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# A Wavelet Approach to Synchronization of Output Cycles\*

Andreas Esser<sup>†</sup>

**Abstract:** The dynamic comovement between time series is a key concept in macroeconomic analysis. The extent to which series are cyclically synchronized is particularly important for evaluating the feasibility of common policy measures for groups of countries. This paper investigates concepts in the time domain and in the frequency domain that have traditionally been used to detect and describe such cyclical comovements in output data. However, methods from the former category cannot account for different cycle lengths, while the statistics from the latter category fail to capture transient relationships.

Therefore, the use of multivariate wavelet analysis and a modification of the cohesion statistic from Fourier analysis is suggested to simultaneously assess comovement at the frequency level and over time for both country pairs and larger aggregates. The main finding from applying this method to output cycles is that synchronization does indeed vary across both dimensions and that important events during the time span of the sample, such as the introduction of the Euro, can be visualized. As a further benefit of the wavelet approach, it turns out to be hardly sensitive to the technique employed to extract the cyclical component from the output series.

**JEL classification:** C40, C65, E32, F15

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\*The views expressed in this paper are those of the author and do not necessarily reflect the opinions of the Deutsche Bundesbank.

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# 1 Introduction

Finding ways to assess comovement of economic time series has been a key issue for economic research, due to its relevance for a broad range of topics. Traditionally, measures to capture such relationships have been based in the time domain, while recently approaches in the frequency domain have gained popularity as well. However, results have often been rather vague and displayed lack of robustness, making it worthwhile to consider alternative approaches. A promising concept widely used in a broad variety of scientific applications, but not very often in economics, is wavelet analysis. The main idea behind this concept is to split up a given function or continuous-time signal into different scale components and study each of them with a resolution matching its scale. In particular, the continuous wavelet transform is well-suited to the issue because it allows to consider both aspects pertaining to the time domain and to the frequency domain simultaneously. While the wavelet approach can be used for many different investigations of comovement in economics, in this paper it is applied to output data. The degree of synchronization will be assessed for various countries representing members of the European common currency zone as well as major economies from outside that group. In particular, a new measure for synchronization of groups of countries which is localized in both time and scale is presented.

The issue of business cycle synchronization has received much attention with the creation and ongoing extension of the European Monetary Union. A lot of research on this topic has focused on the long-run convergence of member countries to common levels for important macroeconomic aggregates. The Maastricht convergence criteria as a prerequisite for admission to the group of countries using the Euro reflect several of these aspects, such as price stability, similar long-term interest rates, and exchange rates limited to a narrow band. These criteria have – among others – also been put forward as requirements for an optimum currency area (OCA) by Mundell (1961).

However, suitability for a monetary union does not only require long-term convergence between countries but also common characteristics in the cyclical components of their economies. That is, a sufficient degree of business cycle comovement is required to conduct a common monetary policy effectively. If asymmetric shocks were affecting member countries of a currency union and as a

consequence business cycles were not synchronized, a common policy measure could not yield favorable outcomes in all member countries; with those adversely affected being bereaved of their own tools to achieve stability for their particular situation. To evaluate whether the benefits of a monetary union – namely lower transaction costs and possibly more transparent pricing – outweigh these costs of giving up the option of individual policy intervention, the linkage of business cycles thus turns out to be an important issue in addition to looking at the long-term convergence goals.

According to the original definition of a business cycle by Burns and Mitchell (1946) it is characterized as a recurrent sequence of expansions and contractions in aggregate economic activity which is not periodic like a seasonal pattern and which cannot be divided into shorter cycles with similar amplitude and features. When using Gross Domestic Product data as a proxy for economic activity, the business cycle thus captures those components of output with higher frequencies than long-term growth components, but lower ones than short-term noise. It is generally agreed that business cycles typically range in length from approximately two to eight years.

Various methods have been introduced to measure business cycles. Approaches in the literature involve categorizing output series into periods of sustained growth and decline as advocated by Bry and Boschan (1971) or Harding and Pagan (2006), considering growth rates over a specific interval, or applying various filtering techniques that are able to extract cycles of a specified length. Among these, the Hodrick-Prescott filter (Hodrick and Prescott, 1997) is most widely known and used in economic applications, yet other methods are able to cut off frequencies corresponding to business cycles more precisely, such as the bandpass filter due to Christiano and Fitzgerald (2003). Results from different synchronization statistics will be compared to assess in which cases the selection of filtering procedure is crucial to the findings. It would be desirable to have synchronization measures that show little dependence on the exact choice of extraction technique, because none of the different approaches can be claimed to be generally superior to the others.

The further setup is as follows. The next section provides a brief overview of previous literature on business cycle synchronization. Afterwards, the characteristics of the output data used for the empirical investigation are presented and the filtering methods to extract the cycle are discussed. Section 4 introduces and

applies several methods to capture synchronization in the time domain, while section 5 does the same for approaches in the frequency domain. Synchronization across the time and the frequency domain using wavelet analysis is investigated in section 6. Finally, section 7 concludes.

## 2 Literature Review

Previous studies have suggested a variety of different measures to capture business cycle synchronization for pairs or groups of countries. Just using regular contemporaneous correlation coefficients between growth cycles of EU-12 members, Fatas (1997) finds higher values in the sub-sample after the creation of the European Monetary System (EMS) than in the pre-EMS period. Döpke (1998) also considers contemporaneous correlation for the cyclical component obtained from applying the Hodrick-Prescott filter to output data of five core Euro countries using a rolling window approach. He observes increases in correlation for most countries, yet with several exceptions pointing in the other direction. Looking at pairwise correlations and employing the band-pass filter introduced by Baxter and King (1999), Wynne and Koo (2000) find that there is some synchronization between founding members of the European Union, while evidence is weaker for newer member states.

Instead of using output data, which is typically only available at a quarterly frequency, several authors also resort to using industrial production data, which is provided as a monthly series. Among these, Artis and Zhang (1997, 1999) find that synchronization has increased for country pairs within the EMS, while it has diverged for others. Their finding, however, cannot be replicated by Inklaar and de Haan (2001) with the same data but a slightly longer sample. Massmann and Mitchell (2004), using rolling windows rather than just a pre-EMS and a post-EMS subsample on the same data, finally conclude that the Euro area has switched between periods of close comovement and phases of divergence throughout the sample. For recent years, they record evidence of increasing synchronization.

Harding and Pagan (2002, 2006) propose a way of modelling the synchronization of cycles that is quite different from the correlation measure in previous papers. They suggest analyzing a constructed binary variable, which is set equal to unity in case of an upward movement in a series and to zero when the direction is downward. This allows them to capture expansions and contractions with much

more emphasis on the turning points of the series and hence the classical view of the business cycle by Burns and Mitchell (1946). However, by discarding all information besides the sign, a lot of detail is lost in this process. Correlations are then computed between these binary series, leading to only low values for individual Euro member countries against the group average.

Another main branch of the literature is based on frequency domain analysis. Several studies have adopted this tool to study relationships at the frequency level (e.g. A'Hearn and Woitek, 2001; Breitung and Candelon, 2006). Measures to quantify a comovement relationship between variables at the frequency level have been suggested by Croux et al. (2001), who introduce the dynamic correlation coefficient and a multidimensional counterpart termed cohesion. Using these statistics, Croux et al. find that cycles of U.S. states are more similar than those of European countries, while Valle e Azevedo (2002) also finds high dynamic correlations between European countries and the Eurozone average. Allowing for time-varying coherence, Hughes Hallett and Richter (2006), however, conclude that the coherence between the United Kingdom and the Eurozone is unstable at best, while it is even decreasing for Germany and the Eurozone.

