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International Diversification Strategies

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Abstract: We estimate a model with country- and industry-specific shocks that extends the dummy variable model used in the portfolio diversification literature by relaxing the restriction that all stocks with exposure to a given shock have the same exposure to that shock. We find that: i) This restriction is strongly rejected by the data. ii) Many industry betas are negative, while almost all country betas are positive. This difference in within-group heterogeneity may explain why country shocks have historically outweighed industry shocks in explaining international return variation. iii) We use the betas to construct portfolios whose volatility is substantially below that of the world market, both in and out of sample.

JEL classification: G11, G15

Key words: diversification, risk, international financial markets, industrial structure
1 Introduction

In these days of high stock market volatility, the question of how to reduce risk is foremost in portfolio managers’ minds. Since Solnik (1974) it is known that international diversification is one way to achieve this goal. More recent papers such as Heston and Rouwenhorst (1994) and Griffin and Karolyi (1998) provide further evidence on the advantages of cross-country diversification. This literature has used as a workhorse a dummy variable or fixed effects model that distinguishes between country-specific and global industry shocks in international stock returns. This fixed effects model imposes a very strong restriction on the data: that the exposure to any given shock, whenever it is non-zero, is the same across stocks. This implies, for example, that all stocks in the US are constrained to have the same exposure to the US country factor, regardless of whether the company in question is a multinational or a firm with only domestic operations. In the case of industry shocks, the restriction implies that all stocks in the financial services industry have the same exposure to that industry factor, no matter whether a firm is a diversified financial services provider or a small bank. It is apparent that this restriction is likely to be violated in the data. In addition, it runs counter to much of the empirical finance literature on the CAPM or the APT, where the key difference across companies is in their betas, i.e. in their exposure to shocks.

This paper builds a model that is similar to fixed effects model in the sense that it has both country- and industry-specific shocks. However, it relaxes the restriction that the exposure to shocks, whenever it is non-zero, is the same across stocks. This model turns out to be a very familiar one in empirical finance: a latent factor model. The key difference between this factor model and those used in the existing literature (see Cho et al. (1986) and Heston et al. (1995) among others) is that here the factors are identified. Specifically, we impose restrictions such that common shocks (or factors) can be characterized as global, country-specific or industry-specific shocks. Using a new solution method, we derive maximum likelihood estimates for the betas of each individual stock on these factors using a dataset that covers US dollar-denominated stock returns for 1965 companies in 21 developed and emerging markets from January 1986 to February
Conditional on a set of auxiliary assumptions, our factor model nests the fixed effects model. As a result, we can test whether the restrictions imposed by the fixed effects model are rejected by the data or not. In the event, we find that they are strongly rejected by the data. We next use our factor model to investigate whether country- or industry-specific shocks are the predominant source of variation in international stock returns. We find that country-specific shocks are the predominant source of variation in returns for the average stock, the global stock market portfolio, the average country portfolio and—to our surprise—also for the average industry portfolio. Why are country-specific shocks so much more important in explaining international return variation than industry-specific shocks? An examination of the betas on both sources of systematic risk shows that the degree of heterogeneity in stocks’ exposure to industry-specific shocks is much greater than it is for country-specific shocks. Indeed, the fraction of stocks that load anti-cyclically on industry shocks is far greater than for country shocks. Country shocks therefore conform much better to our notion of an aggregate-level shock, which through a policy change or a business cycle shock affects all stocks within a country similarly. In contrast, industry affiliation is not generally as informative as country affiliation regarding the extent to which stock returns within a given category actually comove. An additional reason for the small role of industry-specific shocks in our model is that we allow explicitly for a global factor in international stock returns. In contrast, it is possible that in the fixed effects literature the importance of such a factor is subsumed by the global industry effects. This raises the possibility that the recent rise in the importance of global industry effects, reported in Cavaglia et al. (2000) among others, is in fact capturing the changing importance of a global factor over time, rather than actual industry-specific effects.

We next use the loadings of stocks on the different sources of systematic risk to

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1Relative to earlier work by Marsh and Pfleiderer (1997), who use a model similar to ours, our methodological contribution is to develop a maximum likelihood algorithm that works even with a very large cross-section. Marsh and Pfleiderer (1997) use an iterative procedure that has intuitive appeal but where the statistical properties of the resulting estimator are not well-known.
construct large portfolios that are well diversified in their exposure to either global, country-specific or industry-specific shocks. Our rationale for this exercise is simple. If for example the goal of portfolio diversification is to reduce the exposure of a portfolio to country shocks, why not directly pick those stocks that have small country betas? Our variance decomposition suggests that a portfolio that includes stocks with low exposures to country-specific shocks is promising in terms of risk reduction. The in-sample results, the results for different subsamples and the out-of-sample results all suggest that this promise is substantiated. Our candidate portfolio is always sizably less variable than the market portfolio, both in- and out-of-sample. We also find that it outperforms the market both in terms of in-sample and out-of-sample mean returns.

For portfolio managers who follow a top-down approach, typically first diversifying across countries and then choosing the best stocks in each market, our analysis is useful in two regards. First, it shows that, exploiting information on stocks' exposure to different sources of systematic risk, country-specific shocks are by far the most important source of international return variation. International diversification strategies that are based on cross-country diversification therefore still have merit. Second, our approach provides portfolio managers with information on which stocks to pick within countries. Because stocks differ in their exposure to country-specific shocks, not all diversified country portfolios are equal in terms of risk reduction.

The remainder of the paper is organized as follows. Sections 2 and 3 describe the model and the data, respectively. Section 4 contains the results. Section 5 concludes.

2 The Model

For the sake of comparison with the existing literature the section begins by briefly reviewing the fixed effects model used by Heston and Rouwenhorst (1994), Griffin and Karolyi (1998) and others. Let us denote by $R_{nt}$ the return on stock $n$ in period $t$, where $n$ ranges from 1 to $N$ and $t$ from 1 to $T$. Let us index countries with the letter $c$ ($c = 1, .., C$) and industries with the letter $i$ ($i = 1, .., I$). The fixed effects model is
described by the following equation:

\[ R_{nt} = \beta_n f^G_t + \sum_{c=1}^{C} \beta_{nc} f^C_t + \sum_{i=1}^{I} \beta_{ni} f^I_t + \epsilon_{nt}, \] (1)

where \( f^G_t \) is the return to the global market factor, \( f^C_t \) and \( f^I_t \) are the returns to the country factor \( c \) and industry factor \( i \), respectively, and \( \epsilon_{nt} \) represents the idiosyncratic shock to the return of stock \( n \), all in period \( t \). The model is also characterized by the restrictions:

\[ \begin{align*}
\beta^G_n &= 1 \\
\beta_{nc} &= \begin{cases} 
1 & \text{if stock } n \text{ belongs to country } c \\
0 & \text{otherwise}
\end{cases} \\
\beta_{ni} &= \begin{cases} 
1 & \text{if stock } n \text{ belongs to industry } i \\
0 & \text{otherwise}
\end{cases}
\] (2)

which imply that at each point in time \( f^G_t, f^C_t, \) and \( f^I_t \) can be estimated by regressing \( R_{nt} \) on the dummies \( (\beta^G_n, \beta_{nc}, \beta_{ni}) \), after taking into account the multicollinearity among the regressors.

The model used in this paper is essentially the same as (1), with a few important differences. First, and most importantly, we relax the restriction that the \( \beta \)s, when they are not zero, have to be one. In place of (2) we use the restrictions:

\[ \begin{align*}
\beta^G_n &= \text{unconstrained} \\
\beta_{nc} &= \begin{cases} 
\text{unconstrained} & \text{if stock } n \text{ belongs to country } c \\
0 & \text{otherwise}
\end{cases} \\
\beta_{ni} &= \begin{cases} 
\text{unconstrained} & \text{if stock } n \text{ belongs to industry } i \\
0 & \text{otherwise}
\end{cases}
\] (3)

Second, we treat the returns on the global, country and industry factors \( (f^G_t, f^C_t, \) and \( f^I_t) \) as unobservable random variables. This implies that we effectively estimate (1) as a factor model, as opposed to a fixed effects model (in econometric terms, we move from a fixed effects to a random effects model). The restrictions (3) imply that this factor model is different from those commonly used in the international APT literature: this
factor model is identified. As is well known, without the restrictions (3) the factors can be rotated at leisure and hence cannot be separately identified. Here the zero restrictions pin down the rotation matrix and thus we can give economic interpretation to the factors, characterizing them as global, country-specific, and industry-specific factors.

