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**Conference Paper**

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Beiträge zur Jahrestagung des Vereins für Socialpolitik 2014: Evidenzbasierte Wirtschaftspolitik  
- Session: Financial Econometrics, No. C12-V2

**Provided in Cooperation with:**

Verein für Socialpolitik / German Economic Association

*Suggested Citation:* Grammig, Joachim; Sönksen, Jantje (2014) : Consumption-Based Asset Pricing with Rare Disaster Risk: A Simulated Method of Moments Approach, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2014: Evidenzbasierte Wirtschaftspolitik - Session: Financial Econometrics, No. C12-V2, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft, Kiel und Hamburg

This Version is available at:

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# Consumption-Based Asset Pricing with Rare Disaster Risk

## A Simulated Method of Moments Approach

Joachim Grammig and Jantje Sönksen \*

March 1, 2014

### Abstract

The rare disaster hypothesis suggests that the extraordinarily high postwar U.S. equity premium resulted because investors ex ante demanded compensations for unlikely but calamitous risks that they happened not to incur. While convincing in theory, empirical tests of the rare disaster explanation are scarce. We estimate a disaster-including consumption-based asset pricing model (CBM) using a combination of the simulated method of moments and bootstrapping. We consider several methodological alternatives that differ in the moment matches and the way to account for disasters in the simulated consumption growth and return series. Whichever specification is used, the estimated preference parameters are of an economically plausible size, and the estimation precision is much higher than in previous studies that use the canonical CBM. A comparable combination of plausibility and estimation precision has not been delivered in the related literature. Our results thus provide empirical support for the rare disaster hypothesis, and help restore the nexus between real economy and financial markets implied by the consumption-based asset pricing paradigm.

*Key words:* equity premium puzzle, rare disaster risk, asset pricing, simulated method of moments

*JEL:* G10, G12, C58

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# 1 Introduction

It serves as a paradigm in asset pricing theory that positive expected excess returns result as a form of risk compensation. Since the empirical equity premium – the average excess return of equity over the risk-free rate – has been very high for postwar U.S. stocks, there seems to be a considerable amount of risk to be compensated for. The methodological lynchpin of the pertinent literature is Hansen and Singleton’s (1982) consumption-based asset pricing model (CBM). Assuming additive power utility, it implies the asset pricing equation for a gross return  $R_{t+1}$

$$\mathbb{E}_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} \right] = 1, \quad (1.1)$$

where  $\beta$  denotes the subjective discount factor capturing time preference,  $\gamma$  constant relative risk aversion (CRRA), and  $C_t$  consumption in period  $t$ .

As evinced by Mehra and Prescott (1985), the canonical CBM cannot explain the high equity premium at plausible values of relative risk aversion, which led to the widespread belief that the model is strong in theory, but weak in application.<sup>1</sup> It was first noted by Rietz (1988) that the reason for the apparent failure of the CBM may be rare but extreme contractions in consumption. The rationale is that investors demand a compensation for the risk of sharp downturns in their consumption that may occur with a very small probability. This compensation is reflected in high expected returns for assets whose payoffs covary positively with consumption. In a sample without such contractions, average returns of those

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<sup>1</sup> Cochrane (1996) uses Hansen’s (1982) Generalized Method of Moments (GMM) to estimate the parameters of Equation (1.1) and obtains  $\hat{\gamma} = 241$ , which implies an implausibly high risk aversion. Mehra and Prescott (1985), and Rietz (1988) consider a range for  $\gamma$  between one to ten to be plausible, while Cochrane (2005) suggests one to five.

assets can be high. Barro (2006) draws on Rietz’s (1988) approach and provides a model that permits a calibration of equity premia. He uses information about disastrous contractions in GDP, assumes plausible values of relative risk aversion and time preference, and shows that the calibrated equity premia are in the range of their empirically observed counterparts. The seminal contributions by Rietz and Barro have laid the foundation for a growing literature. However, empirical tests of the rare disaster hypothesis are sparse. Whilst calibrations like Barro’s are useful, the question remains, how asset pricing models that account for rare disasters – and in particular the CBM – perform when econometric techniques are applied to estimate the model parameters.

The empirical analysis of asset pricing models that account for the possibility of rare disasters is hampered because extreme consumption contractions are rare by definition. How can the estimation of a disaster-including CBM be accomplished without observing any sharp downturns in the first place? We tackle this epistemological problem by pursuing an econometric approach that is inspired by Cochrane’s (2005, p. 461) remark:

“We had no banking panics, and no depressions; no civil wars, no constitutional crises; we did not lose the Cold War, no missiles were fired over Berlin, Cuba, Korea, or Vietnam. If any of these things had happened, we might well have seen a calamitous decline in stock values, and I would not be writing about the equity premium puzzle.”

In line with this view, we conjecture that the extremely high risk aversion estimates result from a sample selection effect, i.e. the consumption and return data that the U.S. economy produced during the past 65 years represent one lucky

itinerary of the histories that could have been.<sup>2</sup> If this is true, then the apparent empirical deficiencies of the CBM emerge because the available data do not represent the possible – disaster-including – scenarios that investors anticipated. If disastrous contractions in consumption were possible but did not occur, then we have to account for them by traveling (metaphorically) the roads that the U.S. postwar economy did not take. We have to consider histories, in which there were banking panics and depressions, worlds, in which the U.S. did lose the Cold War, in short, alternative histories in which we would not write about the equity premium puzzle.

We combine the simulated method of moments (SMM) and non-parametric and parametric bootstrapping to facilitate such journeys within frequentist statistics’ concept of repeated sampling. All our methodological alternatives rely on simulated disaster-including consumption growth and return data. Adopting the disaster identification scheme proposed by Barro (2006) on GDP data collected by Angus Maddison, we identify contractions that exceed a specified threshold. We draw shrinkage factors from a Double Power Law distribution to allow for sharp contractions of the bootstrapped “regular” consumption growth series. Following Barro and Jin (2011), we use the sample of identified contractions and estimate the distributional parameters by maximum likelihood. We consider three possibilities to simulate financial returns conditional on occurrence and size of a calamitous consumption contraction, and we advocate two types of moment matches that facilitate SMM-type estimation of a disaster-including CBM. In an alternative approach, which we refer to as *Alternative Histories Bootstrap* (a term chosen to echo Cochrane’s quote), we apply GMM to ensembles of bootstrapped disaster-

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<sup>2</sup> By invoking the “peso problem hypothesis”, Veronesi (2004) argues in a similar direction.

including series. Here, parameter estimates are obtained by averaging over those “alternative histories”. All methodological variants deliver comparable, economically plausible and precise estimates of the coefficient of relative risk aversion and the subjective discount factor. Our results thus provide empirical support for the rare disaster hypothesis, and help restore the nexus between real economy and financial markets implied by the consumption-based asset pricing paradigm.

Our study contributes to a growing literature on rare disaster risk in asset pricing. Barro and Jin (2011) present a calibration of the Barro (2006) model with Epstein-Zin preferences and Double Power Law distributed extreme contractions. Aiming at a possible solution to the volatility puzzle, Wachter (2013) extends Barro’s (2006) model by recursive preferences and time-varying disaster probabilities. Gabaix (2012) formulates a model in which the severity of disasters is time-varying, and challenges ten prominent puzzles in macro-finance. Backus et al. (2011) rely on equity index options to obtain the distribution of consumption growth disasters, and Gourio (2012) includes time-varying disaster risk in a business cycle model. Julliard and Ghosh (2012) fit a non-parametric distribution to disastrous GDP contraction data and argue that the equity premium puzzle itself emerges as a rare event. Weitzman (2005) uses a Bayesian approach that focuses on learning about consumption volatility, which implies fat-tailed posterior distributions of future consumption growth. The paper by Posch and Schrimpf (2012) is closest to the present study. They consider an alternative approach to evaluate the rare disaster hypothesis by simulating consumption and return data of economies that are potentially hit by disasters. Based on the simulated data, Posch and Schrimpf analyze the Euler equation errors, and estimate the CBM on

samples conditional on no disasters. They report that the parameter estimates obtained from such a procedure are comparably implausible to those computed from empirical data. Our results complement theirs in that we explicitly focus on disaster-including consumption series and the plausible estimates of relative risk aversion and subjective discount factor that are obtained once rare disaster risk is accounted for.

From a broader perspective, our study contributes to the literature that attempts to vindicate the CBM, while retaining its core paradigm reflected in Equation (1.1). These studies only partially succeed to provide plausible and precise estimates of the CBM preference parameters. Yogo (2006), for example, proposes a structural model that differentiates between consumption of durable and nondurable goods. The model is able to explain cross-sectional and time series variation in expected stock returns but at a still very high level of risk aversion. The smallest relative risk aversion estimate obtained with the unconditional version of his model amounts to  $\hat{\gamma} = 191.4$ . The estimated subjective discount factor is plausible ( $\hat{\beta} = 0.9$ ). Savov (2011) relies on waste data as a measure of consumption, fixes  $\beta = 0.95$  and – using the excess market return as the single test asset – obtains a considerably lower estimate of the parameter of risk aversion,  $\hat{\gamma} = 17.0$ . Estimation precision is moderate (s.e. ( $\hat{\gamma}$ ) = 9.0). Julliard and Parker (2005) analyze the ultimate risk of consumption, which is defined as the covariance of returns and consumption growth aggregated over the current and future periods. They fix  $\beta = 1$ , and estimate  $\hat{\gamma} = 9.1$ . However, the estimate also has a relatively high standard error (s.e. ( $\hat{\gamma}$ ) = 17.2). Savov’s (2011) and Julliard and Parker’s (2005) studies compare to ours in the sense that they modify the consumption data that

is used in the empirical analysis instead of elaborating investors' preferences.

The remainder of the paper is structured as follows. Section 2 motivates a disaster-including CBM and outlines the econometric methodology. Section 3 discusses the estimation results, whilst Section 4 concludes.

## 2 Methodology

### 2.1 Accounting for rare disasters in a consumption-based asset pricing model

There are two rational explanations for the high equity premia measured in U.S. postwar return data, and both are compatible with the CBM. The first is that investors are extremely risk averse and demand large compensations for carrying little risk, a reasoning that is in line with the high CRRA estimates reported by Cochrane (1996) and Yogo (2006). The second is the rare disaster hypothesis, which states that high equity premia result because investors are compensated for the risk of calamitous contractions that (luckily) did not happen. An empirical assessment of this explanation is hampered when the historical data do not contain enough information about disastrous contractions of consumption and asset prices. In the following, we outline our idea to econometrically test the rare disaster hypothesis within the CBM framework.

