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The Growth Drag of Pollution

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The Growth Drag of Pollution

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Abstract

Higher child mortality reduces the willingness of parents to invest in children's education and increases their desired level of fertility. In this context, economic inequality is not only decisive for human capital investments and the emergence of differential fertility, but also for agents' exposure to environmental pollution because wealthier households live in cleaner areas. This is the key mechanism through which environmental conditions may impose a growth drag on the economy. In addition, preferred levels of tax-financed abatement measures differ between population groups with different exposures to pollutants, in the sense that the least affected population group prefers the lowest tax rate. Thus, the adverse effect of inequality and pollution on economic growth is amplified, if the population group that is least affected decides about the level of tax-financed abatement measures.

Keywords: Endogenous Growth, Endogenous fertility, Inequality, Pollution

JEL: O10, Q50, I10

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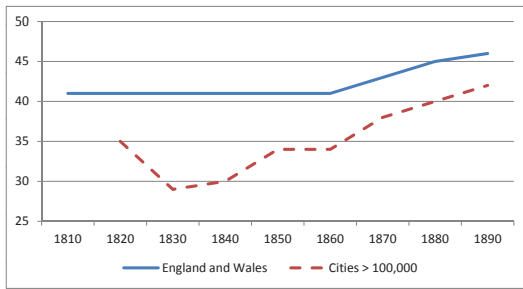
1. INTRODUCTION

The transition from stagnation to growth originated by the industrial revolution caused, via an immense degradation of the environment, adverse effects on individuals' health state in terms of morbidity and life expectancy. Moreover, this transition has been accompanied by a massive shift in demographic variables, the demographic transition. The related decline in fertility rates was mainly the result of increasing parental expenditures for their offspring's human capital which paved ultimately the way for sustained economic growth in per capita terms (Galor and Weil, 2000 and Galor, 2011).

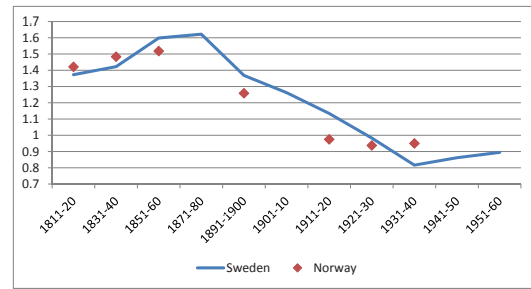
In this paper we analyze the link between child mortality and degradation of the environment through economic development, in the sense that the latter may be conducive for children's survival probabilities, but may also generate adverse impacts on children's probability to survive to adulthood. Higher child mortality reduces the willingness of parents to invest in children's education and increases their desired level of fertility. In this context, economic inequality is not only decisive for human capital investments and the emergence of differential fertility (de la Croix and Doepke, 2003), but also for agents' exposure to environmental pollution because wealthier households live in cleaner areas.¹ The exposure to pollutants triggers again children's probability to survive to adulthood and the willingness of parents to invest in education. This is the key mechanism and the novelty of our approach through which environmental conditions may impose a growth drag on the economy.

The initially adverse impact of economic development on individuals' health is mirrored in the evolution of life expectancies at birth as shown in Figure 1a). Average life expectan-

¹Szreter (1997) argues "*...there is indeed something intrinsically dangerous and socially destabilizing in the wake of economic growth...*". He motivates his statement by the following observations: (1) local authorities were failing the management of their environments, and, (2) as a consequence of it wealthier citizens moved to the periphery of the cities.



a)



b)

Figure 1: a) expectation of life at birth (London excluded) (Szreter, 1997), and b) ratio of infant mortality rates in urban and rural regions (Bairoch, 1988)

cies at birth stagnated during the second phase of the industrial revolution and started to increase only in the last four decades of the 19th century. In cities, life expectancies at birth started even to decline and reached a level passed in the 15th century already, although per capita output was already growing .

Low life expectancies at birth are caused by high infant mortality rates at this time. Bairoch (1988) - see Figure 1b) - documents impressively the initial increase in child mortality rates in cities relative to rural areas during the industrial revolution. Moreover, Figure 1b) shows an inverted u-shaped pattern of infant mortality rates in urban areas compared to rural areas. It is well documented that the gap in mortality rates between cities and rural areas was evoked by environmental degradation and pollution. In this line of argumentation the significance of water as an industrial raw material has been documented by Hassan (1985): fresh water was used for commercial purposes while the new entrepreneurial class saw no point in spending money for sanitation and sewage treatment plants. In addition Hainse (2004) and Komlos (1998) provide evidence for increased

morbidity over the same period of time indicating that physical height of soldiers declined during the 19th century in the U.S. as well as England and the Netherlands.

Even in the presence, externalities of the growth process on the environment are well documented, for example the combined health and non-health cost of outdoor air and water pollution for China's economy comes to around 5.8% of the GDP per year (World Bank, 2007). Chay and Greenstone (2003) provide evidence for the impact of air pollution on infant mortality in the U.S. during the recession period 1981-82 and conclude that a 1-percent reduction in total suspended particulates results in a 0.35-percent decline in infant mortality at the county level.

To our best knowledge Mariani, Perez-Baharona and Raffin (2010) is the only and probably the first paper which relates life expectancy and environmental quality to poverty traps. This paper complements their work in the sense that we relate child mortality and economic inequality to environmental pollution and prospects of future growth. In our overlapping generations framework households decide about the number of children they wish to raise as well as their quality in terms of education and health.² A low survival probability of children, induced by a low level of economic development and/or a high degradation of the environment, reduces parents' willingness to invest in child quality and increases their desired level of fertility. Inequality in terms of relative human capital induces differential fertility a la de la Croix and Doepke (2003, 2004) and regionally differing exposures to pollutants. The latter intensifies the link between inequality and the demographic transition, i.e. more inequality delays the fertility decline, increases the populations' growth rate and reduces growth. In addition, preferred levels of tax-financed abatement measures differ between population groups with different exposures to pollu-

²With respect to the channel connecting fertility and health our work is also related to Strulik (2004, 2008). The difference is that we allow for inequality, pollution, and endogenous growth.

tants, in the sense that the least affected population group prefers the lowest tax rate. Thus, the adverse effect of inequality and pollution on economic growth is amplified, if the population group that is least affected decides about the level of abatement measures. Within this frame, we are able to replicate the historical development path in accordance to empirical patterns: first, in early stages of economic development, there is no abatement since the marginal benefit of abating is low. Moreover, the preferred amount of abatement is inversely related to agents' exposure to pollutants, i.e. wealthier agents prefer less abatement. Second, there is a slow take-off in terms of income per capita growth. Third, the pollution stock increases through economic development. Fourth, the evolution of the populations' growth rate and the evolution of children's regional mortality differentials is hump-shaped.

The remainder of the paper is organized as follows: in Section 2, we introduce our overlapping generations framework with inequality. Section 3 describes the equilibrium structure of the model and in Section 4, we introduce regional differences with respect to the exposure to pollutants. Section 5 performs numerical experiments dealing with the (long-run) effects of different amounts of initial inequality and the growth drag of pollution. Section 6 concludes.

2. THE MODEL

2.1. Human Activities and Pollution

In this setting, time is discrete, indexed by t and ranges from 0 to ∞ . A large number of firms produce aggregate output Y_t using a constant returns to scale technology of Cobb-Douglas type, where K_t denotes aggregate physical capital and L_t aggregate effective

labor, such that

$$Y_t = AK_t^\alpha L_t^{1-\alpha}, \quad (1)$$

with $A > 0$, $\alpha \in (0, 1)$.

