

Grüner, Hans Peter

Working Paper

Optimal institutions for monetary policy: Contracts, shocks and signaling

Diskussionsbeiträge - Serie II, No. 251

Provided in Cooperation with:

Department of Economics, University of Konstanz

Suggested Citation: Grüner, Hans Peter (1995) : Optimal institutions for monetary policy: Contracts, shocks and signaling, Diskussionsbeiträge - Serie II, No. 251, Universität Konstanz, Sonderforschungsbereich 178 - Internationalisierung der Wirtschaft, Konstanz

This Version is available at:

<https://hdl.handle.net/10419/101487>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

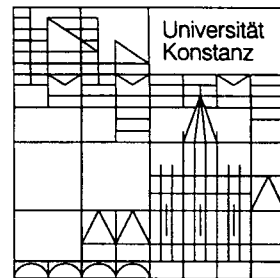
Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



Sonderforschungsbereich 178
„Internationalisierung der Wirtschaft“

Diskussionsbeiträge

Juristische
Fakultät

Fakultät für Wirtschafts-
wissenschaften und Statistik

Hans Peter Grüner

Optimal Institutions for Monetary Policy:
Contracts, Shocks and Signaling

W 113 (251)



27. MRZ. 1995 Weltwirtschaft
K181

W 113 (251)

~~mit~~ bt sig gla

Optimal Institutions for Monetary Policy:

Contracts, Shocks and Signaling

Hans Peter Grüner

596415

Serie II - Nr. 251

Februar 1995

**OPTIMAL INSTITUTIONS FOR MONETARY POLICY:
CONTRACTS, SHOCKS AND SIGNALING**

by **Hans Peter GRÜNER***

University of Konstanz

December 1994

*Paper for German Israeli Foundation Meeting, Mannheim and
Public Choice Society Meeting, 1995, Long Beach*

JEL classification numbers: E50, E 52, E 58

Key words: signaling, monetary target announcements, monetary rules

Abstract: The effects of different institutional arrangements for the central bank are examined in the presence of economic shocks and uncertainty about the central banker's and the median-voter's inflation target. A contract which is based on self-imposed monetary target announcements proves to be superior to the best monetary rule if conflicts about the inflation target within society are relatively small compared to the initial uncertainty about the median voter's objective. It is superior to the laissez faire solution if unemployment exceeds a certain threshold level. The optimal choice of costs of deviations from auto-imposed targets depends on the type of conflict within society, whether the individuals disagree on the weight of the inflation versus the employment target or on the value of the inflation target itself.

*I am indebted to Charles Wyplosz, Michael Burda, Gilles Saint Paul and Antoine d'Autume for helpful discussions and suggestions and to seminar participants in Uppsala, Paris, Louvain La Neuve and Konstanz for their useful comments. Financial support from the EC commission under its SPES program is gratefully acknowledged.

Address: University of Konstanz, Department of Economics and Statistics, PO Box 5560 D 137, D-78434 Konstanz, Phone: +49 753188 2137, 3130 (Fax).

1. Introduction

More than ten years after the appearance of the first time inconsistency models on monetary policy, the question of how to solve the time-inconsistency problem by choice of optimal institutions for monetary policy is still open. Neither the idea of a natural solution for the problem via incentives to build up anti-inflation reputation, nor the popular concept of the "conservative" and independent central banker were entirely convincing. It was shown that the positive results on disciplinary effects of reputation found by Backus and Driffill (1985) were neither robust with respect to assumptions about the policymaker's action space (Vickers (1986)) nor with respect to the assumption that there is hysteresis in unemployment (Grüner (1993,94)). The second prominent concept, the conservative central banker, was criticized by Rogoff (1985) on grounds of the lack of flexibility of such a policymaker when confronted with economic shocks. Giavazzi and Giovannini (1989) additionally criticized the concept of the conservative central bank because, as they claimed, a switch to a conservative policy always induces a costly learning process about the type of central banker in office.