Third, we follow the APT literature and estimate (1) using excess returns over a riskless benchmark. Note that in terms of the fixed effects model, this change in the definition of $R_{nt}$ does not affect the results because of the presence of the fixed time effect $f_g^t$. The rationale for this change in the factor model is that this way the time $t−1$ expected value of the right hand side of (1) can then be interpreted as the risk premium for stock $n$ over the riskless asset. Although the focus of this paper is not on modeling expected returns, we still need to take a stand on how to model such risk premium. Multibeta models would imply the restrictions:

$$E_{t−1}[R_{nt}] = \beta_n^G E_{t−1}[f_g^t] + \sum_{c=1}^C \beta_{nc}^C E_{t−1}[f_c^t] + \sum_{i=1}^I \beta_{ni}^I E_{t−1}[f_i^t] \quad \text{for } n = 1, \ldots, N \quad (4)$$

Under these restrictions we would therefore estimate the following model:

$$R_{nt} = \beta_n^G E_{t−1}[f_g^t] + \sum_{c=1}^C \beta_{nc}^C E_{t−1}[f_c^t] + \sum_{i=1}^I \beta_{ni}^I E_{t−1}[f_i^t] + \beta_n^G (f_g^t − E_{t−1}[f_g^t])$$
$$+ \sum_{c=1}^C \beta_{nc}^C (f_c^t − E_{t−1}[f_c^t]) + \sum_{i=1}^I \beta_{ni}^I (f_i^t − E_{t−1}[f_i^t]) + \epsilon_{nt}, \quad (5)$$

Since the goal of this paper is to model comovement in international stock returns, rather than providing an asset pricing framework, we avoid for the time being imposing the restrictions in (4). The rationale for this choice is that under these restrictions the betas would be chosen to maximize the fit of the model both as a model of expected returns as well as a model of the covariance of stock returns. In this paper we focus on modeling the covariance. In future work, we plan to explore the asset pricing implications of the factor model.\footnote{Following the work of Ferson and Harvey (1993, 1995), we could for example model the time $t−1$ expectations on the mimicking portfolios ($E_{t−1}[f_g^t]$, $E_{t−1}[f_c^t]$, and $E_{t−1}[f_i^t]$) as (possibly linear) functions of time $t−1$ predetermined instruments, such global and local yields, yield curve spreads, and lagged returns.}
Given that we forego for now the goal of explaining expected returns, we model them in the simplest possible way. We assume that the expected excess return for stock $n$ coincides with its estimated sample mean:\(^3\)

$$E_{t-1}[R_{nt}] = \mu_n.$$  

(6)

In conclusion, the model we estimate is:

$$R_{nt} = \mu_n + \beta_n^1(f_t - E_{t-1}[f_t]) + \sum_{c=1}^{C} \beta_{nc}^c(f_t^c - E_{t-1}[f_t^c]) + \sum_{i=1}^{I} \beta_{in}^I(f_t^i - E_{t-1}[f_t^i]) + \epsilon_{nt}, \quad (7)$$

together with restrictions (3) and the assumption $E_{t-1}[\epsilon_{nt}] = 0$ for all $t$ and $n$.

Model (7) is essentially identical to the one postulated in Marsh and Pfleiderer (1997). The restrictions (3) imply that some of the methods for estimating APT models (Connor and Korajczyk 1986) are not immediately applicable, even if one is willing to impose the APT restrictions. Marsh and Pfleiderer (and Cavaglià, Cho, and Singer 2001) estimate this model using an iterative procedure that involves i) estimating the $\beta$s by OLS given the factors, and ii) estimating the factors by OLS given the $\beta$s. They use this procedure on the ground that “.. with the large cross-section of stocks ... we know of no feasible way to estimate the restricted factor model by maximum likelihood methods” (Marsh and Pfleiderer, 1997, page 9). A value-added of this paper is that we provide such a method. The appendix shows that the EM algorithm (Lehman and Modest, 1985) delivers an approach for computing maximum likelihood estimates of model (7) that is computationally feasible even for very large cross-sections.\(^4\)

\(^3\)The motivation for this choice is our prior that the overriding component of comovement in excess returns is driven by comovement in unexpected, rather than expected, excess returns. Ferson and Harvey (1993,1995) show that the adjusted R-squared of a regression of national stock markets returns on predetermined variables is at most about 10%. For individual stock returns the adjusted R-squared is likely to be even lower.

\(^4\)Convergence is reached whenever the mean squared gradient is less than $10^{-4}$. Lehman and Modest (1985) adopt a slightly tighter criterion, namely that the sum of the squared gradients is less than $10^{-4}$. Given that the EM algorithm is notoriously slow to converge close to the summit of the likelihood, and that the results do seem to change as long as the mean squared gradient is less than $10^{-2}$, we adopt a slightly looser convergence criterion.
the same intuition as Marsh and Pfleiderer’s iterative procedure, but unlike Marsh and Pfleiderer’s delivers maximum likelihood estimates. We believe our maximum likelihood approach to be preferable to the one of Marsh and Pfleiderer: The statistical properties of a maximum likelihood estimator are well known, while those of Marsh and Pfleiderer’s iterative method are not. In order to estimate model (7) via maximum likelihood we need to make distributional assumptions, however. Specifically, we assume that i) both the factors and the idiosyncratic shocks are normally distributed - conditional on time \( t - 1 \) information:

\[
f_t^g - E_{t-1}[f_t^g], f_t^c - E_{t-1}[f_t^c], f_t^i - E_{t-1}[f_t^i] \sim N(0, 1), \epsilon_{nt} \sim N(0, \sigma_n^2) \text{ for all } t, c, i, \text{ and } n,
\]

(8)

where the assumption of unitary variance is purely a normalization assumption, and ii) idiosyncratic shocks are cross-sectionally uncorrelated:

\[
E_{t-1}[\epsilon_{nt}\epsilon_{mt}] = 0 \text{ for all } t, n, \text{ and } m.
\]

(9)

We would like to be upfront about the limitations of the model used here. First of all, the maximum likelihood approach used here is applicable to a balanced panel only. This could potentially be an important shortcoming in that it prevents us from including firms that appeared after the start of the sample. However, in this specific case we have evidence that the use of a balanced panel may not distort the results substantially: in a previous paper (Brooks Del Negro, 2002a) we estimate the fixed effects model both for the balanced panel and the full sample of stocks and find that the results are essentially unchanged. Second, the assumptions of normality (8) and cross-sectional uncorrelatedness of the idiosyncratic shocks (9) - while standard in much of the APT literature - are certainly not innocuous. We address the latter assumption by increasing the number of factors, so as to capture the largest possible amount of cross-correlation. Finally, there are a priori reasons to think that the \( \beta \)s may well be

\[5\text{The assumption of unitary variance is standard in factor models: the variance of the factors and the } \beta \text{s are not separately identified.}\]
time-varying (see Giovannini and Jorion, 1989), given that the nature of firms changes over time and so may their exposure to global, country-specific and industry-specific shocks. Yet the current model does not have time-varying $\beta$s. We plan to address these limitations in future work. Nonetheless, we think that model (7) - as is - provides an interesting extension of the fixed effects model used in the literature. In addition, our results indicate that even this simple model potentially contains very useful information for portfolio managers.

3 The data

We use the dataset constructed by Brooks and Del Negro (2002a), which we briefly review here. Their data cover monthly total U.S. dollar-denominated stock returns and market capitalizations from January 1985 to February 2002 for 9,679 companies. They cover all the constituent firms in the Datastream country indices for 42 developed and emerging markets as of March 2002 and augment this list with active and inactive stocks for each market from Worldscope. Each company belongs to one of 39 Level 4 Datastream Global Equity industries (see www.ftse.com for a description of this classification). Table 1 lists these industries and shows how they can be aggregated into the broader (Level 3) FTSE industry sectors.

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$^6$A model with time-varying coefficients may also allow us to capture the change in returns' correlation between bull and bear markets documented in Longin and Solnik (1995) and Erb, Harvey and Viskanta (1995) and as discussed in Bekaert and Ang (2002).

$^7$Using US dollar-denominated returns has the effect of lumping nominal currency influences into country-specific shocks in international stock returns. We investigate the magnitude of this bias by redoing our estimations using returns denominated in foreign countries’ local currency and generally find it to be negligible.