In recent years, some studies have appreciated the advantages wavelet analysis offers for the study of business cycles. Among these, Jagrič and Ovin (2004) compare different wavelet types to measure synchronization of industrial production for Slovenia and Germany and find evidence of increasing synchronization over time. On the other hand, Crowley and Mayes (2008) use wavelet analysis for quarter-on-quarter growth rates of France, Germany, and Italy with the result that cycles continue to differ for each of the pairs. Rua (2010) introduces a refined version of the cross-wavelet spectrum to find that the amount of comovement depends on the frequency and changes over time. Finally, Aguiar-Conraria and Soares (2011) compare industrial production data for each EU-15 country against the weighted EU average and find the highest degree of synchronization for France and Germany, with more peripheral countries being more detached. All wavelet approaches present in the literature so far only allow to check pairs of series for comovement, not larger groups.

### 3 Data and Cycle Extraction

The data set used in this paper comprises output data for a number of key European economies, namely France, Germany, Italy, the Netherlands, Spain, Switzerland, and the United Kingdom. For the following analyses, besides this baseline group of countries, two further groups are considered for comparison purposes. One leaves out the latter two countries of the above list, hence focusing only on countries which share the Euro as a common currency, while the other one adds Japan and the United States representing important economies outside Europe. In summary, and introducing the abbreviations used throughout the paper, the country groups thus are

- *Euro*: FR, DE, IT, NL, ES;
- *European*: FR, DE, IT, NL, ES, CH, UK;
- *all*: FR, DE, IT, NL, ES, CH, UK, US, JP.

All data are obtained from the OECD's Quarterly National Accounts database and represent annual levels of seasonally adjusted output data measured at current prices. Data is available for each quarter from the beginning of 1961 to the end of 2010, covering a total of 50 years.

To investigate the cyclical properties of the data, a variety of measures are computed. These include growth rates as well as the cyclical components obtained from different filtering techniques. A problem of using growth rates is that they severely amplify high-frequency components and consequently attenuate lower frequencies (Baxter and King, 1999). This increases the noise in the extracted cycle and thus limits the usefulness of the approach. Nevertheless, they are widely used as a proxy for business cycles, so that both quarter-on-quarter and year-on-year growth rates will be considered.

In addition to them, two further cycle extraction methods will be used. The Hodrick-Prescott filter (Hodrick and Prescott, 1997) separates a series into a cyclical and a trend component and allows different degrees of smoothing by adjusting a penalty parameter for deviations from the trend. Here, the parameter will be set to 1,600, the standard for quarterly data in the literature. This filter is probably the most widely used in economics, yet King and Rebelo (1993) stress that it may seriously alter measures of persistence, variability, and comovement.

Lastly, Christiano and Fitzgerald (2003) propose a band-pass filter using an asymmetric weighting scheme that avoids having to cut off values at the beginning and end of the sample, which would otherwise be the case for a symmetric version. It is devised as a combination of a low-pass and a high-pass filter and designed to pass through cycles with a length between 8 and 32 quarters without modification, but eliminate movements that have a different frequency.

It is clear that the choice of cycle extraction technique affects any result on cycle synchronization, because the approaches are dissimilar in nature. Yet, as pointed out in section 2, they are often used interchangeably with the same purpose in mind. Also, there is disagreement on how strongly the choice actually influences results. Canova (1998) points out that alternative filters extract different types of information from the original series and asserts that the idea of having just one method corresponding to the exact definition of a business cycle is misleading. As a consequence, he explicitly suggests to subject data to various filtering methods. Burnside (1998) does not consider this to be a problem, yet acknowledges that different filtering techniques may provide different insights concerning the cycle. Massmann and Mitchell (2004) raise the point that the choice of filtering technique can affect the exact shape of what is then termed the “cyclical component”, but may at the same time have no impact regarding convergence or synchronization. Yet the results from the literature that vary greatly with the choice of method imply that this might actually well be the case. To account for these observations, it is worthwhile to compare the different filtering methods when checking for possible synchronization.

## 4 Synchronization Measures in the Time Domain

As outlined in the previous section, many studies on cyclical comovement have considered the standard contemporaneous correlation coefficient as a measure of alignment. That is,

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \quad (1)$$



has been computed for two series  $x_t$  and  $y_t$ , with  $\sigma_{xy}$  denoting the covariance between  $x$  and  $y$ . Exemplarily, this exercise is considered for the cyclical component of the HP filter here. Table 1 reports the resulting correlations for each combination of countries.

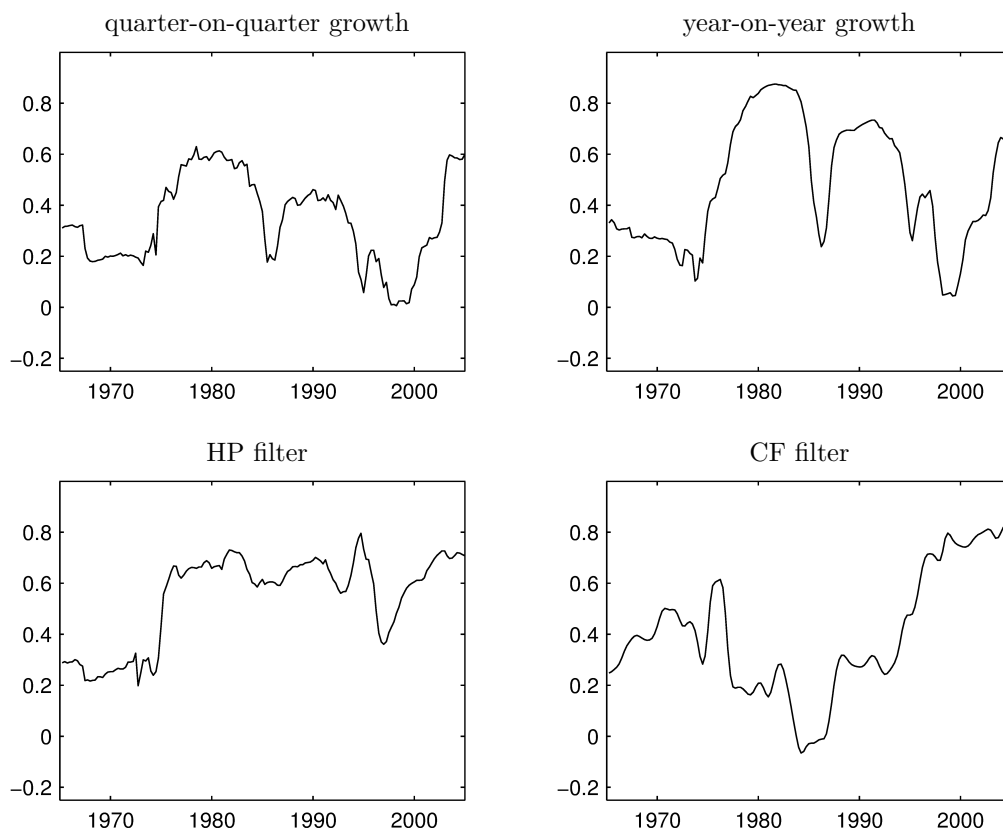
	FR	DE	IT	NL	ES	CH	UK	US	JP
FR	1.000	0.549	0.710	0.699	0.705	0.641	0.472	0.328	0.410
DE		1.000	0.468	0.654	0.523	0.643	0.385	0.393	0.629
IT			1.000	0.619	0.706	0.627	0.362	0.342	0.438
NL				1.000	0.610	0.641	0.439	0.392	0.437
ES					1.000	0.641	0.412	0.299	0.439
CH						1.000	0.295	0.290	0.497
UK							1.000	0.546	0.443
US								1.000	0.352
JP									1.000

**Table 1:** Static correlations for the cyclical component of the HP filter at business cycle frequencies

It is apparent that the correlation is stronger between countries of the Eurozone, as shown in the upper left corner of Table 1 up to the dashed lines. For that group, the average correlation is 0.624, with some subgroups such as France–Italy–Spain showing even higher figures. While across the European country group, delimited by the next set of dashed lines, the average drops to 0.562, this stems notably from the poor alignment of the United Kingdom’s cycle with that of the Euro countries, while Switzerland – being entirely surrounded by Euro members – has much higher correlations with each of them. Finally, looking at the whole set of countries, the data indicate that the United States’ cycle is only correlated weakly with those of all other countries except the U.K. For the combinations involving Japan, correlations are also below 0.5 with the notable exception of the Japan–Germany pair with  $\rho = 0.629$ . The average of correlations for the entire set of countries amounts to 0.501 and is thus lower than for both of the smaller groups.

