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Optimal monetary policy, asset purchases, and credit market frictions

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Optimal monetary policy, asset purchases, and credit market frictions[1]

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Abstract

We examine how borrowing constraints affect monetary transmission and the trade-off of a welfare maximizing central bank. We develop a sticky price model where money serves as the means of payment and ex-ante identical agents borrow/lend among each other. The credit market is distorted as borrowing is constrained by available collateral, while the distortion is amplified under higher nominal interest rates. We show that the central bank cannot implement first best and that optimal monetary policy mainly aims at stabilizing prices. We further demonstrate that central bank purchases of loans can alleviate the borrowing constraint and enhance social welfare.

JEL classification: E4; E5; E32.

Keywords: Secured lending, nominal rigidities, optimal monetary policy, central bank asset purchases

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1 Introduction

Do credit market frictions matter for the conduct of monetary policy? In this paper, we examine how borrowing constraints affect monetary transmission and the trade-off of a central bank that aims at maximizing welfare of a representative agent. For this, we develop a macroeconomic model where prices are sticky and money serves as a mean of payment. Private agents can temporarily differ with regard to their willingness to spend, giving rise to borrowing/lending between ex-ante identical agents, while borrowing is constrained by available collateral. We analyze how monetary policy affects private borrowing/lending, and how the central bank conducts optimal policy depending on the severity of the credit market friction. We further show that the central bank can enhance welfare by easing the latter via purchases of secured loans.

We apply a simple business cycle model where money is essential and private agents borrow/lend among each other. To facilitate aggregation, we consider ex-ante identical agents. In each period, they draw preference shocks from the same time-invariant distribution, i.e. shocks that shift their valuation of the consumption good. Private agents with a high valuation of consumption are willing to consume more, for which they borrow money from other agents. We assume that contract enforcement is limited, such that lending relies on the borrower's ability to pledge collateral, as in Kiyotaki and Moore (1997). We further assume that the central bank supplies money only in exchange for eligible assets, for which we consider treasury securities as well as privately issued securities that are backed by secured loans (corresponding to MBS, which have recently been purchased by the US Federal Reserve).

In this model, loans are assumed to be intraperiod, as in Jermann and Quadrini (2012), which implies that real debt burden cannot be reduced by higher inflation.[3] Here, higher inflation is not desirable since it tends to increase the lending rate and thereby amplifies the credit market friction. For the analysis of optimal policy, we assume that the central bank acts under full commitment (while we neglect the issue of time inconsistency, as usual in the literature). Specifically, it aims at maximizing welfare of a representative agent, taking into account that prices are imperfectly flexible, money is costly, and borrowing is constrained. We find that monetary policy cannot implement first best, regardless of price flexibility, since distortions due to costs of money holdings and due to the borrowing constraint cannot simultaneously be eliminated by the central bank. We first examine a conventional monetary policy regime, where access to central bank money is

[3] This differs from studies on optimal policy under financial market frictions with intertemporal nominal debt (see Monacelli, 2008, or De Fiore et al., 2011).

not effectively constrained by holdings of eligible assets. In this case, central bank asset purchases are neutral and there is just one monetary policy instrument. Under reasonable degrees of price rigidity, we find that an optimizing central bank mainly aims at stabilizing prices, which accords to the results in the literature on optimal monetary policy in sticky price models (see Schmitt-Grohé and Uribe, 2010). If prices were more flexible, the central bank is willing to reduce the inflation rate (implying deflation) as well as the loan rate, which tends to ease the borrowing constraint.

We further consider that the central bank might apply additional instruments by supplying money in a way that induces asset purchases to be effective. Specifically, asset purchases are non-neutral if the central bank rations money supply by restricting the set of assets eligible for open market operations and by setting the price of money, i.e. the policy rate, below its marginal valuation by the lender (see Schabert, 2013). By purchasing secured loans at such a favorable price, it raises the amount of funds/money available for supplying loans and it can induce a downward shift in loan rate. Thus, central bank purchases of secured loans can stimulate private lending. Compared to the latter (conventional) specification of monetary policy where money is supplied in a non-rationed way, the central bank can thus enhance welfare of the representative agent. These welfare gains are, however, considerably small, given that the scope of effective asset purchases is endogenously limited by the valuation of money and by restrictions on policy instruments (like the zero lower bound on interest rates).

The paper relates to studies on optimal monetary policy in sticky price models (see Schmitt-Grohé and Uribe, 2010, for an overview) and under financial market frictions, for example, to Faia and Monacelli (2007), who analyze optimal interest rate rules under agency costs, to Monacelli (2008), who examines optimal monetary policy when borrowing households face a collateral constraint, or to De Fiore et al. (2011), who analyze optimal monetary policy under flexible prices and imperfect monitoring. The central bank asset purchase analysis further relates to studies on unconventional monetary policies like Curdia and Woodford (2011) and Gertler and Karadi (2011), who consider costly financial intermediation and direct lending by the central bank. The possibility to enhance welfare by rationing money supply is shown by Schabert (2013) in a framework with frictionless financial markets.

In Section 2 we present the model. In Section 3, we demonstrate how borrowing is affected by monetary policy. In Section 4, we examine optimal monetary policy considering a (conventional) regime without money rationing and a regime where money supply is effectively rationed. Section 5 concludes.

2 The model

In this Section, we provide an overview of the model, describe the details of the private sector behavior and the public sector, and summarize the first best allocation.

2.1 Overview

There are three sectors: households, firms, and the public sector. Households consists of members who enter a period with money and government bonds and dispose of a constant time endowment. They can further hold a durable good, i.e. housing, which is supplied at a fixed amount. At the beginning of each period, aggregate productivity shocks are realized and open market operations are conducted, where the central bank sells or purchases assets outright or supplies money via repos against eligible assets at the policy rate R_t^m.

Then, idiosyncratic preference shocks are realized.[4] Household members with a high realization of the preference shock (ϵ_b) are willing to consume more than household members with a low realization of the preference shock ($\epsilon_l < \epsilon_b$). Given that purchases of consumption goods rely on cash holdings, the former borrow cash from the latter at the price $1/R_t^L$. Given that loans are not perfectly enforceable, they are collateralized by the market value of borrowers' housing.

After goods are produced, the market for consumption goods opens, where money serves as the means of payment, inducing demand for money and assets eligible for open market operations. In the asset market, borrowing agents repay the secured loans, the government issues new bonds at the price $1/R_t$, and the central bank reinvests payoffs from maturing bonds and leaves money supply unchanged.

The central bank sets the price of money (i.e. policy rate), decides on how many assets it accepts when it supplies money in open market operations, and it transfers interest earnings to the treasury. The government issues risk-free bonds, which mainly back money supply, and has access to lump-sum taxes. Firms produce goods employing labor from households and set prices in an imperfectly flexible way.

2.2 Details

Households There are infinitely many households of measure one. Each household has a unit measure of members i. Following Shi (1997), we assume that assets of all household members are equally distributed at the beginning of each period. Their utility increases with consumption $c_{i,t}$ of

[4] The assumption that preference shocks are realized after treasuries are liquidated is made only to facilitate the analysis for the case where money supply is not effectively rationed.

a non-durable good and holdings of a durable good, i.e. housing $h_{i,t}$, and is decreasing in working time $n_{i,t}$. Like in Iacoviello (2005), we assume that the supply of housing is fixed at h. Members of each household can differ with regard to their marginal valuation of consumption due to preference shocks $\epsilon_i > 0$, which are i.i.d. across members and time. The instantaneous utility function of ex-ante identical members is given by

$$u(c_{i,t}, h_{i,t}, n_{i,t}, \epsilon_{i,t}) = \epsilon_{i,t}(c_{i,t}^{1-\sigma} - 1)\,(1-\sigma)^{-1} + \gamma(h_{i,t}^{1-\sigma_h} - 1)\,(1-\sigma_h)^{-1} - \chi n_{i,t}^{1+\eta}\,(1+\eta)^{-1}, \quad (1)$$

where $\sigma_{(h)} > 0$, $\gamma > 0$, $\chi > 0$, and $\eta \geq 0$ and $h_{i,t}$ denotes end-of-period stock of housing. Note that the end-of-period stock of housing will differ between both types of members. For simplicity, we assume that ϵ_i exhibits two possible realizations, $\epsilon_i \in \{\epsilon_b, \epsilon_l\}$, with equal probabilities, where $\epsilon_l < \epsilon_b$. Household members rely on money for purchases of consumption goods. For this, they hold money $M_{i,t-1}^H$ and can acquire additional money $I_{i,t}$ from the central bank, for which they hold eligible assets, in particular, treasury securities. When household member i draws the realization ϵ_b (ϵ_l), which materializes after treasuries can be liquidated in open market operations,[5] it is willing to consume more (less) than members who draw ϵ_l (ϵ_b). Hence, ϵ_b-type members tend to borrow an additional amount of cash from ϵ_l-type members.

We consider a market for short-term loans, where agents can *b*orrow $L_{b,t} < 0$ and *l*end cash $L_{l,t} > 0$ at the price $1/R_t^L$ and repays it at the end of the period. Thus, loans are assumed to be intraperiod (as e.g. in Jermann and Quadrini, 2012), which greatly simplifies the analysis. We account for the fact that debt repayment cannot always be guaranteed and debt contracts cannot be perfectly enforced. We therefore consider private lending both in form of unsecured and secured lending (as in He et al., 2013). For the latter, we assume that end-of-period value of housing serves as collateral. Specifically, these loans, for which we simplify the notation by using $L_t = L_{l,t}$ and that $L_{l,t} = -L_{b,t}$ will hold in equilibrium, are secured by the liquidation value of the borrower's holdings of housing (see Iacoviello, 2005)

$$L_t \leq z_t P_t q_t h_{b,t}, \quad (2)$$

where q_t denotes the real housing price and z_t a stochastic liquidation value of collateral. A negative (positive) value for L_t indicates borrowing (lending).[6] Household member i can acquire additional money $I_{i,t}$ from the central bank in open market operations, where money is supplied against

[5] Note that this assumption is only relevant for the case, where money is supplied in a non-rationed way.
[6] Given that loans are traded within a period, private sector debt is not directly affected by inflation.

eligible assets discounted with the policy rate R_t^m. Hence, access to money for both members is restricted by

$$I_{i,t} \leq \kappa_t^B \left(B_{i,t-1}/R_t^m \right). \tag{3}$$

The constraint (3) shows that the central bank supplies money against fractions of (randomly selected) bonds κ_t^B. We further assume that after the preference shocks are realized and loan contract are signed, lenders can refinance secured loans L_t at the central bank, which purchases a randomly selected fraction of loans at the price $1/R_t^m$:

$$I_t^L \leq \kappa_t L_t / R_t^m \tag{4}$$

where $I_t^L \geq 0$. By purchasing loans ($\kappa_t > 0$), the central bank can further influence the lenders' valuation of secured loans, and can thereby impact on private borrowing and lending. In the goods market, member i can then use money holdings M_{t-1}^H as well as new injections $I_{i,t}$ net of (positive or negative) loans L_t for consumption expenditures**.** We thereby consider not only for secured loans, but further account for the possibility of refinanced (first liens) loans, measured as a share $\upsilon \geq 0$ of the secured loans (see Greenspan and Kennedy, 2008, for an overview on home equity extraction). These refinanced loans, which we also call unsecured loans, are not eligible, i.e. the central bank can only purchases secured loans (see 4). As a consequence, the goods market constraints for the *l*ender and the *b*orrower read

$$P_t c_{l,t} \leq I_{l,t} + M_{l,t-1}^H - (1+\upsilon) L_t / R_t^L, \text{ and } P_t c_{b,t} \leq I_{b,t} + M_{b,t-1}^H + \left[(1+\upsilon) L_t + L_t^r \right] / R_t^L, \tag{5}$$

Note that the goods market constraint of the borrower contains unsecured loans supplied by the lender, which are funded by the proceeds of central bank purchases, $L_t^r / R_t^L \leq I_t^L$. Notably, these loans L_t^r, which are as well as the share υL_t of original loans, are unsecured and are assumed not to be eligible for central bank operations.

Before, the asset market opens, wages, taxes, and profits are paid, and repos are settled, i.e. agents buy back loans and bonds from the central bank. In the asset market, members repay (intraperiod) loans and invest in treasuries. Thus, the asset market constraint of both types of members is given by

$$\begin{aligned} &M_{i,t-1}^H + B_{i,t-1} + \left[(1+\upsilon) L_t \right] \left(1 - 1/R_t^L \right) + L_t^r \left(1 - 1/R_t^L \right) + P_t w_t n_{i,t} + P_t \delta_{i,t} + P_t \tau_{i,t} \\ &\geq M_{i,t}^H + \left(B_{i,t}/R_t \right) + \left(I_{i,t} + I_t^L \right) \left(R_t^m - 1 \right) + P_t c_{i,t} + P_t q_t \left(h_{i,t} - h_{i,t-1} \right), \end{aligned} \tag{6}$$

where $q_t = P_{h,t}/P_t$ and $P_{h,t}$ is the nominal price of housing. Maximizing $E\sum_{t=0}^{\infty}\beta^t u_{i,t}$ subject to (3), (4), (5), (6), and the borrowing constraints (2), $L_t^r \leq I_t^L R_t^L$, $M_{i,t}^H \geq 0$, and $B_{i,t} \geq 0$, leads to the following first order conditions for consumption, working time, treasuries, and money $\forall i \in \{b, l\}$

$$\epsilon_{i,t} c_{i,t}^{-\sigma} = \lambda_{i,t} + \psi_{i,t}, \tag{7}$$

$$\chi n_{i,t}^{\eta} = w_t \lambda_{i,t}, \tag{8}$$

$$\lambda_{i,t} = \beta R_t E_t \left[\left(\lambda_{i,t+1} + \kappa_{t+1}^B \eta_{i,t+1} \right) / \pi_{t+1} \right], \tag{9}$$

$$\lambda_{i,t} = \beta E_t \left[\left(\lambda_{i,t+1} + \psi_{i,t+1} \right) / \pi_{t+1} \right], \tag{10}$$

Further, $\lambda_{i,t} \geq 0$ is the multiplier on the asset market constraint (6), $\eta_{i,t} \geq 0$ the multiplier on the collateral constraint (3), $\psi_{i,t} \geq 0$ the multiplier on the cash-in-advance constraint (5), and $\zeta_{i,t}$ is the multiplier on the borrowing constraint (2). The cash-constraint implies – for $\psi_{i,t} > 0$ – the usual distortion regarding the optimal choices for consumption and working time (see 7 and 8). Condition (9) indicates that the interest rate on government bonds might be lowered by a liquidity premium, stemming from the possibility to exchange a fraction κ_t^B of bonds in open market operations. Further, the following type specific first order conditions for both types of injections, loans, and housing

$$\psi_{b,t}^e = \left(R_t^m - 1\right) \lambda_{b,t}^e + R_t^m \eta_{b,t}^e, \quad \psi_{l,t} = \left(R_t^m - 1\right) \lambda_{l,t} + R_t^m \eta_{l,t}, \tag{11}$$

$$\varsigma_{l,t} = \lambda_{l,t} \left(R_t^L - R_t^m \right) / R_t^m, \tag{12}$$

$$\psi_{l,t} = \lambda_{l,t} \left(R_t^L - 1 \right) + R_t^L \kappa_t \varsigma_{l,t} / \left(1 + \upsilon\right), \text{ and } \psi_{b,t} = \lambda_{b,t} \left(R_t^L - 1 \right) + \zeta_{b,t} R_t^L / \left(1 + \upsilon\right), \tag{13}$$

$$q_t \lambda_{b,t} = \gamma h_{b,t}^{-\sigma_h} + \zeta_{b,t} z_t q_t + \beta E_t q_{t+1} \lambda_{i,t+1}, \text{ and } q_t \lambda_{l,t} = \gamma h_{l,t}^{-\sigma_h} + \beta E_t q_{t+1} \lambda_{i,t+1}, \tag{14}$$

and $\varkappa_{b,t} = \zeta_{b,t}/(1+\upsilon)$, where $\varsigma_{l,t}$ denotes the multiplier on the money supply constraint (4) and $\varkappa_{b,t}$ the multiplier on the constraint $L_t^r \leq I_t^L R_t^L$. Thus, borrowers demand the maximum amount of refinanced loans ($\varkappa_{b,t} > 0 \Rightarrow L_t^r = I_t^L R_t^L$) when the borrowing constraint is binding ($\zeta_{b,t} > 0$). The superscript "e" indicates expectations based upon information after aggregate shocks and before idiosyncratic shocks are realized (which will only be relevant for the case where money supply is not effectively rationed). Further, the associated complementary slackness conditions and the transversality conditions hold.

