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## Socially excessive bankruptcy costs and the benefits of interest rate ceilings on loans

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**Socially Excessive Bankruptcy Costs and the Benefits of  
Interest Rate Ceilings on Loans**

Marco A. Espinosa-Vega and Bruce D. Smith

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## **Socially Excessive Bankruptcy Costs and the Benefits of Interest Rate Ceilings on Loans**

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**Abstract:** The authors study the capital accumulation and welfare implications of ceilings on loan interest rates in a dynamic general equilibrium model. Binding ceilings on loan rates reduce the probability of bankruptcy. Lower bankruptcy rates result in lower bankruptcy and liquidation costs. The authors state conditions under which the resources freed by this cost-saving result increase the steady state capital stock, reduce steady state credit rationing, and raise the steady state welfare of all agents. The authors also argue that the conditions stated are likely to be satisfied in practice. Finally, their results hold even if initially there is capital over-accumulation.

JEL classification: E44, E58, G28

Key words: interest rate ceilings

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# Interest Rate Ceilings on Loans

## 1. Introduction

Most modern developed economies have, at some point in their past, imposed some kind of legal ceiling on the rates of interest charged on loans.<sup>1</sup> Similar controls have been common in modern developing economies. And, other examples of legal limits on loan rates of interest abound, for instance in the context of Islamic banking. The natural reaction of an economist to loan rate ceilings is to be dubious of this kind of interference with market forces. And, indeed, many countries have dismantled their mechanisms for controlling the rates of interest paid on loans.

However, particularly in the context of the development process, credit markets continue to be plagued by severe informational frictions and the rationing of credit is endemic.<sup>2</sup> Given these observations, and particularly given the prevalence of interest rate ceilings over time and across countries, we propose to reopen the question of whether it is possible for interest rate ceilings on loans to have some beneficial effects. In particular, we explore the possibility that, once the general equilibrium implications of ceiling rates on loans have been fleshed out, it can transpire that such restrictions promote capital accumulation, reduce the rationing of credit, and lead to relatively high steady state welfare for all agents.

More specifically, we pursue the idea that an economy can experience socially excessive bankruptcy costs in the absence of some government intervention. For example, suppose that – in the absence of interest rate regulation – market rates of interest on loans tend to be high. This would be the case, for instance, if credit was rationed. High rates of interest on loans would then imply that loans are relatively difficult for borrowers to repay, so that the incidence of bankruptcies would be high. Since bankruptcies consume resources it is possible that a socially excessive quantity of resources is expended on bankruptcy proceedings.

When this occurs, ceiling rates of interest on loans can reduce the incidence of bankruptcy, and can reduce the quantity of resources expended as part of the costs of bankruptcy. These resources are then freed for other uses, including -but not limited to - capital accumulation. Higher levels of capital accumulation then raise income levels. As a result, ceiling rates of interest on loans can actually raise the level of savings. And, when savings levels increase, more loans can be

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<sup>1</sup>A particularly common example of such ceilings were usury laws.

<sup>2</sup>And, the rationing of credit is not unknown in developed economies like the U.S. See for instance Japelli (1990).

made. Thus ceiling rates of interest on loans can actually lead to an expansion of lending, and a reduction in the extent of credit rationing.

Notice that this possibility stands in marked contrast to conventional wisdom that loan rate ceilings will tend to reduce lending. However, this conventional wisdom is based on partial equilibrium reasoning. In a full general equilibrium context, we provide conditions under which the imposition of some (binding) interest rate ceilings on loans will:

- a) increase short and long-run capital accumulation;
- b) reduce the rationing of credit in a steady state;
- c) raise the steady state welfare of lenders; and
- d) raise the steady state expected utility of borrowers.

In addition we argue that the conditions we provide are quite likely to be satisfied in practice, whenever credit is rationed. Thus there are strong reasons to think that some use of interest rate ceilings on loans can be socially beneficial, at least from a steady state perspective. And, interestingly, all of our results can be obtained even if the economy experiences capital overaccumulation absent any interest rate controls.

Our vehicle for examining these issues is a standard one-sector model of capital accumulation, due originally to Diamond (1965). In addition, we structure the economy so that only a subset of agents can produce capital, and these agents require external finance. Moreover, the provision of external finance is subject to a costly state verification problem of the type considered by Townsend (1979), Diamond (1984), Gale and Hellwig (1985) and Williamson (1986, 1987). Costly state verification constitutes a particularly explicit and simple way of modeling costs associated with bankruptcy. And, as originally noted by Gale and Hellwig (1985) and Williamson (1986, 1987), the presence of a costly state verification problem can easily lead credit to be rationed, and market rates of interest on loans to be high. Thus the presence of a costly state verification problem creates all of the conditions required for loan interest rate ceilings to be potentially beneficial.

Interestingly, while there is a lengthy tradition of considering ceiling rates of interest on deposits in a macroeconomic setting [Friedman (1960), Tobin (1970), Smith (1984) and Hellman, Murdock and Stiglitz (2000)], little attention has been devoted to ceiling rates of interest on loans. This is somewhat paradoxical in view of the recent evidence [see, for example, Brock and Rojas Suarez (2000)] that – at least in some developing countries – rates of interest charged on loans tend to be very high relative to rates of interest paid on deposits. In any event, this paper represents an attempt to bring high rates of interest on loans into the foreground.

The remainder of the paper proceeds as follows. Section 2 sets out the economic environment, and section 3 analyzes trade in credit and factor markets. Sections 4 and 5, describe, respectively, a general equilibrium of our economy without and with interest rate controls. Section 6 studies the welfare implications of imposing a binding loan rate ceiling. Section 7 concludes.

## 2. The Model

### 2.1. Environment

Our economy is populated by an infinite sequence of two-period lived, overlapping generations. Every generation is identical in size and composition, and contains a continuum of agents with unit mass. Within each generation, agents are divided into two types: “potential borrowers” and “lenders”. A fraction  $\alpha \in (0, 1)$  ( $(1 - \alpha)$ ) of the population is potential borrowers (lenders).

Let  $t = 0, 1, \dots$  index time. At date  $t$  a single final good is produced in the economy, using a constant returns to scale technology with capital and labor as factors of production. Let  $K_t$  denote the time  $t$  capital input, and  $L_t$  denote the time  $t$  labor input of a representative firm. Then final output is  $Y_t = F(K_t, L_t)$ . The production function  $F$  satisfies the following conditions: it is increasing in each argument, and  $F(0, L) = F(K, 0) = 0$  holds, for all  $K, L$ . In addition, if  $k \equiv \frac{K}{L}$  is the capital-labor ratio, and if  $f(k) \equiv F(k, 1)$  denotes the intensive production function, then  $f' > 0 > f''$  holds  $\forall k$ . For simplicity we assume that the inherited capital stock at date  $t$  is used in production, and that thereafter it depreciates completely. Moreover, only potential borrowers have access to a technology for producing capital. We describe this technology in more detail below.

Each lender is endowed with one unit of labor when young, which is supplied inelastically. A young lender at date  $t$  then earns the competitively determined real wage rate  $w_t$ .

Let  $c_{1t}^L$  ( $c_{2t}^L$ ) denote the first (second) period consumption of a lender born at date  $t$ . This agent then has the utility function  $U(c_{1t}^L, c_{2t}^L)$ . The utility function  $U$  has standard properties: it is increasing in each argument and is quasi-concave. In addition, we assume that consumption in each period is a normal good for lenders.