The environmental impact of human activity can be captured by population size, affluence and technology.³ Here, human activity injects emission, E_t , through the production process into the eco-system. Emissions may be attenuated by abatement measures, M_t , financed by a proportional tax, τ_t , on production, and may be dampened by the compatibility of human activities with the environment reflected by Π_t , such that the level of emissions in period t is given by

$$E_t = \Pi_t^{-1}(b_2 Y_t - b_3 M_t) = \Pi_t^{-1}(b_2 - b_3 \tau_t) Y_t, \quad (2)$$

with $0 < b_3 < b_2$ and $M_t = \tau_t Y_t$.

For simplicity we assume a one-to-one relationship between the skill level of the working force, i.e. aggregate human capital H_t , and Π_t , such that $\Pi_t = H_t$. Moreover, the environment is adversely affected by population size, \mathcal{N}_t . On the other hand, the environment regenerates at a constant rate b_1 per period of time. Therefore the stock of pollutants reflecting the degree of environmental degradation evolves over time according to

$$P_{t+1} = (1 - b_1)P_t + E_t + b_4 \Pi_t^{-1} \mathcal{N}_t = (1 - b_1)P_t + (b_2 - b_3 \tau_t) \frac{Y_t}{H_t} + b_4 \frac{\mathcal{N}_t}{H_t}, \quad (3)$$

with $0 < b_1 < 1$, $0 < b_3 < b_2$, $b_4 > 0$ and $0 \leq \tau_t < 1$.

2.2. Households

Consider an economy populated by a continuum of overlapping generations and a large number of households indexed by i , where a type- i household is equipped with human

³This refers to the so-called IPAT-identity, where the **impact** is reflected by **population size**, **affluence** and **technology**, see Perman et al. (2003).

capital, h_t^i , and inherited assets, \tilde{b}_{t-1}^i . Households live for two periods: childhood and adulthood. All economically relevant decisions are made in the adult period of life. Adult households care about the level of their own consumption, c_t^i , and the number of surviving offspring $\pi_t^i n_t^i$, where π_t^i denotes the survival probability per child and n_t^i the number of children. Survival of childhood, π_t^i , is influenced by parental expenditures on health and nutrition, d_t^i , and by the economic environment, $\bar{\pi}_t$, i.e. $\pi_t^i = \pi_t^i(d_t^i, \bar{\pi}_t)$, where $\bar{\pi}_t$ is exogenous to the household. Moreover, agents derive utility from the level of human capital per child, h_{t+1}^i , and the amount of bequests per child, b_t^i . To the contrary, agents derive a disutility from the future level of pollution, P_{t+1} , such that preferences of a member i of generation t that is born in $t - 1$ are specified as

$$u_t^i = \ln c_t^i + \rho \ln b_t^i + \gamma \left[\ln \pi_t^i n_t^i + \beta \ln h_{t+1}^i \right] - \mu \ln P_{t+1}, \quad (4)$$

with $\rho, \gamma, \beta, \mu > 0$.

The exogenous survival component $\bar{\pi}_t$ depends on the stage of economic development reflected by the average level of human capital, \bar{h}_t , and the level of pollutants, P_t

$$\bar{\pi}_t = \psi_0 \frac{\bar{h}_t}{1 + \bar{h}_t} - \psi_1 \frac{P_t}{1 + P_t}, \quad (5)$$

with $\psi_0, \psi_1 > 0$.⁴

$\bar{\pi}_t$ has a positive impact on the survival probability of each child, directly, and indirectly through an enhancement of the productivity of private expenditures, d_t^i . Therefore, the probability to survive childhood in household i is determined by

$$\pi_t^i = \min\{1, \underline{\lambda}(\bar{\lambda} d_t^i)^{\bar{\pi}_t}\}, \quad 0 < \underline{\lambda} < 1 < \bar{\lambda}. \quad (6)$$

Human capital per child depends on education, e_t^i , the parental level of human capital,

⁴This formulation implies that $\bar{\pi}_t$ is finite as each term of the function converges to ψ_0 or ψ_1 .

⁵ $\underline{\lambda}$ and $\bar{\lambda}$ are scale parameters implemented for the numerical exercises conducted further below.

h_t^i , and the average level of human capital, \bar{h}_t

$$h_{t+1}^i = (\varepsilon + e_t^i)^\eta (h_t^i)^\nu (\bar{h}_t)^{1-\nu}, \quad (7)$$

where $\eta \in (0, 1)$ reflects the impact of education on the level of human capital. $\nu \in (0, 1)$ denotes the intergenerational transmission of human capital or the intergenerational persistence between parental human capital and the level of human capital per child. The parameter $\varepsilon > 0$ allows for $e_t^i = 0$.

We denote post-tax variables by " $\hat{\cdot}$ ", such that post-tax income of agent i endowed with one unit of time, human capital h_t^i , and assets \tilde{b}_{t-1}^i reads as

$$\hat{y}_t^i = (1 - \tau_t)(w_t h_t^i + R_t \tilde{b}_{t-1}^i), \quad (8)$$

where w_t and R_t represent the wage rate per efficient unit of labor and the return on capital. The budget constraint of agent i is then given by

$$\hat{y}_t^i = (\hat{w}_t h_t^i z + \hat{w}_t \bar{h}_t e_t^i n_t^i + b_t^i + d_t^i) n_t^i + c_t^i. \quad (9)$$

Child rearing costs are captured by: first, forgone wage earnings, $\hat{w}_t h_t^i z n_t^i$, with $0 < z < 1$ denoting the time share necessary to raise one child to adulthood. Second, expenditures for education, $\hat{w}_t \bar{h}_t e_t^i n_t^i$, where education is provided by an educational sector employing teacher equipped with the average level of human capital, \bar{h}_t . Third, expenditures on nutrition, d_t^i , and the level of bequests per child, b_t^i . Maximizing lifetime utility as given by (4) subject to the budget constraint (9), and the evolution of human capital per child

(7), while ignoring the impact on the evolution of the aggregate pollution stock yields

$$\hat{c}_t^i = \frac{1}{1+\gamma} \hat{y}_t^i, \quad (10)$$

$$n_t^i = \frac{\gamma}{1+\gamma} \frac{\hat{y}_t^i}{\hat{w}_t [h_t^i z + \bar{h}_t e_t^i] + b_t^i + d_t^i}, \quad (11)$$

$$e_t^i = \frac{\beta \eta z x_t^i - (\gamma(1 - \bar{\pi}_t) - \rho) \varepsilon}{\gamma(1 - \beta \eta - \bar{\pi}_t) - \rho}, \quad (12)$$

$$d_t^i = \frac{\gamma \bar{\pi}_t (z - \frac{\varepsilon}{x_t^i})}{\gamma(1 - \beta \eta - \bar{\pi}_t) - \rho} \hat{w}_t h_t^i, \quad (13)$$

$$b_t^i = \frac{\rho (z - \frac{\varepsilon}{x_t^i})}{\gamma(1 - \beta \eta - \bar{\pi}_t) - \rho} \hat{w}_t h_t^i, \quad (14)$$

with $\gamma(1 - \beta \eta - \bar{\pi}_t) - \rho > 0$ and $z x_t^i - \varepsilon > 0$, where $x_t^i = \frac{h_t^i}{h_t}$ represents household i 's relative level of human capital.