Intuitively, the money supply constraint (4) will be binding, implying that lenders are willing to refinance loans at the central bank when this allows to extract further rents, i.e. if the policy rate is lower than the loan rate (see 12). Lenders will then refinance the maximum amount

of available loans and use these funds to supply further loans, $L_t^r/R_t^L = I_t^L$. If, however, the policy rate equals the loan rate, $R_t^m = R_t^L$, lenders have no incentive to refinance loans at the central bank. For this case, we assume that lenders do not engage in further lending, $L_t^r = 0$. The money supply constraints in (3) are further binding if the multiplier $\eta_{l,t}$, which is given by $\eta_{l,t} = (\epsilon_{l,t}c_{l,t}^{-\sigma}/R_t^m) - \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}] \geq 0$ (see 7, 10, and 11), is strictly positive. For this, the policy rate has to be lower than the lender's marginal (nominal) rate of intertemporal substitution, $R_t^m < 1/[(c_{l,t}^{\sigma}/\epsilon_{l,t})\beta 0.5 E_t(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]$. It then immediately follows that the multipliers $\eta_{b,t}$ and $\varsigma_{l,t}$ are also strictly positive.

Given that household members are ex-ante identical, their expected valuation of payoffs in the subsequent period are identical, implying that $\lambda_{b,t} = \lambda_{l,t} = \beta E_t \frac{\epsilon_{i,t+1}c_{i,t+1}^{-\sigma}}{\pi_{t+1}}$ (see 10). Hence, both types supply the same amount of working time, $n_{b,t}^{\eta} = n_{l,t}^{\eta} = (w_t/\chi)\,\beta E_t[\epsilon_{i,t+1}c_{i,t+1}^{-\sigma}/\pi_{t+1}]$ (see 8). Combining the first order conditions for housing (14) then gives $\gamma(h_{l,t}^{-\sigma_h} - h_{b,t}^{-\sigma_h}) = \zeta_{b,t}q_t z_t$. Hence, if the collateral constraint is binding $\zeta_{b,t} > 0$, investments in housing differs between both types of members, i.e. $h_{b,t} > h_{l,t}$. Combining the conditions in (13) and substituting out $\lambda_{i,t} + \psi_{i,t}$ with (7), further leads to

$$\epsilon_{b,t}c_{b,t}^{-\sigma} - \epsilon_{l,t}c_{l,t}^{-\sigma} = R_t^L\left(\zeta_{b,t} - R_t^L\kappa_t\varsigma_{l,t}\right)/\left(1+\upsilon\right) \tag{15}$$

which implies that the consumption choice (that would ideally satisfy $\epsilon_{l,t}c_{l,t}^{-\sigma} - \epsilon_{b,t}c_{b,t}^{-\sigma} = 0$, see below) is distorted by the borrowing constraint ($\zeta_{b,t} > 0$) and by the possibility that loans can be liquidated at the central bank ($\varsigma_{l,t} > 0$). This further implies that the central bank can in principle undo the effects of the borrowing constraint by purchasing loans, $\kappa_t > 0$. The borrower demand loans according to $\left(\psi_{b,t} + \lambda_{b,t}\right)/R_t^L = \lambda_{b,t} + \zeta_{b,t}/(1+\upsilon)$ (see 13), which can by using (7) and (10) be rewritten as

$$\frac{1}{R_t^L} = \beta E_t \frac{\epsilon_{i,t+1}c_{i,t+1}^{-\sigma}}{\epsilon_{b,t}c_{b,t}^{-\sigma}\pi_{t+1}} + \frac{\zeta_{b,t}}{\epsilon_{b,t}c_{b,t}^{-\sigma}(1+\upsilon)}. \tag{16}$$

Hence, a positive multiplier $\zeta_{b,t}$ tends – for a given R_t^L – to raise the RHS of (16), implying that current consumption tends to fall, which can be mitigated by a reduction in the loan rate. Put differently, a binding borrowing constraint tends to reduce the loan rate below the borrowers' marginal rate of intertemporal substitution $1/\beta E_t[\epsilon_{i,t+1}c_{i,t+1}^{-\sigma}/(\epsilon_{b,t}c_{b,t}^{-\sigma}\pi_{t+1})]$. The distortion due to the borrowing constraint (2) is obviously less pronounced by a higher share υ of unsecured loans. The lender supplies loans according to $\lambda_{l,t} + \varsigma_{l,t}\kappa_t/\left(1+\upsilon\right) = \left(\psi_{l,t} + \lambda_{l,t}\right)/R_t^L$ (see 13), or – using (7) and (10) – to $\beta E_t \frac{\epsilon_{i,t+1}c_{i,t+1}^{-\sigma}}{\pi_{t+1}} + \varsigma_{l,t}\kappa_t/\left(1+\upsilon\right) = \epsilon_{l,t}c_{l,t}^{-\sigma}/R_t^L$. Eliminating the multiplier $\varsigma_{l,t}$ with

(12), then leads to

$$\frac{1}{R_t^L} = \beta E_t \frac{\epsilon_{i,t+1} c_{i,t+1}^{-\sigma}}{\epsilon_{l,t} c_{l,t}^{-\sigma} \pi_{t+1}} \left[1 + \frac{\kappa_t}{1+\upsilon} \left(\frac{R_t^L}{R_t^m} - 1 \right) \right]. \tag{17}$$

Condition (17) implies that the loan rate is affected by the lender's marginal rate of intertemporal substitution in nominal terms $1/\beta E_t[\epsilon_{i,t+1} c_{i,t+1}^{-\sigma}/(\epsilon_{l,t} c_{l,t}^{-\sigma} \pi_{t+1})]$ as well as by the policy rate R_t^m, if loans are liquidated in open market operations, $\kappa_t > 0$. Hence, by setting the policy rate below the lender's marginal rate of intertemporal substitution (and thus the loan rate) and by purchasing secured loans, the central bank can reduce the loan rate independently of the inflation rate.

Firms There is a continuum of identical intermediate goods producing firms indexed with $j \in [0,1]$. They exist for one period, are perfectly competitive, and are owned by the households. A firm j distributes profits to the owners and hires the aggregate labor input $n_{j,t}$ at a common rate rate w_t. It then produces the intermediate good according to $IO_{j,t} = a_t n_{j,t}^{\alpha}$, where $\alpha \in (0,1)$ and a_t is stochastic with an unconditional mean equal to one, and sells it to retailers. Following related studies (see e.g. Schmitt-Grohé and Uribe, 2010), we allow for a constant subsidy τ^p to eliminate long-run distortions, such that the problem of a profit-maximizing firm j is given by $\max (1+\tau^p) P_{J,t} a_t n_{j,t}^{\alpha} - P_t w_t n_{j,t}$, where $P_{J,t}$ denotes the price for the intermediate good. The first order conditions are given by $(1+\tau^p)(P_{J,t}/P_t)\,\alpha n_{j,t}^{1-\alpha} = w_t$ or $(P_{J,t}/P_t)\, a_t \alpha n_{j,t}^{\alpha-1} = (1-\tau^n)$, where we defined $\tau^n = \tau^p/(1+\tau^p)$ as the production (or wage) subsidy rate. The firms transfer profits to the owners in a lump-sum way.

To introduce sticky prices, we assume that there are monopolistically competitive retailers who re-package intermediate goods $IO_t = \int_0^1 IO_{j,t} dj$. A retailer $k \in [0,1]$ produces one unit of a distinct good $y_{k,t}$ with one unit of the intermediate good (purchased at the common price $P_{J,t}$) and sells it at the price $P_{k,t}$ to perfectly competitive bundlers. They bundle the distinct goods $y_{k,t}$ to a final good $y_t = (\int_0^1 y_{k,t}^{\frac{\varepsilon-1}{\varepsilon}} dk)^{\frac{\varepsilon}{\varepsilon-1}}$ which is sold at the price P_t. The cost minimizing demand for $y_{k,t}$ is then given by $y_{k,t} = (P_{k,t}/P_t)^{-\varepsilon} y_t$. We assume that each period a measure $1-\phi$ of randomly selected retailers may reset their prices independently of the time elapsed since the last price setting, while a fraction $\phi \in [0,1)$ of retailers do not adjust their prices. A fraction $1-\phi$ of retailers sets their price to maximize the expected sum of discounted future profits. For $\phi > 0$, the first order condition for their price $\widetilde{P}_t$ can be written as $Z_{1,t}/Z_{2,t} = \tilde{Z}_t (\varepsilon - 1)/\varepsilon$, where $Z_{1,t} = (1-\tau^n)(\chi/\alpha)\, 0.5^{\eta} n_t^{1+\eta} s_t^{-1} + \phi\beta E_t \pi_{t+1}^{\varepsilon} Z_{1,t+1}$, $Z_{2,t} = (1-\tau^n)(\chi/\alpha)\, 0.5^{\eta} n_t^{1+\eta} (mc_t s_t)^{-1} + \phi\beta E_t \pi_{t+1}^{\varepsilon-1} Z_{2,t+1}$, $\tilde{Z}_t = \widetilde{P}_t/P_t$, and $mc_t = P_{J,t}/P_t$ denotes retailers' real marginal cost. With perfectly competitive bundlers, the price index P_t for the final good satisfies $P_t^{1-\varepsilon} = \int_0^1 P_{k,t}^{1-\varepsilon} dk$.

Using that $\int_0^1 P_{k,t}^{1-\varepsilon} dk = (1-\phi)\sum_{s=0}^{\infty} \phi^s \widetilde{P}_{t-s}^{1-\varepsilon}$ holds, and taking differences, leads to $1 = (1-\phi)(\tilde{Z}_t)^{1-\varepsilon} + \phi\pi_t^{\varepsilon-1}$.

Public sector The central bank transfers seigniorage revenues $P_t\tau_t^m$ to the treasury, which issues one-period bonds and pays a subsidy at a constant rate. The supply of short-term government bonds is specified in a simple way. Specifically, we assume that the total amount of short-term treasuries B_t^T, which are either held by households or the central bank, grows at some exogenously determined rate $\Gamma > \beta$,

$$B_t^T = \Gamma B_{t-1}^T, \tag{18}$$

given $B_{-1}^T > 0$. Note that we do not aim to measure total public debt by the stock of short-term bonds B_t^T, which can be interpreted as t-bills. To abstract from further effects of fiscal policy, we assume that the government has access to lump-sum transfers $P_t\tau_t$, which balance the budget. Its budget constraint is thus given by $(B_t^T/R_t) + P_t\tau_t^m = B_{t-1}^T + P_t\tau_t + P_t\tau^p$.

In open market operations, the central bank supplies money outright and temporarily in repos against treasuries, $M_t^H = \int_0^1 M_{i,t}^H di$ and $M_t^R = \int_0^1 M_{i,t}^R di$, and against loans under repos, $M_t^L = I_t^L$. At the beginning of each period, its stock of treasuries equals B_{t-1}^c and the stock of outstanding money equals M_{t-1}^H. It then receives treasuries and loans in exchange for money. Before the asset market opens, where the central bank rolls over maturing assets, repos are settled. Its budget constraint reads

$$(B_t^c/R_t) - B_{t-1}^c + P_t\tau_t^m = R_t^m\left(M_t^H - M_{t-1}^H\right) + (R_t^m - 1)\left(M_t^L + M_t^R\right). \tag{19}$$

Accounting for common central bank practice, we assume that the central bank transfers its earnings from holding assets and from open market operations to the treasury, $P_t\tau_t^m = (1 - 1/R_t)\, B_t^c + R_t^m\left(M_t^H - M_{t-1}^H\right) + (R_t^m - 1)\left(M_t^L + M_t^R\right)$. Substituting out transfers in (19) shows that central bank asset holdings evolve according to $B_t^c - B_{t-1}^c = M_t^H - M_{t-1}^H$. Assuming that initial values for central bank's assets and liabilities satisfy $B_{-1}^c = M_{-1}^H$, delivers the central bank's balance sheet

$$B_t^c = M_t^H. \tag{20}$$

The central bank has three main instruments. It sets the policy rate $R_t^m \geq 1$ and can decide how much money to supply as a fraction of eligible assets, for which it can adjust the two additional instruments, κ_t and κ_t^B, in a state contingent way. We assume that the central bank sets κ_t^B between zero and one, $\kappa_t^B \in (0, 1]$ and κ_t larger or equal to zero, $\kappa_t \geq 0$. The central bank can further

decide whether money is supplied in exchange for treasuries via repos or outright. Specifically, it can control the ratio of treasury repos to outright sales of bonds $\Omega_t > 0 : M_t^R = \Omega_t M_t^H$, where a sufficiently large value for Ω_t ensures that injections are always positive, $I_{i,t} > 0$.

Equilibrium A definition of a competitive equilibrium is given in Appendix A.1. Whether money supply is effectively rationed or not particularly depends on policy choices. For the analysis of optimal monetary policy, we will therefore distinguish between the two cases where money supply is either effectively rationed or not rationed, which is equivalent to the case where the central bank supplies money in a lump-sum way (as typically assumed in the literature). In this case, the loan rate is identical with the policy rate $R_t^L = R_t^m$ (see 11-13). Before we examine the policy problem of the central bank, we describe the first best allocation, which serves as a benchmark for the subsequent analysis. The following proposition describes the first best allocation.[7]

Proposition 1 *The first best allocation* $\{c_{b,t}^*, c_{l,t}^*, n_{b,t}^*, n_{l,t}^*, h_{b,t}^*, h_{l,t}^*\}_{t=0}^{\infty}$ *satisfies*

$$\epsilon_{b,t}(c_{b,t}^*)^{-\sigma} = \epsilon_{l,t}(c_{l,t}^*)^{-\sigma},\ h_{b,t}^* = h_{l,t}^*,\ n_{b,t}^* = n_{l,t}^*, \tag{21}$$

$\epsilon_{b,t}(c_{b,t}^*)^{-\sigma} = [\chi/(a_t\alpha)]0.5^{\eta}(n_t^*)^{1+\eta-\alpha}$, $h_{b,t}^* + h_{l,t}^* = h$ *and* $c_{l,t}^* + c_{b,t}^* = a_t(n_t^*)^{\alpha}$.

Proof. See Appendix A.1. ■

Under the first best allocation, the marginal utilities of consumption are identical for borrowers and lenders, and their end-of-period stock of housing is the same. This will typically not be the case in a competitive equilibrium where the borrowing constraint is binding. In Section 4.2.2, we examine how the central bank can relax the borrowing constraint by purchases of loans. For this policy to be non-neutral, money supply has to be effectively rationed in the sense that money is supplied at a sufficiently low price such that agents have an incentive to liquidate all eligible assets. For this, the central bank has to set the policy rate below the lender's marginal rate of intertemporal substitution, implying $R_t^m < R_t^L$ (see 17).

3 Constrained borrowing and monetary policy

In this Section, we examine the impact of the existence of the borrowing constraint on the allocation and on prices. We demonstrate how the long-run equilibrium is affected by the borrowing constraint and how changes in the tightness of the borrowing constraint impact on the transmission of aggregate shocks. The parameter values applied for this analysis and in the subsequent

[7] According to the conditions in proposition 1, the solution for $c_{b,t}^*$ and $h_{b,t}^*$ are given by $c_{b,t}^* = a_t^{\frac{1+\eta}{1-\alpha+\eta+\alpha\sigma}}[\alpha\epsilon_{b,t}/(\chi 0.5^{\eta})]^{\frac{\alpha}{1-\alpha+\eta+\alpha\sigma}}[1+(\epsilon_{l,t}/\epsilon_{b,t})^{\frac{1}{\sigma}}]^{-\frac{1-\alpha+\eta}{1-\alpha+\eta+\alpha\sigma}}$ and $h_{b,t}^* = 0.5h$.