A rolling window approach can be used to investigate changes that have occurred in this quantity. Instead of considering the entire time series, the correlation

coefficient is computed for 10-year windows moving across the entire sample from 1960 to 2010. Figure 1 illustrates this development over time for the example of the correlation between the German and the French cycle and compares results for the different methods of cycle extraction. The upper left panel is based on the quarter-on-quarter growth rates and the upper right panel considers the yearly growth data. In the lower half of the figure, the left panel shows the results using the cyclical component of the HP filter and the right panel depicts those based on the CF filter. The years on the  $x$ -axis denote the centers of the respective 10-year-intervals for which the correlation measure is computed.



**Figure 1:** Development of the correlation between the cyclical components of output for France and Germany over time. The years on the  $x$ -axis of each panel denote the centers of rolling ten-year intervals.

An immediate observation from Figure 1 is that the results depend strongly on the method that is used to extract the cycle. Furthermore, the correlation substantially varies over time, following no apparent pattern except for some

increase towards the very end of the sample in all four cases. In any case, from these statistics no inference can be drawn concerning the length of common cycles leading to correlation, so the following section will venture into the frequency domain in order to find measures better suited to capturing periodicities of a particular frequency.

## 5 Synchronization Measures in the Frequency Domain

In order to obtain appropriate comovement indices or measures for common cyclical features, this section shifts the focus to frequency-domain approaches. The goal is to extract short-run and long-run properties of the relationship between series by considering cyclical components of a particular frequency.

Let the spectral density functions of two time series  $x$  and  $y$  be given by  $S_x(\omega)$  and  $S_y(\omega)$ , respectively, while the cross spectrum for the two is denoted by  $S_{xy}(\omega)$ . As a widely-used concept in the literature, coherency between  $x$  and  $y$  is defined as

$$h_{xy}(\omega) = \frac{S_{xy}(\omega)}{\sqrt{S_x(\omega)S_y(\omega)}}. \quad (2)$$

Croux et al. (2001) argue that a slightly different quantity is a better choice for the analysis of comovement. They consider just the real part of coherency, and refer to that quantity as the dynamic correlation between  $x$  and  $y$  at frequency  $\omega$ . Dynamic correlation can also be specified as

$$\rho_{xy}(\omega) = \frac{C_{xy}(\omega)}{\sqrt{S_x(\omega)S_y(\omega)}}, \quad (3)$$

where  $C_{xy}(\omega)$  denotes the cospectrum. A useful feature of dynamic correlation is that it can not only be computed for a specific choice of frequency  $\omega$ , but also be defined for an entire frequency band  $\Lambda = [\omega_1, \omega_2]$  by integrating the spectra over

the individual frequencies; that is,

$$\rho_{xy}(\Lambda) = \frac{\int_{\Lambda} \rho_{xy}(\omega) \sqrt{S_x(\omega) S_y(\omega)} d\omega}{\sqrt{\int_{\Lambda} S_x(\omega) d\omega \int_{\Lambda} S_y(\omega) d\omega}}. \quad (4)$$

If the entire range of frequencies is covered, the dynamic correlation coefficient is the same as the static correlation coefficient from the previous section. By choosing  $\Lambda$  to represent only a sub-interval of the entire range of frequencies, however, a special focus can be put on correlations pertaining to cycles of a particular length. Because interest predominantly lies in the synchronization of series at business cycle frequencies, the frequency band to be considered is chosen to represent cycles with a period between 2 and 8 years. This approximately means  $\Lambda_{bc} = [.20, .79]$ . Results for this choice of  $\Lambda$  using the cyclical component of the HP filter as data input are summarized in Table 2.

	FR	DE	IT	NL	ES	CH	UK	US	JP
FR	1.000	0.561	0.749	0.762	0.799	0.663	0.518	0.364	0.402
DE		1.000	0.497	0.695	0.586	0.660	0.440	0.462	0.667
IT			1.000	0.704	0.735	0.666	0.398	0.365	0.389
NL				1.000	0.699	0.729	0.462	0.398	0.482
ES					1.000	0.681	0.430	0.316	0.430
CH						1.000	0.328	0.332	0.467
UK							1.000	0.571	0.534
US								1.000	0.431
JP									1.000

**Table 2:** Dynamic correlations for the cyclical component of the HP filter at business cycle frequencies.

The results resemble those of the contemporaneous correlation coefficients from the previous section. Those connections between countries that appeared especially weak or strong in Table 1 also do in Table 2. On average, the dynamic correlations at business cycle frequencies are slightly higher than the correlations considering all frequencies. The averages for  $\rho(\Lambda)$  are 0.679 for the Euro group, 0.608 for the European group, and 0.538 when looking at all combinations of

the nine countries. This gives some indication that cyclical comovement may be higher at business cycle frequencies than elsewhere.

While dynamic correlation can only capture a cyclical relationship between two variables, it is possible to construct weighted averages of dynamic correlations for all possible combinations of two countries from a larger set  $y_t = (y_{1t} \dots y_{nt})'$ . The corresponding quantity introduced by Croux et al. is termed cohesion and computed as

$$\text{coh}_y(\omega) = \frac{\sum_{i \neq j} w_i w_j \rho_{y_i y_j}(\omega)}{\sum_{i \neq j} w_i w_j}, \quad (5)$$

where  $w_i$  and  $w_j$  are the weights assigned to variables  $y_i$  and  $y_j$ , respectively. Choosing  $w_i = 1$  for all  $i$  is possible, yet since the countries within the dataset are very dissimilar in terms of inhabitants and economic power, weighting by population or GDP may be more appropriate. For the empirical results in this section, the  $w_i$  represent the population in millions of the countries under consideration.

