. Capital reallocation and accumulation are consequences of investment decisions and depreciation. The investment process is analyzed under a spectrum of expectations which include rational, adaptive, and static expectations as special cases. Only with rational expectations the speed of adjustment is optimal; adjustment is too fast if expectations are biased towards the status quo, and too slow if expectations are biased towards long-run values. Local and global analysis of the adjustment path show sharp qualitative differences. Non-monotonic adjustment of capital stocks and the social product is possible and is not necessarily the result of expectational errors.*

## 1 Introduction

Intertemporal optimizing models emphasizing the role of investment dynamics for adjustment to external shocks have gained considerable interest in trade theory. Mussa (1978) was the first to introduce investment theory in a general equilibrium model of capital reallocation, thereby overcoming the traditional dichotomy of capital reallocation and accumulation characteristic of the specific-factors (SF), the Heckscher-Ohlin (HO) and the Oniki-Uzawa (OU) model. More recent

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contributions by Brock (1988), Murphy (1989,1990), and Sen and Turnovsky (1989a,b) discuss adjustment to changes in relative prices, taxes, interest rates, and investment subsidies. These models analyze adjustment in a small open economy applying the adjustment-cost framework of Hayashi (1982) and Abel and Blanchard (1983) based on the dynamic theory of the firm. Investors and consumers make optimal choices under static or rational expectations and with an infinite time horizon. Persson and Svensson (1985) and Matsuyama (1988) consider investment dynamics and the adjustment process in an overlapping-generations setting. But whenever more than one sector is considered, it is assumed in all the above models that capital is intersectorally mobile in the physical sense. Thus the models are subject to Neary's (1978) critique that in practice investment in most cases is investment in sector-specific capital stocks which cannot be physically reallocated once installed. From this point of view it is impossible to separate capital mobility and investment; mobility is just a metaphor standing for investment and depreciation processes that involve no physical mobility at all.

This point is recognized by Murphy (1988) who analyzes adjustment in a dynamic two-sector SF model with a non-traded good. He focuses on the dynamics of capital stocks, current account, and real exchange rate. However, due to mathematical complications resulting from the Hayashi framework, he cannot solve his model analytically; hence he simulates local dynamics numerically. This means a severe loss of information on the properties of adjustment dynamics.

The present paper discusses adjustment in a simpler dynamic version of the SF model introduced by Albert (1989). The model is tractable analytically since the specification of adjustment costs is much simpler than in Hayashi (1982). It is a two-sector model which allows only for sector-specific capital while labor is perfectly mobile. The temporary equilibrium allocation is described by the static equilibrium of the SF model. The model is that of a small open economy facing constant prices and a constant rate of interest. The economy produces a consumer good and an investment good both of which are traded. Capital accumulation requires the investment good as input and proceeds under convex costs of adjustment. The focus here is on analyzing adjustment dynamics; a complete treatment of comparative-static results is provided by Meckl (1990).

Albert (1989) proves global stability of the adjustment process under static expectations (SE). The present paper extends the analysis to a spectrum of expectation regimes which is defined as a generalization of adaptive expectations

(AE). This spectrum of expectations includes rational (RE), static, and long-run (LRE)<sup>1</sup> expectations as special cases. There is a definite advantage in the analysis of such a broad spectrum. The usual assumptions of SE or RE are both rather unrealistic; in many cases it seems more reasonable to assume that agents correctly anticipate the sign, positive or negative, of a change in prices but not the actual extent or absolute value of the change. For a theoretical analysis it is of course important to set limits to deviations from RE, thereby defining a reasonable spectrum of expectations. In order to find such a spectrum we first consider AE which fulfill our requirements. If AE are specified such that long-run consistency is guaranteed, the expected present value of one unit of capital,  $\lambda^{AE}$ , is a weighted average of the expected present values under SE and LRE,  $\lambda^{SE}$  and  $\lambda^{LRE}$ . The spectrum we consider contains all kinds of expectation formation for which this is also true, i.e. for which  $\lambda^{AE} \in [\lambda^{SE}, \lambda^{LRE}]$ . It is one of our main results that with appropriate simplifications RE fulfill this condition.

Our results generalize Mussa's (1978) findings. He discovered, in a model of physical capital reallocation, that adjustment under SE is too fast while adjustment under RE is socially optimal. The present paper generalizes the analysis to a two-sector model of investment in sector-specific capital and to a range of expectations: RE yield optimal adjustment while a conservative bias, i.e. a bias in the direction of SE, makes adjustment too fast. Expectations biased in the direction of LRE, on the other hand, make adjustment too slow.

Further results are due to the fact that we not only prove global stability of the adjustment process but also analyze properties of the adjustment paths. We show that the global analysis of adjustment differs substantially from local analysis. Great care should be taken in deriving properties of dynamic adjustment solely from local analysis as it is done in most of the literature cited above. Thus the following result is typical for multi-sector models: adjustment will in many cases be accompanied by non-monotonic adjustment of endogenous variables. In the present model we have monotonicity of the wage rate but non-monotonicity of capital stocks. If one only looks at the local analysis, this seems to be impossible, at least for rational expectations. Global analysis shows, however, that if the economy is rather far from the steady state adjustment will be non-monotonic in many cases, depending on the starting point of the system. As the case of

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<sup>1</sup>We adopt the term 'long-run expectations' from Mussa (1976, p. 164).

rational expectations shows, non-monotonic behavior is optimal in these cases.

The paper is organized as follows. Section 2 develops the basic model. Global stability under a spectrum of expectations is analyzed in section 3. Section 4 is concerned with properties of global and local adjustment dynamics. Section 5 offers some concluding remarks. An appendix contains proofs needed for local analysis.

## 2 The Model

### Firm Behavior

The center piece of our two-sector model is the dynamic model of a competitive firm representing one sector of the economy. The firm is a price taker on all relevant markets and operates under convex costs of adjustment. This model of the firm is well-known from the literature (see Treadway 1969, Söderström 1976). The following recapitulation, on the one hand, serves to make the present paper self-contained; on the other hand, it prepares the ground for a thorough discussion of different hypotheses on expectation formation in a two-sector context.