New literature on solution concepts emerged in the nineties based on results originally coming from contract theory. This literature interprets the interaction of government and central bank as a principal agent problem, where, on a constitutional level, the principal can commit to conditional payments to the agent. The trade-off between credibility and flexibility introduced by Rogoff found particular consideration in this literature (see Walsh (forthcoming), Persson and Tabellini (1993), and Lohmann (1992)¹). The results are very encouraging: Persson and Tabellini show that it is possible to achieve the second-best outcome without inflationary bias but with an appropriate reaction to economic shocks if the political principal of the central bank sanctions deviations from auto-imposed target announcements of the central bank. This result of Persson and Tabellini, however, is derived under the

¹In Lohmann's (1992) paper the government is both, principal and agent. It can choose costs at which it can later override the central bank's decision.

assumption that the central bank's preferences are, with the exception of the shock, known and identical to society's preferences². However, inflation has distributive effects and, therefore, conflicts on the rate of inflation are very likely to arise within society. In this case the central banks' preferences may differ from the government's and the median voter's objectives and, if a target announcement is applied, signaling of types occurs.

In this paper, I want to shed light on the question of optimality of contracts for the central bank if both shocks *and* signaling are present, in order to specify conditions for the optimality of either target announcements, rules or the use of no mechanism. I assume that the political principal of the central bank determines the form of the contract and the size of the (conditional) payments at the beginning of the game. She faces a threefold uncertainty at this stage:

1) Each individual has a private inflation target, $\bar{\pi}$. The central banker is drawn randomly from the population, therefore his preferences can differ from those of the median voter, whose preferred inflation rate is π^s .³

2) At the point of time where the contract is imposed on the central bank, the median voter's inflation target π^s is uncertain too. It is distributed uniformly on a closed interval and not verifiable ex-post.

3) A non-verifiable employment shock necessitates an intervention of the central bank. This shock is not verifiable ex-post for the principal.

² In their analysis of central bank independence, Persson and Tabellini also consider a case where the government's utility function differs from the social welfare function while the central bank shares society's objective.

³This assumption shall capture the fact that the type of the central banker is usually not determined in a single vote with informed voters. The choice of the central banker can e.g. be the outcome of logrolling or part of a multidimensional platform of one party.

The second assumption captures the fact that the median voter's objective is not fixed once and for all. It changes e.g. with the need for seignorage revenues or with foreign monetary policy targets. The assumption that π^s is not verifiable ex post does not just mean that it is difficult to measure this variable. Even if there were individuals who were in a position to measure π^s , it could not be excluded that correctly reporting π^s is incompatible with their private objectives. Thus even if π^s was verifiable for some individuals, it would not solve the time inconsistency problem to confine its measurement to them.

It is the political principal's objective to maximize the median voter's ex-ante expected welfare in the first stage of the game. However, in contrast to Walsh (forthcoming), I assume that the principal can not choose freely among all possible functional forms for the contract. This assumption would stipulate that the principal exactly knows how the transfers to the central bank affect utility of the central banker. I here rather assume that the principal knows that the central banker's marginal loss from salary reductions is increasing. This assumption is captured in a quadratic loss function in inflation, unemployment and salary reductions. The principal can choose among three institutional arrangements:

1. no contract
2. A contract which sanctions deviations from auto imposed target announcements
3. A contract which sanctions deviations from a monetary rule.

We will see that under the above assumptions, form and specification of the optimal contract crucially depend on the relative importance of the different types of uncertainty and that the implementation of a contract that will maximize the median-voter's (ex-ante) utility necessitates an analysis of the actual divergence and volatility of the individual's inflation targets. Section 2 presents the signaling game and the perfect Bayesian equilibrium is derived. In section 3 the results on the optimal choice of the contract are presented. Section 4 concludes.

2. The Model

The game between the central bank and the public has the following time structure: in the first (constitutional) stage, the political principal imposes a contract on the central bank, which conditions the remuneration of the central bank on its performance and possibly on its announcement.⁴ In the second stage, nature chooses the median-voter's preferences. Nature then randomly draws the type of central banker from society. His preferences may differ from those of the median voter. In the third stage, the central bank has to announce a target for inflation, if this is foreseen by the contract. Afterwards the public chooses inflation expectations, knowing the actual distribution for the type of central bank. The output shock is then realized and finally the central bank unilaterally chooses the actual rate of inflation. The contract pays off in the last stage of the game.

The median voter's preferences are assumed to be represented by the utility function:

$$v^s(\pi, \pi^e, \pi^s) = E\left(-\frac{a}{2}(\pi - \pi^s)^2 - \frac{b}{2}(\pi - \pi^e - u - \varepsilon)^2\right), \quad (1)$$

where π and π^e are actual and expected inflation, u a measure for frictional unemployment and ε the shock with $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. The second term represents actual unemployment: it can be reduced below the level of u if there is surprise inflation.