Potential borrowers have access to a technology for converting date  $t$  final goods into date  $t + 1$  capital. The capital investment technology has the following

properties. First, it is indivisible and nontradable: each potential borrower has one investment project which can only be operated at the scale  $q$ . In particular,  $q > 0$  units of the final good invested in one project at  $t$  yield  $zq$  units of capital at  $t + 1$ , where  $z$  is an *iid* (across borrowers and periods) random variable, which is realized at  $t + 1$ . We let  $G$  denote the probability distribution of  $z$ , and assume that  $G$  has a differentiable density function  $g$  with support  $[0, \bar{z}]$ . We let  $\hat{z}$  denote the expected value of  $z$ , i.e.,  $\hat{z} \equiv \int_0^{\bar{z}} zg(z)dz$ .

The amount of capital produced by any investment project can be observed costlessly by the project owner. Any agent other than the project owner can observe the project return only by bearing some cost. Following Bernanke and Gertler (1989) and Boyd and Smith (1997, 1998), we assume that verification of a project's return requires some expenditure of capital. In particular the return on any project can be observed by an outsider only by expending  $\gamma > 0$  units of capital.

Potential borrowers have no young period income. Therefore they require external finance in order to operate their projects. The nature of this external finance is described in detail below.

We assume that potential borrowers are risk neutral, and that they care only about second period consumption (income). In addition, any given potential borrower may fail to obtain funding for his project. In this case the agent engages in some outside activity that generates the exogenously given utility level  $\bar{U} \geq 0$ .

The initial old agents in our economy have an aggregate capital endowment of  $K_0 > 0$ . No other agents are endowed with capital or consumption goods at any date.

### 3. Trade

#### 3.1. Factor Markets

We assume that capital and labor are traded in competitive markets at each date. Thus, if  $w_t$  denotes the time  $t$  real wage rate and  $\rho_t$  is the time  $t$  capital rental rate, the standard factor pricing relationships obtain:

$$\rho_t = f'(k_t) \tag{3.1}$$

$$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t). \tag{3.2}$$

Notice that  $w'(k) > 0$  holds and, in addition, we will assume the following.

**Assumption 1.** (A1-1)  $kw'(k)/w(k) < 1$ . (A1-2)  $kf''(k)/f'(k) > -1; \forall k \geq 0$ .

Assumption 1 is satisfied if, for example,  $f$  is any CES production function with elasticity of substitution no less than one.

### 3.2. Credit Markets

The supply of credit in this economy is simply the savings of young lenders. As described by Diamond (1984) and Williamson (1986), we can think of all savings as being intermediated. Let  $r_t$  be the gross real rate of interest on deposits made at  $t$  and withdrawn at  $t + 1$ , and let  $s_t$  be the savings of a young agent at  $t$ . Then  $s_t$  is chosen to maximize  $U(w_t - s_t, r_t s_t)$ . The first order condition for this problem is

$$U_1(w_t - s_t, r_t s_t) = r_t U_2(w_t - s_t, r_t s_t) \quad (3.3)$$

This first order condition implicitly defines a savings function  $s_t = s(w_t, r_t)$ . Our assumption that consumption in both periods is normal implies that

$$0 \leq s_1(w_t, r_t) \leq 1.$$

In addition, we assume that savings is non-decreasing in the rate of return, and that the income elasticity of savings does not exceed unity.

**Assumption 2.** (A2-1)  $s_2(w_t, r_t) \geq 0$ . (A2-2)  $[ws_1(w, r)/s(w, r)] \leq 1$ .

The demand for credit derives from potential borrowers, who require external financing in order to operate their projects. Following Williamson (1986, 1987), we assume that each potential borrower announces a set of loan contract terms. These terms are either accepted or rejected by intermediaries: borrowers whose terms are accepted then receive funding and operate their projects. A loan contract consists of the following objects. First, there is a set of project return realizations  $A_t$  for which verification of the project return occurs at  $t$ . Verification does not occur if  $z \in B_t \equiv [0, \bar{z}] - A_t$ .<sup>3</sup> Second, if  $z \in A_t$ , then the loan repayment can meaningfully be made contingent on the project return. Then, for  $z \in A_t$ , borrowers offer a state contingent repayment schedule  $R_t(z)$ , per unit borrowed. Third, if  $z \in B_t$ ,

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<sup>3</sup>Obviously we abstract from stochastic state verification. Boyd and Smith (1994) show that the potential gains from stochastic state verification are small for realistic parametrizations of the economy.



then the repayment from the borrower to the lender cannot meaningfully depend on the amount of capital yielded by an investment project. Thus, for  $z \in B_t$  borrowers offer an uncontingent payment of  $x_t$  (per unit borrowed).  $x_t$  can be thought of as the gross real rate of interest paid by the borrower.

Any lender can establish an intermediary. Financial intermediaries take deposits and make loans. In the deposit market intermediaries are competitive; they behave as if they can obtain any quantity of deposits at the market rate of interest  $r_t$ . With respect to loans, intermediaries either accept or reject the contracts offered by borrowers, plus they conduct monitoring of project returns as called for by loan contracts. In equilibrium, intermediaries will lend to a large number of agents, and hence will earn a non-stochastic return on their portfolios. Thus they need not be monitored by their depositors. In addition, competition for deposits implies that intermediaries will earn zero profits, in equilibrium.

Since intermediaries believe they can obtain any quantity of funds at the prevailing rate of interest on deposits, it follows that intermediaries are willing to accept loan contract offers yielding an expected return of at least  $r_t$ . Loan contract offers must therefore satisfy the expected return constraint

$$\int_{A_t} [R_t(z)q - \rho_{t+1}\gamma]g(z)dz + x_tq \int_{B_t} g(z)dz \geq r_tq. \quad (3.4)$$

Notice that expected repayments must at least cover the intermediary's cost of funds -  $r_tq$  - plus the real expected monitoring cost

$$\rho_{t+1}\gamma \int_{A_t} g(z)dz.$$

The latter term depends on  $\rho_{t+1}$  because  $\gamma$  units of *capital* are expended when project returns are verified. Finally, project owners must have the proper incentives to correctly reveal when a monitoring state has occurred. The appropriate incentive constraint is

$$R_t(z) \leq x_t; \quad z \in A_t. \quad (3.5)$$

In addition, loan repayments must be feasible, so that

$$0 \leq R_t(z) < z\rho_{t+1}; \quad z \in A_t \quad (3.6)$$

$$\inf_{z \in B_t} z\rho_{t+1} \geq x_t \quad (3.7)$$

must hold.

Borrowers announce loan contract terms to maximize their own expected utility, subject to (3.4)-(3.7). The expected utility of a borrower is simply the expected value of the capital yielded by an investment project, less the expected repayment made on borrowed funds. Thus the expected utility of a funded borrower is

$$\rho_{t+1}\hat{z}q - q \int_{A_t} R_t(z)g(z)dz - x_tq \int_{B_t} g(z)dz = q \left[ \rho_{t+1}\hat{z} - r_t - \left(\frac{\gamma}{q}\right) \int_{A_t} g(z)dz \right],$$

where the equality follows from the fact that (3.4) must hold with equality under an optimal contract.

