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If at first you don't succeed: an experimental investigation of the impact of repetition options on corporate takeovers

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**If at First You Don't Succeed:
An Experimental Investigation of the Impact of
Repetition Options on Corporate Takeovers**

Ann B. Gillette and Thomas H. Noe

Working Paper 2000-9
August 2000

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Abstract: This paper models, and experimentally simulates, the free-rider problem in a takeover when the raider has the option to “resolicit,” that is, to make a new offer after an offer has been rejected. In theory, the option to resolicit, by lowering offer credibility, increases the dissipative losses associated with free riding. In practice, the outcomes of our experiment, while quite closely tracking theory in the effective absence of an option to resolicit, differed dramatically from theory when a significant probability of resolicitation was introduced: The option to resolicit reduced the costs of free riding fairly substantially. Both the raider offers and the shareholder tendering responses generally exceeded equilibrium predictions.

JEL classification: G3, C7

Key words: corporate takeovers, experimental economics, resolicit

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If at First You Don't Succeed: An Experimental Investigation of the Impact of Repetition Options on Corporate Takeovers

1. Introduction

Answering a number of vexing questions relating to collective choice is central to our understanding of how voluntary business, social, economic, and social institutions function. First, consider the governance problem of the modern corporation—ensuring that the strongest possible management team holds the reins of corporate control. The essential problem is this: If, during a takeover attempt, a rival management team seeks to unseat a weaker incumbent, then, because the post-acquisition value of the firm will be higher than the pre-acquisition value, each shareholder of the target firm wants the takeover to succeed. At the same time, the rival team, in order to profit from the acquisition, will offer a price below post-acquisition value. Thus, it follows that no shareholder will want to tender his shares into the offer. However, if no shareholder tenders, then the takeover attempt fails and all shareholders are left worse off. The failure of the outcome to reflect the interests of either the shareholders or the rival team occurs because, while the benefits of control transfer are shared by all shareholders, the “cost” of providing the public good of control transfer is borne only by the tendering shareholders. This allocation of the benefits of control transfer gives individual shareholders an incentive to *free ride*, that is, to reject the offer in the hope that a sufficient number of other shareholders accept the offer, allowing the offer to succeed. In the takeover context, the cost of not free riding is the difference between the post-acquisition share price and the tender price offered by the raider.

Next, consider the problem of soliciting voluntary contributions for the provision of a

public good, e.g., private funds for a public radio or television station. Although this problem is clothed in far different garb from the takeover problem, the incentive structure faced by parties is essentially the same. Each listener to the station benefits from the provision of the public good, yet only those who sponsor the station shoulder the costs of providing the good. All listeners individually will prefer to be “free listener’s,” yet if every one acts like a free listener, no one will be able to listen at all. In the context of this public-goods problem, the cost of not free riding is the contribution provided to the station.

Recently, a number of authors have modeled these sorts of free-rider problems. These authors have noted that when the number of agents is finite, equilibria exist in which free riding is not universal, i.e., individuals “contribute” with positive probability (e.g., Bagnoli and Lipman (1988), Palfrey and Rosenthal (1991)). In fact, even in some formulations of “infinite agent” models, universal free riding is not the only outcome (Noe, 1995). However, in these models, many equilibria exist that feature a partial loss of value from free riding.

This theoretical research raises a natural question: How do its predictions regarding free riding compare with the actual free-riding behavior of economic agents? Experimental research has tried to address this issue.¹ These studies have greatly increased our understanding of how the actual behavior of economic agents reflects the predictions of economic theory. However, one aspect of the free-riding problem that has not been addressed from an experimental perspective, yet is quite salient in many real-world contexts (as any public radio listener can

¹ See Cadsby and Maynes (1998), Issac, Schmidt, and Walker (1988), Kale and Noe (1997), Palfrey and Prisbrey (1997), and Palfrey and Rosenthal (1991).

attest), is the ability of the “solicitor” of the contribution to *resolicit*, that is, to make another solicitation after the first solicitation fails. As well as being ubiquitous in charitable situations, resolicitation is by no means rare in business contexts such as corporate takeovers.² In fact, Betton and Eckbo (2000) found in their comprehensive sample of tender offer contests between 1971-1990, that 38% contained multiple bids of which 41% involved only a single bidder. The aim of this paper is to investigate, from an experimental perspective, the impact of the option to resolicit on the free-rider problem.³

In principle, the option to resolicit can either increase or reduce the economic losses from free riding. The argument that resolicitation will reduce economic losses, which we will term the *direct-effect* argument, is captured by the aphorism, “if at first you don’t succeed, try, try again.”

In other words, because a deal is struck if an offer is accepted even once, the more often offers can be made, the more likely will be their eventual acceptance. Thus, if the direct-effect argument holds, the economic losses from free riding will be reduced by the option to resolicit. However, an argument that the option to resolicit will increase the economic losses from free riding can also be made. This argument, which we term the *credibility* argument, can be summarized by the aphorism, “nothing aids one’s concentration like the sight of the gallows.” In other words, it is

2 For example, Elf Aquitaine for TotalFina, Ashland Inc. for Melamine Chemical, United Dominion Industries for Commercial Intertech, Peco Energy Corporation for PP&L Resources Inc., and Horizon Healthcare Corp. for Hillhaven Corp. Although typically subsequent bids are higher, they can be lower as in the case of National Health Labs who lowered their offer for Allied Clinical Laboratories after learning that federal officials had intensified their investigation.

3 Dorsey (1992) examines how the opportunity to revise contributions in voluntary contribution mechanisms affects the investment level. However, the issues Dorsey addresses are quite different from those we consider. On the one hand, in our analysis, solicitors of contributions can revise the terms of their offer. In Dorsey’s analysis, the terms at which the public good is offered are fixed. On the other hand, in Dorsey’s analysis, contributors can view the contributions of other agents. In our analysis, contributors cannot view the contributions of other agents. Also, in Dorsey, contributing to the public good is a dominant strategy. In our analysis, a positive level of contribution is the outcome of all Nash equilibria.

only the looming threat of economic losses from the rejection of an offer that leads offers to be taken seriously. Attenuating this threat so increases an agent's willingness to free ride that dissipation losses from free riding actually increase in response to the introduction of an option to resolicit.

Which of these arguments, the direct-effect, or the credibility, will most often hold true in actual economic situations? This paper is a theoretical and experimental investigation of this question. We first develop a simple model of resolicitation. This model cast in a corporate takeover setting but isomorphic to a public-goods provision problem, features a raider attempting to take over a firm and implement an improvement in the firm's operations. The takeover game features (i) a conditional tender offer, and (ii) a simple, stationary Markovian structure. By *conditional* we mean that the payments to tendering shareholders are conditional on the raider receiving enough shares to take control of the firm. If insufficient shares are tendered, the raider returns all tendered shares and prepares a new offer. The game is *stationary* in that, after each offer failure, there is a time-stationary probability that the improvement gain generated by the raider will evaporate, and the takeover game will end. We call the complement of this probability, that is, the probability that the gains do not evaporate, the *continuation probability*.

We assume takeovers are conditional for the following reasons: Conditional takeover offers are (a) common (the most common form of takeover offer in the U.K., for example), (b) assumed in most of the classic theoretical papers on shareholder tendering behavior (e.g., Grossman and Hart (1980)), and because, (c) when offers are conditional, takeover games are

isomorphic to (conditional) public-goods-provision games. This isomorphism is not present between unconditional public-goods provision games and unconditional takeover offers.⁴ We assume a stationary probability of negotiation breakdown because this assumption (a) tracks seminal repeated offer models such as Osborne-Rubinstein (1990), (b) is easy to explain to experimental subjects, and (c) when the probability of breakdown is 1, the model is equivalent to a single-period takeover model.⁵

Our theoretical analysis shows that this game has a natural symmetric Nash equilibrium in which all shareholders follow stationary tendering strategies (i.e., strategies that depend only on the raider's current offer) and the raider follows a stationary offer strategy of making the same offer in each period. A numerical investigation of this equilibrium shows that it has the following properties:

- The bid price offered by the raider decreases, at a moderate rate, as the continuation probability increases.
- The shareholders' probability of tendering into the offer falls, at more substantial rate, as the likelihood of continuation increases.
- The likelihood that the game will end in dissipation and a loss of value actually increases in the continuation probability.

This last effect follows the negative effect of changes in shareholder tendering probabilities, caused by an increased probability of continuation, and more than offsets the positive direct effect of the reduction in dissipation caused by a higher continuation probability. In short, the option to

4 See Section 5 for more analysis of this issue.

5 Further examination of these issues is provided in the conclusions and extensions section, Section 5.

resolicit, in our theoretical model, attenuates the incentives for shareholders to tender and for raiders to offer high prices, thereby increasing the economic losses caused by offer failure. Thus, in our model, the credibility effect dominates the direct effect.

Using this model as a benchmark, we performed a series of laboratory experiments. These experiments share many characteristics with the extant experimental literature. First, as in Palfrey and Rosenthal (1991) and in some of the analysis of Kale and Noe (1997), our designs feature conditional takeover offers. We also assume in most of our analysis, consistent with these papers, that each shareholder holds only one share, i.e., we assume indivisible shareholdings. A unique feature of our experimental treatment is the option to resolicit. We implement this option by performing experiments with continuation probabilities of 0.01, 0.75, and 0.95.⁶

As well as an option to resolicit offers, another distinguishing feature of our design is that the raider offers are selected by the experimental subjects. In the extant literature, economic agents respond to offers posted by the researcher. This feature allows us to examine raider behavior, particularly the effect of shareholder rejection on future raider offers. This design also is unique in that it combines both a repeated bargaining and coordination problems. If the experiment featured a single shareholder then the problem would reduce to a one-sided repeated bargaining model. In contrast, if there were no option to repeat, the problem would reduce to a standard coordination problem in the presence of free riding.

In treatments featuring a realistic probability of resolicitation, raiders, on average, offered

prices that exceeded the Nash offer, and played non-stationary, “escalating-offer” strategies, increasing their offers after an initial rejection. When the probability of continuation was 0.75, overbidding was highly significant in a statistical sense; at the 0.95 continuation probability, overbidding was not significant at conventional levels (p-value: 0.157).

Describing shareholder strategies is more difficult because raider strategy is endogenous and because shareholders move after observing the bid price set by the raider. If raider bids deviate from the Nash equilibrium predictions, then the subgame Nash equilibrium response for shareholders may be quite different from the predicted Nash equilibrium response for the overall game. Conformity with the subgame equilibrium may imply lack of conformity with the overall Nash equilibrium predictions and vice versa. For this reason, we consider the strategies of shareholders from two perspectives: the unconditional and the conditional. From the unconditional perspective, we simply compare the shareholders’ tendering behavior with tendering behavior predicted by the Nash equilibrium. From the conditional perspective, we measure the deviation between the observed tendering *proportion given the actual raider offer* and the predicted subgame-equilibrium response to a raider following the stationary strategy of utilizing that offer.