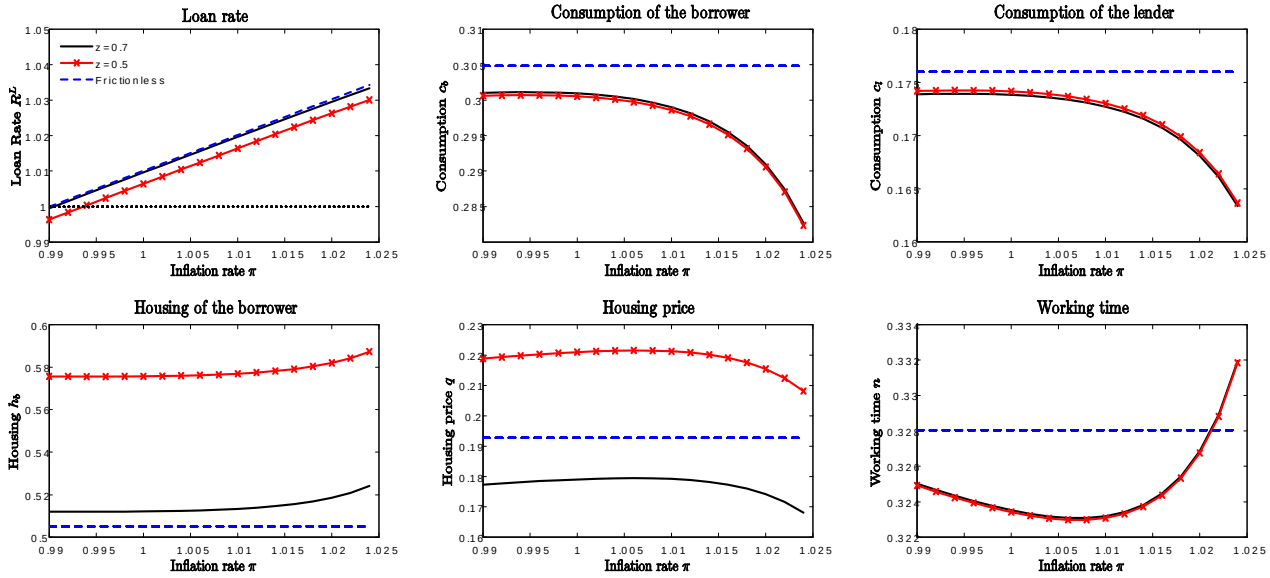


Figure 1: Steady state value for different inflation rates

Sections are given in Appendix A.6. We set most parameter equal to values that are standard in the literature, i.e. $\beta = 0.99$, $\sigma_{(h)} = 2$, $\eta = 1$, $\phi = 0.7$, $\alpha = 0.66$, and $\chi = 98$, the latter implying a first best working time share of roughly one third, and a utility weight on housing of $\gamma = 0.1$ (see Iacoviello, 2005). The housing share qh/y is set at 0.18 and the share of unsecured loans υ at 1/2. The liquidation share is set at $z = 0.8$, which is slightly smaller than in Iacoviello (2005) and will be varied in the subsequent analysis. For the stochastic processes, we assume that the autocorrelation of aggregate shocks equals 0.9 and their standard deviation equals 0.005. The realizations of the idiosyncratic shock, which leads to a highly stylized form of heterogeneity, is assumed to satisfy $\epsilon_l = 0.5$ and $\epsilon_b = 1.5$ with equal probabilities.

Long-run properties Suppose that monetary policy that acts in a non-optimizing way. We examine the long run properties of the economy, where the borrowing constraint is binding and monetary policy is conventional, i.e. money supply is not rationed. In the long-run, the central bank is then endowed with a single choice variable, which is assumed to be the inflation rate (or the inflation target). The loan rate is then determined by the demand and the supply of loans in the private credit market as summarized in (16) and (17). Given that money is assumed not to be rationed ($\varsigma_{l,t} = \eta_{l,t} = 0$), the loan rate thus equals the lender's marginal rate of intertemporal

substitution in nominal terms (see 11 and 13)

$$R^L = (\pi/\beta) \cdot \left(\epsilon_l c_l^{-\sigma} / \overline{c}^{-\sigma}\right), \tag{22}$$

where $\overline{c}^{-\sigma} = 0.5\epsilon_l c_l^{-\sigma} + 0.5\epsilon_b c_b^{-\sigma}$ and variables without a time index denote steady state values. If borrowing were unconstrained (i.e. perfect enforcement) or the borrowing constraint were slack $\zeta_{b,t} = 0$, the borrower's and the lender's marginal utility of consumption would be identical $\epsilon_l c_l^{-\sigma} = \epsilon_b c_b^{-\sigma}$. Thus, consumption of the lender satisfies (see 15 for $\varsigma_{l,t} = 0$):

$$c_l = (\epsilon_l/\epsilon_b)^{1/\sigma} c_b \;\text{ if } \zeta_b = 0 \quad \text{and} \quad c_l > (\epsilon_l/\epsilon_b)^{1/\sigma} c_b \;\text{ if } \zeta_b > 0. \tag{23}$$

Thus, constrained borrowing ($\zeta_b > 0$) increases relative consumption of the lender, which tends to reduce the lending rate. This effect is more pronounced, the tighter the borrowing constraint is, e.g. when the liquidation value of housing z is lower. When the central bank raises the inflation target/rate, the lending rate also increases (see 22). The higher lending rate tends to reduce overall consumption, as lender's consumption falls according to (22) and borrower's consumption falls as the costs of borrowing increase. The impact of a tighter borrowing constraint on consumption of both types is intuitive (though, very small): A lower liquidation value z leads to a larger (smaller) reduction in the borrower's (lender's) consumption. The impact of the borrowing constraint on housing (and its price) is most pronounced. Borrowers are willing to increase investment in housing in order to raise the value of collateral and, thereby, to relax the borrowing constraint. Given that this behavior distorts the allocation of resources (goods and housing), the borrowing constraint raises welfare losses compared to the first best, which is amplified by a higher inflation rate (and thus loan rate).

Based on this line of arguments, the central bank should reduce the inflation rate as much as possible. However, in an environment with imperfectly flexible prices, prices should be stable in the long-run to avoid welfare losses from an inefficient allocation of resources (working time) due to price dispersion. Thus, under sticky prices there exists a trade-off such that a welfare-maximizing central bank will abstain from lowering the loan rate via a low inflation target. If, however, it were able to control the loan rate independently from the inflation rate, it might be able to increase welfare. This is in principle possible under money rationing where the long-run loan rate is not given by (22), but instead by $\frac{1}{R^L} = \frac{\beta}{\pi}\frac{c^{-\sigma}}{\epsilon_l c_l^{-\sigma}}[1 + \frac{\kappa}{1+\upsilon}(\frac{R^L}{R^m} - 1)]$, which shows that the central bank can influence the loan rate by more than one instrument. Specifically, it can control the inflation rate via the money supply constraint (3) and can further manipulate the loan rate by adjusting the

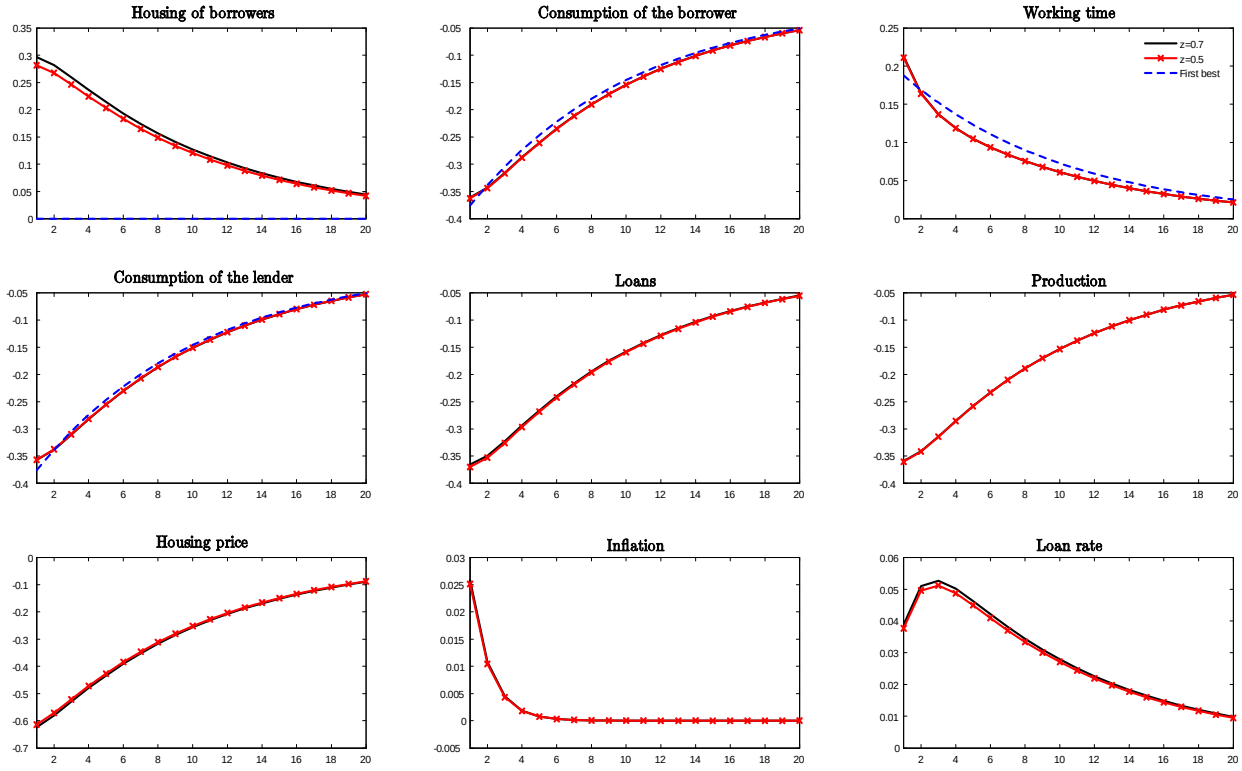


Figure 2: Responses to a contractionary productivity shock for a non-optimizing policy [Note: Steady states are not identical.]

policy rate R^m and the fraction of eligible loans κ. Yet, this strategy is endogenously limited by the requirement that the policy rate has to be set below the lender's marginal rate of intertemporal substitution for the money supply constraint to be binding.

Short-run dynamics To demonstrate how the borrowing constraint affects short-run macroeconomic dynamics, we consider here a simple non-optimizing rule for the policy rate. Specifically, we assume that the central bank supplies money in a non-rationed way ($\eta_{i,t} = 0$, $R_t^m = R_t^L$) and sets the long-run inflation rate equal to one, $\pi = 1$ (price stability), which accords to the view that the price rigidity is the dominant distortion. The central bank further sets the policy rate according to a simple feedback rule: $R_t^m = \left(R_{t-1}^m\right)^{0.9} \left(R^m\right)^{1-0.9} \left(\pi_t/\pi\right)^{1.5(1-0.9)}$, where $R^m = R^L$ and R^L satisfies (22).The Figures 2 and 3 show impulse responses to a contractionary productivity shock a_t and to an increase in the tightness of the borrowing constraint z_t (or the liquidation value of housing). All impulse responses in the paper are given in percentage deviations from the steady state values. The impulse responses refer to two versions of the model which differ with

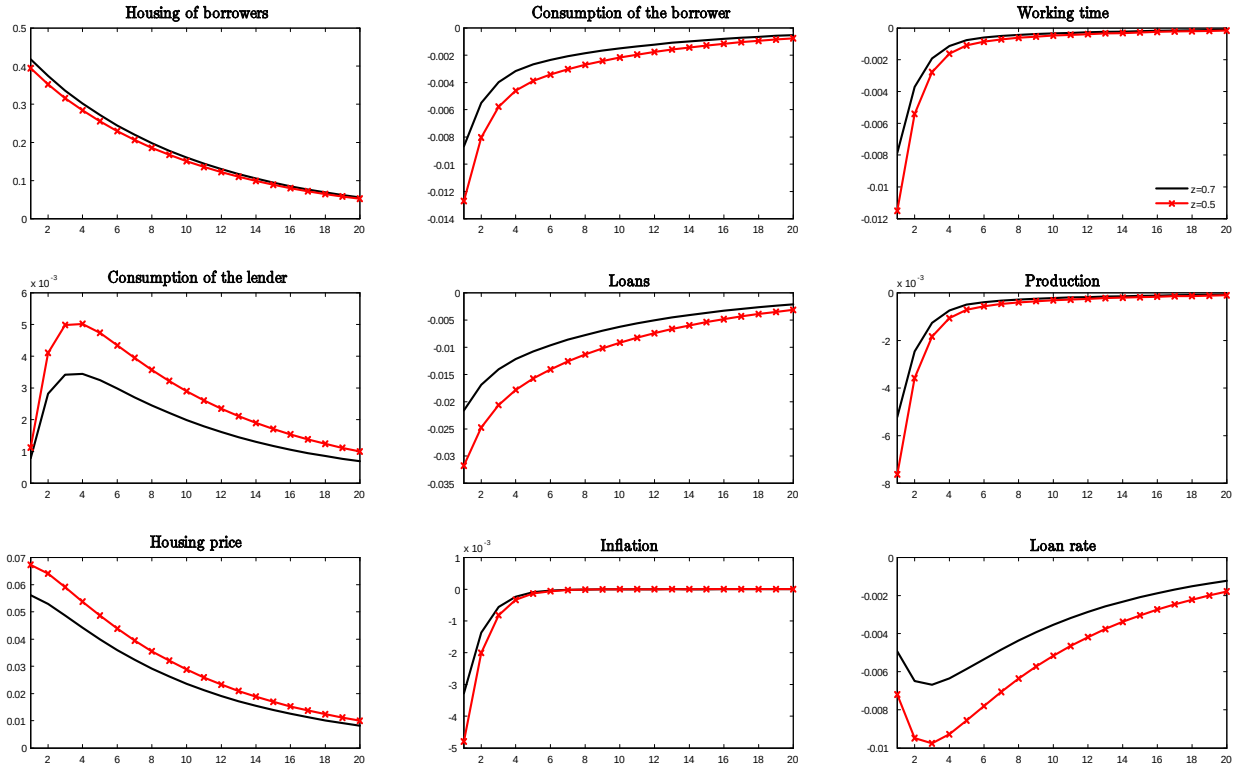


Figure 3: Responses to a lower liquidation value z_t for a non-optimizing policy [Note: Steady states are not identical.]

regard to the mean tightness of the borrowing constraint $z = 0.5$ and $z = 0.7$. The responses to the contractionary productivity shock reveal that the non-optimizing policy leads to deviations of consumption, working time and housing (of borrowers) from their first best values (see Figure 2). Notably, these differences between both versions and the first best allocation are considerably small. This is apparently not the case when the economy is hit by a shock to the tightness of the borrowing constraint. Here, the differences between both versions are sizable, in particular, for the responses of the loan rate (see Figure 3).

The worsening of the borrowing conditions (i.e. the tighter borrowing constraint) reduces loans, production, inflation, and the consumption of the borrower. Consequently, it tends to increase the consumption of the lender, which is associated with a decline in the loan rate. Housing demand of borrowers increases and thereby its price, which both tend to reduce the full impact of the initial shock. Monetary policy accommodates the contractionary shock by reducing the loan rate in accordance with the simple feedback rule. This effect is more pronounced for a lower mean liquidation value z.

4 Optimal monetary policy

In this Section, we examine the policy plan of a central bank that aims at maximizing welfare, for which we assume that it is able to perfectly commit to future policy. We restrict our attention to invariant policies, neglecting the issue of time inconsistency that typically prevails in such a framework. In the first part of this Section, we briefly assess the case of flexible prices and perfect competition and show that first best cannot be implemented (regardless whether money supply is rationed or not). We then examine optimal monetary policy under the assumption that money is supplied in a non-rationed way. In the last part of this Section, we show that once the central bank rations money supply it can implement a more favorable outcome by purchasing loans.

4.1 A flexible price version

In this Section, we examine how the monetary policy decision is affected by the existence of the borrowing constraint. To abstract from less relevant aspects of monetary policy, we assume that – in the case of money rationing – the money supply instrument κ_t^B is residually adjusted by the central bank to support a particular competitive equilibrium. Then, we can restrict our attention to a small set of equilibrium sequences. For the case where prices are perfectly flexible and competition is perfect, an equilibrium can be defined as follows.