It is assumed that at each point  $t$  in time the firm tries to maximize the expected present value of net returns  $NR$  over an infinite horizon. The relevant maximization problem is given in (1); indices pertaining to the sector are dropped at this stage.

$$\begin{aligned} \max_{I(s), L(s)} & \int_t^\infty NR(s) \cdot e^{-i \cdot (s-t)} \, ds & (1) \\ \text{s.t.} & NR(s) := p \cdot x - w^E(s) \cdot L(s) - p_I \cdot h(I(s)) \\ & x = f(K(s), L(s)) \\ & \dot{K}(s) = I(s) - \delta \cdot K(s), \quad I(s) \geq 0 \\ & K(t) = K(0) + \int_0^t \dot{K}(s) \, ds. \end{aligned}$$

$i$  is the interest rate at which the firm can borrow or lend financial capital;  $x$  is the firm's output and  $f$  is a linearly homogeneous production function with arguments capital ( $K$ ) and labor ( $L$ );  $w^E$  is the expected wage rate. With an optimal choice of  $L$  we may write  $p \cdot x - w^E \cdot L$  as  $r^E \cdot K$  where  $r^E$  is the expected marginal value product (rental rate) of capital determined by the expected wage rate. The price of the investment good is given by  $p_I$ .  $h$  is the amount of investment goods

required for the realization of gross investment  $I$ ;  $h$  is strictly convex with<sup>2</sup>

$$h(0) = 0, \quad h'(0) = 0, \quad h'(\infty) = \infty.$$

$\delta$  is a constant rate of depreciation. (1) is to be solved for different kinds of expectations.

We first look at AE. In a discrete-time model the simplest form of AE is given by:

$$r^E(s + \tau) = r(s) + a_\tau \cdot [r(s) - r(s - \tau)]; \quad 0 \leq a_\tau \leq 1 \quad (2)$$

where  $\tau$  is the time interval. This can be written with the help of the forward difference operator  $D_\tau$ , defined by  $D_\tau x(s) := x(s + \tau) - x(s)$ , as

$$D_\tau r^E(s) = a_\tau \cdot D_\tau r(s - \tau).$$

Taking second differences yields

$$D_\tau^2 r^E(s) = (a_\tau - 1) \cdot a_\tau \cdot D_\tau r(s - \tau).$$

If one takes this to the limit for  $\tau$  towards zero the result is

$$\ddot{r}^E(s) = -\beta \cdot \dot{r}(s), \quad 0 \leq \beta \leq \infty.$$

The solution of this second-order differential equation is

$$r^E(s) = [r(t) - r^*] \cdot e^{-\beta \cdot (s-t)} + r^*, \quad s \geq t \quad (3)$$

where  $r^*$  is the long-run value of  $r$ . Except when explicitly stated otherwise, we will assume  $r^*$  to be the actual steady-state value of  $r$  in the complete two-sector model;  $r^*$  will be unique. This assumption, a kind of long-run rationality of expectations, is required for long-run consistency of AE.

We now determine investment under AE. In order to do that we first derive the first-order conditions of (1) which are valid for all kinds of expectations. The current-value Hamiltonian of the problem is given by

$$H(K, I, \lambda, \lambda_0) = \lambda_0 \cdot [r^E \cdot K - p_I \cdot h(I)] + \lambda [I - \delta \cdot K], \quad (4)$$

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<sup>2</sup>Under our assumptions, the non-negativity constraint for gross investment will actually never be active; this will become apparent when we solve the problem.

where  $\lambda_0$  is a constant which might be zero. The necessary condition for the choice of  $I$  maximizing the Hamiltonian is

$$\lambda_0 \cdot p_I \cdot h'(I^{opt}) = \lambda. \quad (5)$$

Inserting the optimal value  $I^{opt}(\lambda_0, \lambda(t))$  into the Hamiltonian yields the maximized Hamiltonian  $H^{max}(K, \lambda, \lambda_0)$ . From  $H^{max}$  we get the necessary condition for the behavior of  $\lambda$ :

$$\begin{aligned} \dot{\lambda} &= i \cdot \lambda - \partial H^{max}(K, \lambda, \lambda_0) / \partial K \\ &= (i + \delta) \cdot \lambda - \lambda_0 \cdot r^E. \end{aligned} \quad (6)$$

The necessary conditions in the case of infinite-horizon problems furthermore state that the vector  $[\lambda_0, \lambda(t)]$  must never vanish<sup>3</sup>, i.e. if  $\lambda_0$  equals zero,  $\lambda(t)$  must not become zero (even in the limit), and if  $\lambda(t)$  either actually becomes zero or goes to zero in the limit,  $\lambda_0$  cannot be zero.  $\lambda_0 = 0$  corresponds to the case of an infinite investment demand since otherwise the necessary optimality condition (5) would be violated. This case can be ruled out because infinite gross investment makes the value of  $r \cdot K - p_I \cdot h(I)$  negative for all times which is clearly suboptimal. Since  $\lambda_0 = 0$  is ruled out, it is possible to set  $\lambda_0 = 1$  as usual.

Solving (6) for  $\lambda(t)$  gives

$$\lambda(t) = A \cdot e^{(i+\delta) \cdot t} + \int_t^\infty r^E(s) \cdot e^{-(i+\delta) \cdot (s-t)} ds.$$

The condition

$$\lim_{t \rightarrow \infty} e^{-i \cdot t} \cdot \lambda(t) = 0 \quad (7)$$

is sufficient for stability. We now use the following theorem<sup>4</sup>: If  $H^{max}$  is concave in  $K$  and the solution  $\bar{\lambda}(t), \bar{K}(t)$  satisfies the first-order conditions and the following transversality condition

$$\lim_{t \rightarrow \infty} e^{-i \cdot t} \cdot \bar{\lambda}(t) \cdot [K(t) - \bar{K}(t)] \geq 0 \quad (8)$$

holds for all admissible  $K(t)$ , then the solution is optimal. The transversality condition (7) is sufficient for stability and therefore for the validity of (8). Since

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<sup>3</sup>Cf. Feichtinger and Hartl (1986), ch. 2, esp. pp. 39–44.

<sup>4</sup>Cf. Feichtinger and Hartl (1986), pp. 42–43.

$H$  is linear in  $K$ ,  $H^{\max}$  is concave in  $K$  and (7) is sufficient for optimality. Hence the optimal solution of (6) is

$$\lambda(t) = \int_t^\infty r^E(s) \cdot e^{-(i+\delta) \cdot (s-t)} ds. \quad (9)$$

(9) shows that  $\lambda$  is the expected present value of one unit of capital. Combining (9) and (2) gives the expected present value of capital under AE:

$$\lambda^{AE}(t) = \theta \cdot \frac{r(t)}{i + \delta} + (1 - \theta) \cdot \frac{r^*}{i + \delta}, \quad (10)$$

where the constant  $\theta$  is determined by  $\beta$ :

$$\theta = \frac{i + \delta}{i + \delta + \beta}. \quad (11)$$

$\theta$  can take on any value in the interval  $[0, 1]$  by appropriate choice of  $\beta$ . With  $\theta = 1$  we have the special case of SE where the firm expects the rental rate  $r$  to stay at its present value.  $\theta = 0$  yields the case of LRE where the firm expects the rental rate to jump to its steady-state value. Because of (5) investment under AE is a weighted average of SE and LRE investment.