Overall, the results of the experiments for shareholders were, in a very rough qualitative way, consistent with the Nash equilibrium predictions. For example, as predicted by theory, the proportion of shares tendered decreased in the continuation probability. However, from an

6 The motivation behind our choice of these three continuation probabilities is explained in Section 3.

unconditional perspective, there was a pattern of increasing overtendering relative to the equilibrium predictions as the continuation probability increased. At the 0.01 continuation probability, shareholders undertendered; at the 0.75 continuation probability, shareholders overtendered in a statistically insignificant fashion; at the 0.95 continuation probability, shareholder overtendering was both highly significant and large in absolute magnitude.

A conditional comparison of the data with theory reveals that much of the overtendering found in the unconditional comparisons can be accounted for by overbidding by shareholders. In fact, from a conditional perspective, shareholders undertendered significantly at the intermediate (0.75) probability of continuation, but overtendered significantly at the very high (0.95) probability of continuation. Overtendering at very high probabilities of continuation is especially surprising given the observed pattern of raider offer escalation in such cases. One might expect that this escalation would make shareholders more willing to hold out than they would if raiders were, in fact, playing stationary strategies.

The net result of deviations by both raiders and shareholders from the predictions of theory is that both efficiency and shareholder welfare are improved by granting raiders the option to resolicit. At the intermediate continuation probability, 0.75, increased efficiency can be attributed to raider overbidding. At the high continuation probability, increased efficiency resulted primarily from shareholder overtendering. In short, in the experimental treatments, in contrast to the theoretical analysis, the direct effect of allowing the raider more chances to make offers to shareholders dominates the credibility effect. Thus, this paper provides the first, to our

knowledge, experimental documentation that the option to resolicit is socially valuable. Given the frequency with which the option to repeat is present in actual economic situations, our results provide a partial explanation for why public goods are frequently provided even in the presence of strong incentives to free ride.

The rest of the paper is organized in the following manner. Section 2 derives and specifies the dynamic bargaining model studied. Section 3 lays out the experimental design and Section 4 reports and analyzes the results. Conclusions and extensions of the analysis are provided in Section 5.

2. Model

In this section, we develop the analytical theory that forms the basis for subsequent experimental investigations. This theory fits two isomorphic economic situations: a profit-maximizing corporate raider making a conditional tender offer to a group of shareholders, and a revenue-maximizing public agency soliciting contributions from a group of citizens for a public good. For clarity we follow the takeover interpretation throughout the analysis. At the end of this section, we outline and reinterpret the model in a public-goods setting.

Consider a firm with N shareholders; let $[N] = \{1, 2, 3, \dots, N\}$ represent the set of shareholders. Each shareholder, $i \in [N]$, holds 1 indivisible share of the firm's stock. At the start of each period, a raider makes a bid b_d , $d = 0, 1, \dots$ for the firm's shares. This bid is a

conditional bid. If the bid is accepted by K or more shareholders, control of the firm is transferred to the raider and the game terminates. In this case, non-tendering shareholders earn a payoff equal to the post-takeover value of the firm, 1 dollar; tendering shareholders earn a payoff equal to the bid price, b ; the raider earns a payoff equal to the number of shares tendered times the difference between the post-takeover value of the shares, 1 dollar, and the bid price he offers, b_d .

If fewer than K shares are tendered, the takeover attempt fails and tendered shares are returned to shareholders. After each failed takeover bid, there is a probability, $1 - \delta$, that the improvement gains which the raider generates evaporate. In this case, the raider and the shareholders earn payoffs of 0, and the takeover game terminates. If evaporation does not occur, the raider will be able to make another bid at the start of the next period. We call δ the “continuation probability” for the game. The comparative statics developed below will focus on the effect of shifts in δ on the outcome of the game.

We next identify the Markovian equilibria of this game. In a Markovian game, shareholders condition their actions on a “state” variable, which summarizes the current state of the system, and on the actions of the other players that they can observe in the current period.⁷

In our game, the choice or “action variable” for each shareholder is the probability with which he will tender his share. We represent the probability with which shareholder i tenders by

p_i ; $p_i \in [0,1]$. A vector of choices for all shareholders is represented by $\mathbf{p} = (p_1, \dots, p_N) \in [0,1]^N$.

A Markovian strategy for the shareholder is a map from the “state variable” of the system and the observed current actions of the other players, into his actions. Since this choice is made only in

the state of the world where the improvement gains from the takeover have not evaporated, and because failed conditional offers have no effect on subsequent agent payoffs, the Markov assumption implies that there is only one non-trivial “state” of the system. Thus, this tendering probability is independent of calendar time and depends only on the current offer made by the raider, $b \in [0,1]$. Thus, a tendering strategy for a shareholder is a map, \mathbf{p}_i^* , from possible current period raider bids, b , to tendering probabilities, $\mathbf{p}_i \in [0,1]$. A vector of strategies for all shareholders is represented by π^* .

Next consider Markovian strategies for the raider. The action for the raider consists of a bid price, $b \in [0,1]$. The raider only makes offers in one state of the world — the state in which evaporation has not occurred. Moreover, the raider never observes any actions of the shareholders in the current period before he makes his offer. Thus, a Markovian strategy for the raider, which we represent by b^* , is simply an element in the interval $[0,1]$. In other words, the raider will make the same offer throughout the game.

Let $w_i^T, (w_i^N)$ be the expected payoff to i , if evaporation has not occurred and i followed the strategy of tendering (not tendering) her share. Let \tilde{S} represent the number of shares tendered in equilibrium and let \tilde{S}^{-i} represent the number of shares tendered conditioned on shareholder i 's tendering 0 shares. Let $n^{-i}(\mathbf{p})$ represent the probability that the takeover succeeds given that i does not tender. That is, $n^{-i}(\mathbf{p}) = P_p[\tilde{S}^{-i} \geq K]$. Let $t^{-i}(\mathbf{p})$ represent the

7 For further analysis of Markovian games, see Friedman (1986). For an application of Markovian game approach to

probability that the takeover succeeds given that i tenders, i.e., $t^{-i}(\mathbf{p}) = P_{\mathbf{p}}[\tilde{S}^{-i} + 1 \geq K]$. The payoff to a shareholder from tendering given that the raider follows strategy b^* is given by

$$w_i^T = t^{-i}(\mathbf{p})b^* + \mathbf{d}(1 - t^{-i}(\mathbf{p}))w_i^T.$$

This equation implies that

$$(1) \quad w_i^T(\mathbf{p}, b^*) = b^* \frac{t^{-i}(\mathbf{p})}{1 - \mathbf{d}(1 - t^{-i}(\mathbf{p}))}.$$

Similarly, the expected payoff from not tendering is given by

$$(2) \quad w_i^N(\mathbf{p}, b^*) = \frac{n^{-i}(\mathbf{p})}{1 - \mathbf{d}(1 - n^{-i}(\mathbf{p}))}.$$

The raider's expected profit, V , for a bid of b , if shareholders follow tendering strategy π^* , is given by

$$V(\mathbf{p}^*, b) = \frac{(1-b)E_{\mathbf{p}^*(b)}[\tilde{S} | \tilde{S} \geq K]P_{\mathbf{p}^*(b)}[\tilde{S} \geq K]}{1 - \mathbf{d}(1 - P_{\mathbf{p}^*(b)}[\tilde{S} \geq K])}.$$

Definition. A stationary subgame perfect Markovian equilibrium consists of an ordered pair $(b^*, \pi^*(\cdot))$, where $b^* \in [0,1]$ and $\pi^*(\cdot)$ is a function mapping $[0,1]$ into $[0,1]^N$, which satisfies the following conditions:

- a. For all $b \in [0,1]$ and $i \in [N]$,

the analysis of repeated unconditional tender offers, see Harrington and Prokop (1995).

$$p_i^*(b) > 0 \Rightarrow w_i^T(\mathbf{p}, b) \geq w_i^N(\mathbf{p}, b) ; \quad p_i^*(b) < 1 \Rightarrow w_i^T(\mathbf{p}, b) \leq w_i^N(\mathbf{p}, b).$$

b. For all $b \in [0,1]$,

$$b^* \in \text{Arg max}_{b \in [0,1]} V(\mathbf{p}^*, b).$$

These conditions have a natural interpretation. Condition (a) simply says that shareholders are sequentially rational. In other words, in response to every possible bid price, shareholders tender with positive probability only if their payoff from tendering is at least as high as their payoff from not tendering, and refrain from tendering with positive probability only if their payoff from refraining from tendering at least equals the payoff from tendering. Condition (b) simply says that given the shareholder-response functions, the raider picks the equilibrium bid price that maximizes his payoff. Henceforth, we will simply call a stationary subgame perfect Markovian equilibrium an “equilibrium.”

This game has a plethora of equilibria. However, by using the concept of perfection (Selten, 1975) we can refine the set of equilibria we consider. First, consider the subgame in which the raider’s offer is fixed at $b \in (0,1)$ in every period. Although there are a number of asymmetric equilibria for each b , there are only two symmetric equilibria. To see this, first note that all shareholders tendering is not a best response to any bid, b , which is less than 1: If all shareholders but one tender, then the payoff from not tendering is 1, while the payoff from tendering is $b < 1$, contradicting that tendering, is a best response. Contrarily, the vector of strategies calling for no shareholder to tender, $\pi = 0$, is a best response to any offer b . This

follows because, the payoff to any one shareholder is 0 “regardless of his tendering strategy” if no other shareholder tenders. However, this equilibrium is not a perfect equilibrium in the sense of Selten (1975). The best response for an individual shareholder to any strictly tendering vector $\pi > 0$ of sufficiently small modulus is to tender with probability 1. This follows because, for sufficiently small tendering probabilities, the ratio of the probability of the shareholder being marginal (exactly $K - 1$ other shareholders tender) to the probability of the shareholder being supermarginal (K or more other shareholders tender) approaches infinity. This implies, as can be seen from inspecting equations (1) and (2), that the payoff from tendering exceeds the payoff from not tendering. The fact that the best response to all “small” tendering probability vectors is to tender with probability 1 implies that the equilibrium strategy of tendering with probability cannot be approached by best responses to any sequence of perturbed strategy vectors. Thus, the equilibrium is not perfect.