Again, it is possible to construct the cohesion measure for a frequency band  $\Lambda$ , yielding

$$\text{coh}_y(\Lambda) = \frac{\sum_{i \neq j} w_i w_j \rho_{y_i y_j}(\Lambda)}{\sum_{i \neq j} w_i w_j}, \quad (6)$$

Table 3 reports cohesion at business cycle frequencies for the three subgroups and all four types of cycle extraction methods.

subgroup	qtr. growth	yr. growth	HP filter	CF filter
all	.516	.555	.467	.484
European	.552	.607	.574	.592
Euro	.602	.664	.646	.648

**Table 3:** Cohesion at business cycle frequencies.

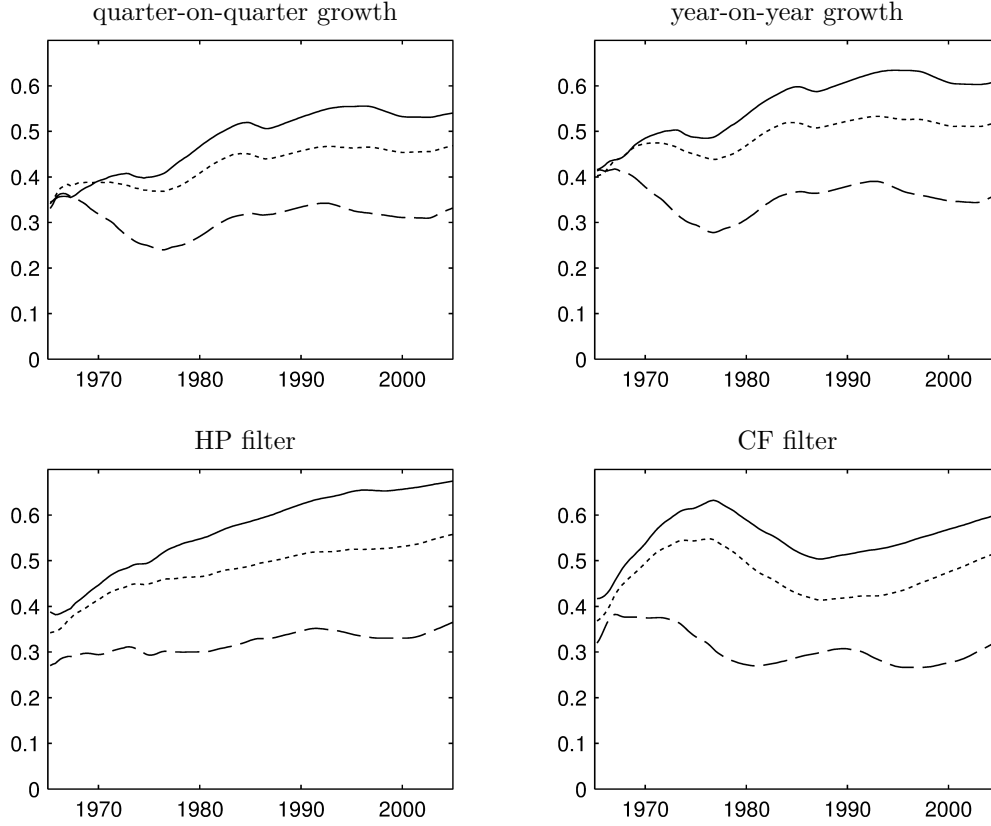
The column corresponding to the cyclical component of the HP filter confirms the result from the pairwise analysis using dynamic correlations. The general pattern, namely cohesion being highest for the Euro countries, followed by the

European group and lastly the whole set of countries, remains the same regardless of the filtering technique considered. However, the actual values for cohesion are somewhat different depending on the detrending method. For the HP and CF filters, they are spread across a bigger interval, with the value for the full group being smaller than those obtained using growth rates, but vice versa for the Euro group.

While the cohesion measure allows to look at the frequencies of interest more closely, it does not provide information concerning the development of comovement patterns over time. To this end, a rolling window approach is used again. The cohesion measure from equation (6) is computed for 10-year windows moving across the entire sample from 1960 to 2010. Figure 2 shows the development of the measure over time. The four filtering techniques are arranged in the same way as before. In each panel, the solid line corresponds to the Euro group, the dotted one represents the European group, and the dashed one shows results for all countries considered together.

According to all filtering methods, there is little evidence of increasing business cycle cohesion for the entire group of countries. Only for the HP-filtered data, a small rise over the years is observable; for the growth rate data, cohesion even falls considerably below its value for the initial window (1965–1975) for some time. For the other two groups, and particularly for the adopters of the Euro, cohesion increases towards the end of the sample. Except for the very early part of the quarterly growth series, cohesion for the Euro countries always exceeds that of the European group, which is in line with the results for the full sample. However, the patterns produced by the different filtering techniques are rather distinct. When looking at either of the growth rate panels, cohesion only rises during the first half of the sample and remains at about the same level afterwards. Using the cyclical component of the HP filter to the contrary yields cohesion values that increase throughout the sample. The result using the CF filter is especially surprising as it indicates a steep rise in cohesion for the first 15 years, followed by a decline during a period of approximately equal length and finally increasing values for the last 20 years again.

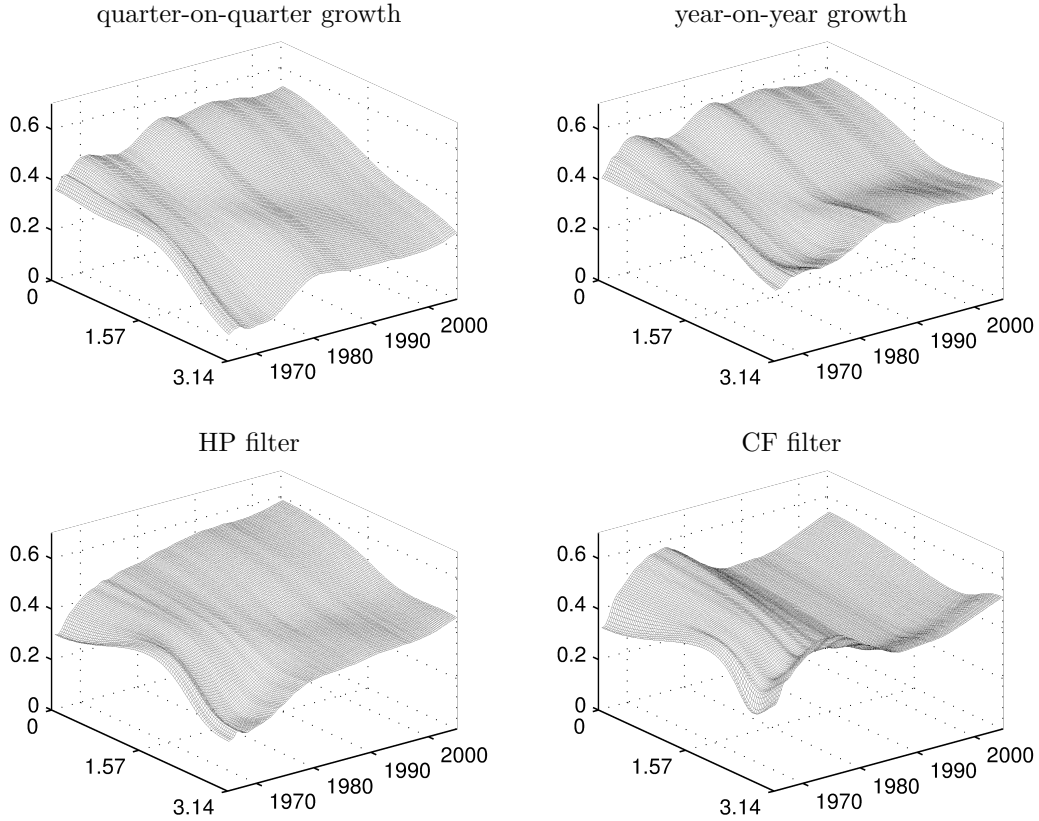
To consider the question whether cohesion is the same for cycles of different length, the previous summary statistic is disentangled. Rather than considering the range of business cycle frequencies as a whole, Figure 3 displays the pattern of cohesion across both time and frequencies in a three-dimensional plot. Again,



**Figure 2:** Development of cohesion at business cycle frequencies over time. The solid lines represent data from the Euro countries, the dotted lines correspond to the European group of countries, while the dashed line is based on data for all countries. The years on the  $x$ -axis of each panel denote the centers of rolling ten-year intervals.

