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Financial Contracting with Tax Evaders

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Abstract

This paper derives the optimal financial contract when a borrowing entrepreneur can evade taxes in a model of costly state verification. In contrast to previous literature on costly state verification and financial contracting, we find that standard debt contracts are not optimal when tax evasion is possible. Instead, the optimal contract is debt-like only for very low and very high profit realizations, and features a constant repayment *and* verification of returns in an intermediate range. This occurs because the entrepreneur has to be given a positive rent even under verification in order to not abuse her limited liability protection for excessive tax evasion activities.

Keywords: financial contracting; security design; corporate tax evasion; costly state verification

JEL Codes: D82; D86; G3; H25; H26

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1 Introduction

Tax evasion poses an interesting challenge for financial contracting. Since it is illegal, the potential gains from evading taxes (a lower tax bill) are typically not contractible. However, its potential losses (fines) may reduce a borrower's ability to repay investors, as long as the government's claims are sufficiently senior. An entrepreneur protected by limited liability, who borrows money to finance a project, may use precisely this asymmetry to her advantage. She might evade more, for instance, than she would if she were fully self-financed, because the gains from evasion are entirely hers if tax evasion goes undetected, while the loss in case of detection is, at least partially, borne by the investors.

This paper analyzes financial contracting under such circumstances. If an entrepreneur may evade taxes, we ask, what will the optimal financial contract between an investor and this potentially tax-evading entrepreneur look like? Using a model of costly state verification, our analysis builds on the seminal work of Townsend (1979) and Gale and Hellwig (1985), who assume verification of an entrepreneur's private information (e.g. about a venture's financial success) to be costly. Their optimal financial contract - standard debt - is optimal because it minimizes the cost of verification. In our model, rather than assuming verification to be costly *per se*, we observe that it influences the entrepreneur's ability to evade taxes. Verification likely delivers information about the true state of the world not only to the investor, but also to the government, as it involves courts and other third parties with informational duties such as auditing firms. Additional information for the government, however, makes tax evasion less profitable in expectation for the entrepreneur. This decreased profitability of tax evasion is the cost of investor verification in our model. An entrepreneur seeking to evade taxes thus proposes a financial contract that minimizes verification, rather like in Townsend (1979) and Gale and Hellwig (1985). However, we find that the optimal contract in this setting is *not* standard debt. Instead, the optimal contract is debt-like only for very low and very high profit realizations. For intermediate realizations, it demands a constant repayment to the investor *and* verification of returns, thus leaving some rent for the entrepreneur. This intermediate range of the contract therefore combines elements of equity - leaving some rent to the entrepreneur despite verifying the true state of the world - with elements of debt, namely a constant repayment.

This is because an optimal contract in this setting, besides minimizing investor verification, faces the particular constraints imposed by tax evasion mentioned in the first paragraph above. Gains from evasion are not contractible, but an entrepreneur's limited liability may make the investor partially liable for fines in case tax evasion is detected. This occurs when fines for evasion exhaust the entrepreneur's funds so that the repayment stipulated in the contract cannot be made in full. An optimal financial contract hence stipulates only repayments that are feasible even if the entrepreneur's tax evasion activities are detected. In particular,

this means that except for very low profit realizations, the repayment is always sufficiently far below the entrepreneur's available funds. This leaves the entrepreneur with a positive rent that prevents her from abusing her limited liability protection for excessive tax evasion activities. The present paper is mainly related to two strands of the economics literature.

First, it builds on and contributes to the large literature on financial contracting and security design. Surveys of this literature include Hart (2001), Harris and Raviv (1995), and Franklin and Winton (1995). More specifically, we use a model of costly state verification pioneered by Townsend (1979) and Gale and Hellwig (1985). Other early contributions include Diamond (1984) and Williamson (1986). Canonical treatments of this model are found in Tirole (2005), Freixas and Rochet (2008), and Bolton and Dewatripont (2005). The basic costly state verification framework has been extended into a variety of directions such as allowing for random auditing (Mookherjee and Png 1989), multiple investors (Winton 1995), and multiple periods (Chang 1990 and Webb 1992). However, no study to date has examined the impact of tax evasion on financial contracting. We propose to extend the basic costly state verification model in this direction. Like for several other extensions, we find that the chief result of the basic costly state verification framework - the optimality of standard debt contracts - is not robust to allowing for tax evasion by borrowers.