The analysis above implies that all perfect symmetric equilibria involve mixed strategies. In such equilibria, sequential rationality requires each shareholder to be indifferent between tendering and not tendering. As we see from the analysis above, this argument implies that the following equilibrium condition must be satisfied:

$$(3) \quad b \frac{\bar{B}_{K-1}(\mathbf{p})}{1 - \mathbf{d}(1 - \bar{B}_{K-1}(\mathbf{p}))} = \frac{\bar{B}_K(\mathbf{p})}{1 - \mathbf{d}(1 - \bar{B}_K(\mathbf{p}))},$$

where, for all $k \in [N - 1]$ and $\pi \in [0,1]$, \bar{B} is defined by

$$\bar{B}_k(\mathbf{p}) = \sum_{j=k}^{N-1} \binom{N-1}{j} \mathbf{p}^j (1-\mathbf{p})^{(N-1)-j}.$$

Next, note that for each offer $b \in (0,1)$ there is a unique associated probability of tendering. This result is recorded below.

Lemma 1. For each raider offer $b \in (0,1)$ there exists a unique probability, $\pi^*(b) \in (0,1)$, such that $\pi^*(b)$ solves equation (3).

Proof. See Appendix I.

2.1 Asymmetric shareholder responses to takeover bids

Equilibrium responses by shareholders also exist which are not symmetric. Associated with each fixed bidding strategy $b \in (0, 1)$, there exist a number of asymmetric best response vectors for shareholders. First, any pure strategy vector in which exactly K shareholders tender and the remaining shareholders do not tender, is a best response for shareholders. To see this, note that each of the tendering shareholders (tenderers) knows that the offer will fail with certainty, given the strategies of the other shareholders, in any round in which she fails to tender. Thus, tendering is a best response for tenderers. Similarly, all nontendering shareholders (nontenderers) know that the offer will succeed even if they do not tender. Thus, nontendering is a best response for nontenderers. By varying the identity of the K shareholders in the tendering group, one can produce $\binom{N}{K}$ pure strategy best responses to a fixed raider strategy.

In addition, there are asymmetric mixed-strategy best responses. As we document in Appendix II, in all asymmetric shareholder best responses, all shareholders that play a mixed strategy use the same randomization probability. This result should not be surprising given a

similar result for the single-shot takeover games found in the Appendix of Holmstrom and Nalebuff (1992). Thus, in any mixed strategy response to a raider offer, a subset of shareholders tender, a subset randomize, and a subset does not tender. Using the same arguments we use in the symmetric equilibrium case, it is easy to show that the tendering probability used by the randomizers is uniquely determined by the number of tenderers and nontenderers. The number of tenderers, nontenderers, and randomizers are restricted by the following conditions: (a) It is not possible for exactly one shareholder to randomize, for, if but one shareholder randomized, that shareholder would conjecture a deterministic tendering distribution coming from the other shareholders and thus she would want to either tender or not tender with probability 1; (b) the number of tenderers plus the number of randomizers must at least equal $K+1$, otherwise a randomizing shareholder would realize that, if she holds out, the offer would surely fail, contradicting the optimality of randomization; (c) the number of tenderers must be less than 4, otherwise a randomizer would know that, even without her participation the offer will succeed. Any quantities of tenderers, nontenderers, and randomizers satisfying (a), (b), and (c) can engender a shareholder best response vector. Moreover, by varying the assignments of shareholder indices to these three classes, many best responses can be associated with each choice of the quantities. Thus, a large number of asymmetric best responses are associated with each fixed raider bid. In fact, simple combinatorics can be used to show that in the 7-shareholder parameterization case utilized in the experiments where 4-votes are required for control, there are 938 asymmetric best responses to a fixed raider offer. This multiplicity of responses is not a product of the dynamics that we introduce into the static takeover models, but is also present in static models themselves.

Like all of the researchers who have previously analyzed static takeover setting, we focus our analysis on the symmetric shareholder best response. There are three reasons for our focus on the symmetric solution. First, this focus facilitates comparisons between our work and other experimental and theoretical research on takeovers and public goods. These papers all focus on the symmetric equilibria, e.g., Holmstrom and Nalebuff (1992), Kale and Noe (1997), and Marks and Croson (1998). Second, we feel that the symmetric outcome is focal because the identically endowed student subjects playing the target shareholders have no reason to conjecture, or coordinate towards, an asymmetric outcome for the game that will produce nonuniform payoffs. Third, the important qualitative comparative statics of the asymmetric equilibria, the relationship between the continuation probability and the severity of the free-rider problem, has the same sign in all of the asymmetric mixed strategy equilibria as it is in the symmetric perfect equilibrium we formally analyzed below. Thus, the interpretation of the differences between our experimental results and Nash predictions is the same under the asymmetric equilibria as it is under the symmetric equilibria.

Theorem 1. In the unique perfect symmetric equilibrium of the game, the raider's bid b^* is a solution to

$$(4) \quad \text{Max}_{b \in (0,1)} \frac{(1-b) E_{P^{*(b)}} [\tilde{S} \mid \tilde{S} \geq K] P_{P^{*(b)}} [\tilde{S} \geq K]}{1 - d(1 - P_{P^{*(b)}} [\tilde{S} \geq K])},$$

where all shareholders tender in each round and $\pi^*(b)$ is implicitly defined, for each $b \in (0,1)$, as

the solution to (3).

Proof. Given Lemma 1, the only thing that remains to be established is that the raider will not make an extremal offer of 0 or 1. Because rejecting an offer of 0 is a dominant strategy, any perfect equilibrium must feature all shareholders rejecting an offer of 0. Thus, the raider's profit from a 0 offer will be 0. Similarly, even if an offer of 1 is accepted, it will provide the raider with a 0 payoff. By Lemma 1, any raider offer strictly between 0 and 1 will be accepted with positive probability. Thus, such an offer produces a higher payoff than any extremal offer. This contradicts the optimality of extremal offers and establishes the result. \square

Theorem 1 and Lemma 1 do not tell us how to compute the tendering probability generated by unique symmetric perfect equilibrium of the game. No closed-form solutions exist. However, numerical calculation of the roots of (2) is straightforward and will be performed for the parameters of the experiment. We also do not have an analytical formula for the raider's optimal bid price. However, for the parameterizations of the model used in the experiment, we compute the optimal raider bidding strategies and compare them with the bids actually made by raiders in the experiments.

2.2 Comparative statics

In this model, the three exogenous parameters are the continuation probability, δ , the number of shares required for the offer to succeed, K , and the total number of shareholders, N . The outcome of the game is described by three variables — the equilibrium bid price, the

equilibrium probability of tendering, and the final division of value between the parties. The division of value between the parties is as follows: total value equals N , the number of shares times their per-unit value. Some of this value will be lost because an agreement is not reached before evaporation occurs. By a simple recursive argument, the expected proportion of this total value lost through dissipation, which we represent by D^* , is given by

$$D^* = \frac{(1 - \mathbf{d})(1 - P^*)}{1 - (1 - P^*)\mathbf{d}}, \quad \text{where } P^* = P_{P^*(b^*)}[\tilde{S} \geq K].$$

The fraction of the gain captured by the raider, which we represent by R^* , is given by the optimized value of the raider's payoff function given by equation (4), divided by the total value, N . Because the equilibrium is symmetric, the fraction of gain captured by the shareholders is the expected payoff to an individual shareholder times the number of shareholders, N , divided by the total value, which is also N . Thus, the fraction of the gain captured by shareholders is just the equilibrium shareholder payoff. Moreover, because in equilibrium shareholders randomize between tendering and not tendering, shareholder payoffs are the same under both of these choices. For this reason, the fraction of the total gain captured by shareholders, which we represent by S^* , equals the equilibrium payoff to any individual shareholder from not tendering.

In the experiments, the only exogenous parameter varied is δ , the continuation probability. The values of N and K were fixed at 7 and 4, respectively. Thus, the numerical comparative statics presented below in Table 1 are focused on the effect of varying δ . The results of numerical variation of δ also are presented in Table 1.

These comparative statics indicate that, as the continuation probability increases, a

shareholder's probability of accepting any given offer falls, as does the optimal raider offer. Dissipation losses from failed tender offers increase and the fraction of the total takeover gain captured by shareholders falls. The effect of increasing the continuation probability on the fraction of surplus captured by the raider, R^* , is not monotone — initially, increasing the continuation probability decreases the ability of the raider to capture takeover gains, but the direction of the effect reverses for higher probabilities of continuation. The intuition behind these comparative statics is as follows. As the continuation probability increases, the responsiveness of shareholders to any fixed raider offer falls. In addition, the elasticity of the tendering probability with respect to the offer falls. This effect is illustrated in Figure 1. Thus the amount that the raider must pay to increase his odds for immediate acceptance increases; at the same time, the cost of evaporation associated with delayed acceptance falls. For this reason, the raider lowers his bid in response to an increase in the continuation probability. This reduction in the bid is recorded in column 2 of Table 1.

As we have seen, increasing the continuation probability both (a) lowers the raider's equilibrium bid, and (b) reduces a shareholder's probability of accepting any fixed bid. The effects (a) and (b) reinforce each other. Thus, an increase in the continuation probability induces a substantial reduction in the probability that a shareholder will tender into the equilibrium offer. This effect is recorded in column 3 of Table 1. The negative effects from a reduction in the probability of acceptance, combined with the lower raider offer price, more than offsets the direct positive effect of an increased probability of continuation. Thus, shareholder gains (S^*) fall and the probability of dissipation (D^*) increases with an increase in the continuation probability. This

effect is evidenced by columns 4 and 6 of Table 1.

2.3 The equivalent public-goods contribution game

Consider a public good that will only be provided if K out of N “beneficiaries” of the good contribute c dollars: in other words, the provision point is $K \times c$. The benefit from the provision of the public good to each of the beneficiaries is 1.00. A “provider” provides the public good at no cost and will reap revenues equal to the sum total of the contributions from the beneficiaries. At the start of each period, the provider makes a solicitation for c ; if fewer than K beneficiaries contribute, then the offer fails and contributions are returned. At this point there is a δ probability that the public good evaporates and a probability of $1 - \delta$ that the public good does not evaporate; thus, the provider has the option of making another solicitation in the next period. If at least K beneficiaries contribute, the provider pockets the contributions, the good is provided, and the game ends.

Note that, in this game, if at least K beneficiaries pay the contribution, the payoff to the paying beneficiaries is $1 - c$, the payoff to the nonpaying or “free-riding” beneficiaries is 1, and the payoff to the provider is $c \times (\# \text{ of contributors})$. If evaporation occurs all parties receive 0. Thus, the payoffs in the public goods game to the beneficiaries, both paying and free-riding for a solicited contribution of c are identical to the payoffs in the tender offer game associated with a bid of $1 - b$. The provider’s payoff for a contribution of c also always equals the raider’s payoff for a bid of $1 - b$. Thus, the public-goods game described here is isomorphic to the takeover game described above.

