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The Wage Effects of Job Polarization: Evidence from the Allocation of Talents

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Abstract This article studies the wage effects of job polarization on 27 year old male workers from the cohorts of the National Longitudinal Survey of Youth. Guided by a Roy model of occupational choice I compare workers who have characteristics that put them into high-, middle-, and low-skill occupations over the two cohorts. Results indicate that the relative wages of middle-skill occupation workers have dropped. The effect of job polarization on the overall wage distribution that is implied by the model explains the increase at the top of the actual distribution but it has difficulty matching the increase at the bottom.

Keywords: Job Polarization; Wage Inequality; Talent Allocation; Roy Model
JEL Classification Numbers: J23, J24, J31
1 Introduction

Over the last two decades, the labor market in the United States has experienced a profound polarization of employment. In particular, the aggregate share of jobs in middle-skill production and clerical occupations has declined by almost seven percentage points since the end of the 1980s while the share of jobs in high-skill professional and managerial occupations as well as low-skill services occupations has surged. This has coincided with a polarization of wages, whereby earnings in the middle of the wage distribution have stagnated or even fallen while earnings at the top and at the bottom have increased substantially (compare Acemoglu and Autor 2011 and see figures 1 and 2 for the data used in this paper).

Pioneering research by Autor, Levy, and Murnane (2003) and others has shown that the main driver of job polarization was a rapid improvement of computer technology which could replace the routine work that is intensively carried out in middle-skill occupations (“routine-biased technological change”). It thus seems natural to hypothesize that the polarization of wages was shaped by the same demand-side forces (e.g. Autor, Katz, and Kearney, 2006; Acemoglu and Autor, 2011; Autor and Dorn, 2013). Indeed, recent empirical studies have examined several aspects of the relationship between job polarization and workers’ wages (Autor and Dorn, 2013; Cortes, 2014; Dustmann, Ludsteck, and Schönberg, 2009; Mishel, Shierholz, and Schmitt, 2013).

The purpose of this paper is to build on the existing work and to employ a unified economic framework in order to estimate the wage effect of job polarization on specific groups of workers as well as its effect on the overall wage distribution. In particular, I use the theory and empirics of the Roy (1951) model to address the following questions: first, have the relative wages of workers in the middle-skill occupations declined as the demand-side explanation for job polarization predicts? Second, how have the wage rates paid for a constant unit of effective labor in the high-, middle-, and low-skill occupations changed with polarization? Third, what was the effect of job polarization on the overall wage distribution and, in particular, could it have generated the polarization of wages that we observe in the data?

In order to answer these questions, I construct two representative cross-sections of 27 years old male workers at the end of the 1980s and at the end of the 2000s from the two cohorts of the National Longitudinal Survey of Youth (NLSY79 and NLSY97). The

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2Note that throughout the paper I write “the wage effect of job polarization” as a shorthand for “the effect on wages of the declining relative demand for middle-skill occupations that leads to job polarization”.

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workers in the sample are born in 1957–1964 and 1980–1982 and they are 27 years old in 1984–1992 and 2007–2009, respectively. Although this is arguably a specific labor market group during a particular time period, the NLSY has substantial advantages over more commonly used data sets: it provides consistent, early-determined, and multidimensional measures of worker talents—such as mathematical, verbal, and mechanical test scores and risky behaviors—which predict occupational choices and wages. This allows me to study the wages of the kind of individuals who are more and less likely to work in the middle-skill compared to the high- and the low-skill occupations over these two decades of rapid job polarization.

The paper’s empirical strategy comparing the same kind of workers across two points in time is informed by the Roy model of occupational choice, which calls for such an approach. In the Roy model, workers have different skills in the high-, middle-, and low-skill occupations that are a function of underlying talents. Once technological change or international trade decrease the demand for labor in the middle-skill occupations, wage rates fall and workers move out of there. Hence we observe job polarization. The Roy model further says that to estimate the wage effects of job polarization one should not compare changes in wages across occupations—because the selection of skills in occupations changes together with the wage rates—but the (kind of) workers who started out across different occupations.

I start by analyzing the sorting of workers into occupations in the end of the 1980s using multinomial choice regressions in the NLSY79. I find that, conditional on the other talents, workers with high math talent are likely to choose the high-skill occupation, workers with high mechanical talent are likely to choose the middle-skill occupation, and workers with high verbal talent are likely to choose the high- or the low-skill occupation but not the middle. This indicates a systematic sorting of workers into occupations according to their relative talent endowments and it enables me to construct predicted probabilities for every worker in both datasets to enter the high-, middle-, and low-skill occupation in the end of the 1980s.

\[^3\]In fact the timing in my dataset is comparable to the 1988–2008 period considered separately in Acemoglu and Autor (2011). For example, refer to their figures 9 and 10.

\[^4\]While some recent papers argue that job polarization has been going on for much longer than the last couple of decades (Bárány and Siegel, 2013; Mishel, Shierholz, and Schmitt, 2013), most studies find it was strongest during the 1990s.

\[^5\]Using a model that is related to the Roy framework, Acemoglu and Autor (2011) have similar concerns about a simple comparison of wages across tasks: “[...] because the allocation of workers to tasks is endogenous, the wages paid to a set of workers previously performing a given task can fall even as the wages paid to the workers now performing that task rise [...]”.

\[^6\]In a recent paper, Speer (2014) conducts a related analysis of worker sorting into jobs. His results on the sorting of math, verbal, and mechanical talents into occupational tasks are similar to the ones described here.
I use these predicted probabilities to study how the wages of workers who were likely to choose to work in the three occupations in the end of the 1980s have changed over time. Effectively, this amounts to a wage regression on the occupational propensities interacted with a dummy for the period at the end of the 2000s. Under the assumption that the distribution of fundamental worker skills conditional on the detailed talents has been constant between the NLSY79 and the NLSY97, these regressions identify the changing returns over time to working in the high- and low-skill occupation versus the middle-skill occupation in the end of the 1980s.\footnote{The fact that the level and cross-correlation of talents have not changed between the NLSY79 and the NLSY97 supports the identification assumption.}

The results from the regressions on occupational propensities indicate that the wages of middle-skill occupation workers declined substantially over the polarization period. I find that the positive wage effect associated with a one percentage point higher propensity to work in the high- compared to the middle-skill occupation almost doubled from .31 to .60 percent. The negative wage effect associated with a one percentage point higher propensity to work in the low-skill occupation attenuated from $-1.65$ to $-0.95$ percent. Moreover, workers with a high propensity to enter the middle-skill occupations in the 1980s suffered even an absolute decline in their expected real wages. These findings are robust to controlling for absolute skill measures, such as educational attainment, which supports the idea that it is relative occupational skills rather than absolute skills whose returns have changed over time.

The above results answer the first question of the paper about how middle-skill workers have fared in terms of relative wages over time. In fact, so far no formal setup of the Roy framework was needed for the analysis. However, applying the Roy model becomes essential in order to answer the second question about the change in the equilibrium wage rates across occupations. This is because wages rates are not directly observed in the data and thus need to be inferred from within the model. To identify the wage rates empirically, I assume that log wages are additively separable into wage rates and skills and that the production function of talents into skills is fixed over time.

Under these assumptions, one can quite straightforwardly estimate the relative change in wage rates across occupations in the Roy model. My estimates indicate that the relative wage rate in the high- compared to the middle-skill occupation increased by 25 percent since the end of the 1980s and the relative wage rate in the low-skill occupation increased by 33 percent. The former rate is estimated relatively precisely while the latter is imprecise and not statistically significant. Nonetheless, overall the results suggest substantially declining relative wage rates for a constant quality of work in the middle-skill occupation.
occupations over time.

The wage rate estimates also help approaching the third question of the paper about the effect of job polarization on the overall wage distribution. I assign each worker in the NLSY79 the estimated change in wage rates according to his occupation. The resulting counterfactual distribution qualitatively matches the increase in wages at the top of the actual distribution compared to the middle. However, it does not reproduce much of the surge of wages in the bottom of the actual distribution.

Where does the asymmetry of rising wages rates for the high- and the low-skill occupations and a rise in the top of the counterfactual wage distribution but not in the bottom stem from? The reason is that low-skill occupation workers now move up in the wage distribution, which lifts not only the (low) quantiles where they started out but also the (middle) quantiles where they end up. The inverse happens for workers in middle-skill occupations but with the same effect on the wage distribution so that it becomes flatter in the lower half. However, the opposite happens at the top of the wage distribution: high-skill occupation workers, who are predominantly in the upper parts of the wage distribution now get another boost to their wages. This not only lifts their original quantiles but also the even higher quantiles that they end up in. These mechanical movements within the wage distribution may be part of the reason why in some countries and time periods we observe rapid job polarization but no “wage polarization”, that is no increase in relative wages at the bottom of the wage distribution.

This paper is related to recent studies that examine the relationship between job polarization and wages. First, Acemoglu and Autor (2011), Mishel, Shierholz, and Schmitt (2013), and Dustmann, Ludsteck, and Schönberg (2009) examine the common occurrence of job polarization and wage polarization across time periods and locations. They do not find that job polarization is generally accompanied by substantial wage polarization. Second, Autor and Dorn (2013) examine the employment and wage trends of local labor markets and find that local labor markets that were initially more specialized in routine tasks experienced a stronger polarization of occupational wages. Third, using longitudinal data, Cortes (2014) compares the wage profiles of workers who move out of the middle-skill occupations into the high- and low-skill occupations to those workers who remain behind. He finds that the movers experience stronger wage growth over the long

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8Another effect on the wage distribution within the Roy model that is omitted here is the wage effect of individuals leaving the middle-skill occupation. In section 7 I examine this effect and find that it needs to be relatively extreme to generate the necessary surge in wages at the bottom of the wage distribution.

9Note that wage polarization in this study refers to the polarization of the individual wage distribution while in some other studies it refers to the polarization of the occupational wage distribution.
The current paper builds on these existing studies by estimating the wage effect of job polarization on the groups of workers who start out in the high-, middle-, and low-skill occupations as well as the effect on the overall wage distribution within a unified economic framework. Moreover, it provides one explanation for why we may observe rapid job polarization that does have an effect on workers’ wages but not a lot of wage polarization in some countries and time periods.

In addition the Roy model employed in this paper is closely related to other theoretical frameworks used in the literature on job polarization. For example, the survey article by Acemoglu and Autor (2011) provides a Ricardian assignment model of skills to tasks where polarization amounts to the substitution of machines for certain tasks previously performed by labor. The model predicts that those workers who have a comparative advantage in tasks for which the relative market price decreases will be displaced and find their relative wages decline. Moreover, the replacement of tasks in the middle of the skill distribution at the same time generates job- as well as wage polarization within their framework. Other models that generate similar predictions include Autor, Levy, and Murnane (2003), Autor, Katz, and Kearney (2006), Jung and Mercenier (2014), and Autor and Dorn (2013).

An aspect in which the Roy model is more general than the above frameworks is that it features an unrestricted distribution of skill. This is empirically relevant as it allows for an overlap in wage distributions across occupations which is a salient feature of the data. It also allows for workers moving up and down in the wage distribution as a result of changes in wage rates across occupations. In a model where skills’ rankings are fixed in terms of the wages that they generate, one cannot have this phenomenon that leads to the flattening of the lower half of the wage distribution under job polarization.

The paper continues as follows. Section 2 constructs a sample of workers in the NLSY

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10In addition, Firpo, Fortin, and Lemieux (2011) use a decomposition method to assess the contribution of different factors to the change in the wage distribution over the last three decades. Their results indicate an important role for technology and de-unionization in the 1980s and the 1990s, and for offshoring from the 1990s onward.

11Feng and Graetz (2013) generate job and wage polarization in a model where firms’ automation choices are endogenous. They argue that these predictions result from automation in general and are thus not a unique consequence of the information and communication technology revolution.

12The exceptions are Autor, Levy, and Murnane (2003) and Firpo, Fortin, and Lemieux (2011) who use the Roy model as well.