$^8$The Datastream Global Equity industry assignments we use differ from the Morgan Stanley Capital International (MSCI) classification which is used by Rouwenhorst (1999) and elsewhere. In a different paper (Brooks and Del Negro, 2002a), we find that our results for the dummy-variables model using the DataStream classification are qualitatively similar to those obtained by authors that use the MSCI classifications, like Cavaglia, Brightman, and Aoked (2000). In that paper we also explore if the results change significantly when we switch from the Datastream industry assignments to the Dow Jones World Stock Index industry classification, which is used by Griffin and Karolyi (1998) for example, and find
The data we use differs in two respects from that in Brooks and Del Negro (2002a). First, we balance their dataset because our maximum likelihood algorithm cannot account for missing observations. The cross-section of firms for which stock returns and market capitalization data are continuously available from January 1985 to February 2002 amounts to 1965 companies in 21 developed and emerging markets. The country composition of this sample, along with the number of firms in each market, are: Australia (41), Austria (9), Belgium (26), Canada (89), Denmark (20), France (54), Germany (100), Hong Kong (51), Ireland (14), Italy (31), Japan (529), Korea (45), Malaysia (18), the Netherlands (56), Norway (16), Singapore (54), South Africa (23), Sweden (16), Switzerland (42), the UK (280) and the US (451).9 Our data set includes firms in 38 (out of 39) Level 4 industries.10 When aggregated into Level 3 industry sectors, the industry composition of the data is: basic industries (287), general industrials (320), cyclical consumer goods (119), non-cyclical consumer goods (226), cyclical services (306), non-cyclical services (42), utilities (92), information technology (73), financials (406) and resources (94). Second, we follow standard practice in the finance literature (see Ferson and Harvey, 1994, and Heston et al., 1995, for example) in estimating our factor model over excess US dollar-denominated stock returns, which we compute by subtracting the monthly total return for a 3-month US Treasury Bill from the individual stock returns.11

Although we lose a large number of firms by balancing the data, our coverage compares favorably to that in papers that estimate the simpler dummy variable model and

9In addition, when a factor (either country and industry) contains only one or two companies we eliminate the factor and the corresponding firms from the analysis. This is because in this case we cannot identify the idiosyncratic component separately from the country or industry factor.

10No companies in the “packaging” Level 4 industry are included in the balanced sample.

11We compute monthly total returns for the 3-month Treasury Bill using the Merrill Lynch 3-month Treasury Bill Index. The 3-month US Treasury Bill Index is comprised of a single issue purchased at the beginning of the month and held for a full month. At the end of the month, that issue is sold and rolled into a newly selected issue. The issue selected at each month-end re-balancing is the outstanding Treasury Bill that matures closest to, but not beyond 3 months from the re-balancing date. To qualify for selection, an issue must have settled on or before the re-balancing (month-end) date. While the index will often hold the Treasury Bill issued at the most recent or prior 3-month auction, it is also possible for a seasoned 6-month or 1-Year Bill to be selected.
are thus not subject to the same computational constrains. For example, Heston and Rouwenhorst (1994) examine data on 829 stocks in 12 European countries. Griffin and Karolyi (1998) collect data on 2,400 firms in 25 developed and emerging markets. Cavaglia et al. (2000) cover 2,645 firms in 21 developed countries.

For illustration, the overall market capitalization of our sample amounts to $15,296 billion in December 2000, which at that point is 47.4 percent of the global market in capitalization terms, according to the 2001 Standard & Poor’s Emerging Stock Markets Factbook. The United States makes up about 51.9 percent of overall market capitalization in our sample. The next biggest markets are Japan, the United Kingdom, Germany, Switzerland and France, which constitute 13.3 percent, 9.6 percent, 4.5 percent, 3.9 percent and 3.4 percent of the sample respectively. In contrast, emerging markets carry very little weight. The largest emerging market, Korea, makes up only 0.25 percent of the sample. In terms of market capitalization, companies in the information technology sector are most heavily represented, making up 22.5 percent of the sample. The next biggest sectors are non-cyclical consumer goods, general industrials, cyclical services, utilities and financials, at 22.1 percent, 12.3 percent, 10.1 percent, 8.5 percent and 7.6 percent respectively. Almost half of all companies in the information technology sector are located in the United States.

We estimate a baseline specification of our factor model using Level 4 industry affiliation to identify our industry-specific factors. In this respect, we follow Griffin and Karolyi (1998) who argue that broad industry classifications (such as Level 3) bias against finding important industry effects because they result in industry portfolios that are larger and therefore more diversified than country portfolios. However, since our factor model allows for heterogeneity within industries - unlike the fixed effects model our factor model does not restrict the factor loadings of firms within industries to be the same - this consideration is less important for us. In the event, we also check the fit of the model using the more aggregated Level 3 industry factors.
4 Results

This section describes the results. In this section we will refer to the baseline model as model (7) estimated with one global factor, 21 country factors (one for each country in our sample) and 38 industry factors (one for each Level 4 industry in the sample). This model is a natural starting point for the investigation, being close to the fixed effects model commonly used in the literature.

The section is composed of three parts. The first part will investigate whether it is worthwhile relaxing the loadings restrictions implicit in the fixed effects model (1). It will also compare the fit of the baseline model with that of alternative specifications. The second part will address the central question in the fixed effects literature: what is the most important source of variation in international stock returns? Is it country or industry shocks? Finally, the third part explores whether the information coming form the factor loadings of the factor model can be exploited for the purpose of constructing well-diversified portfolios.

4.1 Model selection

The likelihood based approach used in this paper provides a formal framework for addressing the issue of model selection. We exploit this framework to look into one of the central questions of the paper: Does it make sense to relax the restriction that the betas, whenever they are not zero, have to be the same across stocks? Specifically, we can test whether a version of model (7) where the restrictions (3) are replaced by the more stringent - restrictions (2) is rejected by the data or not. Given the normalization restrictions on the factor variances we will actually use a looser version of restrictions (2):

\[
\begin{align*}
\beta^G_n &= \beta^G \\
\beta^C_{nc} &= \begin{cases} 
\beta^C_c & \text{if stock } n \text{ belongs to country } c \\
0 & \text{otherwise}
\end{cases} \\
\beta^I_{ni} &= \begin{cases} 
\beta^I_i & \text{if stock } n \text{ belongs to industry } i \\
0 & \text{otherwise}
\end{cases}
\end{align*}
\] (10)
Note however that this is identical to estimating model (7) without normalization restrictions and imposing (2). Because of the auxiliary assumptions discussed in section 2 (distributional assumptions, expected returns) this is not exactly a comparison of model (7) with the fixed effects model in Heston and Rouwenhorst (1994) and Griffin and Karolyi (1998). Yet, conditional on the auxiliary assumption, the test will provide direct evidence of whether the data conform with the restrictions implicit in the fixed effects model - direct evidence that is absent from earlier work such as Marsh and Pfleiderer (1997). The value of the test statistic for the likelihood ratio test, equal to twice the differences between the log-likelihood values at the peak for the unrestricted and the restricted models, is equal to 38910.38. The number of degrees of freedom for the test is $5835 = (\# \text{ of stocks} - 1) \times (\# \text{ of factors})$. In spite of the large number of degrees of freedom, the difference in likelihoods is large enough that the restricted model is soundly rejected: the p-value is essentially zero. This indicates that relaxing the restrictions on the betas implicit in the fixed effects model may be fruitful, in the sense that the fit of the unrestricted model is far superior to that of the restricted one.

Next, we address the issue of model selection. There are many possible alternatives to the baseline model. Table 2 explores the fit of some of these alternatives relative to that of the baseline (model 1). One alternative is the restricted model we just discussed (model 2). Other alternatives are obtained by increasing or reducing the number of factors. Model 3 adds to the baseline an additional world factor. Model 4 augments the baseline with additional regional factors (MSCI Europe, MSCI Americas, and MSCI Pacific) to explore possible sources of comovement at the regional level such as, for instance, the Asian crisis for MSCI Pacific. Model 5 adds two factor to the baseline model. The second world factor is identified from the first by assuming that its loading on the first stock is zero.

