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Monetary Policy Analysis with
Potentially Misspecified Models

Marco Del Negro and Frank Schorfheide

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Abstract: The paper proposes a novel method for conducting policy analysis with potentially misspecified dynamic stochastic general equilibrium (DSGE) models and applies it to a New Keynesian DSGE model along the lines of Christiano, Eichenbaum, and Evans (JPE 2005) and Smets and Wouters (JEEA 2003). We first quantify the degree of model misspecification and then illustrate its implications for the performance of different interest rate feedback rules. We find that many of the prescriptions derived from the DSGE model are robust to model misspecification.

JEL classification: C32

Key words: Bayesian analysis, DSGE models, model misspecification

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1 Introduction

The quantitative evaluation of monetary policy rules plays an important role in the design of stabilization policies. Much of the recent debate about optimal monetary policy rules has been carried out with New Keynesian dynamic stochastic general equilibrium (DSGE) models (see Woodford 2003). DSGE models have the advantage that one can explicitly assess the effect of policy regime changes on expectation formation and decision rules of private agents. Yet, until recently, these models were scarcely used by central banks because they were perceived as being inferior to less structural model in terms of fit. Recent work by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003) has changed this perception. These papers show that the fit of large scale New Keynesian DSGE models is comparable to that of more heavily parameterized models. As a consequence, a number of central banks have begun to show interest in these models as tools for quantitative policy analysis. Despite the success in improving the empirical performance of DSGE models, misspecification remains a concern, as shown in Del Negro, Schorfheide, Smets, and Wouters (2004).

Accounting for misspecification and model uncertainty in general is an important aspect of the assessment of monetary policies. After all, it has long been recognized that model and parameter uncertainty affects optimal policies, e.g., Brainard (1967), Chow (1975), and Craine (1979).¹ This paper proposes a new method for taking model misspecification into account when assessing the performance of alternative interest-rate feedback rules. We apply this approach to evaluate different policies in the context of a New Keynesian DSGE model, including the estimated Volcker-Greenspan rule.

A natural approach in the presence of model uncertainty is to evaluate policy rules within all the model specifications that are under consideration. In choosing the best performing rule one can either follow a Bayesian route, assigning probabilities to models and minimizing the overall posterior expected loss. Alternatively, one can follow a minimax strategy by adopting a policy that minimizes the worst-case loss across models. The literature contains numerous applications of these ideas, e.g., McCallum (1988), Levin, Wieland, and Williams (1999, 2003), Rudebusch (2001, 2003), Onatski and Stock (2002), Onatski and Williams (2003), Brock, Durlauf, and West (2004), Cogley and Sargent (2005), and Hansen and Sargent (2005). All these papers differ with regard to the type of models included in the

¹The distinction between model and parameter uncertainty is somewhat artificial. By enlarging the parameter space appropriately, we can always absorb different model specifications.

model set, and the formulation of the decision problem that leads to the choice of a preferred policy.

Our analysis differs from previous work with respect to the method that is used to construct a model set and assign probabilities to these models so that a Bayesian calculation of posterior expected performance is possible. We start from a DSGE model in which monetary policy is modelled through an interest-rate feedback rule. The DSGE model imposes cross-coefficient restrictions on a vector autoregressive representation of the data. While we are assuming that the data obey a vector autoregressive law of motion, we allow for potential deviations from the cross-coefficient restrictions. At the same time we maintain the assumption that monetary policy follows the same interest-rate rule as in the DSGE model. This collection of identified vector autoregressions forms our model set.

We place a prior distribution on the structural parameters of the DSGE model and on parameters that characterize the discrepancies between the DSGE model restrictions and the vector autoregressive law of motion. In constructing this prior we follow our earlier work in Del Negro and Schorfheide (2004). Roughly speaking, the prior for the discrepancies is centered at zero and has a covariance matrix that is scaled by a hyperparameter which is denoted by λ . If λ is large then the prior for the discrepancies concentrates near zero reflecting the belief that a potential misspecification of the DSGE model restrictions is small. Vice versa, small values of λ induce a prior that implies large misspecification.

Our procedure works in two steps. First, we construct a posterior for the hyperparameter λ , determining the overall extent of misspecification on the basis of the available sample. Conditional on the estimated λ we compute a posterior for the DSGE model and the discrepancy parameters. Next, we evaluate the outcome of alternative policy rules under the simplifying assumption that the public believes the new policy to be in place indefinitely after being announced credibly. We evaluate different policies by studying their impact on the volatility of the three variables we use for estimation: inflation, output gap, and the interest rate. For expositional purposes we sometimes use a summary measure that weights the variances of these three variables. An important feature of our analysis is that we use the historical observations to learn about the degree of DSGE model misspecification. The smaller the estimated degree of misspecification, the more our policy analysis relies on the DSGE model.

Following early work by McCallum (1988) a number of recent papers studies the performance of different interest-rate feedback rules across a variety of macroeconomic models including models that are currently use by the Board of Governors and the European Central

Bank to analyze monetary policy. Examples of this line of research include Levin, Wieland, and Williams (1999, 2003), Taylor (1999), Coenen (2003), Levin and Williams (2003), Brock, Durlauf, and West (2004), and Adalid, Coenen, McAdam, and Siviero (2005). As in our paper, policy performance is measured by a weighted average of unconditional variances of output, inflation, and interest rates. While the class of models considered in these papers is arguably broad and contains both forward-looking as well as backward-looking specifications, with the exception of Brock, Durlauf, and West (2004) little or no attention is paid to fit and forecasting performance when weighting predictions from various models. Hence, potentially too much weight is given to specifications that are clearly at odds with the data. By using likelihood-based measures of fit, the approach proposed in this paper guarantees that model specifications that have performed well historically receive a lot of weight in the policy loss calculation.

Our framework is rich enough to encompass existing approaches to policy analysis such as the evaluation of policy rules directly based on DSGE models or by replacing the policy rule in identified vector autoregressions, e.g. Sims (1999), as polar cases. Specifically, we consider four assumptions about the policy invariance of the misspecification parameters and calculate posterior expected variances as a function of the policy parameters. The first scenario – used as a benchmark – simply ignores misspecification, and computes the variances assuming the DSGE model correctly describes the data. The first scenario corresponds to the type of analysis conducted by Laforte (2003), and Levin, Onatski, Williams, and Williams (2005). These papers estimate New Keynesian DSGE models similar to the one used here with Bayesian methods to study the effects of uncertainty about structural parameters on optimal monetary policy. If uncertainty about taste and technology parameters is certainly important, we find that the uncertainty about the model specification is as at least equally important. Our estimates of λ indeed suggest that policymakers cannot afford to ignore misspecification in the New Keynesian model.²

The remaining three scenarios are an attempt to incorporate the concern about model specification in the policy recommendations derived from DSGE models. The second scenario assumes that the policymaker is willing to learn from historical data about the overall degree of model misspecification, but not about its precise nature. In computing the expected variances for a given policy she therefore uses her prior beliefs about the misspecification parameters conditional on λ . In the third and fourth scenarios the policymaker

²The out-of-sample forecasting results obtained by Del Negro, Schorfheide, Smets, and Wouters (2004) are consistent with this conclusion.

learns from the data about the misspecification parameters, that is, uses the posterior distribution of the misspecification parameters in computing the loss. In the third scenario the policymaker assumes that misspecification is policy invariant, while in the fourth scenario she uses the conditional distribution of the misspecification and policy parameters to let the misspecification vary with policy.

The trade-off between theoretical coherence and empirical fit of econometric models poses a challenge to quantitative policy evaluation. The optimization-based general equilibrium structure of DSGE models lends credibility to its predictions of the effect of policy changes on the behavior of private agents. Moreover, the underlying utility functions can be used to construct welfare-based measures of policy performance. On the other hand, evidence on the DSGE model's inability to fit historical data as well as other specifications casts doubt on the model's quantitative implications. We view the DSGE-VAR framework developed in this paper as a diagnostic tool. If the estimate of λ provides evidence of misspecification, our methods down-weight the DSGE model predictions of changes in private sector behavior and rely more on the estimated reduced-form representation of household and firm behavior. Hence, policy recommendations obtained from the DSGE model can be evaluated under data-driven perturbations of the DSGE model restrictions.

We find that the outcomes associated with different policies change as misspecification is taken into account.³ In particular, the increase in inflation volatility associated with policies that deviate from the DSGE model prescriptions may not be nearly as large as policy analysis under the DSGE model would suggest. At the same time, our results indicate that following those prescriptions, even under misspecification, leads to outcomes that are not substantially inferior to those of rules that perform slightly better under misspecification. In summary, a fairly robust policy recommendation emerges from our analysis: the central bank should avoid strong responses to output gap movements and not react weakly to inflation fluctuations. Moreover, we find that the best-performing rules are those with a substantial degree of inertia in the policy instrument. An implication of these findings is that the gains associated with deviating from the historical Volcker-Greenspan policy, whenever positive, are generally not very large. This suggests that the historical rule, if not always optimal among those we consider, has been reasonably good at least from the perspective of this New Keynesian DSGE model, even taking misspecification into account.

³Preliminary empirical results based on a simple three-equation New Keynesian model without capital accumulation and variable factor utilization were reported in the 2005 Proceedings Volume of the Journal of the European Economic Association, Del Negro and Schorfheide (2005).

The literature is fairly divided between Bayesian and minimax or robust approaches to resolve model uncertainty. Rather than placing a prior distribution on the misspecification parameters, the robustness literature specifies either a static or dynamic two-player zero-sum game in which a malevolent “nature” chooses the misspecification parameters to harm the policymaker. Examples are Tetlow and von zur Muehlen (2001), Giannoni (2002), Onatski and Stock (2002), some of the analysis in Levin and Williams (2003), Onatski and Williams (2003), and the robust control approach developed in the monograph by Hansen and Sargent (2005). The disadvantage of this approach is that the resulting policy performs well in the worst-case but possibly poorly on average. A key difficulty in the use of minimax rules is to constrain the model set and to bound the worst case. In most formulations of the minimax problem the policymaker does not use historical data to learn about the extent of model misspecification. An exception is Onatski and Williams (2003) who bound deviations from the Rudebusch-Svenson (1999) model in a minimax calculation based on a confidence set derived from a posterior distribution. In Hansen and Sargent’s approach the relevant model set is constructed by bounding the Kullback-Leibler discrepancy between the models contained in the set and a reference model, so that it becomes difficult to discriminate among them based on statistical methods. In fact, the minimax approach is arguably most compelling if there is little or no empirical evidence available that can discriminate between model specifications.

While our paper emphasizes a Bayesian resolution of uncertainty, our framework is general enough to enable a risk-sensitive analysis. More specifically, using a result from Jacobsen (1973) we compute posterior expected losses for an exponential transformation of our summary measure of weighted variances. The resulting risk can be interpreted as the Nash-equilibrium of a zero-sum game in which “nature” distorts the probability distribution of the misspecification parameters subject to a penalty that is a function of the Kullback-Leibler discrepancy between the distorted and the non-distorted probabilities.

The paper is organized as follows. The DSGE model is presented in Section 2. This model is based on work by Altig, Christiano, Eichenbaum, and Linde (2002), Smets and Wouters (2003), and Christiano, Eichenbaum, and Evans (2005). Compared to the benchmark New Keynesian models discussed, for instance, in Woodford (2003), our model has been subjected to a number of modifications, all designed to improve its empirical fit. Section 3 discusses the estimation of potentially misspecified DSGE models. Bayesian inference is implemented through Markov Chain Monte Carlo methods described in the Appendix. The framework for policy analysis is introduced in Section 4. Section 5 describes the data

set and discusses our empirical findings, and Section 6 concludes.

2 Model

This section describes the DSGE model, which is based on work by Altig, Christiano, Eichenbaum, and Linde (2002), Smets and Wouters (2003), and Christiano, Eichenbaum, and Evans (2005). The model contains nominal price rigidities, capital accumulation subject to adjustment costs, variable factor utilization, and habit formation. Unlike the above-mentioned authors we restrict our empirical analysis to three variables: nominal interest rates, inflation rates and the output gap. Moreover, we abstract from wage rigidities. The reason for this departure is computational. Bayesian posterior calculations in the framework developed in Section 3 are more difficult to implement than in the direct estimation pursued in Smets and Wouters (2003) and the alternative DSGE-VAR setup used in Del Negro and Schorfheide (2004) and Del Negro, Schorfheide, Smets, and Wouters (2004). Since we do not use wage data in our empirical analysis it is difficult to disentangle price and wage rigidity. Indeed, we did estimate a version of the model that includes wage rigidity but found its fit (adjusted for the additional model complexity) to be worse than the baseline specification with flexible wages.

2.1 Final goods producers

The final good Y_t is a composite made of a continuum of intermediate goods $Y_t(i)$, indexed by $i \in [0, 1]$:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_f}} di \right]^{1+\lambda_f}. \quad (1)$$

The final goods producers are perfectly competitive firms that buy intermediate goods, combine them to the final product Y_t , and resell the final good to consumers. The firms maximize profits

$$P_t Y_t - \int P_t(i) Y_t(i) di$$

subject to (1). Here P_t denotes the price of the final good and $P_t(i)$ is the price of intermediate good i . From their first order conditions and the zero-profit condition we obtain that:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda_f}{\lambda_f}} Y_t \quad \text{and} \quad P_t = \left[\int_0^1 P_t(i)^{\frac{1}{\lambda_f}} di \right]^{\lambda_f}. \quad (2)$$

We define aggregate inflation as $\pi_t = P_t/P_{t-1}$.

2.2 Intermediate goods producers

Good i is made using the technology:

$$Y_t(i) = \max \left\{ K_t(i)^\alpha [\gamma(t) Z_t L_t(i)]^{1-\alpha}, 0 \right\}, \quad (3)$$

where the technology shock Z_t (common across all firms) follows a stationary autoregressive process

$$\ln(Z_t/Z^*) = \rho_z \ln(Z_{t-1}/Z^*) + \sigma_z \epsilon_{z,t} \quad (4)$$

and the function $\gamma(t)$ induces a trend into productivity. All firms face the same prices for their inputs, labor and capital. Hence cost minimization implies that the capital/labor ratio is the same for all firms, and equal to:

$$\frac{K_t}{L_t} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k}, \quad (5)$$

where W_t is the nominal wage and R_t^k is the rental rate of capital. Following Calvo (1983) we assume that in every period a fraction of firms ζ_p is unable to re-optimize their prices $P_t(i)$. These firms adjust their prices mechanically according to

$$P_t(i) = \pi^* P_{t-1}(i), \quad (6)$$

where π^* is the steady state inflation rate of the final good.⁴ Firms that are able to re-optimize prices choose the price level $\tilde{P}_t(i)$ by solving

$$\begin{aligned} \max_{\tilde{P}_t(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \Xi_{t+s}^p \left(\tilde{P}_t(i) (\pi^*)^s - MC_{t+s} \right) Y_{t+s}(i) \\ \text{s.t. } Y_{t+s}(i) = \left(\frac{\tilde{P}_t(i) (\pi^*)^s}{P_{t+s}} \right)^{-\frac{1+\lambda_f}{\lambda_f}} Y_{t+s}, \quad MC_{t+s} = \frac{\alpha^{-\alpha} W_{t+s}^{1-\alpha} R_{t+s}^k \alpha}{(1-\alpha)^{(1-\alpha)} Z_{t+s}^{1-\alpha}}. \end{aligned} \quad (7)$$

where $\beta^s \Xi_{t+s}^p$ is today's value of a future dollar for the consumers and MC_t reflects marginal costs. We consider only the symmetric equilibrium where all firms will choose the same $\tilde{P}_t(i)$. Hence from (2) we obtain the following law of motion for the aggregate price level:

$$P_t = \left[(1 - \zeta_p) \tilde{P}_t^{\frac{1}{\lambda_f}} + \zeta_p (\pi^* P_{t-1})^{\frac{1}{\lambda_f}} \right]^{\lambda_f}. \quad (8)$$

2.3 Households

There is a continuum of households, indexed by $j \in [0, 1]$. The objective function for each household is given by:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\log(C_{t+s}(j) - hC_{t+s-1}(j)) - \frac{\varphi}{1+\nu_l} L_{t+s}(j)^{1+\nu_l} + \frac{\chi}{1-\nu_m} \left(\frac{M_{t+s}(j)}{\gamma(t+s)P_{t+s}} \right)^{1-\nu_m} \right] \quad (9)$$

⁴We also estimated a version of the DSGE model with dynamic price indexation $P_t(i) = \pi_{t-1} P_{t-1}(i)$ but found that the time series fit did not improve.

where $C_t(j)$ is consumption, $L_t(j)$ is labor supply, and $M_t(j)$ are money holdings. Households' preferences display habit-persistence. Real money balances enter the utility function deflated by the trend growth of the economy to make real money demand stationary.

