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**Nominal versus Real Wage Rigidities: A Bayesian Approach**

Pau Rabanal and Juan F. Rubio-Ramírez

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## **Nominal versus Real Wage Rigidities: A Bayesian Approach**

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**Abstract:** This paper explores the capability of a dynamic stochastic general equilibrium model with staggered price setting and real wage rigidities to fit the data with reasonable average durations of price and wage contracts. The authors implement a Bayesian approach for parameter estimation and for model comparison with other models that only incorporate nominal rigidities. Their main results can be summarized as follows: First, the authors find that, on average, prices are fixed for three quarters, nominal wages are fixed for five quarters, and half of the wage setters follow a real wage indexing rule of thumb. Second, when the authors remove real wage rigidities and reestimate the model, the parameter on price duration increases. Hence, the lack of endogenous persistence due to real wage rigidities is substituted by a high degree of price stickiness. Third, the authors find little evidence of backward-looking behavior in price inflation. Finally, using the marginal likelihood as a comparison criterion, their model performs best.

JEL classification: C11, C15, E31, E32

Key words: nominal rigidities, real rigidities, Bayes factors, model comparison

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# Nominal versus Real Wage Rigidities: A Bayesian Approach

## 1 Introduction

Dynamic general equilibrium models with staggered price and wage setting have become increasingly important in the analysis of monetary policy. However, models with nominal rigidities do not generate the persistence in inflation, output and real wages that we observe in the data unless implausible levels of nominal rigidity or some kind of backward looking behavior are assumed<sup>1</sup>. Assuming a high degree of price stickiness is at odds with available evidence, while introducing some kind of “inflation stickiness” is troublesome from a theoretical point of view.

The focus of this paper is to study the role of sticky real wages in explaining persistence, without having to rely on backward looking behavior, either in price or in nominal wage inflation. As Galí and Gertler (1999) point out, in a pure forward looking theory of price setting, inflation persistence is driven by the sluggish adjustment of real marginal costs. Since inflation is a discounted stream of real marginal costs, dampening the behavior of the real marginal cost will dampen the reaction of inflation, generating more persistence. From our point of view, a natural question emerges: does a pure forward looking model with real wage rigidities generate enough inflation persistence while assuming plausible levels of nominal rigidity? In this paper we address this question.

In the first part of the paper, we introduce real wage rigidities in a sticky price model. We build a model with staggered price and wage setting, assuming that prices and wages are changed at random intervals, as in Calvo (1983). In addition, we assume that a fraction of wage setters follow a rule of thumb whenever they are allowed to reset their wage. We

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<sup>1</sup>Fuhrer and Moore (1995) show that a sticky price model generates persistence in the price level but in the inflation rate. Chari, Kehoe and McGrattan (2000) point out that models with nominal rigidities do not generate enough persistence in output. Rabanal (2001) stresses the importance of real wage rigidities to match the behavior of real wages and inflation with the cycle.

consider an indexing rule that takes into account average real wages, the price inflation rate and the average duration of wage contracts. We argue that this type of indexing rule, in addition to staggered price and wage setting, is an important feature of the data.

In the second part, we use a Bayesian approach to perform both parameter estimation and model comparison. On the estimation side, we combine priors and the likelihood function to obtain the posterior distribution of the structural parameters. We face two problems when applying a Bayesian approach to estimate dynamic equilibrium economies: the likelihood function is not tractable analytically and it is not possible to obtain a closed form expression for the posterior distribution. To solve these two problems, we use the Kalman filter to evaluate the likelihood function of a linear approximation<sup>2</sup> of the model and a numerical algorithm (the Metropolis-Hastings algorithm) to draw from the posterior distribution. Then, we use the marginal likelihood to compare the explanatory power of our model with three other models that only incorporate nominal rigidities. In particular, we compare our model to: i) a model with staggered price setting, ii) a model with staggered price setting and backward looking behavior in inflation, and iii) a model with staggered price and wage setting.<sup>3</sup> In doing so, we are able to see how much each rigidity helps in explaining the data.

This paper improves upon the existing literature for several reasons. On the theoretical front, our rule of thumb in real wage setting does not imply backward looking behavior, all wages and prices are reset at random intervals, and we do not depart from a forward looking specification for price inflation. By contrast, Christiano, Eichenbaum and Evans (2001) assume full indexation to lagged inflation within the Calvo lottery for both price and wage setters. Their assumption implies that all prices and wages are reset every period, either optimally or not, and leads to different wage and price dynamics than our model.

On the empirical side, our econometric strategy stands in contrast to other estimation

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<sup>2</sup>See Fernández-Villaverde and Rubio-Ramírez (2001b) for a method to compute the likelihood in the nonlinear case.

<sup>3</sup>A pure forward looking model can be found in Calvo (1983) or Yun (1996). Galí, Gertler and López-Salido (2001) introduce backward looking behavior in inflation. Erceg, Henderson and Levin (2000) develop a model with staggered price and wage setting.

procedures used in the literature. Estimation of reduced form equations or partial equilibrium models suffers from identification problems (see Leeper and Zha (2000)). GMM and maximum likelihood estimation performs poorly in small samples<sup>4</sup>. Estimation of general equilibrium models by minimizing the distance between a structural VAR and the models' predicted impulse responses relies on the identification scheme of the VAR<sup>5</sup>, and does not follow the likelihood principle (see Berger and Wolpert, 1998).

Using a Bayesian approach to parameter estimation has several advantages: it outperforms frequentist approaches in small sample, it takes into account all the implications of the likelihood function, and the beliefs of the researcher about the parameters are introduced by specifying the priors. In addition, Fernández-Villaverde and Rubio-Ramírez (2001a) show that, even in the case of misspecified models, Bayesian estimation and model comparison are consistent. In relation to this last point, and given the amount of plausible competing theories to explain certain features of the data, we would like to remark upon the importance of the marginal likelihood as a tool to perform systematic model comparisons. In this respect, we are not aware of any work comparing between different New Keynesian model specifications.

The main results of our paper are as follows: First, the model presented in this paper delivers reasonable point estimates for the duration of prices and wages, of three and five quarters, respectively. The fraction of wage setters that follow a real wage indexing rule is about one half. Second, when we remove the real wage indexing rule and reestimate the model, the duration of prices increases to a higher value. Third, when we estimate a model that introduces backward looking behavior in the price inflation equation, we find that such behavior is not quantitatively important. Finally, using the marginal likelihood of the data implied by each model, the model presented in this paper performs best.

The remainder of the paper is organized as follows: in Section 2, we present a sticky

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<sup>4</sup>See the articles in the special number of the Journal of Business and Economics Statistics, July 1996

<sup>5</sup>Christiano, Eichenbaum and Evans (2001) and Rotemberg and Woodford (1998) follow this estimation strategy. See Uhlig (1999) for a general criticism of structural VAR's.

price model that allows for real wage rigidities. In Section 3 we present the dynamics of the model. In Section 4 we briefly discuss the other models used for comparison. In Section 5 we explain the Bayesian estimation procedure. In Section 6 we present the main findings, leaving Section 7 for directions of future work.

## 2 A Stylized Model with Price and Wage Rigidities

In this section, we present a dynamic stochastic general equilibrium model, with monopolistic competition in the goods and labor markets, in the spirit of Blanchard and Kiyotaki (1987). The model consists of: i) a continuum of infinitely lived households, indexed by  $j \in [0, 1]$ , each of them selling a type of labor that is an imperfect substitute of the other types, ii) a continuum of intermediate good producers, indexed by  $i \in [0, 1]$ , each producing a specific good that is imperfect substitute for the other goods, and iii) a continuum of competitive final good producers. As a result, each household and each intermediate good producer faces a downward sloping demand for their type of labor or good.

Both households and intermediate goods producers face restrictions in the wage and price setting process. We assume that both prices and wages are set with a Calvo type restriction. All intermediate good producers follow an optimal rule whenever they are allowed to reset their price. Households differ in the way they reset wages: a fraction  $\alpha$  follows a rule of thumb in wage setting, while the remaining fraction  $1 - \alpha$  follows an optimal rule. Prices and wages do not change for those not receiving the Calvo signal. Households have access to complete markets and the government sets a lump sum scheme such that we can abstract from distributional issues between those households who reset wages optimally and those who don't.

In every period, the economy experiences one of finitely many events  $s_t$ . We denote by  $h_t = \{s_0, s_1, \dots, s_t\}$  the history of events up to time  $t$ . Let  $\pi(h_{t+\tau})$  be the probability of event  $h_{t+\tau}$ . The initial realization  $s_0$  is given. Two types of exogenous shocks are considered: a technology shock and a monetary shock.

## 2.1 Preferences and Technology

Household  $j$  maximizes the following lifetime utility function, which is separable in consumption, hours, real money balance holdings, and time:

$$\sum_{t=0}^{\infty} \sum_{h_t} \beta^t \pi(h_t) \left[ \frac{C(h_t, j)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \frac{v}{1-\xi} \frac{M(h_t, j)^{1-\xi}}{P(h_t)} - \frac{N(h_t, j)^{1+\gamma}}{1+\gamma} \right] \quad (1)$$

where  $0 < \beta < 1$  is the discount factor,  $\sigma$  is the elasticity of intertemporal substitution,  $\xi > 1$  is the elasticity of money holdings, and  $\gamma > 0$  is the inverse of the elasticity of labor supply.  $C(h_t, j)$  denotes the consumption of the final good and  $\frac{M(h_t, j)}{P(h_t)}$  denotes holdings of real balances by household  $j$ .  $P(h_t)$  is the price of the final good.  $N(h_t, j)$  is total hours worked by household  $j$ . The constant  $v > 0$  measures the importance of real money balances in the utility function.

Household  $j$ 's resource constraint is given by:

$$P(h_t)C(h_t, j) + M(h_t, j) - M(h_{t-1}, j) + \sum_{\tau=1}^{\infty} \sum_{h_{t+\tau}|h_t} Q(h_{t+\tau}|h_t)D(h_{t+\tau}, j) + \quad (2)$$

$$\frac{B(t+1, j)}{1+R(h_t)} = W(h_t, j)N(h_t, j) + \Pi(h_t, j) + T(h_t, j) + D(h_t, j) + B(t, j)$$

where  $\Pi(h_t, j)$  is an aliquot from firms' profits, since households own firms, and  $T(h_t, j)$  are nominal transfers from the government (or lump-sum taxes paid to the government).  $D(h_{t+\tau}, j)$  denotes holdings of a bond that pays one dollar at time  $t + \tau$  if event  $h_{t+\tau}$  occurs and zero otherwise. Its associated price is  $Q(h_{t+\tau}|h_t)$ .  $B(t+1, j)$  denotes holdings of an uncontingent bond that pays one dollar at time  $t + 1$ . Its associated price is the inverse of the nominal interest rate,  $\frac{1}{1+R(h_t)}$ .  $W(h_t, j)$  is the hourly nominal wage earned by household  $j$ .

Intermediate goods are produced using the following production function:

$$Y(h_t, i) = A(h_t) \left\{ \left[ \int_0^1 N(h_t, i, j)^{\frac{\phi-1}{\phi}} dj \right]^{\frac{\phi}{\phi-1}} \right\}^{1-\delta} \quad (3)$$



$A(h_t)$  is a technology factor, which is common to the whole economy, and  $N(h_t, i, j)$  is the amount of hours of type  $j$  labor used by intermediate good producer  $i$ .  $\phi > 1$  is the elasticity of substitution between different types of labor. The production function is concave in labor, and we neglect the dynamics of capital.

The final good is produced using intermediate goods with the following production function:

$$Y(h_t) = \left[ \int_0^1 Y(h_t, i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (4)$$

where  $\varepsilon > 1$  is the elasticity of substitution between intermediate goods.

## 2.2 The Final Good Producers Problem

Final good producers are competitive and maximize profits:

$$P(h_t)Y(h_t) - \int_0^1 P(h_t, i)Y(h_t, i)di$$

subject to the production function (4), taking as given all intermediate goods prices  $P(h_t, i)$  and  $P(h_t)$ , which is the price of the final good. The input demand functions associated with this problem are:

$$Y(h_t, i) = \left[ \frac{P(h_t, i)}{P(h_t)} \right]^{-\varepsilon} Y(h_t)$$

for every  $i$ . Imposing the zero profit condition in the final good sector delivers the following expression for the price of the final good:

$$P(h_t) = \left[ \int_0^1 P(h_t, i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \quad (5)$$

## 2.3 The Intermediate Goods Producers Problem and Price Setting

Intermediate goods producers operate in a monopolistic competition environment. However, they face a Calvo-type restriction when setting their price: they can only reset their price

whenever they receive a stochastic signal to do so. This signal is received with probability  $1 - \theta_p$ , and is independent across intermediate goods producers and past history of signals. This assumption implies that, on average, firms keep their prices fixed for  $\frac{1}{1-\theta_p}$  periods.

Given this price setting restriction, the profit maximization problem of the intermediate goods producers is divided in two stages. In the first stage, given all wages, firms choose  $\{N(h_t, i, j)\}_{j \in (0,1)}$  to obtain the optimal labor mix. Then, the demand of producer  $i$  for type of labor  $j$  is

$$N(h_t, i, j) = \left[ \frac{W(h_t, j)}{W(h_t)} \right]^{-\phi} \left[ \frac{Y(h_t, i)}{A(h_t)} \right]^{\frac{1}{1-\delta}} \quad (6)$$

where the aggregate wage  $W(h_t)$  is expressed as<sup>6</sup>:

$$W(h_t) = \left[ \int_0^1 W(h_t, j)^{1-\phi} dj \right]^{\frac{1}{1-\phi}} \quad (7)$$

In the second stage, whenever each intermediate goods producer receives the “green light” signal, she chooses the optimal price that maximizes the present valued profit:

$$\sum_{\tau=0}^{\infty} \sum_{h_{t+\tau}|h_t} \theta_p^\tau Q(h_{t+\tau}|h_t) \left\{ P(h_t, i) \bar{Y}(h_{t+\tau}, i) - W(h_t) \left[ \frac{Y(h_t, i)}{A(h_t)} \right]^{\frac{1}{1-\delta}} \right\}$$

subject to the demand for their type of product at  $h_{t+\tau}$  assuming that the price set optimally at  $h_t$  still holds,

$$\bar{Y}(h_{t+\tau}, i) = \left[ \frac{P(h_t, i)}{P(h_{t+\tau})} \right]^{-\varepsilon} Y(h_{t+\tau})$$

$\theta_p^\tau$  is the probability that the optimal price posted at  $t$  is still fixed at  $t + \tau$ . The previous expression is a measure of the discounted stream of profits that would arise in case the firm is not allowed to reset its price, weighted by the corresponding probability.