Conventional arguments [Gale and Hellwig (1985) and Williamson (1986, 1987)] establish that the optimal contract is a standard debt contract. That is, borrowers repay principal plus interest ( $x_tq$ ) if this is feasible. If it is not, the borrower defaults on his loan (declares bankruptcy), and the lender monitors the project and claims the entire return  $\rho_{t+1}zq$ . That is,

$$A_t = \left[ 0, \frac{x_t}{\rho_{t+1}} \right)$$

and

$$R_t(z) = z\rho_{t+1}; \quad z \in A_t. \quad (3.8)$$

In particular, the borrower defaults on his loan iff

$$\frac{x_t}{\rho_{t+1}} > z$$

The expected return to a lender (per unit lent) under the optimal contract is then given by

$$x_t \left[ 1 - G\left(\frac{x_t}{\rho_{t+1}}\right) \right] - \left(\frac{\gamma}{q}\right) \rho_{t+1} G\left(\frac{x_t}{\rho_{t+1}}\right) + \rho_{t+1} \int_0^{\frac{x_t}{\rho_{t+1}}} zg(z)dz$$

$$\begin{aligned}
&= x_t - \rho_{t+1} \int_0^{\frac{x_t}{\rho_{t+1}}} G(z) dz - \left(\frac{\gamma}{q}\right) \rho_{t+1} G\left(\frac{x_t}{\rho_{t+1}}\right) \\
&= \rho_{t+1} \left\{ \frac{x_t}{\rho_{t+1}} - \int_0^{\frac{x_t}{\rho_{t+1}}} G(z) dz - \left(\frac{\gamma}{q}\right) G\left(\frac{x_t}{\rho_{t+1}}\right) \right\} \equiv \rho_{t+1} \pi\left(\frac{x_t}{\rho_{t+1}}\right)
\end{aligned}$$

In order to guarantee the existence of a unique uncollateralized payment  $\hat{x}$  that maximizes the function  $\pi$  we assume the following:

**Assumption 3.**  $g(z) + (\frac{\gamma}{q})g'(z) \geq 0$ ; for all  $z \in [0, \bar{z}]$ .

**Assumption 4.**  $\pi'(0) > 0$ .

Under these assumptions, the function  $\pi$  has the configuration depicted in Figure 1. In addition, the expected return constraint (3.4) at equality reduces to

$$r_t = \rho_{t+1} \pi\left(\frac{x_t}{\rho_{t+1}}\right).$$

It remains to describe the expected utility of a funded borrower under a standard debt contract. Clearly this expected utility is

$$q \left[ \rho_{t+1} \hat{z} - r_t - \left(\frac{\gamma}{q}\right) \rho_{t+1} G\left(\frac{x_t}{\rho_{t+1}}\right) \right] = \rho_{t+1} q \left[ \hat{z} - \pi\left(\frac{x_t}{\rho_{t+1}}\right) - \left(\frac{\gamma}{q}\right) G\left(\frac{x_t}{\rho_{t+1}}\right) \right].$$

Moreover, this quantity must exceed  $\bar{U}$  in order for borrowers to seek funding. Thus

$$\rho_{t+1} q \left[ \hat{z} - \pi\left(\frac{x_t}{\rho_{t+1}}\right) - \frac{\gamma}{q} G\left(\frac{x_t}{\rho_{t+1}}\right) \right] \geq \bar{U} \quad (3.9)$$

is henceforth assume to hold.

### 3.3. Credit Rationing

As is well known, [Gale and Hellwig (1985) and Williamson (1986, 1987)], in this environment it is possible for credit to be rationed. In particular, if all borrowers want to operate their projects at date  $t$ , the total (per capita) demand for funds

is  $\alpha q$ . The total per capita supply of saving at time  $t$  is  $(1 - \alpha)s(w_t, r_t)$ . Thus, aggregate credit demand exceeds aggregate credit supply, and hence credit must be rationed, if the following is true for all  $t \geq 0$ :

$$\alpha q > (1 - \alpha)s(w_t, r_t) \quad (3.10)$$

We will subsequently assume that (3.10) holds, at least at a steady state.

If there is credit rationing, then the rate of interest on loans will be bid up to its return maximizing level, that is

$$\frac{x_t}{\rho_{t+1}} = \eta$$

where

$$\pi'(\eta) = 1 - G(\eta) - \left(\frac{\gamma}{q}\right) g(\eta) = 0. \quad (3.11)$$

It then follows that

$$r_t = \rho_{t+1}\pi(\eta).$$

In addition, the assumption (3.9) reduces to

$$\rho_{t+1} \left[ \widehat{z} - \pi(\eta) - \frac{\gamma}{q} G(\eta) \right] \geq \frac{\overline{U}}{q} \quad (3.12)$$

## 4. A General Equilibrium with No Interest Rate Ceilings

Let  $\mu_t$  be the fraction of borrowers who obtain funding at  $t$ . Then an equality between sources and uses of funds requires that

$$\alpha q \mu_t = (1 - \alpha)s(w_t, r_t) \quad (4.1)$$

Note that the condition under which credit must be rationed (*i.e.*,  $\mu_t < 1$ ) holds if

$$\alpha q < (1 - \alpha)s[w(k_t), \pi(\eta)f'(k_{t+1})] \quad (4.2)$$

Also, in order for borrowers to want funding, we must have

$$f'(k_{t+1})q \left\{ \widehat{z} - \pi(\eta) - \frac{\gamma}{q} G(\eta) \right\} \geq \overline{U}.$$

#### 4.1. Capital Accumulation

Let  $K_{t+1}$  be the aggregate capital stock at  $t + 1$ . Then  $K_{t+1}$  equals  $\hat{z}\alpha q\mu_t$ , less capital expended on monitoring. Capital used in monitoring is

$$\alpha\mu_t\gamma G\left(\frac{x_t}{\rho_{t+1}}\right)$$

Then

$$\begin{aligned} K_{t+1} &= \alpha\mu_t q \left[ \hat{z} - \left(\frac{\gamma}{q}\right) G\left(\frac{x_t}{\rho_{t+1}}\right) \right] \\ &= (1 - \alpha)s \left[ w(k_t), \pi\left(\frac{x_t}{\rho_{t+1}}\right) f'(k_{t+1}) \right] \left[ \hat{z} - \left(\frac{\gamma}{q}\right) G\left(\frac{x_t}{\rho_{t+1}}\right) \right] \end{aligned} \quad (4.3)$$

Since  $k_{t+1} = K_{t+1}/(1 - \alpha)$ , one can express this law of motion in terms of the capital-labor ratio as follows,

$$k_{t+1} = s \left[ w(k_t), \pi\left(\frac{x_t}{\rho_{t+1}}\right) f'(k_{t+1}) \right] \left[ \hat{z} - \left(\frac{\gamma}{q}\right) G\left(\frac{x_t}{\rho_{t+1}}\right) \right] \quad (4.4)$$

Under credit rationing, this law of motion reduces to

$$k_{t+1} = s [w(k_t), \pi(\eta)f'(k_{t+1})] \left[ \hat{z} - \left(\frac{\gamma}{q}\right) G(\eta) \right] \quad (4.5)$$

Figure 2 depicts this law of motion. Under our assumptions, the law of motion is upward sloping, and it is easy to show that a non-trivial steady state ( $k_{t+1} = k_t = k > 0$ ) exists if

$$\left. \frac{dk_{t+1}}{dk_t} \right|_{k_t=0} = \frac{\left[ \hat{z} - \left(\frac{\gamma}{q}\right) G(\eta) \right] s_1(\cdot)w'(0)}{1 - s_2(\cdot)\pi(\eta)f''(0)} > 1 \quad (4.6)$$

holds. We assume throughout that (4.6) is satisfied. It is also easy to show that our assumptions imply the existence of a unique non-trivial steady state, as depicted in Figure 2. Clearly this steady state is asymptotically stable.