Household j 's budget constraint written in nominal terms is given by:

$$P_{t+s}C_{t+s}(j) + P_{t+s}I_{t+s}(j) + B_{t+s}(j) + M_{t+s}(j) \leq R_{t+s}B_{t+s-1}(j) + M_{t+s-1}(j) + A_{t+s-1}(j) + \Pi_{t+s} + W_{t+s}(j)L_{t+s}(j) + (R_{t+s}^k u_{t+s}(j)\bar{K}_{t+s-1}(j) - P_{t+s}a(u_{t+s}(j))\bar{K}_{t+s-1}(j)), \quad (10)$$

where $I_t(j)$ is investment, $B_t(j)$ is holdings of government bonds, R_t is the gross nominal interest rate paid on government bonds, $A_t(j)$ is the net cash inflow from trading state-contingent securities, Π_t is the per-capita profit the household gets from owning firms (households pool their firm shares, and they all receive the same profit), and $W_t(j)$ is the nominal wage earned by household j . The term within parenthesis represents the return to owning $\bar{K}_t(j)$ units of capital. Households choose the utilization rate of their own capital, $u_t(j)$. Households rent to firms in period t an amount of effective capital equal to:

$$K_t(j) = u_t(j)\bar{K}_{t-1}(j), \quad (11)$$

and receive $R_t^k u_t(j)\bar{K}_{t-1}(j)$ in return. They however have to pay a cost of utilization in terms of the consumption good equal to $a(u_t(j))\bar{K}_{t-1}(j)$. Households accumulate capital according to the equation:

$$\bar{K}_t(j) = (1 - \delta)\bar{K}_{t-1}(j) + \left(1 - S\left(\frac{I_t(j)}{I_{t-1}(j)}\right)\right) I_t(j), \quad (12)$$

where δ is the rate of depreciation, and $S(\cdot)$ is the cost of adjusting investment, with $S'(\cdot) > 0, S''(\cdot) > 0$.

We assume that the labor market is perfectly competitive and wages are flexible. Finally, we assume there is a complete set of state contingent securities in nominal terms, which implies that the Lagrange multiplier associated with (10) must be the same for all households in all periods and across all states of the world. This in turn implies that in equilibrium households will make the same choices and can be aggregated into a representative household.

2.4 Government Policies

The central bank follows a nominal interest rate rule by adjusting its instrument in response to deviations of inflation and output from their respective target levels:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_*}\right)^{\psi_1} \left(\frac{Y_t}{Y_{t-1}^p}\right)^{\psi_2} \right]^{1-\rho_R} \sigma_R e^{\epsilon_{R,t}}, \quad (13)$$

where R^* is the steady state nominal rate, Y_t^p is a measure of potential output, and the parameter ρ_R determines the degree of interest rate smoothing. The government budget constraint is of the form

$$P_t G_t + R_{t-1} B_{t-1} + M_{t-1} = T_t + M_t + B_t, \quad (14)$$

where T_t are nominal lump-sum taxes (or subsidies) that also appear in household's budget constraint. Government spending is given by:

$$G_t = (1 - 1/g_t) Y_t, \quad (15)$$

where g_t follows the process:

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \sigma_g \epsilon_{g,t}. \quad (16)$$

2.5 Resource constraint

The aggregate resource constraint

$$C_t + I_t + a(u_t) \bar{K}_{t-1} = \frac{1}{g_t} Y_t \quad (17)$$

can be derived by integrating the budget constraint (10) across households, and combining it with the government budget constraint (14) and the zero profit conditions of the final good producers.

2.6 Model solution and State-Space Representation

Our model economy evolves along a balanced growth path, generated by the trend in technology. Output, consumption, investment, real wage, physical capital and effective capital all grow according to $\gamma(t)$. Nominal interest rates, inflation, and hours worked are stationary. The model can be rewritten in terms of detrended variables. We find the steady states for the detrended variables and use the method in Sims (2002) to construct a log-linear approximation of the model around the steady state.

Our empirical analysis is based on data on nominal interest rates R_t^a (annualized percentages), inflation rates π_t^a (annualized percentages), and the log output gap. Hence, we define the vector of observations as $y_t = [R_t^a, \pi_t^a, 100 \ln Y_t^g]'$. The relationships between the percentage deviations from steady state $\tilde{R}_t, \tilde{\pi}_t, \tilde{Y}_t$ derived from the DSGE model and the observables y_t are given by the following measurement equation:

$$\begin{aligned} y_{1,t} &= r_a^* + \gamma_a + \pi_a^* + 4\tilde{R}_t, \\ y_{2,t} &= \begin{bmatrix} \pi_a^* + 4\tilde{\pi}_t \\ \tilde{Y}_t \end{bmatrix}. \end{aligned} \quad (18)$$

Here, we have partitioned y_t such that $y_{1,t}$ corresponds to the policymaker's instrument (the interest rate), and $y_{2,t}$ is a vector that includes the remaining two observables. The steady state (net) real interest rate in our model is given by $r_a^* + \gamma_a$, where γ_a is the average annual growth rate of the economy. The parameter r_a^* is related to the discount rate β according to $\beta = 1/(1 + r_a^*/400)$. Within the model, \tilde{Y}_t denotes the percentage deviation of output from its trend path $\gamma(t)Y^*$. We interpret the potential output series published by the Congressional Budget Office (CBO) as a measure of $\gamma(t)Y^*$. Hence, the output gap, computed as log difference of real and potential GDP provides us with a measure of \tilde{Y}_t . The monetary policy rule can be rewritten in terms of observables as follows:

$$y_{1,t} = (1 - \rho_R)[(r_a^* + \gamma_a + \pi_a^*) - \psi_1 \ln \pi_a^*] + y_{1,t-1}\rho_R + y_{2,t}' \begin{bmatrix} (1 - \rho_R)\psi_1 \\ 4(1 - \rho_R)\psi_2 \end{bmatrix} + \sigma_R \epsilon_R \quad (19)$$

We collect all the DSGE model parameters in the vector θ and stack the structural shocks in the vector ϵ_t .

3 Setup and Inference

In the subsequent analysis it is assumed that the DSGE model generates a covariance-stationary distribution of the sequence $\{y_t\}$ for all $\theta \in \Theta$. Expectations under this distribution are denoted by $\mathbb{E}_\theta^D[\cdot]$. We will derive an (approximate) vector autoregressive representation for the DSGE model and introduce model misspecifications as deviations from this representation.⁵ Unlike in Del Negro and Schorfheide (2004) and Del Negro, Schorfheide, Smets, and Wouters (2004), we assume that the interest rate feedback rule in the DSGE model is correctly specified and do not relax the restriction generated by the

⁵We are working with vector autoregressive approximations rather than with state-space models to simplify the simulation of the posterior distributions.

policy rule. Finally, we specify a prior distribution for these model misspecifications and discuss posterior inference and policy analysis.

3.1 A VAR Representation of the DSGE Model

Let us rewrite Equation (13), which describes the policymaker's behavior, in more general form as:

$$y_{1,t} = x'_t M_1 \beta_1(\theta) + y'_{2,t} M_2 \beta_2(\theta) + \epsilon_{1,t}, \quad (20)$$

where $y_t = [y_{1,t}, y'_{2,t}]'$ and the $k \times 1$ vector $x_t = [y'_{t-1}, \dots, y'_{t-p}, 1]'$ is composed of the first p lags of y_t and an intercept. The shock $\epsilon_{1,t}$ corresponds to the monetary policy shock $\sigma_R \epsilon_{R,t}$ in the DSGE model. The matrices M_1 and M_2 select the appropriate elements of x_t and $y_{2,t}$ to generate the policy rule. In our application the vector M_1 selects the intercept and the lagged nominal interest rate and the matrix M_2 extracts inflation, and the output gap. The functions $\beta_1(\theta)$ and $\beta_2(\theta)$ are implicitly provided in Equation (19). Considering forecast-based policy rules in this framework would require significant modifications. However, according to the findings reported by Levin, Wieland, and Williams (2003) forecast-based rules do not provide substantial gains in stabilization performance compared with simple outcome-based rules. Hence, we decided not to pursue these modifications at this point.

The remainder of the system for y_t is given by the following reduced form equations:

$$y'_{2,t} = x'_t \Psi^*(\theta) + u'_{2,t}. \quad (21)$$

In general, the VAR representation (21) is not exact if the number of lags p is finite. We define $\Gamma_{XX}(\theta) = \mathbb{E}_\theta^D[x_t x'_t]$ and $\Gamma_{XY_2}(\theta) = \mathbb{E}_\theta^D[x_t y'_{2,t}]$ and let

$$\Psi^*(\theta) = \Gamma_{XX}^{-1}(\theta) \Gamma_{XY_2}(\theta). \quad (22)$$

Since the system is covariance stationary, the VAR approximation of the autocovariance sequence of $y_{2,t}$ can be made arbitrarily precise by increasing the number of lags p . If in addition, the moving-average (MA) representation of the DSGE model in terms of the structural shocks ϵ_t is invertible, then $u_{2,t}$ can also be expressed as a function of ϵ_t for large p . Conditions for invertibility and results on the accuracy of this VAR approximation can be found in Fernandez-Villaverde, Rubio-Ramirez, and Sargent (2004).

The equation for the policy instrument (20) can be rewritten by replacing $y_{2,t}$ with expression (21):

$$y_{1,t} = x'_t M_1 \beta_1(\theta) + x'_t \Psi^*(\theta) M_2 \beta_2(\theta) + u_{1,t}, \quad (23)$$

where $u_{1,t} = u'_{2,t}M_2\beta_2(\theta) + \epsilon_{1,t}$. Define $u'_t = [u_{1,t}, u'_{2,t}]$, $B_1(\theta) = [M_1\beta_1(\theta), 0_{k \times (n-1)}]$, $B_2(\theta) = [M_2\beta_2(\theta), I_{(n-1) \times (n-1)}]$, and let

$$\Phi^*(\theta) = B_1(\theta) + \Psi^*(\theta)B_2(\theta). \quad (24)$$

Hence, we obtain a restricted VAR for y_t

$$y'_t = x'_t\Phi + u'_t, \quad \mathbb{E}[u_t u'_t] = \Sigma^*(\theta) \quad (25)$$

with

$$\Phi = \Phi^*(\theta), \quad \Sigma = \Sigma^*(\theta) = \Gamma_{YY}(\theta) - \Gamma_{YX}(\theta)\Gamma_{XX}^{-1}(\theta)\Gamma_{XY}(\theta).$$

Here the population covariance matrices are $\Gamma_{YY}(\theta) = \mathbb{E}_\theta^D[y_t y'_t]$ and $\Gamma_{XY}(\theta) = \Gamma'_{YX}(\theta) = \mathbb{E}_\theta^D[x_t y'_t]$.

Since the monetary policy rule (13) in the DSGE model is specified so that it can be exactly reproduced by the VAR, see Equation (20), $\Phi^*(\theta)$ equals the population least squares coefficients associated with (25), and the covariance matrix of x_t under the DSGE model and its VAR approximation are identical. We will subsequently ignore the error made by approximating the state space representation of the DSGE model with the finite-order VAR or, in other words, treat (25) as the structural model that imposes – potentially misspecified – cross-equation restrictions on the matrices Φ and Σ . We can do so because we have checked that the impulse responses from the VAR representation of the DSGE model – obtained using the identification scheme discussed in Section 3.3 – match almost exactly those of the DSGE model even when the lag length is four (results are available upon request).

3.2 Misspecification and Bayesian Inference

We make the following assumptions about misspecification of the DSGE model. There is a vector θ and matrices Ψ^Δ and Σ^Δ such that the data are generated from the VAR in (25)

$$\Phi = B_1(\theta) + (\Psi^*(\theta) + \Psi^\Delta)B_2(\theta), \quad \Sigma = \Sigma^*(\theta) + \Sigma^\Delta \quad (26)$$

and there does not exist a $\tilde{\theta} \in \Theta$ such that

$$\Phi = B_1(\tilde{\theta}) + \Psi^*(\tilde{\theta})B_2(\tilde{\theta}), \quad \Sigma = \Sigma^*(\tilde{\theta}).$$

We refer to the resulting specification as DSGE-VAR. A stylized graphical representation of our notion of misspecification can be found in Figure 1. Our econometric analysis is casted

in a Bayesian framework in which initial beliefs about the DSGE model parameter θ and the model misspecification matrices Ψ^Δ and Σ^Δ are summarized in a prior distribution. In order to compare a Bayesian approach to model misspecification to a minimax approach, the reader might find it helpful to think of a fictitious other, “nature”, that draws the misspecification matrices Ψ^Δ and Σ^Δ from a distribution – the prior – rather than maximizing the loss function to harm the policymaker. The remainder of this section describes the choice of this prior.

Our prior is based on the idea that “nature” is more likely to draw smaller than larger misspecification matrices, reflecting the belief that the DSGE model provides a good albeit not perfect approximation of reality. Specifically, we assume that the prior density decreases the larger the size of the discrepancies Ψ^Δ and Σ^Δ . In the spirit of Hansen and Sargent’s (2005) approach to model misspecification and robust control, the size of the discrepancies is determined by the ease with which they can be detected via likelihood ratios. This metric determines the shape of the prior contours (see Figure 1). The mass placed on these contours is determined by the parameter λ . Large values of λ imply that large discrepancies are less likely to occur. Hence, the parameter λ measures the overall degree of misspecification. We will now further motivate and explain the prior distribution using a thought experiment, where for ease of exposition we set $\Sigma^\Delta = 0$ and fix the DSGE model parameter vector θ .

