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Time-Varying Volatility in Canadian and U.S. Stock Index and Index Futures Markets: A Multivariate Analysis

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Abstract: We use a multivariate generalized autoregressive heteroskedasticity model (M-GARCH) to examine three stock indexes and their associated futures prices: the New York Stock Exchange Composite, Standard and Poor's 500, and Toronto 35. The North American context is significant because markets in Canada and the United States share similar structures and regulatory environments. Our model allows examination of dependence in volatility as it captures time variation in volatility and cross-market influences. Estimated time-variation in volatility is significant, and the volatilities are highly positively correlated. Yet, we find that the correlation in North American index and futures markets has declined over time.

JEL classification: G10, G15

Key words: volatility dependence, M-GARCH

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Introduction

Financial markets are interrelated and increasingly global. When making decisions, traders incorporate information pertaining to price movements and volatility in the asset they are trading, including information about related assets. Thus, understanding how markets influence one another is important for pricing, hedging, and regulatory policy. A plethora of studies have examined how price movements are correlated across asset and derivative security markets and international borders, particularly since the market crash in October 1987 (e.g., Hamao, Masulis, and Ng (1990) and King and Wadhwani (1990)). The relation between stock index and stock index futures markets is of special concern because some have suggested that trading in derivative securities causes instability in cash markets (Stoll and Whaley (1990)). However, futures markets can be an important source of information, particularly during periods of market distress. Stock and futures price volatility have definitely caught the attention of researchers, investors, and policy makers.

The purpose of this paper is to further examine dependence in volatility for North American stock and stock index futures markets. We focus on Canada and the United States because markets in the two countries share similar structures and regulatory environments (Mittoo (1992) and Karolyi (1995)). In addition, there are few controls on the movement of capital. Trading times in the countries' eastern markets are concurrent, though futures markets close 15 minutes after stock markets. Earlier research has shown that of the major markets, these two are the most highly correlated (Longin and Solnik (1995)). In fact, regulation in Canada and

the U.S. reflects this inter-market correlation. Recently stock and stock-derivatives markets have again agreed to coordinated circuit breakers.¹ These trading halts are uniformly triggered by large movements in the Dow Jones Industrial Average in the United States *and* Canada.

We adopt the multivariate generalized autoregressive heteroskedasticity model (M-GARCH) developed by Bollerslev (1990) to examine three stock indexes and their associated futures prices. Persistence in volatility has been documented in the U.S. (Akgiray (1989)) and Canada (Calvet and Rahman (1995)). In addition, volatility transmission and correlation has been estimated for international markets (Karolyi (1995) and Longin and Solnik (1995)), as well as cash and futures markets (Chan, Chan, and Karolyi (1991)). This paper contributes to our understanding of market interdependencies by bringing together the literature examining persistence and correlation in price volatility across stock and futures markets. The multivariate model captures time variation in volatility and cross-market influences. In addition, we examine whether the correlation across markets is constant over time.

The results provide insight into daily time variation in price volatility as well as correlation in volatility across stock and futures markets. The estimated time-variation in volatility in stock and stock index futures markets is significant and persistent and these time-varying volatilities are highly positively correlated. Furthermore, we provide evidence of changes in volatility correlation over time. Consistent with results reported by Karolyi (1995) but inconsistent with other studies, we find that volatility correlation across North American markets decreased over time. This finding has important policy implications regarding the coordination of circuit breakers across stock and derivative security markets, as well as across international borders.

Finally, we report evidence concerning day-of-the-week and holiday effects on price movements and volatility. To date, evidence on seasonal patterns in indexes and futures prices is conflicting and inconclusive.² Researchers have documented significant Monday seasonals in stock returns and volatility (Gibbons and Hess, 1981). In his model of daily transmissions of stock return and volatility, Karolyi (1995) also includes weekend and holiday dummies. Significant day-of-the-week effects are reported for his sample which included returns for 1981 through 1989, despite some evidence suggesting that the Monday effect disappeared completely by 1975 (Connolly, 1989). In their examination of seasonality in stock index and stock index futures returns, Khaksari and Bubnys (1992) find greater evidence of day-of-the-week effects in futures prices than in the underlying index. We examine whether seasonal patterns persist in the 1988 through 1993 period in stock and index futures markets. The specification adopted in this paper jointly models first and second moments of the price processes across markets, while allowing for seasonal effects in both prices and price volatility. We find that seasonal effects are not consistently reflected in price movements and volatility.