Definition 1 *A competitive equilibrium under perfectly flexible prices and perfect competition is given by a set of sequences* $\{c_{b,t},\, c_{l,t},\, n_t,\, R_t^L,\, h_{b,t},\, q_t\}_{t=0}^{\infty}$ *satisfying*

$$0 = n_t^{1+\eta-\alpha} - \omega a_t \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}], \tag{24}$$

$$1/R_t^L = \left(c_{b,t}^{\sigma}/\epsilon_{b,t}\right)\{\beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}] + \gamma((1+\upsilon)q_t z_t)^{-1}[(h-h_{b,t})^{-\sigma_h} - h_{b,t}^{-\sigma_h}]\} \tag{25}$$

$$1/R_t^L = \beta\left(c_{l,t}^{\sigma}/\epsilon_{l,t}\right) E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]\{1 + [\kappa_t/(1+\upsilon)][(R_t^L/R_t^m) - 1]\},\ \textit{if}\ \varsigma_{l,t} > 0, \tag{26}$$

$$\textit{or}\ 1/R_t^L = \beta\left(c_{l,t}^{\sigma}/\epsilon_{l,t}\right) E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]\ \ \textit{and}\ R_t^m = R_t^L\ \textit{if}\ \varsigma_{l,t} = 0, \tag{27}$$

$$c_{b,t} = c_{l,t} + z_t q_t h_{b,t}\left[2\left(1+\upsilon\right)\left(R_t^L\right)^{-1} + \kappa_t (R_t^m)^{-1}\right],\ \textit{if}\ \zeta_{b,t} > 0, \tag{28}$$

$$\textit{or}\ \ c_{b,t} \leq c_{l,t} + z_t q_t h_{b,t}\left[2\left(1+\upsilon\right)\left(R_t^L\right)^{-1} + \kappa_t (R_t^m)^{-1}\right],\ \textit{if}\ \zeta_{b,t} = 0$$

$$0 = -q_t n_t^{\eta+1-\alpha}/a_t + \beta E_t[q_{t+1} n_{t+1}^{\eta+1-\alpha}/a_{t+1}] + \gamma\omega(h-h_{b,t})^{-\sigma_h} \tag{29}$$

$$a_t n_t^{\alpha} = c_{l,t} + c_{b,t}, \tag{30}$$

where $\omega = \frac{\alpha}{(1-\tau^n)\chi(0.5)^{\eta}}$ *and* $\tau^n = 0$, $\eta_{l,t} = \chi n_{l,t}^{\eta}/w_t\left(R_t^L - R_t^m\right)/R_t^m \geq 0$ *and* $\zeta_{b,t} = \gamma(q_t z_t)^{-1}(h_{l,t}^{-\sigma_h} - h_{b,t}^{-\sigma_h}) \geq 0$, *and the transversality conditions, for a monetary policy* $\{\kappa_t,\, R_t^m \geq 1\}_{t=0}^{\infty}$ *and exogenous sequences* $\{a_t, z_t\}_{t=0}^{\infty}$, *given* $h > 0$.

As revealed by the conditions in Definition 1, there is one more instrument available for the central bank (κ_t) if it supplies money in a rationed way. In this case, the central bank can, in particular, control the loan rate by setting the policy rate and the fraction of eligible loans (see

26). To effectively ration money supply, it has to set the policy rate below the marginal rate of intertemporal substitution of the lender, such that the multiplier on the money supply constraint is strictly positive. Otherwise, the money supply constraint is slack and the loan rate equals the lender's marginal rate of intertemporal substitution. Thus, rationing money supply endows the central bank with an additional instrument, which can in principle address welfare-reducing frictions in a more favorable way than under a single instrument regime. According to this simple principle, the central bank might be able to enhance welfare by simultaneously controlling money supply and the policy rate.

However, the central bank is not able to implement the long-run efficient allocation (as described in Proposition 1) regardless of whether money supply is rationed or not. This property, which can easily be shown by referring to the model as given in Definition 1, is summarized in the following proposition.

Proposition 2 *Consider a competitive equilibrium as given in definition 1. A long-run efficient allocation can, in general, neither be implemented under rationed money supply nor under non-rationed money supply.*

Proof. See Appendix A.1 ■

The implementation of the long-run efficient allocation would in principle require the central bank to set the inflation rate and, thereby, the loan rate as well as the policy rate according to the Friedman rule to undo the distortion induced by the costs of money holdings. Since both interest rate are identical, money can then not be supplied in a rationed way. Moreover, the credit market is distorted by the borrowing constraint, which, in general, demands a loan rate different from one. It should however be noted that even though money rationing does not matter for the impossibility to implement first best, it can affect the allocation under second best. This will be examined in Section 4.2.2 for the more realistic case where prices are not perfectly flexible.

4.2 Optimal monetary policy under sticky prices

As shown for the flexible price version of the model, monetary policy cannot achieve first best. Here, we examine optimal monetary policy under sticky prices, for which we neglect time inconsistency problems and we restrict our attention to a time-invariant policy plan under commitment. We consider the entire set of conditions that describe the competitive equilibrium (see Definition 3 in Appendix A.1) as constraints to the optimization problem of the central bank. Given that fiscal policy is assumed to have access to lump-sum taxation, we can neglect fiscal policy. The only exception is the supply of treasuries, which can serve as eligible assets for open market operations.

Table 1: Unconditional means under optimal monetary policy w/o money rationing

	First best	Benchmark parameter values	More flexible prices	Tighter borrowing constraint
Consumption of the borrower	0.3018	0.3009	0.3015	0.3003
Consumption of the lender	0.1742	0.1739	0.1742	0.1744
Borrower's housing share	0.5	0.5333	0.5330	0.6368
Working time	0.3248	0.3235	0.3244	0.3234
Loan rate	–	1.0087	1.0016	1.0044
Loans	–	0.1269	0.1273	0.1260
Inflation rate	–	1	0.9926	1

4.2.1 Non-rationed money supply

The central bank faces three frictions: the borrowing constraint, the cash-credit good distortion, and sticky prices. Notably, we assume that the distortion due to the average price mark-up is eliminated by a subsidy, $\tau^n = 1/\varepsilon$, as typically assumed in the literature (see Schmitt-Grohé and Uribe, 2010). The policy problem for the case where money is supplied in a non-rationed way is given Appendix A.4. As mentioned above, asset purchases are then irrelevant for the equilibrium allocation.

The long-run inflation rate turns out to equal one – implying long-run price stability – for a empirically plausible degree of price rigidity ($\phi = 0.7$),[8] while the long-run loan rate is then given by $R^L = 1.0096$. We further consider the case where the degree of price rigidity is extremely small ($\phi = 0.01$) to approximate the case of fully flexible prices. For this case, the central bank implements a long-run inflation rate close to the Friedman rule and a long-run loan rate that is lower than under more rigid prices (see Table 1). The central bank thereby reduces the price of loans and thus the distortion induced by the borrowing constraint. This can be seen from a comparison of the allocation under the optimal policy with the first best allocation, which shows that the gap for the borrower's consumption is particularly reduced. Thus, under more flexible prices an optimal monetary policy is able to reduce the financial distortions in a more successful way.

These principles are confirmed by the impulse responses to aggregate shocks given in the Figures 4 and 5. The responses to a contractionary productivity shock are very similar for both cases (see

[8] Thus, the steady state under the optimal monetary policy is identical to the steady state under the non-optimizing policy described in the previous section.

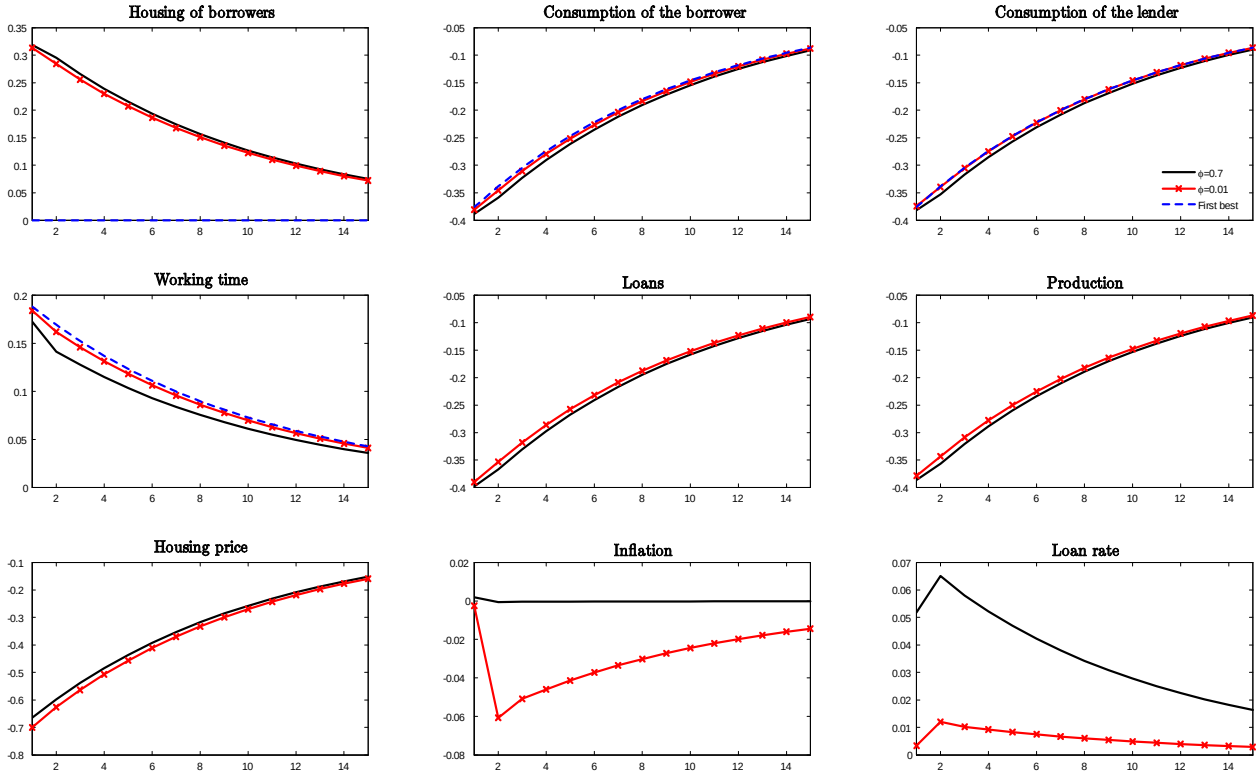


Figure 4: Responses to a contractionary productivity shock under optimal policy w/o money rationing [Note: Steady states are not identical.]

Figure 4), which, of course, exhibit different steady state values. Substantial differences can only be observed for the responses of the inflation rate and the loan rate. The latter is adjusted to a larger extent under more rigid prices, which tends to amplify the adverse borrowing conditions. Hence, in order to stabilize inflation optimal policy is then willing to accept a more pronounced loan contraction than under less rigid prices. Overall, the responses of consumption and working time are closer to the responses under the first best when prices are more flexible, as the central bank is then able to reduce the remaining distortions in a more successful way.

Figure 5 shows responses to a fall in the liquidation value of housing. Again, the inflation response reveals that under a reasonable degree of price stickiness ($\phi = 0.7$), the optimizing central bank mainly aims at stabilizing prices. Under more flexible prices, the central bank strongly reduces the inflation rate. This is associated with a more pronounced reduction in the loan rate, which tends to mitigate the distortion due to the borrowing constraint. Notably, the responses of working time and production substantially differ between both cases, as the inefficiency stemming from price stickiness is reduced.

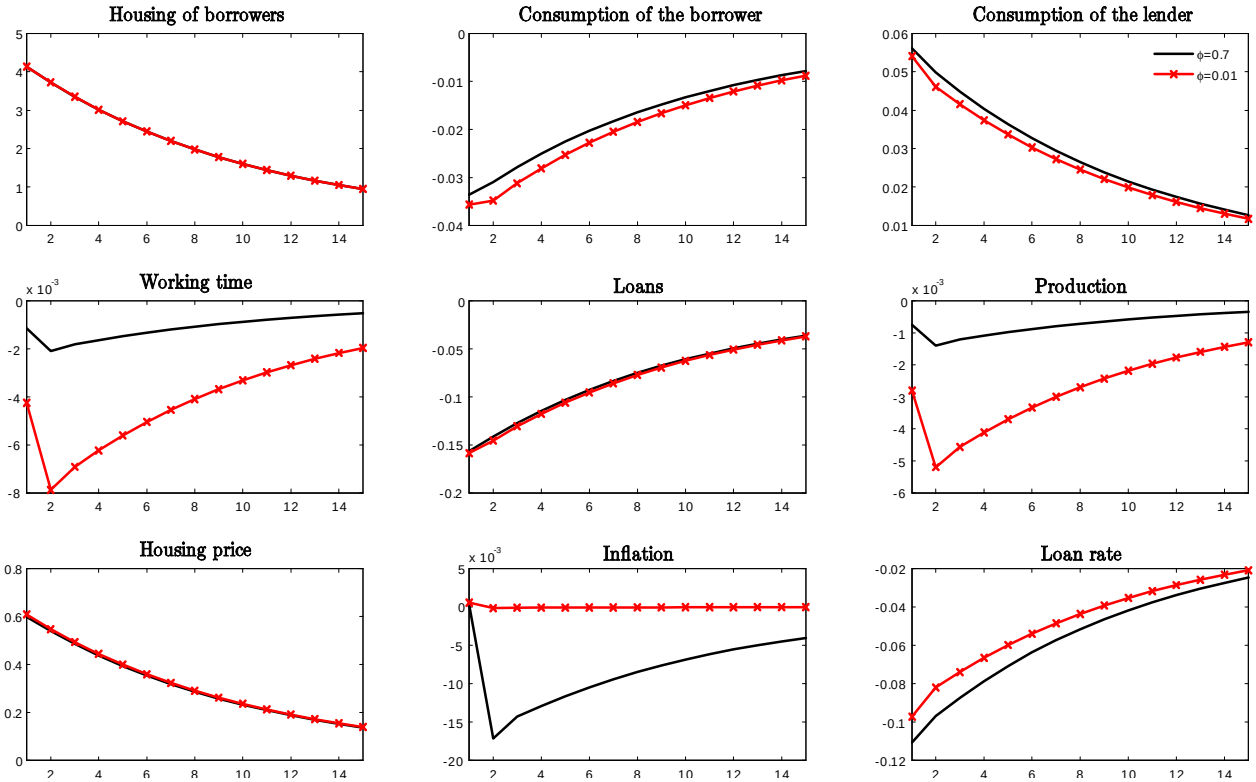


Figure 5: Responses to a lower liquidation value under optimal policy w/o money rationing [Note: Steady states are not identical.]

The last column of Table 1 shows results under an optimal monetary policy for a smaller liquidation value of collateral, $z = 0.5$. Intuitively, the distortion induced by the borrowing constraint is more pronounced, which leads to larger differences from the first best allocation compared to the case with the benchmark parameter values ($z = 0.7$). The exception is the lender's consumption value which is now slightly higher, given that the borrower's consumption is more restricted. Overall, the central bank is not willing to deviate from fully stabilizing prices in favor of reducing distortions due to financial frictions (see also the associated impulse responses in Appendix A.7).

4.2.2 Rationed money supply

When the central bank sets the policy rate R_t^m below the lender's marginal rate of intertemporal substitution, it can effectively ration money supply, i.e. it inducing the money supply constraints (3) and (4) to be binding, $\eta_{l,t} > 0$ and $\varsigma_{l,t} > 0$. The money supply instruments κ_t^B and κ_t can the be non-neutral in the sense that the central bank can affect the private sector behavior by changing the amount of money supplied in exchange for eligible assets, i.e. treasuries and

Table 2: Unconditional means with and w/o money rationing

	Optimal policy w/o money rationing	Optimal policy with money rationing	First best
Consumption of the borrower	0.3009	0.3012	0.3018
Consumption of the lender	0.1739	0.1737	0.1742
Housing of the borrower	0.5333	0.5	0.5
Working time	0.3235	0.3236	0.3248
Loan rate	1.0091	1.0086	–
Inflation rate	1	1	–
Policy rate	1.0091	1.0026	–
Fraction of purchased loans	–	0.6864	–
Relative welfare loss	0.0021	0.0012	–

secured loans. Specifically, the loan rate can be manipulated independently of the lender's marginal rate of intertemporal substitution by central bank purchases of loans, $\kappa_t > 0$. It can then be shown in a straightforward way that the policy problem can be greatly simplified by using that several equilibrium objects are not relevant and that the central bank is equipped with additional instruments (see Appendix A.5). In particular, we use that the central bank can set the fraction of eligible bonds to adjust the amount of money available for household members in a way that is consistent with the optimally chosen allocation, and that the policy rate and the share of purchased loans can be set to implement a favorable loan rate and to ease the borrowing constraint.