### 5. A General Equilibrium with Interest Rate Ceilings on Loans

We now turn attention to our primary task, which is to determine when interest rate ceilings on loans can:

- a) promote capital accumulation;
- b) reduce the rationing of credit;
- c) increase the utility of lenders; and
- d) increase the expected utility of borrowers.

For all of these issues except (a), we focus on steady states.

For simplicity, we assume that interest rate ceilings on loans take the form

$$x_t \leq \bar{x}_t \equiv \phi\eta\rho_{t+1}, \quad (5.1)$$

with  $\phi \in (0, 1]$  chosen by the government. If there is credit rationing in the absence of a loan rate ceiling ( $\phi = 1$ ), then (5.1) says that lenders are only allowed to charge a fraction  $\phi$  of the “market rate of interest on loans.” Note that reductions in  $\phi$  mean tighter loan rate ceilings.<sup>4</sup>

Define

$$\pi(\phi\eta) = \phi\eta - \int_0^{\phi\eta} G(z)dz - \left(\frac{\gamma}{q}\right) G(\phi\eta)$$

Then, if there is credit rationing, the rate of interest on loans must be bid up to its return maximizing level. Thus (5.1) will hold with equality and the market expected return on deposits will satisfy

$$r_t = \pi(\phi\eta)\rho_{t+1} = \pi(\phi\eta)f'(k_{t+1}) \quad (5.2)$$

For future reference it will be useful to remember that, by definition,  $\pi'(\eta) = 0$ .

When an interest rate ceiling is imposed, the law of motion for the capital stock is given by

$$k_{t+1} = s[w(k_t), \pi(\phi\eta)f'(k_{t+1})] \left[ \hat{z} - \left(\frac{\gamma}{q}\right) G(\phi\eta) \right] \quad (5.3)$$

Figure 3 depicts this law of motion.

### 5.1. Loan Rate Ceilings and Capital Accumulation

We are now in a position to study the implications of introducing a loan rate ceiling. We begin by analyzing the consequences of such a policy for capital

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<sup>4</sup>It is our intention to show that some form of interest rate ceilings on loans can be socially beneficial. To do so, it suffices to show that loan rate ceilings of the type given in equation (5.1) can be beneficial.

accumulation. We first consider the vertical shift in the equilibrium law of motion for  $k_t$  associated with a tightening of loan rate ceilings.

To do so, differentiate (5.3) with respect to  $\phi$ , holding  $k_t$  fixed, to obtain;

$$\begin{aligned} \frac{\partial k_{t+1}}{\partial \phi} = s_2(.) \left[ f'(k_{t+1})\eta\pi'(\phi\eta) + \pi(\phi\eta)f''(k_{t+1})\frac{\partial k_{t+1}}{\partial \phi} \right] & \left[ \widehat{z} - \left( \frac{\gamma}{q} \right) G(\phi\eta) \right] \\ - s(.)\eta \left( \frac{\gamma}{q} \right) g(\phi\eta) & \end{aligned} \quad (5.4)$$

(5.4) can be rewritten as

$$\begin{aligned} \frac{\partial k_{t+1}}{\partial \phi} = \frac{s_2}{s(.)}k_{t+1} & \left[ f'(k_{t+1})\eta\pi'(\phi\eta) + \pi(\phi\eta)f''(k_{t+1})\frac{\partial k_{t+1}}{\partial \phi} \right] \\ - \left[ \frac{\eta \left( \frac{\gamma}{q} \right) g(\phi\eta)}{\widehat{z} - \left( \frac{\gamma}{q} \right) G(\phi\eta)} \right] & k_{t+1} \end{aligned} \quad (5.5)$$

Collecting terms in (5.5), evaluating at  $\phi = 1$ , and using  $\pi'(\eta) = 0$ , we get

$$\frac{\phi}{k_{t+1}} \frac{\partial k_{t+1}}{\partial \phi} \Big|_{\phi=1} = - \left[ \frac{\eta \left( \frac{\gamma}{q} \right) g(\eta)}{\widehat{z} - \left( \frac{\gamma}{q} \right) G(\eta)} \right] \left\{ 1 - \frac{s_2}{s(.)}\pi(\eta)f'(k_{t+1})\frac{k_{t+1}f''(k_{t+1})}{f'(k_{t+1})} \right\}^{-1} < 0. \quad (5.6)$$

Thus reducing  $\phi$  (imposing a binding loan interest rate ceiling) shifts the law of motion for  $k_t$  up as shown in Figure 3. For future reference, it will be useful to have an exact expression for the impact on the steady state capital stock of a small change in  $\phi$  (a tightening of the interest ceiling).

Clearly, from (5.3), the steady state capital stock satisfies

$$k = s[w(k), \pi(\phi\eta)f'(k)] \left[ \widehat{z} - \left( \frac{\gamma}{q} \right) G(\phi\eta) \right] \quad (5.7)$$

Therefore,

$$\frac{\partial k}{\partial \phi} = \left[ \widehat{z} - \left( \frac{\gamma}{q} \right) G(\phi\eta) \right] \left\{ s_1(.)w'(k)\frac{\partial k}{\partial \phi} + s_2(.) \left[ \rho\eta\pi'(\phi\eta) + \pi(\phi\eta)f''(k)\frac{\partial k}{\partial \phi} \right] \right\}$$

$$\begin{aligned}
& -s(\cdot)\eta \left( \frac{\gamma}{q} \right) g(\phi\eta) \\
& = \left[ \frac{ws_1(\cdot)}{s(\cdot)} \right] \frac{k w'(k)}{w(k)} \frac{\partial k}{\partial \phi} + \left[ \frac{s_2(\cdot)}{s(\cdot)} \right] \left[ \pi'(\phi\eta) k f'(k) + f'(k) \pi(\phi\eta) \frac{k f''(k)}{f'(k)} \frac{\partial k}{\partial \phi} \right] \\
& \quad - k \left[ \frac{\eta \left( \frac{\gamma}{q} \right) g(\phi\eta)}{\widehat{z} - \left( \frac{\gamma}{q} \right) G(\phi\eta)} \right]
\end{aligned} \tag{5.8}$$

Evaluating this expression at  $\phi = 1$ , using  $\pi'(\eta) = 0$ , and collecting terms gives

$$\left. \frac{\phi}{k} \frac{\partial k}{\partial \phi} \right|_{\phi=1} = - \left[ \frac{\eta \left( \frac{\gamma}{q} \right) g(\eta)}{\widehat{z} - \left( \frac{\gamma}{q} \right) G(\eta)} \right] \left\{ 1 - \left[ \frac{ws_1(\cdot)}{s(\cdot)} \right] \frac{k w'(k)}{w(k)} - \left[ \frac{s_2(\cdot)}{s(\cdot)} \right] \pi(\eta) f'(k) \frac{k f''(k)}{f'(k)} \right\}^{-1} \tag{5.9}$$

We can now state the following result.

**Proposition 5.1.**  $\left. \frac{\partial k}{\partial \phi} \right|_{\phi=1} < 0$ .

Proposition 1 follows immediately from equation (5.9) and assumption 2. It asserts that the imposition of a “small” but binding interest rate ceiling on loans increases steady state capital accumulation, when credit is rationed.