3. Experimental Design

The subjects in this experiment were undergraduate finance majors and MBA students who were currently enrolled in introductory corporate finance classes. These students were told they would have an opportunity to earn money in a research experiment regarding group decision making. Three experimental sessions involving indivisible shareholdings, 1 share for each shareholder, were run: one session for each continuation probability level (0.01, 0.75, 0.95). The 0.01 continuation probability was chosen because we wish to include a base case where, theoretically, continuation effects should be absent. We did not want to use a zero continuation probability because, in this case the instruction sheet to participants would have to be different, not including any discussion of continuation, than the instruction sheet in the other treatments. Such differences could engender framing biases. The 0.75 continuation probability was chosen as the intermediate case since, theoretically, it is at this continuation probability that the raider's profit begins to rise after declining steadily from the 0.01 level. See Table 1. The 0.95 treatment was chosen, as a limiting case, to analyze the situations where the effective cost of resolicitation is very low.

Each experimental session involved three subject groups. Each group consisted of seven shareholders and one raider. The total number of rounds in an experimental session varied as a function of the group's decisions and the continuation probability draw, which allowed the extension of periods within a round for failed takeovers⁸. Subjects participated in only one session and remained the same agent-type throughout the experiment. It is important to note

⁸ In the 0.01 treatment, the total number of rounds was 11 with 33 periods. In the 0.75 treatment, the total number of rounds was 5 with 26 periods. In the 0.95 treatment, the total number of rounds was 3 with 37 periods.

that the instructions use neutral language in describing the game. In particular, raiders were always referred to as Type A participants and shareholders as Type B participants. Each experimental session lasted about 2 ½ hours and salient earnings were paid.

In each experiment, in order to ensure that subjects understood the rules of the game and how their earnings would be generated, the experimenter read the instructions aloud and the subjects worked through the attached worksheets. Part 1 of the instructions explained that the subjects with the highest scores on a “Trivial Pursuit” questionnaire would be selected to be the “Type A” participants (i.e., raiders) for that session. The purpose of the questionnaire was to facilitate aggressive bargaining by allowing the raiders to earn the property right of their position (see Hoffman, McCabe, Shachat, and Smith, 1992). The remaining subjects were designated “Type B” participants (i.e., shareholders), and were randomly assigned to a new group in each round. Subjects were assigned to groups privately and never knew, nor could they find out, who the other members of their group were in each round.

At the beginning of each period, the raiders sent private offer slips to the members of their group, designating the offer price for which they were willing to buy their group’s shares. After receiving the offer slip, each shareholder privately circled on their tendering slip whether or not they wanted to tender their share. The monitor then counted the number of shares tendered per group, displayed the total number of shares tendered for each group on the overhead, and announced whether the offer had succeeded or failed for each group. The offer prices were never posted publicly, and so remained private information among the members of each group. If at

least 4 of the 7 shares were tendered in a group, then the raider of that group was required to purchase all shares offered, at his specified offer price. If fewer than 4 shares were tendered within a group, a draw was made to determine whether the round could continue for another period. The draw was made from a bucket of chips with a continuation probability, c , of 0.01, 0.75, or 0.95, depending on the experimental session. If the draw allowed another offer to be made, then the process repeated itself. Otherwise, the takeover failed and the round terminated.

In many experiments repetition is necessary to allow subjects to learn the game, and the method of repetition can be an important design feature. One can assign subjects to the same group for the entire experiment. In this case, subjects can be viewed as playing a finitely repeated game. Alternatively, one can randomly assign subjects to groups after each period. In this case, subjects can be viewed as playing a series of one-shot games. Since tender offers with a given raider are typically a one-time event, we chose the one-shot mode of repetition.

4. Results

In this section, we present the results of the experimental sessions described in Section 3. First, we compare the raiders' offers to the theoretical predictions for each continuation-probability treatment. The degree of offer escalation in response to rejected offers is then investigated to gain insight into the strategic behavior of raiders. Next, the actual behavior of shareholders is compared with the predicted behavior. At the end of the section, we examine the effect of changes in the continuation probability on welfare.

4.1 Raider behavior

Two features of raider behavior stand out immediately: First, when the chance to resolicit was substantial, raiders made lower offers than they did in the near single-shot game (0.01 chance of continuation), but higher offers than predicted by theory. Second, raiders displayed some tendency to escalate their bids after a failed offer. The evidence for these effects is provided by Tables 2, 3, and 4.

Table 2 compares the raiders' offer prices across the three continuation probability treatments to the Nash equilibrium prices. Average offer prices are reported for (1) all offers in the experimental session, (2) first period offers only, and (3) offers resulting in a successful takeover. The average of all offer prices in the 0.01 treatment is almost identical to the stationary Nash equilibrium offer price. In the 0.75 and 0.95 treatments offer prices were significantly higher than predicted by the Nash equilibrium. Never the less, consistent with the theoretical prediction offers in the 0.95 treatment were substantially lower than they were in the 0.01 treatment.

The results for rejected offers appear in Table 3. The total number of rejected offers, 31, in the 0.95 treatment is almost double the 18 rejected offers found in both the 0.01 and 0.75 treatments. Not surprisingly, the number of rounds ended by the draw declines as the continuation probability increases, from 100 percent in the 0.01 treatment to 9.7 percent in the 0.95 treatment. Insight into the raiders' strategic behavior can be gained by examining the percentage of offers, conditional on the opportunity to resolicit, that are higher, lower, or the

same, as the preceding rejected offer. An examination of these conditional percentages reveals that after a rejected offer, 72.7 percent of the raiders' offers in the 0.75 treatment were higher than their previous offer, while only 50 percent of the raiders' offers increased in the 0.95 treatment. Raiders' offers that were the same as the previous offer totaled 18.2 percent in the 0.75 treatment and 14.3 percent in the 0.95 treatment. Thus, raiders were less likely to raise the offer in the 0.95 treatment than in the 0.75.

Further evidence that differences exist in the raiders' tendency to escalate offers under the two continuation probability treatments can be seen in Table 4, which presents data on the average percentage change in offer prices. Raiders in the 0.75 treatment display a higher geometric average percentage change in offer prices (26.99 percent) than the raiders in the 0.95 treatment (0.54 percent). The median geometric percentage change in offers by raiders is positive in both treatments, but higher in the 0.75 treatment (13.03 percent) than in the 0.95 treatment (1.25 percent). Furthermore, the range of the geometric percentage change in offer prices in the 0.75 treatment (7.34 percent to 84.3 percent) and in the 0.95 treatment (-12.5 percent to 11.7 percent), confirm the above evidence that the raiders facing the higher continuation probability were less likely to escalate their offers.

4.2 Shareholder behavior

In our experiments, just as "overbidding" characterizes raider responses, so "overtendering" characterizes shareholder responses. Less obvious is the extent to which overtendering is a response to raiders' overbidding. We investigate these issues in Tables 5, 6,

and 7 and in Figure 2.

Table 5 compares the shareholders' tendering proportions across the three continuation-probability treatments to the Nash equilibrium expected proportion of shares tendered. The proportion of shares tendered is reported for (1) the average of all periods in the experimental session, (2) the average of the first periods, and (3) the average for successful takeovers. In keeping with the theoretical predictions, the average number of shares tendered fell as the continuation probability rose: However, the decrease was not nearly as great as predicted by theory.

In the 0.01 treatment, shareholders tendered significantly fewer shares than predicted by theory. At the 0.75 continuation probability, shareholders tendered more than predicted by theory, but the difference is not significant. In the 0.95 treatment, the observed average tendering probability exceeds the theoretical prediction by a significant margin.

Next, we investigate how the opportunity to resolicit affected the incidence of successful takeovers. Table 6 presents data on the average number of periods before a successful takeover occurred and on the percentage of successful offers that occurred at the beginning and at the end of a round. The average number of periods before a takeover succeeded increased as the continuation probability increased, from 1 in the 0.01 treatment, to 1.38 in the 0.75 treatment, to 3.6 in the 0.95 treatment. In the 0.01 treatment, the percentage of successful takeovers in the first

period of the first round was 66.7 percent, approximately double that found in the 0.75 treatment, 33.3 percent, and significantly higher than the 0 percent found in the 0.95 treatment. When only the first period of any round is considered, successful takeovers occurred more frequently as the continuation probability decreased; however, the direction of overall success reversed across the continuation probability treatments when all periods of a round are considered. That is, the percentage of successful takeovers before the draw ended a round increased as the continuation probability rose, from 45.5 percent for the 0.01 treatment, to 53.3 percent for the 0.75 treatment, to 62.5 percent for the 0.95 treatment. These results demonstrate that, counter to the theoretical Nash prediction, the direct effect dominates the credibility effect. This implies that the option to resolicit can reduce economic losses and may explain why resolicited offers commonly occur in the real world.

Subject experience in the experimental session may have had an impact on the outcome. In the 0.01 treatment, first period success was much more likely in the first round, 66.7 percent, than in subsequent rounds, 45.5 percent. In contrast, in the 0.75 and in the 0.95 treatments, the likelihood of first period success rose slightly in subsequent rounds.

To more directly examine whether the average overtendering by shareholders is a response to overbidding by raiders, we present data on conditional tendering proportions in Table 7. For each offer in the experimental treatment, we calculate the difference between the actual proportion of shares tendered and the Nash equilibrium expected proportion of shares tendered,

conditioned on the raiders' using that offer as his stationary equilibrium strategy. The differences proved to be -2.299 in the 0.01 treatment, -1.33 in the 0.75 treatment, and 1.18 in the 0.95 treatment. In the 0.01 and 0.75 treatments, shareholders undertendered conditional on the raider's offer. In the 0.95 treatment, shareholders overtendered conditional on the offer.

Figure 2 provides visual confirmation of the results highlighted in Table 7. The fraction of shares tendered is plotted on the vertical axis, while the offer prices are plotted on the horizontal axis. The vertical coordinate line in the figures represents the fraction of shares tendered given that shareholders followed a stationary strategy of offering a tender price equal to the horizontal coordinate. The points on the graphs represent the actual offer price/share tendering responses observed in the experiments. The scatter plot in Panel A emphasizes that shareholders in the 0.01 continuation probability treatment are likely to tender less than predicted by theory. Panel B also confirms a tendency to undertender in the 0.75 treatment, while the scatter plot in Panel C highlights shareholders' tendency, in the 0.95 treatment, to overtender.

4.3 Dissipation and the division of value

How do differences between theory and experimental results affect the welfare of the shareholders? Raider overbidding, for any fixed shareholder response, unambiguously increases shareholder welfare. The effect of overtendering on shareholders welfare is more subtle. *Certis paribus*, overtendering could in principle have either a positive or negative effect on shareholder welfare. Until enough shares are captured to ensure control, more tendering increases the shareholders' welfare. After this point is reached, however, increased tendering transfers value to

the raider. To determine the effect of overbidding on shareholder welfare, we compute, in Table 8, the total value and the division of the total value between the raider and shareholders.