The remainder of this paper is organized as follows. Section I presents the econometric model. Section II discusses the results. Section III contains a summary and concluding remarks.

I. Methodology

Let ΔP_t be the $N \times 1$ vector of interest. The multivariate generalized autoregressive heteroskedasticity model (M-GARCH(J, K)) developed by Bollerslev (1990) can be expressed as

$$\Delta P_t = E(\Delta P_t \mid \Psi_{t-1}) + \epsilon_t, \quad (1)$$

$$\epsilon_t \mid \Psi_{t-1} \sim N(0, H_t), \quad (2)$$

the diagonal elements of H_t are

$$h_{i,i,t} = d_i + \sum_{j=1}^J a_{i,j} h_{i,i,t-j} + \sum_{k=1}^K b_{i,k} \epsilon_{i,i,t-k}^2 + \sum_{l=1}^L c_{i,l} X_{l,t}, \quad (3)$$

the off-diagonal elements of H_t are

$$h_{i,m,t} = \rho_{i,m} (h_{i,i,t} h_{m,m,t})^{1/2}, \quad (4)$$

where $m = 1, \dots, N$, $I = 1, \dots, N$, $I \neq m$, $\Delta P_{i,t}$ represents the change in price for series I at time t , X is a vector of exogenous variables, ϵ_t is a $(N \times 1)$ vector of innovations, Ψ_t is the information set at time t , and $\rho_{i,m}$ indicates the interaction between the variances for I and m . This multivariate, constant correlation model captures time variation in volatility and cross-market influences.³ The appealing feature of the model is that it uses relatively simple estimation procedures yet it

provides a measure of correlation on which inference can be based (Bollerslev, 1990).

Our specific model is

$$\Delta P_{i,t} = \beta_{i,0} + \sum_{j=1}^J \beta_{i,1,j} \Delta P_{i,t-j} + \beta_{i,2} WKND_t + \beta_{i,3} HOL_t + \sum_{k=0}^K \beta_{i,4,k} \epsilon_{i,t-k}, \quad (5)$$

$$\epsilon_t \mid \Psi_{t-1} \sim N(0, H_t), \quad (6)$$

where the diagonal elements of H_t are

$$h_{i,i,t} = d_i + \sum_{j=1}^J a_{i,j} h_{i,i,t-j} + \sum_{k=1}^K b_{i,k} \epsilon_{i,i,t-k}^2 + c_{i,1} WKND_t + c_{i,2} HOL_t. \quad (7)$$

The off-diagonal elements of H_t are given by (4), $WKND_t$ is a weekend dummy variable which takes the value of one each Monday, and HOL_t is a holiday dummy variable which takes the value of one each day following a holiday. Price changes, rather than returns are modeled because with zero investment, futures returns are not well-defined (Black, 1976).⁴ Dummy variables are included in the mean and variance equations because researchers have documented significant seasonals in stock returns and volatility.

Numerical optimization of the log-likelihood function gives maximum likelihood estimates of model parameters. Model estimates provide insight into time variation in the volatility of daily price changes as well as correlation in volatility across stock and futures markets. If $a_{i,j}$ and $b_{i,k}$ are significantly positive then volatility shocks persist and the degree of

volatility persistence is measured by the sum of the parameters of the lag polynomials, $\Sigma a_{i,j} + \Sigma b_{i,k}$. Higher values of the sum indicate greater persistence and if the parameters sum to one, shocks are permanent. In addition, correlation in the volatility of price changes is measured by $\rho_{i,j}$. Statistically significant estimates of $\rho_{i,j}$ indicate that the time-varying volatilities across markets i and j are correlated over time. Finally, our results provide additional evidence concerning day-of-the-week and holiday effects on price movements and volatility. If the coefficients of the dummy variables differ significantly from zero, the mean ($\beta_{i,2}$ or $\beta_{i,3}$) or volatility ($c_{i,1}$ or $c_{i,2}$) of price changes shifts after weekends or holidays.

II. Empirical Results

The sample consists of daily observations for three indexes and the associated futures contract closing prices for January 1988 through March 1993, resulting in 1,302 observations. The total number of observations is less than the total possible for each index or futures contract in isolation because modelling required matching prices.⁵ The nearby futures contract with at least 15 days to maturity is used.⁶ All six series were collected from the *Wall Street Journal* or the *Globe and Mail*.