13Finally, some papers have proposed additional drivers of job polarization over “routine-biased technological change”. For example, Acemoglu and Autor (2011) and Goos, Manning, and Salomons (2014) have examined the role of offshoring and international trade. Mazzolari and Ragusa (2013) and Bárány and Siegel (2013), among others, proposed the demand for consumption of low-skill occupation output coupled with general technological change. Since the Roy model is a partial equilibrium model of the supply side of the labor market, I do not need to take a stand about what are the exact drivers of job polarization in this study.
that has outcomes similar to a comparable sample in the commonly used Current Population Survey (CPS) and which features the qualitatively same job polarization and wage polarization we observe in the U.S. for that time period. Section 3 studies the worker sorting into occupations according to the talent measures available in the NLSY. Section 4 estimates the changes in relative wages for workers who sort themselves into the different occupations according to their talents. These results are interpreted within the Roy model in section 5, while section 6 uses the model to estimate the change in occupation-specific wage rates. With the help of these wage rates, section 7 assesses the impact of job polarization on the overall wage distribution. The last section concludes.

2 Data and Empirical Facts

I use data from the National Longitudinal Survey of Youth (NLSY) cohorts of 1979 and 1997, which contain detailed information on individuals’ fundamental talents that is not available in other datasets. The dataset is constructed such that its distribution of wages and occupations are comparable to the more standard Current Population Survey Merged Outgoing Rotation Groups (CPS) over the same period.

The individuals in my sample are 27 years old in 1984–1992 and 2007–2009, respectively. This timing is similar to the 1988–2008 period considered separately in figures 9 and 10 in Acemoglu and Autor (2011). In fact, the years 2008–09 are a crisis period in the U.S. labor market and a recent paper by Jaimovich and Siu (2014) argues that recessions are closely linked to surges of job polarization. When I split my NLSY and CPS data into 2007 and 2008–09 no systematically different wage- or job polarization facts appear between the two periods. I therefore continue with the pooled data.

The sample selection and attrition weighting for the NLSY sample is done closely in line with a recent paper by Altonji, Bharadwaj, and Lange (2012). Since attrition in the NLSY97 is higher than in the NLSY79, Altonji, Bharadwaj, and Lange (2012) examine it in detail. They conclude that after appropriate weighting any potential biases are not forbidding. I also construct labor supply by hours worked and real hourly wages as in Lemieux (2006). The details of the sample construction can be found in section A of the appendix. Table 1 accounts for how I end up with a sample of 3,054 and 1,207 individuals in the NLSY79 and the NLSY97, respectively.

\[\text{I do not use the 2010 and 2011 samples of the NLSY97 because wages are substantially lower and occupations are worse compared to the CPS and I could not find out what is the reason for this. Also, the AFQT scores of those members of the 1983–84 birth cohorts who work as 27 year olds in 2010–11 are substantially lower than the AFQT scores of the working 1980–82 cohorts.} \]
The overall (male) labor force experienced substantial wage polarization from the end of the 1980s to the end of the 2000s. That is, wages increased substantially at the top of the distribution and somewhat less in the bottom but hardly at all in the middle. Moreover, there was job polarization in the sense that employment in the middle-skill occupations decreased and employment in the high-skill and low-skill occupations increased. For the details of these facts, see the survey paper by Acemoglu and Autor (2011).

I start with the wage distribution in my sample. Figure 1 graphs the change in log real wages by distribution quantile in the NLSY79 and the NLSY97 and for the same years and age group in the CPS. We see that the changes in the NLSY and the CPS align well for both cohorts. Hence there is wage polarization in the NLSY which is similar to what can be found in the CPS.

The second important fact is job polarization. I follow recent studies in the literature which delineate occupation groups along the lines of their routine and non-routine task content (e.g. Acemoglu and Autor, 2011; Cortes, 2014; Jaimovich and Siu, 2014). Specifically, managerial, professional, and technical occupations are grouped as high-skill (or non-routine cognitive); sales, office and administrative, production, and operator and laborer occupations as middle-skill (or routine); and protective, food, cleaning and personal service occupations as low-skill (or non-routine manual).

Figure 2 graphs the percentage point change of employment in the high-, middle-, and low-skill occupations for the NLSY79 and NLSY97 and compared to the CPS. We can see that the employment share of middle-skill occupations is declining substantially while the employment share of the high- and the low-skill occupations is rising. Hence there is job polarization in the NLSY sample and it is close to what can be found for 27 year olds in the CPS.

Before moving on, figure 3 plots the change in average real wages in the high-, middle-, and low-skill occupations in the NLSY and, for comparison again, the CPS. While wages in high-skill occupations have increased robustly in levels and compared to the other two occupations, wages in low-skill occupations have lost somewhat further ground against wages in middle-skill occupations in the NLSY and also slightly in the CPS. One might find this surprising under the demand side explanation for job polarization, which should decrease employment and wages in the middle at the same time. Yet, just as the size of occupations, the composition of skills in occupations does not stay

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15Note that the small differences between wages, occupational employment, and occupational wages in the NLSY and the CPS sample are unlikely to stem from systematic attrition or non-test-taking in the NLSY. This is because sample attrition or non-test-taking are much lower in the NLSY79 than the NLSY97, while the differences between CPS and NLSY are equally large for the two cohorts.

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constant when relative demands change.\textsuperscript{16} If we want to get at the wage effects of job polarization we need to properly account for such changes in skill selection. In order to do this, I study which kind of workers sort into the high-, middle-, and low-skill occupations in the next section.

3 Talent Sorting into Occupations

3.1 Measures of Talent

The NLSY data provide a long array of characteristics of its respondents. Out of these, I focus on variables that are early determined, that are relevant for occupational choices and wages, that should approximate different dimensions of skill, and that can be compared over the two cohorts.\textsuperscript{17}

Table 2 reports labor force averages of NLSY variables that fulfill the four criteria (“early skill determinants”) and some demographic variables and contemporary skill determinants that are available in more standard datasets. In terms of the early skill determinants, I construct composite measures of mathematical, verbal, and mechanical talent by combining test scores on mathematics knowledge, paragraph comprehension and word knowledge, and mechanical comprehension and auto- and shop information, respectively. In addition, I report the AFQT score, which is sometimes taken as a measure of general intelligence.\textsuperscript{18}

There are a couple of advantages of using the early skill determinants—and in particular the composite measures of mathematical, verbal, and mechanical talent—for the purpose of this study: First, the (joint) distribution of test scores is quite stable over the two cohorts while educational attainment increased. In addition, education measures have a lot of bunching at points like high school graduate (12 years of education) or college graduate (16 years of education). Therefore, the test scores should enable me to better capture similar individuals across the two cohorts. Second, early skill determinants should be relatively exogenous to the change in an individual’s occupational choice due

\textsuperscript{16} Also other studies find a further decrease in low-skill compared to middle-skill occupation wages (Goos and Manning, 2007). Autor and Dorn (2013) find that relative wages in clerical occupations rise while quantities fall.

\textsuperscript{17} The popular non-cognitive skill measures of locus of control and self-esteem have to be left out of the analysis because they are not available in the NLSY97.

\textsuperscript{18} All these measures are taken from the Armed Services Vocational Aptitude Battery of tests (ASVAB) which consists of ten components: arithmetic reasoning, word knowledge, paragraph comprehension, mathematics knowledge, general science, numerical operations, coding speed, auto and shop information, mechanical comprehension, and electronics information. The breakup into mathematical, verbal, and mechanical talent is similar to what a factor analysis of test scores suggests. AFQT is essentially the average of arithmetic reasoning, word knowledge, paragraph comprehension, and mathematics knowledge.
to job polarization. This is because test scores are not very malleable, they are measured before entry into the labor market in the NLSY97, and before job polarization was widely known.\footnote{For example, the first academic papers about polarization by Autor, Levy, and Murnane and Goos and Manning were published in 2003 and 2007, respectively.} Finally, the test scores provide proxies for multiple dimensions of individuals’ skills. Thus, they can be used to determine comparative advantage in different occupations.

Before moving on, we see from table 2 that the level of AFQT, which is a measure of IQ, does not change in the male labor force over the two cohorts. In addition, table 3 reports that the cross-correlation of the composite test scores and AFQT remained virtually the same. This supports my identification assumption in the following that the tests measure similar dimensions of talent over the two cohorts and that “within test score groups” individuals can be considered on average the same across cohorts.\footnote{One early determined characteristic that is not constant is race. In particular, the share of Hispanics rose by 8 percentage points. I try to deal with this by controlling for race in all analyses.}

### 3.2 Sorting into Occupations

Figure 4 depicts average mathematical, verbal, and mechanical talent in the three occupation groups in both cohorts. We see that the levels of the three talents are substantially higher in the high-skill occupation than in the middle-skill occupation which, in turn, is higher than the low-skill occupation. Thus, there is a clear ordering of absolute advantage in occupations independent of the talent considered. This underlines the appropriates of terming them high-, middle-, and low-skill.

However, in the absence of restrictions to enter occupations, workers’ choice should not be governed by their absolute but by their comparative advantage and thus depend on their relative skills (for example, compare Sattinger, 1993). We see in figure 4 that average mathematical talent in the high-skill occupation is higher than average verbal or mechanical talent, while average mechanical talent is considerably higher in the middle-skill occupation than mathematical or verbal talent. Verbal talent is higher than mathematical and mechanical talent in the low-skill occupation.

This suggests sorting according to comparative advantage and multidimensional skills as in the well-known Roy (1951) model—with workers who have high math talent choosing the high-skill occupation, workers who have relatively high mechanical talent choosing the middle-skill occupation, and workers who have relatively high verbal talent choosing the low-skill occupation. It is also intuitive, since high analytical skills are required to pursue a career in managerial, professional, or technical jobs while individuals
who have relatively strong mechanical skills or a practical inclination may prefer to work in production or clerical jobs. Verbal skills may be relatively helpful to communicate in personal and protective service occupations. In this case, the uniform absolute ranking of occupations in the three talents should stem from the high cross-correlations between them as seen in table 3.

To test the idea of sorting according to comparative advantage I run multinomial choice regressions. Let \( \{K_{it}\} \) be a set of indicator variables that take the value of 1 when individual \( i \) works in occupation \( K \in \{L, M, H\} \) and zero otherwise. The timing is such that \( t = 0 \) when the members of the NLSY79 are 27 years old and \( t = 1 \) when the members of the NLSY97 are 27 years old. I model the conditional choice probabilities as multinomial logit (MNL):

\[
p_K(x_{it}, t) = \frac{\exp(b_{K0t} + b_{K1t}x_{1it} + \cdots + b_{KJt}x_{Jit})}{\sum_{G=H,M,L} \exp(b_{G0t} + b_{G1t}x_{1it} + \cdots + b_{GJt}x_{Jit})},
\]

where \( p_K(x_{it}, t) \) denotes the probability in point in time \( t \) to enter occupation \( K \) for an individual of talent vector \( x_{it} \), and \( x_{jit} \) represents an element of that talent vector.

Maximum likelihood estimation of equation (1) yields the coefficients of this model and it provides conditional probabilities (“propensities”) to enter each occupation based on the observable talents. As is shown in section 5, these propensities can be interpreted as individuals’ predicted relative skills in an occupation as opposed to the other two occupations.

Table 5 reports the results from the multinomial choice regressions. These extract the marginal effect of an additional unit of each talent on occupational choice when the respective other talents are held constant. For ease of discussion, focus on the first column which gives the sorting into high- and low-skill occupations relative to the omitted middle-skill occupation in the NLSY79. Conditional on the other talents, a one unit higher math score is associated with an about 4.7 percent higher probability to enter the high-skill versus the middle- or the low-skill occupation. A one unit higher mechanical score is associated with a 1.4 and 2.3 percent lower probability to enter the high- and the low-skill occupation as opposed to the middle-skill occupation, respectively. In contrast, a one unit higher verbal score decreases the probability to enter the middle- as opposed to the high- or the low-skill occupation by about two percent. Thus, the idea of sorting according to comparative advantage is strongly supported by these regressions and they are the same when looking at the NLSY97.\(^{21}\)

\(^{21}\)Speer (2014) finds similar results about the sorting of math, verbal, and mechanical talents into occupational tasks.
Finally, the regressions in columns two and four of table 5 are run for creating the propensities to enter occupations based on observables that are used in the following. The test scores are split into terciles in order to also allow for polarization in the demand for skill levels as suggested by one-dimensional skill models. Moreover, normalized measures of illicit activities and engagement in precocious sex are added.