Suppose that a sample of λT observations is generated from the DSGE model (that is, from Equation (25), where $\Phi = \Phi^*$). Here T denotes the size of the actual sample used in the estimation. We will construct a prior that has the property that its density is proportional to the expected likelihood ratio of Ψ evaluated at its (misspecified) restricted value $\Psi^*(\theta)$ versus the true value $\Psi = \Psi^*(\theta) + \Psi^\Delta$. The log-likelihood ratio is

$$\ln \left[\frac{\mathcal{L}(\Psi^*, \Sigma^*, \theta | Y, X)}{\mathcal{L}(\Psi^* + \Psi^\Delta, \Sigma^*, \theta | Y, X)} \right] = -\frac{1}{2} \text{tr} \left[\Sigma^{*-1} \left(B_2' \Psi^{*'} X' X \Psi^* B_2 - 2B_2' \Psi^{*'} X' (Y - X B_1) - B_2' (\Psi^* + \Psi^\Delta)' X' X (\Psi^* + \Psi^\Delta) B_2 + 2B_2' (\Psi^* + \Psi^\Delta)' X' (Y - X B_1) \right) \right].$$

Here Y denotes the $\lambda T \times n$ matrix with rows y_t' and X_t is the $\lambda T \times k$ matrix with rows x_t' . After replacing Y by $X(B_1 + (\Psi^* + \Psi^\Delta)B_2) + U$ the log likelihood ratio simplifies to

$$\ln \left[\frac{\mathcal{L}(\Psi^*, \Sigma^*, \theta | Y, X)}{\mathcal{L}(\Psi^* + \Psi^\Delta, \Sigma^*, \theta | Y, X)} \right] = -\frac{1}{2} \text{tr} \left[\Sigma^{*-1} \left(B_2' \Psi^{\Delta'} X' X \Psi^\Delta B_2 - 2B_2' \Psi^{\Delta'} X' U \right) \right] \quad (27)$$

Taking expectations over X and U using the distribution induced by the data generating process yields (minus) the Kullback-Leibler discrepancy between the data generating process

and the DSGE model:

$$\mathbb{E}_{\Psi^*, \Sigma^*}^{VAR} \left[\ln \frac{\mathcal{L}(\Psi^*, \Sigma^*, \theta | Y, X)}{\mathcal{L}(\Psi^* + \Psi^\Delta, \Sigma^*, \theta | Y, X)} \right] = -\frac{1}{2} \text{tr} \left[\Sigma^{*-1} \left(\lambda T B_2' \Psi^\Delta \Gamma_{XX} \Psi^\Delta B_2 \right) \right]. \quad (28)$$

Here we have used the relationship $\mathbb{E}_{\Psi^*(\theta), \Sigma^*(\theta)}^{VAR} [x_t x_t'] = \mathbb{E}_\theta^D [x_t x_t'] = \Gamma_{XX}(\theta)$, where the first term refers to expectation taken with respect to the probability distribution generated by the VAR. We choose a prior density for Ψ^Δ that is proportional (\propto) to the Kullback-Leibler discrepancy:

$$p(\Psi^\Delta | \Sigma^*, \theta) \propto \exp \left\{ -\frac{\lambda T}{2} \text{tr} \left[\Sigma^{*-1} \left(B_2' \Psi^\Delta \Gamma_{XX} \Psi^\Delta B_2 \right) \right] \right\} \quad (29)$$

The hyperparameter λ determines the length of the hypothetical sample as a multiple of the actual sample size T . This hyperparameter ‘‘scales’’ the overall degree of misspecification. For high values of λ , it is easy to tell the misspecified model and the DSGE model apart even for small values of the misspecification Ψ^Δ . Hence the prior density places most of its mass near the restrictions imposed by the DSGE model when λ is large, and for $\lambda = \infty$ the misspecification disappears altogether. On the contrary, if λ is close to zero the Kullback-Leibler discrepancy can be small even for relatively large values of the discrepancy Ψ^Δ . Hence the prior is fairly diffuse. For computational reasons it is convenient to transform this prior into a prior for Ψ . Using standard arguments we deduce that this prior is multivariate normal

$$\Psi | \Sigma^*, \theta \sim \mathcal{N} \left(\Psi^*(\theta), \frac{1}{\lambda T} \left[(B_2(\theta) \Sigma^{*-1} B_2(\theta)') \otimes \Gamma_{XX}(\theta) \right]^{-1} \right). \quad (30)$$

In practice we also have to take potential misspecification of the covariance matrix $\Sigma^*(\theta)$ into account. Hence, we will use the following, slightly modified, prior distribution conditional on θ in the empirical analysis:

$$\begin{aligned} \Psi | \Sigma, \theta &\sim \mathcal{N} \left(\Psi^*(\theta), \frac{1}{\lambda T} \left[(B_2(\theta) \Sigma^{-1} B_2(\theta)') \otimes \Gamma_{XX}(\theta) \right]^{-1} \right) \\ \Sigma | \theta &\sim \mathcal{IW} \left(\lambda T \Sigma^*(\theta), \lambda T - k, n \right), \end{aligned} \quad (31)$$

where \mathcal{IW} denotes the inverted Wishart distribution. The latter induces a distribution for the discrepancy $\Sigma^\Delta = \Sigma - \Sigma^*$.

The Appendix provides a characterization of the following conditional posterior densities:

$$p(\Psi | \Sigma, \theta, Y), \quad p(\Sigma | \Psi, \theta, Y), \quad \text{and} \quad p(\theta | \Psi, \Sigma, Y).$$

Unfortunately, it is not possible to give a characterization of all conditional distributions in terms of well-known probability distributions. To implement the Gibbs sampler we have

to introduce two Metropolis steps that generate draws from the conditional distributions $p(\Sigma|\Psi, \theta, Y)$ and $p(\theta|\Psi, \Sigma, Y)$. The resulting Markov-Chain-Monte-Carlo (MCMC) algorithm is known as Metropolis-within-Gibbs sampler and allows us to generate draws from the joint posterior distribution of θ , Ψ , and Σ . In addition to the posterior distribution of the parameters we are also interested in evaluating marginal data densities of the form

$$p_\lambda(Y) = \int p(Y|\theta, \Sigma, \Phi) p_\lambda(\theta, \Sigma, \Phi) d(\theta, \Sigma, \Phi) \quad (32)$$

for various choices of the hyperparameter λ and restrictions on the parameter space of the DSGE model. Based on the marginal data densities we can compute Bayes factors and posterior probabilities for the various specifications of our model. Under the assumption of equal prior probabilities, ratios of marginal likelihoods can be interpreted as model odds. While the numerical value of λ itself may be difficult to interpret, a comparison of DSGE and DSGE-VAR impulse response functions will enable us to assess the degree of misspecification implied by a particular value of λ .

3.3 Identification

The model developed in the preceding subsections is of the form

$$\begin{aligned} y_{1,t} &= x_t' M_1 \beta_1 + x_t' \Psi M_2 \beta_2 + u_{1,t} \\ y_{2,t}' &= x_t' \Psi + u_{2,t}' \end{aligned} \quad (33)$$

where $u_{1,t} = u_{2,t}' M_2 \beta_2 + \epsilon_{1,t}$. The policy rule coefficients β_1 and β_2 , and hence the monetary policy shock $\epsilon_{1,t} = \sigma_R \epsilon_{R,t}$ are identifiable. Unlike in a standard VAR, identification is achieved through exclusion restrictions: lagged inflation and output gap do not enter the monetary policy rule by assumption. According to the underlying structural model, the one-step ahead forecast errors $u_{2,t}$ are a function of the monetary policy shock $\epsilon_{1,t}$ and the two other structural shocks $\epsilon_{2,t} = [\epsilon_{g,t}, \epsilon_{z,t}]'$:

$$u_{2,t}' = \epsilon_{1,t} A_1 + \epsilon_{2,t}' A_2.$$

Straightforward matrix algebra leads to the following formulas for the effect of the structural shocks on $u_{2,t}'$:

$$\begin{aligned} A_1 &= \left[\Sigma_{11} - \beta_2' M_2' \Sigma_{22} M_2 \beta_2 - 2(\Sigma_{12} - \beta_2' M_2' \Sigma_{22}) M_2 \beta_2 \right]^{-1} (\Sigma_{12} - \beta_2' M_2' \Sigma_{22}) \\ A_2' A_2 &= \Sigma_{22} - A_1' \left[\Sigma_{11} - \beta_2' M_2' \Sigma_{22} M_2 \beta_2 - 2(\Sigma_{12} - \beta_2' M_2' \Sigma_{22}) M_2 \beta_2 \right] A_1 \end{aligned}$$

Here, Σ denotes the covariance matrix of u_t , and the partitions of Σ conform with the partition of $u'_t = [u_{1,t}, u'_{2,t}]$. Once A_1 is determined, the impulse response function with respect to the monetary policy shock can be calculated.

In order to identify A_2 an additional assumption is needed. We follow the approach taken in Del Negro and Schorfheide (2004). Let $A'_{2,tr}A_{2,tr} = A'_2A_2$ be the Cholesky decomposition of A'_2A_2 . The relationship between $A_{2,tr}$ and A_2 is given by $A'_2 = A'_{2,tr}\Omega$, where Ω is an orthonormal matrix that is not identifiable based on the estimates of $\beta(\theta)$, Ψ , and Σ . However, we are able to calculate an initial effect of $\epsilon_{2,t}$ on $y_{2,t}$ based on the DSGE model, denoted by $A_2^D(\theta)$. This matrix can be uniquely decomposed into a lower triangular matrix and an orthonormal matrix:

$$A_2^{D'}(\theta) = A_{2,tr}^{D'}(\theta)\Omega^*(\theta).$$

To identify A_2 above, we combine $A'_{2,tr}$ with $\Omega^*(\theta)$.⁶ Loosely speaking, the rotation matrix is constructed such that in the absence of misspecification the DSGE's and the DSGE-VAR's impulse responses to $\epsilon_{2,t}$ would coincide. To the extent that misspecification is mainly in the dynamics as opposed to the covariance matrix of innovations, the identification procedure can be interpreted as matching, at least qualitatively, the short-run responses of the VAR with those from the DSGE model.

4 Policy Analysis

We are interested in assessing the effects of changing the existing policy rule between time and on the fluctuations of the interest rate, inflation, and the output gap. We make the simplifying assumption that the public believes the new policy to be in place indefinitely after being announced credibly. The policymaker does not exploit the fact that at the time of the announcement the public has formed its expectations based on the old policy rule. Our assumptions characterize one of the polar cases that arises in a more realistic setting in which there are multiple types of policy shifts (using the terminology of Sims, 1982) ranging from normal policy making that corresponds to unanticipated deviations from a policy rule to rare regime shifts that can be viewed as persistent changes in the central bank's systematic reaction to macroeconomic fundamentals.

⁶The calculation is easily implementable in a Markov Chain Monte Carlo analysis. For every draw of $(\theta, \Psi^\Delta, \Sigma^\Delta)$ from their joint posterior distribution we compute $\Omega^*(\theta)$ and A_2 .

4.1 Evaluating Policies

The goal of our analysis is to study the effects of changing in the policy parameters θ_p on the variability of key macroeconomic variables, namely output gap, inflation, and interest rates. We partition the vector θ into policy rule parameters θ_p and taste-and-technology parameters θ_s and denote the unconditional covariance matrix of the endogenous variables by $\mathbf{V}(\theta_p, \theta_s, \Psi^\Delta, \Sigma^\Delta)$. Consistent with the DSGE model, we assume that the mean of the variables is invariant to changes in the policy parameters ψ_1 , ψ_2 , and ρ_R . In our empirical analysis we examine $tr[\mathcal{W}\mathbf{V}]$ for various choices of the weight matrix \mathcal{W} . Here $tr[\cdot]$ denotes the trace of a matrix. In particular we consider the variances of individual variables which can be obtained by setting all elements of \mathcal{W} to zero, except for one diagonal element which is set equal to one. Moreover, we consider a summary measure of performance by choosing a \mathcal{W} that is diagonal with entries 1/4 (annualized interest rates), 1 (annualized inflation rates), and 1/4 (output gap, percentage deviations from potential output). To ensure that the posterior expected value of the variance is well defined when averaging over θ_s , Ψ^Δ , and Σ^Δ , we truncate the weighted variances at the level \mathcal{B} , that is, we report

$$\min \{\mathcal{B}, tr[\mathcal{W}\mathbf{V}(\theta_p, \theta_s, \Psi^\Delta, \Sigma^\Delta)]\}$$

The upper bound \mathcal{B} is set to 50. This value is substantially larger than the sample variances of the output gap, inflation, and interest rates, which are approximately 2.9, 2.1, and 6, respectively, in our estimation sample.

One advantage of conducting policy evaluation under DSGE models is the use of welfare-based measures. These measures lose some of their appeal in presence of misspecification: To the extent that the discrepancies Ψ^Δ and Σ^Δ are caused by a misspecification of the households' preferences, a representation of consumer welfare derived from the DSGE model is potentially unreliable. Therefore, we choose to focus on the implications of different rules on the variability of the endogenous variables. Arguably this is also the measure used in practice by many central banks in evaluating policies.

We interpret monetary policy shocks as discretionary deviations from the interest-rate feedback rule. Since the monetary policy shocks are assumed to be uncorrelated with the remaining structural shocks in the economy they increase the variability of the endogenous variables. Hence, deviations $\epsilon_{R,t}$ from the policy rule are undesirable. In our empirical analysis we therefore use the procedure described in Section 3.3 to identify the monetary policy shocks and then set their standard deviation to zero, focusing on the fluctuations caused by technology and government spending shocks.

4.2 Taking Misspecification into Account

We assume that there is imperfect knowledge about: (i) the policy invariant private sectors' taste and technology parameters θ_s and (ii) the degree of model misspecification captured by λ , Ψ^Δ and Σ^Δ . The uncertainty is summarized in the posterior distribution.

We consider four different scenarios for the policy invariance of the misspecification matrices Φ^Δ and Σ^Δ . Then we calculate the posterior expectations of the performance measure $\min \{\mathcal{B}, tr[\mathcal{W}\mathbf{V}]\}$ for different policies according to each scenario. If the DSGE model does not suffer from serious misspecification all scenarios collapse to Scenario 1. The goal of the subsequent empirical analysis is to illustrate the sensitivity of policy predictions when the DSGE model is embedded in a collection of identified VARs.

The challenge in the evaluation of monetary policy rules is to predict the private sectors' behavioral responses to policy regime changes. Standard VAR analysis proceeds under the assumption that the behavioral changes are negligible. Our framework creates a link between VAR and DSGE model parameter. Hence, it becomes possible to re-calculate the private sector decision rules with the DSGE model and exploit the link to make non-trivial predictions about the private sector behavior with the identified VAR. If the data contain substantial evidence against the DSGE model restrictions, that is, λ is small, less weight is placed on the DSGE model predictions about the behavior of the private sector.

Scenario 1 – Ignore Misspecification

The DSGE model is estimated directly and its potential misspecification is ignored. The policymaker does, however, take the uncertainty with respect to the non-policy parameters into account when calculating the expected loss. This scenario is explored in detail by Laforte (2003) and Levin, Onatski, Williams, and Williams (2005). If no deviations from the DSGE model restrictions are contemplated then the ad-hoc performance measure introduced in the previous subsection could be replaced by the households' utility function. Unfortunately, the empirical evidence points toward misspecifications. Once we allow for deviations from $\Psi^*(\theta)$ and $\Sigma^*(\theta)$, it is doubtful that the DSGE model-based welfare function remains appropriate. Hence, we study the effects of policy changes on inflation, output gap, and interest rate variability in all four scenarios.

Scenario 2 – Acknowledge Misspecification, Discard the Past

The policymaker believes that the sample (hence the posterior) provides no information about potential misspecification after a regime shift has been implemented. This skepticism

about the relevance of sample information is shared by the robust control approaches of Onatski and Stock (2002), Onatski and Williams (2003), and Hansen and Sargent (2005). However, instead of using a minimax calculation, our Bayesian policymaker relies on her prior distribution $p(\Psi^\Delta, \Sigma^\Delta | \theta, \lambda)$ to cope with uncertainty about model misspecification. The sample is only used to learn about the non-policy parameters θ_s and the overall degree of misspecification λ .

Scenario 3 – Learn about Misspecification (Policy Invariant)

Ψ^Δ and Σ^Δ are assumed to be invariant to changes in policy. The sample information is used to learn about the model misspecification via the posterior distribution. Looking forward, the information is used to adjust the policy predictions derived from the DSGE model by the estimated discrepancies. To implement the analysis, we generate draws from the marginal posterior distribution of θ_s , Ψ^Δ , and Σ^Δ , combine $\tilde{\theta} = [\tilde{\theta}'_p, \theta'_s]'$, and calculate $\Psi^*(\tilde{\theta}) + \Psi^\Delta$ and $\Sigma^*(\tilde{\theta}) + \Sigma^\Delta$. Here, $\tilde{\theta}_p$ is the new set of policy parameters. Since the choice of $\tilde{\theta}_p$ does not affect beliefs about the misspecification matrices we refer to the treatment of misspecification as *policy invariant*. This rather mechanical post-intervention adjustment of $\Psi^*(\theta)$ and $\Sigma^*(\theta)$ has some undesirable properties which will become evident in the empirical analysis.