For each continuation-probability treatment, Table 8 compares the normalized takeover gains observed in the experimental sessions to the Nash equilibrium distribution of gains. The normalized gain to the shareholders is determined in the following way: When the takeover is unsuccessful, a 0 is assigned, and when the takeover is successful, the shareholder gain is calculated as $[(\text{non-tendered shares} \times \text{post-takeover value}) + (\text{tendered shares} \times \text{tender price})] / (\text{total shares} \times \text{post-takeover value})$. The raiders' gain is also 0 if the takeover is unsuccessful, but when it is successful, then the gain is determined to be $(1 - \text{the shareholder gain})$. The dissipation of value is computed by crediting a 1 when a takeover fails and crediting a 0 when a takeover succeeds. The Nash equilibrium prediction holds that, as the continuation probability increases, dissipation increases, the average gain to shareholders decreases, and the average gain to the raider first decreases and then increases. The results of the experiment contrast sharply with the equilibrium predictions: As the continuation probability increases, dissipation is reduced, shareholders' gains increase, and the raiders' gains decrease. Overbidding by raiders when the probability of continuation was high appears on average to have had a positive effect on the welfare of shareholders. In the 0.75 treatment, where the raiders on average escalated their offers more often, the shareholders received more than their predicted share while the raiders received less. The dissipation level was also slightly more than predicted. In the 0.95 treatment, where the shareholders had a tendency to conditionally overtender, we find that the shareholders received more than their predicted share at the expense of the raider. Dissipation was also less than

predicted.

5. Extensions

The goal of this paper was to investigate the effect of the option to repeat solicitations on the magnitude of the welfare losses induced by free riding in takeovers and in the provision of public goods. Resource and time constraints force us to make a number of ancillary assumptions in order to obtain a determinant model of this problem. Notably, we assumed that free riding and repeated takeover solicitation occur in a context in which (a) raiders condition their offers on receiving sufficient shares to acquire control, and (b) all shareholders have identical initial shareholdings. Imposing these ancillary assumptions calls into question the extent to which our results are rooted in these specific assumptions. In other words, are our results robust? We address this question in Section 5.1. Next we address the question of realism. Even if our analysis provides insights into the effect of repetition options on the unadulterated free-riding problem, does it tell us anything about the sort of mixed incentive problems that arise in actual economic situations? This issue is addressed in Section 5.2. Finally, in Section 5.3 we summarize the contribution of this paper.

5.1 Robustness of results

5.1.1 Conditional vs. unconditional offers

How would replacing the conditional offer assumption with an unconditional offer assumption affect on our analyses? First, it would reduce the generality with which we could

interpret our results. As we showed in Section 2, our conditional offers model of takeovers is isomorphic to a natural public goods provision problem. Unconditional takeover offers, however, are not isomorphic to any natural public goods provision problem. With an unconditional offer, the payoff, in the event that the game ends before a successful takeover, is higher for tendering than for non-tendering shareholders. Thus, if the shareholder tenders before failure, he receives the tender price, b , while the shareholder who does not tender before failure, receives 0, the pre-takeover value of the firm.

Because an isomorphism between the two problems would map shareholders into public good beneficiaries making contributions to the provision of a public good, it would require a public goods game that leaves contributors better off when they contribute (by the difference between the value of the public good to them and the requested contribution) and the solicitation fails. It is not easy for us to visualize an economically natural circumstance in which a public goods provision would have this characteristic. Thus, using an unconditional formulation would reduce the scope of applicability of our results.

Other than this change, we do not believe that the treatment of the free-riding problem using unconditional offers would yield radically different results. Indeed, Harrington and Prokop (1995) model repeated unconditional takeover offers. Their results show that, as in our case, with indivisible shareholding, a unique Markovian stationary equilibrium exists. This equilibrium displays features qualitatively similar to those of our equilibrium. Their theoretical research focuses on numerical simulations intended to demonstrate that, as the continuation probability

becomes very large, raider profits become negligible. This result does not appear to be the case in our conditional offer setting. However, this difference is not important from the perspective of our analysis because, unlike Harrington and Prokop, we focus on the welfare losses from free riding, rather than on the division between shareholders and raiders of the gains remaining after welfare losses. Thus, we would not expect an unconditional implementation to radically alter our results. In addition, unconditional tender offers require an additional state variable—the number of shares already acquired by the raider. Incorporating this state variable into the dynamic programming problem greatly complicates the analysis. Thus, an argument for theoretical simplicity buttresses our earlier argument from generalizability, in supporting a conditional, as opposed to an unconditional, formulation.

5.1.2 Asymmetric share allocations

Our analysis and our experiments assume that all shareholders are allocated the same number of shares. This assumption is not required for an analysis of the free rider problem in takeovers, either from a theoretical perspective (see Holmstrom and Nalebuff) or from an experimental perspective. However, if we had modeled asymmetric endowments, we would have lost our ability to refine the set of equilibria by using the symmetry criterion. Since a very large number of asymmetric equilibria exist in takeover games, direct comparison of our experimental results with theory would have been problematic. The central issue raised by asymmetric endowments is the question, Who will tender? In a single-shot offer setting, Holmstrom and Nalebuff argue that, theoretically, larger shareholders will in equilibrium tender a larger proportion of their shares than smaller shareholders. Cadsby and Maynes (1998), again in a

single-shot setting, document that, in fact, shareholders tender the same proportion of shares regardless of the size of their shareholdings. The question addressed by these papers is important and interesting. However, there is no reason to expect that introducing an option to resolicit offers, the focus of this paper, would materially affect the results in those analyses. Thus, our ability to contribute, by studying resolicitation, to the issues raised by asymmetric shareholding is limited. At the same time, the cost of introducing asymmetric shareholdings into our model, the loss of determinant theoretical predictions, is high. For this reason we opted for our symmetric shareholdings approach.

5.2 Realism of results---extensions

In the earlier section, we argued that the conclusions of our theoretical and experimental analysis were not artifacts of some of the ancillary model assumptions. In this section, we consider whether our results, which consider free riding in isolation from other institutional and incentive questions, are relevant to the sort of actual takeover/public goods problems faced by real economic agents. To address this issue, we consider some of the other salient factors present in takeovers/public good situations and how they might affect our conclusions.

5.2.1 Asymmetric information

At least in the context of takeovers, a great deal of research has focused on how information differentials between raiders and shareholders can influence the outcome of takeovers (See, e.g., Hirshleifer and Titman (1990) and Shleifer and Vishny (1986)). This research

undoubtedly reflects the salience of informational effects in actual takeover contests.

First consider the impact of asymmetric information on our results in the case where the raider has private information about the post-takeover value of the firm and the shareholders have no private information. In this case, an increase in the takeover offer not only directly affects shareholder welfare by increasing the benefits of tendering, it also indirectly influences shareholder welfare by changing shareholders' assessment of the private information the raider possesses regarding post-takeover firm value. An increase in the bid increases the shareholders' assessment of post-takeover value. Since shareholders can capture post-takeover value only by free riding, this informational effect reduces the willingness of shareholders to tender. Thus, one would expect asymmetric information to exacerbate the free-rider problem. Allowing for resolicitation would not qualitatively alter this situation. Raiders with very positive information about post-takeover value would be less willing to make high bids because they would fear that such high bids would induce belief revisions by shareholders, both in single-shot and repeated solicitation settings.

Next suppose that raiders have no private information but shareholders have private information about the pre-takeover value of the firm. In a single-shot game, the raider makes his offer before observing any action by shareholders that "signals" their private information. In contrast, in a resolicitation takeover game, raiders can extract shareholder information through

multiple offers, starting with low offers, which will work only when shareholders have adverse private information, and “working up the demand curve” as described in the durable goods monopolist literature (see Gul, Sonnenschein, and Wilson (1986) and Ausubel and Deneckere (1989)). Because this process of “working up the demand curve” does not create any economic value and entails dissipative costs of delay, we would expect the option to resolicit to have even greater adverse consequences, in theory, than it does in our theoretical analysis. We would also expect more dissipation in the experimental treatments. However, whether the increase in dissipation costs would be sufficient to cancel the gains from the option to resolicit documented in our symmetric information experiments remains an open question.

5.2.2 “Noise,” limited rationality

Consider the impact of “noise” on resolicited tender offers. As Hirshleifer (1996) points out, strategic equilibrium models of takeovers with many shareholders, and, by extension, of public goods provisions with many beneficiaries, rely, to perhaps an unreasonable extent, on the assumption of common knowledge of rationality. His argument runs as follows. The marginality of individual shareholders is a *sine qua non* for strategic takeover equilibria in which the offer succeeds with positive probability. However, very small perturbations of individual shareholder tendering strategies, will, if applied to a large number of shareholders, lead to large perturbations in the number of shares tendered. These large perturbations in the number of shares tendered will make negligible the likelihood that any one shareholder is marginal in the takeover. It is clear from this argument that introducing some exogenous random tendering will lower the overall likelihood of takeover success. Our investigation, however, does not center on the overall

likelihood of takeover success. Rather, it focuses on changes in the likelihood of takeover success induced by the introduction of an option to repeat takeover solicitations. Since noise lowers the probability of success both with and without repetition, we have no reason to expect that it would reverse our results.

5.2.3 Norms and framing bias

Although the public-goods provision game and the takeover game studied in this paper are isomorphic, the “framing” of the two games is very different. As Tversky and Kahneman (1986) have shown, framing has an important effect on the actual behavior of experimental subjects. The notable framing difference is that in the public goods setting, a contribution is directly solicited. In the takeover setting the contribution has to be inferred from the difference between post-takeover firm value and the tender price. In addition to framing differences, social norms favor making contributions for goods consumed collectively by one’s citizens or affiliation (e.g., ethnic, religious) groups. However, there is not a corresponding norm favoring aiding one’s fellow shareholders in a corporation. These effects might induce differences in the absolute level of free riding between our experiment, which framed the problem in a takeover setting, and an experiment framed in the public goods contribution setting. However, as pointed out before, the absolute level of free riding is not the focus of our study. Rather, we focus on how the introduction of an option to resolicit offers affects the level of free riding. Since the framing and cultural norm effects discussed above would operate with equal force in a repeated or non-repeated offer setting, we have no reason to expect that the choice of the takeover frame had an important effect on our overall conclusions.