Barro (2006) considers a disaster-including consumption process that he uses to obtain closed form solutions of equity premia conditional and unconditional on disaster periods. We adopt his specification for our purposes, and assume that

consumption evolves as<sup>3</sup>

$$C_{t+1} = C_t e^{u_{t+1}} e^{v_{t+1}}, \quad (2.1)$$

where  $u_{t+1} \sim (\mu, \sigma^2)$  and  $v_{t+1} = \ln(1 - b_{t+1})d_{t+1}$ . The binary disaster indicator  $d_{t+1}$  is one if a disaster occurs in  $t + 1$  and zero otherwise. If  $d_{t+1} = 1$ , consumption contracts by a random factor  $b_{t+1} \in [q, 1]$ , where  $q$  refers to the disaster threshold, such that

$$\frac{C_{t+1}}{C_t} = e^{u_{t+1}} (1 - b_{t+1})^{d_{t+1}}. \quad (2.2)$$

Accordingly,  $e^{u_{t+1}}$  denotes non-disastrous consumption growth and  $(1 - b_{t+1})^{d_{t+1}}$  accounts for the effect of a potential disaster on consumption.

Substituting the right-hand side of Equation (2.2) into Equation (1.1), we can write the basic asset pricing equation as it applies to a gross return:

$$\begin{aligned} \mathbb{E}_t \left[ \beta (e^{u_{t+1}} e^{v_{t+1}})^{-\gamma} R_{t+1} \right] &= p \underbrace{\mathbb{E}_t \left[ \beta (e^{u_{t+1}} (1 - b_{t+1}))^{-\gamma} R_{t+1} | d_{t+1} = 1 \right]}_{\text{expect. cond. on disaster in } t+1} \\ &+ (1 - p) \underbrace{\mathbb{E}_t \left[ \beta (e^{u_{t+1}})^{-\gamma} R_{t+1} | d_{t+1} = 0 \right]}_{\text{expect. cond. on no disaster in } t+1} \quad (2.3) \\ &= 1, \end{aligned}$$

where the disaster probability  $p = \mathbb{P}(d_{t+1} = 1)$  is assumed to be time-invariant. Applying the Law of Total Expectation to Equation (2.3) and re-arranging terms, we obtain

$$\mathbb{E} \left[ \beta (e^{u_t})^{-\gamma} R_t | d_t = 0 \right] = \frac{1}{1 - p} \left[ 1 - p \mathbb{E} \left[ \beta (e^{u_t} (1 - b_t))^{-\gamma} R_t | d_t = 1 \right] \right]. \quad (2.4)$$

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<sup>3</sup> In Barro's (2006) endowment economy, consumption equals output  $A_t$ , where  $\ln A_{t+1} = \ln A_t + \mu + u_{t+1} + v_{t+1}$ , with  $u_{t+1} \sim N(0, \sigma^2)$ . For convenience, we modify Barro's specification by allowing for a non-zero mean of  $u_{t+1}$  and discarding the drift parameter  $\mu$ .

Pricing excess returns, the analogue of Equation (2.4) is

$$\mathbb{E} \left[ (e^{u_t})^{-\gamma} R_t^e | d_t = 0 \right] = -\frac{p}{1-p} \left[ \mathbb{E} \left[ (e^{u_t}(1-b_t))^{-\gamma} R_t^e | d_t = 1 \right] \right]. \quad (2.5)$$

$\beta$  is not identified in this case.

If a sample with disaster observations were available, we could write the sample counterparts of the population moments in Equation (2.4) as

$$\frac{1}{T - D_T} \sum_{t=1}^T \beta c g_{nd,t}^{-\gamma} R_{nd,t} (1 - d_t) = \frac{1}{1 - \frac{D_T}{T}} \left[ 1 - \frac{D_T}{T} \left[ \frac{1}{D_T} \sum_{t=1}^T \beta c g_{d,t}^{-\gamma} R_{d,t} d_t \right] \right], \quad (2.6)$$

where  $D_T = \sum_{t=1}^T d_t$  counts the number of disasters in a series of length  $T$ ,  $c g_{d,t}$  and  $R_{d,t}$  denote consumption growth and gross return in a disaster period, and  $c g_{nd,t}$  and  $R_{nd,t}$  are regular consumption growth and return. As  $T \rightarrow \infty$  and when a Law of Large Numbers (LLN) holds,  $\frac{D_T}{T} \rightarrow p$  and  $\frac{T-D_T}{T} \rightarrow 1-p$ . Furthermore, assuming that a Uniform LLN holds

$$\frac{1}{D_T} \sum_{t=1}^T \beta c g_{d,t}^{-\gamma} R_{d,t} d_t \xrightarrow[p.u.]{} \mathbb{E} \left[ \beta (e^{u_t}(1-b_t))^{-\gamma} R_t | d_t = 1 \right], \quad (2.7)$$

and

$$\frac{1}{T - D_T} \sum_{t=1}^T \beta c g_{nd,t}^{-\gamma} R_{nd,t} (1 - d_t) \xrightarrow[p.u.]{} \mathbb{E} \left[ \beta (e^{u_t})^{-\gamma} R_t | d_t = 0 \right], \quad (2.8)$$

where  $\xrightarrow[p.u.]{} \mathbb{E}$  denotes uniform convergence in probability. Analogous expressions can be given for the sample counterparts of (2.5).

Suppose that you have access to disaster-including data. You could then think of using GMM and matching the sample moments (2.6) for selected test assets with their population counterparts (2.4). However, your strategy would be impeded by

the problem that even for long time series, the quality of the moment matches will be poor and the parameter standard errors huge. Rare disasters are, well, rare, and  $T$  has to be very large to ensure moderate estimation precision.

For the U.S. postwar data, which is used by all the studies mentioned in the introduction, the problem is aggravated. These data do not incorporate any disaster observations, i.e.  $d_t = 0 \forall t$  and thus  $D_T = 0$ , such that  $\hat{p} = 0$ . To apply GMM, we would use the disaster-free consumption growth  $cg_{nd,t}$  and return series  $R_{nd,t}$  (using excess returns:  $R_{nd,t}^e$ ), and match the left-hand side of Equation (2.4) (using excess returns: (2.5)) with their sample counterparts  $\frac{1}{T} \sum_{t=1}^T \beta cg_{nd,t}^{-\gamma} R_{nd,t}$  (using excess returns:  $\frac{1}{T} \sum_{t=1}^T \beta cg_{nd,t}^{-\gamma} R_{nd,t}^e$ ). However, the right-hand side of Equation (2.4) is equal to one, and the right hand side of Equation (2.5) is equal to zero, only if  $p = 0$ . Hence, the usual moment matches for GMM estimation of the CBM

$$\mathbb{G}_T(\beta, \gamma) \equiv \frac{1}{T} \sum_{t=1}^T \beta cg_{nd,t}^{-\gamma} \mathbf{R}_{nd,t} - 1, \quad (2.9)$$

using the gross returns of  $N$  test assets,  $\mathbf{R}_{nd,t} = [R_{nd,t}^1, \dots, R_{nd,t}^N]'$  or

$$\mathbb{G}_T(\gamma) \equiv \frac{1}{T} \sum_{t=1}^T cg_{nd,t}^{-\gamma} \mathbf{R}_{nd,t}^e, \quad (2.10)$$

using excess returns  $\mathbf{R}_{nd,t}^e = [R_{nd,t}^{e1}, \dots, R_{nd,t}^{eN}]'$ , are valid only if disastrous contractions in consumption are impossible. Yet, it is hard to imagine that investors in 1946, after World War II and the Great Depression, and at the wake of the Cold War, should have assigned a probability of zero to states in which their consumption may suffer from extreme contractions.

Does this imply that an empirical assessment of the rare disaster hypothesis

and the estimation of a disaster-including CBM cannot be performed due to a lack of suitable data? That one has to go beyond the *possimpible*?<sup>4</sup> We tackle this problem with an estimation strategy that consists of a mix of parametric and non-parametric bootstrapping and SMM. Our approach is inspired by a quote of Singleton (2006, p.254), with which he advocates the simulated method of moments:

“More fully specified models allow experimentation with alternative formulations of economies and, perhaps, analysis of processes that are more representative of history for which data are not readily available.”

We consider three SMM estimation strategies along that line. Each of them implies matching sample moments and simulated theoretical moments, the latter accounting for the possibility of consumption disasters. None of them requires the availability of disaster-including consumption and return data.

## 2.2 Using SMM to estimate a disaster-including CBM

For the first approach towards estimating a disaster-including CBM, we derive moment matches from Equations (2.4) and (2.5). Using time series of length  $T$  of regular consumption and (excess) returns, sample counterparts of the left-hand side conditional expectations can be computed as  $\frac{1}{T} \sum_{t=1}^T \beta c g_{nd,t}^{-\gamma} R_{nd,t}$  and  $\frac{1}{T} \sum_{t=1}^T \beta c g_{nd,t}^{-\gamma} R_{nd,t}^e$ . On the other hand, the right-hand side moments of Equations (2.4) and (2.5) can neither be expressed as functions of parameters (which would facilitate GMM), nor can sample counterparts be computed using disaster-free data. However, if

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<sup>4</sup> *possimpible*: the verge of the possible, where the possible and the impossible meet. Word creation by Barney Stinson, a popular character of the TV show “How I Met Your Mother”.