Table 4 reports the predicted values from the NLSY79 sorting regression in column two of table 5 in both the NLSY79 and the NLSY97. We see that the distribution of “1980s occupational propensities” is remarkably stable over the two cohorts. Hence, the joint distribution of observable talents which are relevant for occupational choice is virtually unchanged. This is in line with the constant correlations across test scores in table 3 and it supports the identification assumption in the next section.

4 Polarization’s Relative Wage Effect

The prevalent view is that job polarization is a demand-side phenomenon. Thus, the wages of middle-skill workers should fall over time compared to the wages of high- and low-skill workers. I exploit the sorting results of the last section to study the wages of workers who have different probabilities to enter the high-, middle-, and low-skill occupations.

In order to do this I estimate ordinary least squares (OLS) regressions for pooled data of the form

\[ w_{it} = \alpha_0 + \alpha_1 p_H(x_{it},0) + \alpha_2 p_L(x_{it},0) + \alpha_3 \times \text{NLSY97} + \alpha_4 p_H(x_{it},0) \times \text{NLSY97} + \alpha_5 p_L(x_{it},0) \times \text{NLSY97} + \epsilon_{it}, \]  

where NLSY97 is a dummy for whether a particular observation is from the NLSY97 (in fact that \( t = 1 \)), and \( p_H(x_{it},0) \) and \( p_L(x_{it},0) \) are the probabilities to choose the high- and the low-skill occupation in the NLSY79. Hence, the approach is to study the change in average wages for types of workers that have different propensities to work in high-, middle-, and low-skill occupations and it is similar to what Acemoglu and Autor (2011) recommend in their paper.\(^{22}\) The parameters of interest in regression (2) are the changing relative returns to a higher probability in the NLSY97 of working in the high- and the low-skill occupation compared to the middle-skill occupation \( \alpha_4 \) and \( \alpha_5 \).

\[^{22}\text{In Acemoglu and Autor (2011)’s words “[...] the approach here exploits the fact that task specialization in the cross section is informative about the comparative advantage of various skill groups, and it marries this source of information to a well-specified hypothesis about how the wages of skill groups that differ in their comparative advantage should respond [...]”}.\]
Of course, the occupational choice probabilities are not directly available in the data and they have to be estimated in a preceding step in the NLSY79 along the lines of the previous section. The parameter estimates are then used to predict $p_H(x_{it},0)$ and $p_L(x_{it},0)$ for each individual in the NLSY79 and the NLSY97. This makes the estimation of (2) a two-step procedure. In fact, two-step estimation procedures are used throughout this paper since the empirical strategy exploits measuring relative skills in occupations with respect to observable talents and then relates these relative skills to changes in the returns to talents.

In terms of the two-step procedure used here, two clarifications are in order. First, I use the multinomial logit model from the last section to specify $p_K(x_{it},0)$. Alternatively, a multinomial probit or a linear probability model give similar results. Second, I bootstrap the standard errors in the second stage regression (2) to reflect the fact that $p_H(x_{it},0)$ and $p_L(x_{it},0)$ are estimates and thus possess sampling variation.

Table 6 reports the results from wage regression (2). Unsurprisingly, in column one we see that a higher propensity to enter the high-skill occupation compared to the omitted middle-skill occupation is associated with a significantly higher wage. The reverse is true for the propensity to enter the low-skill occupation.

Job polarization should however change the returns to propensities over time, which are indicated in the table by “x NLSY97”. We see that the coefficients change strongly and significantly in the expected direction. For the propensity to enter the high-skill occupation, the coefficient almost doubles (from .31 to .60) while the coefficient for entering the low-skill occupation rises by almost a third (from $-1.65$ to $-0.95$).

For illustration of the effect of different propensities to enter the three occupations, figure 5 plots the predictions from linear wage regressions on each propensity at a time together with their probability densities. In the upper two sub-figures we see that the positive wage effect of a higher propensity to enter the high-skill occupation increases further while the negative wage effect of the propensity to enter the low-skill occupation attenuates. In contrast, for the propensity to enter the middle-skill occupation the already slightly negative wage effect deteriorates substantially. For individuals with a very high propensity to enter the middle, which is quite frequent in the data, expected real wages even decline during the two decades between the NLSY79 and the NLSY97.

The level of the change in the low-skill coefficient is twice that of the high-skill coefficient, which may come as a surprise. However, note that it is also much less precisely estimated. Moreover, when scaling the size of the estimates by the respective standard deviations of the propensities reported in table 4, the change in the effect of the propensity to enter the high-skill occupation is larger: a one standard deviation increase in the high- and low-skill propensities, respectively, is associated with a 11.3 percent higher and 5.2 percent lower wage in the NLSY97 compared to a 5.9 percent higher and 8.4 percent lower wage in the NLSY79.
This is indicated by the crossing of the two lines.

Column two of table 6 adds to the first stage occupational choice regression a dummy for whether the individual completed a four year college or more. We can see that, on top of the talents, this contemporary skill determinant does not alter the conclusions about the changing returns to occupational propensities between the two NLSYs. The results are similar if we add more detailed education dummies in the first stage.

The identification of changes in returns to propensities in regression (2) is based on the assumption that for a given vector of observed talents \( x_{it} \) workers are in expectation the same in terms of their unobserved occupational productivities over the two cohorts. Tables 2 and 3 provided support for this assumption as they showed that the level and cross-correlation of observable early skill determinants is similar in the NLSY79 and NLSY97. In addition, table 4 showed that the distribution of predicted propensities is similar in the NLSY79 and NLSY97. Given this identification assumption, the changes of the propensity coefficients provide the increase in average wages that is associated with relative advantage in the high- or the low-skill occupation compared to the middle.

So far, the result in column one and two of table 6 do not exclude the possible influence of other factors than job polarization on wages of workers with relative advantage in the high- or the low-skill occupations. In particular, skill-biased technological change (SBTC) that is independent of occupational demand constitutes an alternative hypothesis to polarization and may thus have an important effect on talent returns. According to this view, relative advantage in high-, middle-, and low-skill occupations is not important because returns to skills rise across the board. If we allowed for SBTC in regression (2) with all the talents included on top of polarization in the second stage, the identification would have to rely on the functional form of \( p_H(x_{it}, 0) \) and \( p_L(x_{it}, 0) \), because the same variables that are used for estimating the propensities are directly entered into the wage regression. This would lead to near multicollinearity of the explanatory variables in the regression and to imprecise estimates. In additional regressions, I thus use education indicators as absolute skill measures in the second-stage wage regression.

The remaining two columns of table 6 assess the potential importance of the SBTC hypothesis versus polarization. Column three adds to the wage regression a dummy for whether the individual completed a four-year college or more. On the one hand, we see that the level of the coefficient on the propensity to enter the high-skill occupation drops all the way to zero but that the changes in both coefficients are remarkably stable. On the other hand, the level of return to college is large and highly significant while its change does not significantly increase once I control for the propensities. The result is
similar if I control for four different degree dummies (high school dropout and graduate, some college, and at least four year college) in column four. This suggests that Mincerian returns to education are important to explain wages in the cross-section, but that they seem to have less power than relative skills in occupations to explain the change in wages that took place over the twenty years from the NLSY79 to the NLSY97.

5 Theoretical Framework

This section develops a Roy (1951) model of occupational choice in order to interpret the empirical results so far within a more explicit theoretical framework. Moreover, I will estimate key parameters of this framework—the occupation specific wage rates—in the next section.

5.1 General Setup

Let each worker $i$ choose the occupation that offers him the highest wage: \[ W_{it} = \max \{ W_{H\ell t}, W_{M\ell t}, W_{L\ell t} \}, \] (3)

where $\{ H, M, L \}$ again index the high-, middle-, and low-skill occupation, respectively. These wages are composed of the product of $i$'s skill to carry out work in occupation $K \in \{ H, M, L \}$ ($S_{K\ell t}$) and the equilibrium market price (or wage rate) that prevails for that work in point in time $t$ ($\Pi_{K\ell t}$).

As we have seen in section 3, workers choose systematically different occupations according to their talents. This suggests that the $S_{K\ell t}$s depend on workers’ talents in different ways. Thus, the same mixture of talents yields different levels of skill in different occupations. In addition, two workers who have the same level of skill in one occupation will not generally have the same level of skill in the other two occupations. This is different from a one-dimensional model of skill.

An illustrative way to formalize these ideas is Heckman and Sedlacek (1985)'s linear

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24The model can be set up more generally with a decision rule according to utilities instead of wages. Moreover, a richer model could also feature dynamic occupational choice according to (expected) life-time utility, (occupation-specific) skill acquisition through experience, and costs of occupational mobility. The predictions about job polarization’s effect on occupational choices and wages of this richer setup would be qualitatively the same as in the static and purely pecuniary model, while the estimation in my data would only be possible under restrictive assumptions that make it no more realistic than the static and pecuniary model.
factor formulation of log wages:

\[ w_{Kit} = \pi_{Kit} + s_{Kit} = \pi_{Kit} + \beta_{K0} + \beta_{K1}x_{1it} + \ldots + \beta_{KJ}x_{Jit} + u_{Kit}, \]  

(4)

where the small \( s_{Kit} \) and \( \pi_{Kit} \) denote the log of occupation \( K \) specific skill and price, \( x_{it} = [x_{1it}, \ldots, x_{Jit}]' \) are the observed talents, the \( \beta_{Kj} \)'s are the corresponding linear projection coefficients, and \( u_{Kit} \) is an orthogonal regression error which represents the unobserved component of skill in occupation \( K \). Note that this specification for \( s_{Kit} \) is just an intuitive example and that all the results in the following hold for a general dependency of occupation-specific skills on talents.

One can now interpret the sorting results of section 3 within this framework. For the sake of brevity, I only give a crude intuition: Suppose that the productivity of the math talent in the high-skill occupation is high in relative and in absolute terms (i.e. the \( \beta_{Hj} \) corresponding to math is a large number), the productivity of mechanical (verbal) talent is relatively high (low) in the middle-skill occupation, and that the talents are not particularly productive in the low-skill occupation. Moreover, suppose that the intercept \( \beta_{K0} \) in the low-skill occupation is high while it is lower in the middle-skill occupation and lowest in the high-skill occupation. Thus, many workers can do the low-skill occupation decently, while a subset of individuals with relatively high mechanical talent can do the middle-skill occupation well, and only few individuals with high relative and absolute math talent can do the high-skill job well.

Suppose, in addition, that the talents are substantially positively correlated in the population as shown in table 3. Then, we will find the evidence about talent sorting reported in figure 4 and table 5: workers in the high-, middle-, and the low-skill occupations have relatively high math, mechanical, and verbal talents, respectively. In addition, average talents and wages are highest in the high-skill occupation and they are lowest in the low-skill occupation but still there exists substantial dispersion of skills within occupations such that middle-skill (low-skill) occupation workers obtaining higher wages than high-skill (and middle-skill) occupation workers are not uncommon. The latter empirical fact cannot be generated in a one-dimensional skill setup or in a setup with homogenous skills in the low-skill occupation while it is easily explained in the Roy model. Moreover, the Roy model allows for workers moving up or down in the wage distribution if the relative compensation in their occupation increases or decreases, respectively. This turns out to be an important factor in the discussion about the change in the overall wage distribution in section 7.

But how should one think about job polarization within the model? The previous liter-
nature as well as the above empirical findings indicate that job polarization is a demand-side phenomenon. The natural way to model a demand shift for work in occupations, given a more or less constant supply, is that it changes the relative market equilibrium prices for occupation-specific skills:\(^{25}\)

\[
\triangle (\pi_H - \pi_M) > 0 \text{ and } \triangle (\pi_L - \pi_M) > 0.
\]  

(5)

If (the relative) \(\pi_{Mt}\) falls, the wage that every worker could earn in the middle-skill occupation \(w_{Mit}\) will fall. Hence, some of the individuals who previously preferred working in the middle-skill occupation will now switch to either the high- or the low-skill occupation. This immediately generates job polarization as seen in figure 2.