Besides the original contributions of Townsend (1979) and Gale and Hellwig (1985), who provide the basic framework that we extend by allowing for an entrepreneur's tax evasion, a work related to our analysis is Povel and Raith (2004). They consider financial contracting when both an entrepreneur's investment choice and the revenue realization are unobservable and not verifiable to outside investors. Tax evasion can be interpreted as an unobservable investment by the entrepreneur, and in this sense the present analysis is related to Povel and Raith (2004). However, in our model, tax evasion *is* verifiable upon audit. Similarly, revenues can be verified in our model by the investor, and so we operate in a very different framework from that of Povel and Raith, namely a framework of costly state verification.

Second, the present paper is connected to the literature on the economics of tax evasion. For general surveys of this literature, see Slemrod and Yitzhaki (2002), Andreoni et al (1998), or Slemrod (2007). In particular, we consider an entrepeneur's tax evasion choice and its ramifications for corporate financing. Within the economics of tax evasion, our analysis is therefore located in the much smaller part of the literature concerning corporate tax evasion and avoidance, as surveyed by Slemrod (2004) and Nur-Tegin (2008). A key difference between individual and corporate tax evasion is the existence of informational asymmetries between different stakeholders of the corporation, rather than just between taxpayer and government. Crocker and Slemrod (2005) and Chen and Chu (2005) initiated this approach by examining the impact of informational asymmetries between managers and shareholders of the tax evading firm. Surveys of the fairly recent integration of the theory of tax evasion with principal-agent analysis are contained in Hanlon and Heitzman (2010) and Armstrong et al

(2013). While related, our focus is on the interaction of corporate financing and tax evasion, rather than on corporate governance and tax evasion. And indeed, although the general literature on taxation and corporate finance is large (see Auerbach 2002 or Graham 2003 for surveys), the impact of *tax evasion* on optimal security design that we analyze here has not previously been studied.

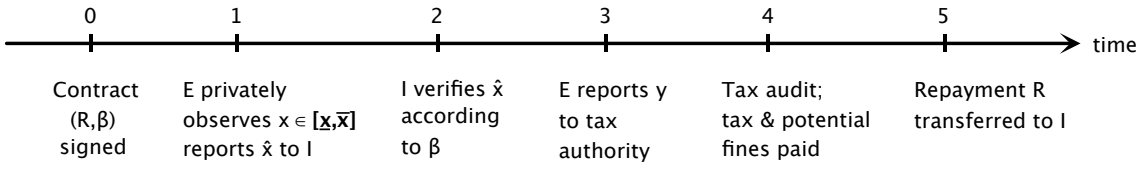
The remainder of this paper is organized as follows. In section 2, we introduce the setup and timing of our formal model. Section 3 presents the analysis. In particular, section 3.1 derives the entrepreneur's tax evasion choice and section 3.2. her reporting behavior toward the investor. In section 3.3 we derive the optimal financial contract in this setting. Section 4 concludes.

2 Model

Consider a risk-neutral, zero-wealth entrepreneur, E, who requires funding from a risk-neutral investor, I, to finance a project. The project turns a unit investment provided by I into a random return x, which is uniformly distributed on the interval $[\underline{x}, \overline{x}]$. The timing of the game is as follows (see Figure 1).

In stage 0 of the game, the entrepreneur offers a take-it-or-leave-it *financial contract,* (R, β), to the investor. This contract combines a repayment function, $R(\hat{x}, x)$, with a verification function, $\beta(\hat{x})$, as is standard in the literature on costly state verification. That is, E proposes to issue a security. I accepts the offer if his expected payoff from it is at least $\underline{v}$, his reservation utility. At the time of contracting, both players have symmetric information.