The median voter's inflation target π^s is distributed uniformly on $[\hat{\pi} - x, \hat{\pi} + x]$. The central banker's preferences may differ from the median voter's preferences. The central banker's private inflation target $\bar{\pi}$ is assumed to be distributed uniformly on $[\pi^s - y, \pi^s + y]$.

⁴I do not consider a stage where the central bank can reject the contract. Like in Persson and Tabellini (1993) the individual rationality constraint is neglected because payments to the central bank are of marginal size compared to the social losses from inflation and unemployment. Thus the contract can always be adjusted in order to ensure participation of the agent.

With a monetary target announcement, we assume that the utility function of the central bank can be written

$$v^{CB}(\pi, \pi^e, \pi^a) = E\left(-\frac{a}{2}(\pi - \bar{\pi})^2 - \frac{b}{2}(\pi - \pi^e - u - \varepsilon)^2 - \frac{c}{2}(\pi - \pi^a)^2\right), \quad (2)$$

where π^a is the announced rate of inflation. The utility functions (1) and (2) are approximations of a convex loss function in inflation, unemployment and revenue losses of the central banker where the payoff of the contract is linear. The loss of $-\frac{c}{2}(\pi - \pi^a)^2$ can, however, also be interpreted as a (quadratic) monetary loss that directly adds to the utility function.

If the contract is a rigid rule, then the utility function can be written analogue

$$v^{CB}(\pi, \pi^e) = E\left(-\frac{a}{2}(\pi - \bar{\pi})^2 - \frac{b}{2}(\pi - \pi^e - u - \varepsilon)^2 - \frac{d}{2}(\pi - \hat{\pi})^2\right). \quad (3)$$

The rule is assumed to be made contingent on $\hat{\pi}$ because the actual realization of π^s is neither known ex-ante nor verifiable ex-post. The third loss term is dropped if there is no contract.

We will consider the politician's problem to impose a contract on the central bank in the first stage of the game that maximizes the ex-ante expected utility of the median voter. Such a policy maximizes the expected utility of an individual who will become a median voter in any state of the world. It also maximizes the ex-ante expected welfare if individual preferences are distributed uniformly on $[\pi^s - y, \pi^s + y]$ and if the welfare function weights each individual's utility equally.

An equilibrium of the game with monetary target announcements consists of expectations $\pi^e = f(\pi^a)$ and announcements $\pi^a = g(\bar{\pi})$ that are compatible. One can show

that there is exactly one equilibrium where f is a continuous function. In this equilibrium, f is linear and the equilibrium is separating.

Proposition 1: Existence of a separating equilibrium with a target announcement

(i) For all $c > 0$, there is a separating equilibrium in pure strategies with

$$\text{expectations } \pi^c = \pi^a + \frac{b}{c}u,$$

$$\text{signals } \pi^a = \bar{\pi} - \frac{b}{c}u,$$

$$\text{and equilibrium-inflation for each central banker } \pi(\pi^c, \pi^a, \bar{\pi}) = \bar{\pi} + \frac{b}{a+b+c}\varepsilon. \quad (4)$$

(ii) There is no separating equilibrium of the game with $\pi^c = f(\pi^a)$ where f is nonlinear and where f has the following properties:

(i) f is continuously differentiable

(ii) There is an interval $A \subset R$ so that A is a set of all the equilibrium announcements of all the $\bar{\pi} \in [\pi^a - y, \pi^a + y]$ types.

Proof 1: (i) Suppose expectations were $\pi^c = \pi^a + \delta$. Given these expectations and π^a , the best choice of the central bank for π is from (2)

$$\pi = \frac{1}{a+b+c} [a\bar{\pi} + (b+c)\pi^a + b(\delta + u + \varepsilon)] \quad (5)$$

Central bank utility in stage 1 of the game is a function of the announcement π^a :

$$v^{CB} = \int_{-\infty}^{\infty} \frac{1}{(a+b+c)^2} \left[-\frac{a}{2} ((b+c)(-\bar{\pi} + \pi^a) + b(\delta + u + \varepsilon))^2 - \frac{b}{2} (a(\bar{\pi} - \pi^a) - (a+c)(\delta + u + \varepsilon))^2 - \frac{c}{2} (a(\bar{\pi} - \pi^a) + b(\delta + u + \varepsilon))^2 d\phi(\varepsilon) \right]. \quad (6)$$

The first-order condition for a maximum of this expression is $\pi^a = \bar{\pi} - \frac{b}{b+c}(\delta + u)$. This means that the announcement π^a is chosen by the central bank with $\bar{\pi} = \pi^a + \frac{b}{b+c}(\delta + u)$.