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<sup>6</sup>We define the aggregate nominal wage as

$$W(h_t, i) \left[ \frac{Y(h_t, i)}{A(h_t)} \right]^{\frac{1}{1-\delta}} = \int_0^1 W(h_t, j) N(h_t, i, j) dj$$

It can be shown that  $W(h_t, i) = W(h_t)$ , for all  $i$  holds.

The first order condition associated with this problem is:

$$\sum_{\tau=0}^{\infty} \sum_{h_{t+\tau}|h_t} \theta_p^\tau Q(h_{t+\tau}|h_t) \left\{ \left[ \frac{P^*(h_t, i)}{P(h_{t+\tau})} - \mu \overline{MC}(h_{t+\tau}, i) \right] \overline{Y}(h_{t+\tau}, i) \right\} = 0 \quad (8)$$

where  $\mu = \frac{\varepsilon}{\varepsilon-1}$  is the steady state mark-up of prices on the real marginal cost of production,  $\overline{MC}(h_{t+\tau}, i)$ . It can be verified that:

$$\overline{MC}(h_{t+\tau}, i) = \frac{W(h_{t+\tau}) \left[ \frac{\overline{Y}(h_{t+\tau}, i)}{A(h_{t+\tau})} \right]^{\frac{\delta}{1-\delta}}}{(1-\delta) P(h_{t+\tau})} \quad (9)$$

Then, in every period,  $1 - \theta_p$  of the intermediate goods producers set  $P^*(h_t, i)$  as their price policy function, while the remaining  $\theta_p$  do not reset their price at all.

## 2.4 The Households Problem and Wage Setting

Every household chooses  $C(h_t, j)$ ,  $D(h_{t+\tau}, j)$ ,  $B(t+1, j)$ ,  $\frac{M(h_t, j)}{P(h_t)}$  and  $W(h_t, j)$ , to maximize (1), subject to (2) and (6). Households also have market power and face a Calvo-type restriction when setting wages. Households differ in the way they reset their wages whenever they are allowed to do so: a proportion  $\alpha$  of households follow a rule of thumb in wage setting, while the remaining fraction  $1 - \alpha$  set their wages optimally. The rule of thumb consists in an indexing rule that takes into account average real wages, price inflation and the average duration of wage contracts. As noted before, we can abstract from distributional issues and separate the consumption/savings and wage setting decisions.

### 2.4.1 Consumption/Savings Decision

The first order conditions with respect to holdings of uncontingent and contingent assets, and money are:

$$C(h_t, j)^{-\frac{1}{\sigma}} = \beta \sum_{h_{t+1}|h_t} \pi(h_{t+1}|h_t) \left\{ C(h_{t+1}, j)^{-\frac{1}{\sigma}} [(1 + R(h_t)) \frac{P(h_t)}{P(h_{t+1})}] \right\} \quad (10)$$

$$Q(h_{t+\tau}|h_t) = \beta^\tau \pi(h_{t+\tau}|h_t) \frac{C(h_{t+\tau}, j)^{-\frac{1}{\sigma}}}{C(h_t, j)^{-\frac{1}{\sigma}}} \frac{P(h_t)}{P(h_{t+\tau})}, \quad \tau = 0, 1, 2, \dots$$

where  $\pi(h_{t+\tau}|h_t) = \frac{\pi(h_{t+\tau})}{\pi(h_t)}$  is the conditional probability of  $h_{t+\tau}$  given  $h_t$ . We neglect the money demand equation because we assume that the monetary authority uses the nominal risk free interest rate as a monetary policy tool.

### 2.4.2 Wage Setting Process

Households face a Calvo type restriction in wage setting, meaning that they can only reset their wage with probability  $(1 - \theta_w)$  every period, and they differ in the way they reset their wage when they are allowed to do so. If household  $j$  belongs to the fraction  $\alpha$  that follows the real wage indexing rule, her wage evolves as:

$$W^{rt}(h_t) = W(h_{t-1}) \left[ \frac{P(h_t)}{P(h_{t-1})} \right]^{\frac{1}{1-\theta_w}} \quad (11)$$

when she receives the “green light” signal.

Note that this rule of thumb implies that households following it catch up with the average nominal wage prevailing the previous period, and they update it by the inflation rate, taking into account the average duration between wage changes,  $\frac{1}{1-\theta_w}$ . For instance, if wages are set for an average duration of one year, the real wage indexing rule says that the worker gets last period’s nominal wage times the annualized inflation rate.

If household  $j$  belongs to the fraction  $1 - \alpha$  of households that set their wages optimally, her wage equals  $W^*(h_t, j)$  when she receives the “green light” signal:

$$\sum_{\tau=0}^{\infty} \sum_{h_{t+\tau}|h_t} (\beta\theta_w)^\tau \left\{ \left[ C(h_{t+\tau})^{-\frac{1}{\sigma}} \frac{W^*(h_t, j)}{P(h_{t+\tau})} - \vartheta \bar{N}(h_{t+\tau}, j)^\gamma \right] \bar{N}(h_{t+\tau}, j) \right\} = 0 \quad (12)$$

where

$$\bar{N}(h_{t+\tau}, j) = \left( \frac{W^*(h_t, j)}{W(h_{t+\tau})} \right)^{-\phi} \int_0^1 \left( \frac{Y(h_t, i)}{A(h_t)} \right)^{\frac{1}{1-\delta}} di$$

is the demand at  $t+\tau$  assuming that the wage set optimally at  $t$  still holds, and  $\vartheta = \frac{\phi}{\phi-1}$  is the steady state mark-up of real wages on the marginal rate of substitution between consumption and labor.

Parallel to the intermediate goods problem, the fraction  $1 - \alpha$  of optimizing households minimize the deviation of real wages and the marginal rate of substitution, taking into account the probability of not being able to reset their wage in the near future. In every period, a fraction  $(1 - \theta_w)\alpha$  of households set  $W^{rt}(h_t)$  as their wage policy function, a fraction  $(1 - \theta_w)(1 - \alpha)$  of households set  $W^*(h_t, j)$  as their wage policy function, while the remaining fraction  $\theta_w$  keep their nominal wage fixed. It can be shown that there are no persistent deviations between the rule of thumb behavior and the optimal one.

There are important differences between our wage equation and other models that introduce staggered wage setting. In Erceg, Henderson and Levin (2000), households that do not receive the Calvo signal update nominal wages according to the unconditional price inflation rate. In our case, there is a nominal and a real wage rigidity, such that even if prices are flexible, real wages are not. In Christiano, Eichenbaum and Evans (2001), households that do not receive the “green light” signal have nominal wages indexed to last period’s price inflation rate. Hence, in their model all wages change every period, either optimally or not. By contrast, we assume that households that do not receive the Calvo signal keep their nominal wage fixed.

## 2.5 The Government

The government cannot run deficits or surpluses, so its budget constraint is:

$$\int_0^1 T(h_t, j) dj = M(h_t) - M(h_{t-1})$$

where  $M(h_t)$  is money creation.

As in Taylor (1993), we assume that the monetary authority conducts monetary policy using the nominal interest rate:

$$R(h_t) = \Psi(h_t, \varepsilon_t^m) \tag{13}$$

where  $\varepsilon_t^m$  is a monetary shock.

## 2.6 The Equilibrium

In equilibrium, all markets clear, consumers maximize utility, and intermediate and final goods producers maximize profits. We restrict our attention to the symmetric equilibrium.

Thus, using (5) the price of the final good evolves as:

$$P(h_t) = [\theta_p P(h_{t-1})^{1-\varepsilon} + (1 - \theta_p) P^*(h_t)^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \quad (14)$$

Similarly, using (7), the aggregate nominal wage evolves as

$$W(h_t) = [\alpha (1 - \theta_w) W^{rt}(h_t)^{1-\phi} + (1 - \alpha) (1 - \theta_w) W^*(h_t)^{1-\phi} + \theta_w W(h_{t-1})^{1-\phi}]^{\frac{1}{1-\phi}} \quad (15)$$

## 3 The Dynamics of the Model

The dynamics of the model are obtained by taking a log-linear approximation of equations (3), and (8) to (15) around a symmetric equilibrium steady state, with zero inflation. The lower case variables denote log-deviations from the steady state value<sup>7</sup>.

The loglinearized resource constraint is simply:

$$y_t = c_t \quad (16)$$

The Euler equation relates consumption growth with the real rate of interest:

$$c_t = E_t c_{t+1} - \sigma(r_t - E_t \Delta p_{t+1}) \quad (17)$$

where  $E_t(\cdot)$  denotes the expectation operator.

The pricing decision of the firm under the Calvo-type restriction delivers the following forward looking equation for price inflation ( $\Delta p_t$ ):

$$\Delta p_t = \beta E_t \Delta p_{t+1} + \kappa_p m c_t \quad (18)$$

where  $\kappa_p = \frac{(1-\delta)(1-\theta_p\beta)(1-\theta_p)}{\theta_p(1+\delta(\varepsilon-1))}$ .

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<sup>7</sup>For every variable  $X(h_t)$ , we define the log linear approximation as  $x_t = \log X(h_t) - \log(X^{SS})$ , where  $X^{SS}$  is the variable's steady state value.

Equation (18) is the so-called “New Keynesian Phillips Curve”, which relates current inflation to expectations of future inflation and to the real marginal cost. It denotes the forward looking behavior of the firms in response to the Calvo-type restriction.

The production function and the real marginal cost of production are:

$$y_t = a_t + (1 - \delta)n_t \quad (19)$$

$$mc_t = w_t - p_t + n_t - y_t \quad (20)$$

The wage setting decision of the workers delivers the following nominal wage growth equation ( $\Delta w_t$ ):<sup>8</sup>

$$\Delta w_t = \frac{\alpha}{\eta_w} \Delta p_t + \frac{\beta \theta_w}{\eta_w} E_t \Delta w_{t+1} - \frac{\alpha}{\eta_w} \beta \theta_w E_t \Delta p_{t+1} + \kappa_w (mrs_t - (w_t - p_t)) \quad (21)$$

where  $\eta_w = \alpha + \theta_w(1 - \alpha)$ , and  $\kappa_w = \frac{(1-\alpha)(1-\theta_w)(1-\beta\theta_w)}{\eta_w(1+\phi\gamma)}$ . The driving force of nominal wage growth is the difference between the marginal rate of substitution and the real wage and expected wage growth (through the optimal wage setters) and current and future inflation (through the rule-of-thumb wage setters).

The marginal rate of substitution between consumption and labor takes the form:

$$mrs_t = \frac{1}{\sigma} c_t + \gamma n_t \quad (22)$$

We choose the following log-linear specification of the Taylor rule:

$$r_t = \rho_i r_{t-1} + (1 - \rho_i) [\gamma_\pi \Delta p_{t-1} + \gamma_y y_{t-1}] + \varepsilon_t^m \quad (23)$$

where  $\gamma_\pi$  and  $\gamma_y$  are the long run responses of the monetary authority to deviations of inflation and output from their steady state values. We include an interest rate smoothing parameter ( $\rho_i$ ) following recent empirical work (as in Clarida, Galí and Gertler, 2000). Finally, the exogenous technology process evolves as:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \quad (24)$$

where  $\rho_a \in [0, 1)$ .

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<sup>8</sup>See Appendix A for a detailed derivation.

## 4 Models Used for Comparison

We use a Bayesian framework to compare the model described in section 2 with other three models that incorporate nominal rigidities. First, we restrict the coefficient on real wage indexing ( $\alpha$ ) to be equal to zero, so we are in fact estimating the staggered price and nominal wage setting model of Erceg, Henderson and Levin (2000, from now on EHL). In that case, the nominal wage growth equation (21) becomes:

$$\Delta w_t = \beta E_t \Delta w_{t+1} + \frac{(1 - \theta_w)(1 - \beta \theta_w)}{\theta_w (1 + \phi \gamma)} [mrs_t - (w_t - p_t)]$$

Second, we remove the assumption of sticky wages, by setting  $\theta_w = 0$ , and we estimate a pure forward looking model for inflation as in Yun (1996, from now on PF). In this case, real wages equal the marginal rate of substitution:

$$w_t - p_t = \frac{1}{\sigma} c_t + \gamma n_t$$

Third, we evaluate the behavior of the RWR model *vis-a-vis* a model that explicitly introduces inflation dynamics in a backward looking way. By doing so we want to study if it is possible to rescue a model of inflation dynamics that does not need to rely in backward looking behavior. Hence, we extend the pure forward looking model with an inflation equation as in Galí, Gertler and López-Salido (2000) (from now on GGLS), where wages are flexible but some price setters follow a rule of thumb in price setting that is backward looking in spirit. Then, the inflation equation (18) becomes:

$$\Delta p_t = \gamma_b \Delta p_{t-1} + \gamma_f E_t \Delta p_{t+1} + \lambda m c_t$$

where:

$$\begin{aligned} \lambda &= \frac{(1 - \omega)(1 - \delta)(1 - \theta_p)(1 - \beta \theta_p)}{\phi [1 + \delta (\varepsilon - 1)]} \\ \gamma_b &= \omega \phi^{-1} \\ \gamma_f &= \beta \theta_p \phi^{-1} \\ \phi &= \theta_p + \omega [1 - \theta_p (1 - \beta)] \end{aligned}$$



In this case,  $\omega$  measures the fraction of price setters that, when they are allowed to reoptimize, follow a pricing rule of thumb that exhibits approximately backward looking indexation<sup>9</sup>.

