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Preliminary and Incomplete

# Corporate Cash Hoarding in a Model with Liquidity Constraints \*

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## Abstract

This paper studies the role of uncertainty in the corporate cash hoarding puzzle. The baseline model is a stochastic neoclassical growth model featuring idiosyncratic and uninsurable productivity shocks and a cash-in-advance constraint on new investments on the individual firm level. Individual agents' choices regarding cash holdings are analyzed. After a wealth threshold is reached, the cash-in-advance constraint ceases to have an effect on the agent's behavior. The resulting aggregate cash holdings of households are non-optimal compared to the complete markets solution and aggregate excess cash increases with uncertainty. Aggregate consumption is also higher, but the added volatility of consumption decreases lifetime utility. Since cash holdings are usually managed by the financial sector, the results suggest a link between firm level risk and the behavior of the banking system.

**Keywords:** Cash-in-advance constraint, idiosyncratic risk, heterogeneous agents, corporate cash hoarding

**JEL Classification:** C63, E21, E41, D81

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# 1 Introduction

Although firms are commonly modeled as maximizing their expected lifetime profits by choosing the input factors of production optimally (Woodford, 2002), these investments are often not constrained by any liquidity considerations of the firm, which implies perfect trust in the future settlement of suppliers' advance. In reality, however, firms carefully manage their cash holdings (and capital structure) to insure adequate cash holdings at every point: liquidity holdings below a certain level will prevent firms from meeting ongoing expenses and, possibly worse, prohibit profitable investments from being undertaken (unless the firm engages in restrictively expensive last minute lines of credit). Too high a cash balance is problematic in its own terms, since the funds could be either spent more profitably by investing in assets necessary for generating future revenue or paid out as dividends to the owners of the firm.

This model studies an agent that chooses her cash balance to balance her motive of profitability with her responsiveness to productivity shocks. The circumstance of a prohibitive minimum cash balance is modeled as a Clower, or cash-in-advance (CIA), constraint. The constraining variable is chosen in the preceding period and this establishes an upper bound for certain expenses, which may be binding, depending on the current choice of those expenses. Establishing a value for this upper bound, which is decided endogenously, is simple in the case of certainty, because it merely has to cover predetermined expenses. In the case of uncertainty, however, the optimal upper bound depends on the expectations of expenses, which depend on the realization of the productivity shock. There is no insurance mechanism apart from self insurance through increased wealth holdings. The non-linear nature of the CIA constraint rules out perturbation techniques. Also, since there is only one law of motion for two state variables, I need to define a grid of possible values for one state variable. This analysis therefore uses value function iteration to solve the model. I receive a policy function for cash holdings that varies with the amount of uncertainty, increasing excess holdings in more risky environments.

The ratio of cash and short term investments to total assets of publicly traded non-financial non-utility US corporations has on average increased from less than 6% in 1990 to more than 12% in 2011 (Bates, Kahle and Stulz, 2009), see Figure 1. There are many reasons for this, ranging from tax reasons to precautionary savings (Sanchez and Yurdugal, 2012). The precautionary motive gains additional weight when considering the answers to a survey of Chief Financial Officers of corporations globally, who state cash flow shortfalls and uncertainty about future investment opportunities among the most relevant reasons for excess cash holdings (Lins, Servaes and Tufano, 2010). In the same survey, the CFOs report that internal cash balances can be cheaper than accessing outside sources of funds. Furthermore, McVanel and Perevalov (2008) find a steady rise in the cash flow variability of Canadian firms from the 1980s to the mid-2000s, a development that tracks the rise of excess cash on company balance sheets.

This is not a problem per se, since cash and short term investments are usually held by financial intermediaries, who will channel the funds to other likely profitable

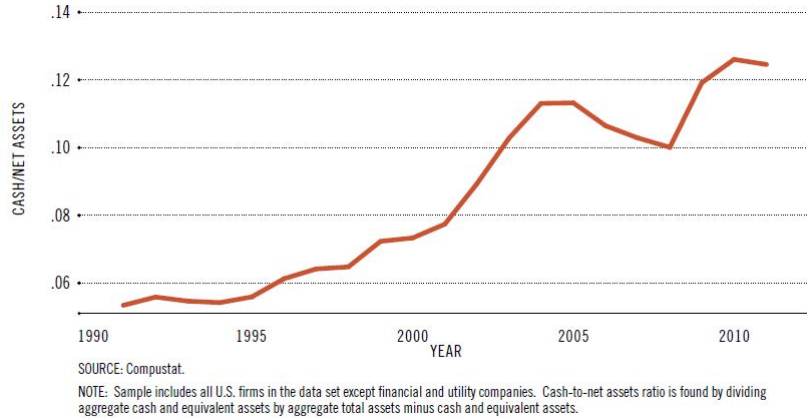


Figure 1: Ratio of Cash to Net Assets. Source: Sachnez and Yurdugal, 2012.

ventures. This allocation of funds increases the interconnectivity between the real and financial sector, however, and may propagate financial risk to the real economy (Gertler and Kiyotaki, 2010). In addition, since monetary policy affects the real economy through the financial sector, different degrees of connectivity may have varying effects on the real economy.