We allow for region-specific factors by dividing our sample into three broad regions: the Americas, Asia and Europe. We follow Morgan Stanley Capital International in allocating the countries in our dataset to each of these regions. Our MSCI Americas region has two markets: Canada and the US. Our MSCI Asia region has six markets: Australia, Hong Kong, Japan, Korea, Malaysia and Singapore. Our MSCI Europe region has 12 countries: Austria, Belgium, Denmark, France, Germany, Ireland, Italy, the Netherlands, Norway, Sweden, Switzerland and the UK. Of course, our country coverage within regions...
model, one for mature and one for emerging markets. Models 6 and 7 use Level 3 industry factors in addition to and in place of, respectively, Level 4 industry factors. The idea is to investigate whether a coarser industry classification helps or not in terms of capturing comovement among international stock returns. Finally, model 8 adds both regional factors (Europe, Americas, and Pacific) and Level 3 industry factors to the baseline model.

Table 2 shows for each model the value of the log-likelihood at the peak, the number of unconstrained parameters, and the Bayesian information criterion (BIC). Since the variance-covariance matrix of excess returns is a sufficient statistic for the likelihood, the value of the likelihood at the peak indirectly measures a model’s ability to capture the in-sample correlations among excess returns. Comparisons of the log-likelihoods shows that, as one would expect, the restricted models always have a worse fit than the unrestricted ones: this result simply indicates that our numerical algorithm seems to deliver reasonable results. We focus on the BIC among the various criteria for model selection for two reasons. First of all, the BIC favors parsimonious models more than other criteria, like the Akaike. We want to avoid models that overfit in sample at the expense of out-of-sample forecasting precision. Second, some of these models are non nested, for example models (1) and (7). The BIC has a Bayesian interpretation as an approximate marginal data density, which allows for comparisons across non-nested models (Kass and Raftery, 1995).


\[ \text{BIC}_m = L_m - \frac{\ln(T)}{2} \times (\# \text{ of free parameters}) \]

where \( L_m \) is the log-likelihood at the peak and the for model \( m \).

does not match exactly that of the MSCI regions. For example, because our sample omits Latin American countries, the MSCI index for the Americas consists only of Canada and the US. See the MSCI coverage matrix (http://www.msci.com/equity/coverage_matrix.pdf) for more information.

14We classify mature markets as comprising the US, the UK, France, Germany, Italy, Japan, Canada, Australia, Austria, Belgium, Denmark, Ireland, the Netherlands, Norway, Sweden and Switzerland. The remaining countries are classified as emerging markets.

15The formula for the BIC for model \( m \) is (see Kass and Raftery 1995):

where \( L_m \) is the log-likelihood at the peak and the for model \( m \).
Table 2 shows that the restricted version of the baseline model is rejected even according to the conservative BIC - further evidence in favor of relaxing the restrictions on the betas implicit in the dummy-variable model. Relative to the other alternatives, the BIC favors the baseline model over models (3), (5), and (7). The addition of an extra world factor, or mature markets/emerging markets factors, and the replacement of Level 4 industry factors with the coarser Level 3 industry factors do not significantly improve the fit of the model. The addition of regional factors (model 4) to the country factors and the Level 3 industry factors to the Level 4 factors (model 6) seem to boost the baseline model’s fit considerably. Adding both regional and Level 3 industry factors to the baseline model (model 8) delivers a model whose BIC is greater than that of the baseline and of model 6, but worse than that of model 4. In the remainder of the paper we will focus on the baseline model for the sake of comparison with the previous literature. In light of the above results, we will also check for the robustness of our findings under models (4) and (8).

4.2 Country versus industry effects

The central issue in the literature on international diversification is whether international stock returns are mainly driven by country or by industry effects. In the former case, portfolio managers should focus on cross-country diversification, while in the latter case they should diversify across industries. This section tackles the issue from the perspective of the factor model described above. Since the factors are orthogonal with respect to each other, one can do an exact variance decomposition exercise. Specifically, from equation (7) it follows that the variance of excess returns for stock \( n \) can be exactly decomposed as the sum of the variances attributed to global, country, and industry shocks:

\[
Var(R_{nt}) = \beta^G_{n}^2 + \beta^C_{n}^2 + \beta^I_{n}^2 + \sigma_n^2,
\]

where \( \gamma \) and \( \iota \) denote the country and the industry stock \( n \) belongs to. Unlike the existing literature, with the exception of L’Her et al. (2002) who allow for global risk factors,

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16This is not the case in the dummy-variable model. See the discussion in Stockman (1988).
our variance decomposition accounts for a global factor.\footnote{L'Her, Sy and Tnami (2002) use a version of the dummy-variable model that controls for the fact that the exposure to the global factor can differ across firms. Specifically, they augment model (1) with the loadings of firms to four different market factors. Our approach has two advantages over theirs: i) we tackle the issue of firm-specific exposures to the global factor directly, without relying on a two step procedure and on an ex-ante specification of the global factors, ii) we let the betas to all factors - as opposed to the global factor only - be firm-specific.} Relative to the fixed effects literature, this is likely to reduce the importance of industry-specific shocks in our model because they are orthogonalized on a global shock.

Table 3 shows the actual variance and the variance decomposition for the following objects: the average across all individual stocks, the world market portfolio, the average across the 21 country portfolios, and the average across the 38 industry portfolios. All the portfolios - as well as the averages - are value weighted. The equal weighted results for the variance decomposition are not very different and hence are not shown for the sake of brevity.\footnote{For instance, the first number on the second line of Table 3 is the value-weighted average of the variances of all (value-weighted) country portfolios. The figures for the variance decomposition are the value-weighted averages of the corresponding figures for the country portfolios. All other figures are similarly constructed. The weights are beginning-of-sample market capitalization. Since the equally weighted results for the variance decomposition are not very different, we are confident that the results are robust to the choice of the time period for the market cap weights. For the equally weighted averages of portfolios, the weights are given by the relative number of stocks in each portfolio.} We will first describe the results for the baseline model, shown in the top panel of Table 3, and then consider the models with added regional factors (model 4) and, in addition, Level 3 industry factors (model 8) in the second and third panel.

The average stock return in the sample has a variance of 81 percent per month-squared. According to the baseline model country specific shocks explain 27% of the variance of the average stock, and are by far the most important factor. Global and industry shocks are roughly of the same importance, and each of them explains around 10% of the variance. The remainder of the variance - more than half of it - is attributed by the model to purely idiosyncratic (stock-specific) shocks.

The variance of the value-weighted world market portfolio for this sample is 18 percent per month-squared. Almost a quarter of this variance is due to the exposure of the portfolio to global shocks. Almost a third of the variance is attributable to country-specific
shocks, indicating that the value-weighted world portfolio is not well diversified in terms of country composition - not surprisingly - in addition to the fact that the country betas for individual stocks are relatively large, as we learned from the previous line of Table 3. Only a small percentage of the variance is due to industry effects: the world portfolio appears to be relatively well diversified in terms of industry composition. The remainder of the variance of the world portfolio, 45%, is in this case not the idiosyncratic variance, which in such a large portfolio is zero, but the fraction of the variance that is not captured by the model. The factor model provides only a parsimonious description of the variance-covariance matrix of stock returns: some of the comovement across stock returns is not captured by the model. This problem is present also in the fixed effects model, and is even more serious there given the further constraint imposed by the loadings restriction. Yet, this finding suggests that further research is needed in order to capture a larger degree of comovement across international stock returns.

The last two lines of the top panel show the variance decomposition for the averages of country and industry portfolio. The table shows that country and industry portfolios have on average the same variance, about 30 percent per month-squared. Actually, for value weighted portfolios the variance of the average industry portfolio is slightly above that of the average country portfolio.\(^{19}\) One might thus jump to the conclusion that diversifying across industries is more promising for risk reduction than diversifying across countries. The results from the variance decomposition indicate that one should be wary of this conclusion. For both the average industry and country portfolio the largest source of variability is the same: country-specific shocks. Of course, the quantitative importance of country-specific shocks is much larger for country portfolios (67%) than for industry portfolios (24%). Yet, even for industry portfolios country-specific shocks play a greater role than industry-specific shocks. This implies that cross-country diversification is more promising for risk reduction than cross-industry diversification.\(^{20}\) A key feature of our model is that the industry-specific shocks are orthogonalized on the global shock, while the global industry effects in the fixed effects literature may be picking up the changing

\(^{19}\) The opposite is true for the equal weighted portfolio.

\(^{20}\) Another interesting fact is that the residual variance is larger for industry portfolio than for country portfolios: the baseline model seems to better capture comovement within countries than across them.
importance of global co-movements, as noted by Marsh and Pfleiderer (1997). It turns out that, once we control for global sources of systematic risk, industry-specific shocks are not important.