## 5 Empirical Analysis

In this section, we explain how to estimate the posterior distribution of the structural parameters and the marginal likelihood of the data implied by each model. Fernández-Villaverde and Rubio-Ramírez (2001a) show that the Bayesian method is a powerful tool to estimate and compare misspecified models: the mean of the posterior distribution converges to its pseudotrue value and the “best” model has asymptotically the highest marginal likelihood.

Three main ingredients are needed to implement the Bayesian analysis. The first one consists in the data we want to explain. The second one is the likelihood function of the data implied by the model. The third one consists in specifying priors about the parameters’ distribution.

Our set of observable variables includes real wages, the nominal interest rate, price inflation, wage inflation and output. Note that we use five variables while just having two perturbation errors in the model. As it is common in the literature, this singularity problem<sup>10</sup> is solved by assuming that we observe all variables but the nominal interest rate with error<sup>11</sup>. Then, we make use of the Kalman filter to evaluate the likelihood function.

Given our priors, that will be specified below, we are not able to obtain a closed form for the posterior of the parameters. We use a numerical algorithm, the Metropolis-Hastings algorithm, to obtain a random draw from the posterior of the parameters and estimate the posterior moments and the marginal likelihood.

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<sup>9</sup>For more details, see Galí, Gertler and López-Salido (2000).

<sup>10</sup>This problem arises because, necessarily, some endogenous variables will be linear combinations of the others. This implies that the variance-covariance matrix of the errors is singular and cannot be inverted, and the log-likelihood cannot be evaluated.

<sup>11</sup>We use this specification because otherwise it was not possible to separate the observation error from the monetary shock.

## 5.1 Estimating the Posterior Distribution

Four steps are needed to estimate the posterior distribution. First, we derive the law of motion for the model economy. Second, we compute the likelihood function of the data. Third, we describe the priors distribution of the parameter. And fourth, we sketch how to use the Metropolis-Hastings algorithm to generate a random draw from the posterior distribution.

### 5.1.1 The Law of Motion

Let  $\theta = (\sigma, \theta_p, \theta_w, \beta, \phi, \alpha, \gamma_y, \gamma_\pi, \rho_i, \rho_a, \delta, \varepsilon, \gamma, \sigma_a, \sigma_m)'$  be the vector of parameters that describe the model,  $x_{t+1} = (w_t - p_t, r_t, \Delta p_{t+1}, \Delta w_{t+1}, y_{t+1})'$  be the vector of observable variables,  $z_t = (a_t, \varepsilon_t^m)'$  the unobserved technology process and the monetary innovation, and  $\varepsilon_t = (\varepsilon_t^a, \varepsilon_t^m)'$  the technology and monetary innovations.

The system of equations (16)-(24) can be written the following way

$$A(\theta) E_t x_{t+1} = B(\theta) x_t + C(\theta) x_{t-1} + D(\theta) z_t \quad (25)$$

$$z_t = N(\theta) z_{t-1} + \varepsilon_t, \quad E(\varepsilon_t \varepsilon_t') = \Lambda(\theta) \quad (26)$$

where matrices  $A(\theta), B(\theta), C(\theta), D(\theta), N(\theta)$  and  $\Lambda(\theta)$  are detailed in appendix B. We use standard solution methods for linear models with rational expectations<sup>12</sup> to obtain the following law of motion to the system (25) and (26):

$$x_t = H(\theta) x_{t-1} + R(\theta) z_t$$

$$z_t = N(\theta) z_{t-1} + \varepsilon_t, \quad E(\varepsilon_t \varepsilon_t') = \Lambda(\theta)$$

### 5.1.2 The Likelihood Function

Once the law of motion of the model is obtained, the Kalman filter is used to evaluate the likelihood function of the data conditional on the parameters. We assume that we observe

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<sup>12</sup>For instance, Blanchard and Kahn (1980).

$\hat{x}_t$  instead of  $x_t$  because of some measurement errors:

$$\hat{x}_{t+1} = x_{t+1} + u_{t+1}$$

where  $u_t$  is the measurement error. We assume that observation errors are *iid* normally distributed with zero mean and a diagonal variance-covariance matrix  $\Omega$ . In addition, observation errors are uncorrelated with the model's innovations at all leads and lags:  $E(u_t \varepsilon'_{t+k}) = 0 \forall t, k$ . Let  $\Theta = (\theta, \Omega)$ . Then, the likelihood function of  $\{\hat{x}_t\}_{t=1}^T$ ,  $L(\{\hat{x}_t\}_{t=1}^T | \Theta)$ , can be evaluated via the Kalman filter.

### 5.1.3 Priors

We now proceed to discuss the prior distributions. We present the list of the structural parameters and its associated prior distributions in Table I. The inverse of the elasticity of intertemporal substitution ( $\sigma^{-1}$ ) follows a gamma distribution, which implies a positive support, with mean 2.5 and standard deviation 1.76, which we view as not particularly tight, given the wide variety of estimates for this parameter. We choose a uniform distribution between  $[0, 0.8]$  for  $\theta_p$  and between  $[0, 0.83]$  for  $\theta_w$ . This choice imply a prior maximum duration for prices and wages of 5 and 6 quarters, respectively. We chose these priors after reviewing the informal evidence presented in Taylor (1999). We use a uniform distribution between  $[0, 1)$  for the fraction of workers following the real wage indexation rule. We do not allow for a value of 1 for this last parameter because it would induce nonstationarities in the model.

Regarding the Taylor rule coefficients, since we do not impose non-negativity restrictions, we opt for Normal distributions. We set the mean of  $\gamma_\pi$  to 1.5 and that of  $\gamma_y$  to 0.5, which is Taylor's original guess. The interest rate smoothing coefficient ( $\rho_i$ ) has a uniform prior between  $[0, 1)$ . For the elasticity of substitution between types of labor,  $\phi - 1$  follows a gamma distribution with mean 5 and standard deviation 2.72, so that  $\phi$  has support  $[1, \infty]$ . This assumption implies that the expected prior real wage mark-up is 20%. We use a normal distribution for the inverse of the elasticity of labor supply,  $\gamma$ , centered at 1 and with a

standard deviation of 0.5. For the autorregressive parameter of the technology we choose a uniform between  $[0.5, 1)$ . We opt for a gamma distribution with mean 0.005 and standard deviation 0.0025 for the standard deviation of both structural shocks.

We imposed dogmatic priors over the parameters  $\beta$ ,  $\delta$  and  $\varepsilon$ . Given that we do not consider capital, we had trouble estimating the first two parameters. On the other hand, there is an identification problem between  $\theta_p$  and  $\varepsilon$ <sup>13</sup>. The values used ( $\beta = 0.99$ ,  $\delta = 0.36$  and  $\varepsilon = 6$ ) are quite conventional in the literature. In addition, we censor the support of  $\theta$  to the region where the model has a unique, stable solution. Therefore, we rule out indeterminacies.

We assume that the diagonal elements of  $\Omega$  follow gamma distributions with mean 0.002 and standard deviation 0.001. The exception is the variance of the observation error of the nominal interest rate equation, that we set to zero. Finally, we assume that all priors are independent across parameters. We denote by  $\pi(\Theta)$  the prior joint density function of  $\Theta$ .

#### 5.1.4 Drawing From the Posterior

From Bayes rule, we obtain the posterior distribution of the parameters

$$\hat{\pi}(\Theta | \{\hat{x}_t\}_{t=1}^T) \propto L(\{\hat{x}_t\}_{t=1}^T | \Theta) \pi(\Theta)$$

The posterior density function is proportional to the product of the likelihood function and the prior joint density function of  $\Theta$ . We are not able to obtain a closed form solution for the posterior distribution. First, because the likelihood function is intractable. Second, because the product of the likelihood function and the specified priors is also intractable. However, we are still able to evaluate numerically both expressions. Hence, we use a numerical algorithm, the Metropolis-Hastings algorithm, to obtain a numerical draw  $\{\hat{\Theta}_i\}_{i=1}^N$  from  $\hat{\pi}(\Theta | \{\hat{x}_t\}_{t=1}^T)$ <sup>14</sup>. We use the draw to estimate the moments of the posterior distribution.

The algorithm works the following way. In order to obtain a draw of size  $N$ :

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<sup>13</sup>The slope of the Phillips Curve,  $\kappa_p$ , is the only one containing  $\varepsilon$  and  $\theta_p$ .

<sup>14</sup>See Geweke (1998).

1. We start with an initial value  $\Theta_0$ . From this value, we evaluate  $L\left(\{\hat{x}_t\}_{t=1}^T \mid \Theta_0\right) \pi(\Theta_0)$
2. For each  $i$ ,

$$\hat{\Theta}_i = \begin{cases} \hat{\Theta}_{i-1} & \text{with probability } 1 - R \\ \Theta_i^* & \text{with probability } R \end{cases}$$

where  $\Theta_i^* = \hat{\Theta}_{i-1} + v_i$ ,  $v_i$  follows an iid normal distribution

$$\text{and } R = \min \left\{ 1, \frac{L(\{\hat{x}_t\}_{t=1}^T \mid \Theta_i^*) \pi(\Theta_i^*)}{L(\{\hat{x}_t\}_{t=1}^T \mid \hat{\Theta}_{i-1}) \pi(\hat{\Theta}_{i-1})} \right\}.$$

A standard result from using this numerical algorithm is that, the sample moments of  $\{\hat{\Theta}_i\}_{i=1}^N$  converge to the ones of  $\hat{\pi}(\Theta \mid \{\hat{x}_t\}_{t=1}^T)$  as  $N$  goes to  $\infty$ .<sup>15</sup> Regardless of the starting value,  $\{\hat{\Theta}_i\}_{i=1}^N$  tends to accept more draws from the regions of the parameter space where the posterior density is high. At the same time, areas of the posterior support with low density are less represented in  $\{\hat{\Theta}_i\}_{i=1}^N$ . Note that, since this is not a maximization procedure, we do not need to maximize the likelihood function, but just be able to evaluate it.

## 5.2 Model Comparison

Let  $M$  be the set of models that we wish to compare. In our case,  $M = \{RWR, EHL, GGLS, PF\}$ .

The Bayesian procedure allows us to obtain the marginal likelihood of the data implied by each model  $m \in M$ . Having specified the conditional likelihood function and the prior distribution, the marginal likelihood of each model  $m \in M$  is:

$$L\left(\{\hat{x}_t\}_{t=1}^T \mid m\right) = \int L\left(\{\hat{x}_t\}_{t=1}^T \mid \Theta, m\right) \pi(\Theta \mid m) d\Theta \quad (27)$$

Using the draw  $\{\hat{\Theta}_i\}_{i=1}^N$  from the posterior distribution of each model  $m \in M$ , we follow Geweke (1998) to estimate (27). We denote the estimated marginal likelihood implied by model  $M$  by  $\hat{L}\left(\{\hat{x}_t\}_{t=1}^T \mid m\right)$ . Once we obtain  $\hat{L}\left(\{\hat{x}_t\}_{t=1}^T \mid m\right)$ , we compute Bayes factor between two distinct models  $n$  and  $m$ :

$$\hat{B}_m^n\left(\{\hat{x}_t\}_{t=1}^T\right) = \frac{\hat{L}\left(\{\hat{x}_t\}_{t=1}^T \mid n\right)}{\hat{L}\left(\{\hat{x}_t\}_{t=1}^T \mid m\right)}$$

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<sup>15</sup>See Geyer (1992).

If  $m^* \in M$  is the best model under the Kullback-Leibler distance, then for any other  $n \in M$ ,  $\widehat{B}_{m^*}^n \left( \{\hat{x}_t\}_{t=1}^T \right)$  converges to zero as  $T$  increases.<sup>16</sup>

## 6 Findings

We use data for the United States, at a quarterly frequency, between 1982:01 and 2000:04. The series for output, prices and wages are obtained from the Bureau of Labor Statistics. We use “output for the non farm business sector” as a measure of output, and its associated price deflator as a measure of prices. We use “hourly compensation for the non farm business sector” as nominal wage. Finally, we use the Federal Funds rate as the relevant instrument for monetary policy. This last series is obtained from the FRED data base. We detrend all variables.

### 6.1 Posterior Distributions and Moments

Table II presents mean and the standard deviation of the posterior distributions for the four models. Figures 1-4 display the estimated posterior distributions of the models. We obtain a draw of size 600,000, and we allow for a warm-up period in order to ensure convergence.

As we can see from the first two columns of Table II, the estimated posterior means of the RWR model are quite reasonable for most parameters. We obtain values for the probabilities of the Calvo lotteries that imply that prices are roughly fixed 3 quarters and wages are fixed for 5 quarters. The mean of  $\alpha$  is 0.56, which implies that roughly half of wage setters follow the real wage setting rule of thumb. The coefficient on price inflation of the Taylor rule is estimated to be 0.82, which is slightly lower than what most studies find. However, the coefficients on the output gap and interest rate smoothing of the Taylor rule are quite similar to those reported by Clarida, Galí, and Gertler (2000).

We obtain an estimate for the elasticity of intertemporal substitution of 0.16, which is close to the result reported in Rotemberg and Woodford (1998) and Christiano, Eichenbaum

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<sup>16</sup>See Fernández-Villaverde and Rubio-Ramírez (2001a) for a detailed explanation on this point.

and Evans (2001). We estimate a elasticity of substitution between types of labor that implies a mark-up on real wages of 5%. The estimate on the inverse of the elasticity of labor supply is 1.56 and the posterior has positive support. We also obtain reasonable parameters for the technology shocks, both for the autocorrelation and the standard deviation. The standard deviation of the monetary shock is also plausible, taking into account that we use quarterly data and that the sample period is post-1982. It is important to note that data is quite informative for almost all the parameters of RWR. In all cases, but  $\gamma_y$  and  $\sigma_a$ , the difference between the prior and posterior means is larger than one posterior standard deviation. This means that the prior and the posterior distributions are quite distinct. In Figure 1 we plot the associated posterior distributions<sup>17</sup>. It is important to note that the posterior probability that  $\theta_p$  is larger than 0.75 is close to zero, while the probability that  $\theta_w$  is larger than 0.75 is almost one.