The use of interest rate ceiling on loans can promote capital accumulation in two ways. First, interest rate ceilings reduce the probability of bankruptcy ( $G(\phi\eta)$ ), and hence they reduce the level of resource expenditures on bankruptcy proceedings (state verification). Second, it is possible that –perhaps contrary to intuition – interest rate ceilings reduce the extent of credit rationing, in a steady state. This can happen because an increase in the steady state capital stock can increase the savings of lenders. We now investigate how steady state credit rationing varies with  $\phi$ .

## 5.2. Loan Rate Ceilings and Credit Rationing

It follows from our previous discussion that, in a steady state,

$$\mu = \frac{k}{\alpha q \left[ \widehat{z} - \left( \frac{\gamma}{q} \right) G(\phi\eta) \right]}, \tag{5.10}$$



where  $\mu$  is the fraction of potential borrowers who obtain funding, in equilibrium. Differentiating (5.10) with respect to  $\phi$  we obtain,

$$\begin{aligned}\frac{\partial \mu}{\partial \phi} &= \frac{\mu}{k} \frac{\partial k}{\partial \phi} + \frac{k \left( \frac{\gamma}{q} \right) \eta g(\phi \eta)}{\alpha q \left[ \hat{z} - \left( \frac{\gamma}{q} \right) G(\phi \eta) \right]^2} \\ &= \frac{\mu}{k} \frac{\partial k}{\partial \phi} + \mu \left[ \frac{\eta \left( \frac{\gamma}{q} \right) g(\phi \eta)}{\hat{z} - \left( \frac{\gamma}{q} \right) G(\phi \eta)} \right]\end{aligned}\quad (5.11)$$

or

$$\frac{\phi}{\mu} \frac{\partial \mu}{\partial \phi} = \frac{\phi}{k} \frac{\partial k}{\partial \phi} + \left[ \frac{\eta \left( \frac{\gamma}{q} \right) g(\phi \eta)}{\hat{z} - \left( \frac{\gamma}{q} \right) G(\phi \eta)} \right] \quad (5.12)$$

Now substitute (5.9) into (5.12) evaluated at  $\phi = 1$  to get

$$\frac{\phi}{\mu} \frac{\partial \mu}{\partial \phi} \Big|_{\phi=1} = \left[ \frac{\eta \left( \frac{\gamma}{q} \right) g(\eta)}{\hat{z} - \left( \frac{\gamma}{q} \right) G(\eta)} \right] \left\{ 1 - \frac{1}{1 - \left( \frac{ws_1}{s(\cdot)} \right) \frac{kw'(k)}{w(k)} - \left( \frac{rs_2}{s(\cdot)} \right) \frac{kf''(k)}{f'(k)}} \right\} \quad (5.13)$$

It then follows from assumption 2 that  $\frac{\partial \mu}{\partial \phi} \Big|_{\phi=1}$  is opposite in sign to the term

$$\left( \frac{ws_1}{s(\cdot)} \right) \frac{kw'(k)}{w(k)} + \left( \frac{rs_2}{s(\cdot)} \right) \frac{kf''(k)}{f'(k)}.$$

Or, in other words,  $\frac{\partial \mu}{\partial \phi} \Big|_{\phi=1} \leq 0$  holds iff

$$\left( \frac{ws_1}{s(\cdot)} \right) \frac{kw'(k)}{w(k)} \geq - \left( \frac{rs_2}{s(\cdot)} \right) \frac{kf''(k)}{f'(k)} \quad (5.14)$$

Since  $w'(k) = -kf''(k)$ , (5.14) is equivalent to

$$\frac{ws_1}{s(\cdot)} \geq - \left( \frac{rs_2}{s(\cdot)} \right) \frac{w(k)}{kf'(k)} \quad (5.15)$$

we can now summarize this result in

**Proposition 5.2.**  $\frac{\partial \mu}{\partial \phi} |_{\phi=1} \leq 0$  holds if (5.15) is satisfied.

Thus, in particular, a small reduction in  $\phi$  (the imposition of a small but binding interest rate ceiling) will reduce steady state credit rationing if the interest elasticity of savings is not too large. Since all empirical evidence suggests that the interest elasticity of savings is quite low, it is empirically very plausible that the use of small interest rate controls can reduce the amount of credit rationing that occurs, in a steady state equilibrium.

Armed with these results we can now turn our attention to our primary task: showing when the use of interest rate ceilings on loans can increase the steady state expected utility of all agents.

## 6. Steady State Welfare

**Lender Expected Utility.** We begin by investigating the impact of ceiling rates on the steady state welfare of lenders. A *lender's utility* in steady state is

$$U \{w(k) - s(.), f'(k)\pi(\phi\eta)s(.)\} \equiv V(\phi), \quad (6.1)$$

where, in (6.1),  $k$  is to be regarded as a function of  $\phi$ .

Now, note that

$$\begin{aligned} V'(\phi) &= U_1 w'(k) \frac{\partial k}{\partial \phi} + U_2 \left[ f'(k)s(.)\eta\pi'(\phi\eta) + \pi(\phi\eta)s(.)f''(k) \frac{\partial k}{\partial \phi} \right] \\ &\quad - [U_1 - f'(k)\pi(\phi\eta)U_2] \frac{\partial s(.)}{\partial \phi} \\ &= U_1 w'(k) \frac{\partial k}{\partial \phi} + s(.)rU_2 \left[ \frac{kf''(k)}{f'(k)} \right] \frac{1}{k} \frac{\partial k}{\partial \phi} + U_2 \eta f'(k)s(.)\pi'(\phi\eta) \end{aligned} \quad (6.2)$$

by the envelope theorem.

Now evaluate (6.2) at  $\phi = 1$ , and use  $\pi'(\eta) = 0$  and (3.3) to get

$$V'(1) = U_1(.) \frac{1}{k} \frac{\partial k}{\partial \phi} |_{\phi=1} \left\{ kw'(k) + s(.) \frac{kf''(k)}{f'(k)} \right\} \quad (6.3)$$

Observing that  $w'(k) = -kf''(k)$ , we can rewrite (6.3) as

$$V'(1) = -U_1(.) \frac{1}{k} \frac{\partial k}{\partial \phi} |_{\phi=1} kf''(k) \left\{ k - \frac{s(.)}{f'(k)} \right\}$$

Now  $\frac{\partial k}{\partial \phi} |_{\phi=1} < 0$  holds, so that  $V'(1) \leq 0$  iff  $kf'(k) \geq s(\cdot)$  or, equivalently, iff

$$\frac{kf'(k)}{f(k)} \geq \frac{s(\cdot)}{f(k)} \quad (6.4)$$

It follows that the steady state utility of lenders is decreasing in  $\phi$  (at  $\phi = 1$ ) if capital's share of total income exceeds the fraction of total income saved. Plausible values for capital's share of income in most economies would be at least 0.25, and few economies save more than 25% of their total production. We can therefore conclude that, in almost all contexts, (6.4) will be satisfied, as a practical matter. Thus the use of small but binding loan rate ceilings can reasonably be expected to increase the steady state welfare of lenders.