it is possible to specify processes that are “more representative of history”, in the sense of Singleton’s (2006) quote, i.e. series that include disaster observations, these moments can be simulated, viz

$$\begin{aligned} \frac{1}{1-p} [1 - p\mathbb{E}[\beta (e^{u_t}(1 - b_t))^{-\gamma} R_t | d_t = 1]] &\approx \frac{1}{1 - \frac{D_{\mathcal{T}}}{\mathcal{T}(T)}} \left( 1 - \frac{1}{\mathcal{T}(T)} \sum_{s=1}^{\mathcal{T}(T)} \beta c g_s^{-\gamma} R_s d_s \right) \\ \frac{p}{1-p} [\mathbb{E}[(e^{u_t}(1 - b_t))^{-\gamma} R_t^e | d_t = 1]] &\approx -\frac{1}{1 - \frac{D_{\mathcal{T}}}{\mathcal{T}(T)}} \frac{1}{\mathcal{T}(T)} \sum_{s=1}^{\mathcal{T}(T)} c g_s^{-\gamma} R_s^e d_s, \end{aligned} \quad (2.11)$$

where  $D_{\mathcal{T}} = \sum_{s=1}^{\mathcal{T}(T)} d_s$  denotes the number of disasters in the simulated sample of size  $\mathcal{T}(T)$ . Using the risk-free rate  $R^f$  and a vector of excess returns  $\mathbf{R}^e$  as test assets, we can utilize the moment matches

$$\mathbb{G}_T(\boldsymbol{\theta}) = \begin{bmatrix} \frac{1}{T} \sum_{t=1}^T \beta c g_{nd,t}^{-\gamma} R_{nd,t}^f - \frac{1}{1 - \frac{D_{\mathcal{T}}}{\mathcal{T}(T)}} \left( 1 - \frac{1}{\mathcal{T}(T)} \sum_{s=1}^{\mathcal{T}(T)} \beta c g_s^{-\gamma} R_s^f d_s \right) \\ \frac{1}{T} \sum_{t=1}^T \beta c g_{nd,t}^{-\gamma} \mathbf{R}_{nd,t}^e + \frac{1}{1 - \frac{D_{\mathcal{T}}}{\mathcal{T}(T)}} \frac{1}{\mathcal{T}(T)} \sum_{s=1}^{\mathcal{T}(T)} \beta c g_s^{-\gamma} \mathbf{R}_s^e d_s \end{bmatrix}, \quad (2.12)$$

where  $\boldsymbol{\theta} = [\beta, \gamma]'$ .<sup>5</sup> SMM estimates of  $\beta$  and  $\gamma$  can then be obtained by solving

$$\hat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\theta} \in \Theta} \mathbb{G}_T(\boldsymbol{\theta})' \mathbf{W}_T \mathbb{G}_T(\boldsymbol{\theta}), \quad (2.13)$$

where  $\mathbf{W}_T$  is a symmetric and positive definite weighting matrix. The analysis has to be based on a large  $\mathcal{T}(T)$  to ensure that the simulated data contain enough disasters, such that the approximations in Equation (2.11) are sufficiently accurate. We will refer to this first SMM approach as MAD-SMM (*Moments Accounting for Disasters*).

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<sup>5</sup>  $\beta$  is identified via the first moment match in Equation (2.12) that invokes the risk-free rate. If you want to use only excess returns,  $\beta$  will not be identified, and you can set it to any non-zero number.

In the next section, we will explain in detail how the simulation of the disaster-including consumption growth,  $\{cg_s\}_{s=1}^{\mathcal{T}(T)}$ , and return series,  $\{R_s\}_{s=1}^{\mathcal{T}(T)}$ , will be performed. But let us first consider an alternative SMM approach, for which the starting point is a set of moment conditions that result from a reformulation of the basic asset pricing equation advocated by Julliard and Parker (2005). They relate the expected excess return to the covariance of the excess return and the stochastic discount factor (SDF),  $m_t = \beta cg_t^{-\gamma}$ ,

$$\text{cov}(m_t, R_t^e) = \mathbb{E}[(\beta cg_t^{-\gamma} - \mu_m)R_t^e], \quad (2.14)$$

where  $\mu_m = \mathbb{E}[\beta cg_t^{-\gamma}]$ , using the moment condition

$$\mathbb{E}\left[R_t^e + \frac{\mathbb{E}[(\beta cg_t^{-\gamma} - \mu_m)R_t^e]}{\mu_m}\right] = 0. \quad (2.15)$$

Again, we want to account for the effect of calamitous, yet unobserved consumption contractions on risk compensations, reflected in  $\text{cov}(m_t, R_t^e)$ . This population moment cannot be expressed analytically as a function of parameters, and using the sample covariance based on non-disastrous data is not helpful either. We therefore resort to an approximation by simulated moments that allows for the possibility of disasters in the generated series, viz

$$\begin{aligned} \mathbb{E}[(\beta cg_t^{-\gamma} - \mu_m)R_t^e] &\approx \frac{1}{\mathcal{T}(T)} \sum_{s=1}^{\mathcal{T}(T)} \left( \beta cg_s^{-\gamma} - \frac{1}{\mathcal{T}(T)} \sum_{s=1}^{\mathcal{T}(T)} \beta cg_s^{-\gamma} \right) R_s^e \\ \mu_m = \mathbb{E}[\beta cg_t^{-\gamma}] &\approx \frac{1}{\mathcal{T}(T)} \sum_{s=1}^{\mathcal{T}(T)} \beta cg_s^{-\gamma}. \end{aligned} \quad (2.16)$$

Recognizing that the risk-free rate and the mean of SDF are related by

$$\mathbb{E} \left[ \frac{1}{R_t^f} \right] = \mu_m, \quad (2.17)$$

estimates of  $\beta$ ,  $\gamma$  and  $\mu_m$  can be obtained using the moment matches

$$\mathbb{G}_T(\beta, \gamma, \mu_m) = \frac{1}{T} \sum_{t=1}^T \begin{bmatrix} \mathbf{R}_t^e + \frac{\frac{1}{T(T)} \sum_{s=1}^{T(T)} (\beta c g_s^{-\gamma} - \mu_m) \mathbf{R}_s^e}{\mu_m} \\ \frac{1}{R_t^f} - \mu_m \\ \mu_m - \frac{1}{T(T)} \sum_{s=1}^{T(T)} \beta c g_s^{-\gamma} \end{bmatrix}, \quad (2.18)$$

in the SMM objective function in Equation (2.13). As pointed out by Julliard and Parker (2005),  $\mathbf{W}_T$  has to be chosen such that the last moment match is exact.<sup>6</sup> We refer to estimates obtained in this fashion as JPM-SMM (*Julliard-Parker Moments*) estimates.

### 2.3 Simulating disaster-including consumption and return data

To apply the two SMM estimation strategies, we need to simulate disaster-including consumption growth,  $\{c g_s\}_{s=1}^{T(T)}$ , risk-free rate,  $\{R_s^f\}_{s=1}^{T(T)}$ , and excess return series,  $\{\mathbf{R}_s^e\}_{s=1}^{T(T)}$ . Our starting point for that task is Barro's (2006) disaster-including consumption process in Equation (2.2), rewritten as

$$c g_s = c g_{nd,s} (1 - b_s)^{d_s}, \quad (2.19)$$

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<sup>6</sup> If you are only interested in the CRRA estimate and an analysis of the equity premium, the second moment match in Equation (2.18) can be omitted as in Julliard and Parker (2005). Then,  $\beta$  is not identified and you can set it to any non-zero number.

which suggests to separate the simulated consumption growth,  $cg_s$ , into a regular component,  $cg_{nd,s}$ , and the factor  $(1-b_s)$ , which contracts  $cg_{nd,s}$  (only) if a disaster occurs ( $d_s = 1$ ). Our empirical strategy thus consists of a two-way bootstrap, where  $cg_{nd,s}$  is drawn with replacement from regular consumption growth data, and  $b_s$  is drawn from a Double Power Law distribution that is fitted to a sample of macroeconomic disasters, as suggested by Barro and Jin (2011). Since the procedure is quite data-dependent, we outline the details on the data sources right here, in the methodology section of our paper.

For the bootstrap of regular consumption growth and returns, we draw on quarterly U.S. postwar data collected by Cochrane (1996), which also contain gross returns of ten size-sorted portfolios and a risk-free rate proxy. Any U.S. postwar data set would do, but Cochrane's (1996) is particularly convenient, as a variety of CBM-type asset pricing models have already been estimated on these data, and the previously reported results provide useful reference points for our study.<sup>7</sup> Table 1 reports descriptive statistics of the data used for bootstrapping regular consumption growth and returns.

[insert table 1 about here]

We also have to provide disaster data to fit the Double Power Law distribution from which we bootstrap contractions. For that purpose, we rely on the procedure proposed by Barro (2006), and identify calamitous GDP contractions in the cross-country panel data collected by Angus Maddison.<sup>8</sup> These data contain annual

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<sup>7</sup> The data are available on John Cochrane's web site:  
[http://faculty.chicagobooth.edu/john.cochrane/research/Data\\_and\\_Programs/JPE\\_cross%20sectional\\_test\\_of\\_investment\\_based/Data/](http://faculty.chicagobooth.edu/john.cochrane/research/Data_and_Programs/JPE_cross%20sectional_test_of_investment_based/Data/) accessed 04/10/2013.

<sup>8</sup> The data are available at:  
<http://www.ggd.net/maddison/oriindex.net> accessed 04/10/2013.

GDP information about 35 countries ranging from 1900 to 2008. Following Barro (2006), we define a disaster as a contraction in GDP that exceeds the threshold  $q = 14.5\%$ . It does not matter whether the contraction accrued over one period or more, and the length of the contraction is measured up to the year before the rebound of the economy. Barro (2006) is not specific about how he deals with short periods of positive growth amidst a disaster. We allow for such one-period intermezzos if that break does not exceed the contraction in the following period in absolute terms. Additionally, the contraction may not decrease in size by ignoring the upward-sloping consumption phase. Following Barro (2006), we also neglect contractions in GDP that are considered aftermaths of war, and which are allegedly not related to a drop in consumption.<sup>9</sup>

[insert figure 1 about here]

Figure 1 illustrates the resulting disaster data. Disastrous contractions in GDP are clustered along WWI, the Great Depression, WWII and then around various turmoils in South America during 1980-2000.

The resulting 67 disaster observations provide the empirical distribution of contraction sizes from which we could bootstrap  $b_s$ . However, the small sample size limits the range and severity of possible contractions. We therefore follow Barro and Jin (2011) and fit a Double Power Law distribution to model the disaster data. The distributional parameters are estimated by maximum likelihood. We collect useful information about density function, mean and quantile function of the Double Power Law distribution in Appendix A.

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<sup>9</sup> The excluded contractions are: Canada (1917-1921): -30%, Italy (1918-1921): -25%, UK (1918-1921): -19%, UK (1943-1947): -15% and USA (1944-1947): -28%.