In the U.S. many index futures contracts are traded. Two have been chosen for analysis. The S&P 500 index is a broad-based index containing five hundred stocks traded on the New York Stock Exchange and the NYSE Composite Index is based on a portfolio of all stocks traded on the NYSE. Both indexes weight by market value. In addition, a Canadian index and futures contract are included. The Toronto Stock Exchange, the nation's largest exchange, introduced the Toronto 35 Index, a modified capitalization weighted index, in May 1987. Larger

capitalization stocks receive more weight but each stock receives a maximum weight of 10% so that no stock or industry dominates the index. We analyse the relationships among these markets to investigate dependencies in second moments across U.S. and Canadian futures and stock index markets. We estimate the model (4) - (7) for pairs of indexes and futures. The full model would require the estimation of a large number of coefficients and convergence would be difficult to obtain.⁷

Table 1 presents summary statistics for each series, including the mean, standard deviation, skewness, and kurtosis for first-differences of each index level and associated futures contract price. The means of the first-differenced series do not differ significantly from zero at the 5% significance level. The first-differenced series are all highly negatively skewed and leptokurtotic.^{8,9} In addition, Jarque-Bera normality tests strongly reject normality for all price changes.¹⁰ Finally, the table reports Ljung-Box serial correlation test statistics for up to 12 lags for price changes (Q(12)) and squared price changes (QS(12)).¹¹ Market inefficiencies are not evident as no significant linear dependencies are reported. However, with the exception of the S&P 500 stock index and the Toronto 35 index futures price change series, significant nonlinear dependencies are indicated. These nonlinear dependencies may result from autoregressive conditional heteroskedasticity which has been well documented in stock returns.¹²

The statistics summarized in Table 1 suggest that temporal dependence in second moments should be investigated. As discussed in the previous section, we model temporal dependence using the constant correlation multivariate GARCH model given by equations (4) - (7). In addition to this specification of the conditional variance process, we examined other specifications in our preliminary analysis.¹³ In order to determine the model that provides the

best fit we investigated the time series dependence in the conditional variances and the correct specification of the conditional variance covariance matrix using Lagrange Multiplier (LM) tests (Ding (1994)). These tests use the ARCH sample covariance normalized residuals, $w_t = (\epsilon_t' H_t^{-1} \epsilon_t) / N$, which is a scalar with mean one, in a regression of $(w_t - 1)$ on a constant and w_{t-q} for $q = 1$ to 5. The LM test for the general validity of the model tests for zero average correlation of the variances over the first five lags and converges in distribution to a χ^2 variable with q degrees of freedom for q lags (Hamilton (1994)). Ceteris paribus, the M-GARCH process with the minimum LM statistic is the preferred model. We also examined Ljung-Box serial correlation test statistics for up to 12 lags for price changes (Q(12)) and squared price changes (QS(12)) of the ARCH normalized residuals. Our initial values were the estimates from univariate GARCH models which were originally based on unconditional variance estimates, though other starting values were used to confirm robustness.

Table 2 reports the results of fitting the M-GARCH model to each stock index and associated futures price change series. For each index pair, the futures price change is modeled in equation one ($\Delta P_{1,t}$) and the stock price index change in equation two ($\Delta P_{2,t}$). The table reports t-statistics using robust standard errors in parentheses below each coefficient estimate (Bollerslev and Wooldridge (1992)). The M-GARCH (1,1) provides the best fit for the index and futures price changes for all three stock index/futures pairs. This result is consistent with findings reported by others (Bollerslev, Chou, Kroner (1992)). In all three models of price and volatility in stock index and associated futures markets, significant autoregressive, conditional heteroskedasticity is reported. Almost every estimated parameter of the conditional variance equations is significantly different from zero.