Up to now we assumed that AE refer to  $r(s)$  and not to the wage rate. If we alternatively specify AE with respect to  $w(s)$ , the expected present value of capital is still given by an equation like (10) but  $\theta$  becomes time dependent. The difference between the two possibilities in defining AE becomes relevant only in a two-sector context. AE defined with respect to  $r(s)$  in both sectors will in general imply sector-specific expectations with respect to the wage rate. This has, however, no further consequences in the present model because there are no future markets. For our purposes the whole question is of no account. As long as AE imply that the expected present value of capital is at each point in time a weighted average of the present values expected under SE and LRE, our analysis will be valid. For this reason we can go one step further and consider a more general hypothesis: we analyze all kinds of expectation formation which result in (10) and an arbitrary time path  $\theta(s) \in [0, 1]$ . This definition covers AE, SE, and LRE; we later prove that in our model it also covers RE.

Let us shortly state the consequences of the above analysis for the resulting differential equation. With  $\theta = 1$ , we get SE investment as

$$\lambda^{SE}(t) = \frac{r(t)}{p_I \cdot (i + \delta)} = h'(I^{SE}(t)).$$

The differential equation to be used in the two-sector model is

$$\dot{K}(t) = I^{SE}(t) - \delta \cdot K(t). \quad (12)$$

With  $\theta = 0$ , we get LRE investment as

$$\lambda^{LRE} = \frac{r^*}{p_I \cdot (i + \delta)} = h'(I^{LRE})$$

. Note that  $I^{LRE}$  is time-independent; the analogue of (12) is an autonomous linear differential equation.

If the time path of  $\theta(s)$  is arbitrary we know at least that gross investment is bounded by gross investment under SE and LRE; the same goes for net investment  $\dot{K}$ . RE imply that the firm correctly anticipates changes in the wage rate and in the rental rate of capital. Thus we cannot solve for the firm's investment without knowledge of the time path of  $r$ . The following solution results from (9) and the condition  $r^E = r$ :

$$\begin{aligned} \dot{K} &= I^{RE} - \delta \cdot K & \text{with } p_I \cdot h'(I^{RE}) = \lambda^{RE} \\ \lambda^{RE}(t) &= \int_t^\infty r(s) \cdot e^{-(i+\delta) \cdot (s-t)} ds. \end{aligned} \quad (13)$$

This solution presupposes stability of the complete model or at least convergence of the integral in (13).

## The Two-Sector Model

We complete the two-sector model by an equation

$$L_I + L_C = L \quad (14)$$

describing the assumption that sectoral labor demand adds up to a fixed total labor supply.  $I$  and  $C$  denote the investment good and the consumer good, respectively. Sectoral labor demand  $L_j$  at any given moment depends on the current stock of capital and fulfills the usual optimality condition

$$w = p_j \cdot \partial f_j(K_j, L_j) / \partial L_j \quad j = I, C \quad (15)$$

where  $f_j$  is sector  $j$ 's production function. In the following we set  $p_I \equiv 1$  and  $p_C / p_I := p$ . We can get rid of all equations pertaining to labor reallocation by making use of the social-product function of the SF model which is defined by

$$y(p, L, K_I, K_C) := \max_{L_I, L_C} \{f_I(K_I, L_I) + p \cdot f_C(K_C, L_C) : L_I + L_C = L\}. \quad (16)$$

This function describes the result of the labor reallocation process at every point in time; it is strictly concave in  $(K_I, K_C)$  and yields the rental rates of capital as  $\partial y / \partial K_j = r_j$ .

As noted before, the rate of interest  $i$  as well as  $p$  are exogenous, i.e. if they change at all this is not anticipated even under RE. We furthermore assume rates of depreciation in both sectors to be identical:  $\delta_I = \delta_C = \delta$ .

The dynamic system is given by:

$$\begin{aligned}\dot{K}_I &= I_I^{opt} - \delta \cdot K_I && \text{with } h'_I(I_I^{opt}) = \lambda_I \\ \lambda_I &= \int_t^\infty r_I^E(s) \cdot e^{-(i+\delta) \cdot (s-t)} ds \\ \dot{K}_C &= I_C^{opt} - \delta \cdot K_C && \text{with } h'_C(I_C^{opt}) = \lambda_C \\ \lambda_C &= \int_t^\infty r_C^E(s) \cdot e^{-(i+\delta) \cdot (s-t)} ds.\end{aligned}\tag{17}$$

### 3 Stability of the Adjustment Process

#### Uniqueness of the Steady State

As shown in Albert (1989), the steady state of the model can be described by the following maximization problem

$$g(p, L, i) = \max_{K_I, K_C} \left\{ y(p, L, K_I, K_C) - \sum_{j=I, C} \frac{i + \delta}{\delta} \cdot h_j(\delta \cdot K_j) \right\}\tag{18}$$

the first-order conditions of which are identical with the steady-state conditions of the dynamic system (17). The dynamic systems resulting from SE, LRE and RE show identical steady states since in the steady state SE and LRE are rational. The maximand in (18) is strictly concave in  $(K_I, K_C)$  due to the strict concavity of  $y$  in  $(K_I, K_C)$  and the strict convexity of the adjustment-cost functions. Thus the solution of (18) and therefore the steady state exists and is unique. We now consider stability.

#### Stability under Static Expectations

In the case of SE, we prove global stability by using the maximand of (18) as a Liapunov function. The time derivative of this function is given by

$$[r_I - (i + \delta) \cdot h'_I(\delta \cdot K_I)] \cdot \dot{K}_I + [r_C - (i + \delta) \cdot h'_C(\delta \cdot K_C)] \cdot \dot{K}_C.\tag{19}$$

As can be seen from the SE form of the differential equations, (19) is strictly positive everywhere except for the steady state where it is zero. Therefore (18) will always converge to its maximum under the dynamic system, where the maximum is identical with the unique steady state. This shows that the system is globally stable.