The paper is structured as follows. In section 2 the individual agent’s problem is presented. The steady state and dynamics are discussed. Section 3 explains the choice of parameters and explains some computational issues. Section 4 presents the global policy functions and aggregates individual decisions to a full distribution of agents. The effects of the CIA constraint and the effects of idiosyncratic risk on aggregate variables are decomposed, giving rise to two different kinds of precautionary savings. The introduction of a financial sector to intermediate deposits between cash rich and cash poor households is discussed in Section 5. The last section concludes.

## 2 The Individual Agent’s Problem

In order to study the role of uncertainty on corporate cash balance sheets, we will build a model of a household that operates a firm. The household behaves like the manager of a firm who needs to invest into capital for the production process to produce consumption goods, which provide her with utility. In order to invest into the production process, she needs to hold a liquid balance to finance the capital expenditure. The connection with the real world is that companies have to continually pay dividends (consumption in this model) that they receive though their production processes. Investing into the production process takes time, which is why capital only becomes productive with a one period lag. Physical capital depreciates and has to be replaced. The replacement is done through investment, which is costly. The investment can ideally be financed through equity or debt issuance. However, profitable investment opportunities often fade quickly and changes to the capital structure can take a long time and are costly (see Bates,

Kahlte and Stulz, 2009). Once the capital is in place, the productivity of the production process can vary. Depending on the realization of productivity, the household is left with more or less resources for either consumption or further investment. The uncertainty about the production process therefore drives the cash balance of the firm.

## 2.1 The infinitely lived household

Consider an infinitely lived household that maximizes discounted utility, which is strictly increasing and strictly concave in consumption:

$$\max_{c_t, k_t, D_t} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \quad (1)$$

The household's behavior is subject to a budget constraint and a CIA constraint:

$$c_t + k_t + D_t = A_t f(k_{t-1}) + (1 - \delta)k_{t-1} + D_{t-1} \quad \{\lambda_t\} \quad (2)$$

$$k_t \leq D_{t-1} + (1 - \delta)k_{t-1} \quad \{\mu_t\} \quad (3)$$

$$A_t = \text{stochastic process} \quad (4)$$

where  $c$  is consumption,  $k$  is the capital stock,  $D$  is deposits at a bank which we will treat as equivalent to cash,  $A$  is the level of technology,  $f$  is the production function,  $\delta$  is the depreciation rate on capital.

The household can either consume or choose to invest into capital or deposit holdings. By investing into capital, the household receives a return from production as well as undepreciated capital the following period. In addition, at the end of the current period the agent can carry over any wealth into the next period by depositing it at a bank at no interest. A CIA constraint motivates the agent to do this: Every period, the maximal amount of investments into capital can not exceed the amount of deposit holdings at the bank. The intuition for this is that the household needs a basket of different goods for investment that it does not itself produce. This basket contains goods that may not necessarily be traded for the household's own produce this period, but instead can be bought using a liquid asset in the form of bank deposits. The household can always sell all of its produce on the market and receive deposits in return. Note that deposits are a real good, which can also be consumed in case it is not used for investment purposes.

Technology  $A$  underlies a stochastic process. The revenue generated from production can therefore not be determined in advance. There are two interpretations for this. In the case of a persistent shock process, technology can be taken as profitable investments. This corresponds to an acquisition of one business by another. However, as in Hopenhayn and Rogerson (1993), this can also be interpreted as a consumer taste shock that shifts the demand curve. This is also true in case of no persistence.

Letting  $\lambda, \mu$  be the Kuhn-Tucker multipliers on the constraints, the resulting First-order conditions for consumption, capital and deposits are, respectively <sup>1</sup>,

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<sup>1</sup>The Lagrangian is

$$L = u(c_t) - \lambda_t [c_t + k_t - (1 - \delta)k_{t-1} + D_t - A_t f(k_{t-1}) - D_{t-1}] - \mu_t [k_t - (1 - \delta)k_{t-1} - D_{t-1}] \\ + \beta \{u(c_{t+1}) - \lambda_{t+1} [c_{t+1} + k_{t+1} - (1 - \delta)k_t + D_{t+1} - A_{t+1} f(k_t) - D_t] - \mu_{t+1} [k_{t+1} - (1 - \delta)k_t - D_t]\}$$

$$U'(c_t) = \lambda_t \quad (5)$$

$$\lambda_t + \mu_t = \beta E_t \{ \lambda_{t+1} [A_{t+1} f'(k_t) + 1 - \delta] + \mu_{t+1} (1 - \delta) \} \quad (6)$$

$$\lambda_t = \beta E_t \{ \lambda_{t+1} + \mu_{t+1} \}. \quad (7)$$

$$[D_{t-1} - k_t + (1 - \delta)k_{t-1}] \mu_t = 0, \quad D_{t-1} - k_t + (1 - \delta)k_{t-1} \geq 0, \quad \mu_t \geq 0 \quad (8)$$