In the third and fourth column of Table II we present the results for the EHL model, and in Figure 2 the associated posterior distributions. In this case, the parameter on the duration of prices increases dramatically, and that on the duration of wages decreases significantly. In particular, prices are fixed almost 5 quarters and wages are fixed for less than 2 quarters. This result indicates that once the partial wage indexation rule is no longer present, we need a large value for the duration of prices to generate enough persistence in inflation. Moreover, we can see in Figure 2 that the posterior probability of  $\theta_p$  being larger than 0.78 is one. This means that the probability that the average duration of prices is lower than 4.7 quarters is basically zero. The estimate for  $\phi$  falls to 4.42, so that the real wage mark up goes up to 30%. The coefficient on price inflation of the Taylor rule increases significantly, up to 1.38, which is a more common value in the empirical literature on interest rate rules. The posterior means for rest of the parameters are similar to those obtained in RWR. In this case, the differences between the prior and posterior means of  $\phi$ ,  $\gamma_\pi$ ,  $\gamma_y$  and  $\sigma_a$  are smaller than one posterior standard deviation, which implies that, given our priors, the data is not

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<sup>17</sup>In order to save space, we only show the posterior distributions for  $\theta_p, \theta_w, \alpha, \gamma_\pi, \gamma_y$  and  $\rho_i$ . The posterior distributions for the rest of the parameters is available at <http://home.nyu.edu/~pr244/research.htm>

so informative about these parameters.

In columns five and six of Table II we present the estimates for the GGLS model, where it is assumed that labor markets are competitive. Hence, we do not estimate  $\theta_w$  and  $\phi$ . As we mentioned previously, in this model  $\omega$  reflects the proportion of price setters that follow a backward-looking rule when setting prices. Hence, it cannot be directly compared to the real wage setting rule of RWR. We now obtain a mean duration of prices of roughly 4.5 quarters. As in the case of EHL, the posterior probability that  $\theta_p$  is between 0.75 and 0.80 is almost one. The estimate on backward looking behavior in inflation is small: we obtain a point estimate of 0.07, and by taking a look at Figure 3 we can see that actually, there is a lot of mass concentrated at zero. This finding is similar to what Galí, Gertler and López-Salido (2000) report: backward looking behavior in inflation is not important. The rest of the posterior means are quite similar to what we obtained before. In this case, and following the same criterion described for RWR and EHL, the data is not so informative about  $\rho_i$ ,  $\gamma$ ,  $\gamma_y$  and  $\sigma_a$ .

Finally, in the last two columns of Table II we present the estimates of the baseline sticky price model (PF). Figure 4 plots the posterior distributions. In this case the duration of prices goes back again to 5 quarters, and by taking a look at Figure 4, we see that the posterior distribution of  $\theta_p$  is almost identical to the one obtained for EHL, with a lot of mass concentrated at 0.80. Hence, the probability that average price duration is less than 4.8 quarters is virtually zero. In this case, we find that data it is not informative about  $\gamma_\pi$  and  $\gamma_y$ .

When comparing the four models, we observe the following four facts. First, data clearly provide support for the existence of real wage rigidities as specified in our model. Second, once real wage rigidities are removed, much more rigidity in price setting is estimated. Third, backward looking behavior in inflation is not important. Fourth, given our priors, data is not very informative about  $\gamma_y$  and  $\sigma_a$ .



## 6.2 Model Comparison

Which model explains best the behavior of our data set? In Table III, we report the log marginal likelihood of each model. The marginal likelihood criterion suggests that the RWR model explains the data best. The log difference in favor of RWR over PF is 4.15. How big is this difference? It is bigger than 3, a bound suggested by Jeffreys (1961), accepted as a strong evidence (Kass and Raftery (1995)) in favor of one model over the other.

As we can see in Table III, the pure forward looking model performs better than EHL and GGLS. PF model is a particular case of EHL and of GGLS, thus it is natural to ask why a richer model ranks worse in terms of marginal likelihood than a simpler model. The reason is simple: richer models have many more hyperparameters and the Bayes factor discriminates against these. This “in-built” Ockham’s Razor is a final and attractive feature of the Bayes factor, that embodies a strong preference for parsimonious modeling.

## 6.3 How Important is the Prior Distribution?

The marginal likelihood depends on both the likelihood function and the prior distribution of the parameters. In order to understand the role of the priors, we reestimate the four models using as priors for  $\theta_p$  and  $\theta_w$  uniform distributions between  $[0, 1]$ . Clearly, these new priors are implausible: they assign the same positive probability to an average duration of four quarters and to an implausible average duration of prices of, let’s say, thirty quarters.

Table IV presents mean and the standard deviation of the posterior distributions for the four models, and figures 5-8 display the estimated posterior distributions of the models. The first thing to note from Table IV is that the estimates for the RWR and the GGLS models do not change significantly. Hence, even when we allow for implausible price and wage durations, the estimates for  $\theta_p$  and  $\theta_w$  do not change too much. On the other hand, the mean of the average duration of prices increases to up to fifteen quarters for EHL and PF. When taking a look at the posterior distributions, we can see that EHL places zero probability for an average duration of prices of less than eight quarters, and PF places zero

probability to an average duration of prices of less than ten quarters. Alternative evidence, as in Taylor (1999), suggests that these events have indeed a probability close to one.

As we can see in Table V, if we use implausible priors on price and nominal wage durations EHL outperforms the other models in terms of the marginal likelihood. This result relies on estimated posterior distributions that imply that the probability that the average duration of prices is lower or equal than 8 quarters is equal to zero. In this case, the PF model also outperforms the RWR model. However, this outcome is obtained with at least doubtful estimated posterior distributions.

## 6.4 Persistence

As we mentioned in the introduction, an important shortcoming of sticky price models is their incapability in generating enough persistence in the endogenous variables when facing exogenous shocks. In Figures 9-12, we compare the implied autocorrelations of output, price inflation, real wages and nominal interest rates of each model with those of US data. We present the posterior mean and bands of two posterior standard deviations for each autocorrelation.

Overall, RWR (Figure 9) mimics the autocorrelations of the four variables best. It slightly underestimates the persistence of output and of nominal interest rates, it gets quite close to the persistence in inflation, and it overstates the persistence of real wages. However, when compared to the other models, it does a fairly good job in capturing the joint autocorrelation of the four variables. The EHL model (Figure 10) underestimates greatly the persistence of all variables, except that of inflation. The GGLS model (Figure 11) replicates quite well the observed autocorrelations of output and inflation, but does quite poorly in explaining the persistence of real wages and nominal interest rates. Finally, the pure forward looking model (Figure 12) performs worse than GGLS, as a consequence of removing backward looking behavior in inflation.

Two main lessons should be taken from these figures. First, even though the estimated fraction of backward looking price setters is small, it can improve greatly the predictions of

the sticky price model in explaining output and inflation. Second, real wage rigidities, on top of nominal wage rigidities, are important in explaining the behavior of all variables.

At this point, we think it is important to remark the convenience of the marginal likelihood to compare models. In order to discriminate among models using Figures 9-12, we would need to specify: i) a *distance* to measure how different are estimated and observed autocorrelations, and ii) a *loss function*, that would determine which autocorrelations, and which lags, are the most important to match. The marginal likelihood criterion solves these two problems for us, and allows to compare between the models. The good news is that the model that gets the highest marginal likelihood, RWR, seems to match the data best.

## 7 Concluding Remarks

In this paper we have estimated a model with staggered price and wage setting, and additional real wage rigidities, for the US economy. We have adopted a Bayesian approach to estimate the models' parameters, and we have compared the results of our model with other three models that only incorporate nominal rigidities. We find that our specification performs best when realistic priors about the duration of prices and wages are used. At the same time, our model with real wage rigidities delivers reasonable parameter values, and fits the autocorrelation of the data fairly well.

We have confirmed several results that have been recently stressed in the literature, applying our methodology. First, models with nominal rigidities need implausible degrees of stickiness to match the data. Second, backward looking behavior in the inflation equation is not important quantitatively. And third, adding staggered wage setting to a sticky price model is not enough to match the data with reasonable parameter values.

Three main issues are left for future research. First, we recognize the fact that the rule of thumb behavior is purely exogenous. Embodying such rule of thumb in a rational expectations framework, as in Gray (1978), is currently in our research agenda. Second, we had to assume degenerate priors on some of the models' parameters. Using a richer model

and data (i.e introducing capital accumulation) should allow us to build a more complete picture of the structure of the economy. Finally, our estimation procedure can also be applied to large scale models that incorporate as many nominal and real rigidities as possible, in the spirit of Christiano, Eichenbaum and Evans (2001) and Smets and Wouters (2001).

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# Appendix

## A Derivation of the Wage Setting Equation

The derivation of this equation follows the literature on Calvo pricing restrictions. We repeat some equations from the main text for convinience.

A fraction  $\alpha(1 - \theta_w)$  follow a rule of thumb:

$$W^{rt}(h_t) = W(h_{t-1}) \left[ \frac{P(h_t)}{P(h_{t-1})} \right]^{\frac{1}{1-\theta_w}} \quad (\text{A.1})$$

The fraction  $(1 - \alpha)(1 - \theta_w)$  of households that set their wages optimally,  $W^*(h_t, j)$ , solve the first order condition:

$$\sum_{\tau=0}^{\infty} \sum_{h_{t+\tau}|h_t} (\beta\theta_w)^\tau \left\{ \left[ C(h_{t+\tau})^{-\frac{1}{\sigma}} \frac{W^*(h_t, j)}{P(h_{t+\tau})} - \vartheta \bar{N}(h_{t+\tau}, j)^\gamma \right] \bar{N}(h_{t+\tau}, j) \right\} = 0 \quad (\text{A.2})$$

where

$$\bar{N}(h_{t+\tau}, j) = \left( \frac{W^*(h_t, j)}{W(h_{t+\tau})} \right)^{-\phi} \int_0^1 \left( \frac{Y(h_t, i)}{A(h_t)} \right)^{\frac{1}{1-\delta}} di$$

Note that the  $j$  superscript in consumption is eliminated because it is fully insured across individuals due to the risk-sharing result. However, agents cannot fully insure the supply of labor.

The evolution of aggregate wages is

$$W(h_t) = \left[ \alpha (1 - \theta_w) W^{rt}(h_t)^{1-\phi} + (1 - \alpha) (1 - \theta_w) W^*(h_t)^{1-\phi} + \theta_w W(h_{t-1})^{1-\phi} \right]^{\frac{1}{1-\phi}} \quad (\text{A.3})$$

and we have imposed symmetry in the wage decision.

To derive an equation for the (log-linear) evolution of nominal wage inflation, we take a loglinear approximation of equations (A.1), (A.2) and (A.3) around the symmetric equilibrium steady state. The first order condition becomes, where lower case letters denote



percent deviations from the steady state:

$$E_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau \left[ \left( w_t^* - p_{t+\tau} - \frac{1}{\sigma} c_{t+\tau} - \gamma n_{t+s}^j \right) \right] = 0$$

In order to derive the wage setting equation, we need to evaluate the marginal rate of substitution at the average household. Note that  $n_{t+s}^j$  is the (log-linear) demand for labor of agents that set their price at  $t$ . It follows that:

$$n_{t+s}^j = n_{t+s} - \phi(w_t^j - w_{t+s})$$

Using this fact and that

$$mrs_t = \frac{1}{\sigma} c_t + \gamma n_t$$

we can reduce the previous expression to:

$$E_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau [(1 + \phi\gamma)w_t^* - \phi\gamma w_{t+\tau} - p_{t+\tau}] = E_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau (mrs_{t+\tau}) \quad (\text{A.4})$$

Now, we proceed by doing the following steps: subtract  $E_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau (w_{t+\tau} - p_{t+\tau})$  from both sides, define  $w_{t+\tau} = w_t + \sum_{j=1}^{\tau} \Delta w_{t+j}$  and  $z_t = w_t^* - w_t$ , and note that:

$$\sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau \sum_{j=1}^{\tau} \Delta w_{t+j} = \frac{1}{1 - \beta\theta_w} \sum_{\tau=1}^{\infty} (\beta\theta_w)^\tau \Delta w_{t+\tau}$$

so that we can rewrite, again, the first order condition as:

$$\begin{aligned} & \frac{1}{1 - \beta\theta_w} z_t - \frac{1}{1 - \beta\theta_w} E_t \sum_{\tau=1}^{\infty} (\beta\theta_w)^\tau \Delta w_{t+\tau} \\ &= \frac{1}{1 + \phi\gamma} E_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau (mrs_{t+\tau} - (w_{t+\tau} - p_{t+\tau})) \end{aligned}$$

Finally, from loglinearizing (A.1) and (A.3), we get that:

$$z_t = \frac{\alpha + \theta_w(1 - \alpha)}{(1 - \theta_w)(1 - \alpha)} \Delta w_t - \frac{\alpha}{(1 - \theta_w)(1 - \alpha)} \Delta p_t \quad (\text{A.5})$$

Combine this with (A.4), and we are ready to write down the final expression which is, after some algebra:

$$\begin{aligned}\Delta w_t &= \frac{\alpha}{\alpha + \theta_w(1 - \alpha)} \Delta p_t + \frac{\beta \theta_w}{\alpha + \theta_w(1 - \alpha)} E_t \Delta w_{t+1} \\ &\quad - \frac{\alpha}{\alpha + \theta_w(1 - \alpha)} \beta \theta_w E_t \Delta p_{t+1} \\ &\quad + \frac{(1 - \alpha)(1 - \theta_w)(1 - \beta \theta_w)}{(1 + \phi \gamma)(\alpha + \theta_w(1 - \alpha))} (m r s_t - (w_t - p_t))\end{aligned}$$