The above are economically intuitive interpretations of the facts about workers’ occupational choices within and across NLSY cohorts. However, the focus of this paper is on the wage effects of polarization about which the Roy model can provide additional empirical predictions. Since the argument in the following is rather involved and the three-occupation case requires complex notation, I use a simplified version of the model with two occupations from now on. The results can be extended to the three-occupation case for the empirical analysis as shown in appendix B.

5.2 A Simplified Model to Study the Wage Effects of Polarization

Assume there are only two occupations, middle \(M\) and nonmiddle \(N\), with \(\triangle (\pi_N - \pi_M) > 0\) under polarization. Moreover, for simplicity and without loss of generality assume \(\beta_{K0} = 0\). I indicate the difference between \(N\) and \(M\) occupation variables by a tilde, i.e. \(\tilde{\pi}_t \equiv \pi_{Ni} - \pi_{Mi}\), \(\tilde{\beta} \equiv \beta_N - \beta_M\), and \(\tilde{u}_i \equiv u_{Ni} - u_{Mi}\). I suppress the index \(t\) for the \(J\) vector of talents \(x_i\) and for \(u_{Ki}\) because the only variables that change in the model are the prices \(\pi_{Ni}\) and \(\pi_{Mi}\) and their functions. Wages in occupations \(K \in \{N, M\}\) become:

\[
w_{Kit} = \pi_{Ki} + s_{Ki} = \pi_{Ki} + \tilde{\beta}_K x_i + u_{Ki}.
\]  

(6)

How do the wages of workers who have a comparative advantage in the middle occupation change over time? Since we do not observe the same individual workers in both points in time (the counter-factual), the prediction from the Roy model will have to be in terms of conditional moments with respect to observable talents. Let \(K_{it}\) be an

\(^{25}\)This assumption (or result) is similar to many other models on job polarization (e.g. Autor, Levy, and Murnane, 2003; Autor, Katz, and Kearney, 2006; Cortes, 2014; Acemoglu and Autor, 2011; Autor and Dorn, 2013).
indicator variable that takes the value of 1 when individual \( i \) works in occupation \( K \) and zero otherwise and consider his expected wage conditional on his observables \( x_i \):

\[
E(w_{it} | x_i) = E(w_{Mit} | x_i, N_{it} = 1) + p_N(x_i, \bar{\pi}_t) \left[ E(w_{Nit} | x_i, N_{it} = 1) - E(w_{Mit} | x_i, N_{it} = 0) \right],
\]

where the notation

\[
p_N(x_i, \bar{\pi}_t) = Pr(\tilde{u}_i > -(\bar{\pi}_t + \tilde{\beta} x_i))
\]

now emphasizes the fact that the probability to enter occupation \( N \) is a function of the differences in price per unit of skill between the two occupations. All of the economics of the Roy model can be found in this equation because the probability \( p_N(x_i, \bar{\pi}_t) \) and the conditional wages \( E(w_{Klt} | x_i, K_{it}) \) are determined by the worker’s optimal choice given his skills and the prices that he faces. Note that \( \tilde{\beta} x_i \) is the expected relative skill given \( x_i \) and, for a fixed \( \bar{\pi}_t \), \( p_N(x_i, \bar{\pi}_t) \) is a monotone function of it. The propensity to enter occupation \( N \) for worker \( i \) estimated from the data can thus be interpreted as a predictor of his relative skill in occupation \( N \).

Analogous to equation (5), job polarization implies \( \Delta (\pi_N - \pi_M) > 0 \). I start by considering the change in worker \( i \)'s wages for a marginal shift in prices:

\[
dw_{it} = \begin{cases} 
  d\pi_N & \text{if } N_{it} = 1 \\
  d\pi_M & \text{if } N_{it} = 0,
\end{cases}
\]

where \( d \) denotes a marginal change. Thus, due to the optimality of workers’ occupational choice and the envelope theorem,\(^{26}\) the effect on wages of a marginal change in \( \pi_{Kl} \) is only the direct price effect

\[
dE(w_{it} | x_i) = d\pi_M + p_N(x_i, \bar{\pi}_t)d(\pi_N - \pi_M).
\]

According to prediction (7), under the polarization hypothesis, workers who are ceteris paribus more likely to enter the nonmiddle occupation are expected to see their relative wages increase.

This qualitative prediction holds beyond the margin as well. That is, the expected overall wage gain from polarization rises with the initial probability to work in the non-middle occupation. Note that the change in worker \( i \)'s expected wage is the sum over his marginal expected wage changes along the adjustment path from \( \pi_0 \) to \( \pi_1 \). Hence, we

\(^{26}\)A version of the envelope theorem also holds for optimization problems where agents’ choices are discreet (e.g. see Milgrom and Segal, 2002).
can integrate prediction (7) from $t = 0$ to $t = 1$ to obtain:

$$E(w_{i1}|x_i) - E(w_{i0}|x_i) = \Delta \pi_M + \int_{\tilde{\pi}_0}^{\tilde{\pi}_1} p_N(x_i, \tilde{\pi}_t) d\tilde{\pi}_t,$$

(8)

where the structure of $p_N(x_i, \tilde{\pi}_t) = Pr(\tilde{u}_i > -(\tilde{\pi}_t + \tilde{\beta}x_i))$ illustrates that on the adjustment path of prices, the ranking of $p_N(x_i, \tilde{\pi}_t)$ with respect to $x_i$ remains unchanged.²⁷

Appendix B shows that the result (8) carries over to the three occupation case analyzed in the empirics. This is the same result as in Acemoglu and Autor (2011) and other papers about the wages of workers who have a comparative advantage in tasks for which the relative market price decreases—obtained from a general model of labor supply with multidimensional skills. Hence, the empirical findings of section 4 on the changing returns to working in the high-, middle-, and low-skill occupations are in line with the predictions of the Roy model: the model predicts that individuals who have a higher propensity to work in the nonmiddle occupations experience a higher increase in average wages over time. Moreover, it illustrates that the changing returns to occupational propensities include the direct price effect as well as the reallocation effect of workers moving into the nonmiddle occupations.

Before moving on, it seems appropriate to discuss in more detail the assumption that polarization amounts to changes in occupation-specific wage rates $\pi_{Ki}$ in equation (5). This assumption has two components. First, the conditional distribution of talents does not change over the two cohorts. In the Heckman and Sedlacek (1985) example of equation (4) it means that the distribution of $u_{Kit}$ conditional on the vector $x_{it}$ does not depend on $t$. This was assumed from the outset of the paper and supporting evidence was reported.

²⁷Another way of deriving equation (8) is illustrative: Concentrate on a specific worker $i$ first and note again that $\tilde{\pi}_1 \equiv \pi_{Ni} - \pi_{Mi}$, $\Delta \tilde{\pi}_1 > 0$, and $N_i$ is an indicator for working in occupation $N$ such that $w_{it} = w_{Mi1} + N_i (w_{Ni1} - w_{Mi1})$. Defining the relative price that makes $i$ indifferent as $\tilde{\pi}_i^t \equiv -\tilde{s}_i = -(s_{Ni} - s_{Mi})$, we get:

$$w_{i1} - w_{i0} = \Delta \pi_M + N_i (w_{Ni1} - w_{Mi1}) - N_i (w_{Ni0} - w_{Mi0})$$

$$= \Delta \pi_M + \left\{ \begin{array}{ll}
\Delta \pi_N - \Delta \pi_M = \tilde{\pi}_1 - \tilde{\pi}_0 & \text{if } N_i = 1, N_i = 1 \\
\tilde{\pi}_1 + \tilde{s}_i = \tilde{\pi}_1 - \tilde{\pi}_i^t & \text{if } N_i = 0, N_i = 1 \\
0 & \text{if } N_i = 0, N_i = 0
\end{array} \right.$$

$$= \Delta \pi_M + \int_{\tilde{\pi}_0}^{\tilde{\pi}_1} N_i d\tilde{\pi}_t.$$

Taking expectations w.r.t. $\tilde{u}_i$ conditional on $x_i$ on the top left and bottom of this equation gives result (8). Hence, since within occupations the wage gain is constant, the overall gain for a specific worker depends solely on the “distance” of the adjustment that the worker is still in the middle ($\pi^t_{Ni} - \pi^t_{Ni0}$) and already in the nonmiddle ($\pi^t_{Ni} - \pi^t_{Ni1}$) occupation. This principle is the same for expected wages and probabilities of being in the nonmiddle occupation.
Second, only the demand levels—and thus market prices—for occupation-specific work but not the production of work are changing. In terms of the Heckman and Sedlacek (1985) example, the $\pi_{Kt}$s are affected by polarization but the $\beta_{Kj}$s are not. This may not entirely be true, since Autor, Levy, and Murnane (2003) and Spitz-Oener (2006) have shown that the task content of occupations has been changing over time. Still, for the purpose of interpreting the reduced-form estimates so far it was largely innocuous. It may also not be too problematic for the estimation in the next section if the production function (or task content) of all occupations is changing in the same direction (i.e. the $\beta_{Kj}$s changing in the same direction for all $K$).28

6 Estimating the Change in Occupational Wage Rates

This section estimates the change in the relative wage rates that are paid across occupations between the NLSY79 and the NLSY97. The wage rates are not only of interest in their own right but they are also crucial to assess the effect of job polarization on the overall wage distribution. To see this consider figure 6. It depicts the counterfactual change in the overall wage distribution when I assign workers in the NLSY79 the changing returns to their observable characteristics that were estimated in section 4. We can see that neither the occupational propensities nor a specification that adds the returns to college can generate much of the change in the actual wage distribution.

Why is this the case? The reason is that observable characteristics in fact cannot explain a large part of the variation in wages or sorting within a given period.29 Hence, they are also unlikely to capture a large part of the changes in wages across periods. Therefore, we need to get hold of the changing returns to unobservable characteristics as well as observables. Estimating the wage rates in the Roy model provides a way to do this because the wage rates apply to both observable and unobservable characteristics in equation (4).

6.1 Estimation Approach

I explain the method with the help of my two-occupation setup. Appendix C shows how this can be extended to the three-occupation model for the actual estimation.

According to result (8), the overall change in worker $i$'s expected relative wage is the

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28Cortes (2014) makes the same restriction in his estimation and he argues in a similar vein.
29With or without education dummies on top of talents, observables can only explain 10-15 percent of the variation in wages for 27 year old males. They also explain 11-13 percent of the variation in sorting in table 4.
integral over his marginal expected wage changes along the adjustment path from \( \pi_0 \) to \( \pi_1 \):

\[
E(w_{i1} | x_i) - E(w_{i0} | x_i) = \Delta \pi_M + \int_{\pi_0}^{\pi_1} p_N(x_i, \tilde{\pi}_t) d\tilde{\pi}_t. \tag{8}
\]

In this equation, I want to estimate the distance between \( \tilde{\pi}_1 \) and \( \tilde{\pi}_0 \) (i.e. \( \Delta \tilde{\pi} \)) and possibly \( \Delta \pi_M \). I know \( E(w_{i1} | x_i) \) and \( p_N(x_i, \pi_t) \) in points in time \( t = 0 \) and \( t = 1 \) in the sense that I can consistently estimate them from my primary data. However, I do not know \( p_N(x_i, \tilde{\pi}_t) \) within the interval \( t \epsilon (0, 1) \) and I will need to make an assumption on it.

The estimation problem can be illustrated in a graph. In figure 7, I want to back out the distance on the x-axis between \( \tilde{\pi}_1 \) and \( \tilde{\pi}_0 \) while I know the start and the end point (the thick dots \( A_1 \) and \( A_2 \)) of the function (the arch) over which I need to integrate and the value of the integral (the shaded area). I thus need to make an assumption about the shape of the curve connecting \( A_1 \) and \( A_2 \). This curve has to be weakly monotonically increasing, as with higher \( \tilde{\pi}_t \) the number of workers in occupation \( N \) will increase, but it can be concave as in the picture or convex or both.