10-year rolling windows are used. For the sake of clarity, only the European country group is considered in this figure, however, the respective graphs for the whole set of countries and for the Euro group are similar. In the three-dimensional plot of Figure 3, the  $x$ -axis denotes the frequencies, while the  $y$ -axis depicts the years representing the centers of the ten-year intervals.

When looking at the changes over time, it is notable that the measure increases during the first 20 years of the sample for most frequencies, but then levels off. It can be seen that cohesion is generally highest for the low frequencies, representing long-term growth effects. Also within the range of business cycle frequencies, that is, for  $\omega$  approximately between 0.20 and 0.79, longer cycles display higher



**Figure 3:** Comparison of cohesion results for the European country group. Cohesion is shown for all possible time-frequency combinations, with the frequencies on the  $x$ -axis and the years denoting the centers of rolling ten-year intervals on the  $y$ -axis.

cohesion than shorter ones. The decline in cohesion during the 1980s when looking at the CF-filtered data is present at all frequencies.

As a whole, the results presented in this section give some indication that comovement between output cycles for key Euro countries is more pronounced compared to other major economies. Furthermore, especially for the Euro countries a tendency towards increasingly strong common cycles is apparent over time. However, the results differ quite a bit depending on the filter used and the rolling window approach is a merely auxiliary technique to get some sense of the temporal evolution.



## 6 Wavelets

A common feature of the approaches discussed so far is their exclusive focus on either the time or the frequency domain. While regular correlation coefficients can capture relationships over time, no indication regarding the length of cycles is given. Similarly, considering dynamic correlation or cohesion, the time information included in the series is lost. Rolling window approaches as presented in the previous sections do provide some insights as to the evolvement of these measures over time, yet it is desirable to have one combined measure involving both the time and the frequency domain. Thus, this section presents a way to reconcile these two dimensions by the use of wavelet analysis.

### 6.1 The Wavelet Transform

The basic notion of a wavelet transform, as outlined in Daubechies (1988, 2006), is to consider a mapping from a given time series into a function of time and frequency. Two main types of wavelet transforms can be distinguished. The Discrete Wavelet Transform (DWT) limits itself to select discrete parameter values which are subsequently considered along the time and frequency dimensions. Because the result of the transformation from one to two dimensions is extremely redundant, this still allows to recover the original series from the DWT. The characteristic of just looking at specific values makes the DWT widely used in applications such as image processing, where compression of data is an essential objective. DWTs have also been considered in economics, e.g. by Ramsey and Lampart (1998) to investigate relationships between money supply and nominal income, or Shik Lee (2004) to analyze price and volatility spillovers in stock markets.

However, for the task of detecting business cycle synchronization, the second type of wavelet transforms – the Continuous Wavelet Transform (CWT) – is more appropriate. It is computationally more demanding, but provides the full redundant outcome of the transformation from a single time series into a set of time and frequency values. Therefore, the interpretation of results is made much easier.

Wavelet analysis addresses the issue that in Fourier analysis the time information included in a series is no longer available after transformation. It is not possible to pinpoint when an event took place or whether any of the cyclical components changed over time. While this is not much of a problem if series

exhibit similar properties throughout the sample, analysis becomes problematic as soon as different regimes and events begin and end within the time span under investigation. For the question at hand, the creation and enhancement of EMU ties constitute numerous such changes during the sample. All these transient dynamics are lost when relying on pure frequency analysis.

The key idea behind using wavelets thus is to look at the spectrum as a function of time and hence capture temporary developments as well. This is achieved by not considering waves of infinite duration, such as the sine or cosine in Fourier analysis, but rather “little waves” (or “ondelettes” in the original French literature). They consist of just a brief oscillation whose amplitude goes towards zero very fast as the function approaches  $\pm\infty$ . Intuitively, a wavelet can be compared to the recording of an earthquake by a seismograph.

To cover the entire real line despite the decay property required for wavelet functions, sets or families of wavelets are considered. They are derived from a mother wavelet  $\psi$  by scaling and shifting. A family  $\psi_{\tau,s}$  of daughter wavelets would then read

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-\tau}{s}\right). \quad (7)$$

The scaling parameter  $s$  influences the width of the wavelet through stretching ( $s > 1$ ) or compressing ( $s < 1$ ) and the translation parameter  $\tau$  controls the location of the wavelet by shifting its position in time. A wavelet created with a specific frequency and duration will then resonate if the signal embedded in the data contains components of this particular frequency.

The mother wavelet has to fulfill several technical conditions as pointed out by Daubechies et al. (1992). Its mean,  $\int_{-\infty}^{\infty} \psi(t)dt$ , must equal zero, while the integral of its square,  $\int_{-\infty}^{\infty} (\psi(t))^2 dt$ , has to be one. The latter requirement yields the limitation of the wavelet to a certain time interval. Additionally, the admissibility condition  $0 < C_{\psi} = \int_0^{\infty} \frac{|\hat{\psi}(\omega)|}{\omega} d\omega < \infty$ , where  $\hat{\psi}(\omega) = \int_{-\infty}^{\infty} \psi(t)e^{-i\omega t} dt$  denotes the Fourier transform of  $\psi$ , has to be met. The simplest choice of wavelet is the Haar

wavelet (Haar, 1910) defined by

$$\psi_H(t) = \begin{cases} 1 & \text{for } 0 \leq t < 0.5, \\ -1 & \text{for } 0.5 \leq t < 1, \\ 0 & \text{otherwise.} \end{cases} \quad (8)$$

which can be obtained by combining two rectangular functions. Its disadvantage is the obvious non-continuity of the function.

The vast majority of recent studies using CWTs instead focus on one group of continuous mother wavelets known as Morlet wavelets (Goupillaud et al., 1984) and given by

$$\psi_{\omega_0}(t) = \pi^{-1/4} e^{i\omega_0 t} e^{-t^2/2}. \quad (9)$$

The Morlet wavelet depends on one parameter,  $\omega_0$ . It is usually chosen to be  $\omega_0 = 6$ , which is also the value that will be used throughout the analysis here.

The popularity of the Morlet wavelets arises from a number of favorable properties. When considering the desired localization in both time and frequency, the Heisenberg uncertainty principle asserting that both cannot be determined to arbitrary precision simultaneously has to be considered. The Morlet wavelet minimizes the size of this Heisenberg window of uncertainty around a point, thereby reaching the lower bound for the inevitable uncertainty. Furthermore, the concentration of  $\psi$  in time is the same as in frequency, providing the best balance with respect to the two dimensions. Finally, as Lilly and Olhede (2009) outline, there are several ways to relate the scale parameter  $s$ , responsible for stretching the wavelet and thus corresponding to the space between oscillations, to Fourier wavelengths. For the Morlet wavelet with  $\omega_0 = 6$ , all these associated frequencies coincide, so the scale parameter can be treated like the frequency in Fourier analysis.

Thus focusing on this particular choice of wavelet, the CWT for some time series  $x(t)$  with respect to the wavelet  $\psi$  is given by

$$W_{\psi,x}(\tau, s) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{s}} \psi^* \left( \frac{t - \tau}{s} \right) dt, \quad (10)$$

where the asterisk denotes complex conjugation. The CWT maps the original

one-variable function into a function of both  $\tau$  and  $s$ , making it possible to conduct inference on both time and frequency simultaneously. The main difference of the CWT compared to a Fourier transform is the use of the wavelet instead of sine and cosine functions and the appearance of  $\tau$  as a localization parameter in the time domain. The wavelet power spectrum corresponding to the CWT is denoted as

$$WPS_{\psi,x}(\tau, s) = |W_{\psi,x}(\tau, s)|^2. \quad (11)$$

## 6.2 Cross-Wavelet Analysis

While the measures described so far aim at detecting time-frequency patterns in a single time series, the question of whether business cycles are aligned requires checking for common patterns in both. Measures for this purpose can be derived from their counterparts in pure frequency analysis.