Throughout the paper, we restricted our attention to a parametrization of the model, for which the borrowing constraint just slightly distorts the decision of the borrower. The additional central bank instruments can then be used to undo the distortions stemming from the borrowing constraint, by setting them according to (26) and (27), without violating the constraints that apply for the instruments $\kappa_t \in [0, 1]$ and $R_t^m \geq 1$ in the neighborhood of the long-run equilibrium. For this case, we find that optimal policy under money rationing can enhance welfare compared to the case where monetary policy without money rationing is conducted in an optimal way (see Section 4.2.1). We compute welfare of the representative agent using $V = E \sum_{t=0}^{\infty} \beta^t u_{i,t}$ for different policy regimes and assume that the initial values are identical with the corresponding steady state values. Deviations from welfare under the first best allocation $(*)$ are then measured as permanent consumption values that compensate for the welfare loss under alternative policy regimes, $(c_{perm} - c^*_{perm})/c^*_{perm}$, where $c_{perm} = ((1-\beta)(1-\sigma)V + 1)^{1/(1-\sigma)}$. The computed welfare gain of money rationing is considerably small, while the loss compared to welfare under the first

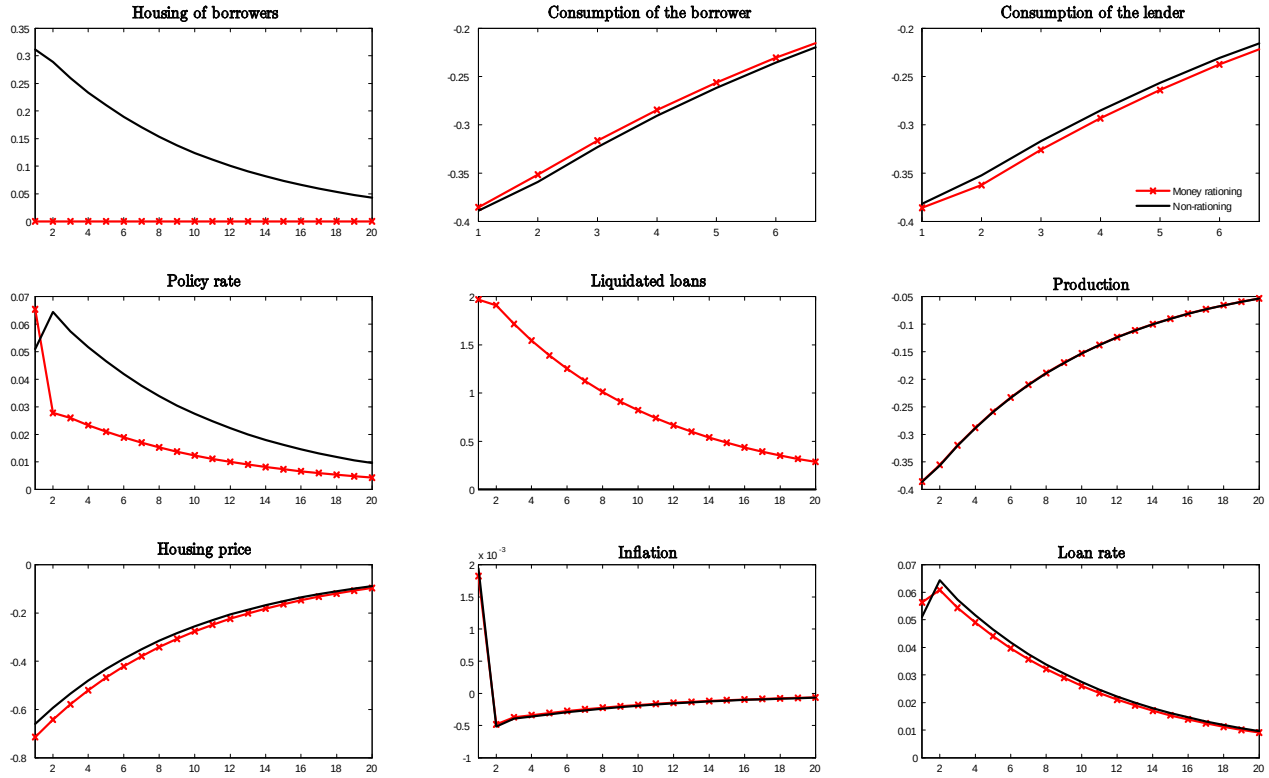


Figure 6: Responses to a contractionary productivity shock under optimizing policies [Note: Steady states are not identical.]

best allocation is almost twice are large as under the non-rationing regime (see Table 2).

The unconditional means given in Table 2 reveal that the difference between the two types of optimal policy regimes are very small under the parameter values applied for the analysis. Nonetheless, they show that an optimal policy under money rationing is able to reduce the differences between the first best allocation and the allocation in a competitive equilibrium. The only exception refers to the lender's consumption, which is lower under both optimal policy regimes than under first best. In the case of non-money rationing its value is slightly larger than under money rationing, given that the borrower's consumption is effectively constrained by its collateral value. The allocation under non-money rationing exhibits the largest difference to first best for the borrower's housing. This, however, does not have a strong impact on welfare, due to the very small utility weight assigned to housing ($\gamma = 0.01$ compared to $\chi = 98$). Notably, the loan rate is even larger under money rationing, which is due to the fact that the market for loans is undistorted in this case (see also Section 3). The policy rate R_t^m, which is well below the lender's marginal rate

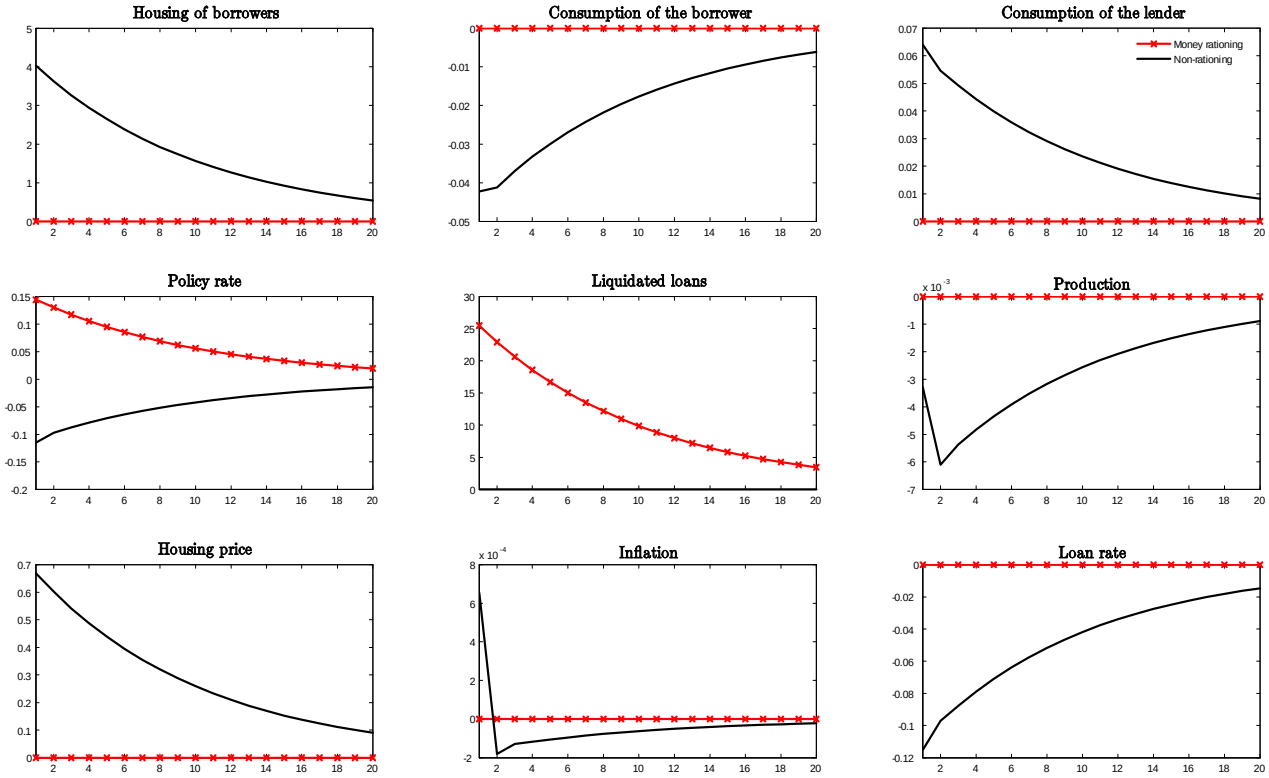


Figure 7: Responses to a lower liquidation value under optimizing policies [Note: Steady states are not identical.]

of intertemporal substitution, and the share of liquidated loans κ_t are identified by assuming that the borrowing constraint is just not binding.

The Figures 6 and 7 further show impulse responses to a contractionary productivity shock and to a reduction in the liquidation value of loans. The responses to the former correspond to the results for the unconditional means, i.e. that the allocation hardly differs between both types of optimal policy regimes (except for the distribution of housing). However, a reduction in the liquidation value of loans is associated with substantial differences between both types of policies. As long as the reduction is not too pronounced, the central bank can off-set this shock by money rationing such that the allocation is unaffected. For this it substantially increases the share of purchased loans (while it increases the policy rate in a consistent way).

5 Conclusion

[to be added...]

6 References

Curdia, V. and M. Woodford, 2011, The Central-Bank Balance Sheet as an Instrument of Monetary Policy, Journal of Monetary Economics 58, 54-79.

De Fiore F., P. Teles, and O. Tristani, 2011, Monetary Policy and the Financing of Firms, American Economic Journal: Macroeconomics 3, 112-142.

Faia, E., and T. Monacelli, 2007, Optimal Monetary Policy Rules, Asset Prices and Credit Frictions, Journal of Economic Dynamics and Control 31, 3228-3254.

Gertler, M. and P. Karadi, 2011, A Model of Unconventional Monetary Policy, Journal of Monetary Economics 58, 17-34.

Greenspan, A., and J. Kennedy, 2008, Sources and Uses of Equity Extracted from Homes, Oxford Review of Economic Policy 24, 120-144.

He, C., R. Wright and Y. Zhu, 2013, Housing and Liquidity, Meeting Papers 168, Society for Economic Dynamics.

Iacoviello, M., 2005, House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle, American Economic Review 95, 739-764.

Jermann, U., and V. Quadrini, 2012, Macroeconomic Effects of Financial Shocks, American Economic Review 102, 238-271.

Kiyotaki, N. and J.H. Moore, 1997, Credit Cycles, Journal of Political Economy 105, 211-248.

Monacelli, T., 2008, Optimal Monetary Policy with Collateralized Household Debt and Borrowing Constraints, in: Asset Prices and Monetary Policy, edited by J. Campbell, Chicago: University of Chicago Press.

Schabert, A., 2013, Optimal Central Bank Lending, unpublished manuscript, University of Cologne.

Schmitt-Grohé, S., and M. Uribe, 2010, The Optimal Rate of Inflation, in: Handbook of Monetary Economics, edited by B. Friedman and M. Woodford, 653–722.

Shi, S., 1997, A Divisible Search Model of Fiat Money, Econometrica 65, 75-102.

A Appendix

A.1 Competitive equilibrium

Definition 3 *A competitive equilibrium is a set of sequences* $\{c_{b,t},\ c_{l,t},\ n_{b,t},\ n_{l,t},\ n_t,\ l_t,\ l_t^r,\ i_{b,t},\ i_{l,t},\ i_t^L,\ m_{b,t}^H,\ m_{l,t}^H,\ m_t^H,\ b_{b,t},\ b_{l,t},\ b_t,\ w_t,\ mc_t,\ \tilde{Z}_t,\ s_t,\ \pi_t,\ R_t^L,\ \zeta_{b,t},\ h_{l,t},\ h_{b,t},\ q_t\ \}_{t=0}^{\infty}$ *satisfying*

$$n_{l,t} = n_{b,t}, \tag{31}$$

$$\chi n_{b,t}^{\eta} = w_t \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}], \tag{32}$$

$$1/R_t^L = \left(c_{b,t}^{\sigma}/\epsilon_{b,t}\right) \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}] + \zeta_{b,t}\left(c_{b,t}^{\sigma}/\epsilon_{b,t}\right)/(1+\upsilon), \tag{33}$$

$$1/R_t^L = \beta\left(c_{l,t}^{\sigma}/\epsilon_{l,t}\right) E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]\{1 + [\kappa_t/(1+\upsilon)][(R_t^L/R_t^m) - 1]\}, \tag{34}$$

$$\begin{aligned} c_{l,t} = i_{l,t} + m_{l,t-1}^H \pi_t^{-1} - (1+\upsilon)\left(l_t/R_t^L\right) \ \text{ if } \psi_{l,t} = (R_t^m - 1)\left(\mu_t \chi {n_{l,t}}^{\eta}/w_t\right) + R_t^m \eta_{l,t} > 0 \\ \text{or } c_{l,t} > i_{l,t} + m_{l,t-1}^H \pi_t^{-1} - (1+\upsilon)\left(l_t/R_t^L\right) \ \text{ if } \psi_{l,t} = 0, \end{aligned} \tag{35}$$

$$\begin{aligned} c_{b,t} = i_{b,t} + m_{b,t-1}^H \pi_t^{-1} + [(1+\upsilon)l_t + l_t^r]/R_t^L \ \text{ if } \psi_{b,t} = \zeta_{b,t} R_t^L + \left(R_t^L - 1\right)\beta E_t[\epsilon_{i,t+1} c_{i,t+1}^{-\sigma}/\pi_{t+1}] > 0 \\ \text{or } c_{b,t} > i_{b,t} + m_{b,t-1}^H \pi_t^{-1} + [(1+\upsilon)l_t + l_t^r]/R_t^L \ \text{ if } \psi_{b,t} = 0, \end{aligned} \tag{36}$$

$$\begin{aligned} R_t^m i_{l,t} = \kappa_t^B b_{l,t-1} \pi_t^{-1} \ \text{ if } \eta_{l,t} = (\epsilon_{l,t} c_{l,t}^{-\sigma}/R_t^m) - \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}] > 0 \\ \text{or } R_t^m i_{l,t} < \kappa_t \kappa_t^B b_{l,t-1} \pi_t^{-1} \ \text{ if } \eta_{l,t} = 0, \end{aligned} \tag{37}$$

$$\begin{aligned} R_t^m i_{b,t} = \kappa_t^B b_{b,t-1} \pi_t^{-1} \ \text{ if } \eta_{b,t} = (\epsilon_{b,t} c_{b,t}^{-\sigma}/R_t^m) - \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}] > 0 \\ \text{or } R_t^m i_{b,t} < \kappa_t^B b_{b,t-1} \pi_t^{-1} \ \text{ if } \eta_{b,t} = 0, \end{aligned} \tag{38}$$

$$R_t^m i_t^L = \kappa_t l_t \ \text{ if } \varsigma_{l,t} = \chi n_{l,t}^{\eta}/w_t\left(R_t^L - R_t^m\right)/R_t^m > 0 \ \text{ or } i_t^L = 0 \ \text{ if } \varsigma_{l,t} = 0, \tag{39}$$

$$\begin{aligned} l_t = z_t q_t h_{b,t}, \ \text{ if } \zeta_{b,t} = \left(\epsilon_{b,t} c_{b,t}^{-\sigma}/R_t^L\right) - \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}] > 0 \\ \text{or } l_t \le z_t q_t h_{b,t}, \ \text{ if } \zeta_{b,t} = 0, \end{aligned} \tag{40}$$

$$l_t^r/R_t^L = i_t^L \ \text{ if } \zeta_{b,t} > 0 \ \text{ or } l_t^r/R_t^L \le i_t^L \ \text{ if } \zeta_{b,t} = 0, \tag{41}$$

$$\zeta_{b,t} q_t z_t = \gamma(h_{l,t}^{-\sigma_h} - h_{b,t}^{-\sigma_h}), \tag{42}$$

$$q_t \chi n_{l,t}^{\eta}/w_t = \gamma h_{l,t}^{-\sigma_h} + \beta E_t[q_{t+1} \chi n_{l,t+1}^{\eta}/w_{t+1}], \tag{43}$$

$$h = h_{l,t} + h_{b,t} \tag{44}$$

$$n_t = n_{l,t} + n_{b,t}, \tag{45}$$

$$m_{b,t}^H = m_{l,t}^H, \tag{46}$$

$$b_t = b_{b,t} + b_{l,t} \tag{47}$$

$$m_t^H = m_{b,t}^H + m_{l,t}^H \tag{48}$$

$$i_{b,t} = (1+\Omega_t) m_{b,t}^H - m_{b,t-1}^H, \tag{49}$$

$$i_{l,t} = (1+\Omega_t) m_{l,t}^H - m_{l,t-1}^H, \tag{50}$$

$$0 = (1-\tau^n)\, w_t - mc_t a_t \alpha n_t^{\alpha-1} \tag{51}$$

$$Z_{1,t}/Z_{2,t} = \tilde{Z}_t\,(\varepsilon-1)/\varepsilon,\ \textit{where}\ Z_{1,t} = (\mu_t \chi n_{b,t}^{\eta}/w_t)(a_t n_t^{\alpha}/s_t)mc_t + \phi\beta E_t \pi_{t+1}^{\varepsilon} Z_{1,t+1} \tag{52}$$

$$\textit{and}\ Z_{2,t} = (\mu_t \chi n_{b,t}^{\eta}/w_t)(a_t n_t^{\alpha}/s_t) + \phi\beta E_t \pi_{t+1}^{\varepsilon-1} Z_{2,t+1},$$

$$1 = (1-\phi)(\tilde{Z}_t)^{1-\varepsilon} + \phi\pi_t^{\varepsilon-1}, \tag{53}$$

$$s_t = (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi s_{t-1}\pi_t^{\varepsilon}, \tag{54}$$

$$a_t n_t^{\alpha}/s_t = c_{l,t} + c_{b,t}, \tag{55}$$

$$0 = \left(b_t + m_t^H\right) - \Gamma\left(b_{t-1} + m_{t-1}^H\right)/\pi_t, \tag{56}$$

the transversality conditions, a monetary policy setting $\{R_t^m \geq 1,\ \kappa_t^B \in (0,1],\ \kappa_t \in [0,1],\ \Omega_t \geq 0\}_{t=0}^{\infty}$*, a subsidy* τ^n*, given* Γ **,** $\{a_t, z_t\}_{t=0}^{\infty}$*,* $m_{b,-1}^H = m_{l,-1}^H > 0$*,* $b_{b,-1} = b_{l,-1} > 0$*,* $b_{-1} = b_{b,-1} + b_{l,-1} > 0$*,* $m_{-1}^H = m_{b,-1}^H + m_{l,-1}^H > 0$*, and* $s_{-1} = 1$*.*