In summary, the findings from the variance decomposition exercise conform with much of the existing literature: country shocks are more important than industry shocks. Is this a sign that industry-specific shocks, counter to evidence on real activity in Stockman (1988) and more recently for the stock market in Cavaglia et al. (2000), are not an important source of systematic risk? Why is this the case? A value added from our analysis is that we can address this question by looking at the distribution of factor loadings across stocks within an industry or country. We find that the degree of heterogeneity in stocks’ exposure to industry-specific shocks is generally much greater than it is for country-specific shocks. Countries are relatively homogeneous clusters of stocks. For all countries in our sample, all stocks within a country have pro-cyclical factor loadings, that is, all country betas have the same sign and are positive (except for the U.S. where .2% of firms have counter-cyclical betas). Country-specific shocks therefore conform better to the notion of a macroeconomic shock, which through a policy change or business cycle shocks affects all stocks within a country similarly. The matter is very different for industries, where stocks are quite heterogeneous in their exposure to industry-specific shocks. On average, 15% of the companies in any Level 4 industry have a negative industry betas (23% when the average is equal weighted). The standard deviation of the cross-sectional distribution of betas (within each country or industry) is on average 1.5% within countries and 2% within industries. These figures indicate that the Level 4 industry classifications are not as informative as country groupings.

\[21\] There is also a high degree of across-industry heterogeneity. The within-industry heterogeneity is larger for some industries than for others.

\[22\] The sign of the betas in model (7) is indeterminate, as in every factor model: One can multiply the betas and the factor by $-1$ and obtain the expression. We solve this indeterminacy by imposing that for each factor the mean of the exposures is positive. It turns out that for all factors the sign of the mean and the median is the same. So a stock has a positive or pro-cyclical country (industry) beta whenever the exposure of that stock to a country (industry) shock has the same sign as the average exposure for that country (industry).
other words, if stocks \( x \) and \( y \) belong to the same country, their returns are more likely to move together than if they belong to the same industry. As a result, many industry portfolios are - paradoxically - relatively well-diversified in their exposure to industry shock and hence are on average more exposed to country than industry shocks.

Should our result be interpreted as a sign that the Level 4 industry sectors are flawed? We follow the literature, for example Baca et al. (2000), in using the Datastream Global Equity indices in assigning stocks to industries. In contrast to Baca et al. (2000), however, our approach to industry classification is much more disaggregated. We assign stocks to one of 38 (Level 4) industries, while they assign stocks to one of 10 (Level 3) FTSE industry sectors. From the outset, our approach therefore allows for more heterogeneity in the nature of industry-specific shocks than is typical. Of course, it is possible that the Datastream Global Equity indices simply get the industry assignment of firms wrong, lumping firms that are actually in very different industries together by accident. In a different paper (Brooks and Del Negro, 2002a), we investigate this possibility. As a robustness test, we explore if the results change significantly when we switch from the Datastream industry assignments to the Dow Jones World Stock Index industry classification, which is used by Griffin and Karolyi (1998) for example. Our results are qualitatively unchanged across classifications, which suggests that there is no systematic error in the way Datastream assigns firms to industries.

We have checked the robustness of these variance decomposition results to the inclusion of regional factors (model 4) and the Level 3 industry factors (model 8). We find that the inclusion of regional factors takes some explanatory power away from country-specific shocks. We also find that the addition of Level 3 industry factors also leads to an increase in the explanatory power of industry factors in general.\(^{23}\) The conclusions

\(^{23}\)Interestingly, the broader Level 3 industry groupings are no more heterogeneous than the Level 4 industry classifications. Whenever Level 3-FTSE groupings replace Level 4 groupings (model 7) the average fraction of negative industry betas is higher for Level 3 groups (19\%) than for Level 4 classifications (15\%), but the average cross-sectional standard deviation for Level 3 is lower than for Level 4 (1.9 versus 2\%). All the above figures are value weighted averages.
from the baseline model, however, are robust: It is always the case that for all portfolios (including the average industry portfolio) the sum of the variance due to regional and country factors exceeds that due to industry factors. \(^24\)

We conclude this section by returning to the analysis of the variance decomposition for the excess returns on the world portfolio. This variance decomposition is particularly interesting, because in much of the international finance literature (see for instance Ferson and Harvey, 1993) the world portfolio is considered the most important risk factor for international stocks.\(^25\) What lies behind such a pervasive risk factor? In a model that allows for a global factors and for global industry factors, we find that the driving force behind movements in the excess return on the world market portfolio are local. This may be a surprising result. The implication of this result is that in order to diversify away exposure to the world market portfolio one needs to select stocks that have low exposure to local risk in the first place. This idea is explored in the next section.

### 4.3 International diversification strategies

Can we use the information coming from the betas for international diversification strategies, that is, to construct portfolios that have lower variance than the world market portfolio? Simple intuition suggests that we can. The whole purpose of cross-country (cross-industry) diversification is to reduce the exposure of a portfolio to country-specific (industry-specific) shocks. This indicates that if we select stocks that have a lower exposure to country (industry) shocks in the first place, we can further reduce the variance of the portfolio. Does this simple intuition hold in practice? Is it quantitatively important

\(^24\)In several cases, the separate identification of regional and country factors, although in principle feasible, may in practice be difficult. As an example, the Americas factor includes only two countries, the U.S. and Canada.

\(^25\)Ferson and Harvey (1993) report that the excess return on the world market portfolio explains between 5 to 71\% of the variance of country indices. They state: "By this measure [the \(R^2\)] the global risk factors explain, ex post, 14 to 18 percent of the variance over the 1975-1989 period. ...[W]e found that the world market portfolio is by far the most important factor in this sense" (Ferson and Harvey 1993, pg. 539-40).
- i.e., by how much can we improve over the world market portfolio in terms of variance reduction? Should portfolio managers choose stocks with low exposure to global, country, or industry shocks? Is the information coming from the betas robust enough that it can be used out-of-sample? These are the issues addressed in this section.

We use the information coming from the baseline factor model to construct three large value- or equal-weighted portfolios, with each portfolio containing half of the sample. The first value-weighted portfolio, a low-global-betas portfolio, is constructed by taking all stocks whose exposure to the global shock is below the median. This portfolio will then have a lower exposure to global shocks than the market and hence possibly a lower variance. The second value-weighted portfolio, a low-country-betas portfolio, is obtained by taking for each country all stocks whose exposure to country-specific shocks is below the median. This portfolio will have a lower exposure to country-specific shocks than the market. The analysis in the previous section suggests that this portfolio is the best candidate in terms of reducing overall variability. The third and last value-weighted portfolio, a low-industry-betas portfolio, is obtained by taking for each industry all stocks whose exposure to the industry-specific shock is below the median. From the previous section we do not expect this portfolio to deliver a substantial reduction in volatility relative to the market. Table 4 provides information on the volatility of these portfolio relative to the global market portfolio. Later in the section we will discuss the ex-post returns of these portfolios in excess of the market’s.

Table 4 shows that our expectations are generally met. The first line of Table 4 shows

\footnote{The value weights are again beginning-of-sample weights.}

\footnote{It is not mechanical that the portfolio will have a lower variance. If the exposure to global shocks were to be inversely correlated to the exposure - say - to country shocks, this portfolio may end up with a higher variance.}

\footnote{We could have in principle used the betas information to find the portfolio that minimizes the expected variance. We choose not to pursue this approach on the ground that it may not be robust. The betas are estimated imprecisely, and a portfolio that minimizes in sample variance may not do so well out-of-sample. The approach we follow here does not rely too much on the point estimates of the betas.}
the market for the full sample period. The left panel shows the value-weighted results, while the right panel shows the equal-weighted results. The table shows that the low global and country betas portfolios have a variance which is 17 percent lower than the market for the value-weighted portfolio, and 18 and 27 percent lower than the market, respectively, for the equally weighted portfolio. Not surprisingly the low-industry-betas portfolio does not perform very differently from the market portfolio, and actually has a 4 percent higher variance. What is more surprising however is that the low-global-betas portfolio performs as well as the low-country-betas portfolio, at least for the value weighted results, even though from the variance decomposition global shocks are not nearly as important as country-specific shocks. The second line shows the variability relative to the market of the residual value- or equal-weighted portfolios, the high global, country, and industry betas portfolios. We expect the variability in these portfolio to be the mirror image of the corresponding low-betas portfolio (the lower the variability of the low-betas, the higher that of the high-betas portfolio) and indeed this is the case. We find that the loss from the high-betas portfolios is generally higher than the gain from the low-betas portfolio. This is because the distribution of betas for many factors (global, country, and industry) is skewed: those stocks with a loading above the median tend to have an exposure further away from the median than those stocks with a betas below the median.