Expectations are unbiased if and only if:

$$\begin{aligned} \pi^e &= \pi^a + \delta = \frac{1}{a+b+c} \left[a \cdot \bar{\pi} + b \cdot (\pi^e + u + \varepsilon) + c\pi^a \right] \\ &= \frac{1}{a+b+c} \left[a \cdot \left(\pi^a + \frac{b}{b+c}(\delta + u) \right) + b(\pi^e + \delta + u) + c\pi^a \right] \end{aligned} \quad (7)$$

Solving for δ yields

$$\delta = \frac{b}{c} \cdot u. \quad (8)$$

This and substitution of (6) into (4) yield the first part of the proposition. For part (ii) the reader should refer to the appendix. ■

Proposition 1 states that the announcement can reveal the type of central banker in office. This is why it permits the central bank to moderate expectations in a way that it can follow its private inflation target. This result is independent of the value of c . A low value of c ensures a fully flexible response to shocks. Suppose now that there was no announcement i.e. that $c=0$. In this case, the rational expectations require that:

$$\pi^e = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{a\bar{\pi} + b(\pi^e + u + \varepsilon)}{a+b} \frac{1}{2y} \cdot d\bar{\pi} \cdot d\phi(\varepsilon) \quad (9)$$

or equivalently: $\pi^e = \pi^s + \frac{b}{a}u$. Inflation of the central bank is

$$\pi = \frac{a\bar{\pi} + b(\pi^e + u + \varepsilon)}{a+b} = \frac{a\bar{\pi} + b(\pi^s + \varepsilon)}{a+b} + \frac{b}{a}u. \quad (10)$$

With a rigid rule, the public's expectations are correct iff:

$$\pi^e = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{a\bar{\pi} - b(\pi^e + u + \varepsilon) + d\hat{\pi}}{a + b + d} \frac{1}{2y} \cdot d\bar{\pi} \cdot d\phi(\varepsilon) \quad (11)$$

$$\text{or } \pi^e = \frac{a\pi^s + d\hat{\pi}}{a + d} + \frac{b}{a + d} u.$$

We are now able to compare the different institutional arrangements for monetary policy and to make our welfare statements.

3. Welfare

The analysis in section 2 permits us to compare the impact of different types of contracts on the median voter's ex-ante utility. We consider the optimal contract with an announcement, the optimal contract which is based on a monetary rule and the laissez faire situation without a contract. Proposition 2 examines the choice for the cost parameter c in the game with an announcement. There is actually no optimal solution: we have already seen that for all positive c , there is a separating equilibrium where $\pi(\pi^e, \pi^s, \bar{\pi}) = \bar{\pi} + \frac{b}{a + b + c} \varepsilon$. Thus, c should be positive but as small as possible in order to permit flexible reaction to economic shocks. Results differ if b but not $\bar{\pi}$ is the uncertain parameter.

Proposition 2: Optimal choice of deviation costs c under different type-spaces

- (i) Consider the game from section 2 with a target announcement. There is no optimal value for c . c should be positive and as small as possible.
- (ii) Suppose instead that the type-space consists of only two potential central banks and that only b is uncertain

$$b^{CB} = \begin{cases} \bar{b} & \text{probability } p \\ \underline{b} & \text{probability } 1-p \end{cases}, \bar{b} > \underline{b} \geq 0, p \in]0,1[.$$

Assume that $\bar{\pi} = \pi^*$ for both central banks and that society's b value is in $[\underline{b}, \bar{b}]$. If the variance of the shock ε , σ_ε^2 , is sufficiently small, then the optimal contract fixes a positive value for c .

Proof 2: See appendix.