## B Matrices to Obtain the Law of Motion

$$A(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \kappa_p & 0 & \beta & 0 & 0 \\ -\kappa_w & 0 & -\frac{\beta \alpha \theta_w}{\eta_w} & \frac{\beta \theta_w}{\eta_w} & 0 \\ 0 & -\sigma & \sigma & 0 & 1 \end{pmatrix}$$

$$B(\theta) = \begin{pmatrix} -1 & 0 & -1 & 1 & 0 \\ 0 & \rho_i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\frac{\delta}{1 - \delta} \kappa_p \\ 0 & 0 & -\frac{\alpha}{\eta_w} & 1 & -(\sigma^{-1} + \frac{\gamma}{1 - \delta}) \kappa_w \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$C(\theta) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (1 - \rho_i) \gamma_\pi & 0 & (1 - \rho_i) \gamma_y \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D(\theta) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ \frac{\kappa_p}{1 - \delta} & 0 \\ \frac{\gamma \kappa_w}{1 - \delta} & 0 \\ 0 & 0 \end{pmatrix}$$

$$N(\theta) = \begin{pmatrix} \rho_a & 0 \\ 0 & 0 \end{pmatrix}$$

and

$$\Lambda(\theta) = \begin{pmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_m^2 \end{pmatrix}$$

Table I: Prior Distributions

Parameter	Prior	Mean	Std.Dev.
$\sigma^{-1}$	Gamma( $\alpha = 2, \beta = 1.25$ )	2.5	1.76
$\beta$	Fixed	0.99	0
$\theta_p$	Uniform[0, 0.8]	0.4	0.18
$\theta_w$	Uniform[0, 0.83]	0.41	0.18
$\alpha / \omega$	Uniform[0, 1)	0.5	0.28
$\gamma_\pi$	Normal(1.5, 0.25)	1.5	0.25
$\gamma_y$	Normal(0.5, 0.125)	0.5	0.125
$\phi - 1$	Gamma( $\alpha = 3.33, \beta = 1.5$ )	5	2.72
$\varepsilon - 1$	Fixed	5	0
$\gamma$	Normal(1, 0.5)	1	0.5
$\delta$	Fixed	0.36	0
$\rho_a$	Uniform[0.5, 1)	0.75	0.20
$\rho_i$	Uniform[0, 1)	0.5	0.28
$\sigma_a$	Gamma( $\alpha = 4, \beta = 0.00125$ )	0.005	0.0025
$\sigma_m$	Gamma( $\alpha = 4, \beta = 0.00125$ )	0.005	0.0025

Table II: Moments of the Posterior Distributions

	RWR		EHL		GGLS		PF	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
$\theta_p$	0.63	0.06	0.79	0.004	0.77	0.03	0.79	0.005
$\theta_w$	0.79	0.02	0.27	0.09	-	-	-	-
Price D.	2.83	0.53	4.91	0.11	4.47	0.39	4.90	0.09
Wage D.	4.95	0.58	1.42	0.28	1	-	1	-
$\alpha$	0.56	0.05	-	-	-	-	-	-
$\omega$	-	-	-	-	0.07	0.06	-	-
$\gamma_\pi$	0.82	0.14	1.38	0.27	1.94	0.16	1.35	0.16
$\gamma_y$	0.55	0.09	0.50	0.12	0.41	0.11	0.52	0.11
$\rho_i$	0.88	0.02	0.81	0.05	0.45	0.08	0.68	0.04
$\sigma$	0.16	0.05	0.20	0.19	0.09	0.02	0.07	0.02
$\phi$	19.4	4.42	5.53	2.41	-	-	-	-
$\gamma$	1.56	0.43	2.02	0.40	1.20	0.29	1.83	0.27
$\rho_a$	0.88	0.03	0.80	0.08	0.88	0.02	0.85	0.02
$\sigma_a$	0.005	0.002	0.007	0.002	0.005	0.0013	0.004	0.0007
$\sigma_m$	0.0017	0.0002	0.0018	0.0002	0.0017	0.0003	0.0018	0.0002

Table III: Log Marginal Likelihood

Model	
RWR	1255.35
EHL	1051.01
GGLS	1160.70
PF	1251.20

Table IV: Moments of the Posterior Distributions

(Implausible Priors)

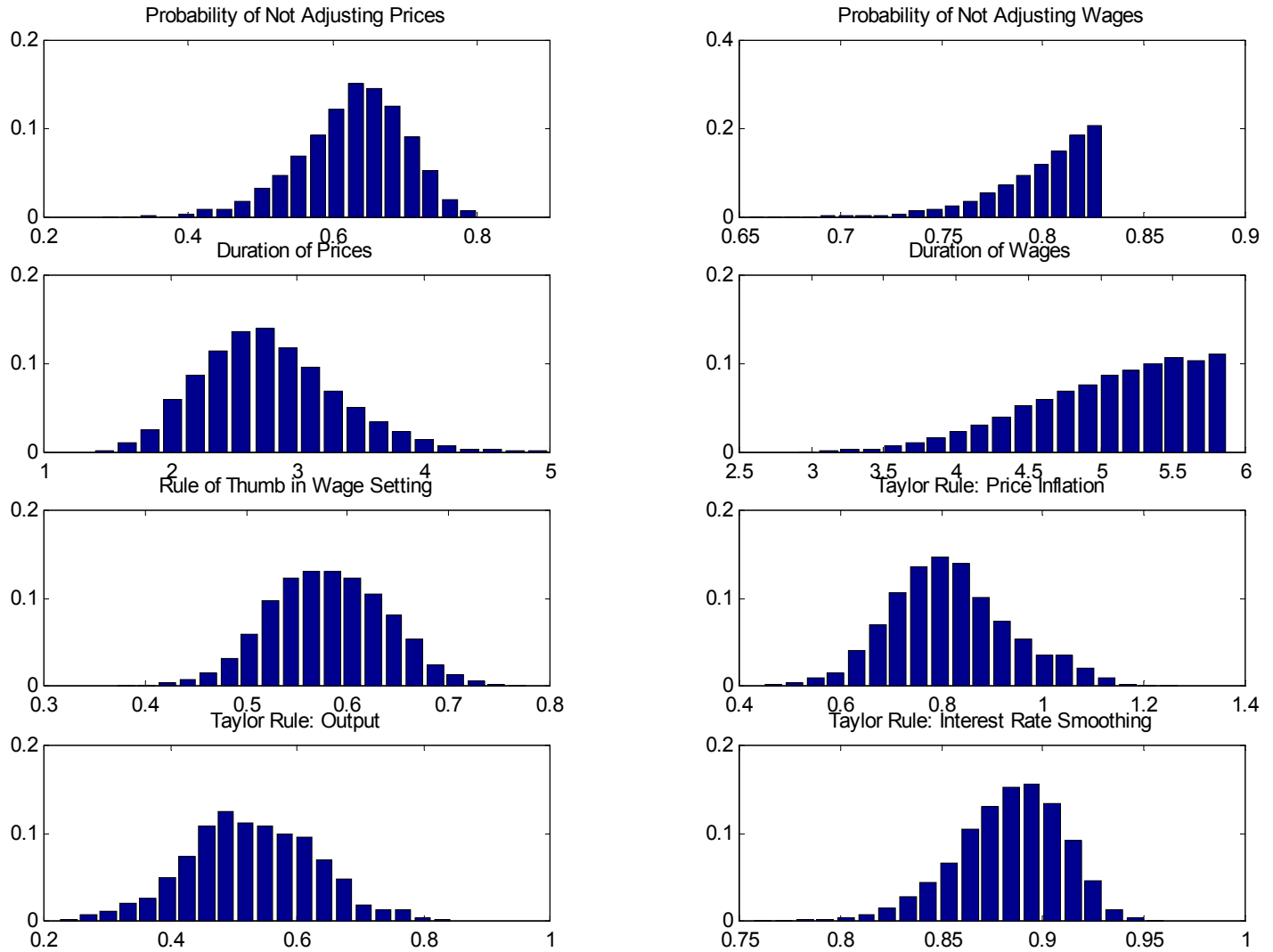
	RWR		EHL		GGLS		PF	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
$\theta_p$	0.66	0.07	0.93	0.01	0.83	0.04	0.93	0.1
$\theta_w$	0.85	0.04	0.32	0.09	-	-	-	-
Price D.	3.16	0.76	15.21	3.26	6.19	1.40	15.3	2.72
Wage D.	7.46	2.30	1.63	0.28	1	-	1	-
$\alpha$	0.68	0.06	-	-	-	-	-	-
$\omega$	-	-	-	-	0.08	0.06	-	-
$\gamma_\pi$	0.76	0.13	1.29	0.28	1.88	0.21	0.74	0.11
$\gamma_y$	0.48	0.11	0.47	0.12	0.35	0.11	0.43	0.08
$\rho_i$	0.88	0.02	0.82	0.03	0.45	0.06	0.68	0.03
$\sigma$	0.14	0.04	0.13	0.04	0.09	0.02	0.14	0.04
$\phi$	15.6	5.80	6.51	2.69	-	-	-	-
$\gamma$	1.28	0.36	1.23	0.37	0.89	0.28	0.60	0.23
$\rho_a$	0.90	0.03	0.94	0.08	0.90	0.02	0.97	0.01
$\sigma(\varepsilon_t^a)$	0.005	0.002	0.008	0.002	0.006	0.002	0.006	0.002
$\sigma(\varepsilon_t^m)$	0.0017	0.0002	0.0017	0.0001	0.0018	0.0002	0.002	0.0001

Table V: Log Marginal Likelihood

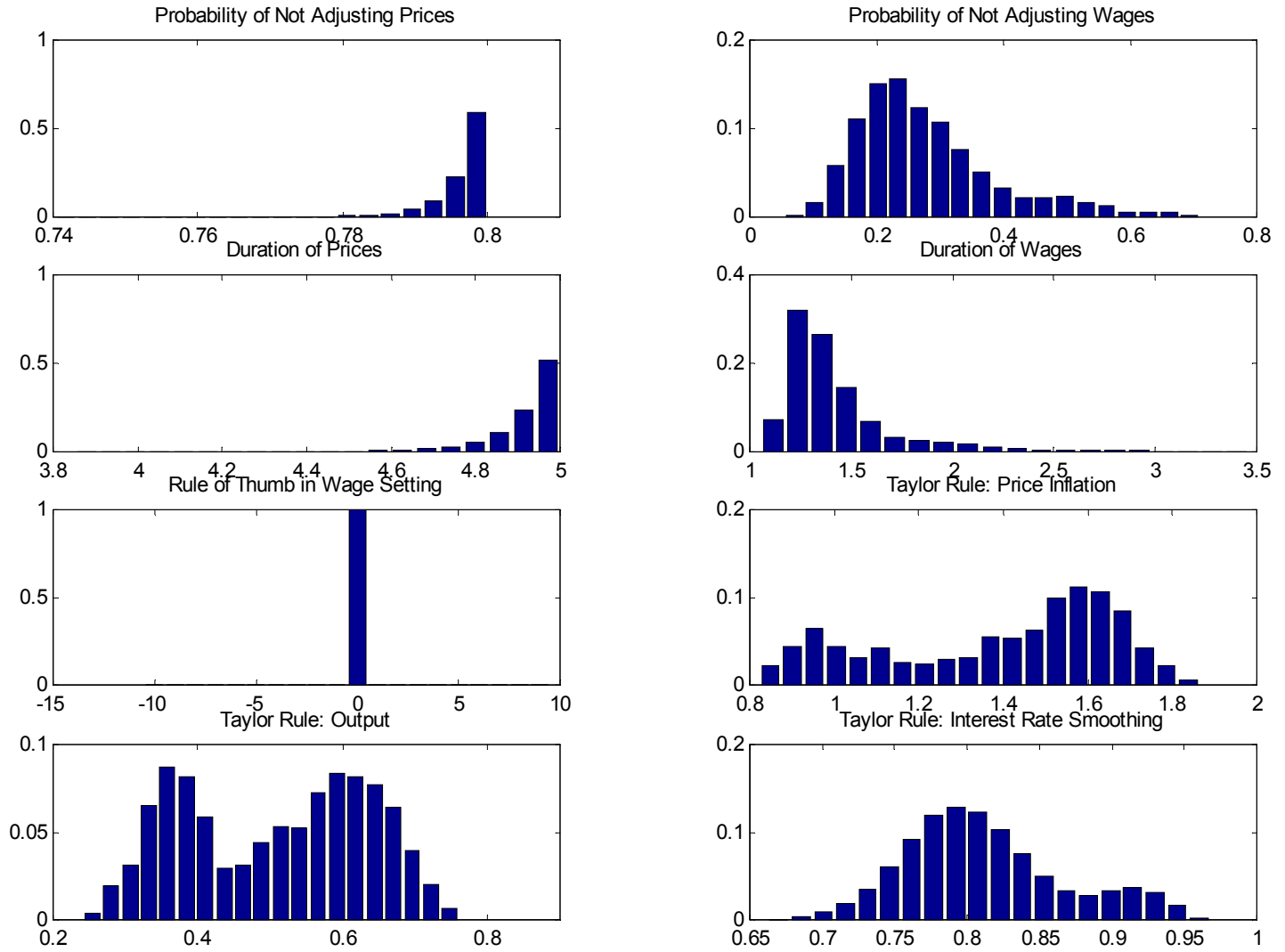
(Implausible Priors)