In order to check for the robustness of these results we have computed the variability relative to the market of the three low-betas portfolios for eight two-year subsamples, starting from the end of the sample. These results are available upon request; they are omitted here for brevity. The gains from the three portfolios relative to the market in terms of variability change across subsamples. Of the three portfolios, the low-country-betas portfolio is the most robust. For all subsamples the variance of the low-country-betas portfolio is less than that of the market portfolio, although the gains range from 27 to 9 percent for the value-weighted portfolio, and from 37 to 16 percent for the equally weighted portfolio. This is not the case for the low-global-betas portfolio: for some subsamples this portfolio has higher variance than the market (up to 16 percent higher for the value-weighted portfolio). While the low-global-betas portfolio performs just-as-
well as low-country-betas portfolio for the full sample, this performance appears to be driven by the later part of the sample, where the variance reduction reaches 60 percent.

Up to this point all the results are in-sample. How robust is the betas information out-of-sample? Are the betas estimated precisely enough, and are they stable enough over time, that they can be used out-of-sample? We leave the last two years, from March 2000 to February 2002, out of the sample and repeat the exercise. Since the factor model is heavily parameterized we cannot afford to leave too much data out of the sample. However, the March 2000-February 2002 period, albeit short, is certainly a trying one in terms of variability in stock returns. We construct the value-weighted low global, country, and industry betas portfolios using only information up to February 2000. The weights used to construct the portfolios are given by the February 2000 capitalization. We then check in the last two lines of Table 4 the out-of-sample performance of these portfolios. We find that the low-industry-betas portfolio performs roughly as well as the market, the low-global-betas portfolio performs considerably worse than the market and, finally, the low-country-betas portfolio performs substantially better than the market. In conclusion, this robustness check is encouraging in that it suggests the information coming from the factor model can be used successfully also out-of-sample.

Table 4 has shown that the low-country-betas portfolio deliver substantial gains in terms of risk reduction relative to the market portfolio. Do we really need to estimate a factor model in order to construct portfolios that have low exposure to country shocks? In other words, can the low-country-betas portfolio be mimicked on the basis of information other than the betas from the factor model? One possibility is that more international firms, either in terms of sales from abroad, income from abroad, or assets held abroad, have a lower exposure to country-specific shocks. Using international sales, income, and assets data from Worldscope we then construct value-weighted portfolios of "international" companies, and check whether these portfolio also deliver a reduction in variance relative to the market portfolio. Specifically, we construct portfolios comprising all firms that are in the highest 25 and 10 percentile for international sales, income, and assets. We find that none of these portfolio actually does better than the market, and
some of them do substantially worse (these figures are available upon request). We also analyze the distribution by industry of the companies that are part of the low-country-betas portfolio. The goal of this exercise is to find out whether most low-country-betas companies belong to a few specific sectors, in which case we could replicate the low-country-betas portfolio by simply constructing a portfolio that includes only companies belonging to these sectors. The three largest sectors in terms of representation in the low-country-betas portfolio are “Electronics and Electrical Equipment” (8.2% of the low-country-betas companies belong to this sector) and two financial sectors, “Banks” (7.25%) and “Investment Companies” (6.74%). When we look at value-weighted portfolios that load exclusively on these industries we find that their volatility is generally higher than that of the market. We conclude that it is not trivial to replicate the information coming from the betas.

We have established that the low-country-betas portfolios deliver a reduction in variance relative to the market, which is a valuable feature to risk-averse investors. We now want to know the relative performance of these portfolios in terms of ex-post mean returns. The first line of Table 5 shows the ex-post mean excess returns for the full sample period relative to the three month T-bill for the world market portfolio, and the low global, country, and industry betas portfolios (returns are month-to-month). The left and right panels of Table 5 show the results for value-weighted and equal-weighted portfolios, respectively. We see that all three portfolios - whether value or equal weighted - outperformed the market in terms of ex-post returns.29 The second line of Table 5 shows the associated Sharpe ratios. For value-weighted results the ranking of the Sharpe ratios sees the low-global-betas portfolio first, the low-country-betas portfolio second (this portfolio has the same in-sample volatility as the low-global-betas portfolio, but a lower ex-post return), followed by the low-industry-betas and the world market portfolios. The first two positions in the Sharpe ratios ranking are reversed for equal weighted

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29 Since we do not use the factor model as an asset pricing model in this paper, we cannot infer too much from this result - for instance, we cannot say whether this result is driven by the lack of financial market integration. However, these results suggests that a static multi-beta model, one where risk premia are not time-varying, would have a hard time fitting the data.
portfolios, on the ground that low-global and low-country betas portfolios have the same ex-post return, but the latter has a lower standard deviation. In conclusion, we see that both the low-global and low-country betas portfolios beat the world market in terms of risk-adjusted ex post returns, in sample.\textsuperscript{30} What about out-of-sample? This question is addressed in the last line of Table 5, which shows the mean excess returns relative to the three month T-bill for the March 2000-February 2002 period. Given that this period is not a march of glory for global stock returns, all ex-post excess returns are negative (for this reason we do not show the \textit{Sharpe} ratios for this period). However, the relative performance of the portfolios is very different. The low-global-betas portfolio performs far worse than the market, while the low-country-betas portfolio performs better than the market, both for value and equal weighted portfolios. As we know from Table 4, the out-of-sample performance in terms of volatility for the low-country-betas portfolio is also better than both the world market’s and the low-global-betas portfolio's.

\section{5 Conclusions}

This paper builds a factor model where the factors are identified. Specifically, we impose restrictions such that the factors can be characterized as global, country-specific or industry-specific shocks. This model improves on a workhorse model of the international diversification literature, a widely used fixed effects model, because it relaxes the restriction that the exposure to shocks, whenever it is non-zero, is the same across stocks. Relative to existing work that attempts to build a similar model (Marsh and Pfeiderer, 1997), our contribution is that we develop an algorithm that makes it possible to compute the maximum likelihood estimates for the parameters even with a very large cross-section.

When we test the restriction imposed by the fixed effects model, we find that it is strongly rejected by the data. We then use our factor model to investigate whether

\textsuperscript{30}We also check for the relative performance of the portfolios across two-year subsamples, and find that the low-global and low-country-betas portfolios fare better than the market for six out of eight subsamples in terms of ex-post mean excess returns.
country- or industry-specific shocks are the predominant source of return variation. We find that country-specific shocks are the predominant source of variation in returns for the average stock, the global stock market portfolio, the average country portfolio and—to our surprise—also for the average industry portfolio. Why are country-specific shocks so much more important in explaining international return variation than industry-specific shocks? An examination of the factor betas on both sources of systematic risk shows that the degree of heterogeneity in stocks’ exposure to industry-specific shocks is much greater than it is for country-specific shocks. Indeed, the fraction of stocks that load anti-cyclically on industry shocks is far greater than for country shocks. Country shocks therefore conform better to our notion of an aggregate-level shock, which through a policy change or a business cycle shock affects all stocks within a country similarly. An additional reason for the small role of industry shocks in our model is that we allow explicitly for a global factor in international stock returns. In contrast, it is possible that in the fixed effects literature the importance of such a factor in explaining return variation is subsumed in the global industry effects.

We next use the betas of stocks on the different sources of systematic risk to construct large portfolios that are well diversified in their exposure to global, country-specific or industry specific shocks. Our variance decomposition suggests that a portfolio that includes stocks with low exposures to country-specific shocks is promising in terms of risk reduction. The in-sample results, the results for different subsamples and the out-of-sample results all substantiate this promise. Our candidate portfolio is always less variable than the market portfolio, although the gain in terms of risk reduction varies across sub periods. We also find that it outperforms the market in terms of both in-sample and out-of-sample mean returns.

For portfolio managers who follow a top-down approach, typically first diversifying across countries and then choosing the best stocks in each market, our analysis is useful in two regards. First, it shows that, using information on stocks’ exposure to different sources of systematic risk, country-specific shocks are by far the most important source
of international return variation. International diversification strategies that are based on cross-country diversification therefore still have merit. Second, our approach provides portfolio managers with information on which stocks to pick within countries.