The parameter estimates reported in Table 2 also provide evidence concerning persistence in volatility.¹⁴ As discussed in the previous section, the sum of the parameters of the lag polynomial measures the degree of volatility persistence. Many of the volatility persistence measures are close to one suggesting that price changes show strong reaction to past information. The largest estimate is 0.9808 for the S&P 500 futures price change and the smallest is 0.8141 for the NYSE Composite futures price change series, with an average over the six series of 0.9073. Thus, although the processes are very slowly reverting, current volatility shocks do not remain important for *all* future forecasts of the conditional variance.¹⁵

Table 2 also reports estimates of the coefficients for the dummy variables representing day-of-the-week and holiday effects on price movements and volatility. With the exception of the NYSE Composite series, the coefficients of the dummy variables in the mean equations ($\beta_{i,2}$ or $\beta_{i,3}$) are not significantly different from zero. NYSE Composite futures and index prices move up, on average, after a weekend. For all index/futures pairs weekends and/or holidays have significant effects on the volatility ($c_{i,1}$ or $c_{i,2}$) of price changes. For the NYSE Composite, lower volatility is observed in the futures price series after weekends. For the S&P 500 and Toronto 35 indexes, futures price volatility decreases after weekends and holidays but stock index price volatility increases after weekends and holidays. Higher volatility in stock markets after non-trading days has been documented (Gibbons and Hess (1981)). Our results show that when the model incorporates interactions between stock and futures markets, volatility actually falls in futures markets. Thus, even though the first and second moments of stock and futures prices are highly correlated, weekends and holidays have opposite effects on volatility in the two related markets.

Finally, Table 2 reports estimates of correlation in the volatility of price changes, $\rho_{i,j}$, for each stock index and futures price change pair. All estimates are large, positive, and differ significantly from zero. These estimates suggest that the time-varying volatilities across the two series modeled are highly positively correlated over time. Across the three indexes, the degree of correlation is roughly similar with estimates ranging from 0.8347 to 0.9425.

Table 3 (4) reports the results of fitting M-GARCH models to pairs of stock index (futures) price change series. For each pair, the series modeled in equation one ($\Delta P_{1,t}$) is listed first, with the second series modeled in equation two ($\Delta P_{2,t}$). Below each coefficient estimate are t-statistics using robust standard errors. With the exception of the NYSE Composite and Toronto 35 stock index pair in Table 3 which is modeled as an M-GARCH (2,1), the M-GARCH (1,1) provides the best fit for the index and futures price changes for all stock index or futures pairs. The coefficient estimates indicate that, in most cases, the lagged conditional variances and past innovations have significant effects on the conditional variances.

The degree of volatility persistence exceeds 0.90 for every series in each pair except for the NYSE Composite and Toronto 35 index futures. This combination also has the lowest correlation (.6529). The persistence of the NYSE futures series, when estimated jointly with the Toronto 35 index futures, is only 0.3318 suggesting little responsiveness to the past. In contrast, for the NYSE/S&P futures pair we have integrated processes, i.e., the sum exceeds one. Current information remains important for forecasts of all horizons and the pricing of long term versus one period contracts may be substantially different (Bollerslev and Engle, 1993). In addition, Chou (1988) shows that stock prices are much more responsive to volatility shocks when the price process is integrated relative to a stationary variance model. In most cases, however,

shocks are not permanent and decay very slowly.

The results reported in Tables 3 and 4 also suggest that day-of-the-week and holiday effects are not consistent influences on price movements and volatility. Although the coefficients of the dummy variables differ significantly from zero in some cases, the NYSE Composite and S&P 500 index pair is the only pair that exhibits a distinct pattern. For this pair, volatility decreases in the NYSE index series after weekends and holidays, whereas volatility increases in the S&P 500. This result is suggestive of a small firm effect where large firms have higher volatility after non-trading days but smaller size firms have lower volatility.

Finally, the results reported in Tables 3 and 4 indicate that there are significant cross-market volatility dependencies. Estimates of correlation in volatility are all high and range from 0.6529 to 0.9545. As expected, the U.S. series are more closely correlated than any U.S. series with a Canadian series. Comparison of Tables 3 and 4 suggests that cross-market correlation is very similar in cash and futures markets. For example, correlation in volatility for the S&P 500 and Toronto 35 stock index pair is 0.6787 which is quite close to the futures index correlation of 0.6771.

In order to provide additional insight into the integration in North American markets, we conducted subperiod analyses. First we simply cut the sample in half and re-estimated the model for each price pair and subsample of 651 observations. Then we systematically investigated whether one or more structural breaks occurred during the sample period. To do so, we examined plots of the data, performed Chow tests, and looked to chronologies of events for major changes in the economy. Only one possible “break” date surfaced. On Friday, October 13, 1989 the DJIA “plunged” 190 points (*Wall Street Journal*, October 16, 1989, page C1).