Scenario 4 – Learn about Misspecification (Conditional)

“Nature” generates a new set of draws from the posterior distribution of Ψ^Δ and Σ^Δ conditional on the post-intervention DSGE model parameters $\tilde{\theta}$. To implement the expected variance calculation we take a draw from the marginal posterior distribution of θ_s , combine it with the policy parameter to obtain $\tilde{\theta} = [\tilde{\theta}'_p, \theta'_s]'$, and generate a draw from $p(\Psi^\Delta, \Sigma^\Delta | Y^T, \tilde{\theta}, \lambda)$ by iterating between the conditional distributions of Ψ^Δ and Σ^Δ provided in the Appendix (see Equations (A.5) and (A.7)). As before, we then calculate $\Psi^*(\tilde{\theta}) + \Psi^\Delta$ and $\Sigma^*(\tilde{\theta}) + \Sigma^\Delta$. In this scenario, the policymaker revises her beliefs about the misspecification matrix as she contemplates different values of the policy parameters. Hence, we use the term *conditional*. Roughly speaking, the calculation can be interpreted as follows: based on the data there is uncertainty about the historical policy rule coefficients. Now suppose one fixes the policy parameters at a particular value $\tilde{\theta}_p$: what information do the data provide about the misspecification parameters? This information is summarized in $p(\Psi^\Delta, \Sigma^\Delta | Y^T, \tilde{\theta}, \lambda)$. If the estimated value of λ is small and hence the perceived DSGE model misspecification is large, the DSGE model’s prediction of the effect of policy rule changes on the reduced form equations for the private sector become less credible. In our

analysis the conditional posterior distribution of Ψ and Σ given θ will effectively become insensitive to θ . As λ tends to zero we effectively analyze monetary policy with a VAR by simply changing the coefficients in the policy rule, ignoring any changes in private-sector behavior that the policy shift might induce as in Sims (1999).

4.3 Risk-Sensitivity

So far, we placed a probability distribution over the misspecification parameters and minimized posterior expected loss. There is a growing literature in economics⁷ that studies the robustness of decision rules to model misspecification. Underlying this robustness analysis is typically a static or dynamic two-person zero-sum game. The decision maker, in our case the central bank, is minimizing a loss function while a malevolent fictitious other, “nature”, chooses the misspecification to harm the decision maker. “Nature’s” choice, in our notation Ψ^Δ and Σ^Δ , is either limited to a bounded set or it is subject to a penalty function that is increasing in the size of the misspecification. The policymaker’s decision is robust, if it corresponds to a Nash equilibrium in the two-person game.

In the Bayesian framework the risk sensitivity that is inherent in the robust control approach can be introduced by transforming a loss function. Let

$$L(\theta_p, \theta_s, \Psi^\Delta, \Sigma^\Delta) = \min \{ \mathcal{B}, \text{tr}[\mathcal{W}\mathbf{V}(\theta_p, \theta_s, \Psi^\Delta, \Sigma^\Delta)] \}.$$

Instead of minimizing the expected value of $L(\theta_p, \theta_s, \Psi^\Delta, \Sigma^\Delta)$, the decision maker is equipped with an exponential utility function. She considers the transformed loss $e^{\tau L}$, and solves

$$\min_{\theta_p} \frac{1}{\tau} \ln \int \exp\{\tau L(\theta_p, \theta_s, \Psi^\Delta, \Sigma^\Delta)\} p(\theta_s, \Psi^\Delta, \Sigma^\Delta) d(\theta_s, \Psi^\Delta, \Sigma^\Delta), \quad (34)$$

where $p(\theta_s, \Psi^\Delta, \Sigma^\Delta)$ denotes the joint density of $\theta_s, \Psi^\Delta, \Sigma^\Delta$. A positive τ makes the policymaker risk averse. It can be shown that the optimization of (34) is the solution to the following zero-sum game

$$\begin{aligned} \min_{\theta_p} \max_{q(\theta_s, \Psi^\Delta, \Sigma^\Delta)} & \int L(\theta_p, \theta_s, \Psi^\Delta, \Sigma^\Delta) p(\theta_s, \Psi^\Delta, \Sigma^\Delta) q(\theta_s, \Psi^\Delta, \Sigma^\Delta) d(\theta_s, \Psi^\Delta, \Sigma^\Delta) \\ & - \frac{1}{\tau} \int \left(\ln q(\theta_s, \Psi^\Delta, \Sigma^\Delta) \right) p(\theta_s, \Psi^\Delta, \Sigma^\Delta) q(\theta_s, \Psi^\Delta, \Sigma^\Delta) d(\theta_s, \Psi^\Delta, \Sigma^\Delta). \end{aligned} \quad (35)$$

The maximization with respect to $q(\cdot)$ is subject to the constraints

$$\int p(\theta_s, \Psi^\Delta, \Sigma^\Delta) q(\theta_s, \Psi^\Delta, \Sigma^\Delta) d(\theta_s, \Psi^\Delta, \Sigma^\Delta) = 1, \quad q(\theta_s, \Psi^\Delta, \Sigma^\Delta) \geq 0.$$

⁷See for instance, the monograph by Hansen and Sargent (2005) or the February 2002 special issue of *Macroeconomic Dynamics*.

The interpretation of this game is that “nature” chooses the function $q(\cdot)$ to distort the probabilities from which the model (misspecification) parameters are drawn. Notice that

$$\begin{aligned} & \int [\ln q(\cdot)] p(\cdot) q(\cdot) d(\theta_s, \Psi^\Delta, \Sigma^\Delta) \\ &= \int [\ln p(\cdot) q(\cdot)] p(\cdot) q(\cdot) d(\theta_s, \Psi^\Delta, \Sigma^\Delta) - \int [\ln p(\cdot)] p(\cdot) q(\cdot) d(\theta_s, \Psi^\Delta, \Sigma^\Delta) \end{aligned}$$

is the Kullback-Leibler discrepancy between the distorted and the undistorted probabilities. The larger τ , the larger the penalty for deviating from $p(\cdot)$. The link between the exponential transformation of the loss function and the zero-sum game representation was pointed out by Jacobsen (1973) in one of the first studies of optimization under a risk-sensitive criterion.

In the subsequent empirical analysis we will also compute posterior expected losses under Scenarios 1 and 2 for the risk sensitive version of the policy problem. Under Scenario 1 the DSGE model itself is assumed to be correctly specified but “nature” is allowed to distort the beliefs about the structural parameters. The larger the uncertainty about a parameter, the easier it is to shift probability mass to create havoc. This analysis is similar in spirit to Giannoni (2002) who assess the robustness of monetary policy to changes in non-policy parameters of a simple three-equation New Keynesian DSGE model. Under Scenario 2 nature can also distort beliefs about the misspecification matrices, that is, the deviation of the “true” law of motion from the DSGE model restrictions and hence our analysis becomes comparable to Onatski and Stock (2002) and Onatski and Williams (2003).

5 Policy Evaluation Under DSGE-VAR

Our empirical analysis is based on interest rate, inflation, and output gap time series. We use the CBO’s potential output series and obtain all other series from Haver Analytics (Haver mnemonics are in italics). The output gap is defined as the log difference of real GDP (nominal GDP divided by the chained-price deflator $JGDP$) and real potential output. The log differences are scaled by 100 to convert them in percentages. Inflation is computed using quarter-to-quarter log-differences of the GDP deflator, scaled by 400 to obtain annualized percentages. The nominal rate corresponds to the effective Federal Funds rate ($FFED$), also in percent. The results reported below are based on a sample from 1983:Q3 to 2004:Q1. We begin with the estimation of the state-space representation of the DSGE model and of the DSGE-VAR for different values of λ . We document the degree of misspecification of the DSGE model. Next, we discuss the estimates of the “deep” parameters and the extent to which they are identified. Finally, we proceed with the policy analysis.

5.1 Estimation

We begin the section by discussing our assumptions on the priors for the “deep” parameters. Since we do not use observations on consumption and investment, it is difficult to identify the capital share and the depreciation rate. Therefore, we let $\alpha = 0.25$ and $\delta = 0.025$. Moreover, in a log-linear approximation the price markup parameter λ_f , which we fix at 0.3, is typically not identifiable. The parameters χ and ν_m only affect the dynamics of the money stock, which is not included in the set of observables. The parameter φ determines the steady state labor supply and does not influence the dynamics of interest rates, inflation, and the output gap. We set the average annualized growth rate of the economy γ_a equal to 1.5%.

Priors for the remaining DSGE model parameters are provided in Table 1. All intervals reported in the text are meant to be 90% intervals. The distribution for ψ_1 and ψ_2 is approximately centered at Taylor’s (1993) values, whereas the smoothing parameter lies in the range from 0.18 to 0.83. The prior mean for the growth adjusted real interest rate, $r_a^* + \gamma_a$, is 2.5% and annualized steady state inflation ranges from 0 to 6.25%, which is consistent with pre-1982 long-run historical averages. The prior mean of g^* implies that that the government share of GDP is 15%. According to our prior the habit persistence parameter h lies between 0.55 and 0.85. Boldrin, Christiano, and Fisher (2001) found that a value of 0.7 enhances the ability of a standard DSGE model to account for key asset market statistics. The interval for ν_l implies that the Frisch labor supply elasticity lies between 0.3 and 1.3, reflecting the micro-level estimates at the lower end, and the estimates of Kimball and Shapiro (2003) and Chang and Kim (2005) at the upper end. According to the prior for ζ_p firms re-optimize their prices on average every 2.5 to 12.5 quarters. This interval encompasses findings in micro-level studies of price adjustments such as Bils and Klenow (2004). The prior for the adjustment cost parameter s'' is consistent with the values that Christiano, Eichenbaum, and Evans (2005) when matching consumption and investment DSGE impulse response functions, among others, to VAR responses. Our prior for a' implies that in response to a 1% increase in the return to capital utilization rates rise by 0.1 to 0.3%. These numbers are considerably smaller than the one used by Christiano, Eichenbaum, and Evans (2005). Finally, the priors for ρ_z and ρ_g are centered around 0.8. The priors for the standard deviation parameters are chosen to obtain realistic magnitudes for the implied volatility of the output gap, inflation, and interest rates. These priors are by and large similar to the ones that have been used elsewhere in the literature, e.g., Smets and Wouters (2003), Del Negro, Schorfheide, Smets, and Wouters (2004), and Levin, Onatski,

Williams, and Williams (2005).

We estimate the state-space representation of the DSGE model using the Bayesian techniques described in Schorfheide (2000) and the DSGE-VARs with the Gibbs sampler discussed in the Appendix for various values of λ . Although λ is in principle a continuous parameter, for computational reasons we consider only 8 values on a grid ranging from 0.25, i.e., large prior variance of the misspecification matrices Ψ^Δ and Σ^Δ , to 10, which implies small misspecification. The DSGE-VAR analysis is based on $p = 4$ lags.

Table 2 summarizes the posterior of the hyperparameter λ . Log marginal data densities are reported in column 2 of the table. Differences of log marginal densities across λ 's can be interpreted as log posterior odds, under the assumption that the prior odds are equal to one. The odds reported in the last column of Table 2 are relative to $\lambda = 0.5$, which is the specification with the largest marginal data density and, according to this likelihood-based criterion, the best fit. The posterior of λ has an inverted U -shape. There is little variation in the marginal data densities for λ values between 0.5 and 1, whereas values outside of this interval lead to a substantial deterioration in fit. In the first row of the Table 2 we report the marginal data density for the DSGE model, which is about 30 points lower than the density of the DSGE-VAR($\lambda = 0.5$) on a log scale. We conclude that over the range of the historical sample the DSGE model is strongly dominated by DSGE-VARs with fairly low values of λ , indicating that the structural model is to some extent misspecified and that its policy predictions should be interpreted with care.⁸ However, the results of the next section indicate that even when λ is as low as 0.5 much of the mechanics of the DSGE model carries over to the DSGE-VAR. All DSGE-VAR results reported subsequently are based on $\lambda = 0.5$.

Parameter estimation results for the DSGE-VAR and the state-space representation of the DSGE model are reported in Table 3. There is a growing literature highlighting parameter identification problems associated with New Keynesian DSGE models, e.g. Beyer and Farmer (2004), Canova and Sala (2005), and Lubik and Schorfheide (2004, 2005). In some cases the rational expectations solution of the DSGE model implies that a subset of structural parameters disappear from the reduced form law of motion of the observables, in other cases the estimation objective function may have little curvature in some directions. Straightforward manipulations of Bayes Theorem can be used to show that priors are not

⁸The estimate of λ is in line with estimates that we obtained for DSGE-VARs fitted to output, inflation, and interest rate data based on a model without capital accumulation, see Del Negro and Schorfheide (2004, 2005), and a DSGE-VAR based on the Smets and Wouters (2003) model fitted to seven variables in Del Negro, Schorfheide, Smets, and Wouters (2004).

updated in directions of the parameter space in which the likelihood function is flat, e.g. Poirier (1998). Hence, to enable a careful comparison of priors and posteriors and an assessment of the information extracted from the sample we also report means and confidence intervals for the prior in Table 3. The summary statistics for the prior reflect the truncation at the boundary of the determinacy region. For the purpose of this study we are mostly interested in the estimation of the degree of misspecification λ and of policy loss differentials. With respect to λ , lack of identification of the deep parameters is not a concern. Therefore, we are only concerned about lack of sample information with respect to those parameters that significantly affect the ranking of policies.

The policy parameter estimates obtained from the DSGE-VAR can be viewed as Bayesian instrumental variable estimates. Since inflation and the output gap are endogenous variables, the estimator of ψ_1 and ψ_2 has to be adjusted for the non-zero conditional expectation of the monetary policy shock. Both the likelihood function associated with the state-space representation of the DSGE model and the DSGE-VAR likelihood generate such an adjustment. The former imposes all cross-coefficient restrictions of the DSGE model whereas the latter relaxes the restrictions. The estimates of ψ_1 are 2.19 for the DSGE-VAR and 1.90 for the DSGE model itself, implying a strong response of the Fed to inflation in the post-Volcker era. The estimated degree of interest rate smoothing is about 0.8. A comparison of prior and posterior means for ψ_2 indicates that the location shift is fairly small. The 90% posterior probability interval is however much tighter than the prior interval reflecting the information about the central bank's response to the output gap contained in the data. The estimated interest-feedback rate rule is admittedly a stylized description of monetary policy in the Volcker-Greenspan years. Yet the R^2 of the policy rule equation, obtained using the parameters that maximize the DSGE model posterior, is 94% for our sample period. This number is not too far from the 97% percent obtained by Blinder and Reis (2005) for the 1987:Q3 to 2005:Q1 period using a more sophisticated rule. We have experimented with a four-quarter moving average of inflation as an argument of the rule. We found that this does not affect the estimates of the other parameters, and slightly worsens the overall fit of the model.

To understand the remaining parameter estimates obtained from the DSGE-VAR it is instructive to consider the likelihood function, which is given by

$$\begin{aligned}
 & p(Y|\Psi, \Sigma, \theta) \\
 & \propto |\Sigma|^{-T/2} \text{etr} \left[\Sigma^{-1} \left(Y - X(B_1(\theta) + \Psi B_2(\theta)) \right)' \left(Y - X(B_1(\theta) + \Psi B_2(\theta)) \right) \right].
 \end{aligned} \tag{36}$$

We show in the Appendix that the mode of the posterior distribution of Ψ conditional on Σ and θ is of the form

$$\tilde{\Psi}(\Sigma, \theta) = (\lambda T T_{XX} + X'X)^{-1} (\lambda T T_{X\tilde{Y}} + X'\tilde{Y}) \Sigma^{-1} B_2' (B_2 \Sigma^{-1} B_2')^{-1},$$

where $\tilde{Y}(\theta) = Y - XB_1(\theta)$. Since we are unable to compute a marginal likelihood function $p(Y|\theta)$ analytically, we consider the concentrated likelihood function instead $p(Y|\tilde{\Psi}(\Sigma, \theta), \Sigma, \theta)$. Notice that as λ approaches zero the non-policy parameters, which do not affect $B_1(\theta)$ and $B_2(\theta)$ vanish from $p(Y|\tilde{\Psi}(\Sigma, \theta), \Sigma, \theta)$ because $\tilde{\Psi}(\Sigma, \theta)$ approaches $(X'X)^{-1} X'\tilde{Y} \Sigma^{-1} B_2' (B_2 \Sigma^{-1} B_2')^{-1}$. Hence, as the value of λ decreases we would expect the posterior distribution of the non-policy parameters to resemble more closely the prior distribution. Indeed, we found this to be the case.