There are several limitations to our model and hence much research lies ahead. A first limitation is that the maximum likelihood algorithm is applicable to a balanced sample only. Secondly, the assumptions of normality and cross-sectional uncorrelatedness of idiosyncratic shocks - while standard in much of the APT literature - are not innocuous. Third, there are a priori reasons to think that the exposures to the factors may well be time-varying, given that the nature of firms changes over time and with it their exposure to global, country and industry-specific shocks. Finally, and most importantly, we plan to explore the asset pricing implications of the factor model.
References


A Computational Appendix

The factor model (7) can be written in compact form as:

\[ R_t = B f_t + \epsilon_t, \quad (A1) \]

where \( R_t \) (the de-meaned returns) and \( \epsilon_t \) are \( N \times 1 \) vectors, \( f_t \) is a \( K \times 1 \) vector (\( K \) is the total number of factors), \( B \) is a \( N \times K \) matrix that includes the \( \beta \) s. The variance-covariance matrix of \( \epsilon_t \), called \( \Phi \), is diagonal, following assumption (8). It is hard to maximize the likelihood for (A1) by brute force when \( N \) is very large, given the large number of parameters (see Table 2). The application of the EM algorithm simplifies the problem considerably from a numerical point of view (see Lehmann and Modest 1985). The EM follows the intuition that if the factors were observable \( B \) could be estimated by means of OLS - equation by equation (given that \( \Phi \) is diagonal).

The EM algorithm is an iterative procedure where at each step the factors \( f_t \) are replaced by their conditional expectations given the observations and the value of the parameters obtained at the end of the previous step, and then applies standard regression tools to obtain a new estimate of the parameters until convergence. Each iteration of the algorithm is bound to increase the likelihood, so that convergence to a possibly local- maximum is guaranteed. The value added of this section relative to Lehman and Modest consists in showing how to estimate the model (7) under the restrictions (3).

Given the distributional assumptions, and assuming a flat prior on all the parameters of interest, the logarithm of the joint posterior distribution of \( B, \Phi \) and \( f \), given the observations \( R \), is equal to the likelihood of the model if the factors were observable

\[ \ln pdf(B, \Phi, f | R) \propto -\frac{T}{2} \ln |\Phi| + \frac{1}{2} \sum_{t=1}^{T} (R_t - B f_t)' \Phi^{-1} (R_t - B f_t) + \frac{1}{2} \sum_{t=1}^{T} f_t' f_t \quad (A2) \]

Let us call \((B^q, \Phi^q)\) the values of the parameters obtained at the end of the previous \((q^{th})\) iteration of the algorithm. The first step of the EM algorithm involves taking the expectation \( (E) \) of (A2) with respect to the conditional distribution of \( f \) given \((B^q, \Phi^q)\) and \( R \). If we denote by \( E_q[\cdot] \) the expectation taken with respect to the conditional distribution of \( f \) given \((B^q, \Phi^q)\) and \( y \), we can write the E-step as:

\[ E_q[\ln pdf(B, \Phi, f | y)] = -\frac{T}{2} \ln |\Phi| + tr \{ \Phi^{-1} [S - 2B (\sum_{t=1}^{T} E_q[f_t]R_t') + B (\sum_{t=1}^{T} E_q[f_t f_t'] / T)]B' \} \quad (A3) \]

where \( S = \sum_{t=1}^{T} R_t R_t' \). One obtains that (see Lehman and Modest 1985):

\[ \sum_{t=1}^{T} E_q[f_t] R_t' / T = (I_k + B' \Phi^{-1} B)^{-1} B' \Phi^{-1} S \equiv N_1 \quad (A4) \]
\[
\sum_{i=1}^{T} \frac{E_q[f_i f_i']}{T} = (I_k + B' \Phi^{-1} B)^{-1}
\]
\[
+ (I_k + B' \Phi^{-1} B)^{-1} B' \Phi^{-1} S \Phi^{-1} B (I_k + B' \Phi^{-1} B)^{-1} \equiv N_0,
\]

where we drop the superscript \(q\) for convenience. The second step consists in maximizing \((M)\) the resulting expression with respect to \((B, \Phi)\). Since the joint maximization is complicated, we split the \((M)\) step into a number of conditional maximization \((CM)\) (see Gelman et al. 1994). Each substep consists in maximizing \((A3)\) with respect to a first set of parameters, keeping all other parameters constant at the level attained at the end of the previous substep. When \(N\) is large and restrictions (3) are present, it is convenient to divide the maximization or \((A3)\) with respect to \(B\) into \(N\) maximizations - equation by equation. In order to see how this can be accomplished, let us note that \((A3)\) can be rewritten as (considering the case \(N = 2\) to simplify the notation):

\[
E_q [\ln pdf(B, \Phi, f[y])] = tr\{\Phi^{-1}(S - 2BN_1 + BN_0B')\}
\]
\[
= tr\{\Phi^{-1}\left[
\begin{bmatrix}
S_{11} & S_{12} \\
S_{12} & S_{22}
\end{bmatrix}
- 2 \begin{bmatrix}
B^1 & B^1 & [N_1 N_2] \\
B^2 & B^2 & 0
\end{bmatrix}
N_0[B'BB']
\right]\}
\]
\[
= tr\left\{
\begin{bmatrix}
\phi_i^{-1} & 0 \\
0 & \phi_i^{2-1}
\end{bmatrix}
\begin{bmatrix}
S_{11} - 2B^1 N_1 + B^1 N_0 B^{1'} & S_{12} - 2B^1 N_2 + B^1 N_0 B^{2'} \\
S_{12} - 2B^2 N_1 + B^2 N_0 B^{1'} & S_{22} - 2B^2 N_2 + B^2 N_0 B^{2'}
\end{bmatrix}
\right\}
\]

where \(B^i\) is a \(1 \times K\) vector. Since the likelihood - as a function of \(B\) - depends only on the trace, we can ignore the off-diagonal terms. If there are no cross-equation restrictions on \(B\) - this is the case for restrictions (3) - each diagonal term depends on \(B^i\) \((i = 1, \ldots, n)\) only. Call \(p^i\) the number of free parameters in \(B^i\), \(b^i\) the \(p^i \times 1\) vector that contains only the unconstrained elements of \(B^i\), and \(M^i\) the \(p^i \times k\) matrix s.t. \(M^i B^i = b^i\). Then:

\[
b^{i,q+1} = [M^i N_0 M'^i]^{-1} M^i N_1^i
\]

Note that \(M^i N_0 M'^i\) simply select the rows and columns of \(N_0\) corresponding to the factors that are relevant to stock \(i\). Call \(B^{q+1}\) the outcome of the \(N\) maximizations. The last \(CM\) step consists in maximizing \((A3)\) with respect to \(\Phi\), for given \(B^{q+1}\), obtaining:

\[
diag(\Phi) = diag(S - 2B^{q+1} N_1 + B^{q+1} N_0 B^{q+1'}).
\]

Note that the \((M)\) step is essentially OLS equation by equation, as in Marsh and Pfeilerer (1997). But the \((E)\) step - where one obtains the estimates of \(f_1\) given the betas - is different. Hence the two algorithms will not in general converge to the same estimator.
Table 1: Industry Sectors