Table 5 reports estimates of correlation in volatility for the overall sample and each subperiod with standard errors below in parentheses. For almost all of the cross-index and cross-futures pairs, the correlation is smaller for the more recent time period.¹⁶ A test for equality of these correlations is reported for each pair of subperiods in brackets. This test statistic has a χ^2 distribution with one degree of freedom. Across all stock index pairs the correlation is significantly lower in the more recent time period.¹⁷ Given the increase in correlation reported in other studies (e.g., Longin and Solnik (1995)), the results in Table 5 seem surprising. However, Karolyi (1995) also examines changes in model estimates over sample subperiods and finds diminishing dependence in Canadian stock market returns to shocks in the U.S. market. Karolyi attributes the declining Canadian dependence on the U.S. market to globalization which is consistent with increasing internationalization of the Canadian market.

IV. Summary and Concluding Remarks

The results we report in this paper suggest that the M-GARCH model provides useful information concerning the movement of asset prices over time. Diagnostic tests indicate that estimates of volatility persistence and correlation are robust across estimated models. Time-variation in volatility is significant in North American stock and stock index futures markets. These time-varying volatilities are highly positively correlated across stock indexes and their associated futures contracts, as well as across markets. Clearly, traders use this information in decision making.

Regulation of securities markets should reflect the dynamics of first and second moments of the price distribution. Following the Brady Commission's *Report to the Presidential Task*

Force on Market Mechanisms (1988), exchanges in the U.S. and Canada have adopted circuit breaker rules wherein trading is halted upon specific downward movements in the Dow Jones Industrial Average. The benefits of circuit breakers have been questioned. If the goal of allowing a cooling-off period in times of market distress is a reasonable one, regulators should scrutinize patterns in volatility in the asset's price in addition to price changes. Furthermore, regulators should closely watch the price volatility for related assets. In fact, in Canada circuit breakers are tied to the Dow Jones Industrial Average, rather than a Canadian measure. Karolyi (1995) questions the effectiveness of this policy because he finds decreasing dependence of TSE 300 returns on U.S. stock market movements over time. However, given the thin trading that has been documented in Canada, the Toronto Stock Exchange's procedures may be wise (Fowler, Rorke, and Jog (1980)).

Circuit breakers in North American stock and derivatives markets are coordinated. This coordination reflects the notion that these markets are closely integrated. We find that although volatility is highly correlated across U.S. and Canadian stock and futures markets, these correlations have declined over time. Thus, our results suggest that regulators of stock and derivatives markets should continue to reassess the rules governing trading interruptions and evaluate their effectiveness in light of evolving market relationships over time.

Endnotes

1. See, for example, "Big Board Shifts Plan on Closing New 'Circuit Breakers' Adopted by Exchange," *Wall Street Journal*, February 6, 1998.
2. However, systematic patterns in return volatility over the trading day have been widely reported. See, for example, Anderson and Bollerslev (1994 and 1997).
3. This model has been used by Bollerslev (1990) and Baillie and Bollerslev (1990) to model exchange rate markets and by Kroner and Sultan (1993) to model foreign currency markets. In addition, Longin and Solnik (1995) fail to reject the null hypothesis of a constant conditional correlation between Canadian and U.S. equity returns.
4. In their study of stock index and index futures seasonality, Khaksari and Bubnys' (1992) conclusions are not altered when other measures of return are used.
5. Because futures markets close 15 minutes after stock markets, closing prices are nonsynchronous. If a great deal of information is released during the 15 minute period when stock markets are closed, we would expect to find significant correlation between futures prices and index prices on the subsequent day. We find that the contemporaneous correlations are large and significant whereas the correlation between index and lagged futures prices are smaller and usually insignificant. Although there is a potential bias toward the finding that futures prices lead index prices, the correlation results provide evidence that the effect of the 15 minute trading time differential is minimal.
6. Some researchers such as Hein, Ma, and MacDonald (1990) suggest the use of nonoverlapping data. They point out that overlapping forecast horizons should be avoided when testing whether futures prices are unbiased predictors of future spot prices. Here the concern is with correlation in volatility across markets resulting from daily information flows, so that there is no overlapping data problem.
7. See Longin and Solnik (1995) on the technical difficulties.
8. For samples exceeding 150 observations the skewness statistic is distributed normally. The standard deviation is approximately equal to the square root of six over the number of observations, 1302. See Snedecor and Cochran (1989).
9. The kurtosis statistic minus three is normally distributed for sample sizes over 1,000. The standard deviation is approximated by the square root of 24 over the number of observations, 1302. See Snedecor and Cochran (1989).
10. The Jarque-Bera (1980) normality test statistic follows a χ^2 distribution with 2 degrees of freedom. The critical value is 9.21 at the 1% significance level.