I decide to make full use of the empirical evidence in \( t = 0 \) and \( t = 1 \) by linearly approximating

\[
p_N(x_i, \tilde{\pi}_t) \approx p_N(x_i, \tilde{\pi}_0) + \frac{p_N(x_i, \tilde{\pi}_1) - p_N(x_i, \tilde{\pi}_0)}{\tilde{\pi}_1 - \tilde{\pi}_0} (\tilde{\pi}_t - \tilde{\pi}_0). \tag{9}
\]

In figure 7, this amounts to approximating \( p_N(x_i, \tilde{\pi}_t) \) as the y-coordinate for the point on the line \( \tilde{A}_1 \tilde{A}_2 \) that corresponds to \( \tilde{\pi}_t \) and by approximating \( E(w_{i1} | x_i) - E(w_{i0} | x_i) \) as the trapezoid \( a + b \). If the shape of \( p_N(x_i, \tilde{\pi}_t) \) in \( \tilde{\pi}_t \epsilon (\tilde{\pi}_0, \tilde{\pi}_1) \) is not too convex or concave, the approximation should be reasonably close.\(^{30}\)

Result (8) now becomes

\[
E(w_{i1} - w_{i0} | x_i) = \Delta \pi_M + \frac{p_N(x_i, \tilde{\pi}_1) + p_N(x_i, \tilde{\pi}_0)}{2} \Delta (\pi_N - \pi_M). \tag{10}
\]

This constitutes an estimable equation. In particular, consider a linear wage regression

\(^{30}\)The alternative to this approximation is to assume that \( \tilde{\pi}_t \) is normally distributed (for simplicity assume \( \sigma = 1 \), which modifies (8) to

\[
E(w_{i1} | x_i) - E(w_{i0} | x_i) = \Delta \pi_M + \int_{\tilde{\pi}_0}^{\tilde{\pi}_1} \Phi(\tilde{\pi}_t + \tilde{\beta}x_i) d\tilde{\pi}_t,
\]

where \( \Phi(.) \) denotes the distribution function of the standard normal. For this to be helpful, one needs to know the structural parameter \( \tilde{\beta} \) from the model. This could in principle estimated from a probit model or a Heckman two stage regression. But then one would be estimating the price change by relying on a distributional assumption in (8) and, in order to implement it, estimating the necessary parameter \( \tilde{\beta} \) relying on the distributional assumption in the first stage. This appears to be no improvement to outright structurally estimating the Roy model with a normality assumption in both cross-sections and comparing the estimated \( \tilde{\pi}_0 \) and \( \tilde{\pi}_1 \).
along the lines of the “reduced-form” estimation (2):

\[ w_{it} = \alpha_0 + \alpha_1 \bar{p}_N(x_i) + \alpha_3 \times NLSY97 + \alpha_4 \bar{p}_N(x_i) \times NLSY97 + \varepsilon_{it}, \quad (11) \]

with \( \bar{p}_N(x_i) \equiv \frac{p_N(x_i, \pi_1) + p_N(x_i, \pi_0)}{2} \). By property of OLS \( \alpha_3 + \alpha_4 \bar{p}_N(x_i) \) provides the best linear predictor of \( E(w_{i1} - w_{i0} \mid \bar{p}_N(x_i)) \). But according to result (10), this is the same as \( E(w_{i1} - w_{i0} \mid x_i) \). Therefore \( \alpha_4 \) identifies \( \Delta(\pi_N - \pi_M) \) and \( \alpha_3 \) identifies \( \Delta\pi_M \).

### 6.2 Empirical Results

Appendix C shows how the estimation approach can be extended to three occupations for the empirical analysis. Equation (12) reports the results from this procedure:

\[ w_{it} = 183.05 + .25 \bar{P}_H(x_{it}) - 1.48 \bar{P}_L(x_{it}) - 4.24 \times NLSY97 + \\
+ 0.25 \bar{P}_H(x_{it}) \times NLSY97 + 0.33 \bar{P}_L(x_{it}) \times NLSY97 + \varepsilon_{it}, \quad (12) \]

where \( \bar{p}_K(x_{it}) = \frac{p_N(x_{it}, \pi_1) + p_N(x_{it}, \pi_0)}{2} \) for \( K \in \{H, L\} \) and the standard errors are bootstrapped again to account for the fact that the \( \bar{p}_K(x_{it}) \)'s are estimates themselves.

Equation (12) indicates that the relative equilibrium log wage rates that are paid for a constant unit of skill across occupations have changed substantially between the two NLSYs. First, the relative wage rate in the high-skill occupation increased by 25 percent compared to the middle-skill occupation. At a standard error of .12 this difference is also statistically significant at the five percent level. Further, the relative wage rate in the low-skill occupation also rose by about 33 percent. However, due to the high standard error this difference is not statistically significant. Finally, the absolute wage rate that is offered in the middle-skill occupation decreased slightly by about 4 percent. Again this is not statistically significant.

Table 7 reports that the results in equation (12) are quite robust across estimation methods and specifications. Estimating (12) including a college dummy analogous to column two of table 6 yields a relative wage rate increase in the high- and the low-skill occupation of 23 and 41 percent, respectively. Using a minimum distance estimation technique applied in earlier versions of this paper gives an optimal minimum distance estimate of 20 and 31 percent, respectively (for details refer to Boehm, 2013).\(^{31}\) This pro-

\(^{31}\)The minimum distance estimator was on the moment conditions for talent returns implied by equation (10). This was somewhat technical and replaced by the more straightforward linear regression estimator in order to improve the readability of the paper. Using different weighting matrices than the asymptotically optimal one, however, led to the estimate for the change in the relative low-skill occupation wage rate to be
cEDURE also provided a test of the restrictions implied by the model which is reported in table 7. We can see that the model is narrowly not rejected at the 10 percent level. Finally, in a recent study Cortes (2014) estimates occupation-specific wage rates using panel data from the Panel Study of Income Dynamics. His results indicate that the relative wage rate for high- and low-skill occupations rose by around 40 and 25 percent from the mid-1980s to 2007, respectively.32

Overall, therefore, the results reported in this section paint a picture of falling demand for work in the middle-skill occupations which manifests itself in substantially declining relative wage rates that are offered there. The next section will use these relative wage rate changes in order to assess the effect that the demand shifts may have had on the overall wage distribution.

7 Polarization’s Effect on the Wage Distribution

This last section assesses the effect that job polarization may have had on the change in overall wage inequality. I start by generating a counterfactual wage distribution that is due to the changes in occupation-specific wage rates estimated in the last section. Then I check whether the remaining difference with the actual distribution may in principle be explained by the wage effects of workers reallocating out of the middle-skill occupations. I conduct these exercises in the NLSY and in the CPS data from section 2. To use the CPS is now possible again because assigning the estimated skill prices only requires knowledge of workers’ occupations and not their talents anymore. Thus note that, conditional on having obtained the “correct” price estimates from the NLSY, the idiosyncracies of either dataset should not drive my conclusions in this section.

I use the price estimates from regression (12) and assign them to every worker in the initial cohort according to his occupation. In figure 8 we can see that in both datasets the increase of wages at the top of the distribution is reproduced quite well by the estimated price changes alone. However, despite a slight increase at the bottom, the lower half of the counterfactual wage distribution is relatively flat. Thus, the counterfactual wage distribution does not match the surge in the actual distribution in the NLSY or CPS. This seems surprising given the substantial relative wage rate increase in the low-skill occupation of 33 percent.

Figures 9 and 10 reveal why the high relative wage rate change and the resulting relative wage increase for low-skill occupation workers do not achieve much in terms of

32These wage rate estimates are read off figure 5 in Cortes’ paper.
lifting the bottom of the counterfactual wage distribution. First, in figure 9 I plot the share of low-, middle-, and high-skill workers into the actual initial wage distribution. We see that in both the NLSY and the CPS the share of low-skill occupation workers declines monotonically with the quantiles of the wage distribution while the share of high-skill occupation workers increases. The share of middle-skill occupation workers features a hump shape—rising up to around the 30th percentile and then slowly declining.

Therefore, a drop in the wage rate for middle-skill occupations will strongest hit workers who already started out in the lower third of the wage distribution. Moreover, all three curves are relatively flat, which indicates that the dispersion of wages within the three occupation groups is large and that they are overlapping substantially. Hence, a decline in the wage rate for middle-skill occupations will drag down the wages of many low earners at the same time as many middle-earners. This smoothes the impact of the price changes in the lower half of the wage distribution.

Second, figure 10 depicts a smoothed version of the counterfactual from figure 8 and an alternative counterfactual where workers’ quantiles in the original distribution of the NLSY79 are kept constant. This alternative counterfactual is related to plots presented in other studies of wage changes in occupations against their initial ranking in the 1980s (e.g. Acemoglu and Autor, 2011; Autor and Dorn, 2013).

In the figure we see that if we force all of the workers’ wage change impacts on their original quantile, and thus shut down that they move up or down the wage distribution depending on their occupations, the rise in the lower half of the counterfactual is stronger while it is weaker in the upper half. Overtaking therefore flattens the wage distribution at the bottom and steepens it at the top. Due to this we do not observe much wage polarization in the data despite the existence of job polarization and an associated substantial decline in middle-skill occupation wage rates.

Overtaking is a potential explanation as to why in some countries and time periods there is rapid job polarization but not a lot of wage polarization. In fact, the overtaking effect can only exist in models like the Roy model which feature a multidimensional distribution of skill. Such models also allow for overlapping wage distributions across occupations as we have seen in figure 9.

Finally, is there a way within the Roy model that job polarization could have generated the wage polarization that we observe in the actual data? So far, the analysis ignored

---

33 The lines are smoothed because for the counterfactual under fixed quantiles the individuals who correspond to these quantiles exclusively determine their change. This makes the non-smoothed counterfactual distribution very spiky.
the wage effect on workers who reallocate out of the middle-skill occupations. The reason is that without additional assumptions about the distribution of workers’ skills, the model and empirical results of the previous sections are silent about this effect. Therefore, the analysis in the following should be considered a calibration exercise that assesses whether one can in principle match the remaining differences between the actual and the counterfactual wage distribution with the reallocation effect.

In the data, there is a net outflow from the middle- to the low-skill and to the high-skill occupation of three and 3.5 percent of the overall workforce, respectively. Therefore, I assume that the lowest earners in the middle who make up three percent of the workforce switch into the low-skill occupation and assign them a fifteen percent wage increase, that is, about half of the maximum wage increase that they could possibly obtain from switching (33% − 4%).

Figure 11 plots the resulting counterfactual wage distribution. This fits the actual quite well, especially in the CPS. Moreover, the reallocation effect that underlies it seems qualitatively plausible. The low-earners in the middle-skill occupations may really have a strong incentive to switch jobs once the relative demand shock hits and it is also conceivable that they could do so gainfully. For example, given probably not too different skill requirements, someone who would have been a low-earning worker in a factory in the 1980s may instead relatively easily become a janitor today.

While qualitatively plausible, the assumptions that are made in order to match the wage distribution in figure 11 are strong. First, the concentrated switching of low-earners from the middle-skill occupation requires that the population distribution of potential wages in the low-skill occupation be condensed, so that the low-earners are the first to find it profitable to “switch down”. This is hard to reconcile with the fact that the empirical wage distributions of the low- and the middle-skill occupation overlap substantially in both cross-sections. Second, the gains from switching that I need to assume are quite large.

34Workers who optimally choose to leave their initial occupations have wage increases compared to staying. This is even true when they “switch down” into low-skill occupations.
35An additional one percent of low earners is assumed to move to the high-skill occupation with the same wage gain.
36Previous versions of the paper in addition reported actual and counterfactual changes in average wages across occupations (e.g. see Boehm, 2013). The calibrated reallocation effect also helped to bring those actual and counterfactual closer together.
8 Conclusion

This article has studied the wage effects of job polarization on 27 year old male workers across the two cohorts of the National Longitudinal Survey of Youth. In order to account for endogenous selection of skill out of the middle-skill occupations, I have examined the wages of groups of workers who are differentially likely to work in high-, middle-, and low-skill occupations in the NLSY79 according to their talents. I have then used the Roy (1951) model of occupational choice in order to estimate the changes in occupation-specific wage rates and to assess the effect that job polarization may have had on the overall wage distribution.