Equation 5 states that the marginal utility of consumption equals the marginal utility of wealth  $\lambda_t$ . Equation 7 states that capital should be invested up until the combined cost of having an additional unit of wealth  $\lambda_t$  and at the same time an additional unit of liquidity bearing assets  $\mu_t$ , which is made up of deposits and undepreciated capital from the previous period, has to be equal to the combined value of the marginal capital return, which is the marginal product from production including next-period's undepreciated capital, weighted by tomorrow's marginal utility plus the future liquidity services the undepreciated future capital will deliver. Equation 7 states that the cost of investing a unit of wealth into deposits today has to equal the value of having that additional unit deliver wealth or liquidity services tomorrow.

## 2.2 The Deterministic Steady State

In the deterministic steady state, the stochastic process regarding technology is turned off and the steady state values for capital, consumption and deposits can be solved from, respectively,

$$\begin{aligned} f'(k) &= \frac{1}{\beta} \left( \frac{1}{\beta} + \delta - 1 \right) \\ c &= f(k) - \delta k \\ D &= \delta k. \end{aligned}$$

Compared to the neoclassical growth model without a CIA constraint, the marginal product of capital is higher by  $\frac{1}{\beta}$ , because the additional liquidity services from cash are costly and have to be financed one period in advance as can be seen in the three period model, where production in period 3 depended on the choice of deposit holdings in period 1.

The CIA constraint is binding, since the first order condition with respect to deposits (7) would otherwise lead to a contradiction: a non-binding CIA constraint implies  $\mu = 0$ , which results in  $\beta = 1$ . This means that the household holds a deposit balance equal to the replacement investment each period due to depreciation. Since the marginal product of capital dominates the marginal product of cash, which does not carry any interest, additional deposit holdings are never profitable. A smaller deposit balance would prevent the household from maintaining its steady state capital level.

### 2.3 Dynamics in the Infinite Time Model

Adding stochastics changes the rational regarding asset holdings. The optimal asset level rises with uncertainty. Assets in the model are comprised of capital and deposits. Interpolating from the two period to the infinite period optimization, the amount of capital holdings increases with risk.

Additionally, the CIA constraint introduces a further motive for precautionary savings as not to hit the liquidity constraint. The household will always prefer to invest into capital, as long as the marginal productivity of capital is above the marginal productivity of deposits, which does not carry any interest and is always 0. However, a positive shock may induce the household to save more deposits as to realize higher capital investments in the following period. Optimal deposit holdings depend on the exact nature of the shock.

The additional CIA constraint changes the household's precautionary savings motive. Not only does self-insurance against adverse shocks play a role. In addition, hitting the CIA constraint is costly in utility terms as well. The household may have an incentive to insure against hitting the liquidity constraint, too.

When no uncertainty is present and the inequality constraint in the model description always binds we can determine deposit holdings each period exactly from the first order condition for next period's optimal capital, which together with our current period capital gives us our deposit holdings exactly. When we do not know whether the inequality constraint is binding or not, because next period's optimal capital can not yet be determined, since the shock has not realized, finding the optimal value for deposit holdings has to be done numerically.

## 3 Model Specification, Parametrization, and Computation

In this section, we will specify the model further and choose parameter values for solving the model. The computational method and the kind of aggregation is discussed.

Household utility is of isoelastic utility form  $U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$  with a coefficient of relative risk aversion  $\gamma$ . The production function is assumed to be concave, i.e.  $f(k) = k^\alpha$  and specified by output elasticity  $\alpha$ . The stochastic process in this model is  $A_t = \bar{A}e^{\rho \log A_{t-1} + \varepsilon_t}$ , where  $\bar{A}$  is the steady state value of technology and  $\varepsilon$  is an idiosyncratic shock to technology that follows a white noise process,  $\varepsilon \sim N(0, \sigma^2)$ .

The parameter values are mostly taken from Aiyagari (1994), who specifies the model period as one year. The utility discount factor  $\beta$  is 0.96. The output elasticity  $\alpha$  equals 0.36. Depreciation  $\delta$  is 0.08. As a coefficient of relative risk aversion  $\gamma$ , I choose 3 out of the ones offered by Aiyagari.

For the shock process, I am choosing no autocorrelation for computational reasons, explained further below. Choosing a standard deviation for the shock process is tricky. In the literature, there are various explanations for standard deviations for idiosyncratic shocks to labor and for aggregate technology shocks. It is harder to find a meaningful parameter for idiosyncratic shocks to a firm's technology in a simple framework like

this. Pesaran and Xu (2011) estimate a  $\sigma = .041$  and Covas (2005) chooses  $\sigma = 0.4$ . Covas has a similarly basic model with a borrowing constraint albeit without a CIA constraint. He performs a sensitivity analysis by varying the standard deviation and persistence and finds a sizeable difference between aggregate capital in a model with uninsurable productivity risk and complete markets depending on the parametrization. With a CIA constraint in place, this effect is likely to vary since not only is there a borrowing constraint in place but an additional constraint on another dimension. I choose  $\sigma = 0.4$  and will perform a sensitivity analysis below.