The DSGE model estimates for the non-policy parameter point toward a large degree of habit persistence, $\hat{h} = 0.92$, and a small elasticity of investment with respect to the value of installed capital $1/s'' = 0.12$ implying fairly large capital adjustment costs. The estimate of the Calvo parameter ζ_p is 0.59, indicating a fairly low degree of price-stickiness: Agents re-optimize their prices on average every 2.5 quarters. We find that all shocks are fairly persistent, but our autocorrelation estimates are not as large as in other studies, e.g. Smets and Wouters (2003). The upper bound of the 90% posterior intervals for ρ_z and ρ_g are well below 1. The likelihood function of the DSGE model provides little information on g^* , and a' . While our values for g^* can be justified by the historical government share, the determination of a' is more difficult. We decided to conduct a robustness analysis by comparing DSGE model-based loss differentials for various values of a' and we found that the losses and the ranking of policies were insensitive to this parameter.

For many parameters, the estimates obtained from the DSGE-VAR are by and large similar to the the DSGE model estimates. One exception is the standard deviation of demand shocks σ_g , which is much lower according to the DSGE-VAR. Other exceptions are h and s'' , for which the DSGE-VAR likelihood function contains little information. We have compared the loss differentials under Scenario 1 for both the DSGE model and the DSGE-VAR, and found that the the differences in the estimates of the deep parameters between the two are quantitatively not important.

5.2 Policy Outcomes Under Misspecification

This section studies how policy outcomes change when misspecification is taken into account. We analyze the effect of different policies on aggregate stability under the four different as-

assumptions regarding misspecification described in Section 4.2. Specifically, we evaluate the variances of inflation, interest rates, and the output gap under each scenario as a function of all the parameters characterizing the interest rate rule (13): ψ_1 and ψ_2 , the central bank’s response to inflation and output, respectively, and ρ_R , the interest rate smoothing parameter. Throughout this section we compute individual variances of the three series as well as a weighted average for each point of a three dimensional grid, where: ψ_1 takes nine values ranging from 1.001 to 3 in intervals of 0.25; ψ_2 takes six different values, computed taking the Taylor’s (1993) value $\psi_2^T = 0.125$ as a reference, namely 0 , $\frac{1}{2}\psi_2^T = 0.062$, $\psi_2^T = 0.125$, $\frac{3}{2}\psi_2^T = 0.188$, $2\psi_2^T = 0.250$, $3\psi_2^T = 0.375$; and ρ_R takes three different values, namely 0.7, 0.8, and 0.95.⁹ Finally, we calculate variance differentials relative to the benchmark $\psi_1 = 2$, $\psi_2 = 0.188$, $\rho_R = 0.8$, and take expectations. The benchmark is chosen by selecting the point in the grid that roughly corresponds to the estimated values for those parameters, i.e., the historical Volcker-Greenspan rule. Negative differentials therefore indicate an improvement, that is, a reduction of variance, relative to the Volcker-Greenspan rule.

The results are summarized in Figures 2 and 3. Figure 2 contains four charts, one for each scenario. Each chart shows three surfaces describing the posterior expected differentials of the weighted variances as a function of ψ_1 and ψ_2 . The three surfaces correspond to the different values for ρ_R (0.7, 0.8, and 0.95), with darkness of the surface being directly proportional to ρ_R . In Figure 3 we decompose the weighted average of variance differentials into the contributions of the three individual series for one particular slice of the three-dimensional plot, obtained by fixing ψ_2 to its benchmark value of 0.188.

Scenario 1 – Ignore Misspecification

The first of these scenarios amounts to evaluating policy as if the New Keynesian model correctly described the data. This first scenario is the natural benchmark for comparing the results obtained once we allow for the presence of misspecification. Although the model considered here has a number of additional rigidities relative to the stylized model considered in Woodford (2003), the policy recommendation emerging from the analysis of the performance differentials are in line with those in Woodford, at least qualitatively. First, a high response of the interest rate to inflation (high ψ_1) is preferred to a low one, regardless of the value taken by ψ_2 and ρ_R . The drop in inflation and interest rate variability is particularly steep as ψ_1 increases from 1.001 to 1.5, but flattens thereafter, as can be appreciated from the two dimensional plot in Figure 3. The mechanism underlying this result is well known: due

⁹We have experimented with a finer grid, especially for ψ_1 and ρ_R , and we have found the loss to be a smooth function of these parameters. Hence, for ease of exposition we focus on the coarser grid.

to the forward looking nature of the model an increase in ψ_1 results in a drop of inflation variability, which in turn implies – for given ρ_R – a lower volatility of interest rates.

The top panel of Figure 4, which shows the impulse responses for the DSGE model as a function of ψ_1 , illustrates this mechanism. The panel displays the impulse responses with respect to the technology shock $\epsilon_{z,t}$ and the demand shock $\epsilon_{g,t}$ for three different values of ψ_1 , 1.25, 2, and 2.75, keeping ψ_2 and ρ_R at their historical values. Recall that these shocks are the only ones that matter for the variance calculations since we set $\sigma_R = 0$ and exclude the monetary policy shocks. The figure shows that the response of inflation and the interest rate to both shocks is less strong the higher the value of ψ_1 . Quantitatively, it is the response to technology shocks that makes the difference, with inflation responding roughly twice as strongly for ψ_1 equal to 1.25 than for ψ_1 equal to 2.75. The response of interest rates to both shocks is slightly stronger on impact the higher the value of ψ_1 , but becomes weaker soon thanks to the fact that inflation is kept under control. The output gap is more volatile for high ψ_1 with respect to technology shocks, but quantitatively the difference is not large. With respect to demand shocks the response of the output gap is strong on impact, but is not affected by the size of ψ_1 .

The expected variance differential as a function of ρ_R depends on the values of ψ_1 and ψ_2 , as shown in Figures 2. For low values of ψ_1 and high values of ψ_2 , a higher interest rate inertia is preferred. As the value of ψ_2 decreases and, especially, that of ψ_1 increases, the expected loss differential as a function of ρ_R becomes flatter. For values of ψ_1 larger than 1.25 the optimal value of ρ_R is 0.8. The individual variance differentials depicted in Figure 3 as well as the impulse responses for the DSGE model as a function of ρ_R , shown in the top panel of Figure 5, provide further insights. A value of ρ_R equal to 0.95 implies a drastic reduction in the variability of interest rates relative to the baseline $\rho_R = 0.8$, but a slightly increased variability of inflation. Given the weights that we used in our calculation of average variance differentials plotted in Figure 2, this trade-off is resolved by choosing a value of ρ_R that is neither too high (here, 0.95) or too low (0.7). It is clear however that the outcome depends on the specific choice of the weights.

Figure 2 shows that the weighted variances as a function of ψ_2 are sharply increasing for low values of ψ_1 and ρ_R : Targeting the output gap in presence of a weak interest rate response to inflation results in high inflation and interest rate variability. The slope of our performance measure with respect to ψ_2 decreases as ρ_R , but especially ψ_1 , increase. For high values of ψ_1 the expected loss differential appears from Figure 2 almost invariant with respect to ψ_2 . In fact, it is slightly U-shaped, with a minimum for the historical value of

$\psi_2 = 0.188$. According to our weighting scheme the variability of the endogenous variables is minimized, ignoring misspecification, if $\psi_1 = 3.00$ (the highest value in the grid) while ψ_2 and ρ_R equal their estimated values of 0.188 and 0.8, respectively. The inference about the misspecification parameter λ in Table 2 casts some doubts on the reliability of DSGE model predictions, however. Hence we move to take potential misspecification into account.

Scenario 2 – Acknowledge Misspecification, Discard the Past

In Scenario 2 the policymaker still uses the DSGE model to compute the mean response of the endogenous variables to the change in the policy parameters, but recognizes that “nature” may be injecting noise around these mean responses using the prior distribution. Under this scenario the policymaker learns from the data about the overall amount of noise (λ) but refuses to learn about the precise nature of the misspecification. We therefore refer to this scenario as “Acknowledge Misspecification – Discard the Past,” where “Discard the Past” refers to the fact that under this scenario the policymaker refuses to use the posterior information regarding Ψ^Δ and Σ^Δ on the ground that it contains no useful information once policy changes. Rather, she uses the prior distribution to generate draws of Ψ^Δ and Σ^Δ in evaluating policy outcomes. In both Figures 2 and 3 we focus on the expected variance differentials computed for $\lambda = 0.5$, which represents the “best-fitting” model. We have computed our measures of aggregate stability for the entire sequence of λ values included in Table 2 and have found that the policy outcomes for intermediate values of λ conform with our expectations. That is, the results are pretty similar to those for the DSGE model when λ equals 5 or 10, and as λ decreases they become more similar to those obtained for $\lambda = 0.5$. Moreover, we find that for λ values between 0.5 and 1 – that is, for all the model that would receive non-negligible weights in a Bayesian posterior averaging calculation – the variance differentials are both qualitatively and quantitatively very similar. Therefore we focus on $\lambda = 0.5$.

Under Scenario 2 the qualitative policy implications obtained under the DSGE model remain valid. Figures 2 show that a strong response of the interest rate to inflation movements is preferred. Moreover, for high enough ψ_1 the best choice of ρ_R equals the historical value of 0.8. Unlike in Scenario 1 we find that under Scenario 2 the policymaker should not respond at all to the output gap if performance is measured by the weighted average of variances. Indeed, the best performing policy according to this measure under Scenario 2 is identical to that under Scenario 1, except that ψ_2 equals 0. Quantitatively, the expected

variance differentials also appear very similar under Scenarios 1 and 2.¹⁰ Under Scenario 2 (as under the prior) the misspecification matrices Ψ^Δ and Σ^Δ have zero mean. Because of non-linearities in the impulse-response functions, the zero mean in the misspecification matrices does not imply that the expected variance differentials are the same with and without misspecification. But in our application they are roughly the same. This does not imply that introducing misspecification under Scenario 2 bears no consequences for the policymaker. The impact of misspecification can be felt in the uncertainty surrounding a given policy outcome. This uncertainty averages out in Figures 2 and 3, but can play a substantial role in a risk-sensitive analysis. We will return to this point later.

Scenario 3 – Learn About Misspecification (Policy Invariant)

In Scenario 3 the policymaker uses sample information to learn about the precise nature of misspecification, unlike in the previous scenario. In addition, she believes that the historically observed discrepancies Ψ^Δ and Σ^Δ are policy invariant. Figure 2 shows that under this scenario the variance surface is quite different than under Scenarios 1 and 2. First of all, there is much more curvature with respect to ψ_2 . Second, the variance profile is no longer strictly decreasing in ψ_1 , at least for values of ρ_R less than 0.95, as can be seen from Figure 3. For small ρ_{RS} the loss differential is a U-shaped function of ψ_1 , with the minimum attained at the value of 1.5 for $\rho_R = 0.8$.

The contours of the variance differentials under Scenario 3 are almost exclusively driven by the presence of explosive roots, at least for values of ρ_R less than 0.95. Explosiveness is not a concern at all for Scenarios 1 (mechanically), and 2 (as is non-uniqueness of the rational expectation equilibrium). For Scenario 4 the fraction of explosive draws is very small. For Scenario 3 explosiveness is ubiquitous. Hence, the policy recommendation under Scenario 3 is largely driven by the desire to avoid explosiveness.

We view this policy recommendation with suspicion, because it results from ignoring the correlation between the policy parameters and the size of misspecification in models that have a backward-looking component. We now elaborate on this point for the case of

¹⁰This is a bit misleading, however, as the underlying draw of the deep parameters θ is different under the two scenarios – one corresponds to the DSGE model and the other to DSGE-VAR (the values are given in Table 3). Whenever we computed the loss under Scenario 1 using the DSGE-VAR draws for θ , we find that the shape of the loss functions is exactly the same, but the expected loss differentials is smaller, largely because the estimated standard deviation of demand shocks is smaller. When we use the same set of draws for the deep parameters under both scenarios, we see that allowing for misspecification actually enhances the loss differentials, particularly those associated with small values of ψ_1 . Quantitatively, the difference is not very large.

DSGE-VAR. Recall that the estimated VAR parameters $\hat{\Psi}$ can be decomposed into the sum of the parameters implied by the DSGE restrictions $\Psi^*(\theta)$ and of the misspecification Ψ^Δ . Roughly speaking, under Scenario 3, the new set of VAR parameters is computed as the sum of $\Psi^*(\tilde{\theta})$, which changes with policy, and Ψ^Δ , which is assumed to be invariant. If the policy parameters are close to estimated ones, the sum of $\Psi^*(\tilde{\theta})$ and Ψ^Δ returns the estimated VAR parameters, which do not have explosive roots. But as we move away from the estimated policy parameters, or as we ignore the correlation between the policy parameters and Ψ^Δ , the sum of $\Psi^*(\tilde{\theta})$ and Ψ^Δ can deliver new VAR parameters whose roots are explosive. Here, this is particularly true for low values of ρ_R .¹¹ Inserting a policy rule different from the estimated one into a backward-looking model can often produce explosiveness.¹² This explosiveness is driven by the fact that the backward looking component of the system (here, Ψ^Δ) is not allowed to change with policy. Here, for instance, we find that a number of policies that are not too different from the historical one (say, ψ_1 between 1.75 and 2.5, $\psi_2 = 0.188$, and $\rho_R = 0.8$) delivers a non-negligible probability of explosiveness. We see this as a warning against having the backward-looking component of the model invariant with policy, rather than a warning against these policies.

Scenario 4 – Learn About Misspecification (Conditional)

Scenario 4 is an attempt to address this concern. Under this Scenario the policymaker again uses sample information to learn about potential model misspecification. As in Scenario 3, the misspecification is backward looking. But unlike in Scenario 3, the policymaker takes into account the in-sample correlation between the policy parameters and the misspecification matrices Ψ^Δ and Σ^Δ . Specifically, the policymaker now asks the question: What would the estimates of the discrepancies Ψ^Δ and Σ^Δ be if the new policy had been in place during the sample period? Explosiveness is no longer an overriding concern in Scenario 4, as it was in Scenario 3. The reason for this result is that now as $\tilde{\theta}$ changes both $\Psi^*(\tilde{\theta})$ and $\Psi^\Delta(\tilde{\theta})$ change in such a way that the sum of the two is not too different from the estimated VAR parameters. Indeed, for λ close to zero the dynamics for all equations other than the interest rate rule are approximately independent of the policy parameters.

¹¹Whenever λ is large, however, Ψ^Δ is negligible and the new VAR parameters roughly coincide with $\Psi^*(\tilde{\theta})$, which is non explosive.

¹²There are several examples in the literature. For instance, Levin and Williams (2003) find that many of the rules they consider generate explosiveness in the backward-looking Rudebusch-Svenson model. Likewise, Cogley and Sargent (2005) find that the optimal policy computed under their rational expectations “Lucas” model leads to explosive behavior when plugged into the backward-looking “Solow-Samuelson” model.

Figure 2 shows that under Scenario 4 the variance profile becomes flatter. In particular, the drop in inflation variability that characterized the increase in ψ_1 from 1.001 to 1.5 under the DSGE model is much smaller under Scenario 4, as can be appreciated from Figure 3. Under the DSGE model the mechanics of the rational expectations equilibrium imply that high values of ψ_1 help to anchor inflationary expectations. Since in equilibrium inflation moves less than under high values of ψ_1 , interest rates need to move less as well. The presence of substantial misspecification changes these dynamic responses, as shown in the bottom panel of Figure 4. As ψ_1 increases the response of inflation to both technology and demand shocks is more subdued, as in the DSGE model. Unlike in the DSGE model, the interest rate becomes more volatile as ψ_1 increases, not only on impact but also in the medium run. Figure 3 indeed confirms that interest rate variability rises as the central bank responds more strongly to inflation. Moreover, compared to Scenario 1 the increase in the output gap volatility is amplified. If the policymaker only cared about inflation, she would still choose a high response to inflation even under misspecification. However, if interest rate and output gap variability are also a concern a large value of ψ_1 is undesirable. Indeed, if performance is measured by our weighted average of variances the preferred value of ψ_1 is 1.75.