We follow Barro (2006) and estimate the annualized disaster probability  $p_a$  by dividing the number of extreme contractions in GDP by the overall number of years for which data is available. The quarterly disaster probability is then estimated by

$$\hat{p}_q = 1 - (1 - \hat{p}_a)^{1/4}. \quad (2.20)$$

Maddison's data do not contain information about financial returns associated with (disastrous) GDP contractions. To simulate such returns, we transfer the notion of a disaster-including consumption growth process (cf. Equation (2.19)) to gross returns, viz

$$R_s = (1 - \tilde{b}_s)^{d_s} R_{nd,s}, \quad (2.21)$$

and bootstrap  $R_{nd,s}$  from the regular data. We consider two choices for the contraction factor  $\tilde{b}_s$ . The first is to equate the contractions in consumption with contractions in returns,  $\tilde{b}_s = b_s$ . This approach draws its motivation from the empirical evidence that the correlations between consumption growth and returns increases in the tails of the joint distribution. The second choice is to draw  $\tilde{b}_s$  independently, but from the same distribution as  $b_s$ . We distinguish the two procedures to generate disaster-including returns by referring to them as *MaxCorr* (*Maximum Correlation*) and *InDraw* (*Independent Draws*).

A third strategy to simulate disaster-including returns is based on the assumption that log-consumption growth,  $c_d$ , and log-return,  $r_d$ , in the disaster state can be described by a bivariate Gaussian distribution:

$$\begin{bmatrix} c_d \\ r_d \end{bmatrix} \sim \mathcal{N} \left( \begin{bmatrix} \mu_{c,d} \\ \mu_{r,d} \end{bmatrix}, \begin{bmatrix} \sigma_{c,d}^2 & \sigma_{cr,d} \\ \sigma_{cr,d} & \sigma_{r,d}^2 \end{bmatrix} \right), \quad (2.22)$$

which implies that

$$\begin{aligned}\mathbb{E}[r_d|c_d] &= \mu_{r,d} + \rho_d \sigma_{c,d} \sigma_{r,d} [c_d - \mu_{c,d}] \\ \text{Var}[r_d|c_d] &= (1 - \rho_d^2) \sigma_{c,d}^2\end{aligned}\tag{2.23}$$

where  $\rho_d = \frac{\sigma_{cr,d}}{\sigma_{c,d}\sigma_{r,d}}$ . If the parameters of the bivariate normal distribution in Equation (2.22) were known, and given simulated disaster log-consumption  $c_{d,s} = \ln[(1 - b_s)cg_{nd,s}]$ , we could simulate a disaster log-return  $r_{d,s}$  by drawing from a Gaussian distribution with mean and variance given in Equation (2.23).

In order to estimate the five distributional parameters ( $\mu_{r,d}$ ,  $\mu_{c,d}$ ,  $\sigma_{c,d}$ ,  $\sigma_{r,d}$ ,  $\rho_{r,d}$ ), we proceed as follows.  $\mu_{c,d}$  and  $\sigma_{c,d}$  are estimated by the sample mean and standard deviation of a very long simulated disaster-including consumption growth series that is obtained as described previously. To estimate  $\rho_d$ , we assume that the correlation of log-consumption growth and log-returns conditional on  $d = 1$  is the same as conditional on  $d = 0$  ( $\rho_{nd}$ ):

$$\rho_d = \rho_{nd} = \frac{\sigma_{cr,nd}}{\sigma_{c,nd}\sigma_{r,nd}}.\tag{2.24}$$

$\sigma_{cr,nd}$ ,  $\sigma_{c,nd}$  and  $\sigma_{r,nd}$  denote covariance and standard deviations conditional on  $d = 0$ , which can be estimated using the regular consumption and return data.

To provide estimates of  $\mu_{r,d}$  and  $\sigma_{r,d}$ , we further assume that the expected value of a gross return in the disaster state equals the expected value of that gross return in the regular state, scaled by one minus the mean contraction size:

$$\mathbb{E}[R_d] = (1 - \mathbb{E}[b])\mathbb{E}[R_{nd}].\tag{2.25}$$

By the properties of the log-normal distribution, we then have

$$\mu_{r,d} = \ln(1 - \mathbb{E}[b]) + \ln(\mathbb{E}[R_{nd}]) - \frac{\sigma_{r,d}^2}{2}. \quad (2.26)$$

To estimate the mean contraction size, we replace in the analytical expression for  $\mathbb{E}[b]$  (see Appendix A) the Double Power Law parameters by their ML-estimates.

The final parameter to account for is  $\sigma_{r,d}^2$ . We do that by assuming constant ‘‘Sharpe ratios’’,

$$\frac{\mathbb{E}[R_d]}{\sqrt{\text{Var}(R_d)}} = \frac{\mathbb{E}[R_{nd}]}{\sqrt{\text{Var}(R_{nd})}}, \quad (2.27)$$

and using the properties of the log-normal, which imply that

$$\sigma_{r,d}^2 = \ln \left( 1 + \frac{\text{Var}(R_{nd})}{\mathbb{E}[R_{nd}]^2} \right). \quad (2.28)$$

$\text{Var}(R_{nd})$  and  $\mathbb{E}[R_{nd}]$  can be estimated by sample moments of the regular gross return data.

We can now replace all right-hand side parameters of Equation (2.23) by their estimates, and simulate log-returns,  $r_{d,s}$ , conditional on log-consumption growth in the disaster state,  $c_{d,s}$ . We refer to this third procedure of simulating disaster-including returns as *G-Draw* (*Gaussian Draws*).

Summing up, the simulation procedure to generate  $\{cg_s\}_{s=1}^{\mathcal{T}(T)}$ ,  $\{R_s^f\}_{s=1}^{\mathcal{T}(T)}$  and  $\{\mathbf{R}_s^\varepsilon\}_{s=1}^{\mathcal{T}(T)}$  works as follows:

- For every  $s = 1, \dots, \mathcal{T}(T)$ , decide by drawing from a Bernoulli distribution with probability  $\hat{p}_q$  whether  $d_s = 0$  or  $d_s = 1$ . Regardless of the outcome, draw  $cg_{nd,s}$ ,  $R_{nd,s}^f$  and  $\mathbf{R}_{nd,s}$  with replacement from the regular consumption and return data. In order to maintain the covariance structure of consumption

and returns, draws have to be performed simultaneously.

- If  $d_s = 0$ , set  $cg_s = cg_{nd,s}$ ,  $R_s^f = R_{nd,s}^f$  and  $\mathbf{R}_s^e = \mathbf{R}_{nd,s} - R_{nd,s}^f$
- If  $d_s = 1$ , draw  $b_s$  from the fitted Double Power Law distribution and compute  $cg_s = (1 - b_s)cg_{nd,s}$  and three variants of disaster returns,  $\mathbf{R}_{d,s}$ , using the *MaxCorr*, *InDraw* and *G-Draw* procedures. Finally,  $\mathbf{R}_s^e = \mathbf{R}_{d,s} - R_{nd,s}^f$  and  $R_s^f = R_{nd,s}^f$ .

## 2.4 Alternative Histories Bootstrap

The two-way bootstrap described in the previous section suggests an alternative approach towards estimating the structural parameters of the disaster-including CBM. Since the simulated consumption growth and return series include disaster observations, the moment matches in Equations (2.9) and (2.10), which have been discarded because of the sample selection problem, can be reconsidered. To complement the two SMM procedures described in Section 2.2, we therefore propose a third approach that we refer to as *Alternative Histories Bootstrap* (AHB), a term that echoes Cochrane’s remark from the introduction.

The starting point for the AHB-procedure are  $H$  independent disaster-including samples (“alternative histories”) of size  $\mathcal{T}(T)$ , which we generate as described in Section 2.3. Let  $\{cg_s^{(h)}\}$ ,  $\{R_s^{f(h)}\}$  and  $\{R_s^{e(h)}\}$  denote the simulated data from replication  $h$ . For each replication  $h = 1, \dots, H$ , we estimate  $\beta$  and  $\gamma$  by GMM, using the moment matches

$$\mathbb{G}_T^{(h)}(\beta, \gamma) = \begin{bmatrix} \frac{1}{\mathcal{T}(T)} \sum_{s=1}^{\mathcal{T}(T)} \beta \left( cg_s^{(h)} \right)^{-\gamma} R_s^{f(h)} - 1 \\ \frac{1}{\mathcal{T}(T)} \sum_{s=1}^{\mathcal{T}(T)} \beta \left( cg_s^{(h)} \right)^{-\gamma} \mathbf{R}_s^{e(h)} \end{bmatrix}. \quad (2.29)$$

AHB-estimates of  $\beta$  and  $\gamma$  are obtained from averaging the resulting estimates across ensembles, viz

$$\hat{\beta} = \frac{1}{H} \sum_{h=1}^H \hat{\beta}^{(h)} \quad \text{and} \quad \hat{\gamma} = \frac{1}{H} \sum_{h=1}^H \hat{\gamma}^{(h)}, \quad (2.30)$$

where  $\hat{\gamma}^{(h)}$  and  $\hat{\beta}^{(h)}$  refer to the estimates obtained in the  $h^{\text{th}}$  replication. In fact, we obtain three sets of estimates using the alternative ways to generate disaster-including returns discussed in Section 2.3. We use the empirical distribution of  $\hat{\gamma}^{(h)}$  and  $\hat{\beta}^{(h)}$  to provide standard errors, quantiles and kernel densities. By varying  $\mathcal{T}(T)$ , we can quantify the considerations in Section 2.1 regarding the size of disaster-including samples and the implications for estimation precision.

### 3 Empirical results

All variants to estimate a disaster-including CBM rely on simulated disaster sizes drawn from a Double Power Law distribution that is fitted to the sample of identified disasters shown in Figure 1. Maximum likelihood estimates of the distributional parameters and their standard errors are reported in the caption of Figure 2, which depicts the empirical distribution function of the disaster sizes and the fitted Double Power Law c.d.f. The point  $\kappa$  indicates the switch from one Power Law to the other one. The figure shows that the Double Power Law fits the empirical distribution function well for small contraction sizes, but also leaves more room for severe downturns.

[insert figure 2 about here]

For the estimation of a disaster-including CBM, we use the same test assets as for the bootstrap simulations, i.e. the ten size-sorted portfolios and the risk-free rate used by Cochrane (1996) (descriptive statistics are contained in Table 1).<sup>10</sup> We also consider a set of test assets that consists of excess returns only. In this case, the moment condition for the risk-free rate is excluded from the respective GMM and SMM moment matches. As discussed previously, this entails that the remaining moment conditions cannot identify  $\beta$ , which we set equal to one.