A.2 First best

Proof of proposition 1. Using $n_t = n_{l,t} + n_{b,t}$ and $n_{l,t} = n_{b,t}$, the social planer problem can be summarized as

$$\begin{aligned}
\max_{\{c_{l,t},c_{b,t},h_{l,t},h_{b,t},n_t,n_{j,t},y_{k,t}\}_{t=0}^{\infty}} \quad & E\sum_{t=0}^{\infty}\beta^t\Big\{0.5\left[\epsilon_{b,t}(c_{b,t}^{1-\sigma}-1)+\epsilon_{l,t}(c_{l,t}^{1-\sigma}-1)\right](1-\sigma)^{-1} \\
& -\chi\,(0.5n_t)^{1+\eta}\,(1+\eta)^{-1} + 0.5\gamma(h_{b,t-1}^{1-\sigma_h}-1)\,(1-\sigma_h)^{-1} \\
& +0.5\gamma(h_{l,t-1}^{1-\sigma_h}-1)\,(1-\sigma_h)^{-1}\Big\} \\
\text{s.t.}\quad & a_t\int_0^1 n_{j,t}^{\alpha}dj = \int_0^1 y_{k,t}dk, \\
& \int_0^1 n_{j,t}dj = n_t, \\
& h = \int_0^1 h_{b,t}di + \int_0^1 h_{l,t}di \\
& \int_0^1 y_{k,t}^{\frac{\varepsilon-1}{\varepsilon}}dk = \left(\int_0^1 c_{b,t}di + \int_0^1 c_{l,t}di\right)^{\frac{\varepsilon-1}{\varepsilon}}.
\end{aligned}$$

The first order conditions can easily be simplified to $\epsilon_{b,t}c_{b,t}^{-\sigma} = \epsilon_{l,t}c_{l,t}^{-\sigma}$, $\chi\,(0.5n_t)^{\eta} = a_t\alpha n_t^{\alpha-1}\epsilon_{b,t}c_{b,t}^{-\sigma}$, $h_{b,t}^{-\sigma_h} = h_{l,t}^{-\sigma_h}$, $h_{b,t} + h_{l,t} = h$, and $c_{l,t} + c_{b,t} = a_t n_t^{\alpha}$. These conditions immediately lead to the following values for the first best allocation

$$\begin{aligned}
c_{b,t} &= a_t^{\frac{1+\eta}{1-\alpha+\eta+\alpha\sigma}}\left[\alpha\epsilon_{b,t}/(\chi 0.5^{\eta})\right]^{\frac{\alpha}{1-\alpha+\eta+\alpha\sigma}}\left[1+(\epsilon_{l,t}/\epsilon_{b,t})^{\frac{1}{\sigma}}\right]^{-\frac{1-\alpha+\eta}{1-\alpha+\eta+\alpha\sigma}}, \\
c_{l,t} &= (\epsilon_{l,t}/\epsilon_{b,t})^{\frac{1}{\sigma}}\,c_{b,t},\ h_{b,t} = h_{l,t}, n_t = (c_t/a_t)^{1/\alpha}.
\end{aligned}$$

which characterize the first best allocation. ■

A.3 Flexible prices

Proof of proposition 2. Consider the long-run competitive equilibrium as given in definition 1. The long-run equilibrium values $\{c_b,\, c_l,\, n,\, R^L,\, h_b,\, q\}$ then satisfy

$$1/R^L = (c_b^{\sigma}/\epsilon_b)\,\beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1} + (c_b^{\sigma}/\epsilon_b)\,\gamma\,((1+\upsilon)qz)^{-1}\left[(h-h_b)^{-\sigma_h} - h_b^{-\sigma_h}\right], \quad (57)$$

$$1/R^L = [(c_l^{\sigma}/\epsilon_l)\,\beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1}\{1 + [\kappa/(1+\upsilon)][(R^L/R^m) - 1]\}, \text{ if } \varsigma_l > 0, \quad (58)$$

$$\text{or } 1/R^L = \beta\,(c_l^{\sigma}/\epsilon_l)\,\beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1} \text{ and } R^m = R^L \text{ if } \eta_l = 0,$$

$$n^{1+\eta-\alpha}/\omega = \beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1}, \quad (59)$$

$$c_b - c_l = zqh_b\left[2(1+\upsilon)(R^L)^{-1} + \kappa(R^m)^{-1}\right], \text{ if } \zeta_b > 0, \quad (60)$$

$$\text{or } \; c_b - c_l \leq zqh_b\left[2(1+\upsilon)(R^L)^{-1} + \kappa(R^m)^{-1}\right], \text{ if } \zeta_b = 0,$$

$q = \gamma\omega(h-h_b)^{-\sigma_h}/\left(n^{\eta+1-\alpha}\,(1-\beta)\right)$ and $n^{\alpha} = c_l + c_b$. Given that the long-run first best allocation satisfies $\epsilon_b\,(c_b^*)^{-\sigma} = \epsilon_l\,(c_l^*)^{-\sigma}$, and $\epsilon_i\,(c_i^*)^{-\sigma} = (n^*)^{1+\eta-\alpha}/\omega$, (59) implies that the implementation of a long-run efficient allocation would require $\pi = \beta$. Using $h_b^* = h_l^*$ and $\epsilon_b\,(c_b^*)^{-\sigma} = \epsilon_l\,(c_l^*)^{-\sigma}$ as well as (57) and (58), shows that long-run efficiency would further require $R^L = R^m = 1$, implying $\eta_l = 0$. Eliminating q in the borrowing constraint (60), and again using $\epsilon_b\,(c_b^*)^{-\sigma} = \epsilon_l\,(c_l^*)^{-\sigma}$ and $h_b^* = h_l^*$, then gives

$$c_b^* = \frac{2z}{1-(\epsilon_l/\epsilon_b)^{\frac{1}{\sigma}}}\frac{\gamma\omega(0.5h)^{1-\sigma_h}}{n^{\eta+1-\alpha}\,(1-\beta)}/R^L.$$

Further, substituting out n with $n = (2c_b^*)^{1/\alpha}$ and c_b^* with $c_b^* = (\epsilon_b\omega)^{\frac{\alpha}{1-\alpha+\eta+\alpha\sigma}}\,[1+(\epsilon_l/\epsilon_b)^{\frac{1}{\sigma}}]^{-\frac{1-\alpha+\eta}{1-\alpha+\eta+\alpha\sigma}}$, implies that the implementation of a long-run efficient allocation requires the loan rate to equal

$$\Lambda = \frac{2z}{1-(\epsilon_l/\epsilon_b)^{\frac{1}{\sigma}}}\frac{\gamma\omega(0.5h)^{1-\sigma_h}}{0.5\,(1-\beta)}\left(2[\epsilon_b\omega]^{\frac{\alpha}{1-\alpha+\eta+\alpha\sigma}}[1+(\epsilon_l/\epsilon_b)^{\frac{1}{\sigma}}]^{-\frac{1-\alpha+\eta}{1-\alpha+\eta+\alpha\sigma}}\right)^{-(\eta+1)/\alpha}. \quad (61)$$

Given that the expression in (61) only consists of exogenously given terms, the two conditions $R^L = \Lambda$ and $R^L = 1$ are in general inconsistent, implying that the long-run efficient allocation cannot be implemented. ■

For example, using the benchmark parameter values (see Appendix A.6), the term Λ as given in (61) is substantially smaller than one. As a long-run efficient relation of consumption and housing between both types of agents would require $R^L = \Lambda$, the long-run first best allocation cannot be implemented, given that the cash-credit distortion further requires the loan rate to be set according to the Friedman rule.

A.4 Optimal monetary policy under non-rationed money supply

In this Apendix, we consider the policy problem of the central bank that neglects the possibility of effectively ratoning money supply. Hence, the money supply constraints (3) and (4) are disregarded for the derivation of the optimal policy plan, which can – by accounting for the remaining constraints imposed by a competitive equilibrium (see Definiton 3) – be summarized as

$$\max_{\{c_{b,t},c_{l,t},n_t,mc_t,\tilde{Z}_t,Z_{1,t},Z_{2,t},s_t,\pi_t,h_{b,t}\, q_t,R_t^L\}_{t=0}^{\infty}} \min_{\{\lambda_{0,t},\dots\lambda_{10,t}\}_{t=0}^{\infty}} \tag{62}$$

$$E\sum_{t=0}^{\infty}\beta^t\begin{bmatrix}0.5\epsilon_{b,t}(c_{b,t}^{1-\sigma}-1)(1-\sigma)^{-1}+0.5\epsilon_{l,t}(c_{l,t}^{1-\sigma}-1)(1-\sigma)^{-1}-\chi(0.5n_t)^{1+\eta}(1+\eta)^{-1}\\ +0.5\gamma(h_{b,t}^{1-\sigma_h}-1)(1-\sigma_h)^{-1}+0.5\gamma((h-h_{b,t})^{1-\sigma_h}-1)(1-\sigma_h)^{-1}\end{bmatrix},$$

$$+\lambda_{0,t}\left[0.5\epsilon_{l,t}c_{l,t}^{-\sigma}-0.5\epsilon_{b,t}c_{b,t}^{-\sigma}\quad+0.5R_t^L(\gamma/(1+\upsilon)q_tz_t)\left((h-h_{b,t})^{-\sigma_h}-h_{b,t}^{-\sigma_h}\right)\right]$$

$$+\lambda_{1,t}\left[(1-\tau^n)\chi 0.5^{\eta}n_t^{\eta+1-\alpha}/(mc_ta_t\alpha)-\beta E_t[0.5(\epsilon_bc_{b,t+1}^{-\sigma}+\epsilon_lc_{l,t+1}^{-\sigma})/\pi_{t+1}]\right]$$

$$+\lambda_{2,t}\left[a_tn_t^{\alpha}/s_t-c_{l,t}-c_{b,t}\right]+\lambda_{3,t}\left[s_t-\phi s_{t-1}\pi_t^{\varepsilon}-(1-\phi)^{\frac{1}{1-\varepsilon}}\left(1-\phi\pi_t^{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]$$

$$+\lambda_{4,t}\left[(1-\phi)(\tilde{Z}_t)^{1-\varepsilon}+\phi\pi_t^{\varepsilon-1}-1\right]+\lambda_{5,t}\left[\tilde{Z}_t(\varepsilon-1)/\varepsilon-Z_{1,t}/Z_{2,t}\right]$$

$$+\lambda_{6,t}\left[Z_{1,t}-(1-\tau^n)(\chi/\alpha)0.5^{\eta}n_t^{1+\eta}s_t^{-1}-\phi\beta E_t\pi_{t+1}^{\varepsilon}Z_{1,t+1}\right]$$

$$+\lambda_{7,t}\left[Z_{2,t}-(1-\tau^n)(\chi/\alpha)0.5^{\eta}n_t^{1+\eta}(mc_ts_t)^{-1}-\phi\beta E_t\pi_{t+1}^{\varepsilon-1}Z_{2,t+1}\right]$$

$$+\lambda_{8,t}\left[2\left((1+\upsilon)z_tq_th_{b,t}/R_t^L\right)-c_{b,t}+c_{l,t}\right]+\lambda_{9,t}\left[\beta E_t[0.5(\epsilon_bc_{b,t+1}^{-\sigma}+\epsilon_lc_{l,t+1}^{-\sigma})/\pi_{t+1}]-\epsilon_{l,t}c_{l,t}^{-\sigma}/R_t^L\right]$$

$$+\lambda_{10,t}\left[\frac{q_tn_t^{\eta+1-\alpha}}{mc_ta_t}-\beta E_t\frac{q_{t+1}n_{t+1}^{\eta+1-\alpha}}{mc_{t+1}a_{t+1}}-\gamma\omega(h-h_{b,t})^{-\sigma_h}\right]$$

Neglecting the conditions for $t=0$, the solution to the policy problem (62) has to satisfy the following first order conditions

$$\begin{aligned}
h_{b,t}:0&=0.5\gamma h_{b,t}^{-\sigma_h}-0.5\gamma(h-h_{b,t})^{-\sigma_h}+\lambda_{0,t}0.5R_t^L(\gamma/(1+\upsilon)q_tz_t)\left(\sigma_h(h-h_{b,t})^{-\sigma_h-1}+\sigma_hh_{b,t}^{-\sigma_h-1}\right)\\
&\quad+\lambda_{8,t}2\left((1+\upsilon)z_tq_t/R_t^L\right)-\lambda_{10,t}\gamma\omega\sigma_h(h-h_{b,t})^{-\sigma_h-1},\\
R_t^L:0&=\lambda_{0,t}0.5(\gamma/(1+\upsilon)q_tz_t)\left((h-h_{b,t})^{-\sigma_h}-h_{b,t}^{-\sigma_h}\right)-\lambda_{8,t}2(1+\upsilon)z_tq_th_{b,t}\left(R_t^L\right)^{-2}+\lambda_{9,t}\epsilon_{l,t}c_{l,t}^{-\sigma}\left(R_t^L\right)^{-2},\\
q_t:0&=-\lambda_{0,t}0.5R_t^L\gamma/\left((1+\upsilon)q_t^2z_t\right)\left((h-h_{b,t})^{-\sigma_h}-h_{b,t}^{-\sigma_h}\right)+\lambda_{8,t}2(1+\upsilon)z_th_{b,t}/R_t^L\\
&\quad+\lambda_{10,t}n_t^{\eta+1-\alpha}/(mc_ta_t)-\lambda_{10,t-1}n_t^{\eta+1-\alpha}/(mc_ta_t),\\
mc_t:0&=-[\lambda_{1,t}(1-\tau^n)\chi 0.5^{\eta}n_t^{\eta+1-\alpha}/\left(mc_t^2\alpha a_t\right)]+\lambda_{7,t}(1-\tau^n)(\chi/\alpha)0.5^{\eta}n_t^{1+\eta}mc_t^{-2}s_t^{-1}\\
&\quad-\lambda_{10,t}q_tn_t^{\eta+1-\alpha}/(mc_t^2a_t)+\lambda_{10,t-1}q_tn_t^{\eta+1-\alpha}/(mc_t^2a_t),\\
s_t:0&=-(\lambda_{2,t}a_tn_t^{\alpha}/s_t^2)+\lambda_{3,t}-\beta E_t\lambda_{3,t+1}\phi\pi_{t+1}^{\varepsilon}+\lambda_{6,t}(1-\tau^n)(\chi/\alpha)0.5^{\eta}n_t^{1+\eta}s_t^{-2}\\
&\quad+\lambda_{7,t}(1-\tau^n)(\chi/\alpha)0.5^{\eta}n_t^{1+\eta}mc_t^{-1}s_t^{-2},
\end{aligned}$$