### Stability under Long-Run Expectations

With  $I_j = h_j'^{-1}(\lambda_j^{LRE}) = h_j'^{-1}(r_j^*/(i + \delta))$  we set investment constant and equal to its steady-state rate. Thus adjustment to the steady state is globally stable. Integration of the equations of motion yields

$$K_j(t) = [K_j(0) - K_j^*] \cdot e^{-\delta \cdot t} + K_j^* \quad (20)$$

where an asterisk denotes steady-state values. Differentiating (20) shows that the system moves on a straight line with slope

$$\frac{dK_I}{dK_c} = \frac{K_I(0) - K_I^*}{K_c(0) - K_c^*}.$$

### Stability under Adaptive Expectations

Stability under AE is proved by reference to stability under SE and LRE. This follows from the fact, noted before, that net investment  $\dot{K}$  under AE is a weighted average of net investment under SE and LRE. By the same argument global stability follows for all kinds of expectation formation with  $\lambda(t) \in [\lambda^{SE}(t), \lambda^{LRE}(t)]$ . In the following we refer to these kinds of expectations as "expectations between SE and LRE".

### Wage-Rate Adjustment under Static and under Long-run expectations

Our proof of stability under RE will make use of the monotonicity of the wage rate under SE and LRE adjustment. In order to prove monotonicity we first consider the iso-wage lines in the  $(K_I, K_c)$  plane. Given the wage rate, the capital intensities in both sectors are fixed because goods prices are given. Therefore an iso-wage line has the form

$$K_c = k_c(w) \cdot L - [k_c(w)/k_I(w)] \cdot K_I$$

where  $k_j$  denotes the capital intensity in sector  $j$  which is determined by  $w$ . Lines corresponding to higher wages lie more outward in the plane since the wage rises with capital stocks.

We now consider the line where the wage rate is equal to its steady-state value. On that line rental rates of capital are also equal to their respective long-run values. The LRE path which starts on this line coincides with this line since the path is a line. The SE path coincides with the LRE path because initial values of investment coincide and do not change during adjustment since the wage rate does not change. Thus SE prove to be rational and this path is also a RE path. The plane accordingly is divided into three sections (see fig. 1):

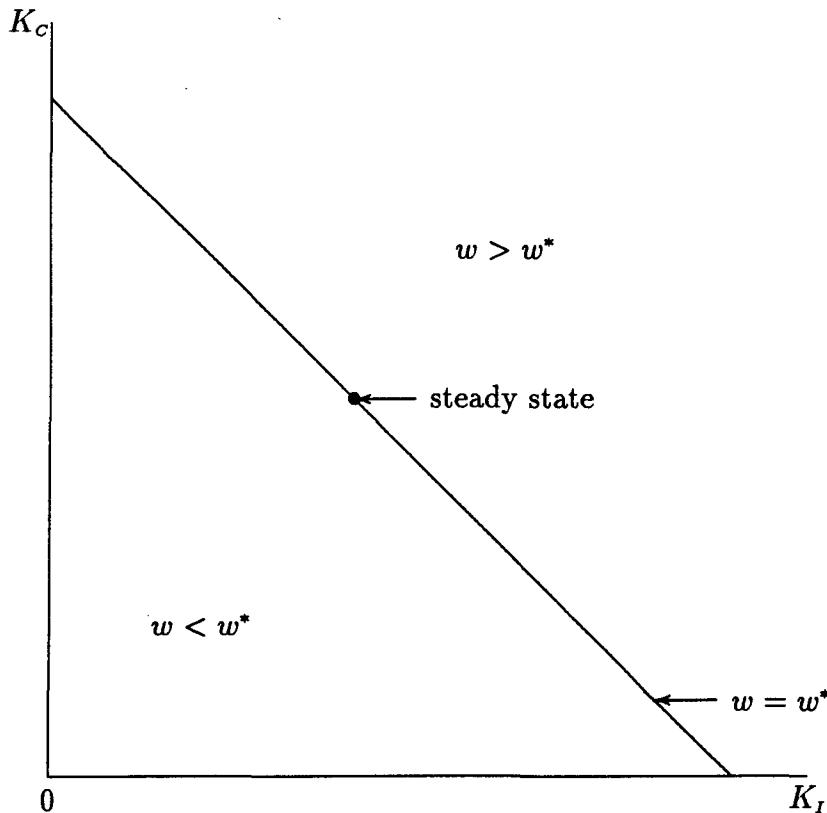


Figure 1: Long-Run Iso-Wage Line.

- I. The long-run iso-wage line where all modes of adjustment considered here coincide.
- II. The section below the long-run iso-wage line where the wage rate is below its long-run value.

### III. The section above the long-run iso-wage line where the wage rate is higher than in the steady state.

For any point in the plane but not on the long-run wage line the SE and the LRE gradients, i.e. the vectors of net investment, will have different directions. Whenever expectations are between SE and LRE, net investment in each sector lies strictly between SE and LRE investment. Therefore the gradient resulting from expectations in the spectrum between SE and LRE is always in the convex cone of the SE and LRE gradients. As stated above, it is obvious that all adjustment processes resulting from this expectational spectrum are globally stable; this is established by reference to the global stability of the extremes. The non-trivial point is to establish that RE belong to this spectrum. In order to prove this we first demonstrate strict monotonicity of the wage rate under both LRE and SE adjustment. Strict monotonicity of the wage rate under LRE is obvious since the adjustment path as well as the iso-wage curves are linear; slopes coincide only for the long-run iso-wage line which is excluded from the present considerations. Consequently the LRE path is never tangential to an iso-wage line.

Strict monotonicity of the wage rate for SE adjustment is to be expected: if the path was ever tangential to an iso-wage line, it would stay on that line since, whenever the wage-rate is fixed, the path becomes linear due to identical rates of depreciation. This can be proved by differentiating  $dK_I/dK_C = [I_I - \delta \cdot K_I] / [I_C - \delta \cdot K_C]$  with respect to time:

$$\left( \frac{d\dot{K}_I}{dK_C} \right) = \left( \frac{\dot{I}_I - \delta \cdot \dot{K}_I}{\dot{K}_I} - \frac{\dot{I}_C - \delta \cdot \dot{K}_C}{\dot{K}_C} \right) \cdot \frac{dK_I}{dK_C}.$$

Whenever the direction of movement is tangential to an iso-wage line the time derivative of the wage rate is zero. Under these circumstances the time derivative of  $dK_I/dK_C$ , i.e. the change in the direction of movement, is also zero since  $\dot{I}_I = 0$ ; the system stays on the iso-wage line. Global stability of the SE path together with the fact that different adjustment paths can have only the steady-state combination  $(K_I^*, K_C^*)$  in common ensures that the wage rate must behave strictly monotonically.