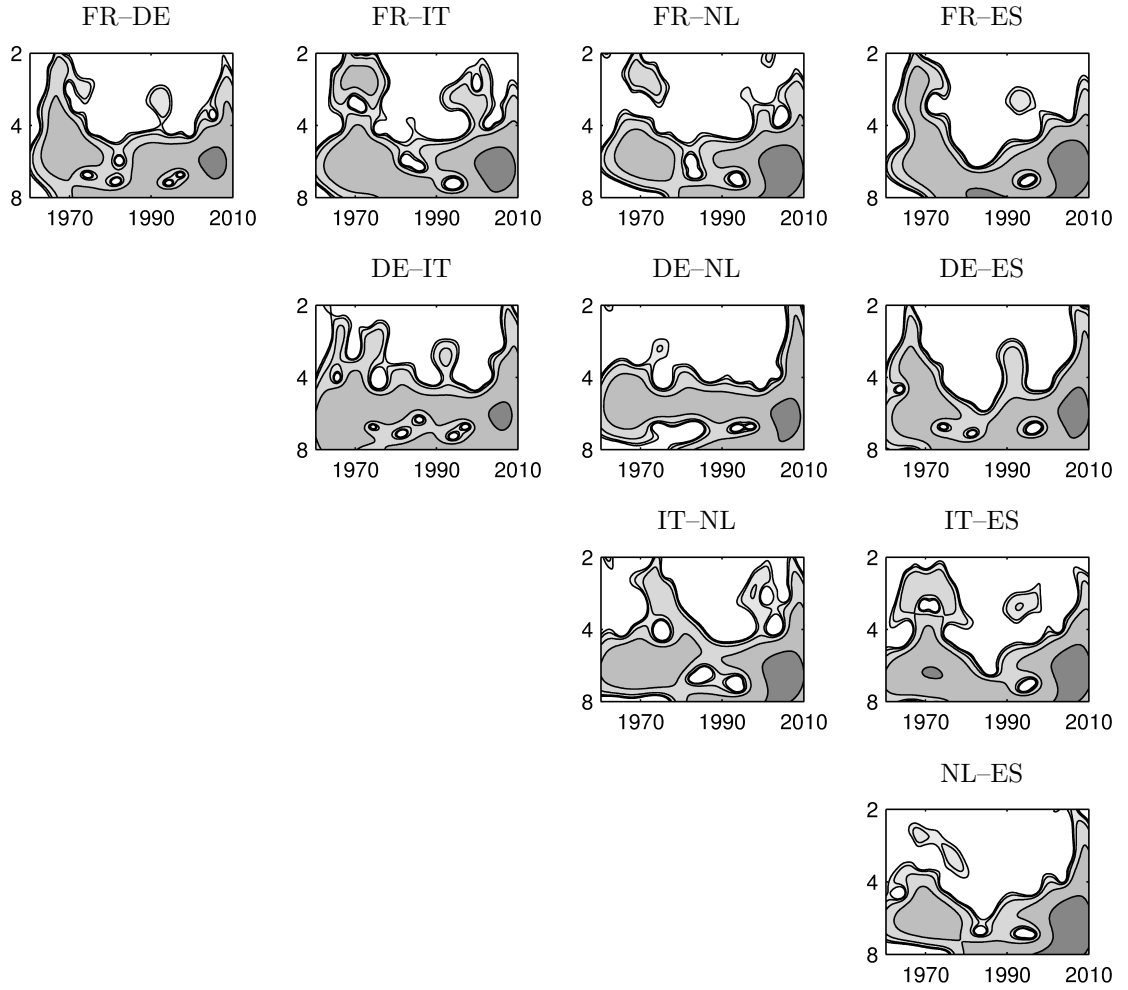
For two time series  $x$  and  $y$ , the cross-wavelet transform (XWT) has been introduced by Hudgins et al. (1993) as

$$W_{xy}(\tau, s) = W_x(\tau, s)W_y(\tau, s)^*, \quad (12)$$

suppressing the  $\psi$  in the index from now on, since the investigation is only concerned with the Morlet wavelet. It represents the covariance between two series at each possible combination of time and frequency and can hence serve as an indication of how similarities are distributed in this two-dimensional space.

The result is exemplarily visualized in Figure 4 for the pairs that can be constructed from the HP-filtered cyclical components of the members in the Euro country group. Dark areas depict time-frequency combinations spotting a high cross wavelet transform, while lower values are presented in lighter shades. For convenience, the  $y$ -axis denotes the length of the cycles in years rather than the corresponding scale parameter. While the results differ for the individual pairs, there is a general tendency that higher cross-wavelet transforms can be observed for cycles with a period of at least four years. Concerning the development over time, cross-wavelet transforms are generally lower in the middle of the sample than in the initial or final years. The highest values can be observed for the last decade, after the introduction of the Euro. Only for a few country pairs, such as Germany–Italy or Germany–Netherlands, the indication of comovement for cycle

lengths around five years is uninterrupted throughout the sample. Furthermore, there is some evidence of synchronization at higher frequencies, that is, for shorter cycles, during the 1970s and towards the end of the sample for most country pairs. The graphs in Figure 4 also illustrate the deficiencies of approaches which are either purely time- or purely frequency-based. The former would miss all differences depicted along parallels to the vertical axis, while the latter would disregard all changes over the horizontal axis.



**Figure 4:** Cross Wavelet Transforms (XWT) at business cycle lengths for pairs of Euro countries. For each panel, the  $y$ -axis shows the length of the cycle in years and the  $x$ -axis shows the localization in time. Time-scale combinations with a higher XWT are shaded darker.

A serious drawback of the cross-wavelet transform as introduced in equation (12) is the lack of normalization. The wavelet coherency measure, due to Torrence and Webster (1999), addresses this concern through normalizing  $W_{xy}(\tau, s)$  by the spectrum of each series. The resulting statistic is very similar to the concept of coherency in pure frequency analysis and is defined as

$$R_{xy}(\tau, s) = \frac{|S(W_{xy}(\tau, s))|}{\sqrt{S(|W_x(\tau, s)|^2)S(|W_y(\tau, s)|^2)}}, \quad (13)$$

where  $S(\cdot)$  denotes a smoothing operator. The smoothing operator is applied with regard to both frequencies and time. Suitable choices for the operator are discussed in Torrence and Webster (1999) and Torrence and Compo (1998). Basically, smoothing can be obtained by a convolution with a window function along both the time and scale dimensions:

$$S(W(\tau, s)) = \int_{\tau-\Delta_1/2}^{\tau+\Delta_1/2} \int_{s-\Delta_2/2}^{s+\Delta_2/2} W(t, \varsigma) f_{\Delta_1, \Delta_2}(t, \varsigma) dt d\varsigma,$$

where  $f_{\Delta_1, \Delta_2}$  satisfies  $\int \int f_{\Delta_1, \Delta_2}(t, \varsigma) dt d\varsigma = 1$ . Wavelet coherency thus is unity if at a particular time and scale, a perfect linear relation exists between the two time series; while at the other extreme, it is zero if the series are independent. Although the exact choice of the smoothing function  $f$  is somewhat arbitrary, this is not different to the situation in Fourier analysis, where coherency is based on the smoothed periodogram.<sup>1</sup>

Just as for the coherency measure in pure frequency analysis, the appeal of the statistic in equation (13) is its similarity to the standard correlation coefficient, so that it can be considered as a correlation coefficient localized in time-frequency space (Grinsted et al., 2004).