Model	
RWR	1256.35
EHL	1315.77
GGLS	1174.47
PF	1280.59

# Figure 1: Posterior Distributions, RWR model

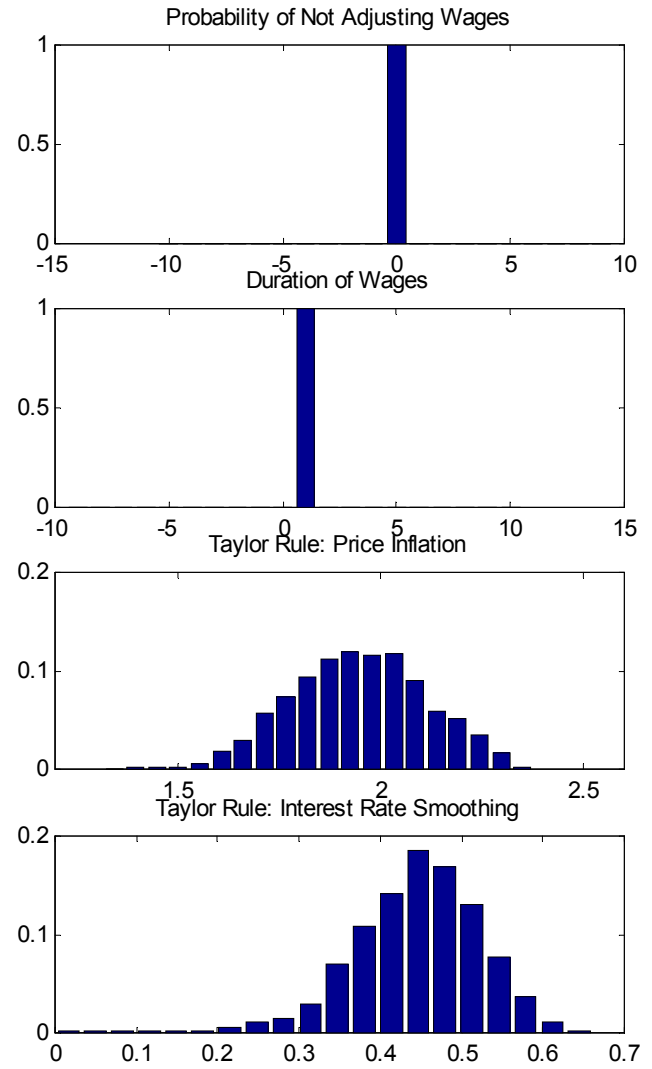
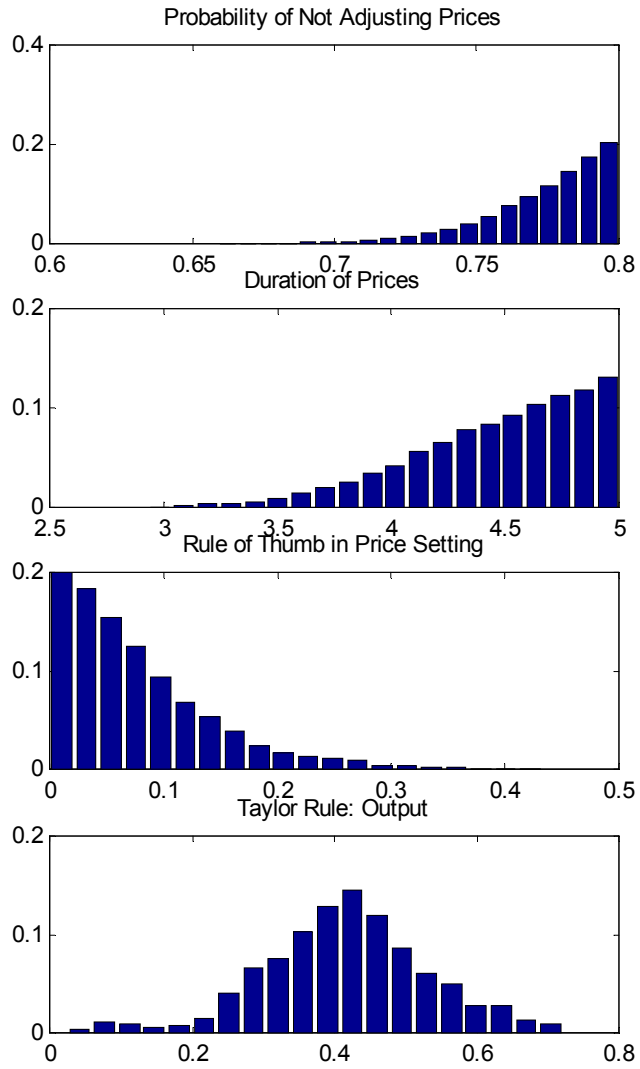


## Figure 2: Posterior Distributions, EHL Model

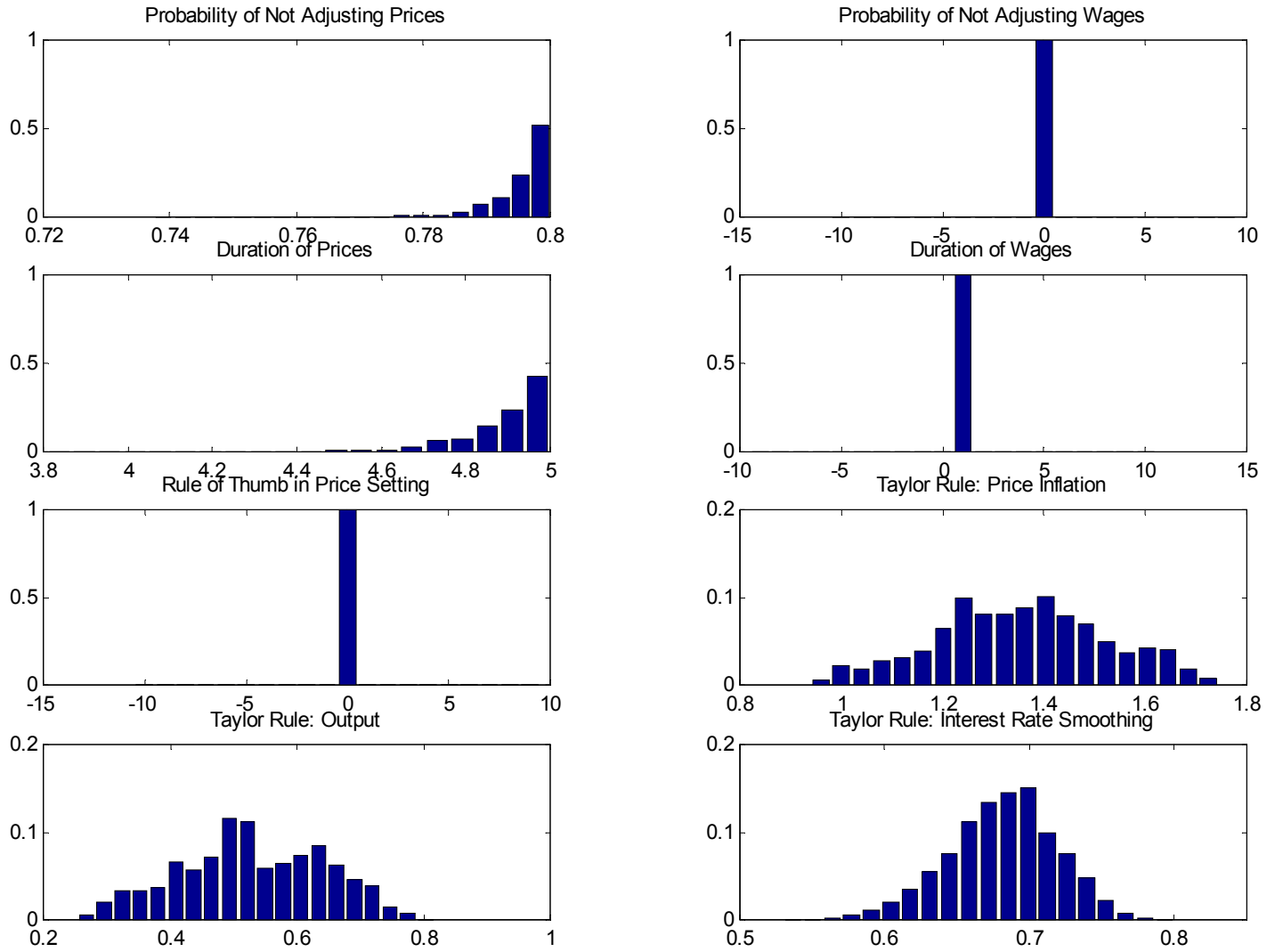




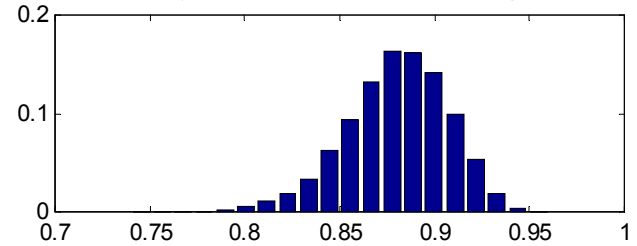
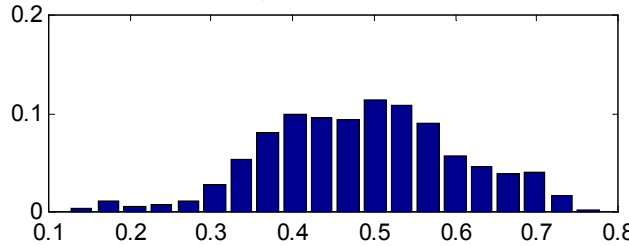
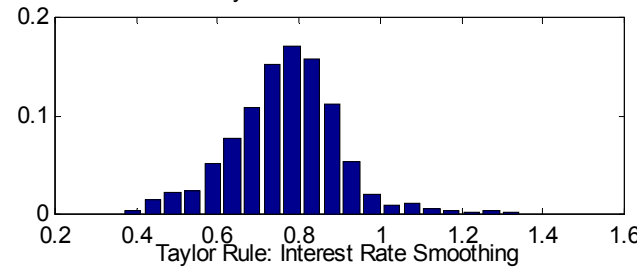
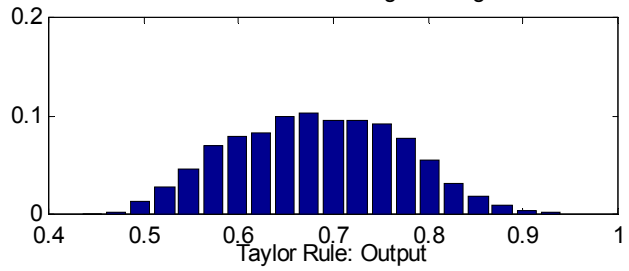
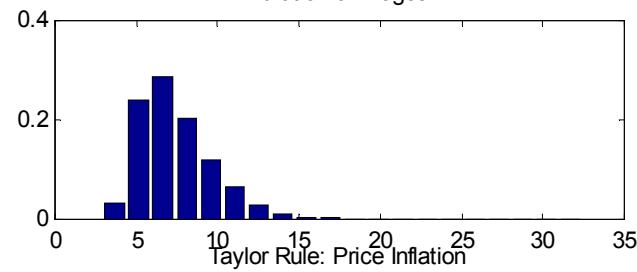
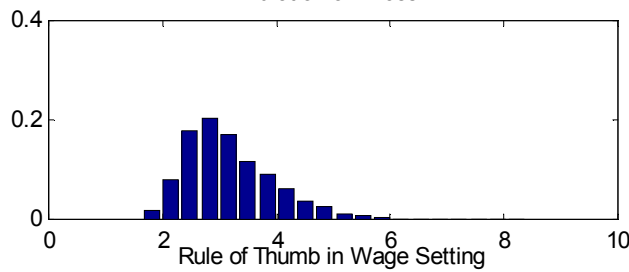
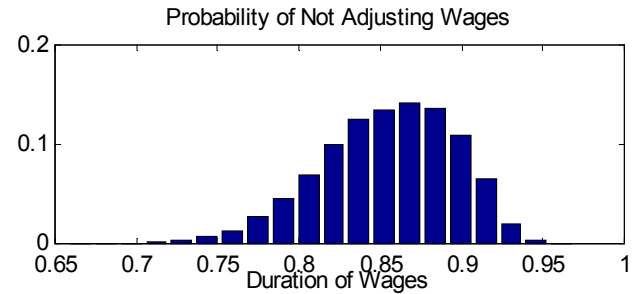
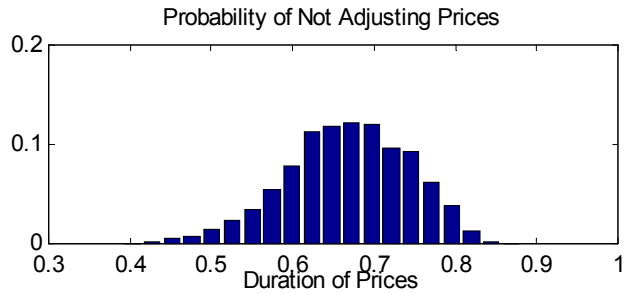
### Figure 3: Posterior Distributions, GGLS Model