Figure 1 - Timing of the game



In stage 1, the entrepreneur *privately* observes the realized return x and makes a report, $\hat{x}$, about it to the investor, which may or may not be truthful.

In stage 2 of the game, the investor either verifies E's report and learns the true return x, or he accepts the report and does not verify. We assume deterministic and contractible verification, so that the verification probability $\beta(\hat{x})$ is a function $\beta : [\underline{x}, \overline{x}] \to \{0, 1\}$.

In stage 3, the entrepreneur needs to make a tax report, y, to the tax authority, G.[1] The true return x is subject to taxation at rate $\tau \in [0, 1]$. We denote by $e = x - y$ the amount

[1]The tax authority G is assumed to be a static entity, not a player of the game.

of underreporting, or evasion. Tax evasion is costly to the entrepreneur, and we denote this cost by $c(e)$, assumed to take the quadratic form $c(e) = \frac{k}{2}e^2$, where $k \geq 0$ is a given cost parameter.

In stage 4, the tax authority may learn about the true realization x. Without verification (i.e. if $\beta(\hat{x}) = 0$), it does so with a fixed probability, p, its baseline audit probability. If, however, a report is verified (i.e. if $\beta(\hat{x}) = 1$) by the investor, this entails public revelation[2] and so the tax authority learns the true x with certainty. If the tax authority learns the true x, then in addition to the tax payment τy based on E's *reported* profit, it collects a fine $\lambda\tau e$ on evaded taxes, where $\lambda > 1$.

In the last stage of the game, the repayment $R(\hat{x}, x)$ stipulated in the financial contract is transferred to the investor.[3] Under non-verification of a report, this repayment can only depend on the report $\hat{x}$, whereas under verification, since the investor learns the true x, the repayment can depend on both $\hat{x}$ and x. To clearly distinguish these cases, we denote by $R_{nv}(\hat{x})$ the repayment function if $\beta(\hat{x}) = 0$ and by $R_v(\hat{x}, x)$ the repayment function in case $\beta(\hat{x}) = 1$. So the contractual repayment specification $R(\hat{x}, x)$ can be written as

$$R(\hat{x}, x) = \begin{cases} R_{nv}(\hat{x}) & if \quad \beta(\hat{x}) = 0 \\ \\ R_v(\hat{x}, x) & if \quad \beta(\hat{x}) = 1 \end{cases}$$

R_{nv} maps reports onto real numbers, $R_{nv} : [\underline{x}, \overline{x}] \to \mathbb{R}_0$, and R_v is a mapping from the set of report-realization combinations onto the real numbers, $R_v : [\underline{x}, \overline{x}] \times [\underline{x}, \overline{x}] \to \mathbb{R}_0$.

The entrepreneur's utility is quasi-linear in her monetary payoff and the cost of tax evasion. Her utility function, further specified below, therefore generally takes the form

$$U = \Pi - c(e)$$

where Π is an expected monetary payoff and $c(e)$ is the cost of tax evasion. The investor's utility is just his monetary payoff.

[2] In the classic interpretation due to Gale and Hellwig (1985), verification is interpreted as the initiation of bankruptcy proceedings. Involving the authorities, courts, and third parties with strictly regulated informational duties such as auditing firms, it is plausible to assume that such proceedings deliver information about the true state of the world to the government. For simplicity, we assume this information to be perfect. Our results remain qualitatively unaffected as long as verification increases the probability of a tax audit over the baseline probability p.

[3] Note that this implies absolute priority of government claims over creditor claims as was the case, for instance, in Germany until 1994. A reintroduction of absolute government priority is currently being discussed in Germany. The US Bankruptcy Code, Art. 501, prioritizes some taxes absolutely over creditor claims. Most jurisdictions treat government and creditor claims equally (*par conditio creditorum*), which still entails a potential loss to the investor, and does not qualitatively change the analysis presented here.

3 Analysis

We want to characterize the financial contract offered by the entrepreneur E in the initial stage of this game. So we analyze the decisions made by E backwards, beginning with his tax reporting decision in stage 3 and continuing with E's report to I in stage 1 of the game. These stages will imply a set of constraints on contracting required for I's acceptance of the entrepreneur's proposal at the initial contracting stage.