Households spent a fraction $\frac{1}{1+\gamma}$ of their post-tax income on consumption. The remaining part, $\frac{\gamma}{1+\gamma}$, is spent on child rearing. Fertility, n_t^i , is positively related to the level of inherited wealth, but it is negatively related to expenditures on child quality as captured by e_t^i, d_t^i and the level of bequests, b_t^i , per child. These variables depend in turn positively on the level of parental relative human capital, x_t^i , and the exogenous component of the survival probability, $\bar{\pi}_t$. This means that a favorable environment which increases the number of surviving offspring reduces the desired level of fertility. Consequently, more resources are available for education, nutrition and bequests. Conversely, education may be zero if either x_t^i is low and/or the exogenous component of the mortality risk per child, $(1 - \bar{\pi}_t)$, is high, i.e. if

$$\beta \eta z x_t^i < (\gamma(1 - \bar{\pi}_t) - \rho) \varepsilon, \quad (15)$$

then $e_t^i = 0$ and

$$n_t^i = \frac{\gamma}{1+\gamma} \frac{\hat{y}_t^i}{\hat{w}_t h_t^i z + b_t^i + d_t^i}, \quad (16)$$

with

$$b_t^i = \frac{\rho z}{(\gamma - \rho)(1 - \bar{\pi}_t) - \rho \bar{\pi}_t} \hat{w}_t h_t^i, \quad (17)$$

$$d_t^i = \frac{\bar{\pi}_t z}{1 - \bar{\pi}_t} \hat{w}_t h_t^i + b_t^i. \quad (18)$$

At the beginning of the second period of life (adulthood), bequests of children that didn't survive to adulthood are redistributed equally within the family among the surviving offspring. Thus, wealth per adult at the beginning of period $t + 1$ is

$$\tilde{b}_t^i = \frac{b_t^i}{\pi_t^i}. \quad (19)$$

2.3. Policy

The government sets a tax rate $0 \leq \tau_t < 1$ so as to maximize lifetime utility (4) of a representative adult, j , given optimal decisions (10)-(14) and the evolution of the pollution stock, (3)

$$u_t^j = \max_{\{0 \leq \tau_t^j < 1\}} \left\{ \begin{array}{l} \ln c_t^j + \rho \ln b_t^j + \gamma \left[\ln \pi_t^j n_t^j + \rho \ln h_{t+1}^j \right] \\ - \mu \ln \left[(1 - b_1) P_t + H_t^{-1} (b_2 Y_t - b_3 \tau_t Y_t + b_4 \mathcal{N}_t) \right] \end{array} \right\}, \quad (20)$$

such that the first-order condition equilibrating marginal cost and benefits reads as

$$-\frac{1 + \rho + \gamma \bar{\pi}_t}{1 - \tau_t^j} + \frac{\mu b_3 Y_t}{H_t \left[(1 - b_1) P_t + \frac{(b_2 - b_3 \tau_t^j) Y_t + b_4 \mathcal{N}_t}{H_t} \right]} = 0. \quad (21)$$

In light of the last expression, we can thus establish the following results

Proposition 1

- (i) *Since the preferred tax rate of an agent j is in this case independent from her level of income and wealth, the preferred tax rate is the same for all agents i , such that the government implements the tax rate $\tau = \tau^j$, where*

$$\tau_t = \frac{(1 - b_1)(1 - \rho - \gamma \bar{\pi}_t) P_t \frac{H_t}{Y_t} + (1 + \rho + \gamma \bar{\pi}_t)(b_2 + b_4 \frac{\mathcal{N}_t}{Y_t}) - \mu b_3}{b_3(1 + \rho + \gamma \bar{\pi}_t - \mu)}. \quad (22)$$

(ii) Defining marginal benefits of the tax as $MBT = \frac{\mu b_3 Y_t}{H_t[(1-b_1)P_t + \frac{(b_2 - b_3 \tau_t)Y_t + b_4 N_t}{H_t}]}$ yields

a) $\lim_{K_t \rightarrow 0} MBT = \lim_{H_t \rightarrow 0} MBT = 0$.

b) MBT is monotonously increasing in K_t .

c) MBT is hump-shaped in H_t .

d) In the long run, the tax rate is asymptotically constant, if the long-run growth rate of aggregate output is at least as high as the population's net-growth rate,

i.e. $g_*^Y \geq g_*^N$.

It follows that the tax rate is potentially zero in earlier stages of economic development characterized by low levels of K_t and H_t (Item (ii) a)). If this is the case it needs economic development in terms of capital formation and or human capital accumulation which both increases pollution but also increases marginal benefits from taxation, i.e. MBT . Because of Item (ii) c), the tax rate may be hump-shaped. The ambiguous sign of MBT with respect to changes in H_t stems from the twofold effect of human capital on the environment. On the one hand H_t increases production and on the other hand the compatibility of production to the environment. In the long-run, the tax rate is realistically constant since the population's growth rate is around the reproduction level in developed countries and projected to be zero for the world as whole (see United Nations 1998).

3. EQUILIBRIUM

Definition 1

Given a large number of households $i \in [1, \dots, N_0]$, in period $t = 0$, an initial distribution of human capital determining the average stock of human capital, \bar{h}_0 , an (equal) initial distribution of wealth with $b_{-1}^i = b_{-1} \forall i$ implying K_0 , and given an initial stock of pollution P_0 ,

an equilibrium consists of a sequence of aggregate quantities $\{K_t, K_{t+1}, L_t, P_t, P_{t+1}, \mathcal{N}_t, \mathcal{N}_{t+1}, H_t, H_{t+1}, \bar{h}_t, \bar{h}_{t+1}\}_{t=0}^{\infty}$, a sequence of factor prices and tax rates $\{w_t, R_t, \tau_t\}_{t=0}^{\infty}$, and a sequence of optimal decisions $\{c_t^i, n_t^i, e_t^i, d_t^i, b_t^i\}_{t=0}^{\infty}$ that maximize lifetime utility.

Perfect competition implies that equilibrium rates of reward read as

$$w_t = (1 - \alpha)Ak_t^\alpha, \quad (23)$$

$$R_t = r_t + \delta = \alpha Ak_t^{\alpha-1}, \quad (24)$$

where δ is the depreciation rate of physical capital and $k_t = \frac{K_t}{L_t}$.