5.3 Conclusions

In this paper we developed a theoretical model of takeovers and the provision of public goods in the face of free-rider problems. In the context of this model, we investigated the option to resolicit shares. We showed that in the stationary perfect Nash equilibrium of the game, the option to solicit again, by reducing the penalty from offer failure, increased free riding to such an extent that the option to repeat solicitation actually increased dissipation costs associated with free riding. Next, we simulated our takeover model in laboratory experiments. We found that, in contrast to the results of our theoretical analysis, the costs of free riding were reduced by the introduction of an option to resolicit offers. In fact, the dissipation losses from free riding were uniformly decreasing in the likelihood of another solicitation. This divergence between theory and experiment arose primarily because the offer made by the raider (or, in the public goods context, the solicitor of contributions) did not fall nearly as quickly in the probability of being able to resolicit, as theory predicts.

One implication of our analysis is that options to resolicit may produce economically significant reductions in the costs of free riding both in takeovers and in public goods solicitation problems. Thus, lowering the costs of resolicitation, in the takeover context for example, by reducing the filing costs associated with making another offer for the same firm's stock, will improve social welfare. In addition, our results explain the frequency with which actual tendering and public goods solicitations feature repeat solicitations. Finally, our results show that the option to resolicit offers may be as important as the more frequently cited impact of contribution

divisibility, for mitigating the costs of free riding. That is, if at first you don't succeed you can gain by "trying, trying again".

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Appendix I

Proof of Lemma 1. We need to show that equation (3) has a unique solution. To this end, define

$$h(\mathbf{p}) = \frac{\binom{N-1}{K-1} \mathbf{p}^{K-1} (1-\mathbf{p})^{N-K}}{\bar{B}_K(\mathbf{p})}.$$

Next, note that through simple algebra we can rearrange (3) to obtain the equivalent equality

$$\left(b - \frac{\bar{\mathbf{c}}_K(\mathbf{p})}{1 - \mathbf{c}(1 - \bar{B}_K(\mathbf{p}))} \right) h(\mathbf{p}) - (1-b) = 0.$$

Because the left-hand side of (2) is strictly positive for all π sufficiently small, strictly negative for all π sufficiently large, and continuous in π , a root must exist. Multiple roots are impossible by the following argument. First, note that

$$b - \frac{\bar{\mathbf{c}}_K(\mathbf{p})}{1 - \mathbf{c}(1 - \bar{B}_K(\mathbf{p}))}$$

is strictly decreasing in π and is strictly positive in any sufficiently small neighborhood of a root of (2). Similarly, h is strictly decreasing and positive. This implies that in a neighborhood of any root of (2), the expression on the left-hand side of (2) is decreasing. Thus, no root can cut the horizontal axis from below. This implies that once the left-hand side expression turns negative, it stays negative. Uniqueness follows.

Appendix II

Asymmetric Mixed Strategy Equilibria

Lemma 1. In every asymmetric mixed strategy equilibria, all shareholders who play mixed strategies use the same mixed strategy.

Proof. Consider an asymmetric mixed strategy equilibrium. Let i and j be two shareholders that are playing mixed strategies in this equilibrium. Equations (1) and (2) imply that the following equalities hold:

$$(1) \quad b^* \frac{t^{-i}(\mathbf{p}^*)}{1-\mathbf{d}(1-t^{-i}(\mathbf{p}^*))} - \frac{n^{-i}(\mathbf{p}^*)}{1-\mathbf{d}(1-n^{-i}(\mathbf{p}^*))} = 0,$$

$$(2) \quad b^* \frac{t^{-j}(\mathbf{p}^*)}{1-\mathbf{d}(1-t^{-j}(\mathbf{p}^*))} - \frac{n^{-j}(\mathbf{p}^*)}{1-\mathbf{d}(1-n^{-j}(\mathbf{p}^*))} = 0.$$

Simple algebra shows that (1) and (2) are equivalent to

$$(3) \quad t^{-i}(\mathbf{p}^*) = \frac{(1-\mathbf{d})n^{-i}(\mathbf{p}^*)}{b^*(1-\mathbf{d}) - n^{-i}(\mathbf{p}^*)\mathbf{d}(1-b^*)},$$

$$(4) \quad t^{-j}(\mathbf{p}^*) = \frac{(1-\mathbf{d})n^{-j}(\mathbf{p}^*)}{b^*(1-\mathbf{d}) - n^{-j}(\mathbf{p}^*)\mathbf{d}(1-b^*)}.$$

Next let $Q = P_p [\tilde{S}^{-ij} \geq K - 2]$; $R = P_p [\tilde{S}^{-ij} \geq K - 1]$; $\tilde{S}^{-ij} = P_p [\tilde{S}^{-ij} \geq K]$, where \tilde{S}^{-ij} represents the

total number of shares tendered by shareholders other than i and j . Next define the function $\mathbf{p} \in [0, 1]$

by

$$(5) \quad T[\mathbf{p}] = \mathbf{p}Q + (1-\mathbf{p})R,$$

$$(6) \quad N(\mathbf{p}) = \mathbf{p}R + (1-\mathbf{p})S.$$

Note that $t^{-i}(\mathbf{p}^*) = T[\mathbf{p}_j^*]$, $n^{-i}(\mathbf{p}^*) = N[\mathbf{p}_j^*]$; $t^{-j}(\mathbf{p}^*) = T[\mathbf{p}_i^*]$, $n^{-j}(\mathbf{p}^*) = N[\mathbf{p}_i^*]$. Define the following

function of \mathbf{p} :

$$(7) \quad F[\mathbf{p}] = (b^* T[\mathbf{p}](1 - d(1 - N[\mathbf{p}])) - (N[\mathbf{p}](1 - d(1 - T[\mathbf{p}])))).$$

Note that F is quadratic in \mathbf{p} and that $F[\mathbf{p}_j^*]$ has the same sign as the left hand side of (1) and $F[\mathbf{p}_i^*]$ has the same sign as the left hand of (2). Thus, there exists a mixed strategy equilibrium in which two shareholders randomize using different probabilities if and only if F has two distinct roots on the interval $[0, 1]$.

Because i is indifferent to tendering when j tenders with a positive probability, it must be the case that i prefers tendering to not tendering if j is tendering with probability 0, for the same reason, i must prefer not tendering if j tenders with probability 1. These arguments imply that $F[0] > 0$ and that $F[1] < 0$. However, a quadratic function that is below the x-axis at zero and above the x-axis at 1, crossed the x-axis at exactly 1 point between 0 and 1, thus the root of F is unique, implying that $\mathbf{p}_i^* = \mathbf{p}_j^*$. □

Appendix III

Instructions

GENERAL

You are about to participate in an experiment of group decision making. There are two parts to this experiment. If you understand the instructions and make careful decisions, you may earn a considerable amount of money. These earnings will be paid to you, in cash, at the end of the experimental session.

PART I

This first part of the experiment involves answering a questionnaire which will be used to designate participant types for the second part of the experiment. Individuals with the highest scores on the questionnaire will be designated Type A participants, while the other participants will be, Type B. If there is a tie, then a drawing will be held to determine who will be a Type A participant.

[Stop and please answer the questionnaire]

PART II

Three groups of eight participants have been randomly formed where one member is a Type A participant. Other than the Type A participant, the identities of the other members in your group will not be known to you. There will be multiple rounds to this part of the experiment and in each round you will be assigned to a different group. The groups were formed randomly, that is the assignment was not the result of any individual or other characteristic.

Within each round there **may or may not** be multiple decision-periods. The outcome of your group's decision each period and a draw will determine whether there will be additional periods to a given round. Your payoff in each round will be determined by your decision and the outcome of your

group's decision. There will be no interaction between groups nor will any member's choices in another group affect your payoffs.

At the start of every round, each Type B participant will be given a token. Each AType A participant will send a **private offer slip** to the seven members of **his group**. The offer message will simply designate a price, in whole numbers, for which the Type A participant is willing to purchase their tokens; the offer price will be the **same** for all Type B participants in his group. Each Type B participant will **privately** respond with a sales slip marking, Accept, if they want to accept the offer, or Reject, if they want to reject the offer. A monitor will collect the sales slips and report the total number of accept responses for each group on the overhead at the front of the room.

If at least 4 tokens in total, **within a given group**, are offered for sale then the round is over for that particular group. The Type A participant must then purchase **all** tokens offered for sale within his group at his designated offer price. If less than 4 tokens are offered for sale within a given group, **none** of the tokens in that group will be sold in that period. In this case, a drawing to determine whether there will be another decision-period to this round will be held. If there is another decision-period then the Type A participant will make a new offer, **which can be the same, lower, or higher than the previous offer**.

The drawing for another decision-period in a given round will affect only those groups who do not have at least 4 tokens offered for sale. The drawing will be held from a bucket containing 100 poker chips, of which 75 are white and 25 are red. If a **red chip** is drawn the round is over. However, if the **white chip** is drawn then there will be another decision-period to that round.

[Stop for demonstration of drawing of chips from the bucket]

Thus, there is a 75% chance another decision-period will occur and a 25% chance the round will

be over. This process repeats itself until the round is over by either a draw of a red chip or each group sells at least 4 tokens.

EARNINGS

The experiment uses fictional currency called francs. All transactions are denominated in that currency as are the individual payoffs. The total number of francs you earn in each round will convert to your real dollar payoff at the end of the experiment. The conversion factor is **0.02**. That is, **one hundred francs** of earnings will equal **two dollars**.

Type A Participants Round Earnings:

At the end of each round,

If total tokens offered for sale ≥ 4 then,

$$\text{Earnings} = [(100 \text{ francs} - \text{offer price}) * (\text{total tokens offered for sale})]$$

If total tokens offered for sale < 4 then,

$$\text{Earnings} = 0 \text{ francs.}$$

Type B Participants Round Earnings:

At the end of each round,

If total tokens offered for sale ≥ 4 then,

Earnings if **you accept** offer = offer price

Earnings if **you reject** offer = 100 francs

If total tokens offered for sale < 4 then,

Earnings if you **accept or reject** offer = 0 francs

WORKSHEET I

Please work through the following worksheet. Please raise your hand if you have any questions and a monitor will come by to help you.

<u>Your Decision</u>	<u>Other Type B's #Accept; #Reject</u>	<u>Total Tokens Offered for Sale</u>	<u>End of Round or Draw for Another Period</u>
<u>Accept</u>	<u>3;3</u>	_____	_____
<u>Accept</u>	<u>1;5</u>	_____	_____
<u>Reject</u>	<u>3;3</u>	_____	_____
-			
<u>Reject</u>	<u>6;0</u>	_____	_____
-			

WORKSHEET II

Please work through this worksheet to make sure you understand how to calculate your earnings. For this worksheet assume the round has ended; either at least 4 tokens were offered for sale or a red chip was drawn. If you have any questions please raise your hand and a monitor will come by to assist you.