My findings indicate that workers who have talents that would have made them likely to work in middle-skill occupations in the 1980s have experienced a substantial decline in their relative wages and possibly even a decline in their absolute wages. Further, the workers in the NLSY97 are young enough such that they are unlikely to have had acquired much occupational experience when rapid job polarization occurred during the 1990s. Since occupation- or task-specific experience makes it more costly to change occupations in response to job polarization (Gathmann and Schönberg, 2010), the relative wage effects on the workers in my dataset might in fact be a lower bound of what happened to the more experienced overall workforce.

My findings further indicate that the equilibrium wage rates that are paid for a constant unit of skill across occupations have shifted in favor of the high- and of the low-skill occupations. Job polarization can explain the rise of inequality in the upper half of the actual wage distribution but it has a hard time explaining the increase of wages at the bottom of the actual wage distribution. Whether this last result is due to the data and assumptions used in this paper or whether job polarization does not by itself generate substantial wage polarization seems to be an important question for further research.

References


Figure 1: Percentage Growth of the Quantiles of the Wage Distribution

Notes: The figure depicts the change in log real wages along the quantiles of the wage distribution between the two cohorts for the NLSY and the comparable years and age group in the CPS.

Figure 2: Change in Employment Shares by Occupations

Notes: The figure depicts the percentage point change in employment in the low-, middle-, and high-skilled occupations in the NLSY and the comparable years and age group in the CPS. The high-skill occupations contain managerial, professional services, and technical occupations. The middle-skill occupations contain sales, office / administrative, production, and operator and laborer occupations. The low-skill occupation contain protective, food, cleaning and personal service occupations.
Figure 3: Wage Changes by Occupations

Notes: The figure depicts the change in average real wages in low-, middle-, and high-skilled occupations for the NLSY and the comparable years and age group in the CPS. The high-skill occupations contain managerial, professional services, and technical occupations. The middle-skill occupations contain sales, office / administrative, production, and operator and laborer occupations. The low-skill occupations contain protective, food, cleaning and personal service occupations.

Figure 4: Average Talents in Occupations, NLSY 1979 and 1997

Notes: The figures display the average math, verbal, and mechanical test scores in the three occupation groups for the NLSY79 and the NLSY97.
Figure 5: Predicted Relative Skill Returns and their Changes

(a) Propensity High-Skill Occupation
(b) Propensity Low-Skill Occupation
(c) Propensity Middle-Skill Occupation

Notes: The figures plot the returns to propensities of entering the respective occupation in the NLSY79 and the NLSY97 together with the empirical density of these propensities in the NLSY79. The returns are estimated in regressions of log wages on a constant and the respective propensity together with an interaction term for the NLSY97.
Figure 6: Actual and Predicted Wage Distribution Change

(a) Returns to Propensities

(b) Returns to Propensities and College

Notes: The figures plot the actual and the predicted change in the wage distribution when workers in the NLSY79 are assigned the change in the returns to their observable characteristics between the two cohorts estimated in columns one and three of table 6.

Figure 7: The Estimation Problem

\[ p_N(x_i, \tilde{\pi}_t) \]

\[ E(w_{i1}|x_i) - E(w_{i0}|x_i) = \Delta \pi_M + a + b + c \]
Figure 8: Actual and Counterfactual Wage Distribution Change, NLSY and CPS

Notes: The figure plots the actual and the counterfactual change in the wage distribution when workers in the initial period are assigned the estimated wage rate changes in their occupations from equation (12).

Figure 9: Share of Occupations in the Wage Distribution, 1979 Cohort

Notes: The figure plots the smoothed employment share of the low-, middle-, and high-skill occupations within the quantiles of the initial wage distribution. Smoothing is done using the predicted values from a fourth order polynomial regression of the employment shares on the quantiles.
Figure 10: Smoothed Counterfactual Wage Distribution Change, Flexible and Fixed Quantiles

(a) NLSY

(b) CPS

Notes: The solid line in this figure depicts the growth of the wage distribution along the quantiles under the counterfactual. The dashed line depicts the growth of the original quantiles under the counterfactual, i.e., individuals are fixed at their quantiles in the original wage distribution and the growth of these fixed quantiles is computed. The lines are smoothed because for the counterfactual under fixed quantiles the individuals who correspond to these quantiles exclusively determine their change. This would make the counterfactual very spiky. Smoothing is done using the predicted values from a fourth order polynomial regression of average wage changes on the quantiles.

Figure 11: Actual and Counterfactual Wage Distribution Change with Reallocation, NLSY and CPS

(a) NLSY

(b) CPS

Notes: The figure plots the actual and the counterfactual change in the wage distribution when workers in the initial period are assigned the estimated wage rate changes in their occupations from equation (12) plus a reallocation effect: the lowest-earning three percent are assumed to move out of the middle- to the low-skill occupation with a 15 percent relative wage increase and the next low-earning one percent is assumed to move to the high-skill occupation with the same relative wage gain.
Table 1: From the full NLSY to the analysis sample

<table>
<thead>
<tr>
<th>Reason for exclusion</th>
<th>NLSY79 (Birthyears 1956-1964)</th>
<th>NLSY97 (Birthyears 1980-1984)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total males</td>
<td>6,403</td>
<td>4,599</td>
</tr>
<tr>
<td>Excluded oversampled white and older arrivers in US than age 16</td>
<td>4,585</td>
<td>4,599</td>
</tr>
<tr>
<td>Birthyear &gt; 1982</td>
<td>4,585</td>
<td>2,754</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type of attrition</th>
<th>NLSY79 (Birthyears 1956-1964)</th>
<th>NLSY97 (Birthyears 1980-1984)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ought to be present with ASVAB at age 27</td>
<td>4,585</td>
<td>2,754</td>
</tr>
<tr>
<td>No ASVAB excluded</td>
<td>4,299</td>
<td>2,081</td>
</tr>
<tr>
<td>%</td>
<td>94</td>
<td>76</td>
</tr>
<tr>
<td>Not present at age 27 excluded</td>
<td>3,939</td>
<td>1,737</td>
</tr>
<tr>
<td>%</td>
<td>86</td>
<td>63</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Conditioned on working</th>
<th>NLSY79 (Birthyears 1956-1964)</th>
<th>NLSY97 (Birthyears 1980-1984)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excluded who report no or farm occupation, self-employed, and those with no wage income</td>
<td>3,054</td>
<td>1,207</td>
</tr>
</tbody>
</table>

Notes: The table reports how I get from the full NLSY 1979 and 1997 to my analysis sample and where observations are lost or need to be dropped.
Table 2: Labor Supply with Respect to Average Demographics, Early, and Contemporary Skill Determinants

<table>
<thead>
<tr>
<th></th>
<th>NLSY79</th>
<th>NLSY97</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nbr of observations</td>
<td>3051</td>
<td>1210</td>
</tr>
<tr>
<td>Percentage of observations</td>
<td>71.60</td>
<td>28.40</td>
</tr>
<tr>
<td><strong>Demographics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>27.00</td>
<td>27.00</td>
</tr>
<tr>
<td>White</td>
<td>0.80</td>
<td>0.72</td>
</tr>
<tr>
<td>Black</td>
<td>0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>Hispanic</td>
<td>0.06</td>
<td>0.14</td>
</tr>
<tr>
<td><strong>Early skill determinants</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AFQT</td>
<td>167.31</td>
<td>167.65</td>
</tr>
<tr>
<td>Low AFQT Tercile</td>
<td>0.34</td>
<td>0.33</td>
</tr>
<tr>
<td>Middle AFQT Tercile</td>
<td>0.33</td>
<td>0.34</td>
</tr>
<tr>
<td>High AFQT Tercile</td>
<td>0.33</td>
<td>0.32</td>
</tr>
<tr>
<td>Math Score (NCE)</td>
<td>50.45</td>
<td>50.73</td>
</tr>
<tr>
<td>Verbal Score (NCE)</td>
<td>50.26</td>
<td>50.49</td>
</tr>
<tr>
<td>Mechanical Score (NCE)</td>
<td>50.41</td>
<td>50.69</td>
</tr>
<tr>
<td>Illicit Activities (NCE, Measured 1980)</td>
<td>49.98</td>
<td>50.01</td>
</tr>
<tr>
<td>Precocious Sex (NCE, Measured 1983)</td>
<td>49.91</td>
<td>50.24</td>
</tr>
<tr>
<td>Mother’s Education (Years)</td>
<td>11.86</td>
<td>13.11</td>
</tr>
<tr>
<td>Father’s Education (Years)</td>
<td>10.83</td>
<td>13.09</td>
</tr>
<tr>
<td><strong>Contemporary skill determinants</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High School Dropout (HSD)</td>
<td>0.12</td>
<td>0.07</td>
</tr>
<tr>
<td>High School Graduate (HSG)</td>
<td>0.43</td>
<td>0.58</td>
</tr>
<tr>
<td>Some College (SC)</td>
<td>0.20</td>
<td>0.06</td>
</tr>
<tr>
<td>College Graduate (CG)</td>
<td>0.19</td>
<td>0.24</td>
</tr>
<tr>
<td>Advanced Degree (AD)</td>
<td>0.06</td>
<td>0.04</td>
</tr>
<tr>
<td>North East</td>
<td>0.22</td>
<td>0.17</td>
</tr>
<tr>
<td>North Central</td>
<td>0.29</td>
<td>0.25</td>
</tr>
<tr>
<td>South</td>
<td>0.32</td>
<td>0.35</td>
</tr>
<tr>
<td>West</td>
<td>0.17</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Notes: The table shows average demographics and skill proxies in the NLSY79 and NLSY97 for all individuals weighted by hours worked. NCE indicates variables in the population (including non-workers) are standardized to “normal curve equivalents” with mean 50 and standard deviation 21.06. This is done when absolute values of these variables cannot be compared over the two cohorts.
Table 3: Pairwise Correlations between Composite Test Scores

<table>
<thead>
<tr>
<th></th>
<th>NLSY79</th>
<th>NLSY97</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AFQT</td>
<td>Math</td>
</tr>
<tr>
<td>AFQT (NCE)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Math Score (NCE)</td>
<td>0.82</td>
<td>1</td>
</tr>
<tr>
<td>Verbal Score (NCE)</td>
<td>0.93</td>
<td>0.71</td>
</tr>
<tr>
<td>Mechanical Score (NCE)</td>
<td>0.63</td>
<td>0.53</td>
</tr>
<tr>
<td>Nbr Observations</td>
<td>2936</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The table shows the pairwise correlations between composite test scores after standardizing to normal curve equivalents with mean 50 and standard deviation 21.06.