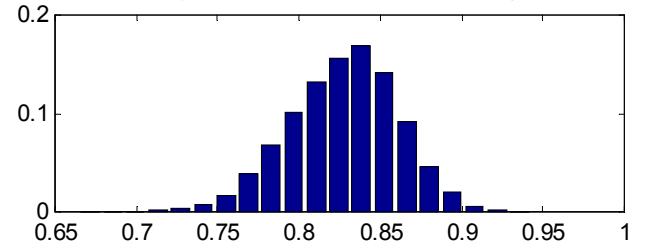
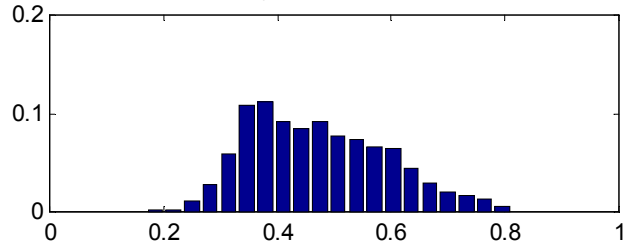
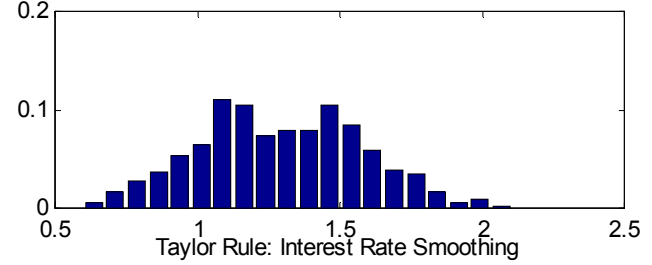
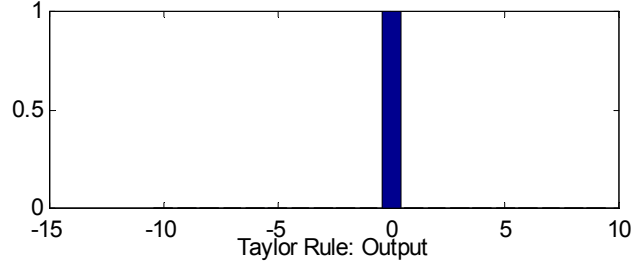
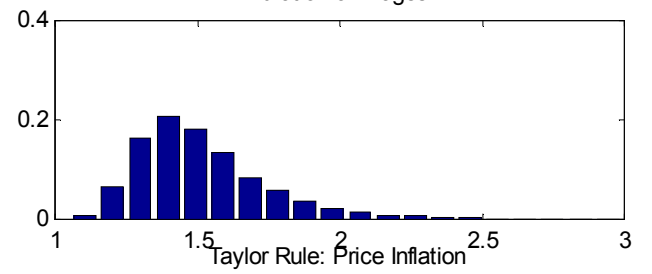
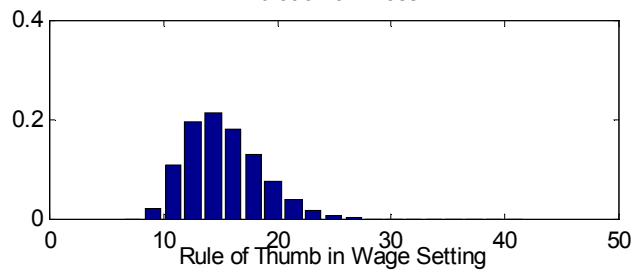
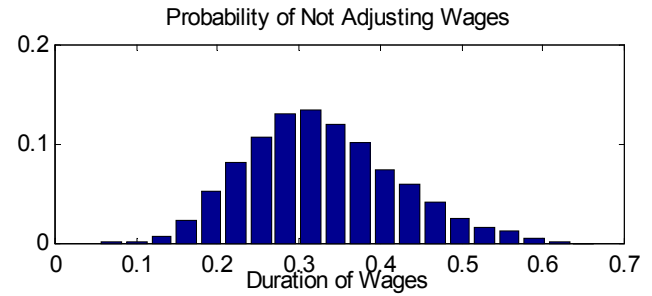
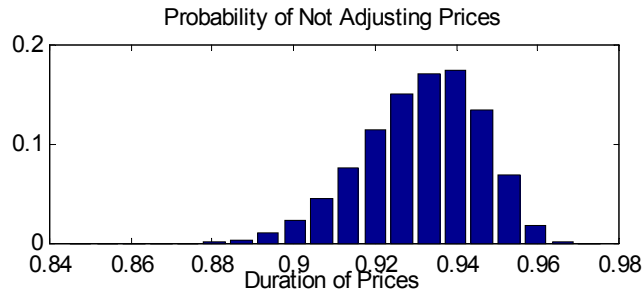
# Figure 4: Posterior Distributions, PF Model



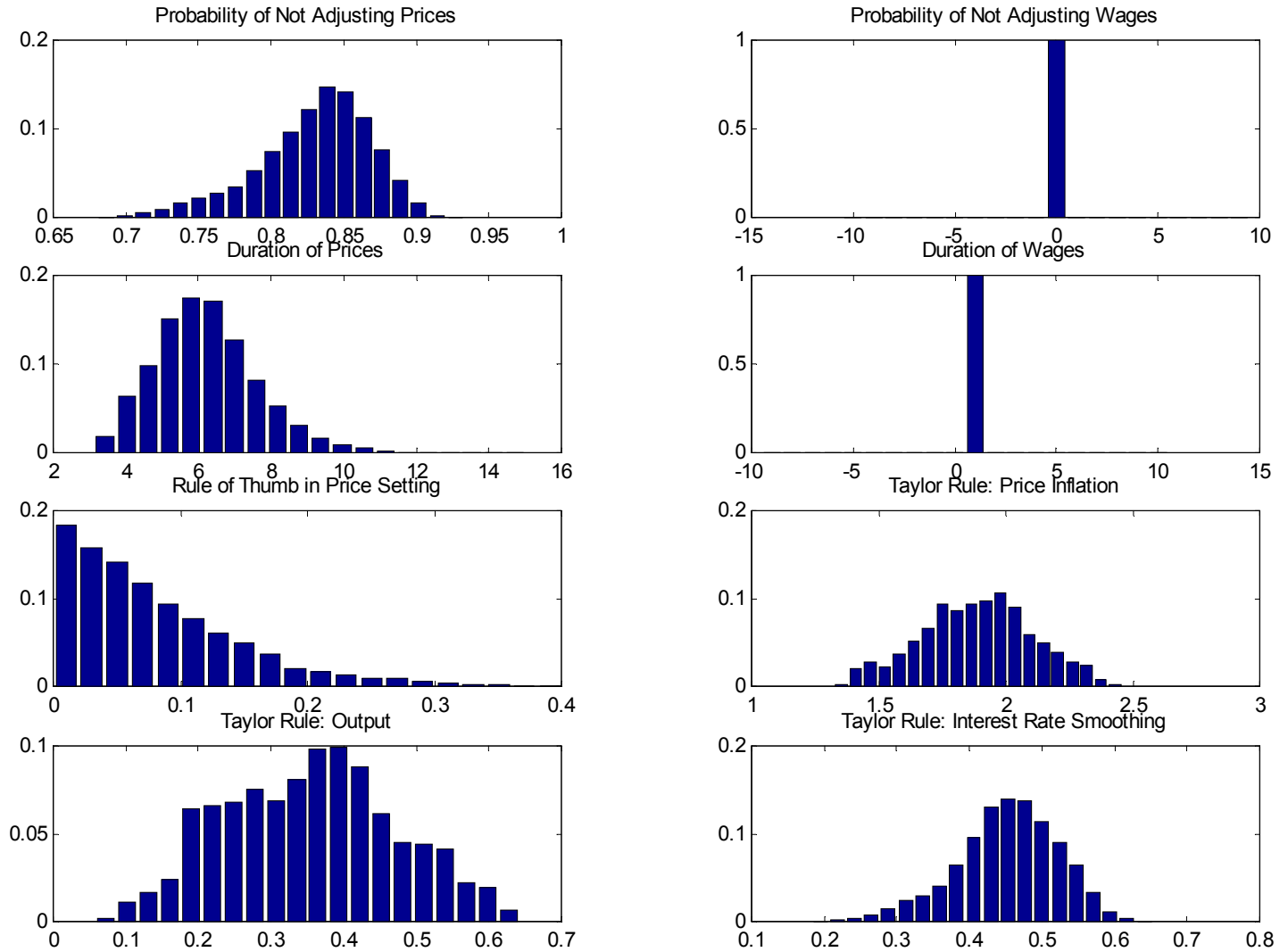
**Figure 5: Posterior Distributions, RWR model  
(Implausible Priors)**



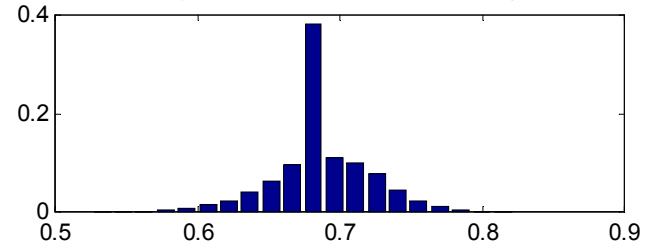
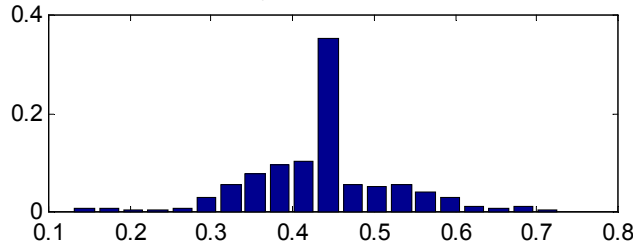
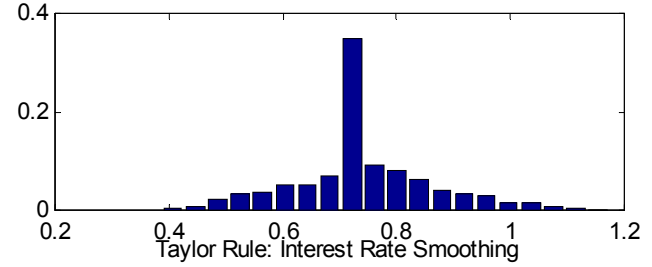
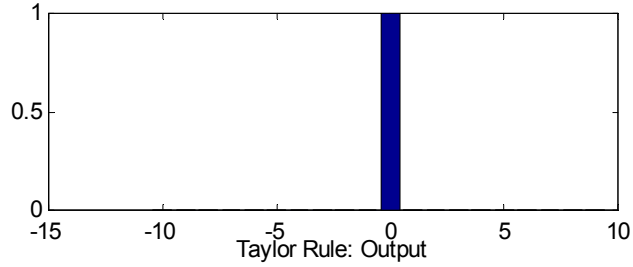
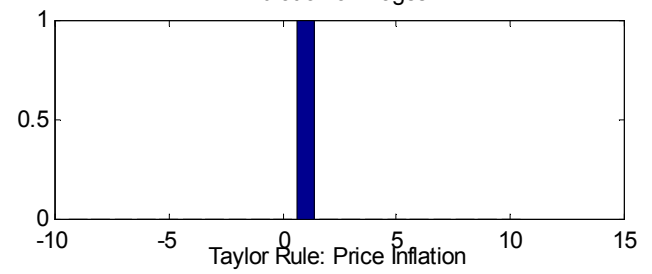
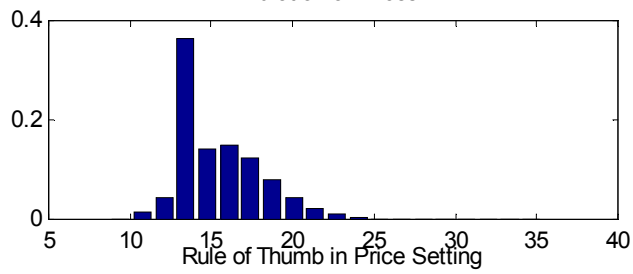
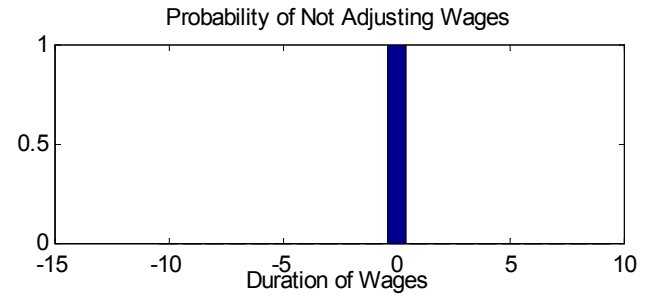
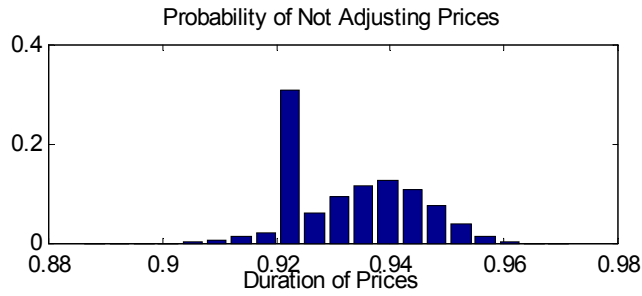
## Figure 6: Posterior Distributions, EHL Model (Implausible Priors)