11. The Ljung-Box (1978) serial correlation test statistic for q lags follows a χ^2 distribution with q degrees of freedom. The critical value is 26.217 (21.026) at the 1% (5%) significance level for 12 lags.
12. See the comprehensive review by Bollerslev, Chou, and Kroner (1992).
13. One model we examined was the GARCH-BEKK process used by Karolyi (1995) to model international transmissions of stock returns and volatility. The BEKK model has two disadvantages. The model is not parsimonious and estimated unconditional covariance matrices may not be positive definite during estimation. This is the “internal inconsistency problem” discussed by Ding (1994). The constant correlation model was proposed in the literature because it does not suffer from these two deficiencies.
14. Persistence may be overestimated when a structural break is present but not taken into account (Lamoureux and Lastrapes, 1990). In their examination of the persistence in stock return volatility in Canada, Calvet and Rahman (1995) estimate persistence in volatility for three distinct monetary regimes. They find that persistence is smaller for subperiods determined by the different regimes, though the changes are minor. Thus, structural breaks are expected to have little effect on measures of persistence. Furthermore, our sample period lies completely within Calvet and Rahman’s third regime
15. In a previous study of time variation and persistence in volatility in futures prices and stock indexes, Ackert and Racine (1996) report univariate GARCH estimates of persistence. They find that the Toronto 35 index and futures price exhibit weak persistence. In this study we find that persistence for the Canadian series is similar to the U.S. experience. This difference highlights the importance of including cross-market effects in properly modeling the time variation in volatility.
16. We also estimated the model using other subperiods and the results were unchanged. For example, we examined whether the business cycle impacts our correlation estimates. Erb, Harvey, and Viskanta (1994) suggest that correlations are related to business cycles with higher correlation during recessions. Our correlation estimates declined in more recent subperiods with no apparent relation to the business cycle. We also examined whether patterns in correlation were related to changes in futures price volatility. Kawaller, Koch, and Koch (1993) suggest that the correlation between cash and futures prices increases as futures volatility increases. Correlations consistently declined over later time periods, regardless of the pattern in futures volatility.
17. We also tested whether the correlations in subperiods equal the overall correlation and inferences are unaffected.

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Table 1
Summary Statistics

The table reports summary statistics for changes in daily stock indexes and futures prices for the NYSE Composite, Standard and Poor's 500 (S&P 500), and Toronto 35 stock indexes for the January 1, 1988 through March 31, 1993 time period, a total of 1302 observations.

	Stock Index			Stock Index Futures		
	NYSE Composite	S&P 500	Toronto 35	NYSE Composite	S&P 500	Toronto 35
Mean	0.0818	0.1503	-0.0006	-0.0007	0.1499	0.0117
Standard Deviation	1.5161	3.0321	1.2381	1.9267	3.2530	1.4032
Skewness	-0.7120**	-0.6360**	-0.5040**	-0.8450**	-0.7590**	-0.3580**
Kurtosis	8.5500**	7.8100**	7.8700**	13.2600**	11.7500**	6.4900**
Normality	1775.0000**	1338.0000**	1337.0000**	5838.0000**	4261.00**	683.9000**
Q(12)	12.6100	15.0900	10.7600	16.1600	13.1300	15.4600
QS(12)	28.3000**	17.2000	49.5000**	35.9000**	25.4000*	7.1000