The two SMM variants (MAD-SMM and JPM-SMM) use  $\mathcal{T}(T) = 1\,000\,000$ . For the Alternative Histories Bootstrap, we vary  $\mathcal{T}(T)$  from 187 to 16\,000, and perform  $H = 400$  replications. The  $\mathcal{T}(T) = 16\,000$  variant is used to compare SMM and AHB results. The other simulated time series lengths will be used to study the effect of an increasing sample size on parameter estimates and their precision. The GMM and SMM objective functions use the identity matrix for  $\mathbf{W}_T$ ; for JPM-SMM, we make sure that the moment condition that invokes the mean of the SDF is exactly matched as requested by Julliard and Parker (2005).

Table 2 (using excess returns and the risk-free rate as test assets) and Table 3 (using excess returns as test assets only) collect SMM and AHB estimation results. Panels A (*G-Draw*), B (*InDraw*) and C (*MaxCorr*) break down the results by the three alternative disaster return simulation procedures. We report the preference parameter estimates and asymptotic standard errors (for SMM) or standard deviations across replications (for AHB), the  $p$ -values of the  $J$ -statistics (for SMM)

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<sup>10</sup> In principle, the test assets and the data used to bootstrap regular consumption growth and returns may come from different sources, but the natural choice is to use the same data for both purposes.

and the root mean squared errors computed as

$$RMSE = \sqrt{\frac{1}{N} \mathbb{G}_T(\hat{\beta}, \hat{\gamma})' \mathbb{G}_T(\hat{\beta}, \hat{\gamma})}, \quad (3.1)$$

where  $N$  denotes the number of test assets. Figure 3 illustrates the estimation precision using kernel density estimates, based on the  $H = 400$  AHB-ensembles.

[insert tables 2 and 3 about here]

[insert figure 3 about here]

The key finding is that all variants to estimate a disaster-including CBM yield economically plausible estimates for the preference parameters. For the CRRA parameter  $\gamma$  one to ten is considered a sensible range (more rigorous scholars cap the interval at five); the subjective discount factor  $\beta$  should be less than one. Only for MAD-SMM with *MaxCorr* simulated returns do we get a  $\beta$ -estimate slightly above one. However, the hypothesis that  $\beta$  is smaller than one cannot be rejected at conventional levels of significance. Both asymptotic inference (for SMM) and the bootstrap inference (for AHB) yield small parameter standard errors, i.e. narrow confidence bounds for the preference parameters. The kernel densities in Figure 3 illustrate and confirm these conclusions. The  $p$ -values of the  $J$ -statistics computed for the two SMM approaches indicate that the disaster-including CBM cannot be rejected at conventional levels of significance. Furthermore, the quality of moment matches (measured by the RMSE) does not differ much across estimation methods and disaster return simulation procedures. The *G-Draw* variant tends to deliver the smallest RMSE.

While the general picture is that neither disaster return simulation procedure nor estimation philosophy (SMM vs. AHB) yield qualitatively different results, there is some variation across the estimates. The MAD-SMM approach tends to deliver the highest, the JPM-SMM specification the smallest  $\hat{\gamma}$ . Moreover, the *MaxCorr* return simulation implies somewhat smaller  $\gamma$ -estimates than the *InDraw* and *G-Draw* procedures. Hence,  $\hat{\gamma} = 3.538$  (JPM-SMM/*MaxCorr*) and  $\hat{\gamma} = 7.240$  (MAD-SMM/*G-Draw*) span the range of CRRA parameter estimates. Yet overall, the risk aversion estimates do not change much with respect to the procedure to simulate disaster returns. For example, comparing the JPM-SMM estimates, we have  $\hat{\gamma} = 4.167$  when using the *InDraw* return simulation,  $\hat{\gamma} = 4.178$  when relying on the *G-Draw* simulation and  $\hat{\gamma} = 3.538$  with the *MaxCorr* procedure. Eight out of nine specifications end up with estimates of the subjective discount factor between  $\hat{\beta} = 0.93$  (AHB/*InDraw*) and  $\hat{\beta} = 0.98$  (JPM-SMM/*MaxCorr* and AHB/*MaxCorr*). The estimates of  $\beta$  tend to be higher and more precise for the *MaxCorr* procedure. The kernel density estimates in Figure 3 illustrate the effect of the choice of return simulation procedures on estimated preference parameters.

By comparing the AHB estimates for increasing sample sizes, we can quantify the effects that we have discussed in Section 2. We have argued there that the quality of the GMM moment matches will be affected by using short time series that contain too few if any disaster observations in order to be representative of the possible paths of history that investors had in mind. Using the AHB approach, we can assess which sample size would be needed for standard GMM to achieve a reasonable estimation precision. We can also study the distribution of the estimates when the simulated sample size is as small as in the empirical data, but when

some simulated histories include disaster observations. A comparison with the empirical results using disaster-free data serves as a plausibility check for the AHB methodology.

In addition to  $\mathcal{T}(T) = 16\,000$ , we therefore also perform AHB estimations with simulated histories of lengths  $\mathcal{T}(T) = 187, 1\,000$  and  $5\,000$ . These choices are motivated as follows. Cochrane’s (1996) data consist of 187 quarterly observations, so the ensembles have the same length but are potentially disaster-including. For  $\mathcal{T}(T) = 1\,000$ , the simulated data span roughly three successive generations, assuming a life-span of 80 years.<sup>11</sup> For  $\mathcal{T}(T) = 5\,000$ , they overlap approximately fifteen generations.

[insert tables 4 and 5 about here]

[insert figures 4 - 6 about here]

For each  $\mathcal{T}(T)$ , we perform separate AHB estimations using the *MaxCorr*, *InDraw* and *G-Draw* return simulation procedures. The results are reported in Table 4 (test assets include the risk-free rate) and Table 5 (only excess returns as test assets). Figures 4 - 6 illustrate the findings using kernel densities.

The AHB estimates using the small simulated sample size of  $\mathcal{T}(T) = 187$  have the properties well-known from empirical applications. Regardless of the procedure used to simulate disaster returns,  $\hat{\beta}$  is greater than one and  $\hat{\gamma}$  is far beyond the upper plausibility limit. Furthermore, the estimates are very imprecise, as indicated by the huge standard errors and the shape of the kernel densities.

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<sup>11</sup> “The days of our years are threescore and ten; and if by reason of strength they be fourscore years, yet is their strength labor and sorrow; for it is soon cut off, and we fly away.” Psalms 90:10, King James Bible, Pure Cambridge Edition.

Increasing the sample size to  $\mathcal{T}(T) = 1\,000$ , the point estimates take on more plausible values. The estimation precision improves, but is still low as indicated by the standard errors and the kernel densities. At  $\mathcal{T}(T) = 5\,000$ , the estimation results are satisfactory in the sense that the estimation precision is sufficient and the  $\beta$  and  $\gamma$ -estimates are economically plausible. This simulation exercise shows that the failure of the CBM using conventional estimation techniques and the currently available information about returns and consumption comes at no surprise, and is not at odds with the rare disaster hypothesis. If the rare disaster hypothesis is true, and if conventional estimation techniques are used, one has to wait for a long time – with potentially very unpleasant intermezzos of consumption contractions – before a sufficient estimation precision can be expected. Our simulation-based methods thus provide a shortcut.

## 4 Conclusion

Financial economics and econometrics alike use Hansen and Singleton's (1982) consumption-based asset pricing model with additive power utility SDF as reference point and springboard for theoretical extensions and methodological developments. When taking the canonical CBM to data, however, its empirical performance has been notoriously disappointing. The estimates of the CBM preference parameters, subjective discount factor and relative risk aversion, tend to be both highly implausible and quite imprecise. However, the CBM framework is not easily discarded: it represents the only rational link between the real economy (consumption) and financial markets (prices of future risky payoffs). Accordingly, attempts to vindicate the CBM have been manifold. Scaled factors have been employed

to account for time varying risk aversion, alternative measures for the errors-in-variables-prone macroeconomic consumption data have been used, investor heterogeneity has been accounted for, and more flexible specifications of intertemporal utility functions have been tested. Whilst these studies have had some empirical success, the problem of imprecise and implausible preference parameter estimates is at best mitigated.

Our study has probed an alternative explanation to vindicate the CBM: the rare disaster hypothesis that is associated with the seminal work by Rietz and Barro. We retain the stochastic discount factor of Hansen and Singleton's CBM, but we account for the disturbing suspicion that the U.S. data, which have been used extensively to test the CBM, may not be representative. Consumers and investors born and living after WWII in the U.S. and other western countries that have collected consumption and financial data for 60+ years have experienced an unprecedented period of peace, prosperity and progress. A lucky path of history spared them from calamitous contractions of GDP and aggregate consumption. Those investors and data tell the story of survivors, which is always a pleasant, but often a misleading narrative. Statistics and econometrics classes center around sample selection problems and the danger of interpreting self-selected data. In empirical macro-finance, we sometimes ignore the presence of a pink elephant in our laboratory.

Adopting Barro's (2006) specification of a disaster-including consumption process, we motivate two alternative moment matches that we use to estimate the CBM preference parameters by SMM. To simulate disaster-including consumption growth and return processes, we perform a non-parametric bootstrap from regular

U.S. postwar data, which is combined with a parametric bootstrap from a distribution that is fitted to calamitous contractions data. An alternative estimation strategy entails repeating the bootstrap simulation and applying GMM to the simulated “alternative histories”. Here, point estimates of the preference parameters result from averaging over the replications. The bootstrap distributions enable us to judge the estimation precision that can be expected using varying sample sizes. The three methods to estimate a disaster-including CBM rely on three specifications to simulate disaster-including financial returns. We perform the estimation with and without the risk-free rate included in the test assets.

Whichever approach is used, the result remains qualitatively the same. The estimated preference parameters are of an economically plausible size and the estimation precision is much higher than in previous studies that use the canonical CBM. In particular, the estimate of the relative risk aversion parameter is smaller than five in most specifications and always smaller than ten, i.e. the estimates are in the range that is considered to be consistent with reasonably risk-averse investors. The parameter standard errors are small, the confidence bounds narrow. A comparable combination of plausibility and estimation precision has not been delivered in the related literature (cf. the empirical results discussed in the introduction).

We also show that size and precision of the CBM estimates reported in previous studies are quite realistic under the rare disaster hypothesis. Decades have to pass before standard empirical techniques would yield precise estimation results on empirical data. The simulation-based estimation techniques that we apply in our study provide a shortcut to empirically assess the effect of consumption disasters

on asset prices. They come at the cost of assumptions, which may be questioned but can be modified, and one can study the sensitivity of the estimation results. In our study, the variation of assumptions did not change the results qualitatively.