The presence of misspecification also affects the policy prescription with respect to the degree of inertia ρ_R . Figure 5 shows that under the DSGE model high interest rate inertia leads to slightly higher inflation variability, but quantitatively the effect is small. This is because in the DSGE model agents are aware that the eventual increase in interest rates following a sustained increase in inflation would be large, even if today's response is small. Under misspecification, this mechanism does not come into play: The milder response of interest rates to shocks under high inertia leads to a substantial increase in the volatility of inflation. Finally, Scenario 4 shares with Scenario 2 an aversion against high responses to the output gap. Indeed, the best performing policy under Scenario 4 is $\psi_1 = 1.75$, $\psi_2 = 0.06$ and $\rho_R = 0.8$.

The results in Figures 2 and 3 depend on the somewhat arbitrary choice of the bound. For this reason, we have recomputed all figures using a bound that is double (100) or ten times larger (500) than the one used so far. Although the variance differentials that are affected by explosive behavior, particularly in Scenario 3, change substantially with the bound, we find that the overall shape of the contours, and hence the gist of our conclusions, stay roughly the same.

Risk Sensitive Analysis

Finally, Figure 6 compares the variance differentials that we just analyzed with those obtained under the risk-sensitive version of our problem. We focus in particular on Scenario 2, which is the closest in spirit to the robustness literature, but we also show the benchmark, Scenario 1. For both scenarios the Figure shows the risk-sensitive loss (black) as well as the risk-neutral loss (light grey).¹³ For Scenario 1, where the risk-sensitivity is only with respect to the deep parameters θ_s , we find that the risk sensitive loss is generally not too different from the plain-vanilla one. The only difference is quantitative: The loss stemming from a weak response to inflation is much larger under the risk-sensitive calculations. In Scenario 2, risk-sensitivity would induce the policymaker to avoid not only low values of ψ_1 , but also high values of ψ_2 as long as ψ_1 is not above 2. Recall that misspecification alone had little effect on the expected loss differential in Scenario 2. However, when combined with a concern for robustness it leads to a starker recommendation relative to the DSGE model: Avoid the combination of moderate interest rate responses to inflation and strong responses to the output gap.

Performance of Alternative Rules

In Table 4 we report the implications in terms of interest rate, inflation, and output gap volatility for the best performing rule under each Scenario, as well as for a set of alternative policy rules that have been considered in the literature. The best performing rules under each Scenario are computed according to the summary measure defined in Section 4 and reported in the last 3 rows of the table. The remaining rules include four of the five benchmark policy rules described in Table 1 of John Taylor’s (1999) volume. Taylor I and II are interest rate smoothing rules with a coefficient $\rho_R = 1$. These rules were favored in the simulations by Levin, Wieland, and Williams (1999) who studied the performance of interest-rate rules in four different structural macroeconomic models of the U.S. economy: the Federal Reserve Board staff model, the Monetary Studies Research model, the Fuhrer-Moore model, and Taylor’s multi-country model. In our DSGE model $\rho_R = 1$ rules make interest rates non-stationary and the calculation of the model-based autocovariance matrices that is needed for the construction of the prior distribution would have to be adjusted. We decided to replace the coefficient of unity by $\rho_R = 0.95$.¹⁴ Taylor III is Taylor’s (1993) rule, which

¹³A caveat of our analysis so far is that we do not distinguish between uncertainty in the deep parameters θ and in the misspecification parameters Ψ^Δ and Σ^Δ . In principle we want to be robust gains the latter, but not necessarily the former.

¹⁴Equation (19) provides the basis for the conversion of policy rule coefficients into our setup. The interest rate and output gap coefficients are converted according to $\psi_1 = g_\pi/0.05$ and $\psi_2 = g_y/(4 \cdot 0.05)$.

does not entail interest rate smoothing. Taylor IV rule is the same as Taylor III except that the central bank responds twice as strongly to the output gap. The fifth rule in Taylor (1999) is a super-inertia rule ($\rho_R = 1.3$) which we omit from our analysis. The last two rules considered in Table 4 correspond to the robust Bayesian and minimax rules reported in Levin and Williams (2003).¹⁵ Levin and Williams' (2003) calculation of robust rules is based on three models, a New Keynesian DSGE model, the Rudebusch-Svenson (1999) model, and Fuhrer's (2000) habit persistence model.

The first panel of Table 4 displays for each policy the coefficients ψ_1 , ψ_2 , and ρ_R . The next three panels show the implication in terms of volatility for the interest rate, inflation, and the output gap, respectively, for the ten different policies we consider. For each panel we report the change in the volatility of the relevant variable relative to the baseline rule, computed under Scenarios 1, 2, and 4. We omit Scenario 3 from the Table 4. The analysis conducted under this Scenario is not particularly interesting for the reasons discussed before: The presence of explosive draws is driving the results for most of the rules.

The best-performing rule under the DSGE model (rule 1) not surprisingly leads to good outcomes for all the variables under Scenarios 1 and 2. But under Scenario 4, rules that are less sensitive to the output gap, such as rule 2 for instance, are preferred. The high inertia of rule 3, combined with a non-negligible value of ψ_2 , also leads to high volatility for all variables under Scenario 4. Rule 4, which displays a more moderate response to inflation, performs slightly worse than the baseline in terms of inflation if the DSGE model is correct. Under misspecification however it performs well for all variables. The rules Taylor I and Taylor II perform very well under Scenarios 1 and 2 since they imply a strong response to inflation movements. In fact, the Taylor I rule reduces inflation volatility more than our preferred rules for Scenarios 1 and 2 because we restrict the value of ψ_1 to be less than 3 in our calculations. Quantitatively the difference is not large, however. Because of their high output sensitivity, these rules lead to increased variability in the output gap relative to the baseline under Scenario 4. Based on our previous discussion it is not surprising that the non-inertial ($\rho_R = 0$) rules Taylor III and Taylor IV perform poorly across all four Scenarios. Levin and Williams' (2003) robust rules also perform well under Scenarios 1 and 2. This may stem from the fact that the authors included a New Keynesian DSGE model in their analysis of monetary policy rules. However, Rules 5 and 6 are dominated by Taylor I across all Scenarios. Both the Levin-Williams rules display high output gap sensitivity and hence

¹⁵Taken from the $\lambda = 0.5$ and $\phi = 0.1$ entry in Tables 4 and 7 of Levin and Williams (2003) and converted according to $\psi_1 = 1 + \alpha/(1 - \rho)$ and $\psi_2 = \beta/(4 \cdot (1 - \rho))$.

perform somewhat poorly in the “learn about misspecification” Scenarios compared to the baseline. This is the case for all three variables.

In summary, the qualitative implications of our findings are as follows: (i) Policies that mainly aim at clamping down on inflation actually succeed in reducing inflation variability, even in presence of misspecification. Misspecification however implies that the cost of these policies in terms of interest rate variability is higher than under the DSGE model. (ii) Policies that also respond to the output gap lead to comparable (and sometimes slightly better) outcomes if the DSGE model is correct. However, these policies lead to worse outcomes under Scenario 4 not only in terms of inflation, but also of output variability. (iii) Non-inertial policies lead to a high degree of interest rate volatility under all scenarios. Policies that are extremely inertial ($\rho_R = 0.95$) lead to worse outcomes in terms of inflation under scenario 4. Quantitatively we find that the risks and the gains associated with deviating from the historical Volcker-Greenspan policy are sensitive to the misspecification assumptions considered. For instance, when the policymaker chooses to learn from the historical data about misspecification (Scenario 4), she finds that the increase in inflation volatility associated with policies that deviate from the DSGE model prescriptions are not as large as policy analysis under the DSGE model would suggest. In the end, a fairly robust policy recommendation emerges from our analysis: the central bank should avoid strong responses to output gap movements and not react weakly to inflation fluctuations. Also, we find that the gains associated with deviating from the historical Volcker-Greenspan policy, whenever positive, are generally not very large. This suggests that the historical rule, if not always optimal among those we consider, has been reasonably good at least from the perspective of this sticky-prices DSGE model, even taking misspecification into account.

6 Conclusion

Encouraged by the work of Smets and Wouters (2003) many central banks are in the process of developing and estimating DSGE models usable for quantitative monetary policy analysis. The empirical results in this paper and in Del Negro, Schorfheide, Smets, and Wouters (2004) document that model misspecification remains a concern as less restrictive vector autoregressive specifications attain a better time series fit than the DSGE model itself. In this paper we developed and applied techniques to conduct quantitative monetary policy analysis with DSGE models while explicitly taking account of their potential misspecification.

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A Implementation of the Posterior Simulation

A.1 Draws from the Posterior

We adopt the notation that $\tilde{Y}(\theta) = Y - XB_1(\theta)$ which leads to the definitions

$$\Gamma_{\tilde{Y}\tilde{Y}} = \Gamma_{YY} - \Gamma_{YX}B_1(\theta) - B_1(\theta)'\Gamma_{XY} - B_1(\theta)'\Gamma_{XX}B_1(\theta), \quad \Gamma_{X\tilde{Y}} = \Gamma_{XY} - \Gamma_{XX}B_1(\theta).$$

Let $\text{etr}[A] = \exp[-\frac{1}{2}\text{tr}[A]]$. The likelihood function for the VAR representation is given by

$$\begin{aligned} p(Y|\Psi, \Sigma, \theta) & \\ \propto |\Sigma|^{-T/2} \text{etr} \left[\Sigma^{-1} \left(Y - X(B_1(\theta) + \Psi B_2(\theta)) \right)' \left(Y - X(B_1(\theta) + \Psi B_2(\theta)) \right) \right]. & \end{aligned} \quad (\text{A.1})$$

Using Lemma 1(i) we can rewrite the prior mean of Ψ as

$$\Psi^*(\theta) = \bar{\Psi}(\Sigma, \theta) = \Gamma_{XX}^{-1}(\theta)\Gamma_{X\tilde{Y}}(\theta)\Sigma^{-1}B_2'(\theta)[B_2(\theta)\Sigma^{-1}B_2'(\theta)]^{-1}.$$

The prior density for Ψ conditional on Σ is of the form

$$p(\Psi|\Sigma, \theta) \propto \text{etr} \left[\Sigma^{-1}\lambda T \left(-2B_2'\Psi'\Gamma_{X\tilde{Y}}(\theta) + B_2'\Psi'\Gamma_{XX}(\theta)\Psi B_2 \right) \right]. \quad (\text{A.2})$$

The prior density for Σ is given by

$$p(\Sigma|\theta) \propto |\Sigma|^{-\frac{1}{2}(\lambda T - k + n + 1)} \text{etr} \left[\Sigma^{-1} \lambda T \Sigma^*(\theta) \right] \quad (\text{A.3})$$

To simplify notation the (θ) -argument of the functions B_1 , B_2 , \tilde{Y} , Γ_{XY} , Γ_{XX} , and Γ_{YY} is omitted.

Conditional Posterior of Ψ : Combining the the prior density (A.2) with the likelihood function (A.1) yields

$$\begin{aligned} p(\Psi|\Sigma, \theta, Y) & \\ & \propto p(Y|\Psi, \Sigma, \theta) p(\Psi|\Sigma, \theta) \quad (\text{A.4}) \\ & \propto \text{etr} \left[\Sigma^{-1} \lambda T \left(\Gamma_{YY} - 2B_2' \Psi' \Gamma_{X\tilde{Y}} + B_2' \Psi' \Gamma_{XX}(\theta) \Psi B_2 \right) + (\tilde{Y} - X \Psi B_2)' (\tilde{Y} - X \Psi B_2) \right] \\ & \propto \text{etr} \left[\Sigma^{-1} \left(-2B_2' \Psi' (\lambda T \Gamma_{X\tilde{Y}} + X' \tilde{Y}) + B_2' \Psi' (\lambda T \Gamma_{XX} + X' X) \Psi B_2 \right) \right] \end{aligned}$$

Define

$$\tilde{\Psi}(\Sigma, \theta) = (\lambda T \Gamma_{XX} + X' X)^{-1} (\lambda T \Gamma_{X\tilde{Y}} + X' \tilde{Y}) \Sigma^{-1} B_2' (B_2 \Sigma^{-1} B_2')^{-1}.$$

The previous calculations show that

$$\Psi|\Sigma, \theta, Y \sim \mathcal{N} \left(\tilde{\Psi}(\Sigma, \theta), \left[(B_2 \Sigma^{-1} B_2') \otimes (\lambda T \Gamma_{XX} + X' X) \right]^{-1} \right). \quad (\text{A.5})$$

Conditional Posterior of Σ : Combining the the prior densities (A.2) and (A.3) with the likelihood function (A.1) yields

$$\begin{aligned} p(\Sigma|\Psi, \theta, Y) & \propto p(Y|\Psi, \Sigma, \theta) p(\Psi|\Sigma, \theta) p(\Sigma|\theta) \quad (\text{A.6}) \\ & \propto |\Sigma|^{-\frac{1}{2}((\lambda+1)T - k + n + 1)} |(B_2 \Sigma^{-1} B_2')^{-1}|^{-\frac{k}{2}} \\ & \quad \text{etr} \left[\Sigma^{-1} \left(\lambda T (\Gamma_{\tilde{Y}\tilde{Y}} - \Gamma_{\tilde{Y}X} \Gamma_{XX}^{-1} \Gamma_{X\tilde{Y}}) + (\tilde{Y} - X \Psi B_2)' (\tilde{Y} - X \Psi B_2) \right) \right. \\ & \quad \left. + \lambda T (B_2 \Sigma^{-1} B_2') (\Psi - \bar{\Psi})' \Gamma_{XX} (\Psi - \bar{\Psi}) \right]. \end{aligned}$$

Using the definition of $\bar{\Psi}$, the last term can be manipulated as follows

$$\begin{aligned} & \text{etr} \left[\lambda T B_2 \Sigma^{-1} B_2' (\Psi - \bar{\Psi})' \Gamma_{XX} (\Psi - \bar{\Psi}) \right] \\ & = \text{etr} \left[\lambda T \Sigma^{-1} \left(B_2' \Psi' \Gamma_{XX} \Psi B_2 - 2B_2' \Psi' \Gamma_{X\tilde{Y}} \right) \right. \\ & \quad \left. + \lambda T \Sigma^{-1} B_2' (B_2 \Sigma^{-1} B_2')^{-1} B_2 \Sigma^{-1} \Gamma_{X\tilde{Y}}' \Gamma_{XX}^{-1} \Gamma_{X\tilde{Y}} \right] \end{aligned}$$

Hence,

$$\begin{aligned}
p(\Sigma|\Psi, \theta, Y) &\propto |\Sigma|^{-\frac{1}{2}((\lambda+1)T-k+n+1)} |(B_2\Sigma^{-1}B_2')^{-1}|^{-\frac{k}{2}} & (A.7) \\
&\times \text{etr} \left[\Sigma^{-1} \left(\lambda T \Gamma_{\tilde{Y}\tilde{Y}} + \tilde{Y}'\tilde{Y} - 2B_2'\Psi'(\lambda T \Gamma_{X\tilde{Y}} + X'\tilde{Y}) \right. \right. \\
&\quad \left. \left. + B_2'\Psi'(\lambda T \Gamma_{XX} + X'X)\Psi B_2 \right) \right] \\
&\times \text{etr} \left[\lambda T (\Sigma^{-1}B_2'(B_2\Sigma^{-1}B_2')^{-1}B_2\Sigma^{-1} - \Sigma^{-1}) \Gamma'_{X\tilde{Y}} \Gamma_{XX}^{-1} \Gamma_{X\tilde{Y}} \right].
\end{aligned}$$

If the DSGE model satisfies Equation (20) and the error $u_{1,t}$ is orthogonal to x_t then

$$\Gamma_{X\tilde{Y}} = \Gamma_{XX}\Psi_0(\theta)B_2$$

and

$$(\Sigma^{-1}B_2'(B_2\Sigma^{-1}B_2')^{-1}B_2\Sigma^{-1} - \Sigma^{-1})\Gamma'_{X\tilde{Y}}\Gamma_{XX}^{-1}\Gamma_{X\tilde{Y}} = 0. \quad (A.8)$$

While the conditional posterior distribution of Σ given our prior distribution is not of the \mathcal{IW} form use an \mathcal{IW} distribution as proposal distribution in a Metropolis-Hastings step.