** Significant at the 1% level

* Significant at the 5% level

Table 2
Multivariate GARCH Model:
Futures and Associated Stock Index Prices

	NYSE Composite	S&P 500	Toronto 35
Mean Equation (5)			
$\beta_{1,0}$	-	0.2243 (2.02)*	-0.0050 (-0.10)
$\beta_{1,1,1}$	-0.1250 (-6.37)**	-0.2480 (-15.25)**	-
$\beta_{1,2}$	0.2559 (2.41)*	0.2233 (1.17)	0.0161 (0.18)
$\beta_{1,3}$	0.3203 (1.41)	0.2032 (0.54)	0.1576 (0.91)
$\beta_{2,0}$	-	0.2428 (2.39)*	-0.0049 (-0.12)
$\beta_{2,1,1}$	-0.1208 (-6.09)**	-0.1882 (-11.33)**	-
$\beta_{2,2}$	0.2262 (2.83)**	0.0459 (0.25)	-0.0396 (-0.52)
$\beta_{2,3}$	0.2612 (1.75)	0.2020 (0.58)	0.0977 (0.59)
Variance Equation (7)			
d_1	0.9693 (14.75)**	1.0294 (10.87)**	0.2135 (3.75)**
$a_{1,1}$	0.6569 (26.59)**	0.9604 (177.35)**	0.9484 (46.73)**
$b_{1,1}$	0.1572 (10.37)**	0.0204 (6.08)**	0.0024 (0.61)
$c_{1,1}$	-0.8624 (-9.17)**	-4.9962 (-10.16)**	-0.7240 (-5.09)**
$c_{1,2}$	-0.1511 (-1.13)	-3.5927 (-6.65)**	-0.3724 (-4.78)**
d_2	0.4433 (5.73)**	0.7632 (6.95)**	0.2329 (3.87)**
$a_{2,1}$	0.7114 (18.36)**	0.9546 (173.27)**	0.8556 (21.76)**
$b_{2,1}$	0.1267 (7.96)**	0.0254 (9.39)**	0.0243 (4.30)**
$c_{2,1}$	0.2699 (1.20)	5.0923 (13.75)**	0.7817 (5.81)**
$c_{2,2}$	0.0694 (0.42)	3.9895 (8.60)**	0.5335 (5.07)**
$\rho_{1,2}$	0.8975 (307.51)**	0.9425 (465.38)**	0.8347 (135.74)**

** Significant at the 1% level

* Significant at the 5% level

Table 3
Multivariate GARCH Model:
Across Stock Indexes

	NYSE Composite and S&P 500	NYSE Composite and Toronto 35	S&P 500 and Toronto 35
Mean Equation (5)			
$\beta_{1,0}$	0.1684 (3.47)**	-	-
$\beta_{1,2}$	-0.1253 (-1.65)	-0.0242 (-0.28)	-0.0178 (-0.24)
$\beta_{1,3}$	0.2335 (1.51)	0.1648 (0.98)	0.0935 (0.61)
$\beta_{2,0}$	0.2956 (3.08)**	-	-
$\beta_{2,2}$	-0.1003 (-0.66)	0.0863 (0.85)	0.3149 (1.84)
$\beta_{2,3}$	0.3993 (1.26)	0.1996 (0.98)	0.1402 (0.40)
Variance Equation (7)			
d_1	0.2770 (14.14)**	0.0364 (0.80)	0.0875 (2.58)**
$a_{1,1}$	0.9581 (241.56)**	0.2839 (1.39)	0.9231 (38.94)**
$a_{1,2}$	-	0.6091 (3.17)**	-
$b_{1,1}$	0.0270 (11.88)**	0.0415 (3.76)**	0.0282 (3.75)**
$c_{1,1}$	-1.4419 (-14.30)**	0.2490 (2.66)**	-0.1112 (-1.38)
$c_{1,2}$	-6.3943 (-15.49)**	0.2128 (1.29)	-2.34 (-3.54)**
d_2	1.2563 (14.26)**	-0.0304 (-1.00)	0.3906 (3.11)**
$a_{2,1}$	0.9569 (248.37)**	0.4484 (2.21)*	0.9892 (471.36)**
$a_{2,2}$	-	0.5011 (2.55)	-
$b_{2,1}$	0.0275 (14.63)**	0.0286 (4.31)**	0.0070 (5.00)**
$c_{2,1}$	1.4045 (18.96)**	0.1374 (1.05)	0.1733 (1.93)
$c_{2,2}$	5.4236 (11.39)**	0.7666 (3.70)**	2.9325 (7.84)**
$\rho_{1,2}$	0.9545 (762.26)**	0.7019 (55.52)**	0.6787 (54.00)**