Our findings should encourage those who believe that rational investor behavior prevails in financial markets. Yes, the CBM can explain the equity premium at reasonable levels of risk aversion, once the latent risk of rare disasters is accounted for. The nexus between finance and real economy postulated by the CBM is, after all, empirically not refuted.

## A Power Law distribution: useful results

Following Barro and Jin (2011), we use a Double Power Law distribution to model the distribution of disastrous contraction sizes  $b$ . For that purpose, we use the transformation of disaster sizes into the random variable  $z = \frac{1}{1-b}$ , for which we assume the Double Power Law density

$$f_Z(z) = \begin{cases} 0 & \text{if } z < z_0 \\ Bz^{-\theta} & \text{if } z_0 \leq z < \delta, \\ Az^{-\alpha} & \text{if } \delta \leq z \end{cases} \quad (\text{A.1})$$

where  $B = A\delta^{(\theta-\alpha)}$  and  $A = \left[ \frac{\delta^{(\theta-\alpha)}}{\theta-1} \left( z_0^{(1-\theta)} - \delta^{(1-\theta)} \right) + \frac{\delta^{(1-\alpha)}}{\alpha-1} \right]^{-1}$ .  $z_0$  is defined as  $z_0 = \frac{1}{1-q}$ , where  $q$  denotes the disaster threshold.

A draw from the Double Power Law density can be performed by drawing a standard uniform random variable,  $\nu$ , and inserting it in the quantile function, which is given by

$$z_{[\nu]} = \begin{cases} \sqrt[\theta]{z_0^{-\theta} - \frac{\theta}{B}\nu} & \text{if } \nu \leq F_{\delta \leq z}(\delta) \\ \sqrt[\alpha]{\delta^{-\alpha} - \frac{\alpha}{A} \left( \nu - \frac{B}{\theta} (z_0^{-\theta} - \delta^{-\theta}) \right)} & \text{if } \nu > F_{\delta \leq z}(\delta). \end{cases} \quad (\text{A.2})$$

The realizations of the random variables  $z$  that are drawn using the quantile function (A.2) have to be re-transformed into contraction sizes by  $b = 1 - \frac{1}{z}$ .

Using the density for  $z = \frac{1}{1-b}$ , the expected value of the contraction size  $b$  (which we need for the *G-Draw* return simulation) is given by:

$$\mathbb{E}[b] = \mathbb{E} \left[ 1 - \frac{1}{z} \right] = 1 + A\delta^{-(\alpha+1)} \left( \frac{1}{\theta+1} - \frac{1}{\alpha+1} \right) - \frac{A}{(\theta+1)} \delta^{(\theta-\alpha)} z_0^{-(\theta+1)}. \quad (\text{A.3})$$

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## Tables and figures

Table 1: **Descriptive statistics of regular consumption growth and return data used for bootstrapping**

The table holds descriptive statistics on Cochrane's (1996) consumption growth and gross return data. The data range from the second quarter in 1947 to the end of 1993. Consumption growth is denoted as  $\frac{C_{t+1}}{C_t}$  and  $1^{st}$ ,  $2^{nd}$ , etc. refer to the respective return deciles for ten size-sorted portfolios of NYSE stocks.  $R^f$  is the risk-free rate proxy. The column titled  $\rho$  gives information on the autocorrelation of the variables and std refers to the standard deviation. The last eleven columns picture the correlations between the different variables.

	mean	std	$\rho$	correlation										
$\frac{C_{t+1}}{C_t}$	1.004	0.006	0.206	$\frac{C_{t+1}}{C_t}$	$R^f$	$10^{th}$	$9^{th}$	$8^{th}$	$7^{th}$	$6^{th}$	$5^{th}$	$4^{th}$	$3^{rd}$	$2^{nd}$
$1^{st}$	1.031	0.100	0.290	0.288	0.090	0.754	0.858	0.893	0.910	0.936	0.939	0.948	0.960	0.971
$2^{nd}$	1.027	0.090	0.293	0.288	0.131	0.797	0.895	0.928	0.946	0.965	0.968	0.975	0.982	
$3^{rd}$	1.026	0.087	0.307	0.285	0.132	0.812	0.905	0.940	0.955	0.969	0.976	0.981		
$4^{th}$	1.026	0.083	0.313	0.267	0.129	0.836	0.923	0.956	0.969	0.980	0.979			
$5^{th}$	1.024	0.080	0.308	0.259	0.158	0.848	0.936	0.966	0.973	0.979				
$6^{th}$	1.025	0.078	0.288	0.238	0.144	0.863	0.952	0.974	0.979					
$7^{th}$	1.024	0.074	0.332	0.242	0.165	0.896	0.973	0.985						
$8^{th}$	1.023	0.070	0.291	0.239	0.173	0.906	0.976							
$9^{th}$	1.022	0.067	0.302	0.201	0.193	0.930								
$10^{th}$	1.018	0.058	0.376	0.232	0.224									
$R^f$	1.002	0.008	0.700	0.130										

Table 2: **SMM and AHB estimation results using excess returns and the risk-free rate as test assets**

The table presents the SMM and AHB estimates of the preference parameters  $\beta$  and  $\gamma$ . Asymptotic standard errors of the SMM estimates are reported in parentheses. The numbers in brackets are standard deviations of the AHB estimates across  $H = 400$  replications. The table also reports the  $p$ -values (in percent) of Hansen's  $J$ -statistic. The RMSE is computed using the average pricing errors,  $\text{RMSE} = \sqrt{\frac{1}{N} \mathbb{G}_T(\hat{\beta}, \hat{\gamma})' \mathbb{G}_T(\hat{\beta}, \hat{\gamma})}$ , where  $N$  denotes the number of test assets. For the AHB approach, the reported RMSE (in basis points) is obtained by averaging over the 400 replications. Panels A, B and C break down the results by procedure used to simulate disaster-including returns (*G-Draw*, *InDraw* and *MaxCorr*).

<b>Panel A – <i>G-Draw</i></b>				
	$\hat{\beta}$	$\hat{\gamma}$	$J$	RMSE
MAD-SMM	0.956 (0.017)	7.240 (0.128)	57.4	18
JPM-SMM	0.938 (0.018)	4.178 (0.184)	60.6	21
AHB	0.943 [0.011]	4.778 [0.891]		21
<b>Panel B – <i>InDraw</i></b>				
	$\hat{\beta}$	$\hat{\gamma}$	$J$	RMSE
MAD-SMM	0.961 (0.016)	7.199 (0.127)	34.0	24
JPM-SMM	0.939 (0.018)	4.167 (0.181)	41.2	28
AHB	0.932 [0.029]	4.740 [0.843]		23
<b>Panel C – <i>MaxCorr</i></b>				
	$\hat{\beta}$	$\hat{\gamma}$	$J$	RMSE
MAD-SMM	1.005 (0.006)	6.646 (0.128)	39.0	28
JPM-SMM	0.979 (0.007)	3.538 (0.176)	41.7	28
AHB	0.979 [0.002]	4.046 [0.844]		20

Table 3: **SMM and AHB estimation results using only excess returns as test assets**

The table presents the SMM and AHB estimates of the coefficient of relative risk aversion  $\gamma$ . Asymptotic standard errors of the SMM estimates are reported in parentheses. The numbers in brackets are standard deviations of the AHB estimates across  $H = 400$  replications. The table also reports the  $p$ -values (in percent) of Hansen's  $J$ -statistic. The RMSE is computed using the average pricing errors,  $\text{RMSE} = \sqrt{\frac{1}{N} \mathbb{G}_T(\hat{\gamma})' \mathbb{G}_T(\hat{\gamma})}$ , where  $N$  denotes the number of test assets. For the AHB approach, the reported RMSE (in basis points) is obtained by averaging over the 400 replications. Panels A, B and C break down the results by procedure used to simulate disaster-including returns (*G-Draw*, *InDraw* and *MaxCorr*).

<b>Panel A – <i>G-Draw</i></b>			
	$\hat{\gamma}$	$J$	RMSE
MAD-SMM	7.240 (0.128)	60.1	20
JPM-SMM	4.178 (0.184)	60.6	22
AHB	4.769 [0.888]		25
<b>Panel B – <i>InDraw</i></b>			
	$\hat{\gamma}$	$J$	RMSE
MAD-SMM	7.198 (0.127)	38.2	26
JPM-SMM	4.167 (0.181)	41.2	29
AHB	4.745 [0.848]		28
<b>Panel A – <i>MaxCorr</i></b>			
	$\hat{\gamma}$	$J$	RMSE
MAD-SMM	6.646 (0.128)	40.4	29
JPM-SMM	3.538 (0.176)	41.7	30
AHB	4.051 [0.840]		23

Table 4: **Effect of varying  $\mathcal{T}(T)$  on AHB parameter estimates**

The table reports the AHB estimates of the subjective discount factor and the coefficient of relative risk aversion for a varying  $\mathcal{T}(T)$ . It also shows the standard deviation (std) and the 95%-quantile of the parameter estimates from the  $H = 400$  simulated histories. The RMSE is computed using the average pricing errors,  $\text{RMSE} = \frac{1}{H} \sum_{h=1}^H \sqrt{\frac{1}{N} \mathbb{G}_T^{(h)}(\hat{\beta}^{(h)}, \hat{\gamma}^{(h)})' \mathbb{G}_T^{(h)}(\hat{\beta}^{(h)}, \hat{\gamma}^{(h)})}$ , where  $N$  denotes the number of test assets. It is expressed in basis points. Panels A, B and C break down the results by procedure used to simulate disaster-including returns (*G-Draw*, *InDraw* and *MaxCorr*).