### *Computation*

The computational method is a value function iteration that was performed using Matlab. The algorithm is standard and I only highlight some parameter values and one divergence from my own model.

Although the model is specified in terms of capital and deposits, both variables play the role of savings devices. Instead of evaluating the model at absolute values of these variables, I have rewritten the model such that the first variable is combined wealth, equal to capital and deposits, and the second variable is the percentage of wealth that is held in deposits.

$$k_t + D_t \equiv x_t$$

$$\frac{D_t}{k_t + D_t} \equiv d_t$$

This is done for computational reasons: In VFI it is a problem when the derived policy functions hit the grid boundaries since I can not be sure whether the agent would have preferred a value outside the grid. The grid is usually chosen in such a way that most of the dynamics take place towards the middle. When choosing absolute values for both variables, the method evaluates extreme areas as well: those with low and high values for both variables at the grid end points. Since an agent with a high level of combined capital and deposits would choose a very high capital value that is not within the grid bounds, this may distort the policy functions, although an agent would never visit these areas given a steady state and a stochastic process. Including this higher capital value on a successive larger grid exaggerates the problem, since the reaction to this grid point including deposits is yet higher. For absolute values there is no upper bound. Instead, the absolute variable wealth can be distributed optimally by a relative variable that is bound between 0% and 100% and if these grid end point are reached, they are a naturally occurring extreme.

The shock process is modeled using Tauchen's (1986) method.

### *Aggregation*

The economy is populated with a continuum of households all of the type discussed in Section 2. Denoting individual variables with  $j \in [0,1]$ , we can now write aggregate variables as



$$\begin{aligned}
c^A &= \int_0^1 c^j dj \\
k^A &= \int_0^1 k^j dj \\
D^A &= \int_0^1 D^j dj \\
y^A &= \int_0^1 (k^j)^\alpha dj
\end{aligned}$$

Importantly, aggregate production consist of adding all the individual production functions, instead of assuming aggregate capital as the input in the production function.

Every period, the household's technology is hit with a shock and the household reacts to it depending on its state. Since the grids are discrete, eventually all possible state combinations are visited by the agent. Equivalently, we can look at a distribution of agents, starting with the total mass concentrated on one state, and look at the reaction to each possible shock. Since the shock realizations are on a discrete grid, the first such reaction will be control vectors for wealth and deposit percentage of a size equal to or less than the number of shock realizations. Each of these control vectors can be regarded as possible states in the following period and again hit with all possible realizations of the shock. Since all grids are discretized, the maximal combination of states and shocks is the number of grid points of both state variables multiplied with each other and with the number of realizations of the idiosyncratic shock. Eventually, if further shocks only change individual agents' reaction but not the distribution across state combinations, we arrive at a stationary distribution. Fortunately, this is the case for this model<sup>2</sup>. This stationary distribution from idiosyncratic shocks can then be analyzed in comparison to the complete markets case.

Since the grids in this model are discretized, I am receiving a stationary distribution that has mass concentrated on specific state combinations. Since the discretization is only a computational tool to solve the policy functions and the policy functions would themselves be continuous, I will present the resulting stationary distribution after fitting a probability distribution to the data. I am using the MATLAB command `fitdist` with a normal kernel smoother along both dimensions and a window size of 1 for th wealth dimension and a window size of 0.05 for deposit percentage.

Aggregation for comparing the results is done using the discretized results. Since the distribution is stationary, I simply aggregate the state variables capital and deposits with regard to their mass. For consumption the realization of the shock matters. I therefore aggregate over their mass conditional on the shock they receive.

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<sup>2</sup>I have yet to prove whether this is the case in general and for which parameter space