3.1 The entrepreneur's tax reporting decision

Consider first the entrepreneur's choice of tax evasion, e, in stage 3 of the game,[4] given a contract (R, β), a realization x, and a report $\hat{x}$ made to I in stage 1 of the game. Two cases require distinction.

First, the case where the entrepreneur's report has been verified, i.e. $\beta(\hat{x}) = 1$. This entails public revelation of the true x and so, in particular, $\beta(\hat{x}) = 1$ means the government learns the true project success x. As should be expected, this will imply no tax evasion takes place.

Second, if the report $\hat{x}$ has not been verified by the investor, i.e. $\beta(\hat{x}) = 0$, evasion is detected only with probability p, which will imply some tax evasion is profitable in expectation.

But let us formally analyze the two cases of verification and non-verification in turn.

3.1.1 Tax evasion when the investor verifies, i.e. when $\beta(\hat{x}) = 1$

If the investor verifies E's report, the true x is publicly revealed. So the entrepreneur's utility in the final stage of the game is given by

$$U_v(e\,;\hat{x}, x) = max\,\{x - \tau(x - e) - \lambda\tau e - R_v(\hat{x}, x)\,,\,0\,\} - c(e) \qquad (1)$$

The subscript v is meant to indicate the case of investor verification examined here. Utility U_v weighs the entrepreneur's payoff, which is non-negative because of her limited liability, against the utility cost of tax evasion. But tax evasion in case of verification is always detected and implies a fine $\lambda\tau e$, which is larger than the tax savings from underreporting τe, since $\lambda > 1$. So it is seen immediately from (1) that the entrepreneur's maximizing choice is to not evade taxes at all, which we denote as

$$e_v^* = 0\ .$$

[4] Since $e = x - y$, choosing a level of evasion e implies a tax report $y = x - e$. The analysis is more intuitive, though of course equivalent, when E's choice is modeled in terms of e, rather than y.

3.1.2 Tax evasion when the investor does not verify, i.e. when $\beta(\hat{x}) = 0$

By assumption, a tax audit now occurs with probability p, since the report $\hat{x}$ has not been verified by the investor. We first state the entrepreneur's respective utilities in case of a tax audit and in absence of a tax audit. Her expected utility is then the sum of these two utilities weighted by the audit probability p. The entrepreneur chooses a level of tax evasion that maximizes this expected utility.

With a tax audit, E's utility in the final stage of the game is given by

$$U_{nv,a}(e\,;\hat{x},x) = max\,\{x - \tau(x-e) - \lambda\tau e - R_{nv}(\hat{x})\,,\,0\,\} - c(e) \qquad (2)$$

The subscript nv indicates the case of non-verification that we analyze in this section, and the subscript a indicates the case of a tax audit. E's payoff is then the true realization x, minus the tax payment on reported profits, $\tau(x-e)$, minus a fine for evaded taxes, $\lambda\tau e$, and the contractual repayment under non-verification, $R_{nv}(\hat{x})$. Bearing in mind that the entrepreneur is protected by limited liability, this difference is her payoff only if it is non-negative, however. Otherwise it is just 0. This payoff is weighed against the utility cost of tax evasion, $c(e)$, to yield the utility $U_{nv,a}$.

Without a tax audit, E's utility in the final stage of the game is given by

$$U_{nv,na}(e\,;\hat{x},x) = max\,\{x - \tau(x-e) - R_{nv}(\hat{x})\,,\,0\,\} - c(e) \qquad (3)$$

Again, the subscript nv denotes the case of non-verification and the subscript na indicates the absence of an audit by the tax authority. The utility in this case differs from $U_{nv,a}$ only in that no fine for evasion is levied, because the tax authority does not learn the true x and so tax evasion goes undetected.