<u>Offer Price</u>	<u>Total Tokens Offered for Sale</u>	<u>Type B's - Accept Earnings</u>	<u>Type B's - Reject Earnings</u>	<u>Type A's Earnings</u>
<u>45</u>	<u>4</u>	<u> </u>	<u> </u>	<u> </u>
<u>85</u>	<u>5</u>	<u> </u>	<u> </u>	<u> </u>
<u>85</u>	<u>2</u>	<u> </u>	<u> </u>	<u> </u>
<u>06</u>	<u>4</u>	<u> </u>	<u> </u>	<u> </u>
<u>28</u>	<u>7</u>	<u> </u>	<u> </u>	<u> </u>

RECORD KEEPING PROCEDURES

The Type A and B participants have slightly different record sheets. Please look at your record sheet now.

Each round the Type B participants must fill in GROUP #, PERIOD, YOUR DECISION, # GROUP TOKENS OFFERED, OFFER PRICE, YOUR ROUND EARNINGS IN FRANCS, AND YOUR CUMULATIVE EARNINGS IN FRANCS. After each round the Type A participants must fill in GROUP #, PERIOD, YOUR OFFER PRICE, # GROUP TOKENS OFFERED, YOUR ROUND EARNINGS IN FRANCS, YOUR CUMULATIVE EARNINGS IN FRANCS. It is important that you keep accurate records throughout the experiment.

Once the experiment begins, remember the following:

- 1) **Do NOT talk, signal, or make noises to others in the class.**
- 2) **Do NOT show anyone else your offer price, response message, or record sheet.**
- 3) **Try to ensure that you will not need to leave the room until the exercise is over.**

THIS IS THE END OF THE INSTRUCTIONS. IF YOU HAVE ANY QUESTIONS PLEASE RAISE YOUR HAND AT THIS TIME.

QUESTIONNAIRE

- _____ 1. The last Triple Crown Winner in horse racing was
- A. Seattle Slew C. Winning Colors
B. Citation D. Affirmed
- _____ 2. Pearl Harbor was attacked on
- A. December 7, 1941 C. May 7, 1941
B. December 7, 1942 D. May 7, 1941
- _____ 3. In 1980, President Jimmy Carter and Brezhnev signed the SALT II Treaty.
- A. True B. False
- _____ 4. A colon punctuates the salutation in formal correspondence.
- A. True B. False
- _____ 5. Who was the first wife of Napoleon Bonaparte?
- A. Josephine C. Hortense
B. Marie Louise D. Joan
- _____ 6. Who is credited with naming May 30, 1868, as the first Memorial Day?
- A. Ulysses S. Grant C. John Logan
B. William Smith D. Abraham Lincoln
- _____ 7. A rule of thumb is that, on any government multiple-choice test the shortest answer is usually the correct one.
- A. True B. False
- _____ 8. Shivering produces as much heat as eating two medium-sized chocolate bars per hour.
- A. True B. False
- _____ 9. As a rule, 20% of a product line accounts for 90% of the profit.
- A. True B. False
- _____ 10. The minimum hourly wage was raised from \$1.00 to \$1.25 in
- A. 1937 C. 1954
B. 1966 D. 1961

TABLE 1
Numerical Comparative Statics

d	b^*	$p^*(b^*)$	S^*	R^*	D^*
0.05	0.621836	0.564011	0.482817	0.166699	0.350484
0.10	0.620561	0.556126	0.480158	0.165539	0.354303
0.15	0.619039	0.547720	0.477221	0.164385	0.358393
0.20	0.617258	0.538757	0.474017	0.163243	0.362741
0.25	0.615161	0.529167	0.470489	0.162117	0.367394
0.30	0.612726	0.518901	0.466646	0.161017	0.372337
0.35	0.609898	0.507883	0.462454	0.159951	0.377595
0.40	0.606630	0.496036	0.457902	0.158933	0.383164
0.45	0.602862	0.483269	0.452982	0.157978	0.389041
0.50	0.598531	0.469478	0.447691	0.157104	0.395205
0.55	0.593556	0.454531	0.442034	0.156338	0.401628
0.60	0.587852	0.438271	0.436039	0.155713	0.408248
0.65	0.581289	0.420475	0.429723	0.155274	0.415003
0.70	0.573720	0.400847	0.423150	0.155085	0.421765
0.75	0.564918	0.378934	0.416391	0.155241	0.428368
0.80	0.554530	0.354006	0.409553	0.155893	0.434554
0.85	0.541935	0.324749	0.402800	0.157310	0.439890
0.90	0.525836	0.288386	0.396419	0.160056	0.443525
0.95	0.502530	0.236927	0.391109	0.165761	0.443131

Table 1. *Numerical comparative statics.* Table 1 presents numerical simulations of the equilibrium raider bid (b^*), the equilibrium shareholder response to the equilibrium raider bid ($\pi^*(b^*)$), and the fraction of total value expected to be captured by the shareholders (S^*), the raider (R^*), and dissipation (D^*). All of the results in Table 1 assume that there are seven shares outstanding and that the raider requires four shares to obtain control.

TABLE 2
Offer Prices

Continuation Probability	Equilibrium	Average Price of All Offers	Average Price of First Period Offers	Average Price of Takeover Offers
0.01				
mean (p-value)	0.623	0.623 [†] (0.4995)	0.623	0.701
variance	0	0.0380	0.0380	0.0172
n		33	33	15
0.75				
mean (p-value)	0.565	0.634 (0.0325)	0.608	0.738
variance	0	0.0328	0.0515	0.0491
n		26	15	8
0.95				
mean (p-value)	0.503	0.537 [‡] (0.157)	0.68 [‡]	0.81 [‡]
variance	0	0.0405	0.0517	0.0195
n		36	8	5

[†] All offers are first period offers since the draw always ended the round after the first period.

[‡] The outlier observation of the offer price 3.00 is omitted. With outlier included, the average price of all offers is 0.604, the average price of first period offers is 0.938, and the average price of takeover offers is 1.17.

Table 2. Offer prices. Table 2 compares the average offer price to the theoretical Nash equilibrium price. *Average price of all offers* includes offers made during all periods and rounds of the experimental session. *Average price of first period offers* includes only the first offer made in a given round to a new group of shareholders. *Average price of takeover offers* includes only those offers when the takeover succeeded. Variances and number of observations are also reported.

Note: P-value is for a one-tailed test of the null hypothesis that the average of all offers is equal to the predicted price.

TABLE 3
Raider Response to Rejected Offers

<u>Continuation Probability</u>	<u>0.01</u>	<u>0.75</u>	<u>0.95</u>
Total Number Rejected Offers	18	18	31
Draw Ended the Round	18	7	3
Subsequent Offer:	0	11	28
Was Higher	--- †	8 (72.7%)	14 (50%)
Was Lower	--- †	1 (9.1%)	10 (35.7%)
Was the Same	--- †	2 (18.2%)	4 (14.3%)

† The draw ended the round in all of the first periods: There was not an opportunity for a subsequent offer to the same shareholder group.

Table 3. *Raider response to rejected offers.* Table 3 identifies how the raiders responded to rejected offers. *Total number of rejected offers* represents all offers rejected in the experimental session. *Draw ended the round* gives the number of rejected offers where the raider was unable to make a subsequent offer because the draw ended the round. “*Subsequent offer*” represents the total number of offers made in periods after the first period. *Was higher* represents the number of rejected offers where the subsequent bid price was higher. *Was lower* represents the number of rejected offers where the subsequent bid price was lower. *Was the same* represents the number of rejected offers where the subsequent bid price was identical. The percentage numbers are conditional on the opportunity of a subsequent offer.

TABLE 4
Offer Escalation

Continuation Probability	Geometric Average % Change in Offer	Median Change in Offer	Range of Change in Offer
0.01	--- [†]	---	---
0.75	26.99% (0.0675) [‡]	13.03%	7.43% to 84.3%
0.95	0.54% (0.438) [‡]	1.25%	-12.5% to 11.7%

[†] The draw did not allow for any subsequent offers to be made.

[‡] P-value.

Table 4. *Offer escalation.* Table 4 highlights the degree of escalation in offers after an offer was rejected. *Geometric average percentage change in offer* is the geometric average percentage change for all groups for a given round over all rounds in a given experimental session. *Median change in offer* is the median of the geometric percentage change in offers for all groups over all rounds. *Range of change in offer* is the range of the geometric percentage change in offers for all groups over all rounds.

Note: The p-values were computed using a t-statistic under the null hypothesis that the change in offers, measured by the geometric average, equals 0.

TABLE 5
Shares Tendered

Continuation Probability	Equilibrium	Average Proportion of All Shares Tendered	Average Proportion of First Period Shares Tendered	Average Proportion of Shares Tendered in Takeovers
0.01				
mean (p-value)	0.569	0.494 (0.0258)	0.494 [†]	0.686
variance	0.2452	0.322	0.322	0.0448
n		33	33	15
0.75				
mean (p-value)	0.379	0.401 (0.3158)	0.419	0.679
variance	0.2354	0.3773	0.5404	0.1939
n		26	15	8
0.95				
mean (p-value)	0.237	0.306 [‡] (0.0264)	0.411 [‡]	0.686 [‡]
variance	0.1808	0.2954	0.14	0.1001
n		36	8	5

[†] The draw did not allow for any subsequent offers to be made in a given round.

[‡] The outlier observation with the offer price 3.00 is omitted. With the outlier included, the average proportion of all shares tendered is 0.317, the average proportion of first period shares tendered is 0.444, and the average proportion of shares tendered in takeovers is 0.691.

Table 5. Shares tendered. Table 5 compares the proportion of shares tendered to the respective Nash equilibrium expected proportion of shares tendered. The proportion of shares tendered is used instead of actual shares in order to facilitate comparisons between the 1-share-per-shareholder indivisible case and the divisible control case where shareholders have 7 shares each. *Average proportion of all shares tendered* includes shares tendered during all periods and all rounds of the experimental session. *Average proportion of first period shares tendered* includes only the first period shares tendered in all rounds to a new group of shareholders. *Average proportion of shares tendered in takeovers* includes only those shares tendered when the takeover succeeded.

Note: The p-values come from a one-tailed z-test of the null hypothesis that the proportion of shares tendered is given by the Nash equilibrium prediction.

TABLE 6**Successful Takeovers**

<u>Continuation Probability</u>	<u>0.01</u>	<u>0.75</u>	<u>0.95</u>
Average Number of Periods For a Successful Takeover	1	1.38	3.6 [†]
Percentage of Successful First Period Offers in the First Round of a Session	66.7%	33.3%	0% [†]
Percentage of Successful First Period Offers in any Round	45.5%	40%	12.5% [†]
Percentage of Successful Offers in any Period Before Draw Ends the Round	45.5%	53.3%	62.5% [†]

[†] The outlier observation with the offer price 3.00 is omitted. With the outlier included, the average number of periods for a successful takeover is 3.17, the percentage of successful first period offers in the first round of a session is 33.3 percent, the percentage of successful first period offers in any round is 22.2 percent, and the percentage of successful offers in any period before draw ends the round is 66.7 percent.