Because of the similarity to its Fourier counterpart, the approach can also easily be extended to provide insight for entire groups of countries, allowing to compare the group of Euro countries with other sets. Consider a vector  $y_t = (y_{1t} \dots y_{nt})'$  with  $n \geq 2$  and positive weights  $w = (w_1 \dots w_n)'$  attached to each element of  $y_t$ . The proposed measure is motivated in the same manner as Croux et al.'s (2001) measure of cohesion in the frequency domain and will hence be referred to as

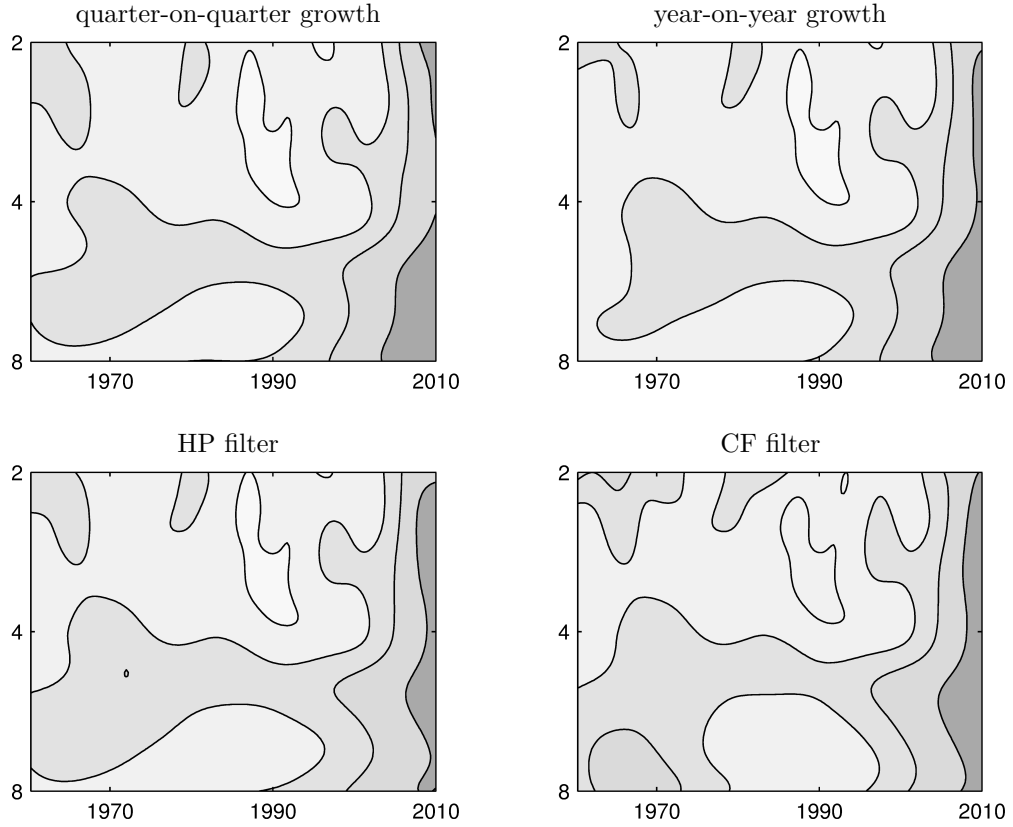
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<sup>1</sup>Both in the Fourier and in the wavelet case, the coherency measure would be unity everywhere without smoothing.

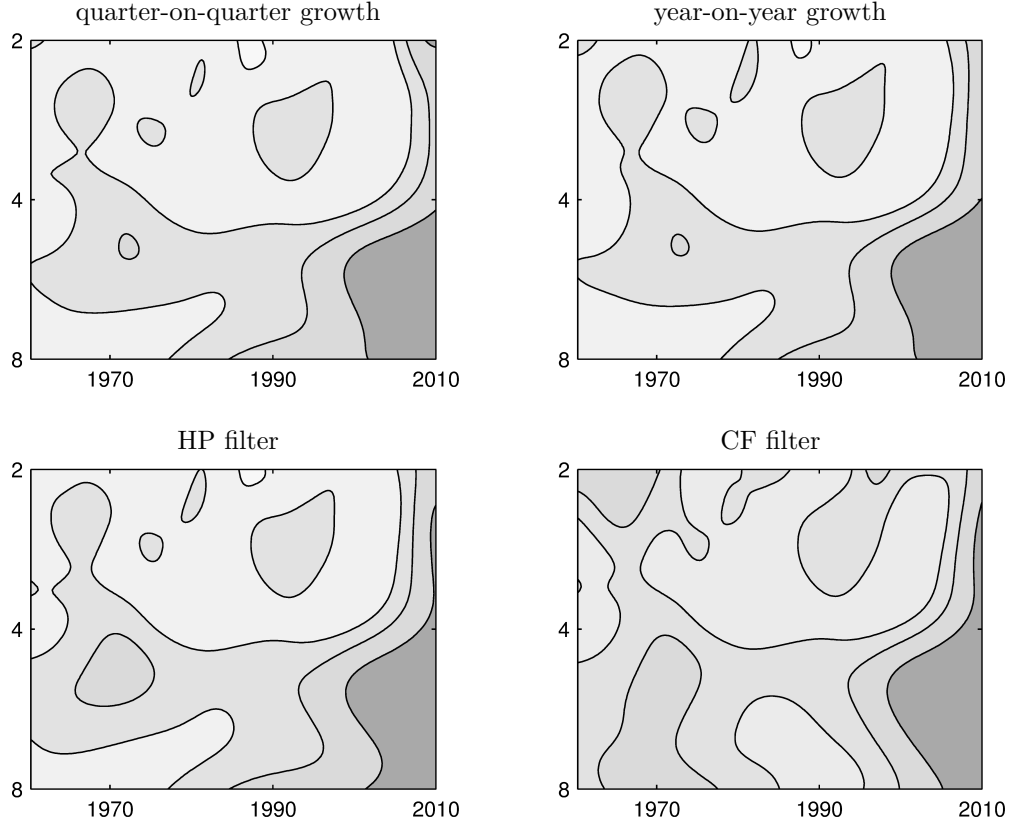
wavelet cohesion. It equals

$$\text{wavecoh}_y(\tau, s) = \frac{\sum_{i \neq j} w_i w_j R_{y_i y_j}(\tau, s)}{\sum_{i \neq j} w_i w_j}. \quad (14)$$

To simplify the measure, weights can be chosen as  $w_i = 1$  for all  $i$ . While this may be appropriate if each element in  $y_t$  were of the same importance, with countries in the dataset greatly varying in size, it is more reasonable to take account of this fact by a suitable weighting scheme. Hence, like previously for the cohesion measure in pure frequency analysis, the  $w_i$  are chosen to be the population of the countries in 2010, i.e. at the end of the sample.