### Stability under Rational Expectations

As a first step in our proof of global stability of the RE system we simplify our task by showing that the respective dynamic system is identical with a system

resulting from an optimal-control problem for the entire economy. It can be shown that the optimal solution of the control problem is the stable path of the system if a stable path exist. In addition we prove the local existence of a stable (and hence optimal) path in a neighborhood of the steady state. The latter proof is given in the appendix since it is only supplementary to the analysis.

Given the existence of a globally stable and socially optimal path, the initial value of capital's shadow price  $\lambda_c(0)$  is just the present value of a unit of capital if the economy develops along this path. Thus the socially optimal path is an equilibrium path in the sense that it is not rational for competitive firms to deviate from it: if firms expect the economy to develop along this path, they invest according to  $\lambda_c(0)$  thereby holding the economy on the path. As always in this kind of models, the same argument applies to socially non-optimal paths if they are expected to be chosen. There is a conventional element in the assumption that the socially optimal path is chosen from all possible RE paths; the RE approach involves no hypothesis on expectation formation or learning. In the present context, however, this weakness of the approach does not matter: we use RE only to discriminate between effects resulting from expectational errors and effects which result from coping successfully with given constraints.

Optimal adjustment in a model of a small open economy means optimization of the present value of the income stream for all factor owners. This yields the following control problem.

$$\max_{I_j(t)} \int_0^\infty [y(p, L, K_I, K_C) - h_I(I_I) - h_C(I_C)] \cdot e^{-i \cdot t} dt \quad (21)$$

$$\begin{aligned} \text{s.t.} \quad & \dot{K}_j = I_j - \delta_j \cdot K_j \\ & K_j(0) = K_{j0} \quad j = I, C. \end{aligned}$$

The Hamiltonian of (21) is

$$\begin{aligned} H(K_I, K_C, I_I, I_C, \lambda_I, \lambda_C) = & y(p, L, K_I, K_C) - h_I(I_I) - h_C(I_C) \\ & + \lambda_I \cdot (I_I - \delta \cdot K_I) + \lambda_C \cdot (I_C - \delta \cdot K_C). \end{aligned} \quad (22)$$

The necessary conditions are given by (13). By the same theorem already used in section 2 we now prove that stability is a sufficient condition for optimality.

We note that the Hamiltonian  $H$  and therefore the maximized Hamiltonian  $H^{\max}$  is always concave in  $(K_I, K_C)$ . Now consider an admissible trajectory

$\bar{K}(t) := (\bar{K}_I(t), \bar{K}_C(t))$  which fulfills the necessary conditions. If the corresponding  $\bar{\lambda}(t) := (\bar{\lambda}_I(t), \bar{\lambda}_C(t))$  is such that for all the other admissible trajectories  $K(t) := (K_I(t), K_C(t))$  the transversality condition

$$\lim_{t \rightarrow \infty} \bar{\lambda}(t) \cdot [K(t) - \bar{K}(t)] \cdot e^{-i \cdot t} \geq 0. \quad (23)$$

is fulfilled, then  $\bar{I}(t)$  is optimal. Under the concavity condition for  $H^{\max}$ , (23) is a sufficient condition for an optimal solution of (21). The shadow price of capital  $\lambda$  is nonnegative since an exogenous increase in capital never reduces income.  $K(t)$  is of course nonnegative for all  $t$ . If this is the case, then we know that a stable path if it exists is always optimal since on the stable path  $\bar{\lambda} \cdot \bar{K} \cdot e^{-i \cdot t}$  goes to zero.

In the appendix local existence of a stable path is proved. This local result will be used in the following global analysis.

Let us assume for a start that the system is globally stable under RE. Then  $\lambda_j(t)^{\text{RE}}$  as the present value of a unit of capital in sector  $j$  is equal to

$$\int_t^{\infty} r_j(s) \cdot e^{-(i+\delta) \cdot (s-t)} ds \quad (24)$$

as stated in section 2. Assume further that the wage rate  $r_j$ , and with it  $r_j$ , behaves strictly monotonically under RE. Under these circumstances we have the following properties of  $\lambda_j(t)$  in the three sections of the  $(K_I, K_C)$  plane defined above:

I. If the wage rate is equal to its long-run value, we have

$$\frac{r_j(t)}{(i + \delta)} = \lambda_j^{\text{SE}}(t) = \lambda_j^{\text{RE}}(t) = \lambda_j^{\text{LRE}} = \frac{r_j^*}{(i + \delta)}.$$

All adjustment paths coincide.

II. If the wage rate is above its long-run value,  $r_j(t)$  will grow over time. SE are too pessimistic, and we have

$$\lambda_j^{\text{SE}}(t) < \lambda_j^{\text{RE}}(t) < \lambda_j^{\text{LRE}}.$$

III. If the wage rate is below its long-run value,  $r_j(t)$  will fall over time. SE are too optimistic, and we have

$$\lambda_j^{\text{SE}}(t) > \lambda_j^{\text{RE}}(t) > \lambda_j^{\text{LRE}}.$$

Strict inequalities follow from the assumption of strict monotonicity and hold for every finite  $t$ .

This shows that except for region I, where stability is no problem, RE investment in both sectors will be strictly between SE and LRE investment. Given our as yet unproved assumption of strictly monotone behavior of the wage rate, the gradient of RE adjustment will always be in the convex cone of the gradients resulting from SE and LRE.

The crucial point, then, is to establish the monotonicity of the wage rate under RE which we have assumed up to now. Local analysis (cf. appendix) shows that locally a stable path exists. Since locally the equilibrium is a stable node in the stable manifold we know that at least the last part of the path shows a strictly monotone behavior of the wage rate. We now argue backwards from the equilibrium to show that if the last part of the path has this property, the whole path must have it. We prove this by deriving a contradiction from the assumption of non-monotonicity.

Consider the last extremal point of the wage rate before the steady state. At this point the path is tangential to some iso-wage line. After this point of tangency the wage rate behaves monotone on the path. At the point of tangency, however, the strict inequalities above pertaining to the present values of capital units must hold since (i) the wage rate now is different from its long-run value and (ii) will change monotonically. Hence the RE gradient is in the convex cone of the SE and the LRE gradients. Neither the SE nor the LRE path, however, is ever tangential to an iso-wage line if the actual wage differs from its long-run value. Therefore the RE path cannot be tangential to an iso-wage line, contrary to our assumption.

The assumption of non-monotonic behavior of the wage-rate under RE leads to a contradiction. This establishes that the RE path runs between the SE and the LRE path and hence must be globally stable.