At $t = 0$, population size \mathcal{N}_t equals the number of households (dynasties). The size of a type- $i \in [1, \dots, \mathcal{N}_0]$ household with relative human capital endowment x_t^i evolves from one period to another according to $N_{t+1}^i = \pi_t^i n_t^i N_t^i$. Hence population size in $t + 1$ is obtained as

$$\mathcal{N}_{t+1} = \sum_{i=0}^{\mathcal{N}_0} N_{t+1}^i. \quad (25)$$

Average human capital in t is given by $\bar{h}_t = \sum_{i=1}^{\mathcal{N}_0} \frac{N_t^i}{\mathcal{N}_t} h_t^i$ and evolves according to

$$\frac{\bar{h}_{t+1}}{\bar{h}_t} = \frac{\sum_{i=1}^{\mathcal{N}_0} \frac{N_{t+1}^i}{\mathcal{N}_{t+1}} h_{t+1}^i}{\sum_{i=1}^{\mathcal{N}_0} \frac{N_t^i}{\mathcal{N}_t} h_t^i}. \quad (26)$$

From (7) and the fact that a household endowed with average human capital is characterized by $x_t^i = 1$, the dynamics of relative human capital of household i is described by

$$x_{t+1}^i = \left(\frac{zx_t^i - \varepsilon}{z - \varepsilon} \right)^\eta (x_t^i)^\nu. \quad (27)$$

Time devoted to child rearing and education is not available for production, such that

$$L_t = \sum_{i=1}^{\mathcal{N}_0} N_t^i \left[(1 - zn_t^i) h_t^i - e_t^i n_t^i \bar{h}_t \right]. \quad (28)$$

Next period's stock of aggregate capital is determined by the sum of wealth per child surviving to adulthood

$$K_{t+1} = \sum_{i=1}^{N_0} N_{t+1}^i \tilde{b}_t^i. \quad (29)$$

4. INEQUALITY AND REGIONAL SURVIVAL DIFFERENTIALS

Consider a population that inhabits two areas denoted by A and B . Both areas differ with respect to the residents' exposure to pollutants in the sense that children of region B suffer a higher impact of environmental pollution on their exogenous component of the survival probability, i.e.

$$\bar{\pi}_t^j = \psi_0 \frac{\bar{h}_t}{1 + \bar{h}_t} - \psi_1^j \frac{P_t}{1 + P_t}, \quad j = A, B, \quad (30)$$

with $\psi_1^A < \psi_1^B$, such that $\bar{\pi}_t^B < \bar{\pi}_t^A$.

Since exposure to pollutants will be negatively associated to housing prices we can expect that richer agents live in healthier areas. We implement a shortcut of a housing market, in the sense that agents are allocated to region A or B according their relative level of human capital x_t^i : agents with relative human capital $\tilde{x} \geq x_t^i$ live in region A and type- B agents are characterized by $x_t^i < \tilde{x}$.

Proposition 2

(i) $x_t^{i,A}$ evolves according to

$$x_{t+1}^{i,A} = \left(\frac{z x_t^{i,A} - \varepsilon}{z - \varepsilon} \right)^\eta (x_t^{i,A})^\nu, \quad (31)$$

with a stationary and stable solution at $x_t^{i,A} = x_*^{i,A} = 1$ for all t and $i \in \{A\}$.

(ii) The evolution of relative human capital of region- B agents is triggered by the differential between the external component of children's survival probability, $\bar{\pi}_t^j$, $j =$

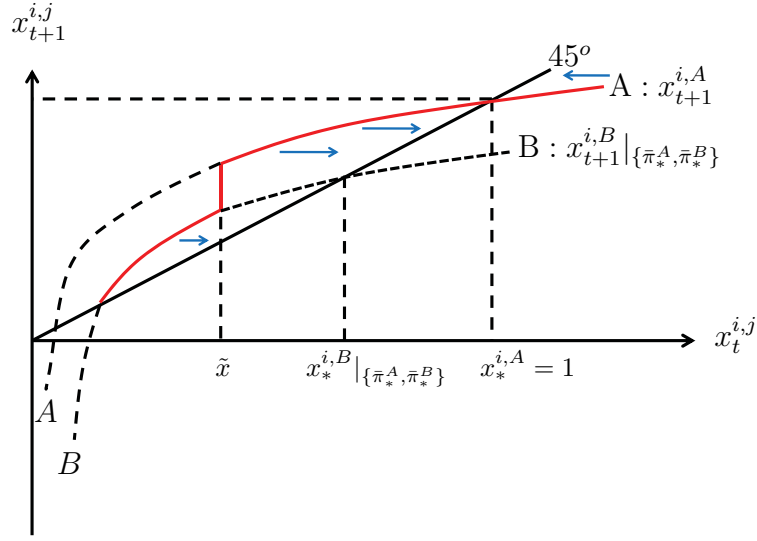


Figure 2: Evolution of relative human capital x_t^i . *AA*-locus: evolution of $x_{t+1}^{i,A}$. *BB*-locus: evolution of $x_{t+1}^{i,B}$ given $\{\bar{\pi}_t^A, \bar{\pi}_t^B\}$.

A, B, in the sense that

$$x_{t+1}^{i,B} = \left(\frac{z x_t^{i,B} - \varepsilon}{z - \varepsilon} \right)^\eta \left(\frac{\gamma(1 - \beta\eta - \bar{\pi}_t^A) - \rho}{\gamma(1 - \beta\eta - \bar{\pi}_t^B) - \rho} \right)^\eta (x_t^{i,B})^\nu. \quad (32)$$

Since $\bar{\pi}_t^A > \bar{\pi}_t^B$, it follows that $\frac{\gamma(1 - \beta\eta - \bar{\pi}_t^A) - \rho}{\gamma(1 - \beta\eta - \bar{\pi}_t^B) - \rho} < 1$ which implies that relative human capital of type-*B* agents evolves at a slower pace as compared to type-*A* agents.

The evolution of $x_t^{i,j}$, $j = A, B$ is depicted in Figure 2 and follows the solid gray line. Agents with $x_t^i \geq \tilde{x}$ are allocated to region *A* and the evolution of their relative human capital stock follows the solid part of the *A*-locus. Agents with $x_t^i < \tilde{x}$ are allocated to region *B* and the evolution of their relative human capital stock follows the solid part of the *B*-locus.⁶ Region-*B* agents face a higher impact of pollutants to their external component of the survival probability, i.e. $\bar{\pi}_t^B < \bar{\pi}_t^A$. Therefore, type-*B* agents exhibit a

⁶Note that the location of the *B*-locus is conditional on the state of the external component of children's survival probability $\bar{\pi}_t^j$, $j = A, B$, but fixed along the BGP with $\bar{\pi}_t^j = \bar{\pi}_*^j = \text{const.}$, $j = A, B$. Moreover, both, the *A* and the *B*-locus exhibit an unstable steady state to the left of \tilde{x} because of $\varepsilon > 0$, which would imply zero human capital for some dynasties in the long-run. Empirically this region is irrelevant since it exceeds the maximal possible number of children over the life course by far. The assumption that agents with average human capital live in region *A* is not harmful: if they were allocated to the *B*-region, $x_t^{i,B}$ would evolve according to (31) and $x_t^{i,A}$ would then be positively influenced by the survival differential between the *A* and the *B* region.

higher fertility and lower investments in education per child. Furthermore, the forces of the quality quantity trade-off are amplified via a below average level of human capital. Therefore, relative human capital $x_t^{i,B}$ is converging with a lower speed to its lower long-run value.

Contrary to Section 2.3, the magnitude of the tax rate depends now on the location of the agent whose preferences are decisive for the government, i.e. $j = A, B$.

Proposition 3

The tax rate is declining in the external component of children’s survival probability, given $P_t, H_t, K_t, \mathcal{N}_t$ and the sufficient condition $\mu < 2$, i.e.

$$\frac{\partial \tau^j}{\partial \bar{\pi}_t^j} = \underbrace{\frac{-\gamma(b_1 - 1)(\mu - 2)}{b_3(1 + \rho + \gamma\bar{\pi}_t^j - \mu)^2} \frac{P_t H_t}{Y_t}}_{<0 \text{ if } \mu < 2} - \underbrace{\frac{\gamma\mu(b_2 - b_3 + b_4 \frac{\mathcal{N}_t}{Y_t})}{b_3(1 + \rho + \gamma\bar{\pi}_t^j - \mu)^2}}_{>0} < 0, \tag{33}$$

with $0 < b_1 < 1$ and $b_2 > b_3 < 0$.