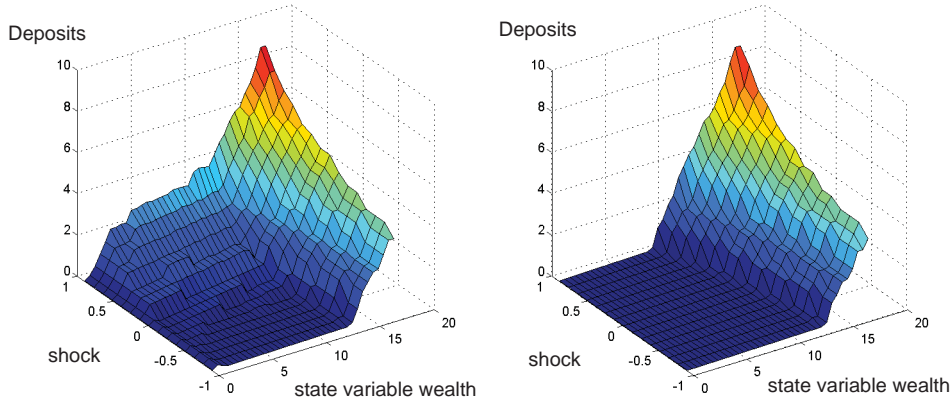


Figure 2: Policy function for Deposits, constrained (left) and unconstrained

$$\begin{aligned}
 c^A &= \sum_0^1 c^j dj \\
 k^A &= \int_0^1 k^j dj \\
 D^A &= \int_0^1 D^j dj \\
 y^A &= \int_0^1 (k^j)^\alpha dj
 \end{aligned}$$

It is important to note that every agent produces using their own capital holdings as input for the production function. Therefore each agent will have a different marginal productivity, depending both on individual capital holdings and idiosyncratic technology shock.

## 4 Results

### 4.1 Policy Functions

The household reacts to the current technology shock  $\varepsilon_t$  and state variables  $x_{t-1}$  and  $d_{t-1}$  by choosing current consumption  $c_t$ , current wealth  $x_t$  and percentage invested in deposits  $d_t$ . Although all policy functions are highly non-linear, the key result is the difference in the policy function for deposits in the case of a constrained vs an unconstrained agent.

For wealthy firms, the value of additional consumption and additional production does not exceed the value of additional deposits anymore. As can be seen from figure 2,

the behavior of a constrained wealthy firm is identical to the behavior of an unconstrained wealthy firm. Wealthy firms are those that own state variable wealth of more than 12. Once wealth exceeds a boundary, the CIA constraint does not play any role anymore. In both cases, investments into deposits increase with the size of the shock and the amount of wealth.

This can be interpreted using the first order conditions derived earlier [**TO BE CONTINUED**]

Wealthy firms value the certain return of cash over the uncertain but possibly higher return of additional production.

For less wealthy firms, the policy function for deposits increases mostly in the size of the shock: higher available resources lead to additional investments into deposits because the firm insures against hitting the CIA constraint. This is clearly not a motivation in the unconstrained case: all available resources will be either consumed or invested into production, because the marginal product of capital exceeds the marginal product of deposits, and uncertain returns are more valuable than uncertain ones.

## 4.2 Aggregation

The heterogeneous agent economy converges to a stationary distribution after some time. As can be seen from figure 3 the distribution of agents is spread across the wealth and deposit percentage dimensions. Two things stand out. First, the distribution is bound by zero deposit holdings to the north west of the figure. Since agents can not borrow, the distribution can not spread out across the zero threshold. Without a borrowing constraint we might be able to see the distribution extend across the threshold.

This is a problem for many households at the zero deposit threshold because zero deposit holdings mean that they can not make any net new capital expenditures beyond their undepreciated capital. Even if a favorable shock occurs, they will need to allocate the additional return from production to deposits first and can only invest into capital the following period. The difference between the possible marginal return on capital and the marginal return on deposits is a loss to the household that can not be regained.

At the same time, there are many agents that do not need all their deposit holdings, because their realized investment is smaller. By investing into deposits they are insuring against the case that they are going to be constrained by the CIA constraint and lose the difference between marginal productivity of capital and deposits themselves. All agents could therefore benefit by allocating some of the deposits of the deposit-rich agents to the deposit-poor agents that would like to invest. The additional marginal productivity could be split in some way that makes all agents better off. Such a financial intermediary does not exist in this model.

The second observation is the distribution of agents that is continually hit with positive shocks and therefore keeps accumulating wealth. It is visible in figure 3 as the mass branching off from the probability distribution to the north east. Wealth holdings increase and the agents allocate more of their wealth into deposits. These are the agents that are holding wealth higher than 12 in this model. Their value from liquidity services is greater than their marginal productivity of additional capital or the additional marginal