Our first welfare result is therefore that the optimal implementation of the contract depends on the type of uncertainty about the central bank's preferences. We now turn to the comparison of welfare under two different institutions, a rigid rule and the optimal contract which is based on a target announcement. Both of them have their specific disadvantages: the rule is more inflexible than the announcement-contract because it does not react to changes in the median voter's preferences. The announcement on the other hand permits the central bank to follow its private policy objective at lower costs. The first effect is stronger if the initial uncertainty about π^S is sufficiently large.

Proposition 3: Comparison of announcement and optimal monetary rule

For every distribution of preferences in society $y \in \mathbb{R}^+$ there is a value for initial uncertainty about the median voter's inflation target $x^* \geq 0$ so that:

- (i) For $x > x^*$, the optimal contract based on a self imposed monetary target announcement will outperform the optimal contract based on a rule. $x^* = h(y)$ is nondecreasing.
- (ii) For $0 \leq x \leq x^*$, the optimal contract based on a self-imposed monetary target announcement will outperform the optimal contract based on a rule.
- (iii) There is a $y^* > 0$ so that $h(y) > 0$ for $y > y^*$.

Proof 3: Call $w^A(x, y)$ the maximal value of welfare, obtainable with a contract based on a monetary announcement. Call $w^R(x, y)$ the maximal value of welfare, obtainable with a contract which is based on a rigid rule. (i) and (ii) follow from $\frac{\partial w^A(x, y)}{\partial x} = 0 \forall x, y \in \mathbb{R}^+$,

$\frac{\partial w^R(x,y)}{\partial x} < 0 \forall x,y \in \mathbb{R}^+$ and the fact that W^R is not bounded from below. In order to prove (iii), it is necessary to consider $w^A(0,y)$ and $w^R(0,y)$. First note that $w^A(0,0) > w^R(0,0)$ because from (3) with an announcement society faces no trade-off between credibility and flexibility. Note further that $\frac{\partial w^A(0,y)}{\partial y}$ and $\frac{\partial w^R(0,y)}{\partial y}$ are negative. $\frac{\partial w^A(0,y)}{\partial y}$ is negative because of increasing costs of deviation from π^S . $\frac{\partial w^R(0,y)}{\partial y}$ is negative because the central bank would have to increase disorder to maintain the costs of deviation. But this would increase costs of inflexibility. However, considering the case with $d = \infty$, we see that $w^R(0,y)$ is bounded from below while $w^A(0,y)$ is obviously not. (iii) follows. ■

Having established the conditions for the superiority of the announcement to the rule, we finally compare the institution of a target announcement with a laissez-faire policy. Surprisingly, proposition 4 tells us that the target announcement is not always a welfare-improving institution.

Proposition 4: Announcent vs. no announcement

The optimal contract which is based on the announcement of a monetary target is a welfare-improving institution if $\left(\frac{b}{a}\right)^2 + \frac{b}{a} > \frac{\sigma_{\bar{\pi}}^2}{u^2}$ with $\sigma_{\epsilon}^2 = \int_{\pi^{t-y}}^{\pi^{t+y}} \frac{1}{2y} (\bar{\pi} - \pi)^2 d\bar{\pi}$. No announcement is better if $\left(\frac{b}{a}\right)^2 + \frac{b}{a} < \frac{\sigma_{\bar{\pi}}^2}{u^2}$. The value of σ_{ϵ}^2 is irrelevant here because the optimal value of c is near zero.

Proof 4.: Expectations without announcement must satisfy:

$$\pi^e = \int_{\pi^{t-y}}^{\pi^{t+y}} \frac{1}{2y} \frac{1}{a+b} (a\bar{\pi} + b(\pi^e + u)) d\bar{\pi}$$

$$\Leftrightarrow \pi^e = \pi^S + \frac{b}{a} u,$$

in order to be unbiased. Inflation is:

$$\pi = \frac{1}{a+b} \left[a\bar{\pi} + b(\pi^e + \varepsilon) \right],$$

and welfare without announcement is:

$$\begin{aligned} W^N &= \int_{\pi^s-y}^{\pi^s+y} \frac{1}{2y} \left[-\frac{a}{2} \left(\frac{1}{a+b} \left(a\bar{\pi} + b \left(\pi^s + \frac{b}{a} u + u + \varepsilon \right) \right) - \pi^s \right)^2 - \frac{b}{2} \left(\frac{1}{a+b} \left(a\bar{\pi} + b \left(\pi^s + \frac{b}{a} u + u + \varepsilon \right) \right) - \left(\pi^s + \frac{b}{a} u \right) - u - \varepsilon \right) \right] d\bar{\pi} d\varepsilon \\ &= \int_{\pi^s-y}^{\pi^s+y} \frac{1}{2y} \left[-\frac{a}{2} \left(\frac{a}{a+b} (\bar{\pi} - \pi^s) + \frac{b}{a} u \right)^2 - \frac{ab^2}{(2(a+b))^2} \int \varepsilon^2 - \frac{b^3}{2((a+b))^2} \int \varepsilon^2 - \frac{b}{2} \left(\frac{a}{a+b} \pi - \pi^s \right)^2 \right] d\bar{\pi} \\ &= \frac{a^3 + a^2b}{2(a+b)^2} \sigma_\varepsilon^2 + \left(-\frac{a b^2}{b a^2} - \frac{b}{2} \right) u^2 - \frac{ab^2 + b^3}{2(a+b)^2}. \end{aligned}$$

We know that inflation with announcement is: $\pi = \bar{\pi} + \frac{b}{a+b+c} \varepsilon$. c can be neglected as we

know from proposition 1. Expected welfare with announcement is:

$$W^A = -\frac{a}{2} \sigma_\pi^2 - \frac{b}{2} u^2 - \frac{ab^2}{2(a+b)^2} \int_{-\infty}^{\infty} \varepsilon^2 d\varepsilon - \frac{b^3}{2(a+b)^2} \int_{-\infty}^{\infty} \varepsilon d\varepsilon. \quad (12)$$

The introduction of an announcement makes the median voter better off if and only if

$$\begin{aligned} \frac{a}{2} - (a+b) \frac{a^2}{(a+b)} &< \frac{b^2 u^2}{a \sigma_\pi^2} \\ \Leftrightarrow \left(\frac{a}{b} \right)^2 - \frac{a}{b^2} \frac{a^2}{a+b} &< \frac{u^2}{\sigma_\pi^2} \Leftrightarrow \left(\frac{a}{b} \right)^2 - \left(\frac{a}{b} \right)^2 \frac{a}{a+b} < \frac{u^2}{\sigma_\pi^2} \Leftrightarrow \left(\frac{a}{b} \right)^2 \frac{b}{a+b} < \frac{u^2}{\sigma_\pi^2} \\ \Leftrightarrow \left(\frac{b}{a} \right)^2 + \frac{b}{a} &> \frac{\sigma_\pi^2}{u^2}. \blacksquare \end{aligned} \quad (13)$$

The introduction of the target announcement permits the central banker to moderate inflation expectations. Thus, an announcement only makes the median voter better off, if the ex-ante variance of the policymaker's objective is not too large. Otherwise the inflexibility of expectations without announcement has a desirable disciplinary effect on the central bank.

Proposition 2 and 3 and the fact that the optimal rule with $d > 0$ is better than $d = 0$ if and only if x is sufficiently small, permits to distinguish three regimes in the x - y space where one of the three institutions is the optimal one. The regimes are depicted in figure 1. Proposition 2 states that higher unemployment increases the boundary value for y , y^* below which an announcement is a welfare-improving institution.

4. Conclusion

Among the known solution concepts for the problem of time inconsistency in monetary policy, contracts are one of the most promising ones. In particular, hysteresis in the unemployment rate and the impossibility to exploit credibility of other players via a pegged exchange rate should make the solution via contracts an interesting option for a possible European Central Bank. However, the correct choice and specification of such contracts is not a trivial problem. The correct choice of the contract crucially depends on the variability of society's inflation target and on the extent to which conflicts concerning the inflation rate are prevalent within society. The optimal choice of costs of deviations from auto-imposed targets by contrast depends on the type of conflict in society, whether it is on the weight of the inflation versus the employment target or the value of the inflation target itself. Further empirical research in this domain is therefore necessary in order to find appropriate proposals for the implementation of optimal contracts.