Table 4: Occupational Propensities from the NLSY 1979 Sorting Regressions

<table>
<thead>
<tr>
<th></th>
<th>Prop High</th>
<th>Prop Middle</th>
<th>Prop Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979</td>
<td>26.6</td>
<td>63.9</td>
<td>9.5</td>
</tr>
<tr>
<td></td>
<td>19.1</td>
<td>17.3</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>6.2</td>
<td>37.6</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>20.9</td>
<td>69.5</td>
<td>7.9</td>
</tr>
<tr>
<td></td>
<td>55.7</td>
<td>82.7</td>
<td>16.7</td>
</tr>
<tr>
<td>1997</td>
<td>26.2</td>
<td>63.9</td>
<td>10.0</td>
</tr>
<tr>
<td></td>
<td>18.8</td>
<td>16.9</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>6.0</td>
<td>39.4</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>20.7</td>
<td>69.4</td>
<td>8.1</td>
</tr>
<tr>
<td></td>
<td>54.3</td>
<td>82.4</td>
<td>18.1</td>
</tr>
<tr>
<td>N</td>
<td>4146</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The table reports propensities to enter the high-, middle-, and low-skill occupations in the NLSY79 and NLSY97. The propensities are from the NLSY79 only and they are from multinomial logit regressions of occupational choice including mathematical, verbal, and mechanical talent terciles, illicit activities, precocious sex and dummies for respondents’ race.
Table 5: Sorting into Occupation Groups, Multinomial Logit Regressions

<table>
<thead>
<tr>
<th></th>
<th>NLSY79</th>
<th>NLSY79</th>
<th>NLSY97</th>
<th>NLSY97</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>High</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-4.024***</td>
<td>-1.710***</td>
<td>-3.176***</td>
<td>-1.384***</td>
</tr>
<tr>
<td>Black</td>
<td>0.235</td>
<td>0.159</td>
<td>-0.152</td>
<td>-0.106</td>
</tr>
<tr>
<td>Hispanic</td>
<td>0.03</td>
<td>-0.031</td>
<td>-0.472*</td>
<td>-0.456*</td>
</tr>
<tr>
<td>Math (NCE)</td>
<td>0.047***</td>
<td>0.034***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Verbal (NCE)</td>
<td>0.023***</td>
<td>0.032***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mechanic (NCE)</td>
<td>-0.014***</td>
<td>-0.019***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Middle Math Tercile</td>
<td>1.144***</td>
<td>0.441*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>High Math Tercile</td>
<td>2.315***</td>
<td>1.426***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Middle Verbal Tercile</td>
<td>0.207</td>
<td>0.670**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>High Verbal Tercile</td>
<td>0.750***</td>
<td>1.445***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Middle Mechanic Tercile</td>
<td>-0.269</td>
<td>-0.258</td>
<td></td>
<td></td>
</tr>
<tr>
<td>High Mechanic Tercile</td>
<td>-0.552***</td>
<td>-0.618**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Illicit Activities (NCE)</td>
<td>-0.009***</td>
<td>-0.003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Precocious Sex (NCE)</td>
<td>-0.004</td>
<td>-0.006</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Low</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-1.689***</td>
<td>-1.608***</td>
<td>-1.339***</td>
<td>-2.053***</td>
</tr>
<tr>
<td>Black</td>
<td>0.636***</td>
<td>0.762***</td>
<td>0.473*</td>
<td>0.658**</td>
</tr>
<tr>
<td>Hispanic</td>
<td>0.201</td>
<td>0.243</td>
<td>-0.216</td>
<td>-0.114</td>
</tr>
<tr>
<td>Math (NCE)</td>
<td>-0.002</td>
<td></td>
<td></td>
<td>-0.009</td>
</tr>
<tr>
<td>Verbal (NCE)</td>
<td>0.018***</td>
<td>0.021**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mechanic (NCE)</td>
<td>-0.023***</td>
<td>-0.017**</td>
<td></td>
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</tr>
<tr>
<td>Middle Math Tercile</td>
<td>-0.381**</td>
<td></td>
<td>-0.07</td>
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<tr>
<td>High Math Tercile</td>
<td>0.128</td>
<td>-0.395</td>
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</tr>
<tr>
<td>Middle Verbal Tercile</td>
<td>0.342</td>
<td>0.27</td>
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<td></td>
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<tr>
<td>High Verbal Tercile</td>
<td>0.471*</td>
<td>0.790**</td>
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<tr>
<td>Middle Mechanic Tercile</td>
<td>-0.319</td>
<td>-0.281</td>
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<td></td>
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<tr>
<td>High Mechanic Tercile</td>
<td>-0.908***</td>
<td>-0.608*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Illicit Activities (NCE)</td>
<td>-0.002</td>
<td>0.013*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Precocious Sex (NCE)</td>
<td>-0.003</td>
<td>-0.003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pseudo R-Squared</td>
<td>0.132</td>
<td>0.123</td>
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</tr>
<tr>
<td>N</td>
<td>2936</td>
<td>2936</td>
<td>1210</td>
<td>1210</td>
</tr>
</tbody>
</table>

Notes: Each column presents the results from a multinomial logit regression of occupational choice on demographics and talent proxies. The omitted group is the middle occupation. The first column uses only linear test scores in the NLSY79. The second column, which is the specification to estimate occupational propensities in the following, uses terciles of test scores and adds measures of risky behavior. The last two columns repeat these estimations for the NLSY97. In order to save space, standard errors are not reported but statistical significance is indicated: * p<0.1, ** p<0.05, *** p<0.01.
Table 6: Returns to Occupational Propensities over the Two Cohorts

<table>
<thead>
<tr>
<th></th>
<th>(1) Log Wage</th>
<th>(2) Log Wage</th>
<th>(3) Log Wage</th>
<th>(4) Log Wage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>181.15***</td>
<td>180.16***</td>
<td>185.17***</td>
<td>176.66***</td>
</tr>
<tr>
<td></td>
<td>(3.10)</td>
<td>(2.83)</td>
<td>(3.11)</td>
<td>(3.76)</td>
</tr>
<tr>
<td>Const x NLSY97</td>
<td>-7.90</td>
<td>-9.18</td>
<td>-10.27</td>
<td>5.60</td>
</tr>
<tr>
<td></td>
<td>(6.74)</td>
<td>(6.35)</td>
<td>(6.57)</td>
<td>(7.97)</td>
</tr>
<tr>
<td>Prop High Occ</td>
<td>0.31***</td>
<td>0.35***</td>
<td>0.03</td>
<td>-0.06</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.05)</td>
<td>(0.08)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>Prop H Occ x NLSY97</td>
<td>0.29***</td>
<td>0.27***</td>
<td>0.25**</td>
<td>0.30**</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.10)</td>
<td>(0.13)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>Prop Low Occ</td>
<td>-1.65***</td>
<td>-1.64***</td>
<td>-1.80***</td>
<td>-1.75***</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.16)</td>
<td>(0.17)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>Prop L Occ x NLSY97</td>
<td>0.70*</td>
<td>0.83**</td>
<td>0.86**</td>
<td>0.91**</td>
</tr>
<tr>
<td></td>
<td>(0.39)</td>
<td>(0.38)</td>
<td>(0.38)</td>
<td>(0.38)</td>
</tr>
<tr>
<td>College</td>
<td></td>
<td></td>
<td>19.23***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2.92)</td>
<td></td>
</tr>
<tr>
<td>Coll x NLSY97</td>
<td></td>
<td></td>
<td>4.04</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(5.20)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>4154</td>
<td>4149</td>
<td>4149</td>
<td>4149</td>
</tr>
<tr>
<td>R²</td>
<td>0.09</td>
<td>0.11</td>
<td>0.11</td>
<td>0.12</td>
</tr>
<tr>
<td>College 1st-stage</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Degree dummies</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: The table reports OLS wage regressions of 100 times the deflated log wage on propensities to enter occupations (predicted relative occupation-specific skills) and the change in the coefficient between the NLSY79 and the NLSY97. The propensities are from the NLSY79 only and they are from multinomial logit regressions of occupational choice including mathematical, verbal, and mechanical talent terciles, illicit activities, precocious sex and dummies for respondents’ race. “x NLSY97” stands for the interaction between the variable and an NLSY97 dummy, i.e. the change in the coefficient between the NLSY79 and the NLSY97. The specification in column two adds a college dummy to the first-stage propensity estimation. Columns three and four take the propensities from column one but add in the wage regression a dummy for college degree and the talents that were used in the estimation of the propensities, respectively. Standard errors are from bootstrapping the first (estimating the propensities) and second stage regressions together 500 times and they are reported below the coefficients. * p<0.1, ** p<0.05, *** p<0.01.
Table 7: Estimates of Wage Rate Changes across Occupations

<table>
<thead>
<tr>
<th>Method</th>
<th>( \Delta (\pi_H - \pi_M) ) (in % (s.e.))</th>
<th>( \Delta (\pi_L - \pi_M) ) (in % (s.e.))</th>
<th>( \Delta \pi_M ) (in %)</th>
<th>Model Test (p-value in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS on Propensities</td>
<td>25.1 (12.3)</td>
<td>32.9 (38.4)</td>
<td>-4.24 (7.7)</td>
<td></td>
</tr>
<tr>
<td>OLS on Propensities (1st-stage college)</td>
<td>22.9 (10.9)</td>
<td>41.3 (38.6)</td>
<td>-5.26 (7.8)</td>
<td></td>
</tr>
<tr>
<td>Opt. Min. Distance</td>
<td>20.1 (9.7)</td>
<td>31.4 (35.2)</td>
<td>-2.4 (10.7)</td>
<td>13.1</td>
</tr>
<tr>
<td>Fixed Effects in PSID (from Cortes, 2014)</td>
<td>40</td>
<td>25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The table presents estimated relative wage rate changes in the high- and the low-skill occupation compared to the middle-skill occupation across methods and data. OLS on propensities restates the result presented in equation (12) in the main text. The next set of wage rate estimates adds a college dummy into to first-stage occupational choice regression along the lines of column two in table 6. The third set of wage rate estimates is from a minimum distance estimation carried out in earlier versions of this paper (e.g. Boehm, 2013). This estimation approach also provides a test of the restrictions on talent returns implied by the model. The final set of wage rate estimates is from a recent paper by Cortes (2014) using PSID panel data and a fixed effects regression strategy. The wage rates are crude and don’t have standard errors because they are read off from figure of 5 in this paper for the period mid-1980s to 2007.
Appendix

A Detailed Sample Construction

I use data from the National Longitudinal Survey of Youth (NLSY) cohort of 1979 and 1997. As is necessary for this paper, the NLSY studies provide detailed information about individuals’ background, education, and labor market outcomes.

I restrict my attention to males for several reasons. Firstly, polarization seems to have had the most dire effect on males (Acemoglu and Autor, 2011). Secondly, female hours worked and thus the type of selection of females into the labor market (see Mulligan and Rubinstein, 2008) changed substantially over the two NLSYs. In addition, females made strides in educational attainment, their wages rose across the whole distribution, and attitudes towards them and discrimination against them in the labor market seem to have changed drastically. Thus, there are diverse changes in (the structure of) female labor supply and demand that are likely to work aside from the forces of polarization. Restricting the analysis to males provides a cleaner comparison of workers across the two decades between the NLSY79 and the NLSY97.

I evaluate individuals’ labor market outcomes at age 27 with the NLSY79 birth cohorts of 1957–64 reaching that age in 1984–92 and the NLSY97 birth cohorts of 1980–82 reaching it in 2007–09. Table 1 summarizes how the sample restrictions, attrition, and labor market participation for males reduce my sample size from 6,403 to 3,054 and from 4,599 to 1,207 males in the NLSY79 and the NLSY97, respectively. I restrict the sample to individuals who participated in the Armed Services Vocational Aptitude Battery of tests (ASVAB) in the first survey year. This restriction is necessary because the ASVAB provides measures of different dimensions of talent for each individual that are comparable over the two cohorts.

The participation in ASVAB is substantially lower in the NLSY97 than the NLSY79 where almost everyone participated. Moreover, sample attrition at age 27 is higher in the NLSY97 than the NLSY79 and overall only 63 percent of the NLSY79 participated in ASVAB and are also present at age 27. This problem is known (e.g. Altonji, Bharadwaj, and Lange, 2012; Aughinbaugh and Gardecki, 2007) and the attrition and non-test-participation rates in my data closely line up with those reported in the study by Altonji, Bharadwaj, and Lange (2012, henceforth ABL). The only difference is that ABL consider outcomes at the younger age of 22 and thus have slightly lower attrition rates.

37 Other papers in the literature that do this include Firpo, Fortin, and Lemieux (2011) and Cortes (2014).
In their paper, ABL note that the higher attrition rate in the NLSY97 may be partly due to NLSY97 respondents being first interviewed at ages 12–16 versus ages 14–21 for the NLSY79 and thus had more time to attrit. ABL further extensively examine the potential non-randomness of attrition and non-test-participation and its likely impact in biasing important labor market outcomes. Aughinbaugh and Gardecki (2007) do a similar exercise but focus on social and educational outcomes. Both studies find evidence that attrition is not random with respect to youths’ outcomes and their backgrounds. However, Aughinbaugh and Gardecki (2007) conclude that attrition from the NLSY97 does not appear to affect inference when estimating the three outcomes at age 20 that they are considering and ABL decide that the differences between non-attriters and the whole sample are not forbidding.