Define

$$\begin{aligned}
\tilde{S}(\Psi, \theta) &= \lambda T \Gamma_{\tilde{Y}\tilde{Y}} + \tilde{Y}'\tilde{Y} - (\lambda T \Gamma_{X\tilde{Y}} + X'\tilde{Y})'\Psi B_2 - B_2'\Psi'(\lambda T \Gamma_{X\tilde{Y}} + X'\tilde{Y}) & (A.9) \\
&\quad + B_2'\Psi'(\lambda T \Gamma_{XX} + X'X)\Psi B_2
\end{aligned}$$

Our proposal distribution for Σ is

$$\mathcal{IW}(\tilde{S}(\Psi, \theta), (\lambda + 1)T, n).$$

Conditional Posterior of θ : The posterior distribution of θ is irregular. Its density is proportional to the joint density of Y , Ψ , Σ , and θ , which we can evaluate numerically since the normalization constants for $p(\Psi|\Sigma, \theta)$ and $p(\Sigma|\theta)$ are readily available.

$$p(\theta|\Psi, \Sigma, Y) \propto p(Y, \Psi, \Sigma, \theta) = p(Y|\Psi, \Sigma, \theta)p(\Psi|\Sigma, \theta)p(\Sigma|\theta)p(\theta). \quad (A.10)$$

To obtain a proposal density for $p(\theta|\Psi, \Sigma, Y)$ we (i) maximize the posterior density of the DSGE model with respect to θ and (ii) calculate the inverse Hessian at the mode, denoted by $V_{\tilde{\theta}, DSGE}$. (iii) We then use a random-walk Metropolis step with proposal density

$$\mathcal{N}(\theta_{(s-1)}, cV_{\tilde{\theta}, DSGE})$$

where $\theta_{(s-1)}$ is the value of θ drawn in iteration $s - 1$ of the MCMC algorithm, and c is a scaling factor that can be used to control the rejection rate in the Metropolis step.

Table 1: PRIOR DISTRIBUTION

Name	Domain	Density	P(1)	P(2)
ψ_1	\mathcal{R}^+	Gamma	1.500	0.400
ψ_2	\mathcal{R}^+	Gamma	0.200	0.100
ρ_R	$[0, 1)$	Beta	0.500	0.200
r_a^*	\mathcal{R}^+	Gamma	1.000	0.400
π_a^*	\mathcal{R}	Normal	3.000	2.000
g^*	\mathcal{R}^+	Beta	0.150	0.050
h	$[0, 1)$	Beta	0.700	0.100
ν_l	\mathcal{R}^+	Gamma	2.000	0.750
ζ_p	$[0, 1)$	Beta	0.750	0.100
s'	\mathcal{R}^+	Gamma	4.000	1.500
a''	\mathcal{R}^+	Gamma	0.200	0.075
ρ_z	$[0, 1)$	Beta	0.800	0.050
ρ_g	$[0, 1)$	Beta	0.800	0.050
σ_z	\mathcal{R}^+	InvGamma	0.400	2.000
σ_g	\mathcal{R}^+	InvGamma	0.300	2.000
σ_R	\mathcal{R}^+	InvGamma	0.200	2.000

Notes: P(1) and P(2) list the means and the standard deviations for Beta, Gamma, and Normal distributions; the upper and lower bound of the support for the Uniform distribution; s and ν for the Inverse Gamma distribution, where $p_{IG}(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. See Section 2 for a definition of the DSGE model's parameters. We are reporting annualized values for π^* and r^* (a -subscript). The following parameters were fixed: $\alpha = 0.25$, $\delta = 0.025$, $\gamma = 1.5$, $\lambda_f = 0.3$. The effective prior is truncated at the boundary of the determinacy region.

Table 2: LOG MARGINAL DATA DENSITIES AND POSTERIOR ODDS

Specification	$\ln p(Y)$	Post Odds
DSGE Model	-305.44	5E-13
DSGE-VAR, $\lambda = 10.0$	-297.16	2E-09
DSGE-VAR, $\lambda = 5.0$	-291.28	7E-07
DSGE-VAR, $\lambda = 2.0$	-283.70	0.001
DSGE-VAR, $\lambda = 1.5$	-281.23	0.017
DSGE-VAR, $\lambda = 1.0$	-278.46	0.278
DSGE-VAR, $\lambda = 0.75$	-277.22	0.968
DSGE-VAR, $\lambda = 0.50$	-277.18	1.000
DSGE-VAR, $\lambda = 0.25$	-283.34	0.002

Notes: The marginal data densities are obtained by integrating the likelihood function with respect to the model parameters, weighted by the prior density conditional on λ . The difference of log marginal data densities can be interpreted as log posterior odds under the assumption of that the two specifications have equal prior probabilities.

Table 3: PARAMETER ESTIMATION RESULTS

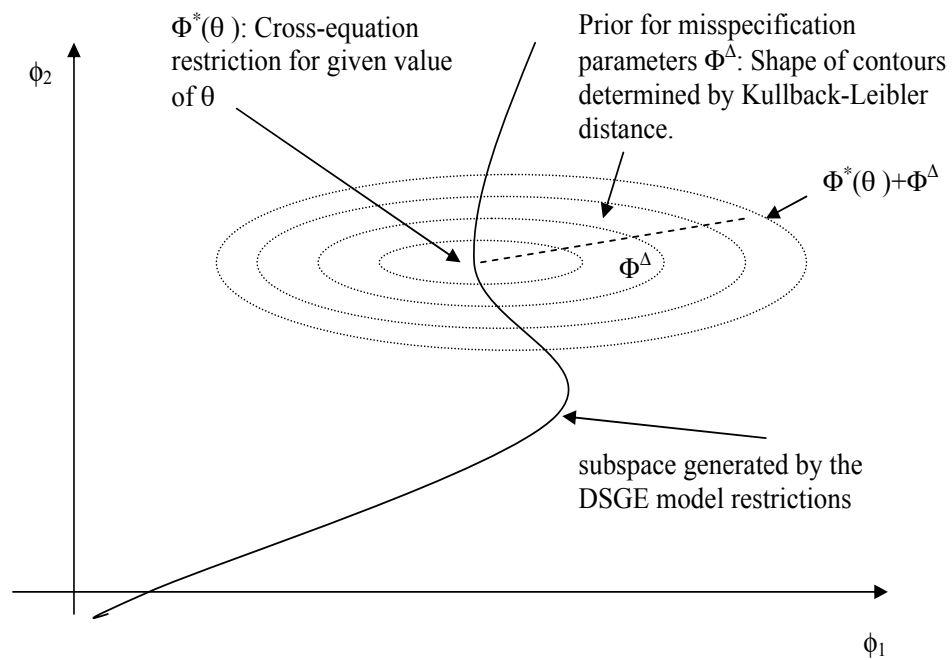
	Prior		Posterior			
	Mean	90% Interval	DSGE-VAR ($\lambda = 0.5$)		DSGE (State Sp)	
			Mean	90% Interval	Mean	90% Interval
ψ_1	1.56	[1.00 , 2.07]	2.19	[1.68 , 2.71]	1.90	[1.43 , 2.37]
ψ_2	0.20	[0.05 , 0.35]	0.15	[0.07 , 0.23]	0.18	[0.11 , 0.26]
ρ_R	0.50	[0.18 , 0.83]	0.83	[0.79 , 0.88]	0.80	[0.75 , 0.85]
r_a^*	1.00	[0.37 , 1.61]	0.86	[0.36 , 1.34]	0.88	[0.47 , 1.27]
π_a^*	3.00	[-0.31 , 6.26]	3.10	[2.46 , 3.71]	3.02	[2.44 , 3.59]
g^*	0.15	[0.07 , 0.23]	0.15	[0.07 , 0.23]	0.13	[0.06 , 0.20]
h	0.70	[0.54 , 0.86]	0.73	[0.59 , 0.89]	0.92	[0.87 , 0.96]
ν_l	2.00	[0.78 , 3.12]	1.55	[0.72 , 2.35]	1.19	[0.76 , 1.60]
ζ_p	0.75	[0.59 , 0.92]	0.66	[0.54 , 0.78]	0.59	[0.46 , 0.71]
s''	4.00	[1.61 , 6.30]	4.39	[1.95 , 6.77]	8.15	[5.09 , 11.15]
a'	0.20	[0.08 , 0.32]	0.22	[0.10 , 0.34]	0.23	[0.11 , 0.35]
ρ_z	0.80	[0.72 , 0.88]	0.83	[0.76 , 0.90]	0.83	[0.78 , 0.88]
ρ_g	0.80	[0.72 , 0.88]	0.83	[0.75 , 0.90]	0.89	[0.86 , 0.93]
σ_z	0.71	[0.16 , 1.24]	0.83	[0.47 , 1.18]	0.85	[0.59 , 1.09]
σ_g	0.53	[0.12 , 0.93]	0.37	[0.27 , 0.47]	0.66	[0.56 , 0.76]
σ_R	0.36	[0.08 , 0.63]	0.13	[0.10 , 0.16]	0.16	[0.13 , 0.18]

Table 4: COMPARATIVE PERFORMANCE OF SELECTED POLICY RULES

		Preferred in Scenario				Taylor (1999)				Levin-Williams	
		1	2	3	4	I	II	III	IV	Bayes	Minimax
Parameters											
	ψ_1	3.00	3.00	3.00	1.75	60.00	24.00	1.50	1.50	4.20	6.50
	ψ_2	0.19	0.00	0.19	0.06	4.00	5.00	0.125	0.25	0.80	1.375
	ρ_R	0.80	0.80	0.95	0.80	0.95	0.95	0.00	0.00	0.40	0.80
Interest Rate Variance											
Scenario:	1	0.13	0.03	-1.38	-0.23	1.02	0.50	9.48	10.31	3.42	1.20
	2	0.07	-0.34	-1.16	-0.48	1.79	1.56	8.31	11.13	5.89	2.49
	4	2.53	0.49	3.80	-2.76	1.50	5.91	3.56	6.43	5.55	6.19
Inflation Variance											
Scenario:	1	-0.80	-0.66	0.39	0.42	-1.61	-0.92	2.85	3.44	-0.86	-0.81
	2	-0.89	-1.12	-0.12	-0.23	-1.79	-1.01	2.73	4.77	-0.54	-0.70
	4	0.04	-0.78	4.76	-0.67	-0.16	6.75	1.66	3.71	3.92	6.18
Output Gap Variance											
Scenario:	1	0.01	0.04	0.05	0.04	-0.02	-0.07	-0.02	-0.08	-0.08	-0.08
	2	0.06	0.12	0.05	0.05	0.14	0.03	-0.11	-0.18	-0.01	0.01
	4	1.16	-0.76	3.45	-1.68	3.10	6.98	0.57	2.37	4.51	6.48
Weighted Variances											
Scenario:	1	-0.77	-0.64	0.05	0.37	-1.36	-0.81	5.21	6.00	-0.02	-0.53
	2	-0.86	-1.17	-0.40	-0.34	-1.31	-0.61	4.78	7.50	0.93	0.07
	4	0.62	-0.72	4.79	-1.46	0.35	6.85	2.51	5.10	4.64	6.54

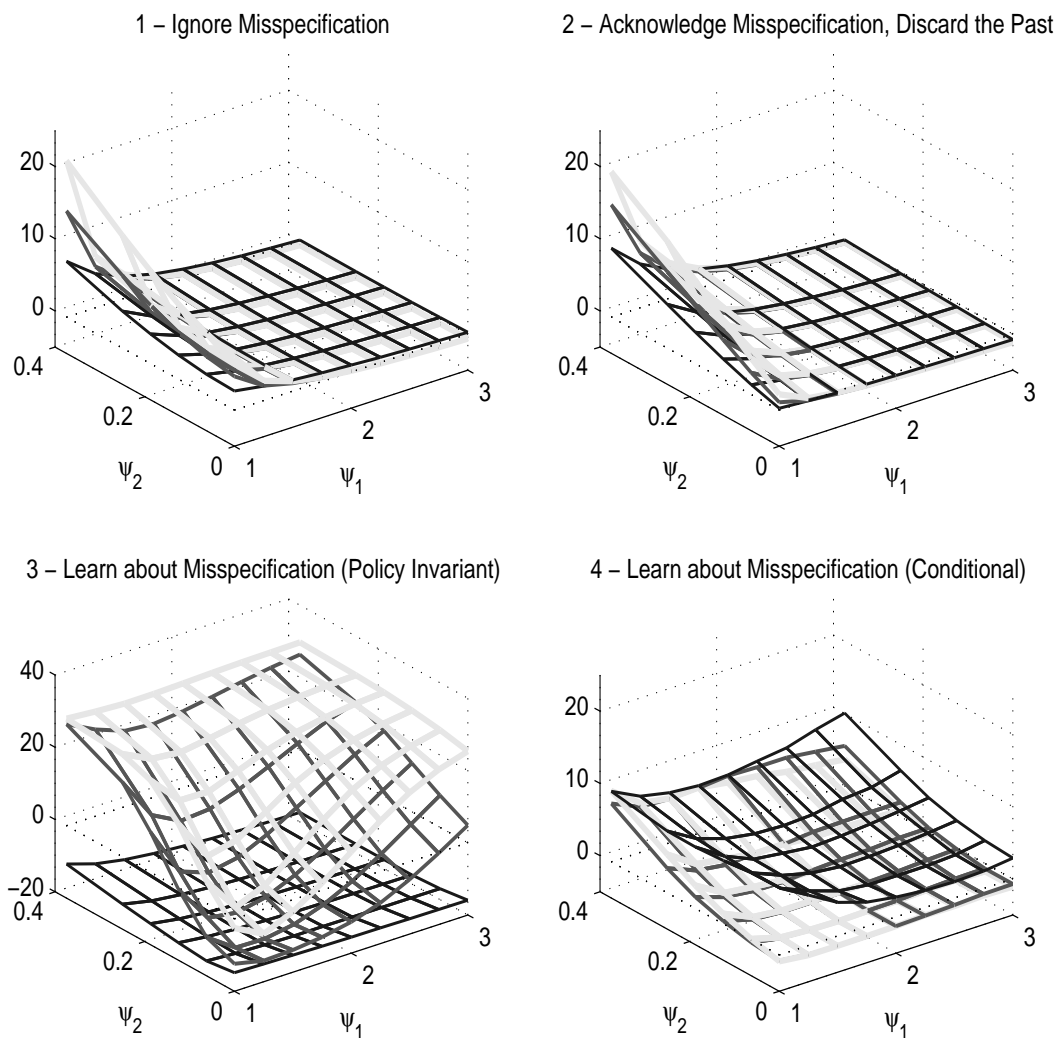
Notes: Posterior expected variance differentials as a function of ψ_1 and ψ_2 relative to baseline policy rule $\psi_1 = 2.0$, $\psi_2 = 0.188$, $\rho_R = 0.80$. Negative differentials signify a variance reduction relative to baseline rule.

Figure 1: STYLIZED VIEW OF DSGE MODEL MISSPECIFICATION



Notes: $\Phi = [\phi_1, \phi_2]'$ can be interpreted as the VAR parameters, and $\Phi^*(\theta)$ is the restriction function implied by the DSGE model.