$$
\begin{aligned}
\pi_t: 0 &= \left[\lambda_{1,t-1}\left(0.5\epsilon_b c_{b,t}^{-\sigma}+0.5\epsilon_l c_{l,t}^{-\sigma}\right)/\pi_t^2\right]+\lambda_{4,t}\left(\varepsilon-1\right)\phi\pi_t^{\varepsilon-2}-\lambda_{6,t-1}\phi\varepsilon\pi_t^{\varepsilon-1}Z_{1,t}\\
&\quad -\lambda_{7,t-1}\phi\left(\varepsilon-1\right)\pi_t^{\varepsilon-2}Z_{2,t}+\lambda_{3,t}\left[-\phi s_{t-1}\varepsilon\pi_t^{\varepsilon-1}+\left(1-\phi\right)^{\frac{1}{1-\varepsilon}}\varepsilon\left(1-\phi\pi_t^{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}-1}\phi\pi_t^{\varepsilon-2}\right]\\
&\quad -\lambda_{9,t-1}\left(0.5\epsilon_b c_{b,t}^{-\sigma}+0.5\epsilon_l c_{l,t}^{-\sigma}\right)/\pi_t^2,\\
n_t: 0 &= -\chi 0.5^{1+\eta}n_t^{\eta}+\left[\lambda_{1,t}\left(\eta+1-\alpha\right)\left(1-\tau^n\right)\chi 0.5^{\eta}n_t^{\eta-\alpha}/\left(mc_t\alpha a_t\right)\right]+\left(\lambda_{2,t}a_t\alpha n_t^{\alpha-1}/s_t\right)\\
&\quad -\lambda_{6,t}\left(1+\eta\right)\left(1-\tau^n\right)\left(\chi/\alpha\right)0.5^{\eta}n_t^{\eta}s_t^{-1}-\lambda_{7,t}\left(1+\eta\right)\left(1-\tau^n\right)\left(\chi/\alpha\right)0.5^{\eta}n_t^{\eta}\left(mc_ts_t\right)^{-1}\\
&\quad +\lambda_{10,t}\left[\left(\eta+1-\alpha\right)q_tn_t^{\eta-\alpha}/(mc_ta_t)\right]-\lambda_{10,t-1}\left[\left(\eta+1-\alpha\right)q_tn_t^{\eta-\alpha}/(mc_ta_t)\right],\\
c_{b,t}: 0 &= 0.5\epsilon_{b,t}c_{b,t}^{-\sigma}+0.5\lambda_{0,t}\epsilon_{b,t}\sigma c_{b,t}^{-\sigma-1}+\lambda_{1,t-1}\epsilon_{b,t}0.5\sigma\left(c_{b,t}^{-\sigma-1}/\pi_t\right)-\lambda_{2,t}-\lambda_{8,t}\\
&\quad -\lambda_{9,t-1}\epsilon_{b,t}0.5\sigma\left(c_{b,t}^{-\sigma-1}/\pi_t\right),\\
c_{l,t}: 0 &= 0.5\epsilon_{l,t}c_{l,t}^{-\sigma}-0.5\lambda_{0,t}\sigma\epsilon_{l,t}c_{l,t}^{-\sigma-1}+\lambda_{1,t-1}\epsilon_{l,t}0.5\sigma\left(c_{l,t}^{-\sigma-1}/\pi_t\right)-\lambda_{2,t}+\lambda_{8,t}\\
&\quad -\lambda_{9,t-1}\epsilon_{l,t}0.5\sigma\left(c_{l,t}^{-\sigma-1}/\pi_t\right)+\lambda_{9,t}\sigma\epsilon_{l,t}c_{l,t}^{-\sigma-1}/R_t^L,\\
Z_{1,t}: 0 &= -(\lambda_{5,t}/Z_{2,t})+\lambda_{6,t}-\lambda_{6,t-1}\phi\pi_t^{\varepsilon},\\
Z_{2,t}: 0 &= (\lambda_{5,t}Z_{1,t}/Z_{2,t}^2)+\lambda_{7,t}-\lambda_{7,t-1}\phi\pi_t^{\varepsilon-1},\\
\tilde{Z}_t: 0 &= \lambda_{4,t}(1-\phi)\left(1-\varepsilon\right)\left(\tilde{Z}_t\right)^{-\varepsilon}+\lambda_{5,t}\left(\varepsilon-1\right)/\varepsilon,
\end{aligned}
$$

as well as the constraints

$$
\begin{aligned}
0 &= 0.5\epsilon_{l,t}c_{l,t}^{-\sigma}-0.5\epsilon_{b,t}c_{b,t}^{-\sigma}\ +0.5R_t^L\left(\gamma/(1+\upsilon)q_tz_t\right)\left((h-h_{b,t})^{-\sigma_h}-h_{b,t}^{-\sigma_h}\right),\\
0 &= \left(1-\tau^n\right)\chi 0.5^{\eta}n_t^{\eta+1-\alpha}/\left(mc_ta_t\alpha\right)-\beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma}+\epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}],\\
0 &= a_tn_t^{\alpha}/s_t-c_{l,t}-c_{b,t},\\
0 &= s_t-\phi s_{t-1}\pi_t^{\varepsilon}-\left(1-\phi\right)^{\frac{1}{1-\varepsilon}}\left(1-\phi\pi_t^{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}},\\
0 &= (1-\phi)(\tilde{Z}_t)^{1-\varepsilon}+\phi\pi_t^{\varepsilon-1}-1,\\
0 &= \tilde{Z}_t\left(\varepsilon-1\right)/\varepsilon-Z_{1,t}/Z_{2,t},\\
0 &= Z_{1,t}-\left(1-\tau^n\right)\left(\chi/\alpha\right)0.5^{\eta}n_t^{1+\eta}s_t^{-1}-\phi\beta E_t\pi_{t+1}^{\varepsilon}Z_{1,t+1},\\
0 &= Z_{2,t}-\left(1-\tau^n\right)\left(\chi/\alpha\right)0.5^{\eta}n_t^{1+\eta}\left(mc_ts_t\right)^{-1}-\phi\beta E_t\pi_{t+1}^{\varepsilon-1}Z_{2,t+1},\\
0 &= 2\left((1+\upsilon)z_tq_th_{b,t}/R_t^L\right)-c_{b,t}+c_{l,t},\\
0 &= \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma}+\epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]-\epsilon_{l,t}c_{l,t}^{-\sigma}/R_t^L,\\
0 &= \frac{q_tn_t^{\eta+1-\alpha}}{mc_ta_t}-\beta E_t\frac{q_{t+1}n_{t+1}^{\eta+1-\alpha}}{mc_{t+1}a_{t+1}}-\gamma\omega(h-h_{b,t})^{-\sigma_h},
\end{aligned}
$$

and the transversality conditions, given τ^n, Γ, $\{a_t, z_t\}_{t=0}^{\infty}$, $h>0$, and $s_{-1}=1$.

A.5 Optimal monetary policy under rationed money supply

In this Appendix, we consider the policy problem for the case where the central bank takes the possibility of money rationing into account. To identify the solution to the optimal policy problem we proceed as follows. We first set-up the policy problem including the money supply constraints (3) and (4). We then examine if the central bank is able to undo several constraints imposed by the private sector equilibrium behavior (see Definiton 3) by using its instruments, R_t^m, κ_t, and κ_t^B. We thereby ignore further restrictions on these instruments, like $R_t^m \geq 1$ and $\kappa^{(B)} \in [0,1]$. After solving for the optimal policy plan and the associated sequences for all instruments, we verify (numerically) that the restrictions on the policy instruments are not violated for the chosen set of parameter values and in the neighborhood of the long-run equilibrium. The policy problem can then be summarized as

$$\max_{\{c_{b,t},c_{l,t},n_t,m_t^H,b_t,b_t^T,l_t,mc_t,\tilde{Z}_t,Z_{1,t},Z_{2,t},s_t,\pi_t,R_t^L,\kappa_t^B,\kappa_t,R_t^m,h_{b,t}q_t\}_{t=0}^{\infty}} \quad \min_{\{\theta_{1,t},\ldots\theta_{16,t}\}_{t=0}^{\infty}} \tag{63}$$

$$E\sum_{t=0}^{\infty}\beta^t\left[\begin{array}{c}0.5\epsilon_{b,t}(c_{b,t}^{1-\sigma}-1)(1-\sigma)^{-1}+0.5\epsilon_{l,t}(c_{l,t}^{1-\sigma}-1)(1-\sigma)^{-1}-\chi(0.5n_t)^{1+\eta}(1+\eta)^{-1}\\ +0.5\gamma(h_{b,t}^{1-\sigma_h}-1)(1-\sigma_h)^{-1}+0.5\gamma((h-h_{b,t})^{1-\sigma_h}-1)(1-\sigma_h)^{-1}\end{array}\right],$$

$$+\theta_{1,t}\left[\epsilon_{b,t}c_{b,t}^{-\sigma}/R_t^L-(1-\tau^n)\chi(0.5n_t)^{\eta}/(mc_ta_t\alpha n_t^{\alpha-1})-\gamma((1+\upsilon)q_tz_t)^{-1}((h-h_{b,t})^{-\sigma_h}-h_{b,t}^{-\sigma_h})\right]$$

$$+\theta_{2,t}\left[(1-\tau^n)\chi(0.5n_t)^{\eta}/\left(mc_ta_t\alpha n_t^{\alpha-1}\right)-\beta E_t[0.5(\epsilon_bc_{b,t+1}^{-\sigma}+\epsilon_lc_{l,t+1}^{-\sigma})/\pi_{t+1}]\right]$$

$$+\theta_{3,t}[\left(1/R_t^L\right)-\beta\left(c_{l,t}^{\sigma}/\epsilon_{l,t}\right)E_t[0.5(\epsilon_bc_{b,t+1}^{-\sigma}+\epsilon_lc_{l,t+1}^{-\sigma})/\pi_{t+1}]\{1+[\kappa_t/(1+\upsilon)][(R_t^L/R_t^m)-1]\}]$$

$$+\theta_{4,t}\left[0.5(1+\Omega_t)m_t^H+\{(1+\upsilon)l_t/R_t^L\}+(\kappa_tl_t/R_t^m)-c_{b,t}\right]+\theta_{5,t}\left[b_t-b_t^T+m_t^H\right]$$

$$+\theta_{6,t}\left[0.5(1+\Omega_t)m_t^H-(1+\upsilon)\left(l_t/R_t^L\right)-c_{l,t}\right]$$

$$+\theta_{7,t}\left[a_tn_t^{\alpha}/s_t-c_{l,t}-c_{b,t}\right]+\theta_{8,t}\left[\tilde{Z}_t(\varepsilon-1)/\varepsilon-Z_{1,t}/Z_{2,t}\right]+\theta_{9,t}\left[(1-\phi)(\tilde{Z}_t)^{1-\varepsilon}+\phi\pi_t^{\varepsilon-1}-1\right]$$

$$+\theta_{10,t}\left[s_t-(1-\phi)\tilde{Z}_t^{-\varepsilon}-\phi s_{t-1}\pi_t^{\varepsilon}\right]+\theta_{11,t}\left[Z_{1,t}-(1-\tau^n)(\chi/\alpha)0.5^{\eta}n_t^{1+\eta}s_t^{-1}-\phi\beta E_t\pi_{t+1}^{\varepsilon}Z_{1,t+1}\right]$$

$$+\theta_{12,t}\left[Z_{2,t}-(1-\tau^n)(\chi/\alpha)0.5^{\eta}n_t^{1+\eta}(mc_ts_t)^{-1}-\phi\beta E_t\pi_{t+1}^{\varepsilon-1}Z_{2,t+1}\right]+\theta_{13,t}\left[b_t^T-\Gamma b_{t-1}^T/\pi_t\right]$$

$$+\theta_{14,t}\left[\kappa_t^Bb_{t-1}/\left(R_t^m\pi_t\right)-(1+\Omega_t)m_t^H+m_{t-1}^H\pi_t^{-1}\right]+\theta_{15,t}\left[z_tq_th_{b,t}-l_t\right]$$

$$+\theta_{16,t}\left[q_tn_t^{\eta+1-\alpha}/(mc_ta_t)-\beta E_tq_{t+1}n_{t+1}^{\eta+1-\alpha}/(mc_{t+1}a_{t+1})-\varrho(h-h_{b,t})^{-\sigma_h}\right]$$

We first examine the optimal choices for policy related variables and, in particular, for the monetary policy instruments. We thereby show that the set of relevant constraints of the original policy problem (63) can be reduced, if the central bank rations money supply. Once we have shown that several constraints in (63) are not binding, we continue with the simplified policy problem. The first order condition for κ_t^B, $\theta_{14,t}b_{t-1}/\left(R_t^m\pi_t\right)=0$, immediately leads to $\theta_{14,t}=0$, such

that the first order conditions for b_t, $\theta_{5,t} + \beta E_t \theta_{14,t+1} \kappa^B_{t+1} / R^m_{t+1} \pi_{t+1} = 0$, and for b^T_t, $\theta_{5,t} = \theta_{13,t} - \Gamma \beta E_t \theta_{13,t+1} / \pi_{t+1}$, imply $\theta_{5,t} = 0$ and $\theta_{13,t} = 0$. Then, the first order condition for m^H_t, $\theta_{5,t} + \theta_{4,t} 0.5 (1 + \Omega_t) + \theta_{6,t} 0.5 (1 + \Omega_t) - (1 + \Omega_t) \theta_{14,t} + \beta E_t \theta_{14,t+1} / \pi_{t+1} = 0$, leads to $\theta_{4,t} = -\theta_{6,t}$.

The optimal choices for the policy rate R^m_t and κ_t depend on whether the policy rate is set below the loan rate or not. If $R^m_t = R^L_t$, the constraint (17) reduces to $\left(1/R^L_t\right) - \beta (c^{\sigma}_{l,t} / \epsilon_{l,t}) E_t [0.5 (\epsilon_b c^{-\sigma}_{b,t+1} + \epsilon_l c^{-\sigma}_{l,t+1}) / \pi_{t+1}]$ as in the case of non-rationed money supply (see Appendix A.4). Here, we consider the case where money supply rationing is induced by

$$R^m_t < R^L_t,$$

such that the first order condition for R^m_t is given by $\theta_{3,t} \beta (c^{\sigma}_{l,t} / \epsilon_{l,t}) E_t [0.5 (\epsilon_b c^{-\sigma}_{b,t+1} + \epsilon_l c^{-\sigma}_{l,t+1}) / \pi_{t+1}] [\kappa_t / (1 + \upsilon)] R^L_t = \theta_{4,t} \kappa_t l_t + \theta_{14,t} \kappa^B_t b_{t-1} \pi^{-1}_t$, which can by using $\theta_{14,t} = 0$ be further simplified to $\theta_{3,t} \beta (c^{\sigma}_{l,t} / \epsilon_{l,t}) E_t [0.5 (\epsilon_b c^{-\sigma}_{b,t+1} + \epsilon_l c^{-\sigma}_{l,t+1}) / \pi_{t+1}] / (1 + \upsilon) = \theta_{4,t} l_t / R^L_t$. Combining the latter with the first order condition for κ_t, $-\theta_{3,t} \beta (c^{\sigma}_{l,t} / \epsilon_{l,t}) E_t [0.5 (\epsilon_b c^{-\sigma}_{b,t+1} + \epsilon_l c^{-\sigma}_{l,t+1}) / \pi_{t+1}] [1 / (1 + \upsilon)] [(R^L_t / R^m_t) - 1] + \theta_{4,t} (l_t / R^m_t) = 0$, then leads to $\theta_{4,t} l_t / R^L_t = 0$ and thus $\theta_{4,t} = 0$. The first order conditions for l_t, $\theta_{4,t} [(1 + \upsilon) / R^L_t + (\kappa_t / R^m_t)] - \theta_{6,t} (1 + \upsilon) / R^L_t - \theta_{15,t} = 0$, then implies $\theta_{15,t} = 0$. We can therefore conclude that the constraints associated with the multiplier $\theta_{3,t}$, $\theta_{4,t}$, $\theta_{5,t}$, $\theta_{6,t}$, $\theta_{13,t}$, $\theta_{14,t}$, and $\theta_{15,t}$, which are all equal to zero, are not binding for the policy choice. Then, the loan rate can be set to ensure that the constraint associated with the multiplier θ_{1t} is satisfied, while the constraint associated with the multiplier $\theta_{16,t}$ can be used to residually determine the sequence of q_t for a given allocation. Thus, the policy problem (63) can be reduced to

$$\begin{aligned}
&\max_{\{c_{b,t}, c_{l,t}, n_t, mc_t, \tilde{Z}_t, Z_{1,t}, Z_{2,t}, s_t, \pi_t, h_{b,t}\}_{t=0}^{\infty}} \ \min_{\{\lambda_{1,t}, \ldots \lambda_{7,t}\}_{t=0}^{\infty}} \qquad (64)\\
&E \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} 0.5 \epsilon_{b,t} (c^{1-\sigma}_{b,t} - 1) (1 - \sigma)^{-1} + 0.5 \epsilon_{l,t} (c^{1-\sigma}_{l,t} - 1) (1 - \sigma)^{-1} - \chi (0.5 n_t)^{1+\eta} (1 + \eta)^{-1} \\ + 0.5 \gamma (h^{1-\sigma_h}_{b,t} - 1) (1 - \sigma_h)^{-1} + 0.5 \gamma ((h - h_{b,t})^{1-\sigma_h} - 1) (1 - \sigma_h)^{-1} \end{array} \right],\\
&+ \lambda_{1,t} \left[(1 - \tau^n) \chi 0.5^{\eta} n^{\eta+1-\alpha}_t / (mc_t a_t \alpha) - \beta E_t [0.5 (\epsilon_b c^{-\sigma}_{b,t+1} + \epsilon_l c^{-\sigma}_{l,t+1}) / \pi_{t+1}] \right] + \lambda_{2,t} \left[a_t n^{\alpha}_t / s_t - c_{l,t} - c_{b,t} \right]\\
&+ \lambda_{3,t} \left[s_t - \phi s_{t-1} \pi^{\varepsilon}_t - (1 - \phi)^{\frac{1}{1-\varepsilon}} \left(1 - \phi \pi^{\varepsilon-1}_t \right)^{\frac{\varepsilon}{\varepsilon-1}} \right] + \lambda_{4,t} \left[(1 - \phi) (\tilde{Z}_t)^{1-\varepsilon} + \phi \pi^{\varepsilon-1}_t - 1 \right]\\
&+ \lambda_{5,t} \left[\tilde{Z}_t (\varepsilon - 1) / \varepsilon - Z_{1,t} / Z_{2,t} \right] + \lambda_{6,t} \left[Z_{1,t} - (1 - \tau^n) (\chi / \alpha) 0.5^{\eta} n^{1+\eta}_t s^{-1}_t - \phi \beta E_t \pi^{\varepsilon}_{t+1} Z_{1,t+1} \right]\\
&+ \lambda_{7,t} \left[Z_{2,t} - (1 - \tau^n) (\chi / \alpha) 0.5^{\eta} n^{1+\eta}_t (mc_t s_t)^{-1} - \phi \beta E_t \pi^{\varepsilon-1}_{t+1} Z_{2,t+1} \right]
\end{aligned}$$