## 4 Properties of the Adjustment Process

We analyse the different adjustment paths resulting from alternative assumptions on the investors' expectations using the phase-diagram technique in the  $(K_I, K_c)$  plane. We contrast the results from a purely local analysis with our results on

$\bar{K}(t) := (\bar{K}_I(t), \bar{K}_C(t))$  which fulfills the necessary conditions. If the corresponding  $\bar{\lambda}(t) := (\bar{\lambda}_I(t), \bar{\lambda}_C(t))$  is such that for all the other admissible trajectories  $K(t) := (K_I(t), K_C(t))$  the transversality condition

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## 4 Properties of the Adjustment Process

We analyse the different adjustment paths resulting from alternative assumptions on the investors' expectations using the phase-diagram technique in the  $(K_I, K_c)$  plane. We contrast the results from a purely local analysis with our results on

global behavior. This serves to show that the usual analysis of local behavior may be quite misleading. In many cases a local analysis is used to discuss qualitative properties of adjustment processes. In the present case such an analysis would lead to the conclusion that RE and SE adjustment differs qualitatively in that under RE overshooting of capital stocks does not occur. As the global analysis shows, however, this is not true.

## Static Expectations

Implicit differentiation of the dynamic system

$$\begin{aligned}\dot{K}_I &= I_I^{SE} - \delta \cdot K_I \\ \dot{K}_C &= I_C^{SE} - \delta \cdot K_C.\end{aligned}\tag{25}$$

yields the slopes of the isokines illustrated in fig. 2 as

$$\begin{aligned}\frac{dK_I}{dK_C} \bigg|_{\dot{K}_I=0} &= -\frac{(\partial I_I^{SE}/\partial r_I) \cdot (\partial r_I/\partial K_C)}{(\partial I_I^{SE}/\partial r_I) \cdot (\partial r_I/\partial K_I) - \delta} < 0 \\ \frac{dK_I}{dK_C} \bigg|_{\dot{K}_C=0} &= -\frac{(\partial I_C^{SE}/\partial r_C) \cdot (\partial r_C/\partial K_C) - \delta}{(\partial I_C^{SE}/\partial r_C) \cdot (\partial r_C/\partial K_I)} < 0.\end{aligned}\tag{26}$$

(26) states that both isokines are negatively sloped. Furthermore, the dynamic behavior of the system implies that the  $\dot{K}_C=0$ -curve must be flatter than the  $\dot{K}_I=0$ -curve (cf. appendix). As indicated by the set of directional arrows that illustrate the model's dynamic behavior in fig. 2 the steady state *A* is a stable node.

The trajectories illustrated in fig. 2 indicate that all paths starting from  $(K_I, K_C)$  combinations in regions *a* and *c* converge to the steady state without leaving their respective region; i.e. adjustment of both capital stocks is monotonic. On the other hand, overshooting of one sectors' capital stock is possible for starting points in regions *b* or *d*.

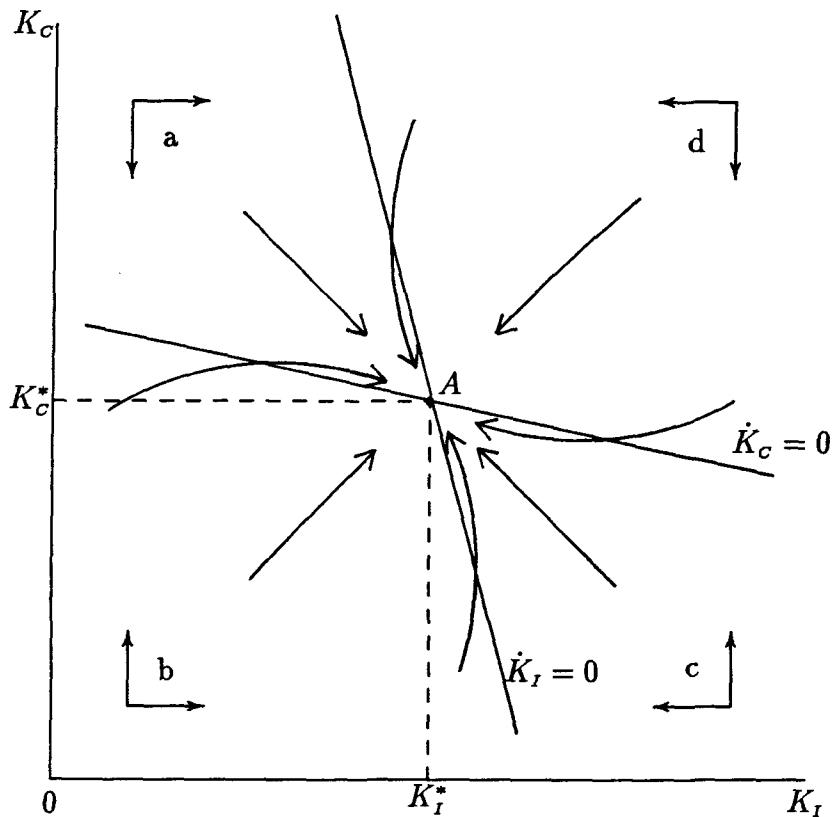


Figure 2: Dynamics of the Two-Sector Model with Static Expectations.

### Long-Run Expectations

In this case the differential equations are independent as we proved above; the isokines therefore are orthogonal. The system is linear; local and global analysis coincide. Since the isokines are trajectories themselves, overshooting of capital stocks is not possible with LRE.

### Rational Expectations

The local stability analysis in the appendix shows that the long-run equilibrium of the dynamic system with RE is a saddle point in general, and a stable node in the stable manifold. As with LRE, the isokines are orthogonal in the  $(K_I, K_C)$  plane and coincide with the LRE isokines. There are three linear paths: the two isokines and the long-run iso-wage line. In the case of two dimensions this

implies that all paths are linear<sup>5</sup>; the system coincides with the LRE system. Thus overshooting of capital stocks is ruled out locally. The global analysis, however, shows that RE adjustment is always strictly between SE and LRE adjustment. The trajectories of the system will cross the LRE isokines; overshooting of capital stocks is optimal and not a result of expectational errors, as one might conclude from the local comparison of RE and SE. Note that rational overshooting implies that it may even be profitable to adjust in the wrong direction, i.e. to further enlarge a capital stock which is already too large compared with its long-run size.