5. NUMERICAL EXPERIMENTS

Before we turn to the numerical evaluation of our model, we present the underlying set of parameters and a sketch of the method in the next subsection. In our first numerical experiment (Section 5.2), we abstract from differences in regional exposures to pollutants in order to analyze the link between child mortality, inequality, and pollution in isolation. In Section 5.3, we introduce regional survival differentials of children into the model and conduct the following numerical experiments: First, we investigate changes in the initial distribution of the population over the regions, due to different amounts of initial inequality, while population group A decides about the tax rate (Section 5.3.1) . Second, we examine the long-run effects of changes in this initial distribution on the long-run performance of the economy, given that either population group A ’s or B ’s preferred tax rate is implemented (Section 5.3.2).

5.1 Calibration and Method

We choose parameters of the model such that the balanced growth path of the model fits to empirical observations of the US economy and United Nations long-run projections. One period in our model has a length of 30 years. We fix the capital income share in the production of Y_t , α at 0.3. Moreover, capital depreciates within 30 years entirely, i.e. $\delta = 1$. As regards child-rearing time, we fix the time share necessary to raise one child to adulthood, z , at 0.07 which implies opportunity costs about 15 percent of parents' time endowment (see de la Croix and Doepke, 2003 and Knowles, 1999). The remaining parameters are fixed in an iterative way. In order to match a long-run interest rate of

Technology	$\alpha = 0.3; \delta = 1; A = 0.25$
Pollution	$b_1 = 0.85; b_2 = 1.42; b_3 = 1.365; b_4 = 0.035$
Human capital	$B = 3.3; \eta = 0.25; \nu = 0.5; \varepsilon = 0.01; \tilde{x} = 0.7$
Preferences	$\rho = 0.315; \gamma = 0.92; \beta = 0.45; \mu = 1.95$
Survival	$\psi_0 = 0.45; \psi_1^A = 0.1; \psi_1^B = 0.17; \underline{\lambda} = 0.25; \lambda = 1000$
Child-rearing	$z = 0.07$

Table 1: Parameters

4% per year, an investment share in the vicinity of 14% fitting the 10 year average of US private fixed capital formation as a share of GDP, we set $\rho = 0.315$ and $A = 0.25$. The long-run growth rate per year should not be larger than 2% and not be lower than 1.7% while population growth is zero. This implies $\gamma = 0.92$, $\beta = 0.45$ and $B = 3.3$, while $\varepsilon = 0.01$, $\eta = 0.25$ and $\nu = 0.6$ are chosen such that implied fertility differential between the wealthiest and the poorest households does not exceed three surviving children and the average expenditure share for education is in between 5-7%. Since, we consider the evolution of a single pollution stock and abstract therefore from any interaction between pollutants, we assume a rather risk averse calibration as far as the absorptive capacity of the environment is concerned, i.e. we set $b_1 = 0.85$. In order to take account for the

observation of an environmental Kuznets curve, which is apparently more realistic as far as local pollutants are concerned, we set $b_2 = 1.42$, $b_3 = 1.365$, $b_4 = 0.035$ and $\mu = 1.95$, $\psi_0 = 0.45$, $\psi_1^A = 0.1$, $\psi_1^B = 0.17$.⁷ This also implies an upper boundary for the pollution tax of 0.12 compatible with the afore mentioned long-run interest rate of 4% and the investment share of 14%. In order to get a reasonable fit of the transition period we set $\underline{\lambda} = 0.25$ and $\bar{\lambda} = 1000$. $\tilde{x} = 0.7$ implies that (depending on initial inequality) between one and two third of the population live in the less healthy region B .

For the simulation of the model we generate a large number of households $\mathcal{N}_0 = 1000$ and draw for each of them an initial level of human capital h_0^i from a log-normal distribution $F(\mu_{h,0}, \sigma_{h,0}^2)$.⁸ Given an equal distribution of wealth, i.e. $\tilde{b}_0^i = \tilde{b}_0$, the initial capital stock, K_0 is known. Given P_0 , the only unknown is aggregate labor supply to production which in turn depends on households's decisions. We therefore estimate labor supply by means of the delta method⁹ using the moments of the log-normal distribution and households' optimal decisions (10)-(14), the tax rate (22) and factor prices (23),(24). Now the solutions for our artificial sample of households are obtainable and the state of the economy for the next period is calculated from the equilibrium conditions as described in Section 3. The next step of the iteration starts again with the delta method and the stopping criterion is 10^{-5} .

5.2 Transitory Dynamics and Initial Inequality

For the moment, we abstract from differences in regional exposures to pollutants and analyze the link between child mortality, inequality, and pollution in isolation. Under this

⁷Whether or not the evolution of environmental quality follows an inverted u-shaped pattern at a global level is debatable. Since we focus rather on local pollution, the emergence of a Kuznets curve may be a reasonable fit, see for example Perman et al (2003). Note also that the qualitative results of the paper are independent from the emergence of an environmental Kuznets curve.

⁸In the baseline scenario, we set $\mu_h = 0.2$ and $\sigma_h = 0.25$.

⁹See for example Oehlert (1992).