<table>
<thead>
<tr>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>BASIC Basic Industries</td>
<td>CHMCL Chemicals</td>
</tr>
<tr>
<td></td>
<td>CNSBM Construction &amp; Building Materials</td>
</tr>
<tr>
<td></td>
<td>FSTPA Forestry &amp; Paper</td>
</tr>
<tr>
<td></td>
<td>STLOM Steel &amp; Other Metals</td>
</tr>
<tr>
<td>GENIN General Industrials</td>
<td>AERSP Aerospace &amp; Defense</td>
</tr>
<tr>
<td></td>
<td>DIVIN Diversified Industrials</td>
</tr>
<tr>
<td></td>
<td>ELTNC Electronic &amp; Electric Equipment</td>
</tr>
<tr>
<td></td>
<td>ENGEN Engineering &amp; Machinery</td>
</tr>
<tr>
<td>CYCGD Cyclical Consumer Goods</td>
<td>AUTMB Automobiles &amp; Parts</td>
</tr>
<tr>
<td></td>
<td>HHOLOD Household Goods &amp; Textiles</td>
</tr>
<tr>
<td>NCYCG Non-Cyclical Consumer Goods</td>
<td>BEVES Beverages</td>
</tr>
<tr>
<td></td>
<td>FOODS Food Producers &amp; Processors</td>
</tr>
<tr>
<td></td>
<td>HLTHC Health</td>
</tr>
<tr>
<td></td>
<td>PCKGN* Packaging</td>
</tr>
<tr>
<td></td>
<td>PERSH Personal Care &amp; Household Products</td>
</tr>
<tr>
<td></td>
<td>PHARM Pharmaceuticals</td>
</tr>
<tr>
<td></td>
<td>TOBAC Tobacco</td>
</tr>
<tr>
<td></td>
<td>BIOTE Biotechnology</td>
</tr>
<tr>
<td>CYSER Cyclical Services</td>
<td>DISTR Distributors</td>
</tr>
<tr>
<td></td>
<td>RTAIL Retailers, General</td>
</tr>
<tr>
<td></td>
<td>LESUR Leisure, Entertainment &amp; Hotels</td>
</tr>
<tr>
<td></td>
<td>MEDIA Media &amp; Photography</td>
</tr>
<tr>
<td></td>
<td>SUPSV Support Services</td>
</tr>
<tr>
<td></td>
<td>TRNSP Transport</td>
</tr>
<tr>
<td>NCYSR Non-Cyclical Services</td>
<td>FDRET Food &amp; Drug Retailers</td>
</tr>
<tr>
<td></td>
<td>TELCM Telecom Services</td>
</tr>
<tr>
<td>UTILS Utilities</td>
<td>ELECT Electricity</td>
</tr>
<tr>
<td></td>
<td>GASDS Gas Distribution</td>
</tr>
<tr>
<td></td>
<td>WATER Water</td>
</tr>
<tr>
<td>ITECH Information Technology</td>
<td>INFOH Information Tech. Hardware</td>
</tr>
<tr>
<td></td>
<td>SFTCS Software &amp; Computer Services</td>
</tr>
<tr>
<td>TOTLF Financials</td>
<td>BANKS Banks</td>
</tr>
<tr>
<td></td>
<td>INSUR Insurance</td>
</tr>
<tr>
<td></td>
<td>LIFEA Life Assurance</td>
</tr>
<tr>
<td></td>
<td>INVSC Investment Companies</td>
</tr>
<tr>
<td></td>
<td>RLEST Real Estate</td>
</tr>
<tr>
<td></td>
<td>SPFIN Speciality &amp; Other Finance</td>
</tr>
<tr>
<td>RESOR Resources</td>
<td>MNING Mining</td>
</tr>
<tr>
<td></td>
<td>OILGS Oil &amp; Gas</td>
</tr>
</tbody>
</table>

Notes: The asterisk indicates that the sector is not included in the balanced sample.
Table 2: Model Selection

<table>
<thead>
<tr>
<th>Model</th>
<th>Log-Lik</th>
<th># parameters</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) baseline</td>
<td>842258.00</td>
<td>7860</td>
<td>821319.45</td>
</tr>
<tr>
<td>(2) restricted</td>
<td>822802.81</td>
<td>2025</td>
<td>817408.33</td>
</tr>
<tr>
<td>(3) with additional world factor</td>
<td>846373.40</td>
<td>9824</td>
<td>820202.87</td>
</tr>
<tr>
<td>(4) with regional factors</td>
<td>848570.68</td>
<td>9802</td>
<td>822458.76</td>
</tr>
<tr>
<td>(5) with mature/emerging mkts factors</td>
<td>846999.37</td>
<td>9825</td>
<td>820526.17</td>
</tr>
<tr>
<td>(6) with level 3 industry factor</td>
<td>848292.40</td>
<td>9825</td>
<td>822119.21</td>
</tr>
<tr>
<td>(7) with level 3 industry factor (replacing level 4 factors)</td>
<td>839017.37</td>
<td>7860</td>
<td>818078.81</td>
</tr>
<tr>
<td>(8) with regional &amp; level 3 industry factor</td>
<td>853472.57</td>
<td>11767</td>
<td>822126.01</td>
</tr>
</tbody>
</table>

Notes: The first column indicates the value of the likelihood \( L \) at the peak. The second column indicates the number of parameters for each model (excluding the means). The last column shows the Bayesian Information Criterion (BIC), computed as \( BIC = L - \frac{\ln(T)}{2} \times (\# \text{ of free parameters}) \), where \( T \) is the sample size.

Table 3: Variance Decomposition

<table>
<thead>
<tr>
<th></th>
<th>variance</th>
<th>global</th>
<th>country</th>
<th>industry</th>
<th>residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>average across stocks</td>
<td>88.5</td>
<td>8.3</td>
<td>27.3</td>
<td>9.4</td>
<td>55.0</td>
</tr>
<tr>
<td>world market portfolio</td>
<td>16.5</td>
<td>22.5</td>
<td>36.7</td>
<td>1.6</td>
<td>39.3</td>
</tr>
<tr>
<td>average across country portfolios</td>
<td>30.3</td>
<td>12.8</td>
<td>72.4</td>
<td>2.2</td>
<td>12.6</td>
</tr>
<tr>
<td>average across industry portfolios</td>
<td>30.4</td>
<td>18.8</td>
<td>28.0</td>
<td>16.9</td>
<td>36.3</td>
</tr>
</tbody>
</table>

Notes: All figures are in percent. Portfolios are value-weighted using beginning-of-sample capitalization.
Table 4: Variance Ratio Relative to the Market for the Low and High Betas Portfolios

<table>
<thead>
<tr>
<th></th>
<th>global</th>
<th>country</th>
<th>industry</th>
<th>global</th>
<th>country</th>
<th>industry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>value</td>
<td>weighted</td>
<td></td>
<td>equal</td>
<td>weighted</td>
<td></td>
</tr>
<tr>
<td>low loadings portfolio - full sample</td>
<td>83.13</td>
<td>83.26</td>
<td>104.61</td>
<td>82.27</td>
<td>73.23</td>
<td>100.96</td>
</tr>
<tr>
<td>high loadings portfolio - full sample</td>
<td>155.46</td>
<td>150.49</td>
<td>101.12</td>
<td>144.51</td>
<td>135.18</td>
<td>101.82</td>
</tr>
<tr>
<td>low loadings portfolio - out-of-sample</td>
<td>225.77</td>
<td>60.25</td>
<td>104.47</td>
<td>152.60</td>
<td>67.17</td>
<td>101.66</td>
</tr>
<tr>
<td>high loadings portfolio - out-of-sample</td>
<td>42.97</td>
<td>195.69</td>
<td>103.21</td>
<td>73.30</td>
<td>145.58</td>
<td>102.15</td>
</tr>
</tbody>
</table>

Notes: The numbers are in percent. They represent the ratio of the ex-post variance of excess return relative to the monthly return on a three-month T-Bill for the appropriate low or high betas portfolio relative to the ex-post variance of the excess returns for the world market portfolio for the same period.

Table 5: Ex Post Returns and Sharpe Ratios - Market versus Low Betas Portfolios

<table>
<thead>
<tr>
<th></th>
<th>market</th>
<th>global</th>
<th>country</th>
<th>industry</th>
<th>market</th>
<th>global</th>
<th>country</th>
<th>industry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>value</td>
<td>weighted</td>
<td></td>
<td>equal</td>
<td>weighted</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean return - full sample</td>
<td>0.38</td>
<td>0.49</td>
<td>0.43</td>
<td>0.41</td>
<td>0.35</td>
<td>0.41</td>
<td>0.41</td>
<td>0.37</td>
</tr>
<tr>
<td>Sharpe ratio - full sample</td>
<td>0.09</td>
<td>0.13</td>
<td>0.12</td>
<td>0.10</td>
<td>0.08</td>
<td>0.10</td>
<td>0.11</td>
<td>0.08</td>
</tr>
<tr>
<td>mean return - out-of-sample</td>
<td>-1.56</td>
<td>-2.50</td>
<td>-1.54</td>
<td>-1.59</td>
<td>-0.71</td>
<td>-1.32</td>
<td>-0.59</td>
<td>-0.67</td>
</tr>
</tbody>
</table>

Notes: For each portfolio (world market, low-global, country, and industry betas portfolios), the return figures are the ex-post month-to-month mean excess return relative to the monthly return on a three-month T-Bill. The Sharpe ratio is equal to the return divided for the standard deviation of excess returns for the same time-period.