The entrepreneur's expected utility in stage 3 of the game under non-verification is thus

$$U_{nv}(e\,;\hat{x},x) = p\,U_{nv,a} + (1-p)\,U_{nv,na} \qquad (4)$$

The entrepreneur chooses a level of tax evasion to maximize this expected utility, and we define

$$e^*_{nv} \in \underset{e}{argmax}\,\{\,U_{nv}\,\} \qquad (5)$$

as the entrepreneur's utility-maximizing choice of tax evasion under non-verification.

Let us now deduce e^*_{nv}.

U_{nv} from (4) can be written as

$$U_{nv}(e\,;\hat{x},x)=\begin{cases}(1-\tau)x+(1-p\lambda)\tau e-R_{nv}(\hat{x})-c(e) & if \quad (a)\\ \\ (1-p)[(1-\tau)x+\tau e-R_{nv}(\hat{x})]-c(e) & if \quad (b)\end{cases}$$

where

$$x-\tau(x-e)-\lambda\tau e-R_{nv}(\hat{x})\geq 0 \qquad (a)$$

and

$$x-\tau(x-e)-\lambda\tau e-R_{nv}(\hat{x})<0\leq x-\tau(x-e)-R_{nv}(\hat{x}) \qquad (b)$$

Writing out expected utility in this way reveals two distinct ranges for U_{nv}, associated with conditions (a) and (b). They arise depending on whether the payoff terms of the form $max\{\cdot\,,0\}$ in (2) and (3) above bind at zero, or are slack and thus positive. That is, on whether the repayment R_{nv} can be made in full, or the entrepreneur's limited liability binds and prevents a full repayment of R_{nv}.

Condition (a) implies that the payoff terms in *both* (2) and (3) are slack and so the repayment to I can be made both with and without a tax audit. As will become clear, an evasion choice satisfying condition (a) does not cause any problems for the investor, as even after a tax audit and fines, he will be repaid in full.

Condition (b) says that the payoff term in (2) binds at 0 while the payoff term in (3) is slack. This means the repayment to I is only feasible if there is no tax audit, whereas if there is a tax audit, the fine for evasion exhausts the entrepreneur's funds and prevents a full repayment of R_{nv} due to the entrepreneur's limited liability protection. We will argue below that an evasion choice satisfying condition (b) cannot be supported in an optimal contract, as with probability p of a tax audit, the investor will not be repaid what was stipulated in the contract.[5]

We are now able to characterize the entrepreneur's tax evasion choice in stage 3 of the game as follows.

[5]Technically, U_{nv} has a third region where both payoff terms bind at 0, i.e. where $x-\tau(x-e)-R_{nv}(\hat{x})<0$. Then expected utility is just the utility cost of tax evasion, $-c(e)$. This range will obviously never be attained since its maximizing choice would be $e=0$ yielding utility 0, which is always dominated by a choice of e that satisfies conditions (a) or (b).

Proposition 1. [Entrepreneur's best response tax evasion choice]

Given any contract (R, β), realization x, and report $\hat{x}$, the entrepreneur's best response tax evasion choice in stage 3 of the game is given by

$$\begin{cases} e^*_{nv,(a)} = (1-p\lambda)\frac{\tau}{k} & if \quad \beta(\hat{x}) = 0 \quad and \quad R_{nv}(\hat{x}) \leq (1-\tau)x - \phi_{nv} \\ e^*_{nv,(b)} = (1-p)\frac{\tau}{k} & if \quad \beta(\hat{x}) = 0 \quad and \quad R_{nv}(\hat{x}) > (1-\tau)x - \phi_{nv} \\ e^*_v = 0 & if \quad \beta(\hat{x}) = 1 \end{cases}$$

where

$$\phi_{nv} = [\,(1-p)^2 - (1-p\lambda)^2\,]\,\frac{\tau^2}{2pk}\,.$$

Proof. See appendix A.1. □

We already stated in section 3.1.1 above that under verification of the entrepreneur's report, her best response tax evasion choice is zero, $e^*_v = 0$, since verification entails public revelation of the true project success x.

The entrepreneur's best response tax evasion choices under non-verification reflect the two regions of her expected utility U_{nv} characterized by conditions (a) and (b) above. If condition (a) holds, the entrepreneur's limited liability condition does not bind even after a tax audit. So in deciding how much to evade, E marginally weighs the expected tax savings against both the expected fine in case of audit and the cost of evasion, yielding $e^*_{nv,(a)} = (1-p\lambda)\frac{\tau}{k}$.