Neglecting the conditions for $t = 0$, the solution to the policy problem (64) has to satisfy the

following first order conditions:

$$
\begin{aligned}
0 &= h_{b,t} - 0.5h \\
0 &= 0.5\epsilon_{b,t} c_{b,t}^{-\sigma} + \lambda_{1,t-1}\epsilon_{b,t} 0.5\sigma \left(c_{b,t}^{-\sigma-1}/\pi_t \right) - \lambda_{2,t}, \\
0 &= 0.5\epsilon_{l,t} c_{l,t}^{-\sigma} + \lambda_{1,t-1}\epsilon_{l,t} 0.5\sigma \left(c_{l,t}^{-\sigma-1}/\pi_t \right) - \lambda_{2,t}, \\
0 &= -\chi 0.5^{1+\eta} n_t^{\eta} + \left[\lambda_{1,t} \left(\eta + 1 - \alpha \right) \left(1 - \tau^n \right) \chi 0.5^{\eta} n_t^{\eta-\alpha} / \left(m c_t \alpha a_t \right) \right] + \left(\lambda_{2,t} a_t \alpha n_t^{\alpha-1} / s_t \right) \\
&\quad - \lambda_{6,t} \left(1 + \eta \right) \left(1 - \tau^n \right) \left(\chi/\alpha \right) 0.5^{\eta} n_t^{\eta} s_t^{-1} - \lambda_{7,t} \left(1 + \eta \right) \left(1 - \tau^n \right) \left(\chi/\alpha \right) 0.5^{\eta} n_t^{\eta} \left(m c_t s_t \right)^{-1}, \\
0 &= \left[\lambda_{1,t-1} \left(0.5\epsilon_b c_{b,t}^{-\sigma} + 0.5\epsilon_l c_{l,t}^{-\sigma} \right) / \pi_t^2 \right] + \lambda_{4,t} \left(\varepsilon - 1 \right) \phi \pi_t^{\varepsilon-2} \\
&\quad - \lambda_{6,t-1} \phi \varepsilon \pi_t^{\varepsilon-1} Z_{1,t} - \lambda_{7,t-1} \phi \left(\varepsilon - 1 \right) \pi_t^{\varepsilon-2} Z_{2,t} \\
&\quad + \lambda_{3,t} \left[-\phi s_{t-1} \varepsilon \pi_t^{\varepsilon-1} - \left(1 - \phi \right)^{\frac{1}{1-\varepsilon}} \frac{\varepsilon}{\varepsilon - 1} \left(1 - \phi \pi_t^{\varepsilon-1} \right)^{\frac{\varepsilon}{\varepsilon-1}-1} \left(- \left(\varepsilon - 1 \right) \phi \pi_t^{\varepsilon-2} \right) \right], \\
0 &= -\left(\lambda_{2,t} a_t n_t^{\alpha} / s_t^2 \right) + \lambda_{3,t} - \beta E_t \lambda_{3,t+1} \phi \pi_{t+1}^{\varepsilon} \\
&\quad + \lambda_{6,t} \left(1 - \tau^n \right) \left(\chi/\alpha \right) 0.5^{\eta} n_t^{1+\eta} s_t^{-2} + \lambda_{7,t} \left(1 - \tau^n \right) \left(\chi/\alpha \right) 0.5^{\eta} n_t^{1+\eta} m c_t^{-1} s_t^{-2}, \\
&\quad - \left[\lambda_{1,t} \left(1 - \tau^n \right) \chi 0.5^{\eta} n_t^{\eta+1-\alpha} / \left(m c_t^2 \alpha a_t \right) \right] + \lambda_{7,t} \mu_t \left(1 - \tau^n \right) \left(\chi/\alpha \right) 0.5^{\eta} n_t^{1+\eta} m c_t^{-2} s_t^{-1}, \\
0 &= -\left(\lambda_{5,t} / Z_{2,t} \right) + \lambda_{6,t} - \lambda_{6,t-1} \phi \pi_t^{\varepsilon}, \\
0 &= \lambda_{5,t} \left(Z_{1,t} / Z_{2,t}^2 \right) + \lambda_{7,t} - \lambda_{7,t-1} \phi \pi_t^{\varepsilon-1}, \\
0 &= \lambda_{4,t} (1 - \phi) \left(1 - \varepsilon \right) \left(\tilde{Z}_t \right)^{-\varepsilon} + \lambda_{5,t} \left(\varepsilon - 1 \right) / \varepsilon,
\end{aligned}
$$

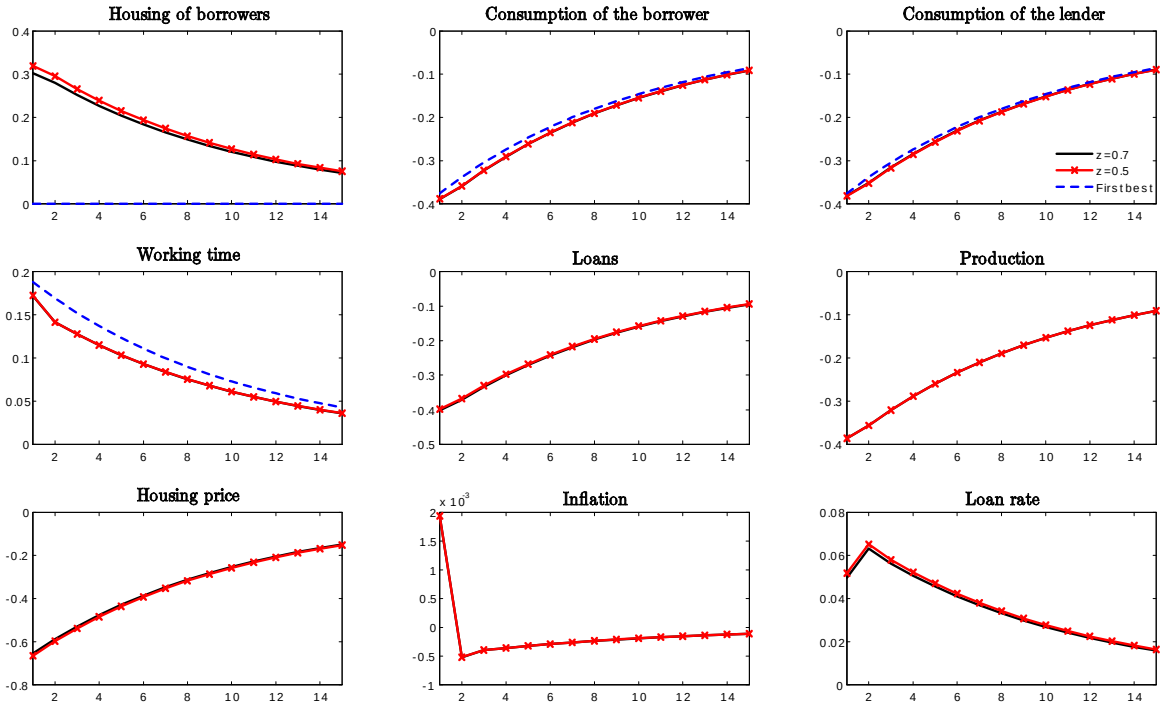
as well as the constraints to the policy problem (64). The steady state of the solution, where all exogenous and endogenous variables satisfy $x_t = x_{t-1} = x_{t+1} = x$, can be reduced to a set $\{c_b, c_l, n, \pi, s, \lambda_1, \lambda_3, h_b\}$ satisfying

$$
\begin{aligned}
0 &= \epsilon_l c_l^{-\sigma} \left(1 + \sigma c_l^{-1} \lambda_1 / \pi \right) - \epsilon_b c_b^{-\sigma} \left(1 + \sigma c_b^{-1} \lambda_1 / \pi \right), \\
0 &= \frac{\pi}{\beta} 0.5 \left(1 + \sigma c_b^{-1} \lambda_1 / \pi \right) \left(\alpha n^{\alpha} / s \right) + \lambda_1 \frac{0.5\epsilon_b c_b^{-\sigma} + 0.5\epsilon_l c_l^{-\sigma}}{\epsilon_b c_b^{-\sigma}} \left(\eta + 1 - \alpha + \left(1 + \eta \right) \Phi(\pi) \right) \\
&\quad - \frac{\chi n^{1+\eta} 0.5^{1+\eta}}{\epsilon_b c_b^{-\sigma}} \frac{\pi}{\beta}, \quad \text{where} \quad \Phi(\pi) = \frac{1 - \phi\beta\pi^{\varepsilon}}{1 - \phi\beta\pi^{\varepsilon-1}} \frac{1 - \phi\pi^{\varepsilon-1}}{1 - \phi\pi^{\varepsilon}} - 1. \\
0 &= \lambda_1 \frac{0.5\epsilon_b c_b^{-\sigma} + 0.5\epsilon_l c_l^{-\sigma}}{\epsilon_b c_b^{-\sigma}} \Phi(\pi) + \frac{\pi}{\beta} 0.5 \left(1 + \sigma c_b^{-1} \lambda_1 / \pi \right) \left(n^{\alpha} / s \right) - \frac{\pi}{\beta} s \lambda_3 \frac{c_b^{\sigma}}{\epsilon_b} \left(1 - \beta\phi\pi^{\varepsilon} \right), \\
0 &= -\lambda_1 + \lambda_3 \phi \varepsilon \pi^{\varepsilon} \frac{s}{0.5\epsilon_b c_b^{-\sigma} + 0.5\epsilon_l c_l^{-\sigma}} \frac{\pi - 1}{1 - \phi\pi^{\varepsilon-1}} + \lambda_1 \beta \frac{\varepsilon\phi\pi^{\varepsilon-1}}{1 - \phi\beta\pi^{\varepsilon-1}} \frac{1 - \pi}{1 - \phi\pi^{\varepsilon}}, \\
0 &= \left(1 - \tau^n \right) \frac{\varepsilon}{\varepsilon - 1} \frac{\pi}{\beta} \left(\frac{1 - \phi\pi^{\varepsilon-1}}{1 - \phi} \right)^{\frac{1}{\varepsilon-1}} \frac{\left(1 - \phi\beta\pi^{\varepsilon-1} \right)}{\left(1 - \phi\beta\pi^{\varepsilon} \right)} - \frac{0.5\epsilon_b c_b^{-\sigma} + 0.5\epsilon_l c_l^{-\sigma}}{\left(\chi/\alpha \right) 0.5^{\eta} n^{\eta+1-\alpha}}, \\
0 &= (1 - \phi)^{\frac{1}{1-\varepsilon}} \frac{\left(1 - \phi\pi^{\varepsilon-1} \right)^{\frac{\varepsilon}{\varepsilon-1}}}{\left(1 - \phi\pi^{\varepsilon} \right)} - s, \; 0 = c_l + c_b - n^{\alpha}/s, \; h_b = 0.5h.
\end{aligned}
$$

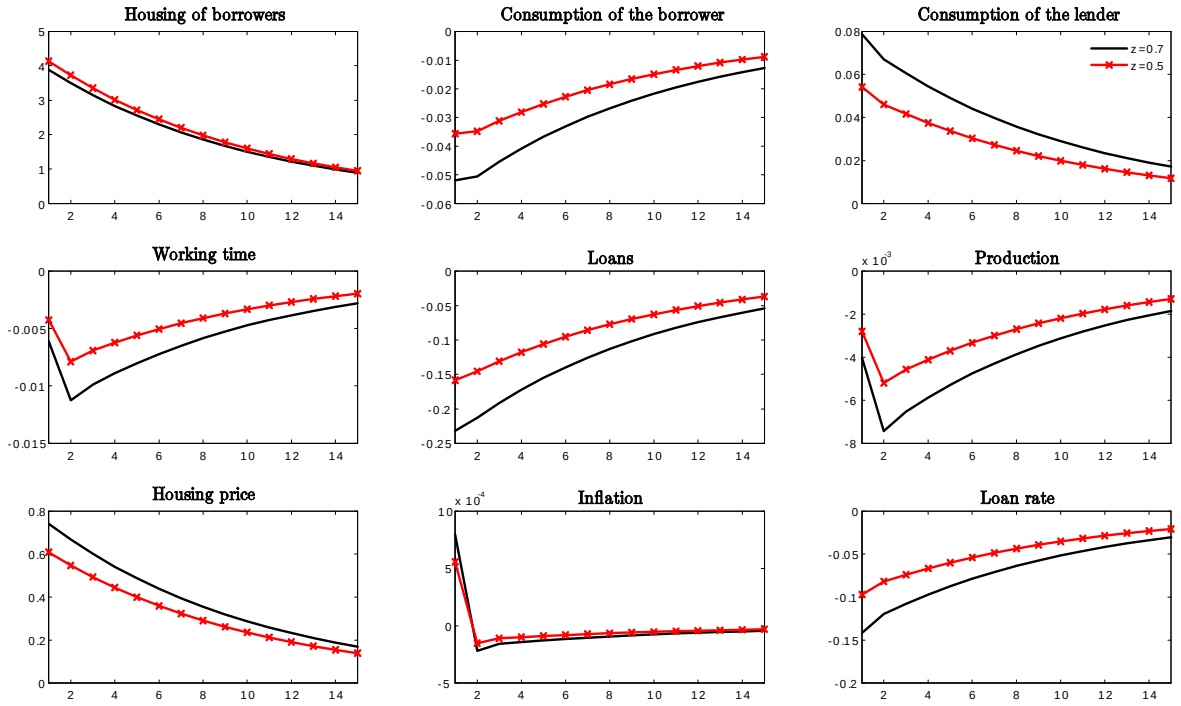
A.6 Parameter values

Table A1 Benchmark parameters			
Subjective discount factor	$\beta = 0.99$	Share of unsecured loans	$\upsilon = 0.5$
Inverse int. elasticity of substitution	$\sigma_{(h)} = 2$	Utility weight on housing	$\gamma = 0.1$
Inverse of Frisch elasticity	$\eta = 1$	Utility weight on working time	$\chi = 98$
Substitution elasticity	$\varepsilon = 10$	Housing supply	$h = 28$
Degree of price stickiness	$\phi = 0.7$	Stochastic consumption weights	$\Delta\epsilon_i = 1$
Labor income share	$\alpha = 0.66$	Mean liquidation share of collateral	$z = 0.8$
Autocorrelation of shocks	$\rho_{a,z} = 0.9$	Standard deviation of shocks	$sd_{a,z} = 0.005$

A.7 Additional figures



Responses to a contractionary productivity shock under optimal policy w/o money rationing



Responses to a lower liquidation value under optimal policy w/o money rationing