**Borrower Expected Utility.** We now investigate the impact of interest rate ceilings on the steady state expected utility of borrowers. Recall that  $\mu$  is the probability of being funded for a borrower. If a borrower gets funded, his expected payoff is

$$\begin{aligned} \widehat{z}q\rho_{t+1} - qx_t \left[ 1 - G\left(\frac{x_t}{\rho_{t+1}}\right) \right] - q\rho_{t+1} \int_0^{\frac{x_t}{\rho_{t+1}}} zg(z)dz &= \widehat{z}q\rho_{t+1} - qx_t + q\rho_{t+1} \int_0^{\frac{x_t}{\rho_{t+1}}} G(z)dz \\ &= q\rho_{t+1} \left\{ \widehat{z} - \phi\eta + \int_0^{\phi\eta} G(z)dz \right\} = qf'(k) \left\{ \widehat{z} - \pi(\phi\eta) - \left(\frac{\gamma}{q}\right) G(\phi\eta) \right\}. \end{aligned}$$

Thus, the expected utility of a borrower, in steady state, is

$$\mu qf'(k) \left[ \widehat{z} - \pi(\phi\eta) - \left(\frac{\gamma}{q}\right) G(\phi\eta) \right] + (1 - \mu)\overline{U} \equiv W(\phi) \quad (6.5)$$

Clearly, then,

$$\begin{aligned} W'(\phi) &= f'(k) [\widehat{z}q - q\pi(\phi\eta) - \gamma G(\phi\eta)] \frac{\partial \mu}{\partial \phi} - \overline{U} \frac{\partial \mu}{\partial \phi} \\ &\quad + \mu [\widehat{z}q - q\pi(\phi\eta) - \gamma G(\phi\eta)] f''(k) \frac{\partial k}{\partial \phi} \\ &\quad - \mu q\eta f'(k) \pi'(\phi\eta) - \mu \eta \gamma f'(k) g(\phi\eta) \end{aligned} \quad (6.6)$$

Now substitute (5.12) into (6.6) to get

$$\begin{aligned}
W'(\phi) = & \mu f'(k) [\hat{z}q - q\pi(\phi\eta) - \gamma G(\phi\eta)] \left\{ 1 + \frac{k f''(k)}{f'(k)} \right\} \frac{1}{k} \frac{\partial k}{\partial \phi} \\
& - \bar{U} \frac{\partial \mu}{\partial \phi} - \mu \eta q f'(k) \pi'(\phi\eta) - \mu \eta \gamma f'(k) g(\phi\eta) \\
& + \mu f'(k) [\hat{z}q - q\pi(\phi\eta) - \gamma G(\phi\eta)] \left[ \frac{\eta \left( \frac{\gamma}{q} \right) g(\phi\eta)}{\hat{z} - \left( \frac{\gamma}{q} \right) G(\phi\eta)} \right]
\end{aligned} \tag{6.7}$$

Collecting terms in (6.7) gives

$$\begin{aligned}
W'(\phi) = & \left[ 1 + \frac{k f''(k)}{f'(k)} \right] \mu f'(k) [\hat{z}q - q\pi(\phi\eta) - \gamma G(\phi\eta)] \frac{1}{k} \frac{\partial k}{\partial \phi} - \bar{U} \frac{\partial \mu}{\partial \phi} \\
& - \mu q f'(k) \eta \pi'(\phi\eta) - \mu f'(k) \gamma \eta g(\phi\eta) \left\{ 1 - \frac{\hat{z} - \pi(\phi\eta) - \left( \frac{\gamma}{q} \right) G(\phi\eta)}{\hat{z} - \left( \frac{\gamma}{q} \right) G(\phi\eta)} \right\}
\end{aligned} \tag{6.8}$$

Since  $\pi'(\eta) = 0$ , since

$$1 > \frac{\hat{z} - \pi(\phi\eta) - \left( \frac{\gamma}{q} \right) G(\phi\eta)}{\hat{z} - \left( \frac{\gamma}{q} \right) G(\phi\eta)}$$

holds, and since

$$\frac{k f''(k)}{f'(k)} > -1,$$

and  $\frac{\partial k}{\partial \phi} |_{\phi=1} < 0$  both hold, we therefore have  $W'(1) < 0$  if  $\bar{U}$  is not large. Or, to summarize, small binding loan rate ceilings improve the steady state welfare of a borrowers if the value of a borrowers' external option,  $\bar{U}$ , is not too large.

If we are willing to impose additional assumptions on the production technology, then it is possible to show that  $W'(1) < 0$  holds, independently of the magnitude  $\bar{U}$ , so long as (3.12) and (5.15) are satisfied. In particular, the following result is proved in the appendix.

**Proposition 6.1.** *Suppose that the production function has the Cobb -Douglas form  $f(k) = Bk^\beta$ , with  $\beta \in (0, 1)$ , and suppose that (3.12) and (5.15) hold. Then  $W'(1) < 0$ .*

Thus binding loan rate ceilings will increase borrower expected utility, in a steady state, if the final goods production technology is Cobb-Douglas, and if such ceilings reduce credit rationing, in a steady state.

### 6.1. Discussion

We have now established that a small binding interest rate ceiling on loans increases steady state capital accumulation, and reduces steady state credit rationing under quite plausible conditions. In addition, such loan rate ceilings increase the steady state welfare of lenders if the economy's savings rate is not too large. And, they increase the steady state welfare of borrowers if their utility is not too high when they fail to receive funding. Or alternatively, interest rate ceilings on loans raise the steady state welfare of borrowers if the final goods production technology is Cobb-Douglas, and if such ceilings reduce credit rationing, in a steady state

Thus, if credit is rationed, it is quite plausible that bankruptcy (state verification costs) are socially excessive, from the perspective of steady state welfare. These bankruptcy costs can be reduced, and steady state welfare can be improved by the use of interest rate ceilings.

### 6.2. Steady State Capital Underaccumulation

It is well-known that, in overlapping generations models, there can be capital underaccumulation, from a steady state perspective. In conventional overlapping generations models, capital will be underaccumulated, in a steady state, if the real rate of interest on savings exceeds the steady state real rate of growth.

It follows that a natural question, in our context, is whether any or all of the results stated thus far depend on there being capital underaccumulation [ $\pi(\eta)f'(k) > 1$ ] in a steady state? Or, in other words, do the conditions under which  $V'(1) < 0$  and  $W'(1) < 0$  imply capital underaccumulation?

A steady state with  $\phi = 1$  has

$$\begin{aligned} kf'(k) &= s(.) \left[ \hat{z}q - \left( \frac{\gamma}{q} \right) G(\eta) \right] f'(k) \\ &= s(.) \left[ \frac{\hat{z} - \left( \frac{\gamma}{q} \right) G(\eta)}{\pi(\eta)} \right] \pi(\eta)f'(k) \end{aligned} \tag{6.9}$$

Thus  $\pi(\eta)f'(k) \leq 1$  holds in a steady state with  $\phi = 1$  iff

$$\frac{k f'(k)}{f(k)} \leq \left[ \frac{s}{f(k)} \right] \left[ \frac{\hat{z} - \left( \frac{\gamma}{q} \right) G(\eta)}{\pi(\eta)} \right] \quad (6.10)$$

But clearly (6.10) does not imply the violation of any of our previously stated conditions since  $\hat{z} - \left( \frac{\gamma}{q} \right) G(\eta) > \pi(\eta)$  holds. Thus a binding loan rate ceiling can raise the steady state utility of all agents, even if there is capital overaccumulation [ $\pi(\eta)f'(k) < 1$ ] in the steady state.

## 7. Conclusions

During their process of economic development, many currently developed economies had a history of legal ceilings on loan rates of interest. A particularly common manifestation of such ceilings took the form of usury laws. In modern times, a number of developing economies have also imposed an array of interest rate controls.