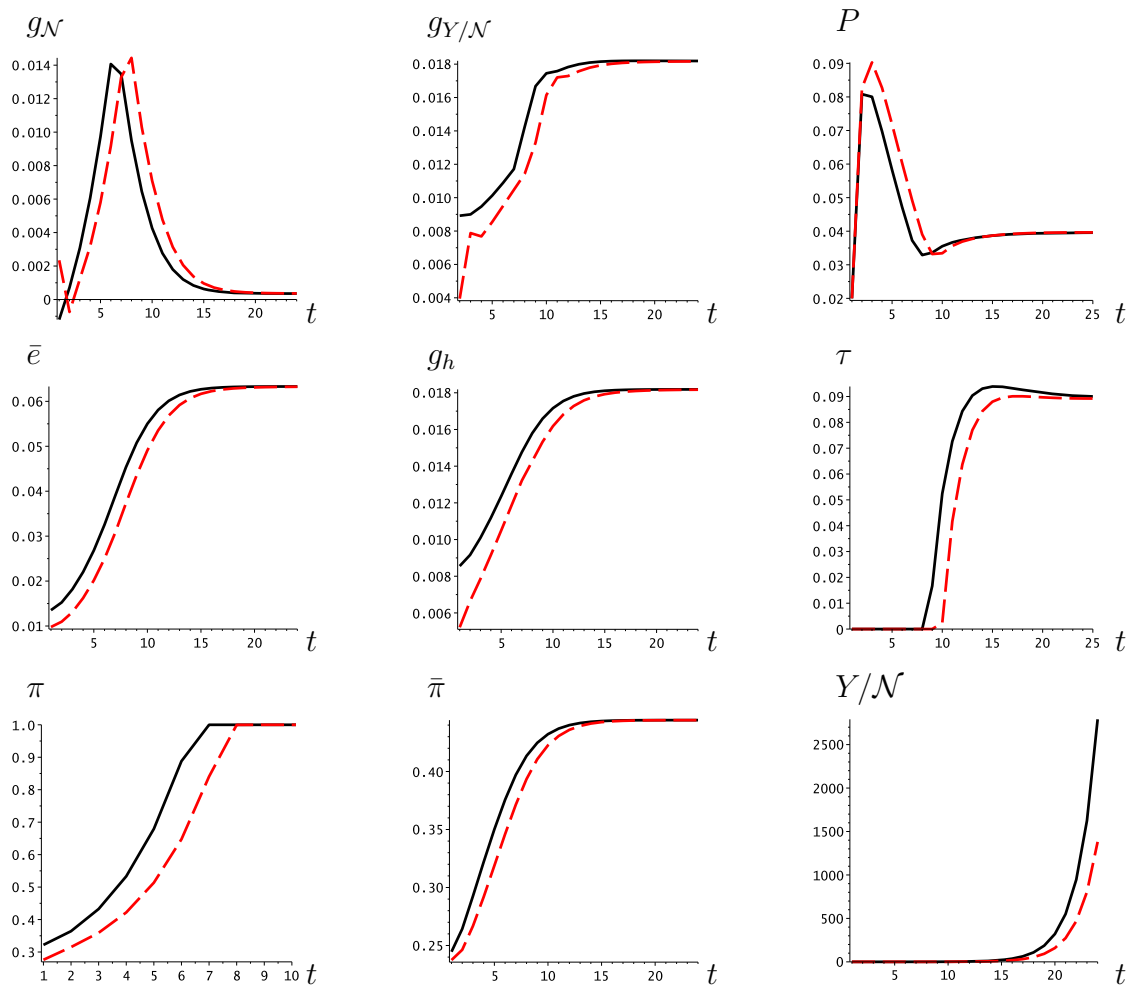


Figure 3: Baseline scenario: $\sigma_0 = \text{low}$ (solid line) and $\sigma_0 = \text{high}$ (dashed line)

circumstances, inequality in terms of initial human capital endowment has no long-run effect. The evolution of relative human capital follows (27), such that relative human capital converges to $x_*^i = 1$ for all i and t . The results are depicted in Figure 3. Note also that growth rates of variables between two periods denoted by g are adjusted to their 30 years average. So far, our model is able to capture several empirical regularities of economic development: a hump-shaped behavior of the (net-) population's growth rate, a slow take-off, i.e. increasing growth rates in output per capita, an environmental Kuznets curve, increasing expenditures on abatement measures, and increasing survival probabil-

ities of children.

Initially, the number of births is high while the net population's growth rate is low due to low survival probabilities of children. In this stage, low survival probabilities of children are owed to a low exogenous component of children's survival probability, $\bar{\pi}_t$, caused by a low stage of economic development reflected by the average stock of human capital, \bar{h}_t . A low $\bar{\pi}_t$ induces, while the majority of households exhibits a low x_t^i , a high number of births with little resources left for expenditures on education and health. Some households are even at the corner solution $e_t^i = 0$, whereas other households start to invest in education. Thus, the growth rate of output per capita and human capital is low. A slowly increasing level of aggregate production increases pollution. At the same time households investing in human capital contribute to an increase in the average level of human capital, \bar{h}_t . If the latter effect overcompensates the adverse effect of increasing pollution on the external survival component, $\bar{\pi}_t$ starts to increase. The increase in $\bar{\pi}_t$ strengthens the forces of the quality quantity trade-off, i.e. average expenditures on health and education increase, lower the number of births and reduce child mortality. Since most households are characterized by a low x_t^i , the reduction in the number of births is not very strong. Thus the net-population's growth rate starts to increase. An increase in the level of human activities captured by population size and the level of aggregate production induces a fast increase in the pollution stock. When the government starts to raise taxes and the level of human capital is high enough, the pollution stock may decline. Eventually, the probability to survive childhood approaches one such that the forces responsible for an increase in the net-population's growth rate disappear. Net-population growth reaches a maximum and starts to decline.

Higher initial inequality (dashed line)¹⁰ lowers average expenditures on child quality, (ed-

¹⁰We keep the mean of the initial distribution of human capital constant.

ucation and health). Since this reduces the growth rate of human capital and the accumulation of capital, the implementation of abatement measures is delayed and τ_t declines compared to baseline scenario (see Prop. 1, Item (ii)), such that the external component of children's survival probability, $\bar{\pi}_t$, declines as well which reinforces the negative effect of inequality on child quality and thus increases the number of births. Due to an increased mortality risk of children in response to an increase in initial inequality, the net-population's growth rate may fall below the one of the baseline scenario before net-population growth reaches its maximum. Since $\bar{\pi}$ increases at a slower pace the peak of the population's growth rate is delayed. Once children's survival probability reaches one, however, net population growth equals the number of births, such that from now on a more unequal society must be characterized by a higher population's growth rate. Due to a lower growth rate of human capital per-capita, output growth falls during the transition below the level realized in the baseline scenario.

5.3 Regional Survival Differentials

Let's consider now a population which inhabits two regions, A and B , where population group B suffers a higher exposure to pollutants than population group A . Therefore, initial inequality in human capital endowments triggers the distribution of the population over the two regions and affects the evolution of the economy not only during the transition but also in the long-run (see Figure 2). In the following two experiments, we analyze therefore: (1) changes in the initial distribution of the population over the two regions given that population group A decides about the tax rate, and, (2) the long-run effects of initial inequality given that either population group A 's or population group B 's preferred tax rate is implemented.

5.3.1 Changes in the initial inequality while group A decides about the tax rate

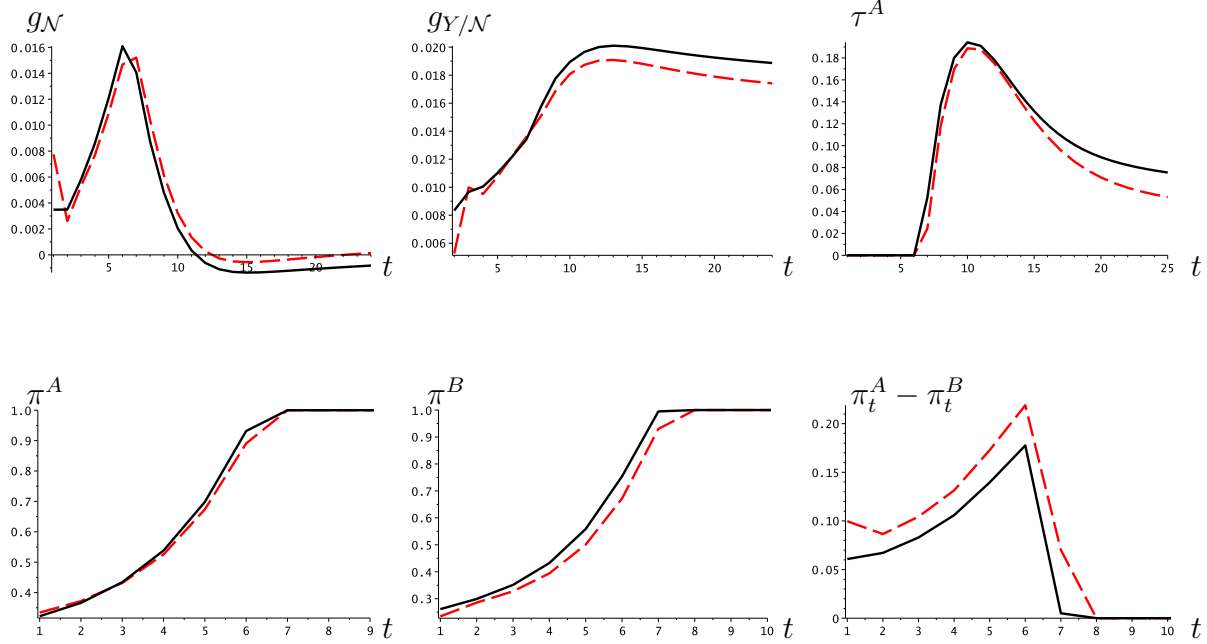


Figure 4: Changes in the initial distribution of the population over the two regions and implementation of τ^A . Baseline: solid line; increased inequality: dashed line

We change the initial distribution of the population over the two regions by changing the initial distribution of relative human capital while we keep the mean of the distribution constant. The results of this experiment are depicted in Figure 4.