Appendix

Proof 1, Part (ii):

(ii) Suppose that the π^* central bank chooses to announce π^a . Take the linear approximation of f around π^a : $f^{\pi^a} = g_1 + g_2\pi^a$. Nonlinearity implies that there must be a π^{a*} so that $g_2 \neq 1$. Substitution of the expected value of the optimal inflation rate for a given announcement π^a and expectations π^e

$$\pi = \frac{1}{a+b+c} [a\bar{\pi} + b(g_1 + g_2\pi^a + u + \varepsilon) + c\pi^a] \quad (14)$$

into v^{CB} and derivation with respect to π^a yields

$$\begin{aligned} \frac{\partial v^{CB}}{\partial \pi^a} &= \frac{1}{2} \left(\frac{1}{a+b+c} \right)^2 \cdot [-a(-(b+c)\bar{\pi} + b(g_1 + g_2\pi^a + u) + c\pi^a)(bg_2 + c) \\ &\quad - b(a\bar{\pi} - (a+c)(g_1 + g_2\pi^a + u) + c\pi^a)(-(a+c)g_2 + c) \\ &\quad - c(a\bar{\pi} + b(g_1 + g_2\pi^a + u) - (a+b)\pi^a) \cdot (bg_2 - (a+b))]. \end{aligned} \quad (15)$$

The optimal choice of π^a can be obtained from $\frac{\partial v}{\partial \pi^a} = 0$:

$$\begin{aligned} F(\pi^a, \bar{\pi}, u) &= \bar{\pi} \cdot [(bg_2 + c) \cdot (-a)(-b-c) + (-(a+c)g_2 + c) \cdot (-ba) + (bg_2 - (a+b))(-ca)] \\ &\quad + \pi^a \cdot [-a(bg_2 + c) \cdot (c + bg_2) + -b(-(a+c)g_2 + c)(c - (a+c)g_2) + -c(bg_2 - (a+b))(-(a+b) + bg_2)] \\ &\quad + u \cdot [-ab(bg_2 + c) + b(a+c)(-(a+c)g_2 + c) - cb(bg_2 - (a+b))] = 0. \end{aligned} \quad (16)$$

g_1 and g_2 have to take on the particular values so that given a marginal change of the announcement, $\pi^e = E(\pi)$ is maintained and, consequently,

$$d\pi^e = \left[a \cdot d\bar{\pi} + b \cdot d\pi^e + c \cdot d\pi^a \right] \cdot \frac{1}{a+b+c}, \quad (17)$$

where $d\pi^e = g_2 d\pi^a$ and from (16) $d\bar{\pi} = -\frac{\partial F / \partial \bar{\pi}}{\partial F / \partial \pi^a} \cdot d\pi^a$. This necessitates that

$$(a+c)g_2 d\pi^a = a \cdot \left(-\frac{\partial F / \partial \bar{\pi}}{\partial F / \partial \pi^a} \right) \cdot d\pi^a + c \cdot d\pi^a \quad (18)$$

or

$$\frac{\partial F / \partial \bar{\pi}}{\partial F / \partial \pi^a} = \frac{c}{a}(1-g_2) - g_2. \quad (19)$$

From (16) one obtains that (19) constitutes a linear equation in g_2 with the only solution $g_2=1$.

■

Proof 2: (i) We first derive the optimal value for c if $\bar{\pi}$ is uncertain. Welfare is

$$w = E\left(-\frac{a}{2}(\pi(\bar{\pi}, \varepsilon, c) - \pi^s)^2 - \frac{b}{2}(\pi(\bar{\pi}, \varepsilon, c) - \pi^s - u - \varepsilon)^2\right). \quad (20)$$

Where $\pi(\bar{\pi}, \varepsilon, c)$ can be derived from $\pi^e = \pi^s + \delta$ and $\pi^s = \bar{\pi} - \frac{b}{b+c}(\delta + a)$.

Substitution of this into:

$$\pi = \frac{1}{a+b+c} \left(a\bar{\pi} + b(\pi^e + u + \varepsilon) + c\pi^s \right) \quad (21)$$

yields

$$\pi = \bar{\pi} + \frac{b}{a+b+c} \varepsilon. \quad (22)$$

The welfare function can therefore be written as:

$$w = \bar{w} + \int_{-\infty}^{\infty} -\frac{a}{2} \left(\frac{b}{a+b+c} \varepsilon \right)^2 - \frac{b}{2} \left(\frac{-(a+c)}{a+b+c} \varepsilon \right)^2 d\Phi(\varepsilon) = \bar{w} - \frac{1}{2} \frac{ab^2 + b(a+c)^2}{(a+b+c)^2} \sigma_{\varepsilon}^2 \quad (23)$$

so that

$$\frac{\partial w}{\partial c} = -\frac{\sigma_{\varepsilon}^2}{2} \frac{2(a+b+c)^2 b(a+c) - 2(ab^2 + b(a+c)^2)(a+b+c)}{(a+b+c)^4}. \quad (24)$$