<b>Panel A – <i>G-Draw</i></b>								
$\mathcal{T}(T)$	187		1 000		5 000		16 000	
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
	1.072	82.630	0.959	12.745	0.944	5.407	0.943	4.778
std	0.182	93.005	0.037	30.781	0.014	1.574	0.011	0.891
95%-quantile	1.409	249.919	1.005	20.527	0.963	8.410	0.959	6.341
RMSE	65		33		23		21	
<b>Panel B – <i>InDraw</i></b>								
$\mathcal{T}(T)$	187		1 000		5 000		16 000	
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
	1.064	83.016	0.948	12.860	0.936	5.498	0.932	4.740
std	0.190	92.710	0.051	30.822	0.033	1.592	0.029	0.843
95%-quantile	1.409	249.919	1.012	21.563	0.972	8.343	0.965	6.267
RMSE	61		28		24		23	
<b>Panel C – <i>MaxCorr</i></b>								
$\mathcal{T}(T)$	187		1 000		5 000		16 000	
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
	1.068	83.149	0.973	12.509	0.977	4.659	0.979	4.046
std	0.185	92.645	0.030	30.946	0.003	1.610	0.002	0.844
95%-quantile	1.409	249.919	0.989	23.738	0.982	7.486	0.982	5.621
RMSE	61		26		20		20	

Table 5: **Effect of varying  $\mathcal{T}(T)$  on the AHB estimate of  $\gamma$  using excess returns**

The table reports the AHB estimates of the coefficient of relative risk aversion for a varying  $\mathcal{T}(T)$ . It also shows the standard deviation (std) and the 95%-quantile of the parameter estimates from the  $H = 400$  simulated histories. The RMSE is computed using the average pricing errors,  $\text{RMSE} = \frac{1}{H} \sum_{h=1}^H \sqrt{\frac{1}{N} \mathbb{G}_T^{(h)}(\hat{\gamma}^{(h)})' \mathbb{G}_T^{(h)}(\hat{\gamma}^{(h)})}$ , where  $N$  denotes the number of test assets. It is expressed in basis points. Panels A, B and C break down the results by procedure used to simulate disaster-including returns (*G-Draw*, *InDraw* and *MaxCorr*).

<b>Panel A – <i>G-Draw</i></b>					
<b><math>\mathcal{T}(T)</math></b>	<b>187</b>	<b>1 000</b>	<b>5 000</b>	<b>16 000</b>	
$\hat{\gamma}$	75.779	12.233	5.405	4.769	
std	82.122	27.586	1.573	0.888	
95%-quantile	216.105	20.524	8.408	6.332	
RMSE	61	37	26	25	
<b>Panel B – <i>InDraw</i></b>					
<b><math>\mathcal{T}(T)</math></b>	<b>187</b>	<b>1 000</b>	<b>5 000</b>	<b>16 000</b>	
$\hat{\gamma}$	76.164	12.350	5.493	4.745	
std	81.820	27.633	1.592	0.848	
95%-quantile	216.105	21.554	8.338	6.280	
RMSE	58	32	30	28	
<b>Panel C – <i>MaxCorr</i></b>					
<b><math>\mathcal{T}(T)</math></b>	<b>187</b>	<b>1 000</b>	<b>5 000</b>	<b>16 000</b>	
$\hat{\gamma}$	76.301	12.000	4.659	4.051	
std	81.754	27.763	1.609	0.840	
95%-quantile	216.105	23.728	7.485	5.620	
RMSE	56	28	23	23	

Figure 1: **Disastrous GDP contractions identified from Maddison's data (1900-2008)**

The figure illustrates the 67 cases for which the contraction in GDP exceeded  $q = 14.5\%$ . The analysis was based on per capita GDP data for 35 countries: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, Colombia, Denmark, Finland, France, Germany, Greece (1901-1912), India, Indonesia (1942-1948), Italy, Japan, Malaysia (1900-1910 and 1943-1946), Mexico, Netherlands, New Zealand, Norway, Philippines (1900-1901 and 1941-1945), Peru, Portugal, South Korea (1900-1910), Spain, Sri Lanka, Sweden, Switzerland, Taiwan (1900), U.K., U.S.A., Uruguay, Venezuela. Values in parentheses indicate missing data. Black lines refer to European countries, red ones to South America, Green belongs to Western Offshores and Blue denotes Asian countries. The average contraction size is 27.88%. The standard deviation of contractions is 14.24%. The smallest disaster found in the data is 14.52% (India, 1916-1918), whilst the biggest one equals 66.14% (Greece, 1937-1945). Computed as proposed by Barro (2006), these data imply an estimated quarterly disaster probability of  $\hat{p}_q = 0.43\%$ .

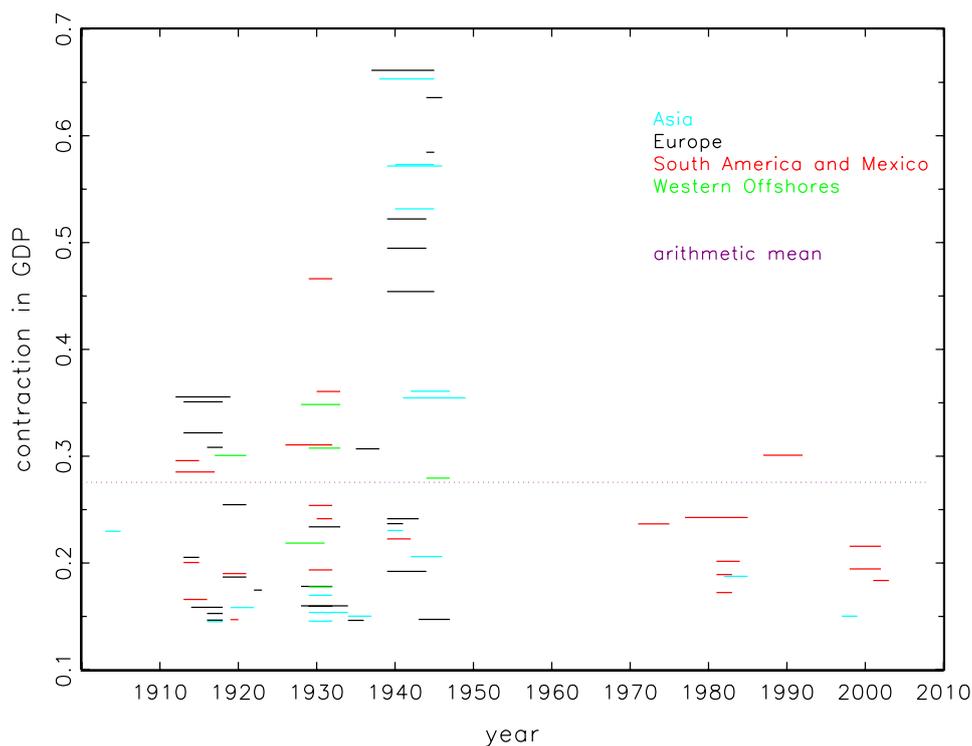


Figure 2: **Comparison of Double Power Law and empirical distribution function for disaster sizes**

The figure illustrates the empirical distribution function (blue bumpy line) and the fitted cumulative distribution function (red smooth line) of the disastrous contractions identified in Maddison's macroeconomic data using a disaster threshold of 14.5%. We estimate the parameters by means of maximum likelihood as  $\hat{\alpha} = 3.627$  (0.394),  $\hat{\theta} = 12.250$  (1.656) and  $\hat{\delta} = 1.366$  (0.024). Standard errors are reported in parentheses.  $\kappa$  denotes the threshold at which the one power law morphs into the other one and is linked to  $\hat{\delta}$  by means of  $\kappa = 1 - \frac{1}{\hat{\delta}} = 0.268$ .

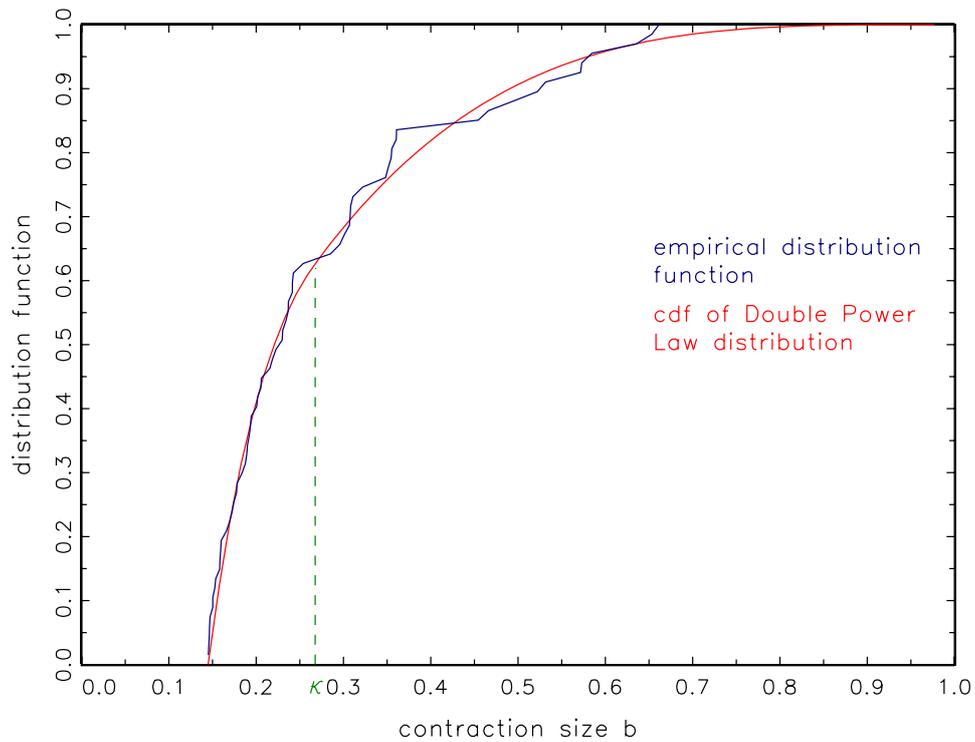


Figure 3: **Kernel densities for AHB estimates**

The two figures depict kernel densities of the AHB estimates of the subjective discount factor,  $\hat{\beta}$ , and the coefficient of relative risk aversion,  $\hat{\gamma}$ . We use  $H = 400$  and  $\mathcal{T}(T) = 16\,000$ . The green (solid) density is for estimates based on the *G-Draw* return simulation procedure. The gray density (short dashes) is for parameter estimates that rely on the *InDraw* procedure and purple density (long dashes) refers to estimates that use the *MaxCorr* procedure. The AHB point estimates are indicated by vertical lines.