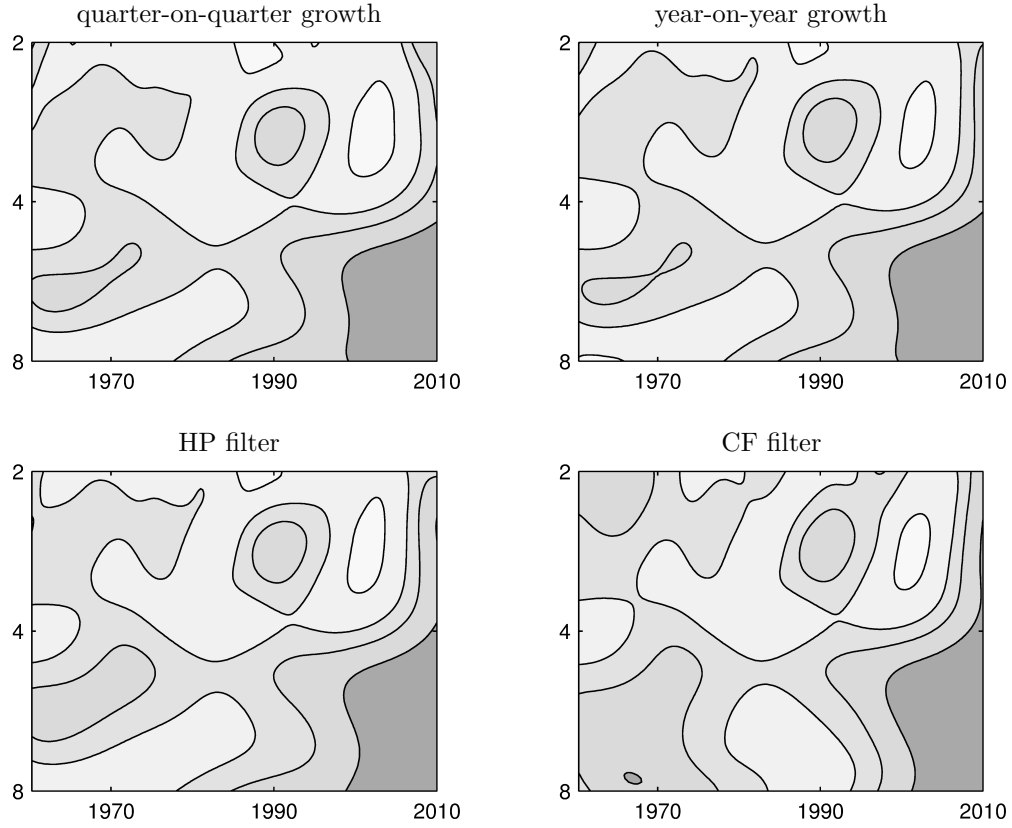
**Figure 5:** Wavelet cohesion for country group “all” at business cycle lengths. The  $y$ -axis shows the length of the cycle in years and the  $x$ -axis shows the localization in time. Time-scale combinations with higher wavelet cohesion are shaded darker.



**Figure 6:** Wavelet cohesion for country group “Europe” at business cycle lengths. The  $y$ -axis shows the length of the cycle in years and the  $x$ -axis shows the localization in time. Time-scale combinations with higher wavelet cohesion are shaded darker.

With this new wavelet cohesion measure, it is possible to capture synchronization of business cycles for groups of countries. Figures 5 to 7 show graphical representations of the  $\text{wavecoh}_y(\tau, s)$  statistic using the county groups consisting of all, the European, and the Euro member states, respectively. The contour plots indicate wavelet cohesion for the entire time span of the sample and for cycle lengths between two and eight years. Areas colored in the darkest shade of gray indicate a cohesion statistic between 0.8 and 1.0 and thus the highest possible degree of synchronization. Each contour line and switch to a lighter shade then represents a decrease in cohesion by a 0.2 increment, leaving areas with little cohesion –  $\text{wavecoh}_y(\tau, s)$  between 0 and 0.2 – entirely white.





**Figure 7:** Wavelet cohesion for country group “Euro” at business cycle lengths. The  $y$ -axis shows the length of the cycle in years and the  $x$ -axis shows the localization in time. Time-scale combinations with higher wavelet cohesion are shaded darker.

Figure 5 shows cohesion for all countries together. It indicates that except for the very beginning of the sample, synchronization is present for cycles with a length of approximately five years, as cohesion at this scale exceeds surrounding higher and lower frequencies. During the late 1980s and early 1990s, the link is least substantial. Also within that time span, cohesion is particularly low for shorter cycles between two and four years. More generally, cohesion is low for cycles shorter than four years everywhere except at the very end of the sample, where all frequencies show a clear increase in cohesion. Another notable aspect is that, unlike for the purely time- and frequency-based measures of the previous sections, there is only very little discrepancy between the results for the different cycle extraction methods and the conclusions concerning cyclical comovement

behavior hold regardless of the filtering method.

For the European country group, whose wavelet cohesion results are depicted in Figure 6, the overall picture is rather similar to that for the complete set of countries. Evidence of cohesion is a bit stronger than before and again it is most pronounced for cycles of around five years length. Since around 2000, those five-year cycles have very high cohesion values of above 0.8. Interestingly, the time-scale combination of two- to three-year cycles around 1990, which spotted the smallest cohesion in Figure 5 now exhibits a somewhat stronger synchronization link than its surroundings.

Finally Figure 7, considering only the Euro member countries, confirms the presence of synchronization for five-year cycles throughout the sample. Unlike for the other groups, however, there is considerable cohesion already during the early years. As in the previous groups, cohesion is generally somewhat lower in the 1980s than before and after. The “island” of high cohesion for shorter cycles around 1990 is again present and more substantial than for the European group. For the years subsequent to the introduction of the Euro in 1999 and for cycles longer than five years, there is very extensive cohesion throughout. The results of the different filtering approaches are not as similar as for the group consisting of all countries, yet unlike for the purely time- or frequency-based approaches, all important features are qualitatively the same across methods.

The use of wavelets thus does not only introduce the advantage of combining the analysis of time and frequency dimensions, but it also turns out that the statistics are much more robust to the choice of filtering technique. While both the contemporaneous correlation coefficients and the different approaches in the frequency domain yield results that vary depending on which measure is used to extract the cycle, this is not the case for wavelet analysis.

## 7 Conclusion

This paper has compared a variety of methods targeted at measuring business cycle synchronization. It has shown that it is insufficient to consider the time domain and the frequency domain separately and established the sensitivity of present statistics to the choice of cycle extraction technique. As a more refined approach, the paper has pointed out that wavelets are a valuable tool for the analysis of business cycles, because they allow to consider a localization of common

periodicities in scale and time. In particular, a new measure of comovement between several – rather than just a pair of – series has been introduced for a wavelet setting.

The wavelet cohesion statistic asserts that cyclical components in output with a length of approximately five years are synchronized to a certain degree for various sets of countries. This lies well within the range typically considered as business cycles and corresponds exactly to the finding of Artis et al. (1997) who pinpoint the typical business cycle length to be between five and six years. Cohesion is stronger for countries sharing the Euro, in particular since the actual introduction of the currency, but also in earlier years already. These findings align with those of previous studies such as Rua (2010), who considers fewer countries and just bivariate relationships over a shorter sample. Furthermore, using the wavelet approach, Canova's (1998) criticism of arbitrariness in the choice of cycle extraction technique is remedied, because the deliberate choice of filter does not affect the result eventually obtained for synchronization.

The wavelet-based measures turn out to be very useful because results show comovements both hinge on the frequency of cycles and develop over time. These two dimensions are easily incorporated in the wavelet cohesion statistic, while other approaches have to resort to auxiliary tools to provide insight beyond a single one of the dimensions. By virtue of the simple extendibility to groups and the possibility to obtain an intuitive visualization of the synchronization relationships, the technique should also prove very useful and warrant more widespread use in further areas of economics.

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