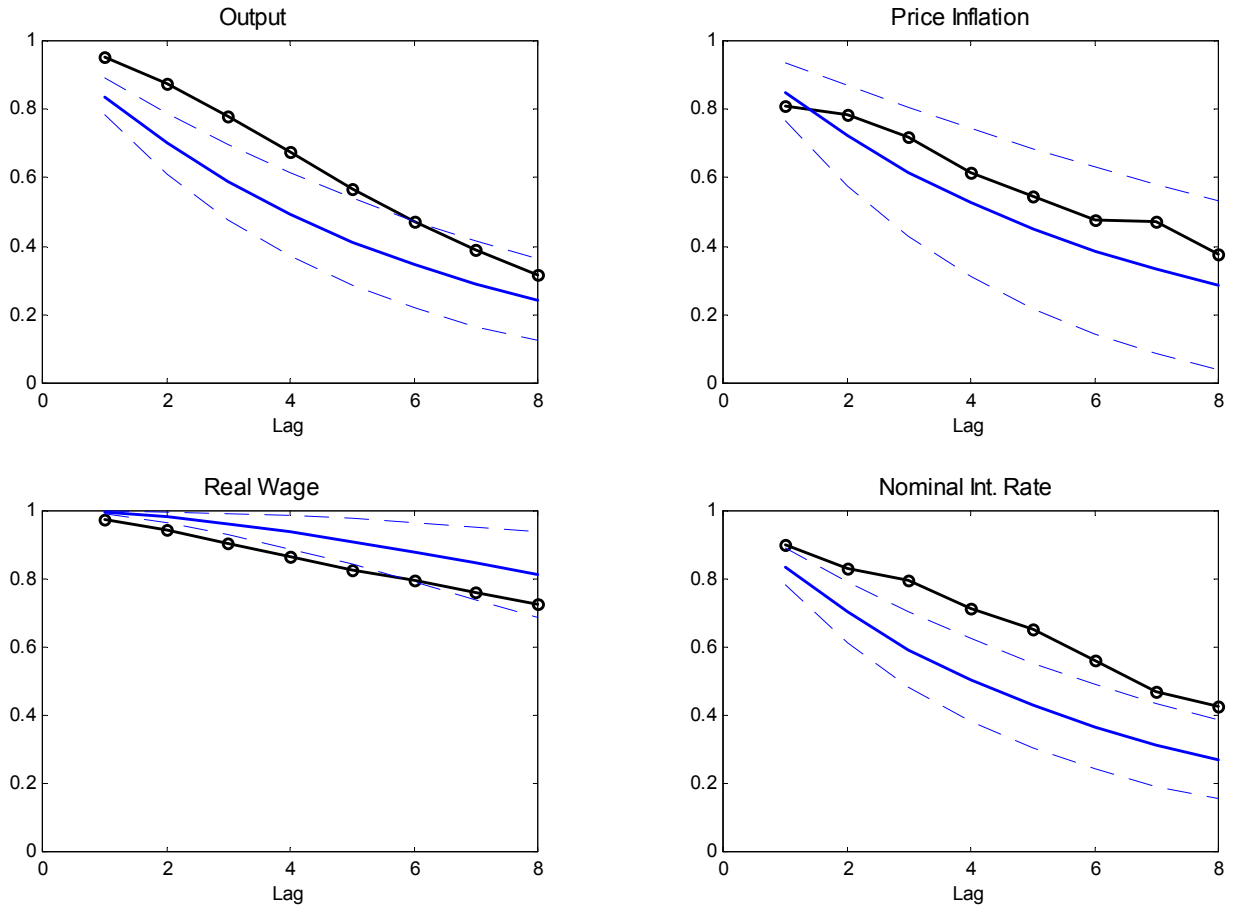
**Figure 7: Posterior Distributions, GGLS Model  
(Implausible Priors)**



**Figure 8: Posterior Distributions, PF Model  
(Implausible Priors)**

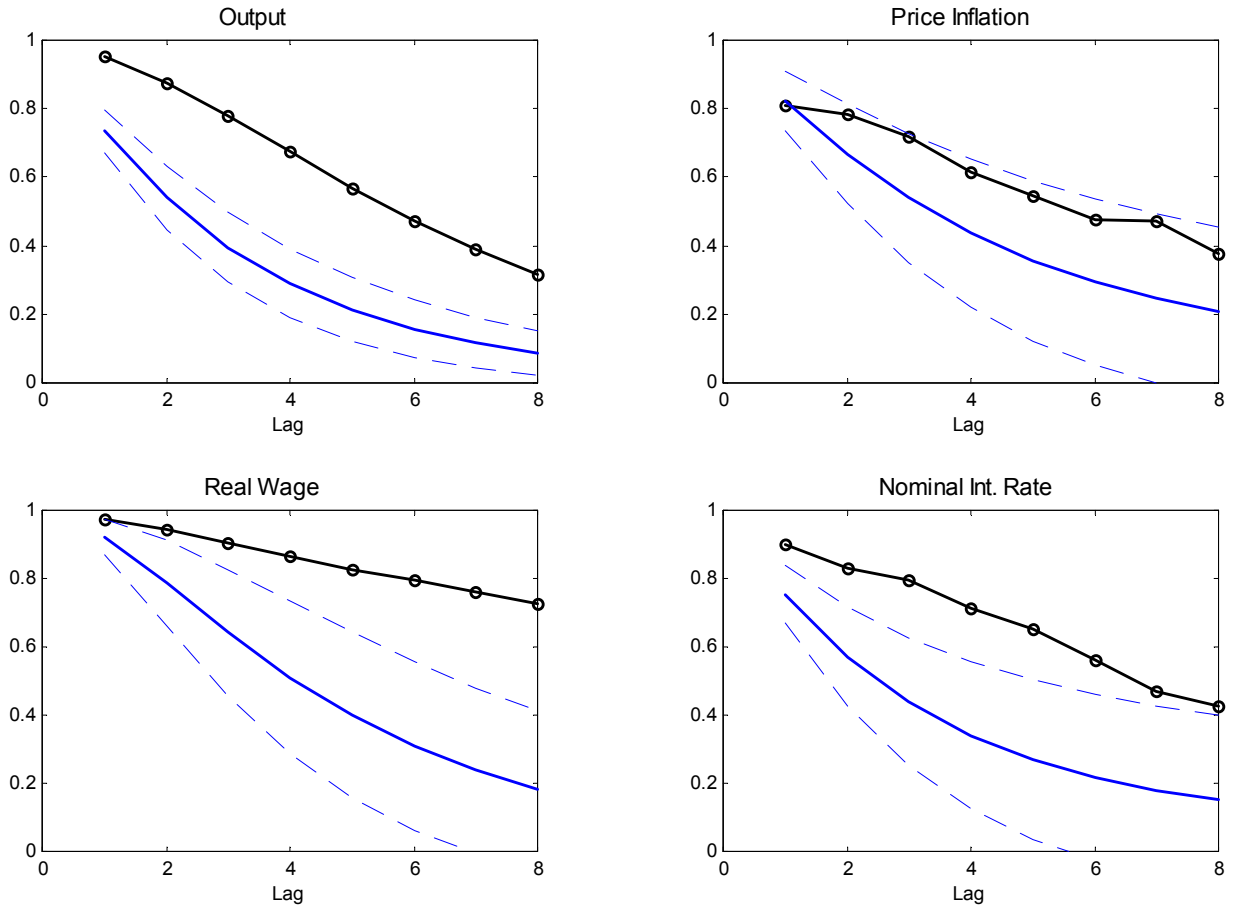


**Figure 9: Autocorrelations, RWR Model**



**Solid Line=Mean Posterior, Dashed Lines=+/-2 Std. Dev Posterior, Circle= US Data**

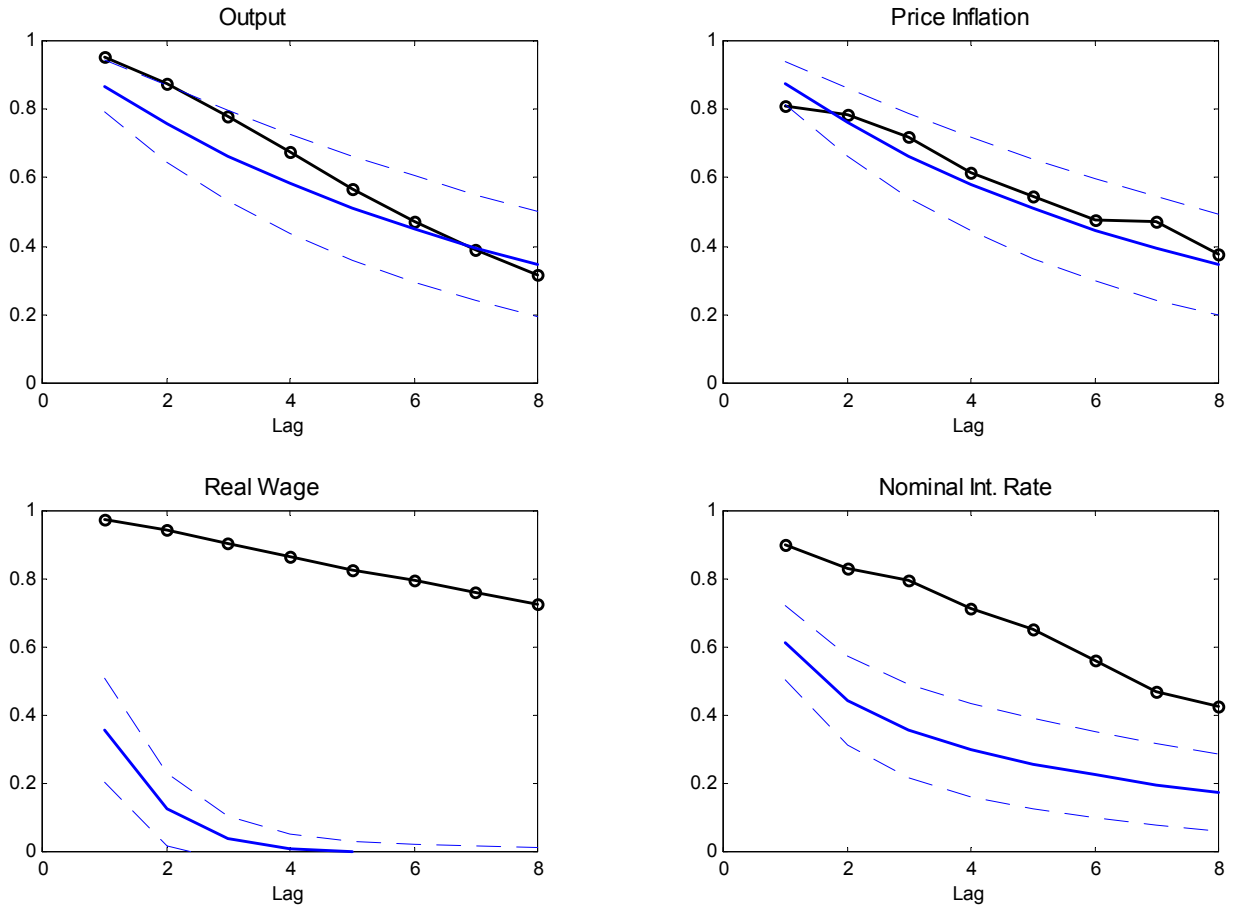
**Figure 10: Autocorrelations, EHL Model**



**Solid Line=Mean Posterior, Dashed Lines= $\pm 2$  Std. Dev Posterior, Circle= US Data**

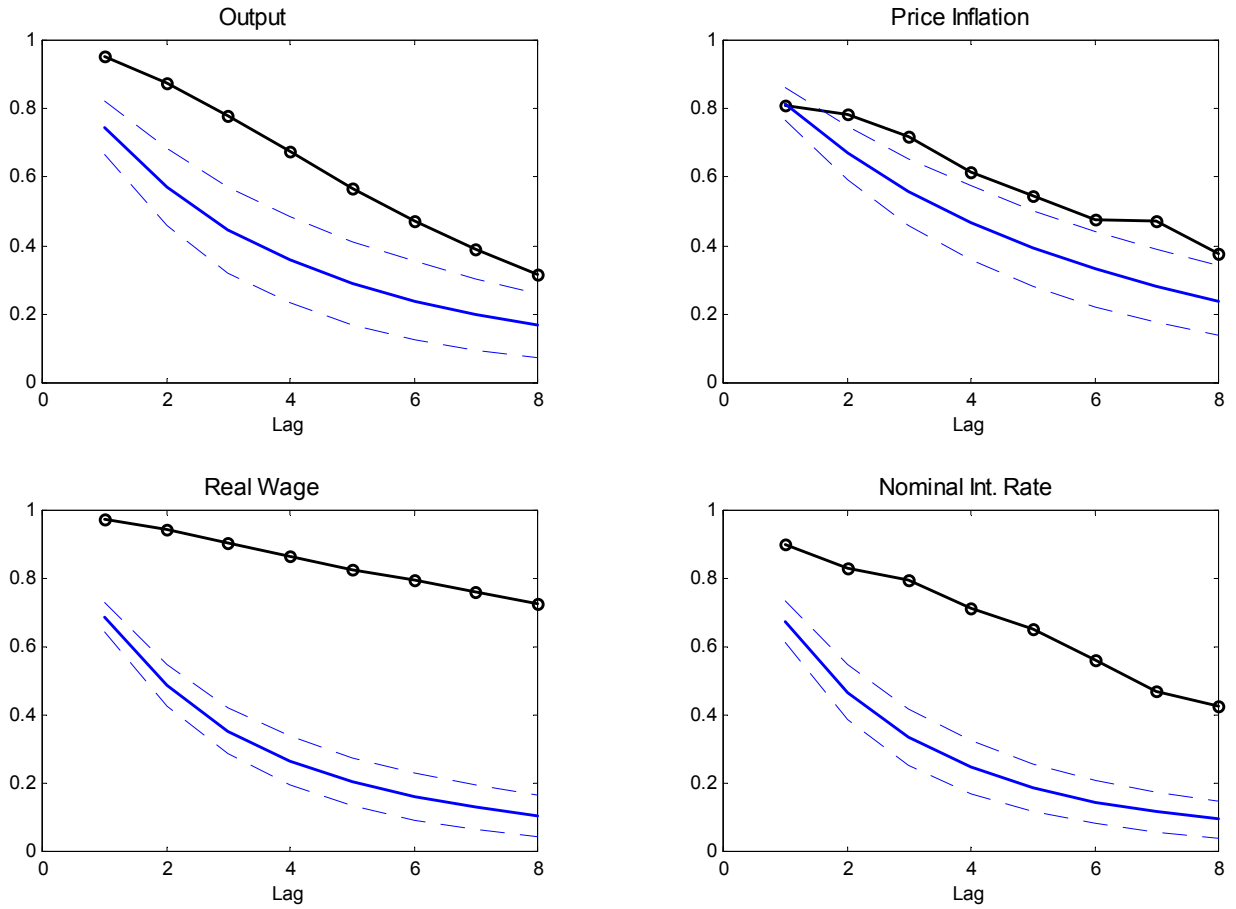


**Figure 11: Autocorrelations, GGLS model**



**Solid Line=Mean Posterior, Dashed Lines= $\pm 2$  Std. Dev Posterior, Circle= US Data**

**Figure 12: Autocorrelations, PF model**



**Solid Line=Mean Posterior, Dashed Lines= $\pm 2$  Std. Dev Posterior, Circle= US Data**