As $\bar{\pi}_t^A > \bar{\pi}_t^B$ and since type- A agents are equipped with a higher relative human capital stock than type- B agents, type- B agents invest less in child quality and exhibit a higher number of births. Lower expenditures on health and nutrition in conjunction with a lower external component of children's survival probability induce a slower increase in children's survival probability in region B as compared to region A . Consequently, the mortality differential between region B and A as expressed by $\pi_t^A - \pi_t^B = (1 - \pi_t^B) - (1 - \pi_t^A)$ must increase. Since the increase in children's survival probability is subject

to declining marginal returns with respect to expenditures on health and nutrition, and since $\bar{\pi}_t^j$, $j = A, B$ is also concave in all its arguments, mortality differentials must decline again, eventually accelerated if region A reaches the upper boundary of one. Higher initial inequality (dashed line) implies that more households are living in region B , such that the forces of initial inequality are aggravated by a lower external survival component in region B . Due to high child mortality, per capita growth may even be temporarily above the level of the baseline scenario which could be referred to as the dark side of economic development. Higher initial inequality reduces the growth rate of human capital accumulation through fertility differentials and a higher exposure to pollutants of children living in region B . A lower stock of aggregate human capital reduces therefore the tax rate which dampens the increase in the external survival component in both regions implying lower private expenditures on child quality and a higher number of births in the economy. Therefore per capita growth is reduced, and survival probabilities of children increase in both regions at slower pace, while the mortality differential between the regions is amplified.

5.3.2 Long-run effects of initial inequality while either group A or B decides about the

tax rate

Again we increase initial inequality in human capital while we keep the average of the distribution constant. The results of the long-run effects are presented in Table 2. There we consider two scenarios in the sense that either population group A (left-hand side panel) or B (right-hand side panel) is decisive for the tax rate, for example because either the median-voter is located in the region A (B) or the political process is biased towards the rich (poor). Apparently population group B which is more affected by pollution prefers a higher tax rate ($\tau_*^B > \tau_*^A$, since in light of Prop. 3: $\frac{\partial \tau_*^j}{\partial \bar{\pi}_*^j} < 0$). Higher initial

	$\sigma_{h,0} = 0.25$	$\sigma_{h,0} = 0.5$	$\sigma_{h,0} = 0.75$		$\sigma_{h,0} = 0.25$	$\sigma_{h,0} = 0.5$	$\sigma_{h,0} = 0.75$
τ_*^A	0.07798	0.06723	0.05999	τ_*^B	0.11569	0.10789	0.09819
$g_*^{Y/N}$	0.01829	0.01784	0.01753	$g_*^{Y/N}$	0.01840	0.01802	0.01761
P_*	0.07117	0.07299	0.07423	P_*	0.06754	0.06891	0.07063
$\bar{\pi}_*^A$	0.44335	0.44319	0.44308	$\bar{\pi}_*^A$	0.44367	0.44355	0.44340
$\bar{\pi}_*^B$	0.43873	0.43843	0.43824	$\bar{\pi}_*^B$	0.43924	0.43907	0.43878

Table 2: Long-run effects of increasing initial inequality. Left panel: population group A 's preferred tax rate is implemented. Right panel: population group B 's preferred tax rate is implemented.

inequality reduces the tax rate and induces a higher stock of pollution because the economy accumulates human capital at a slower pace. Therefore the exogenous components of children's survival probabilities are lower. A lower $\bar{\pi}_*^j, j = A, B$ is the second channel that reduces human capital accumulation (and increases population growth). Hence, the long-run growth rate is inversely related to initial inequality due to the adverse effect of pollution on human capital investments. The effects are weakened, if the population group that is most affected by pollutants decides about the tax rate, since this group prefers the highest tax rate.

6. SUMMARY AND CONCLUSIONS

In this paper we analyze the link between child mortality and degradation of the environment through economic development. Higher child mortality reduces the willingness of parents to invest in children's education and increases their desired level of fertility. In this context, economic inequality is not only decisive for human capital investments and the emergence of differential fertility, but also for agents' exposure to environmental pollution because wealthier households live in cleaner areas. The exposure to pollutants triggers again children's probability to survive to adulthood and the willingness of parents to invest in education. This is the key mechanism and the novelty of our approach

through which environmental conditions may impose a growth drag on the economy. Higher initial inequality lowers average expenditures of parents on child quality, in terms of education and health. Since this reduces the growth rate of human capital, the technological impact on the environment aggravates while tax-financed abatement measures shrink. Hence, the pollution stock increases which lowers the survival probability of children. The latter reinforces the negative effect of inequality on child quality and thus increases the number of births with a lower survival probability per child. With less children surviving to adulthood, the growth rate of per-capita income may be temporarily above a reference scenario with lower inequality. This phenomenon may be coined as the dark side of economic development.

If a society is characterized by regional differentials of citizens' exposure to pollutants, in the sense that wealthier agents live in cleaner areas, the link between inequality and pollution affects the long-run growth rate of the economy through regional differences in child mortality rates. The evolution of the regional differential of child mortality rates follows the empirically observed hump-shaped pattern. The long-run growth rate is inversely related to initial inequality due to the adverse effect of pollution on human capital investments. A slower take-off in growth rates shifts the pace of the demographic transition and is accompanied by an increase in children's regional mortality differential. The effects are weakened, if the population group that is most affected by pollutants decides about the level of tax-financed abatement measures, since this population group prefers the highest tax rate.

REFERENCES

- Bairoch, R., 1988. *Cities and Economic Development - From the Dawn of History to the Present*. The University of Chicago Press, Chicago.
- Galor, O., 2011. *Unified Growth Theory*, Princeton University Press, Princeton and Oxford.
- Galor, O., Weil, D.N., 2000. Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond, *American Economic Review* 90, 806-824.
- Chay, K.Y, M. Greenstone, 2003. The Impact of Air Pollution on Infant Mortality: Evidence from Geographic Variation in Pollution Shocks Induced by a Recession, *Quarterly Journal of Economics*, August 2003, 1121-1167.
- de la Croix, D., Doepke, M., 2003. Inequality and Growth: Why Differential Fertility Matters, *American Economic Review* 93, 1091-1113.
- de la Croix, D., Doepke, M., 2004. Public versus Private Education when Fertility Differentials Matter, *Journal of Development Economics* 73, 607-629.
- Haines, M.R., 2004. Growing Incomes, Shrinking People - Can Economic Development be Hazardous to Your Health?, *Social Science History* 28, 249-270.
- Knowles, J., 1999. Can Parental Decisions Explain U.S. Income Inequality?, Mimeo, University of Pennsylvania.
- Komlos, J., 1998. Shrinking in a Growing Economy? The Mystery of Physical Stature During Industrial Revolution, *Journal of Economic History* 58, 779-802.