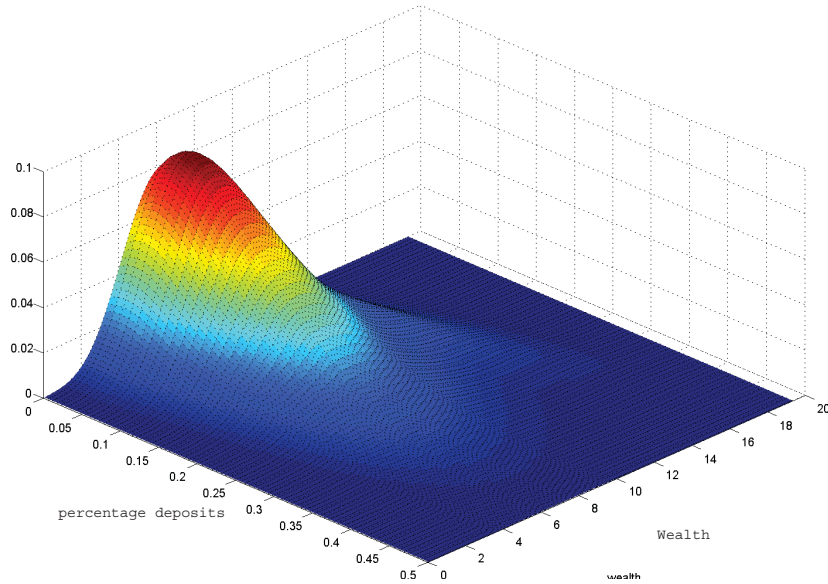


Figure 3: Stationary distribution over states wealth and deposit percentage

utility from consumption, and this value increases quickly, inducing ever higher deposit holdings.

This fact can be observed well in figure 4, a Lorenz curve for deposit holdings. The distribution of deposit holdings is distributed very unequally across the population. A financial intermediary could allocate this liquidity from deposit-rich to deposit-poor households to meet the CIA constraint.

### 4.3 Decomposition into Liquidity and Risk Motives

We have introduced two mechanisms into the model that cause precautionary savings of two different types. One is the well known precautionary motive for wealth holdings to insure against idiosyncratic risk. The other one is precautionary savings against hitting the liquidity constraint and not being able to invest optimally into capital. I will call these two different precautionary motives PS1 and PS2 after Xu (1995) who studied precautionary savings in the income fluctuation problem from idiosyncratic risk and a borrowing constraint. By solving a model with and without CIA constraint (but borrowing constraints) I can compare the values of the aggregate economy with their steady state counterparts. Precautionary savings are then defined as:

$$\begin{aligned}
 PS1 &= wealth^{noCIA} - wealth^{SS} \\
 PS2 &= wealth^{CIA} - wealth^{noCIA}
 \end{aligned}$$

Table 1 compares aggregate values from the stationary distribution with their steady

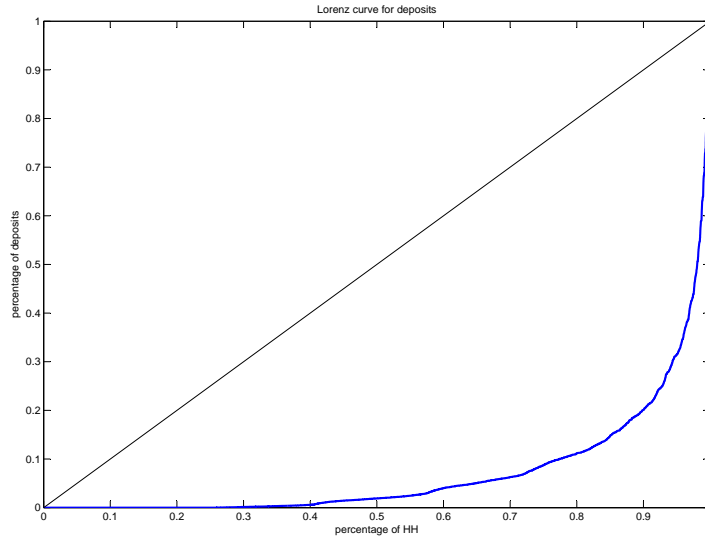


Figure 4: Lorenz curve for deposits

	Consumption	Capital	Deposits	Wealth
Steady State	1.39	5.11	0	5.11
Idiosyncratic Risk	3.2	14.16	0.21	14.37
Deviation	230%	277%	NA	281%

Table 1: Comparison between steady state and aggregate values without CIA constraint

state counterparts in a model of idiosyncratic risk but no CIA constraint. Agents insure mostly by accumulating more capital up until the point at which the marginal productivity of capital becomes less than the marginal productivity of deposits. This is indeed the case in this model under our current parameterization. The aggregate values for the economy deviate significantly from their steady state values. This deviation is obviously related to the standard deviation of the shock.

Table 2 compares steady state values of a model with idiosyncratic risk and a CIA constraint with their aggregate realization. The difference in the steady states is only in deposits, since the agent in the CIA constraint model needs deposits each period that cover the investment expenses for depreciated capital. The aggregate realizations are even higher, which is unsurprising given that the households have to insure along two dimensions. Especially of interest is the fact that firms hoard enormous amounts of the liquid asset without even planning on spending it. One motive is the uncertainty about investment opportunities. Secondly, however, since uncertainty already increases their capital holdings, firms' marginal productivity of capital is further reduced. This reduction brings the low marginal productivity of deposits less unattractive. The result

	Consumption	Capital	Deposits	Wealth
Steady State	1.39	5.11	0.41	5.52
CIA constraint	6.36	26.76	2.79	29.55
Deviation	458%	524%	680%	535%

Table 2: Comparison between steady state and aggregate values with CIA constraint

CRRRA	Consumption	Capital	Deposits	Wealth
Steady State	1.39	5.11	0.41	5.52
0.5	1.57	5.91	0.55	6.46
1	1.57	6.04	0.58	6.62
2	3.16	12.7	1.26	13.95
3	6.36	26.76	2.79	29.55

Table 3: Comparison of aggregate values with CIA constraint for different values of  $\gamma$

is a further investment into these assets. Excess cash hoarding is the result.