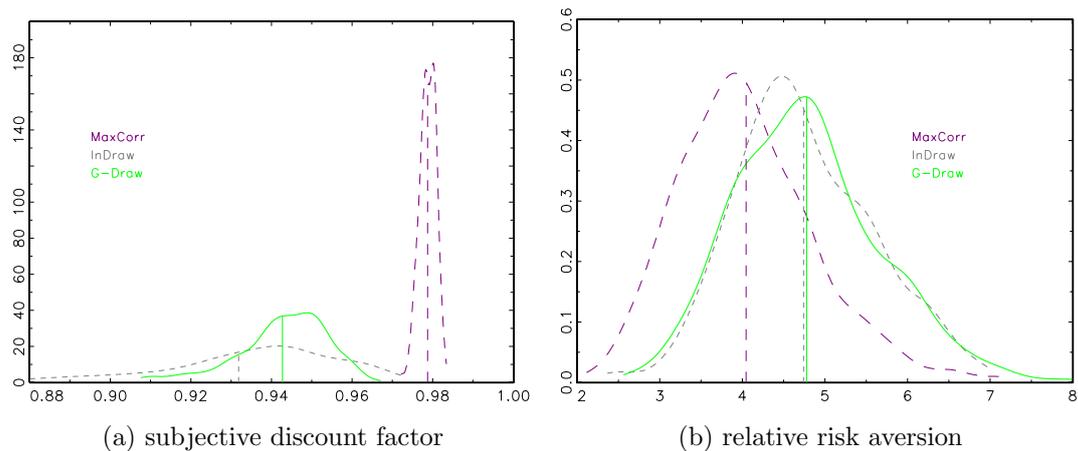
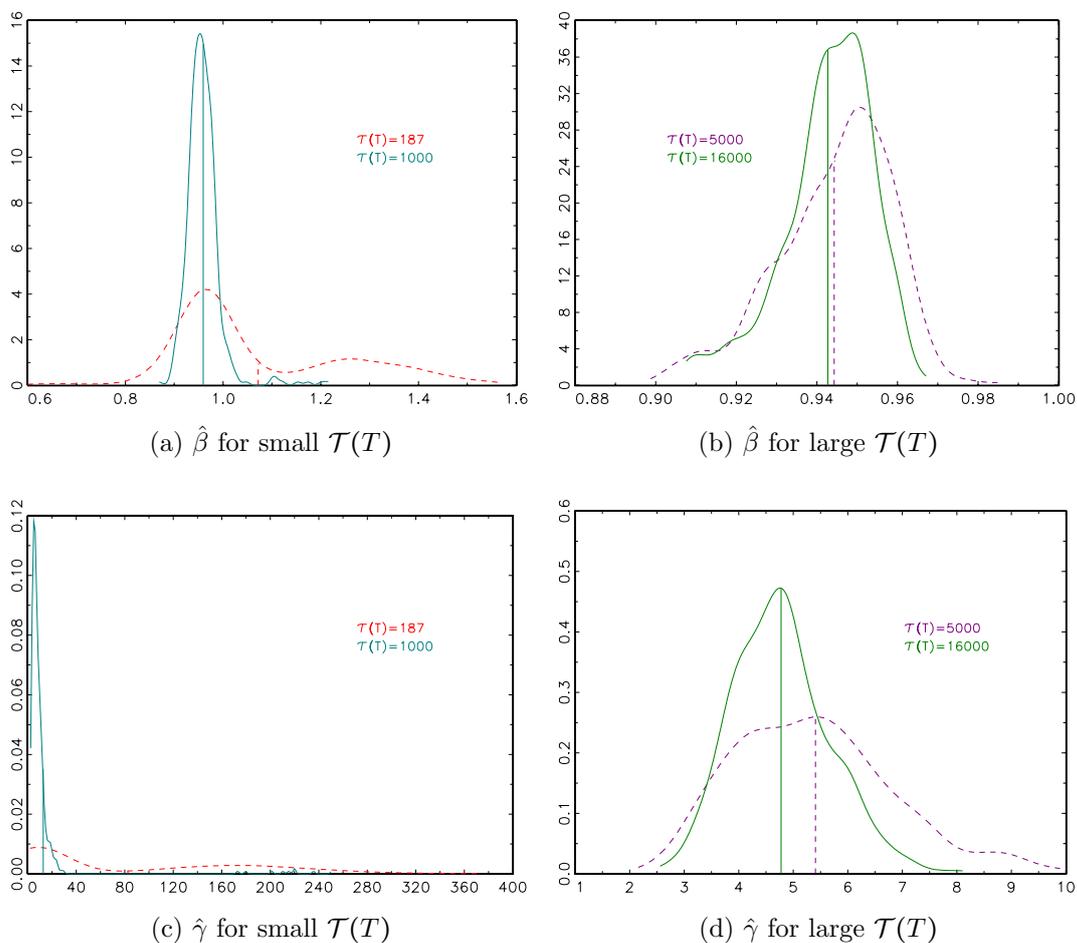


Figure 4: **Effect of varying  $\mathcal{T}(T)$  on AHB parameter estimates using the *G-Draw* return simulation procedure**

The four panels depict kernel densities of the AHB/*G-Draw* estimates of the subjective discount factor,  $\hat{\beta}$ , (upper panels) and the coefficient of relative risk aversion,  $\hat{\gamma}$  (lower panels). We use  $H = 400$ , and  $\mathcal{T}(T)$  varies between 187 and 1000 (left panels) and 5000 and 16000 (right panels). The dashed red densities in the left panels are for  $\mathcal{T}(T) = 187$ , whilst the solid cyan colored densities refer to  $\mathcal{T}(T) = 1000$ . The dashed purple densities are associated with  $\mathcal{T}(T) = 5000$  and green (solid) relies on  $\mathcal{T}(T) = 16000$ . The AHB point estimates are indicated by vertical lines in the respective colors.



**Figure 5: Effect of varying  $\mathcal{T}(T)$  on AHB parameter estimates using the *InDraw* return simulation procedure**

The four panels depict kernel densities of the AHB/*InDraw* estimates of the subjective discount factor,  $\hat{\beta}$ , (upper panels) and the coefficient of relative risk aversion,  $\hat{\gamma}$  (lower panels). We use  $H = 400$ , and  $\mathcal{T}(T)$  varies between 187 and 1000 (left panels) and 5000 and 16000 (right panels). The dashed red densities in the left panels are for  $\mathcal{T}(T) = 187$ , whilst the solid cyan colored densities refer to  $\mathcal{T}(T) = 1000$ . The dashed purple densities are associated with  $\mathcal{T}(T) = 5000$  and green (solid) relies on  $\mathcal{T}(T) = 16000$ . The AHB point estimates are indicated by vertical lines in the respective colors.

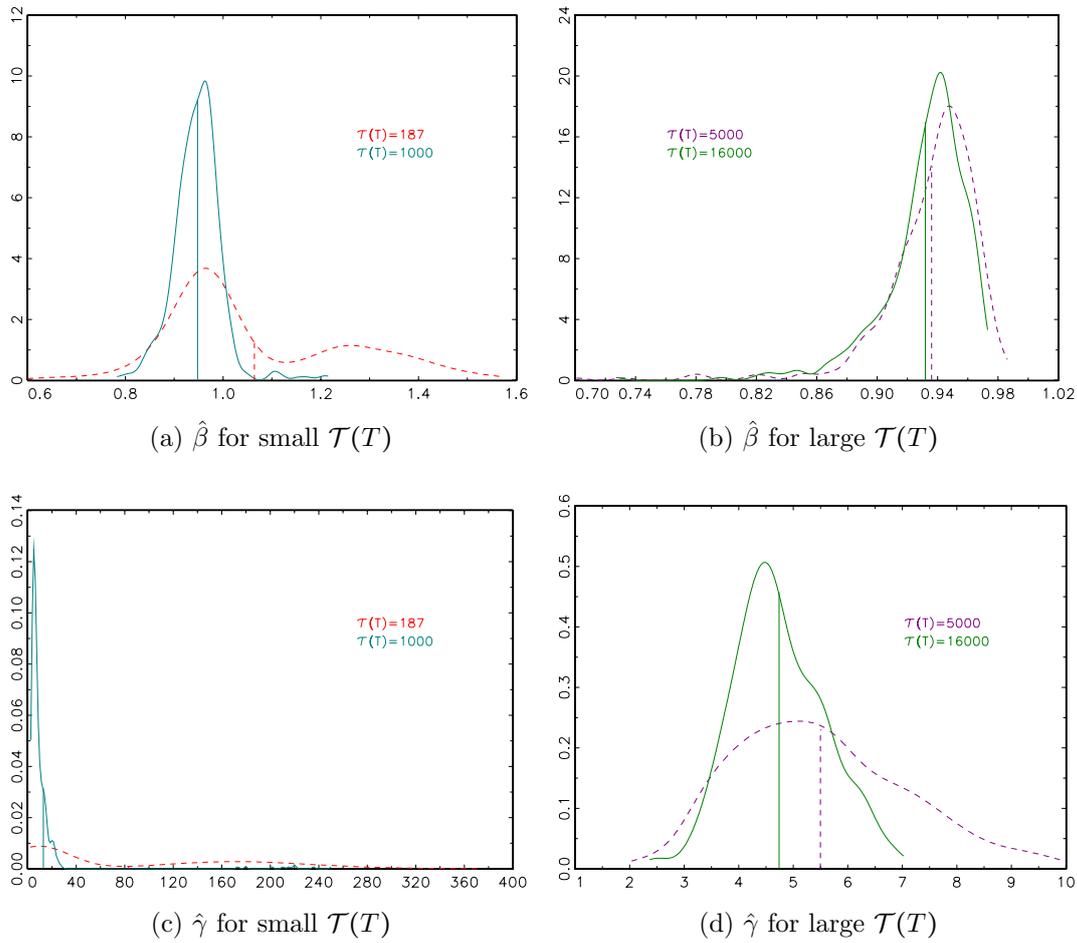


Figure 6: **Effect of varying  $\mathcal{T}(T)$  on AHB parameter estimates using the *MaxCorr* return simulation procedure**

The four panels depict kernel densities of the AHB/*MaxCorr* estimates of the subjective discount factor,  $\hat{\beta}$ , (upper panels) and the coefficient of relative risk aversion,  $\hat{\gamma}$  (lower panels). We use  $H = 400$ , and  $\mathcal{T}(T)$  varies between 187 and 1 000 (left panels) and 5 000 and 16 000 (right panels). The dashed red densities in the left panels are for  $\mathcal{T}(T) = 187$ , whilst the solid cyan colored densities refer to  $\mathcal{T}(T) = 1 000$ . The dashed purple densities are associated with  $\mathcal{T}(T) = 5 000$  and green (solid) relies on  $\mathcal{T}(T) = 16 000$ . The AHB point estimates are indicated by vertical lines in the respective colors.