** Significant at the 1% level

* Significant at the 5% level

Table 4
Multivariate GARCH Model:
Across Stock Index Futures

	NYSE Composite and S&P 500	NYSE Composite and Toronto 35	S&P 500 and Toronto 35
Mean Equation (5)			
$\beta_{1,0}$	-	-0.0192 (-0.47)	-
$\beta_{1,2}$	0.6647 (12.40)**	0.0756 (0.77)	0.0287 (0.32)
$\beta_{1,3}$	0.5339 (3.35)**	0.2764 (1.23)	0.2000 (1.25)
$\beta_{2,0}$	-	-0.0535 (-0.96)	-
$\beta_{2,2}$	1.3580 (13.94)**	0.1607 (1.23)	0.4953 (2.59)**
$\beta_{2,3}$	0.9684 (3.76)**	0.3960 (1.37)	0.2144 (0.57)
Variance Equation (7)			
d_1	0.2754 (6.88)**	1.2378 (3.99)**	0.0776 (2.12)*
$a_{1,1}$	0.6746 (66.18)**	0.2468 (1.60)	0.9604 (49.44)**
$b_{1,1}$	0.4060 (17.73)**	0.0850 (2.93)**	0.0152 (2.37)*
$c_{1,1}$	-0.0008 (-0.01)	0.1764 (1.11)	-0.1820 (-1.34)
$c_{1,2}$	0.0078 (0.02)	-0.1363 (-0.69)	-2.5516 (-4.84)**
d_2	0.8331 (5.68)**	0.5264 (5.17)**	0.4629 (4.64)**
$a_{2,1}$	0.7169 (42.28)**	0.7574 (26.23)**	0.9815 (322.70)**
$b_{2,1}$	0.3026 (14.33)**	0.0994 (7.82)**	0.0108 (5.58)**
$c_{2,1}$	0.0326 (0.14)	1.0652 (2.74)**	0.1628 (1.46)
$c_{2,2}$	0.4959 (0.62)	1.4641 (4.98)**	3.2771 (9.03)**
$\rho_{1,2}$	0.9515 (578.30)**	0.6529 (48.04)**	0.6771 (46.09)**

** Significant at the 1% level

* Significant at the 5% level

Table 5
Subperiod Estimates of Correlation

Series	Full sample	First Half	Last Half	Before Break	After Break
Futures and Associated Stock Index Prices					
NYSE Composite	0.8945 (0.0028)	0.8933 (0.0042)	0.8960 (0.0053)	0.9114 (0.0076)	0.8962 (0.0048)
		[0.06]		[2.01]	
S&P 500	0.9406 (0.0021)	0.9252 (0.0040)	0.9404 (0.0018)	0.9470 (0.0044)	0.9380 (0.0027)
		[4.47]**		[1.89]	
Toronto 35	0.8326 (0.0065)	0.8710 (0.0075)	0.7990 (0.0100)	0.8745 (0.0081)	0.7992 (0.0096)
		[18.88]***		[18.97]***	
Across Stock Indexes					
NYSE and S&P 500	0.9493 (0.0014)	0.9558 (0.0024)	0.9457 (0.0025)	0.9814 (0.0012)	0.9528 (0.0023)
		[3.61]*		[64.99]***	
NYSE and Toronto 35	0.7008 (0.0121)	0.7580 (0.0150)	0.6270 (0.0177)	0.7832 (0.0187)	0.6478 (0.0181)
		[21.07]***		[23.13]***	
S&P 500 and Toronto 35	0.6728 (0.0125)	0.7164 (0.0176)	0.6152 (0.0233)	0.7612 (0.0221)	0.6349 (0.0203)
		[10.85]***		[18.09]***	
Across Stock Index Futures					
NYSE and S&P 500	0.9453 (0.0018)	0.9571 (0.0025)	0.9587 (0.0023)	0.9712 (0.0022)	0.9454 (0.0026)
		[0.12]		[30.96]***	
NYSE and Toronto 35	0.6495 (0.0135)	0.6703 (0.0189)	0.6238 (0.0237)	0.6826 (0.0227)	0.6390 (0.0192)
		[2.08]		[0.74]	
S&P 500 and Toronto 35	0.6730 (0.0143)	0.7244 (0.0172)	0.6030 (0.0251)	0.7190 (0.0197)	0.6404 (0.0207)
		[15.53]***		[6.23]***	

*** Significant at the 1% level.

** Significant at the 5% level.

* Significant at the 10% level.