Curiously, aggregate consumption is higher in both models than in the steady state. This may be surprising, since the agents only value consumption in utility and a higher aggregate level in a stationary distribution would mean that risk and even a constraint increase utility. This is only true of the aggregate level, however. In time, agents in both models visit all of the different levels of consumption eventually. This fluctuation in consumption is costly: Utility of an (aggregate) agent living in the steady state, the idiosyncratic but unconstrained model, and the CIA constrained model are, respectively -0.26, -0.42 and -0.85. Agents would therefore always prefer the steady state to the idiosyncratic and to the constrained model.

#### 4.4 Sensitivity Analysis

In order to study how sensitive the model results are to parameter values, I am showing key results from different parameterizations. Most of the parameters are directly taken from Aiyagari (1994). However, no counter part from that paper exists for a technology shock. In addition, the economic interpretation of this shock process can differ. I therefore present aggregate variables of the economy for different values of the standard deviation as shown in Figure 5.

In addition, since this study is concerned with precautionary savings, a variation of the coefficient of relative risk aversion is in order. Table 3 shows values for aggregate variables of the economy for different values of the coefficients of relative risk aversion. The higher the coefficient of relative risk aversion the more negatively a household is affected by changing consumption. Because volatility in consumption is bad, a household will self insure against negative consumption outcomes by having a higher consumption level in expectations.

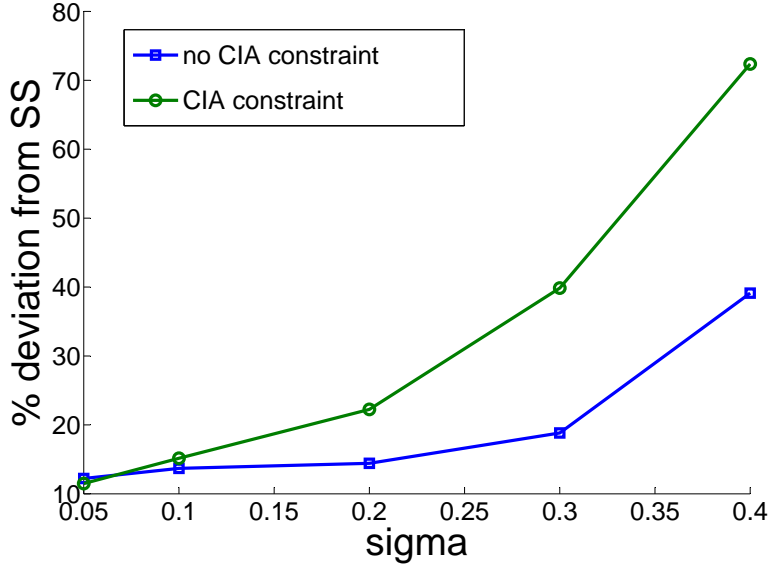


Figure 5: Deviation of deposits from steady state value for different standard deviation of the shock

## 5 Introducing a Financial Sector

As explained above, households need liquid assets to finance investments. These liquid assets take the form of deposits, which are simply created by an exogenously assumed financial sector. This sector holds the liquid assets for the households and trades them for investments when so demanded. In reality, the banks are themselves economic entities that maximize profits by finding profitable investment opportunities. Investment for them takes the form of loans into businesses that are cash constrained. This model features many agents that are cash constrained, namely those whose CIA constraint is binding. We will now discuss whether and how the financial sector can distribute funds from households with available funds to households with demand for available funds.

What are available funds? Available funds are all deposits that sit idly on the household's balance sheet, i.e. that have not been used for investment or consumption this period and are instead rolled over into the next period. We can calculate this amount for each household but need to first introduce another variable, excess cash, which is defined as the cash not used for investments:

$$xsc_t \equiv \min\{D_{t-1} + (1 - \delta)k_{t-1} - k_t, D_{t-1}\}$$

The first argument in the min-operator is the residual of the CIA constraint, which is necessarily bigger or equal to 0. The second argument is introduced to allow for the case that the household invests less than his depreciated capital, i.e. if it dismantles and