The necessary condition for an optimum is:

$$(a+b+c)b(a+c) - ab^2 - b(a+c)^2 = 0$$

$$\Leftrightarrow (a+b)ba - ab^2 - ba^2 = (-(a+b)b - ab + 2ba)c - bc^2$$

$$\Leftrightarrow 0 = -b^2c - bc^2$$

$$\Leftrightarrow c^2 + bc = 0. \quad (25)$$

The solution is $c_{1,2} = 0, -b$. $\frac{\partial w}{\partial c} = -\sigma_{\varepsilon}^2 \frac{b^2c}{(a+b+c)^3}$ is negative for $c > 0$. The proposition follows .

(ii) See Grüner (1994, pp 21-31) for the proof that this game has a unique pooling equilibrium at $\pi^s = \bar{\pi} = \pi^s$ which satisfies the criterion of unchanged out-of equilibrium beliefs. Equilibrium inflation is $\pi = \pi^s + \frac{b}{a+c}(u + \varepsilon)$. We need to show that $c > 0$ is optimal if σ_{ε}^2 is sufficiently small. The gains from smaller deviations from π^s are an increasing function of c , $r(c, u, \sigma_{\varepsilon}^2)$, with $r_c > 0, r_{cu} > 0, r_{\sigma_{\varepsilon}^2} < 0, \lim_{c \rightarrow \infty} r(c, u, \sigma_{\varepsilon}^2) = r^{\max} > 0$. The second effect is the loss which is due to smaller reactions of both types with respect to the shock, $l(c, u, \sigma_{\varepsilon}^2)$. We have $l_{\sigma_{\varepsilon}^2} > 0, \lim_{c \rightarrow \infty} l(c, u, \sigma_{\varepsilon}^2) = l^{\max} > 0, \frac{\partial l^{\max}}{\partial \sigma_{\varepsilon}^2} > 0$. The proposition follows from the fact that l^{\max} is an unbounded function of the variance σ_{ε}^2 . ■

References

- Backus, David and John Driffill (1985): "Inflation and reputation", *American Economic Review* 75, pp. 530-538.
- Giavazzi, Francesco and Alberto Giovannini (1989): *Limiting exchange rate flexibility: the European Monetary System*, MIT Press, Cambridge, Massachusetts.
- Grüner, Hans P. (1993): "Signaling in a one period game with monetary target announcements and a repeated game with hysteresis in unemployment", Document 93-19, DELTA, Paris.
- Grüner, Hans P. (1994): "Monetary target announcement, reputation and hysteresis", University of Konstanz, SFB-discussion paper# 222, Paper presented at EEA 1994 congress.
- Lohmann, Susanne (1992): "Optimal commitment in monetary policy: credibility versus flexibility", *American Economic Review* pp 273-268.
- Persson, Torsten and Guido Tabellini (1993): "Designing institutions for monetary stability", Paper presented at the Carnegie-Rochester Conference, Pittsburgh.
- Rogoff, Kenneth (1985): "The optimal degree of commitment to an intermediate monetary target" *Quarterly Journal of Economics* 100, pp. 1169-1190.
- Walsh, Carl (forthcoming) "Optimal contracts for central bankers", *American Economic Review*.

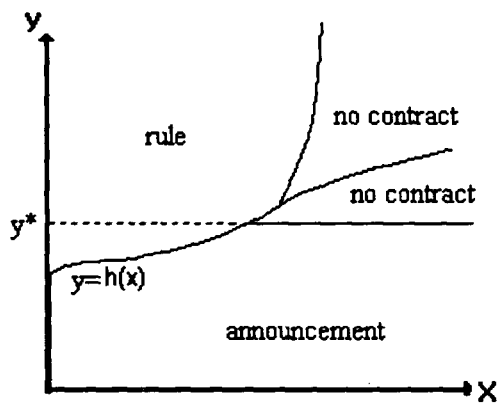


Figure 1: The three regimes.
Above $h(x)$ the rule is better than the announcement.
Above y^* no contract is better than the announcement.