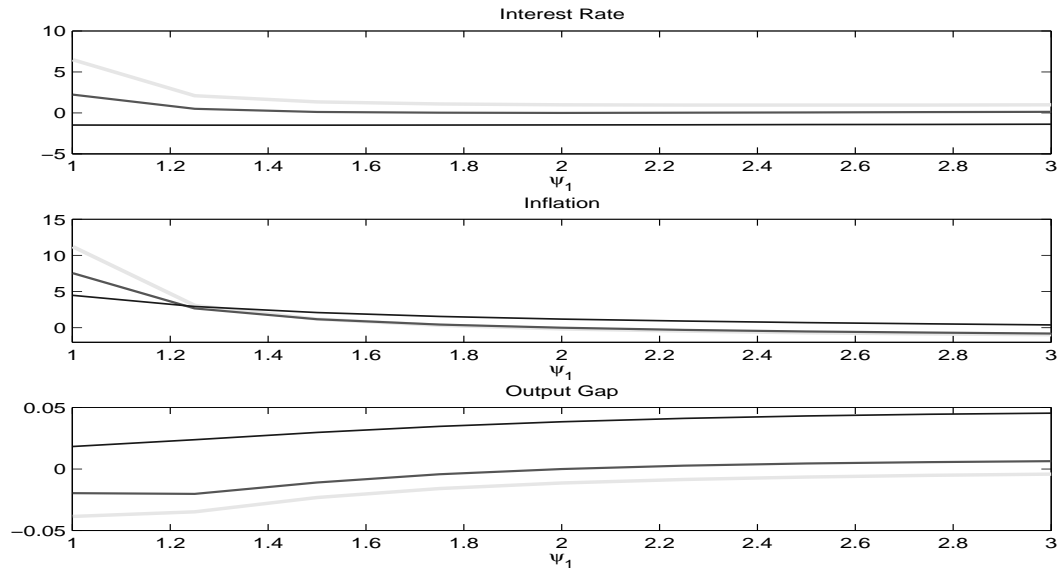
Figure 2: COMPARATIVE PERFORMANCE OF DIFFERENT RULES: SUMMARY MEASURE



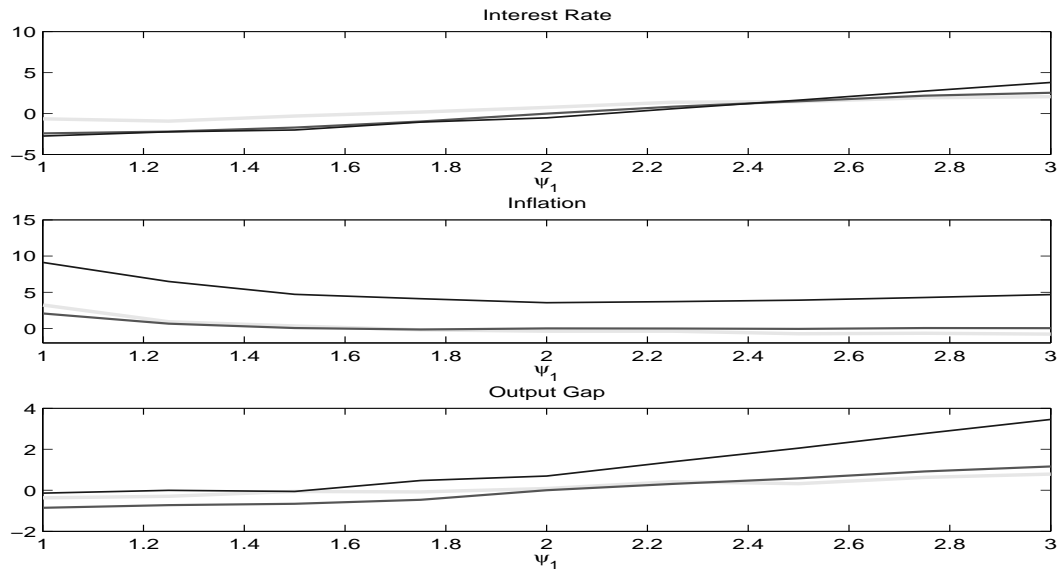
Notes: Posterior expected variance differentials as a function of ψ_1 and ψ_2 relative to baseline policy rule $\psi_1 = 2.0$, $\psi_2 = 0.188$, $\rho_R = 0.80$. The [weighted variances under the baseline policy rule](#) for the four scenarios are 2.46, 2.54, 20.13, and 4.77 respectively. Negative differentials signify a variance reduction relative to baseline rule. Surfaces' color ranges from very light grey ($\rho = 0.7$) to dark grey ($\rho = 0.95$), with the darkness of the surface being directly proportional to ρ .

Figure 3: COMPARATIVE PERFORMANCE OF DIFFERENT RULES: INDIVIDUAL VARIABLES

Scenario 1 – Ignore Misspecification



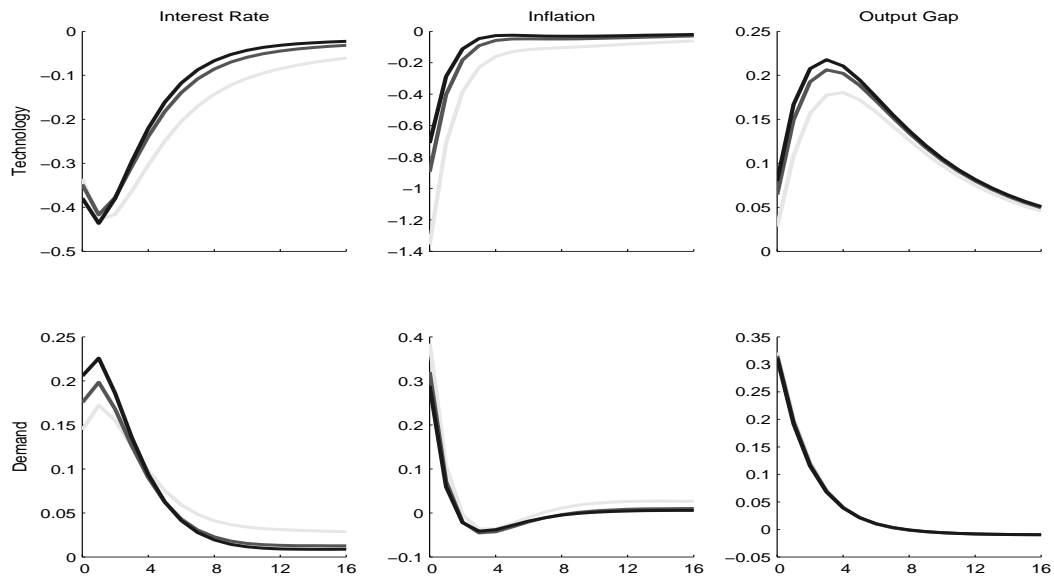
Scenario 4 – Learn about Misspecification (Conditional)



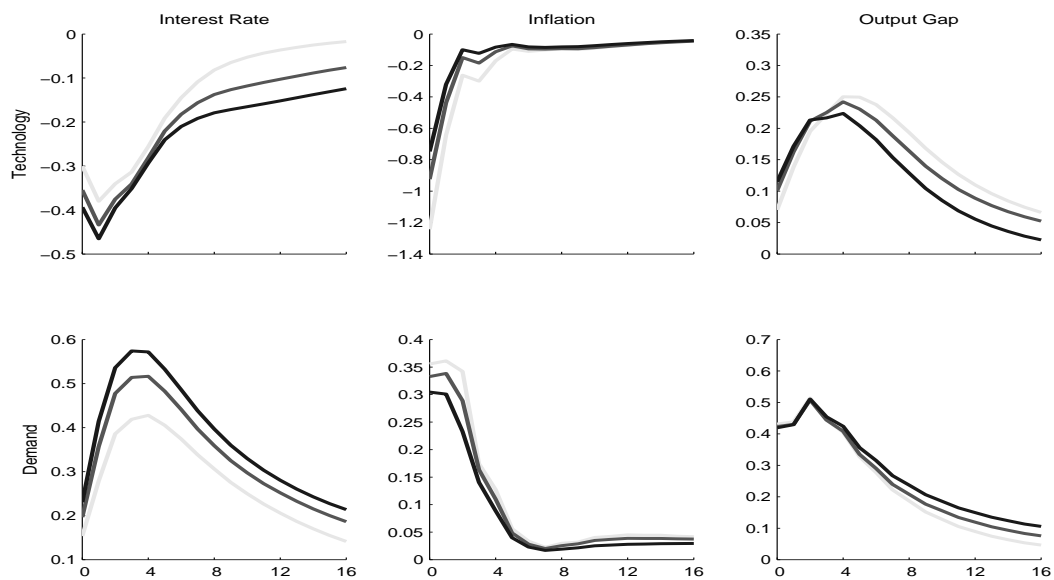
Notes: Posterior expected variance differentials as a function of ψ_1 and ψ_2 relative to baseline policy rule $\psi_1 = 2.0$, $\psi_2 = 0.188$, $\rho_R = 0.80$. Negative differentials signify a variance reduction relative to baseline rule. Lines' color ranges from very light grey ($\psi_1 = 1.25$) to dark grey ($\psi_1 = 2.75$), with the darkness of the line being directly proportional to ψ_1 .

Figure 4: IMPULSE RESPONSES AS FUNCTION OF ψ_1

Scenario 1 – Ignore Misspecification



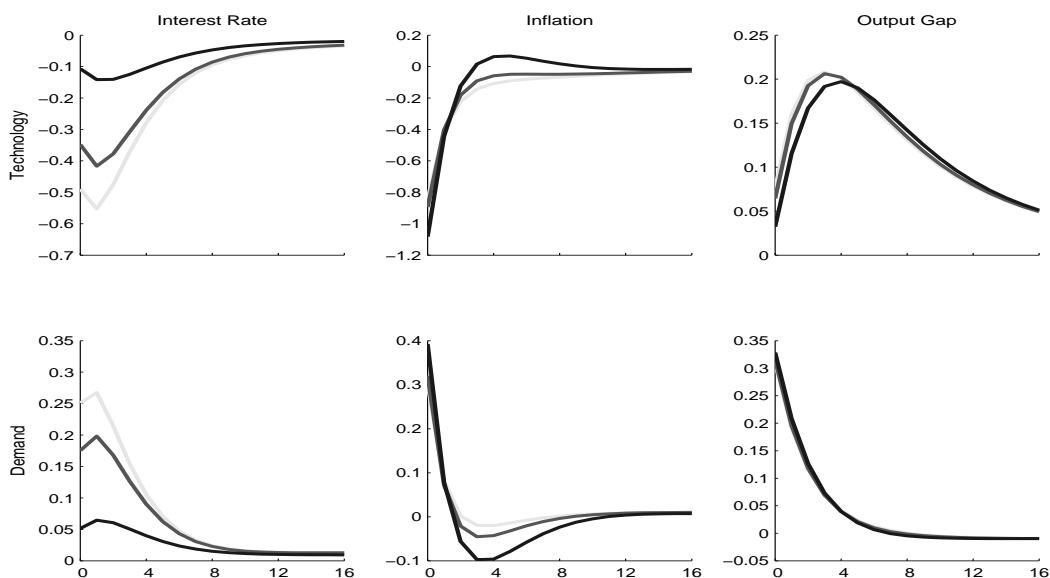
Scenario 4 – Learn about Misspecification (Conditional)



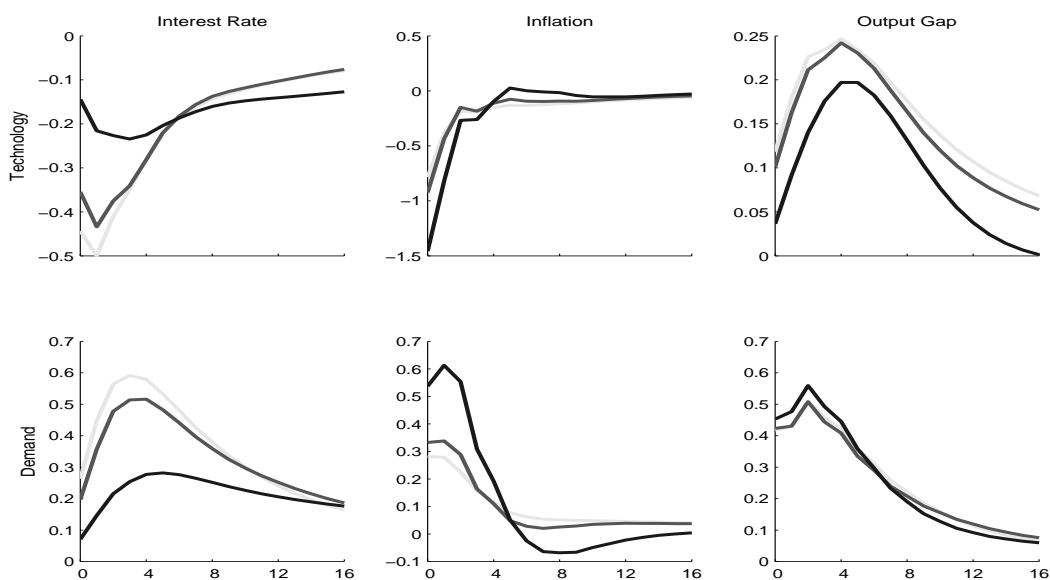
Notes: Lines' color ranges from very light grey ($\psi_1 = 1.25$) to dark grey ($\psi_1 = 2.75$), with the darkness of the line being directly proportional to ψ_1 .

Figure 5: IMPULSE RESPONSES AS FUNCTION OF ρ_R

Scenario 1 – Ignore Misspecification

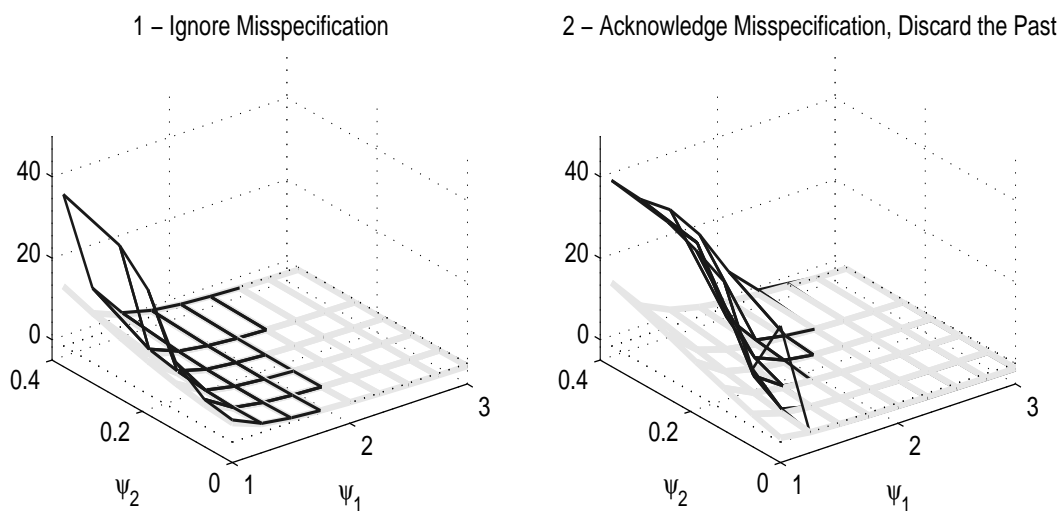


Scenario 4 – Learn about Misspecification (Conditional)



Notes: Lines' color ranges from very light grey ($\rho_R = .7$) to dark grey ($\rho_R = .95$), with the darkness of the line being directly proportional to ρ_R .

Figure 6: EXPECTED POLICY LOSS DIFFERENTIALS - RISK-NEUTRAL VERSUS RISK-SENSITIVE



Notes: Policy loss differentials relative to baseline policy rule $\psi_1 = 2.0$, $\psi_2 = 1.88$, $\rho_R = 0.80$. All numbers are computed fixing the value of $\rho_R = 0.8$. Negative differentials signify an improvement relative to baseline rule. For each scenario, the expected loss differential is shown in light grey, and the risk-sensitive loss differential in black.