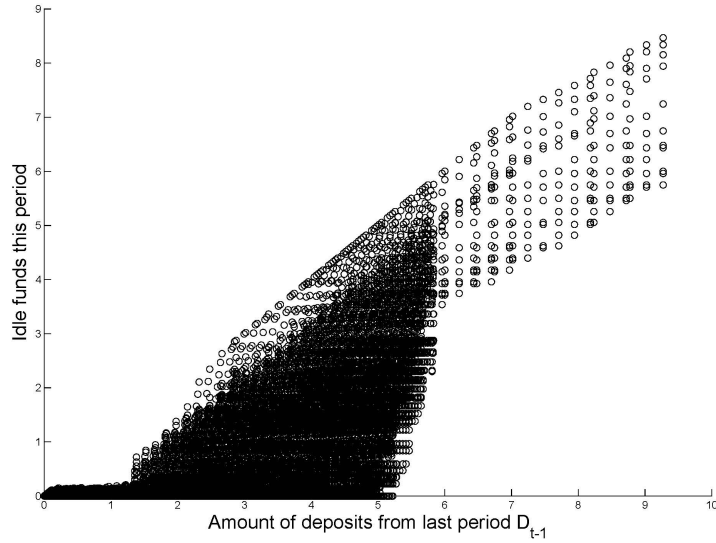


Figure 6: Scatter Plot Deposits Last Period  $D_{t-1}$  to Available Funds

consumes or sells some of its capital. Excess cash will not rise in this case. The funds are only idle if the household does not consume this excess cash and instead transfers it to the next period in the form of new deposits. We can therefore define the available funds this period as

$$funds_t \equiv \min\{xsc_t, D_t\}.$$

These funds are not being used at all and remain in the form of deposits throughout the whole period. The reason being that households value the future liquidity services of these funds more than they value additional consumption or additional investments. Who are the households with the additional funds? Figure 6 shows the amount of available funds all the different groups of deposit holders have. Unsurprisingly, the households that carried over more deposits from the last period tend to have more available funds. However, there is high heterogeneity among the households that are in the middle of the deposit distribution. Those that have had a positive technology shock use their production windfall for consumption and spend all of their deposits on investments and are bound by the constraint. The households in the middle of the deposit distribution that have had an adverse technology shock consume and invest. Interestingly, none of the households to the very right use up their deposits for investments and all of them are thus left with available funds. They are never bound by the CIA constraint and their deposit taking motive.

A financial intermediary could lend these funds to households that are constrained by the CIA constraint for a fee. The question is how the lending decisions would take place.



Since households operate their own production, there is a distribution of capital holdings and a resulting distribution of marginal productivities. It would be socially most efficient to deploy the deposits with the households that have the highest expected marginal productivity, i.e. the households with the lowest capital holdings, since the additional capital returns the highest additional marginal productivity. However, the households with low capital values might want to consume additional funds before investing into additional capital, e.g. because their marginal utility of consumption is very high for very poor households.

The Kuhn Tucker multiplier on the CIA constraint,  $\mu$ , implies the value of liquidity services. We can calculate this value and deploy the available funds with the households that most value liquidity. The lending and borrowing rationale has to be incorporated into the households maximization problem and the exact functioning of the interest rate for lending has to be discussed next. I am developing this section at the moment.

## 6 Preliminary Conclusion

Firms faced with idiosyncratic shocks regarding their cash flows that cannot be fully insured against and that have to react to investment opportunities on the spot and without outside financing amass liquid holdings well in excess of the amount they actually are likely to spend in the next period.

Non wealthy firms invest into deposits in excess of their expected investment next period because an additional unit in deposits tomorrow will enable an additional investment into capital tomorrow. This is valuable, if the twice discounted marginal return on capital in two periods time is higher than the marginal return on capital tomorrow.

Wealthy firms invest into deposits because they value certain returns over risky returns. Importantly, wealthy firms' policy function for deposits is unaffected by a CIA constraint. The liquidity motive does not play a role.

Two additional mechanisms that drive higher cash holdings are at work. Firms increase their capital holdings because of their precautionary savings motive. This overinvestment into capital decreases average marginal productivity across the economy. Because the opportunity costs of deposits decline, the low marginal productivity of deposits becomes more attractive. Additionally, higher precautionary capital levels make higher replacement investments necessary. In order to finance these additional investments, precautionary deposits have to be held in advance.

Aggregate consumption in the case of uncertainty is higher than in the complete markets case. The reason for this is the high variability of productivity and the resulting volatility of marginal productivity. This volatile consumption, which is higher in the aggregate, is unfavorable to lifetime utility. Agents would prefer a less risky environment.

There exists a small number of deposit-rich and a high number of deposit-poor households. All households could benefit from redistributing deposits temporarily in order to allow deposit-poor households to reap high marginal productivity. However, no mechanism for this temporary reallocation of funds exists. A distribution of wealth and liquid asset holdings is the result. Introducing a financial intermediary may improve upon this

result.

The increase in aggregate variables, partly attributable to the choice of parameter values, has effects on the general equilibrium. Once endogenous interest rates and prices are introduced, the absolute level of deposit holdings will influence these macro variables and vice versa.

The high level of liquid assets, which is usually managed by the financial sector, increases the connection between the real and the financial sector. Idiosyncratic firm risk has a direct effect on funding opportunities for banks and other types of financial intermediaries. By modeling the financial sector explicitly, we can study the exact connection next.

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