### Global Comparison of Alternative Expectation Regimes

In section 3 we have shown the relations between the wage differential  $w(t) - w^*$  and the shadow prices of capital under different expectations. From the first-order condition (5) determining the sectors' investment demand we can derive the relations:

$$\begin{aligned} w(t) > w^* &\iff I_j^{SE}(t) < I_j^{RE}(t) < I_j^{LRE}(t) \\ w(t) = w^* &\iff I_j^{SE}(t) = I_j^{RE}(t) = I_j^{LRE}(t) \\ w(t) < w^* &\iff I_j^{SE}(t) > I_j^{RE}(t) > I_j^{LRE}(t). \end{aligned} \tag{27}$$

According to (27) adjustment to the long-run equilibrium is always faster with SE than with RE (except for the case where all regimes coincide). Rational investors correctly anticipate future changes in capital rentals which reduce incentives to (dis-)invest, whereas investors with SE myopically respond to current values. Hence Mussa's (1978) result concerning adjustment speeds under alternative expectation regimes also holds in our context of total immobility of physical capital stocks and endogenous capital supply. On the other hand, investment decisions based solely on steady-state values reduce adjustment speed relative to RE adjustment since temporary higher returns are not taken into account. LRE always generate slowest adjustment in the spectrum of expectations considered here. Qualitative behavior of the system under all forms of expectations is simi-

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<sup>5</sup>Since the equilibrium is a stable node both eigenvalues must be negative and real; hence the differential equations are independent. Additionally all paths (with the exception of the eigenspaces) must behave qualitatively similar: if the adjustment speeds of the independent equations are equal, all paths are linear; otherwise all paths (other than the eigenspaces) are nonlinear. Since there are only two eigenspaces, three linear paths imply linearity of all paths.

lar: capital–stock overshooting is rational; it becomes more pronounced, however, if expectations are biased towards the status quo.

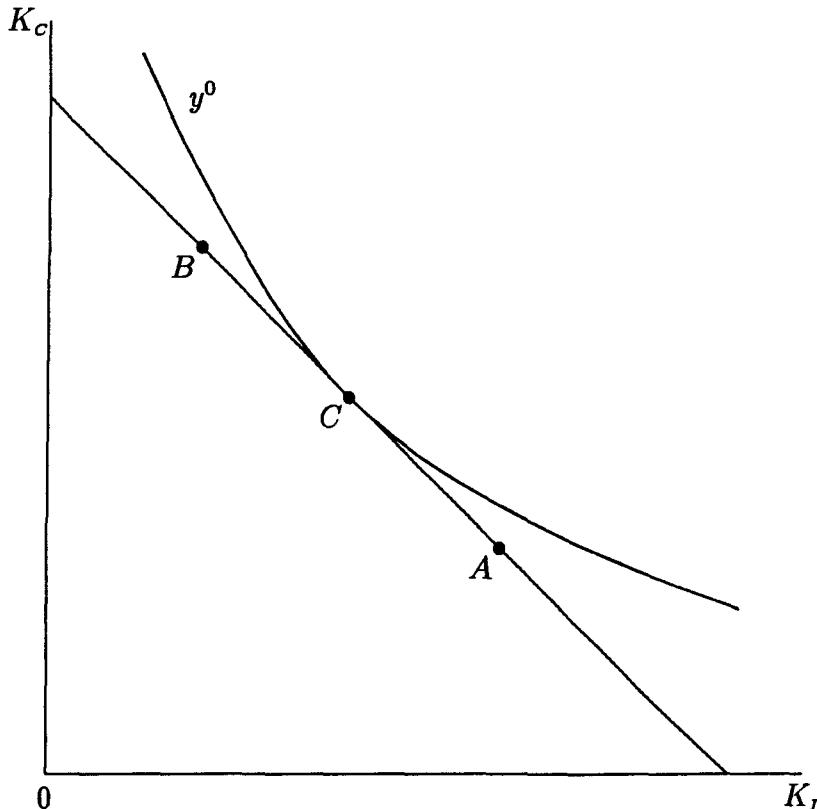


Figure 3: Overshooting of the Social Product.

### Social Product Adjustment

Adjustment of social product may also be characterized by overshooting. This can be seen by differentiating the social–product function (16) with respect to time:

$$\dot{y} = r_I \cdot \dot{K}_I + r_C \cdot \dot{K}_C. \quad (28)$$

(28) shows that  $y$  may adjust non–monotonically as  $\dot{K}_I$  and  $\dot{K}_C$  differ in sign<sup>6</sup>. In the following we demonstrate this possibility by looking at the special case of adjustment along the long–run iso–wage line in fig. 3. The iso–social–product lines are strictly convex towards the origin in the  $(K_I, K_C)$  plane; their slope is

<sup>6</sup>Note that capital stock overshooting is neither necessary nor sufficient for overshooting of the social product.

equal to  $-r_I/r_C$ . Differentiation of (16) shows that the slope of an iso- $y$  line goes to infinity (zero) as  $K_I$  ( $K_C$ ) goes to zero. Hence there is exactly one iso- $y$  line that is tangential to the iso- $w^*$  line establishing the  $(K_I, K_C)$  combination giving the greatest social product for given the wage  $w^*$ . In fig. 3 the maximum social product which is possible on the iso- $w^*$  line is  $y^0$ ; it is reached at  $C$ . Consider adjustment to the long-run equilibrium  $A$  lying right to  $C$ . If the initial allocation lies to the left of  $C$  (say, in  $B$ ) adjustment initially drives up social product until it reaches a maximum in  $C$ . As the economy eventually moves from  $C$  to  $A$  social product decreases. Since all modes of adjustment coincide in that example, social product overshooting can be a consequence of rational decisions.

## 5 Conclusions

In this paper we have analyzed the dynamics of capital-stock adjustment in a temporary equilibrium model, emphasizing the role of agents' expectations. Four different kinds of expectations have been considered explicitly : static expectations, long-run expectations, adaptive expectations, and rational expectations. Global stability of the dynamic system has been proved for all of them. We do not suggest that these types of expectations are necessarily of special interest. Instead we stress the qualitative robustness of the results. There is a broad spectrum of expectations for which the qualitative behavior of the system is alike; this spectrum is bounded by static expectations on the one and long-run expectations on the other side, and includes adaptive and rational expectations.

The analysis of the different adjustment paths provides two main general conclusions. The first is that there exists exactly one path where adjustment is identical under all expectation regimes considered whereas for all other paths adjustment is always faster under static expectations than under rational expectations, and is slowest under long-run expectations. Thus our analysis generalizes Mussa's (1978) result that physical capital reallocation under static expectations is too fast compared with optimal adjustment.