It is natural to suspect that legal controls on interest rates interfere with the market allocation of funds and, therefore, that they have largely negative consequences for an economy. However, we have presented a model in which, under empirically plausible conditions, legal ceilings on rates of interest that can be charged on loans a) promote steady state capital accumulation; b) decrease the rationing of credit, in a steady state; and c) increase the steady state (expected) utility of all agents.

The intuition underlying these results is simple. In economies where unregulated market rates of interest are quite high - as might transpire when credit is rationed - it will be relatively difficult for borrowers to repay loans. As a consequence, bankruptcy rates will also be high. Since bankruptcies generate social as well as private costs, it is possible that a socially excessive quantity of resources is expended on bankruptcy proceedings; monitoring, a verification of asset values, etc. If ceilings are imposed on loan rates of interest, bankruptcy rates can be reduced and a resource saving can result. Some of the resources saved can be redirected to capital formation. And, as the capital stock rises, so will incomes and savings (if the interest elasticity of savings is not too high). As a result, credit rationing will become less severe. This, along with lower rates of interest, is the source of the expected utility gains for borrowers that arise from interest rate regulation.

It bears emphasis that none of our results depends in any way on the magnitude of bankruptcy costs - the verification cost parameter  $\gamma$  plays no particular role in any of the results described above. It is also the case that - with two possible exceptions - we have made no special assumptions to obtain our results. In particular, we have used an absolutely conventional model of capital accumulation [Diamond (1965)], and an absolutely conventional model of bankruptcy costs: the costly state verification model of Townsend (1979), Diamond (1984), Gale and Hellwig (1985), and Williamson (1986, 1987).

Our analysis does make use of the two special features. First, credit is rationed. This feature of the model does not seem central to our results, although it does substantially simplify their proofs. Moreover, credit rationing seems to be a common characteristic of credit markets, particularly in development contexts, and so it is not an empirically implausible aspect of the analysis.

Second, we have assumed that state verification uses capital. We note first that this is a common assumption in the relevant literature.<sup>5</sup> Moreover, it too is not essential to our results. The main point is that resources are freed when bankruptcy costs are reduced. We do not believe that it matters whether it is capital or labor that is freed, or a combination of both, although the assumption that state verification consumes capital substantially simplifies the analysis. What does matter is that the factors of production freed by lower bankruptcy rates can be redirected to other, socially more productive uses. Hence the potential benefits of legal limitations on loan rates of interest.

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<sup>5</sup>See for instance, Bernanke and Gertler (1989) or Boyd and Smith (1997, 1998).

## 8. Appendix: Proof of Proposition 6.1.

Equation (6.6) implies that

$$W'(1) = \left\{ qf'(k) \left[ \hat{z} - \pi(\eta) - \left( \frac{\gamma}{q} \right) G(\eta) \right] - \bar{U} \right\} \frac{\partial \mu}{\partial \phi} \Big|_{\phi=1} \quad (8.1)$$

$$- \mu q \eta f'(k) \pi'(\eta) - \mu \left\{ \eta \gamma f'(k) g(\eta) - q \left[ \hat{z} - \pi(\eta) - \left( \frac{\gamma}{q} \right) G(\eta) \right] f''(k) \frac{\partial k}{\partial \phi} \Big|_{\phi=1} \right\}$$

It follows from proposition 5.2 that, if (5.15) is satisfied, then  $\frac{\partial \mu}{\partial \phi} \Big|_{\phi=1} < 0$  holds. In addition, it follows from  $\pi'(\eta) = 0$  and

$$f'(k) q \left\{ \hat{z} - \pi(\eta) - \frac{\gamma}{q} G(\eta) \right\} \geq \bar{U}$$

that

$$W'(1) \leq -\mu \left\{ \eta \gamma f'(k) g(\eta) - q \left[ \hat{z} - \pi(\eta) - \left( \frac{\gamma}{q} \right) G(\eta) \right] f''(k) \frac{\partial k}{\partial \phi} \Big|_{\phi=1} \right\},$$

if (5.15) is satisfied. Therefore,  $W'(1) < 0$  holds if the hypotheses of the proposition are satisfied, and if

$$\frac{\eta \left( \frac{\gamma}{q} \right) g(\eta)}{\hat{z} - \pi(\eta) - \left( \frac{\gamma}{q} \right) G(\eta)} > \left[ \frac{k f''(k)}{f'(k)} \right] \frac{\phi}{k} \frac{\partial k}{\partial \phi} \Big|_{\phi=1}. \quad (8.2)$$

Then, upon substituting (5.9) into (8.2), it follows that  $W'(1) < 0$  holds if

$$\left[ \frac{\hat{z} - \left( \frac{\gamma}{q} \right) G(\eta)}{\hat{z} - \pi(\eta) - \left( \frac{\gamma}{q} \right) G(\eta)} \right] \left\{ 1 - \left[ \frac{ws_1(\cdot)}{s(\cdot)} \right] \left[ \frac{k w'(k)}{w(k)} \right] - \left[ \frac{s_2(\cdot)}{s(\cdot)} \right] \pi(\eta) k f''(k) \right\} > \left[ \frac{k f''(k)}{f'(k)} \right]. \quad (8.3)$$

Now with  $f(k) = Bk^\beta$ ,  $\frac{k w'(k)}{w(k)} = \beta$  and  $\left[ \frac{k f''(k)}{f'(k)} \right] = (1 - \beta)$ . It is then apparent that (8.3) holds if

$$1 - \alpha \left[ \frac{ws_1(\cdot)}{s(\cdot)} \right] \geq 1 - \alpha. \quad (8.4)$$