Table 6. *Successful takeovers.* Table 6 examines how the continuation probability affects the chance of a successful takeover. *Average number of periods for a successful takeover* is a simple average of the period numbers in which a successful takeover occurred in any round of the experimental session. *Percentage of successful first period offers in the first round of a session* is the frequency with which a successful takeover occurred in the first period of the first round. *Percentage of successful first period offers of any round* is the frequency with which a successful takeover occurred in the first period of a round in the experimental session. *Percentage of successful offers in any period before draw ends the round* is the frequency with which a successful takeover occurred relative to the total number of opportunities (number of groups x number of rounds) in the experimental session.

TABLE 7

Conditional Tendering Proportions

Continuation Probability	Average Overtendering
0.01	- 0.0697 (0.000) [†]
0.75	- 0.0512 (0.000) [†]
0.95	0.0304 [‡] (0.055) [†]

[†] P-value.

[‡] The outlier observation with the offer price 3.00 is omitted. With the outlier included, then the average overtendering is 0.0319.

Table 7. *Conditional tendering proportion.* Table 7 presents the average difference between the actual tendering proportions and the theoretically predicted, for a given raider offer. *Average overtendering* provides the sum of all the individual differences for every offer across the entire experiment divided by the number of periods of observation in that experimental session. The difference in each raider offer is the actual proportion of shares tendered minus the Nash equilibrium expected proportion of shares tendered.

Note: The p-values come from a one-tailed z-test of the null hypothesis that the proportion of shares tendered is given by the Nash equilibrium prediction.

TABLE 8**Division of Gains**

Continuation Probability		Average Shareholder Gain	Average Raider Gain	Average Dissipation
0.01	Actual	0.361	0.093	0.545
	Nash	0.485	0.168	0.348
0.75	Actual	0.45	0.083	0.467
	Nash	0.416	0.155	0.428
0.95	Actual	0.549 [†]	0.076 [†]	0.375 [†]
	Nash	0.391	0.166	0.443

[†] The outlier observation with an offer price of \$3.00 is omitted. With the outlier included, the average shareholder gain is 0.599, the average raider gain is 0.067, and the average dissipation is 0.333: This calculation truncates shareholder gain to the total value of the firm.

Table 8. *Division of gains.* Table 8 shows the average normalized takeover gains disbursed among the shareholders, raiders, and dissipation. A comparison is made to the Nash equilibrium distribution of gains for each continuation probability. The shareholders' gain is 0 if the takeover is unsuccessful. When the takeover is successful, the shareholder gain is calculated for each group in each round as [(non-tendered shares x post-takeover value) + (tendered shares x tender price)] / (total shares x post-takeover value). The raider's gain is also 0 if the takeover is unsuccessful; but, if it is successful, then the gains are calculated for each group in each round as (1 – shareholder gains). For each group in each round, dissipation is credited a 0 when the takeover is successful, but credited a 1 when it is unsuccessful. The gains are averaged over each group for all rounds in each experimental session.

Figure 1

Optimal Bid Responses and the Continuation Probability

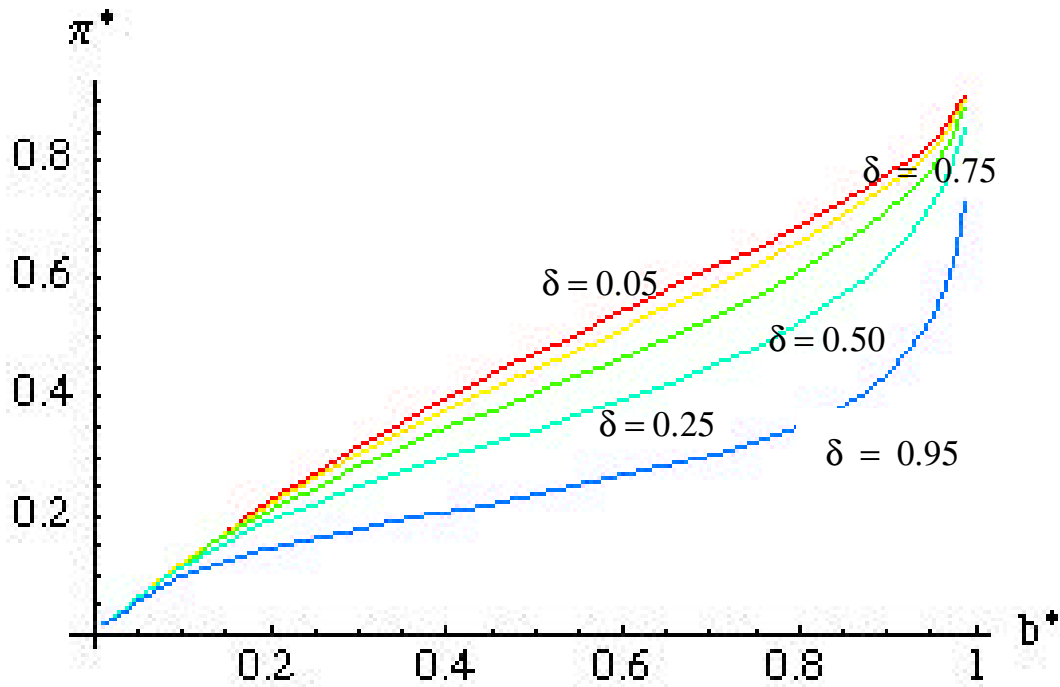
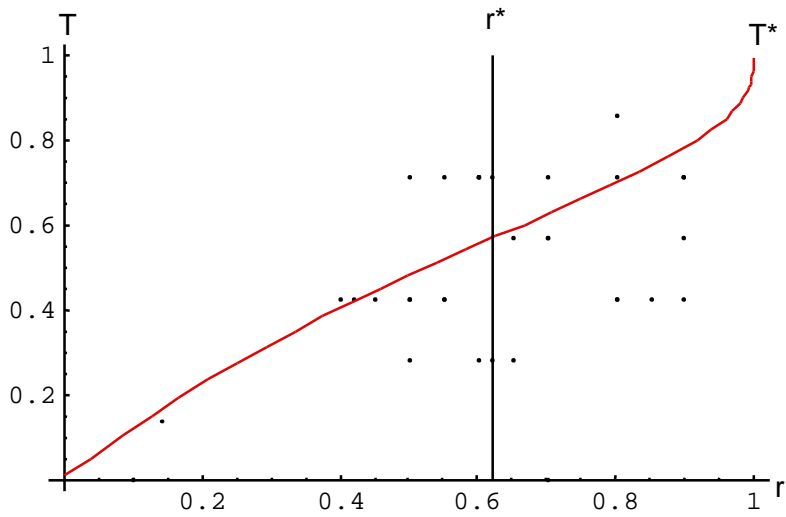


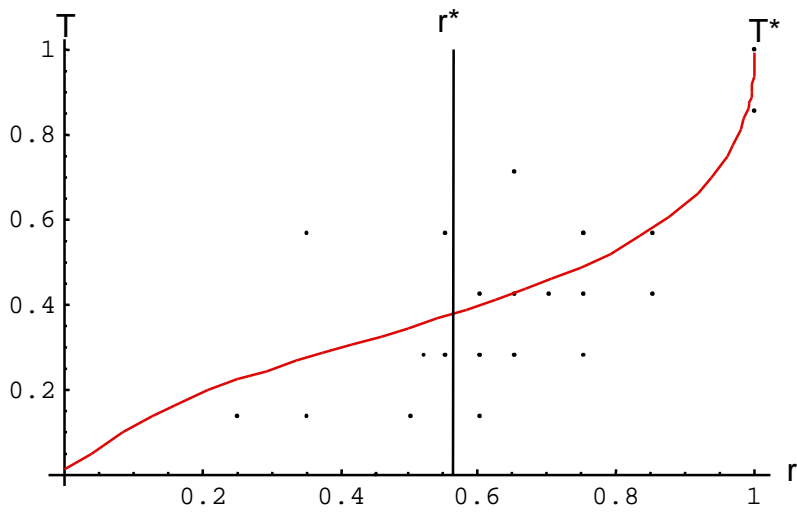
Figure 1. *Optimal bid responses and the continuation probability.* Figure 1 presents the optimal bid response and associated shareholder tendering probability given that bid, for each continuation probability. The horizontal axis, b^* , represents the equilibrium bid. The vertical axis, π^* , represents the equilibrium fraction of shares tendered to any fixed raider offer, given the continuation probability.

Figure 2

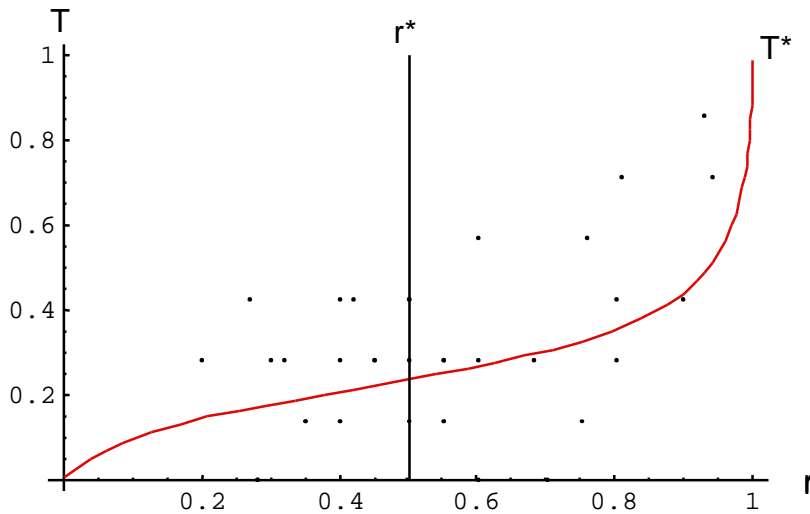
Equilibrium Fraction of Shares Tendered vs. Offer Price



Panel A. Continuation probability equal 0.01.



Panel B. Continuation probability equal 0.75.



Panel C. Continuation probability equal 0.95.

Figure 2. *Equilibrium fraction of shares tendered vs. offer price.* Figure 2 presents the fraction of shares tendered plotted and the tender price. In each figure, the horizontal axis, r , represents the normalized offer price. The vertical axis, T , represents the fraction of shares tendered. The line, T^* , represents the equilibrium fraction of shares tendered given that raiders follow the tendering strategy of offering a tender price equal to r . The line labeled by r^* represents the equilibrium offer. The scatter of points on the graphs represents the actual offer price/share tendering responses observed in the experiments. The outlier 3.00 bid was dropped because it was not in the range of theoretical values predicted.