Moreover, ABL carefully select the samples of NLSY79 and NLSY97 to make them comparable to one another and compute weights that adjust for attrition and non-test-participation on observable characteristics. I closely follow their procedures for constructing my own sample. First, I follow ABL in excluding from the NLSY79 immigrants who arrived in the United States after age 16. This is done because the scope of the NLSY97 (age 12–16) also doesn’t include older than age 16 arrivals. Second, I exclude the economically disadvantaged whites and military supplemental samples from the NLSY79 because they were discontinued early on in the survey and thus don’t provide labor market outcomes at age 27 (or for ABL’s purposes). Table 1 reports that 1,818 observations are dropped by making these restrictions to the sample. For each individual I retain the observation that is closest to 27 years and 6 months of age and then measure labor market and final educational outcomes from this observation.

ABL use a probit model to adjust the NLSY79 and NLSY97 base year sample weights to account for attrition and non-test-participation according to several observable characteristics, such as parental education, parental presence at age 14, indicators by birth-year, urban and SMSA residence status, indicator variables for race and gender, and an interviewer coded variable describing the attitude of the respondent during the interview. I also employ a probit model to adjust weights for attrition and non-test-participation and use the same specification and variables as ABL apart from leaving out parental presence at age 14. Alternatively, a fully stratified set of indicators for birthyear, year, sex, and race, as employed by the Bureau of Labor Statistics for weighting, yields very similar

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38Thus, for more information on the sample construction and for statistics on the effects of attrition, please refer to ABL in addition to the description provided here. I would like to thank Prashant Bharadwaj for providing me with their data and do-files.
results. As ABL do in their paper, I proceed from this point with the assumption that, after attrition weighting, my two NLSY samples are representative of the population of young Americans that they are supposed to cover. These samples have the size of 3,939 and 1,737 individuals in the NLSY79 and the NLSY97, respectively.

I follow Lemieux (2006), who uses CPS May Outgoing Rotation Group data, in how I compute wages and in defining the sample of working individuals (henceforth labor supply). I use hourly wages reported for the current main job instead and normalize them to 1979 real values by adjusting with the PCE deflator provided by the St. Louis Federal Reserve Bank. While Lemieux (2006) removes outliers with 1979 real hourly wages below $1 and above $100, I remove the high wages from $40 onward because my NLSY wage data is very inaccurate for values above this threshold.

Finally, in order to condition on the sample of working individuals, I keep all individuals who report not to be self-employed, and who are employed in a non-farm, non-fishing and non-forestry occupation according to the Census 1990 three-digit occupation classification. This leaves me with an analysis sample of 3,054 and 1,207 males in the NLSY79 and NLSY97, respectively (compare table 1 again). As in Lemieux (2006) I weight all of those individuals by the number of hours that they work per week on top of the sample weights that are adjusted for test-participation and attrition.

B Generalization of the Theory to Three Occupations

In the following I derive predictions (7) and (8) for the three-occupation case. For ease of exposition, wages in occupations (4) are reproduced here:

\[ w_{K_{it}} = \pi_{K_{it}} + s_{K_{it}}. \]

Note that \( s_{K_{it}} \) can be a general function of observable talents \( x_{it} \) and unobservables. From equation (3) and the wages in occupations we have:

\[
  w_{it} = \begin{cases} 
  w_{H_{it}} = \pi_{H_{it}} + s_{H_{it}} & \text{if } H_{it} = 1 \\
  w_{M_{it}} = \pi_{M_{it}} + s_{M_{it}} & \text{if } M_{it} = 1 \\
  w_{L_{it}} = \pi_{L_{it}} + s_{L_{it}} & \text{if } L_{it} = 1 
  \end{cases}
\]

\(^{39}\)I thank Steve McClaskie and Jay Zagorsky for providing me with the official attrition-adjusted sample weighting program for the NLSY.

\(^{40}\)Source: “Personal Consumption Expenditures: Chain-type Price Index (PCECTPI)”, accessed 2012-8-14, http://research.stlouisfed.org/fred2/series/PCECTPI
When occupational wage rates change, by the envelope theorem, the marginal change in worker \( i \)’s wage becomes

\[
dw_{it} = \begin{cases} 
  d\pi_H & \text{if } H_{it} = 1 \\
  d\pi_M & \text{if } M_{it} = 1 \\
  d\pi_L & \text{if } L_{it} = 1.
\end{cases}
\]

Thanks to its linearity, the expected change in wage given talents \( x_{it} \) and initial prices \( \pi_t \) can be written as

\[
E(dw_{it}|x_{it}, \pi_t) = p_H(x_{it}, \pi_t)d\pi_H + p_M(x_{it}, \pi_t)d\pi_M + p_L(x_{it}, \pi_t)d\pi_L,
\]

where \( p_K(x_{it}, \pi_t) \) is the probability for an individual of talent vector \( x_{it} \) to enter occupation \( K \) under prices \( \pi_t \). Exploiting that the three probabilities sum to one gives the three-occupation analogue to equation (7):

\[
dE(w_{it}|x_{it}, \pi_t) = d\pi_M + p_H(x_{it}, \tilde{\pi}_{HMi}, \tilde{\pi}_{LMi})d\tilde{\pi}_{HMi} + p_L(x_{it}, \tilde{\pi}_{HMi}, \tilde{\pi}_{LMi})d\tilde{\pi}_{LMi}, \tag{13}
\]

where \( \tilde{\pi}_{KMi} = \pi_{Ki} - \pi_{Mi} \) for \( K \in \{H, L\} \),

\[
p_H(x_{it}, \tilde{\pi}_{HMi}, \tilde{\pi}_{LMi}) = Pr[s_{Hi} - s_{Mi} > -(\pi_{Hi} - \pi_{Mi})],
\]

and similarly for \( p_L(x_{it}, \tilde{\pi}_{HMi}, \tilde{\pi}_{LMi}) \). According to prediction (13), workers who are ceteris paribus more likely to enter the high- or the low-skill occupation are expected to experience higher increases in wages.

Holding constant \( \tilde{\pi}_{HMi} \) and \( \tilde{\pi}_{LMi} \) at \( t = 0 \) and integrating equation (13) with respect to \( \pi_{Mi} \) we get

\[
E(w_{it}|x_{it}, \pi_{M1}, \tilde{\pi}_{HMI}, \tilde{\pi}_{LMI}) - E(w_{it}|x_{it}, \pi_{M0}, \tilde{\pi}_{HMI}, \tilde{\pi}_{LMI}) = \Delta\pi_M.
\]

Similarly,

\[
E(w_{it}|x_{it}, \pi_{M1}, \tilde{\pi}_{HMI}, \tilde{\pi}_{LMI}) - E(w_{it}|x_{it}, \pi_{M1}, \tilde{\pi}_{HMI0}, \tilde{\pi}_{LMI0}) = \int_{\pi_{HMI0}}^{\pi_{HMI1}} p_H(x_{it}, \tilde{\pi}_{HMI}, \tilde{\pi}_{LMI0})d\tilde{\pi}_{HMI},
\]

\[
E(w_{it}|x_{it}, \pi_{M1}, \tilde{\pi}_{HMI}, \tilde{\pi}_{LM1}) - E(w_{it}|x_{it}, \pi_{M1}, \tilde{\pi}_{HMI1}, \tilde{\pi}_{LM0}) = \int_{\tilde{\pi}_{LM0}}^{\pi_{LM1}} p_L(x_{it}, \tilde{\pi}_{HMI}, \tilde{\pi}_{LM1})d\tilde{\pi}_{LM1}.
\]
Summing these three expressions gives the three-occupation analogue to equation (8):

\[
E(w_{11} - w_{10}|x_{it}) = \triangle \pi_M + \frac{\pi_H(x_{it}, \tilde{\pi}_{HMI}, \tilde{\pi}_{LM0})}{\tilde{\pi}_{HMI} - \tilde{\pi}_{HMO}} \triangle (\pi_H - \pi_M) + \frac{\pi_L(x_{it}, \tilde{\pi}_{HMI}, \tilde{\pi}_{LM1})}{\tilde{\pi}_{LM1} - \tilde{\pi}_{LM0}} \triangle (\pi_L - \pi_M).
\]  

Hence, the overall wage effect of polarization for workers of talent \(x_{it}\) includes the direct price effect as well as the reallocation effect of moving into the high- and low-skill occupations.

C Estimating the Price Changes for Three Occupations

Analogous to the estimation of the relative price change using equation (8), we want to estimate the relative price changes \(\triangle (\pi_H - \pi_M)\) and \(\triangle (\pi_L - \pi_M)\) using equation (14) for the three-occupation case. Since the choice probabilities on the adjustment path are unknown, they have to be approximated analogously to equation (9) and figure 7:

\[
\begin{align*}
\pi_H(\tilde{\pi}_{HMI}, \tilde{\pi}_{LM0}) &\approx \pi_H(\tilde{\pi}_{HMO}, \tilde{\pi}_{LM0}) + \frac{\pi_H(\tilde{\pi}_{HMI}, \tilde{\pi}_{LM1}) - \pi_H(\tilde{\pi}_{HMO}, \tilde{\pi}_{LM0})}{\tilde{\pi}_{HMI} - \tilde{\pi}_{HMO}} (\tilde{\pi}_{HMI} - \tilde{\pi}_{HMO}) \\
\pi_L(\tilde{\pi}_{HMI}, \tilde{\pi}_{LM0}) &\approx \pi_L(\tilde{\pi}_{HMO}, \tilde{\pi}_{LM0}) + \frac{\pi_L(\tilde{\pi}_{HMI}, \tilde{\pi}_{LM1}) - \pi_L(\tilde{\pi}_{HMO}, \tilde{\pi}_{LM0})}{\tilde{\pi}_{LM1} - \tilde{\pi}_{LM0}} (\tilde{\pi}_{LM1} - \tilde{\pi}_{LM0}).
\end{align*}
\]

Plugging these linear approximations into (14) and picking up the dependence on the talent vector \(x_{it}\) again, we get:

\[
E(w_{11} - w_{10}|x_{it}) = \triangle \pi_M + \frac{p_H(x_{it}, \pi_1) + p_H(x_{it}, \pi_0)}{2} \triangle (\pi_H - \pi_M) + \frac{p_L(x_{it}, \pi_1) + p_L(x_{it}, \pi_0)}{2} \triangle (\pi_L - \pi_M).
\]

A linear wage regression of the type

\[
w_{it} = a_0 + a_1 \overline{p}_H(x_{it}) + a_2 \overline{p}_L(x_{it}) + a_3 \times NLSY97 + a_4 \overline{p}_H(x_{it}) \times NLSY97 + a_5 \overline{p}_L(x_{it}) \times NLSY97 + \epsilon_{it},
\]

with \(\overline{p}_K(x_{it}) \equiv \frac{p_K(x_{it}, \pi_1) + p_K(x_{it}, \pi_0)}{2}\), provides \(a_3 + a_4 \overline{p}_H(x_{it}) + a_5 \overline{p}_L(x_{it})\) as the best linear predictor of \(E(w_{11} - w_{10}|\overline{p}_H(x_{it}), \overline{p}_L(x_{it})) = E(w_{11} - w_{10}|x_{it})\). Therefore, \(a_3\), \(a_4\), and \(a_5\)

\[\text{Note that one might prefer using } p_H(\tilde{\pi}_{HMI}, \tilde{\pi}_{LM0}) \text{ instead of } p_H(\tilde{\pi}_{HMI}, \tilde{\pi}_{LM1}) \text{ in the first approximation and } p_L(\tilde{\pi}_{HMI}, \tilde{\pi}_{LM0}) \text{ instead of } p_L(\tilde{\pi}_{HMO}, \tilde{\pi}_{LM0}) \text{ in the second, which are not observable in the data. Yet, } p_H(\tilde{\pi}_{HMI}, \tilde{\pi}_{LM0}) > p_H(\tilde{\pi}_{HMI}, \tilde{\pi}_{LM1}) \text{ while } p_L(\tilde{\pi}_{HMI}, \tilde{\pi}_{LM0}) < p_L(\tilde{\pi}_{HMO}, \tilde{\pi}_{LM0}), \text{ so this additional approximation error should not be too large.}\]
identify the wage rate changes $\Delta \pi_M$, $\Delta (\pi_H - \pi_M)$, and $\Delta (\pi_L - \pi_M)$, respectively.