Secondly, sectoral capital stocks may adjust non-monotonically. Whereas local analysis might be taken to imply that this results from expectational errors, global analysis proves that overshooting is rational. This generalizes Murphy's (1988) findings that the aggregate capital stock may overshoot its long-run value.

These overshooting effects depend on the multi-sectoral structure of our model: in a one-sector model as it is typically used in macroeconomics all variables (with the exception of investment demand) adjust monotonically.

Future research should introduce the demand side of the economy in order to study the consequences of social-product overshooting for adjustment of the current account. Analysis of the case of balanced trade<sup>7</sup> endogenizes the determination of the interest rate and will be a first step to analyse a two-country model of trade in goods and factors.

## Appendix: Local Analysis of the Adjustment Process

### Static Expectations

In order to get the slopes of the isokines under SE we linearize (25) around the steady state:

$$\begin{bmatrix} \dot{K}_I \\ \dot{K}_C \end{bmatrix} = \begin{bmatrix} \frac{\partial I_I^{SE}}{\partial r_I} \cdot \frac{\partial r_I}{\partial K_I} - \delta & \frac{\partial I_I^{SE}}{\partial r_I} \cdot \frac{\partial r_I}{\partial K_C} \\ \frac{\partial I_C^{SE}}{\partial r_C} \cdot \frac{\partial r_C}{\partial K_I} & \frac{\partial I_C^{SE}}{\partial r_C} \cdot \frac{\partial r_C}{\partial K_C} - \delta \end{bmatrix} \cdot \begin{bmatrix} \delta \cdot K_I - I_I^{SE} \\ \delta \cdot K_C - I_C^{SE} \end{bmatrix}. \quad (29)$$

Stability requires that  $tr(J^{SE})$  is negative and  $|J^{SE}|$  is positive where  $J^{SE}$  is the Jacobi Matrix with static expectations of (31). Calculating  $tr(J^{SE})$  and  $|J^{SE}|$  yields

$$tr(J^{SE}) = \left( \frac{\partial I_I^{SE}}{\partial r_I} \cdot \frac{\partial r_I}{\partial K_I} + \frac{\partial I_C^{SE}}{\partial r_C} \cdot \frac{\partial r_C}{\partial K_C} - 2 \cdot \delta \right) < 0 \quad (30)$$

and

$$|J^{SE}| = \delta \cdot \left( \frac{\partial I_I^{SE}}{\partial r_I} \cdot \frac{\partial r_I}{\partial K_I} - \delta \right) \cdot \left( \frac{\partial I_C^{SE}}{\partial r_C} \cdot \frac{\partial r_C}{\partial K_C} - \delta \right) - \frac{\partial I_I^{SE}}{\partial r_I} \cdot \frac{\partial r_I}{\partial K_C} \cdot \frac{\partial I_C^{SE}}{\partial r_C} \cdot \frac{\partial r_C}{\partial K_I} \cdot \delta^{-2}. \quad (31)$$

$|J^{SE}|$  must be positive since otherwise the system would show a saddle point; that, however, is incompatible with global stability. Additionally  $|J^{SE}| > 0$  implies that the  $\dot{K}_C=0$ -curve must be flatter than the  $\dot{K}_I=0$ -curve in the  $(K_I, K_C)$  plane.

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<sup>7</sup>Albert and Meckl (1991) provide first results on this topic; instead of using an intertemporal optimizing approach to savings a fixed rate of saving is assumed.

## Rational Expectations

Existence of a stable path is established *via* a local stability analysis where we show that the steady state is a saddle point. For this purpose we linearize the system (17) around the steady state. The result is the following system:

$$\begin{bmatrix} \dot{K}_I \\ \dot{K}_C \\ \dot{\lambda}_I \\ \dot{\lambda}_C \end{bmatrix} = \begin{bmatrix} -\delta & 0 & 1/h''_I(I^*_I) & 0 \\ 0 & -\delta & 0 & 1/h''_C(I^*_C) \\ -\partial r_I/\partial K_I & -\partial r_I/\partial K_C & i + \delta & 0 \\ -\partial r_C/\partial K_I & -\partial r_C/\partial K_C & 0 & i + \delta \end{bmatrix} \cdot \begin{bmatrix} K_I - K_I^* \\ K_C - K_C^* \\ \lambda_I - \lambda_I^* \\ \lambda_C - \lambda_C^* \end{bmatrix}. \quad (32)$$

It can be shown that for the steady state to be a saddle point with a two-dimensional stable manifold it is necessary and sufficient that  $\kappa < 0$  and  $0 < |J| \leq \kappa^2/4$ , where  $|J^{RE}|$  is the determinant of system (32)'s Jacobian with rational expectations and  $\kappa := M - i^2$ , with  $M$  the sum of  $J^{RE}$ 's principal minors of order 2. This surprisingly simple result is due to the fact that in systems of this kind many terms in the computation of the eigenvalues cancel out<sup>8</sup>. We compute both  $|J^{RE}|$  and  $\kappa$ .

The determinant of  $J^{RE}$  is given by

$$|J^{RE}| = \frac{1}{h''_I \cdot h''_C} \cdot |\mathcal{H}| \quad (33)$$

where  $\mathcal{H}$  is the Hessian of the optimization problem (16). The determinant of  $\mathcal{H}$  is almost always positive since  $g(p, L, i)$  is strictly concave in capital stocks. With strictly convex costs of adjustment the determinant of  $J^{RE}$  must have the same sign as the determinant of  $\mathcal{H}$ .

The value of  $\kappa$  is given by

$$\frac{\partial r_I}{\partial K_I} \cdot \frac{1}{h''_I} + \frac{\partial r_C}{\partial K_C} \cdot \frac{1}{h''_C} - 2 \cdot \delta \cdot (i - \delta) < 0. \quad (34)$$

Hence  $\kappa^2/4 - |J^{RE}| \geq 0$  always holds since

$$\frac{1}{4} \cdot \left( \frac{\partial r_I}{\partial K_I} \cdot \frac{1}{h''_I} - \frac{\partial r_C}{\partial K_C} \cdot \frac{1}{h''_C} \right)^2 + \left( \frac{\partial r_C}{\partial K_I} \right)^2 \cdot \frac{1}{h''_I \cdot h''_C}$$

must be positive. The long-run equilibrium is a saddle point in the stable manifold  $(K_I, K_C)$ . However, the signs become ambiguous if the rates of depreciation are not identical between industries.

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<sup>8</sup>Feichtinger and Hartl (1986), pp. 134–135.

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