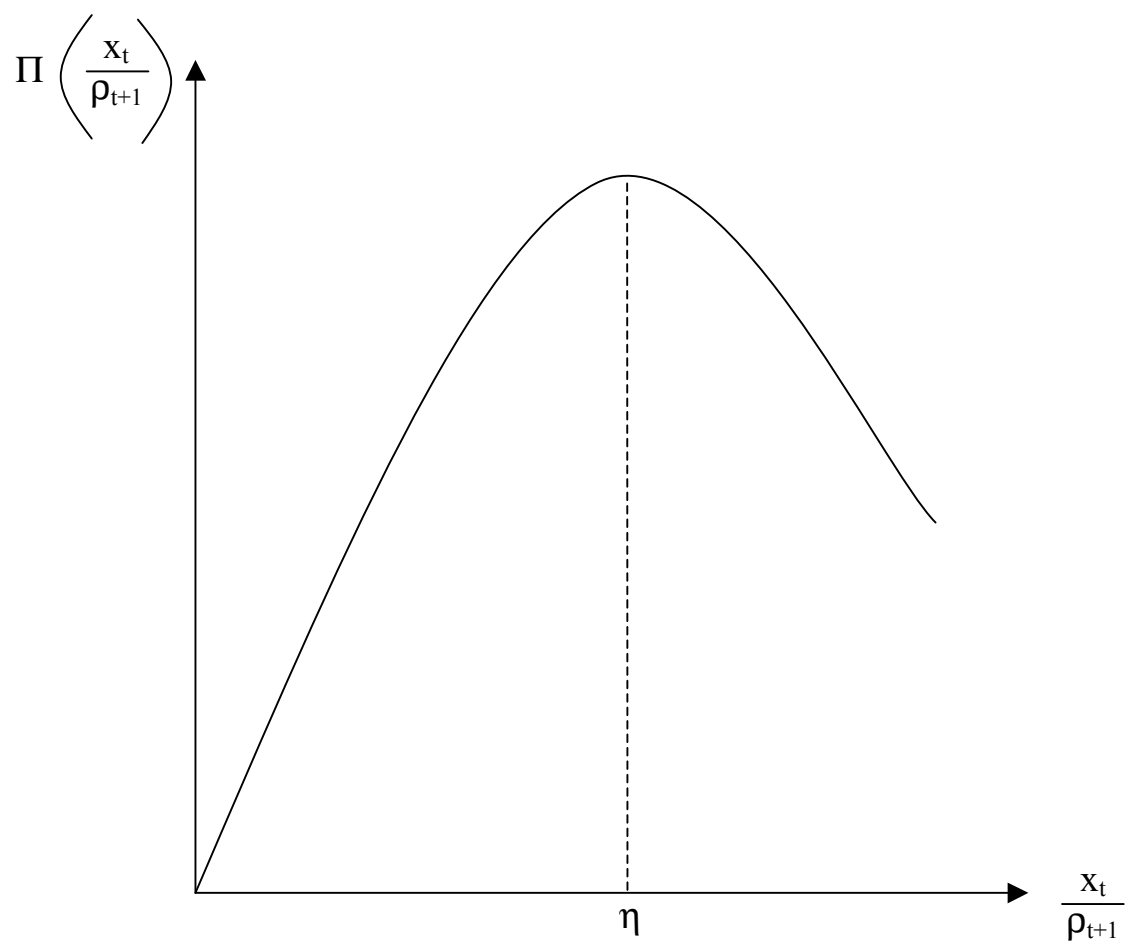
But this follows from assumption 2. This establishes the claim.  $\square$



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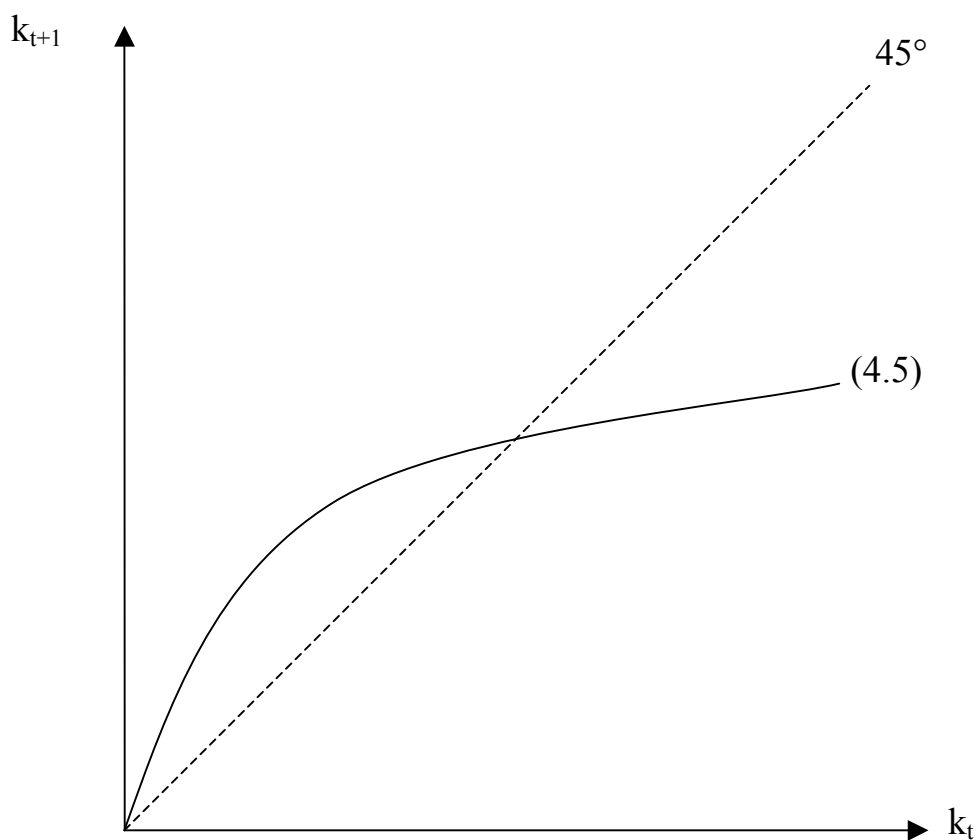
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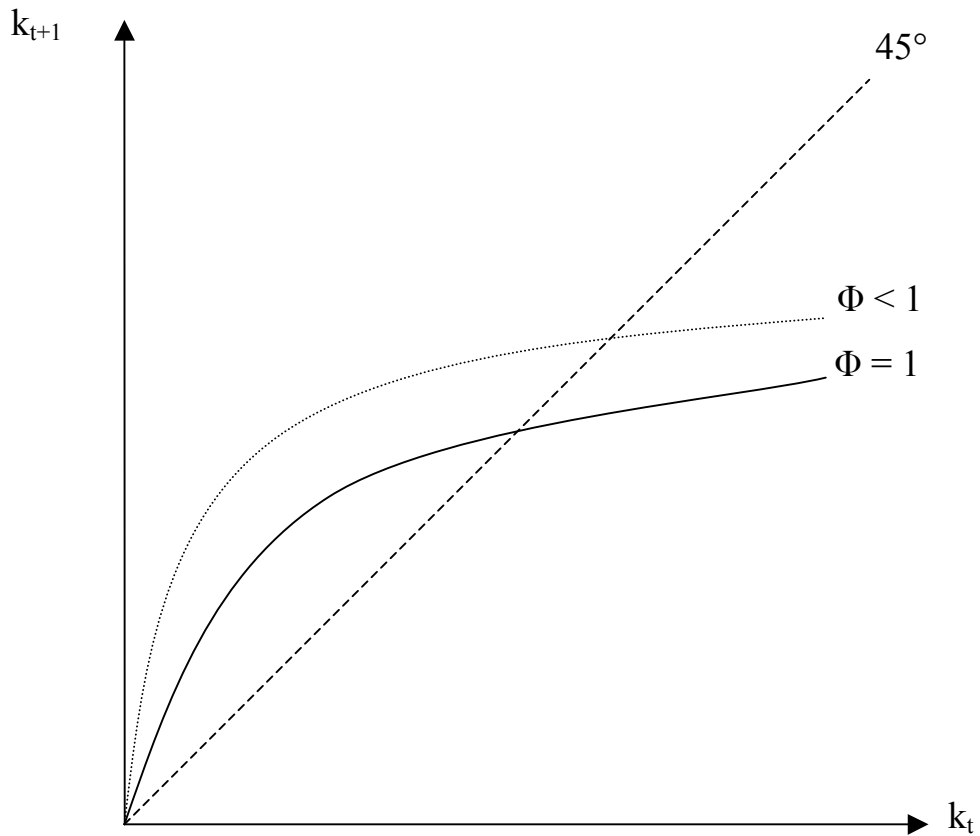
**FIGURE 1**

**THE EXPECTED RETURN FUNCTION**



**FIGURE 2**

**LAW OF MOTION FOR THE CAPITAL STOCK: NO  
INTEREST RATE CONTROLS**



**FIGURE 3**

**LAW OF MOTION FOR THE CAPITAL STOCK  
WITH AND WITHOUT INTEREST RATE  
CONTROLS**