- Mariani, F, A. Perez-Barahona, N. Raffin, 2010. Life Expectancy and the Environment, *Journal Economic Dynamics and Control* 34, 798-815.
- Oehlert, G.W., 1992. A Note on the Delta Method, *The America Statistician* 46, 27-29.
- Perman, R., Y. Ma, J. McGilvray, M. Common, 2003. *Natural Resource and Environmental Economics*, 3rd edition, Pearson, Harlow.
- Strulik, H., 2004. Endogenous Growth and Stagnation with Endogenous Health and Fertility, *Journal of Population Economics* 17, 433-453.
- Strulik, H., 2008. Geography, Health, and the Pace of Demo-Economic Development, *Journal of Development Economics* 86, 61-75.
- Szreter, S., 1997. Economic Growth, Disruption, Disease and Death: on the Importance of the Politics of Public Health for Development, *Population and Development Review*, 23, 693-728.
- Szreter, S., G. Mooney, 1998. Urbanization, Mortality, and the Standard of Living Debate: New Estimates of the Expectation of Life at Birth in Nineteenth-Century British cities, *Economic History Review* 1, 84-112.
- Worldbank (2007), <http://www.worldbank.org/en/news/press-release/2007/07/11/statement-world-bank-china-country-director-cost-pollution-china-report>.

Households

Maximizing (4) subject to (9) by recognizing (7) but ignoring the impact on P_{t+1} yields the following set of first-order conditions with λ_t denoting the shadow price,

$$\frac{1}{c_t^i} = \lambda_t^i, \quad (34)$$

$$\frac{\gamma}{n_t^i} = \lambda_t^i [(1 - \tau_t) w_t h_t^i z + (1 - \tau_t) w_t \bar{h}_t e_t^i + b_t^i + d_t^i], \quad (35)$$

$$\frac{\gamma \beta \eta}{e_t^i + \varepsilon} = \lambda_t^i (1 - \tau_t) w_t \bar{h}_t n_t^i, \quad (36)$$

$$\frac{\beta}{b_t^i} = \lambda_t^i n_t^i, \quad (37)$$

$$\frac{\gamma \frac{\partial \pi_t^i}{\partial d_t^i}}{\pi_t^i} = \lambda_t^i n_t^i. \quad (38)$$

Equating (36) and (35) yields

$$e_t^i = \frac{\beta \eta}{1 - \beta \eta} \left(z x_t^i + \frac{b_t^i + d_t^i}{(1 - \tau_t) w_t \bar{h}_t} \right) - \frac{\varepsilon}{1 - \beta \eta}. \quad (39)$$

Equating (37) and (35) and recognizing that $x_t^i = \frac{h_t^i}{h_t}$ yields together with (39)

$$\frac{\rho}{b_t^i} = \frac{\gamma}{(1 - \tau_t) w_t h_t^i \left(z + \frac{e_t^i}{x_t^i} + \frac{b_t^i + d_t^i}{(1 - \tau_t) w_t h_t^i} \right)} \quad (40)$$

$$\Rightarrow b_t^i = \frac{\delta}{\gamma(1 - \beta \eta) - \rho} \left[\left(z - \frac{\varepsilon}{x_t^i} \right) (1 - \tau_t) w_t h_t^i + d_t^i \right]. \quad (41)$$

Equating (38) and (35) using (39) and (41) yields

$$d_t^i = \frac{\gamma \bar{\pi}_t \left(z - \frac{\varepsilon}{x_t^i} \right)}{\gamma(1 - \beta \eta - \bar{\pi}_t) - \rho} \hat{w}_t^i h_t^i, \quad (42)$$

which implies with (41) and (41)

$$b_t^i = \frac{\rho \left(z - \frac{\varepsilon}{x_t^i} \right)}{\gamma(1 - \beta \eta - \bar{\pi}_t) - \rho} \hat{w}_t^i h_t^i, \quad (43)$$

$$e_t^i = \frac{\beta \eta z x_t^i - (\gamma(1 - \bar{\pi}_t) - \rho) \varepsilon}{\gamma(1 - \beta \eta - \bar{\pi}_t) - \rho}. \quad (44)$$

Proposition 1

Item (ii):

Define the production function as $Y_t = AK_t^\alpha(\nu_t H_t)^{1-\alpha}$ with ν_t representing the share of human capital devoted to the production sector, such that MBT writes as

$$MBT = \frac{\mu b_3 AK_t^\alpha (\nu_t H_t)^{1-\alpha}}{H_t \left[(1-b_1)P_t + \frac{(b_2-b_3\tau_t)AK_t^\alpha (\nu_t H_t)^{1-\alpha} + b_4 \mathcal{N}_t}{H_t} \right]} \quad (45)$$

$$\Rightarrow MBT = \frac{\overbrace{\mu b_3 A \nu_t^{1-\alpha}}^{\Lambda_1}}{\underbrace{[(1-b_1)P_t(H_t/K_t)^\alpha]}_{\Lambda_2} + \underbrace{(b_2-b_3\tau_t)A\nu_t^{1-\alpha}}_{\Lambda_3} + b_4 \mathcal{N}_t / (K_t^\alpha H_t^{1-\alpha})}. \quad (46)$$

Apparently,

$$\lim_{K_t \rightarrow 0} MBT_t = \lim_{H_t \rightarrow 0} MBT_t = 0, \quad (47)$$

$$\lim_{H_t \rightarrow \infty} MBT_t = 0, \quad (48)$$

$$\lim_{K_t \rightarrow \infty} MBT_t = \frac{\Lambda_1}{\Lambda_3} > 0. \quad (49)$$

From (47) it follows Item (ii)a).

Item (iii)b) follows from

$$\frac{\partial MBT_t}{\partial K_t} = \frac{\Lambda_1 \left(\frac{\Lambda_2 \alpha P_t H_t^\alpha}{K_t^{1+\alpha}} + \frac{b_4 \alpha \mathcal{N}_t}{K_t^{1+\alpha} H_t^{1-\alpha}} \right)}{\left(\frac{\Lambda_2 P_t H_t^\alpha}{K_t^\alpha} + \Lambda_3 + \frac{b_4 \mathcal{N}_t}{K_t^\alpha H_t^{1-\alpha}} \right)^2} > 0, \quad (50)$$

(47) and (49).

Item (ii)c) follows from

$$\frac{\partial MBT_t}{\partial H_t} = - \frac{\Lambda_1 \left(\frac{\Lambda_2 \alpha P_t H_t^\alpha}{K_t^\alpha H_t} - \frac{b_4 (1-\alpha) \mathcal{N}_t}{K_t^\alpha H_t^{2-\alpha}} \right)}{\left(\frac{\Lambda_2 P_t H_t^\alpha}{K_t^\alpha} + \Lambda_3 + \frac{b_4 \mathcal{N}_t}{K_t^\alpha H_t^{1-\alpha}} \right)^2} \begin{matrix} \geq \\ \leq \end{matrix} 0, \quad (51)$$

with $\frac{\partial MBT_t}{\partial H_t} = 0$ at $\tilde{H} = \frac{(1-\alpha)b_4 \mathcal{N}_t}{\Lambda_2 \alpha P_t} > 0$. Then it follows immediately in light of (47) and

(48) that MBT is hump-shaped in H_t .