If, however, condition (b) holds, the entrepreneur is protected by limited liability in case of a tax audit. This means she will evade only with a view to the non-audit case. She then marginally weighs expected tax savings against only the cost of tax evasion. The fine does not matter in this case, since it is only incurred in case of a tax audit, where E's payoff is 0 anyway. This yields a higher level of tax evasion, $e^*_{nv,(b)} = (1-p)\frac{\tau}{k}$.[6]

Under non-verification, the entrepreneur thus chooses between $e^*_{nv,(a)}$ and $e^*_{nv,(b)}$ the level of evasion that yields a higher expected utility. The lower level $e^*_{nv,(a)}$ is chosen if the entrepreneur is always left a sufficiently high rent under non-verification so that the fine for evasion still marginally matters to her, i.e. if $R_{nv}(\hat{x}) \leq (1-\tau)x - \phi_{nv}$. If the rent left to the entrepreneur is less than that, i.e. if $R_{nv}(\hat{x}) > (1-\tau)x - \phi_{nv}$, she is better off in expectation by choosing the high level of tax evasion $e^*_{nv,(b)}$ that implies limited liability protection in case a tax audit occurs.

But in that latter case, the contractually agreed upon repayment, $R_{nv}(\hat{x})$, can only be made if there is no audit by the tax authority, i.e. with probability $(1-p)$. Such a contract is

[6] For simplicity, let us assume that $\underline{x} > (1-p)\frac{\tau}{k}$, so that the entrepreneur never evades the full amount of taxes due, but only a share thereof.

therefore not feasible, for with probability p, its stipulations cannot be met. The investor would not sign such a contract. We conclude that a feasible contract must satisfy the the constraints summarized in the following corollary.

Corollary 1. [Feasibility constraints on financial contracting]

A contract (R, β) is feasible if it satisfies

$$R_{nv}(\hat{x}) \leq (1-\tau)x - \phi_{nv} \qquad (F_{nv})$$

$$R_v(\hat{x}, x) \leq (1-\tau)x \qquad (F_v)$$

where

$$\phi_{nv} = [\,(1-p)^2 - (1-p\lambda)^2\,]\,\frac{\tau^2}{2pk}$$

Proof. Follows from Proposition 1. □

Note that condition (F_{nv}) is stricter than condition (F_v) imposed when the investor verifies the entrepreneur's report, in which case the entire statutory after-tax income $(1-\tau)x$ is contractible.[7]

Tax evasion thus imposes a constraint on financial contracting if it is such that the entrepreneur's limited liability binds in case of a tax audit. Because the entrepreneur is protected by limited liability, she will evade as if there were no fine for evasion when the contractual repayment under non-verification exceeds $(1-\tau)x - \phi_{nv}$.

A simple illustration of the problem that arises in this case is to consider $R_{nv}(\hat{x}) = (1-\tau)x$, clearly a violation of (F_{nv}). Then with no evasion at all, E's utility will be 0, since she pays τx in taxes and $(1-\tau)x$ to the investor.

But now consider a minimally positive amount of evasion, say $e = \epsilon > 0$. Now in case of a tax audit, E's utility is $-c(\epsilon)$. If there is no tax audit, E's utility is $\tau\epsilon - c(\epsilon)$. The entrepreneur's expected utility is therefore $(1-p)(\tau\epsilon - c(\epsilon)) - pc(\epsilon) = (1-p)\tau\epsilon - c(\epsilon) = (1-p)\tau\epsilon - \frac{k}{2}\epsilon^2 > 0$ for $\epsilon < (1-p)\frac{2\tau}{k} > 0$. So the entrepreneur is better off in expectation when evading the positive amount ϵ. But this means that in case of a tax audit, the funds available for repaying the investor are only $(1-\tau)x - (\lambda-1)\tau\epsilon < (1-\tau)x = R_{nv}(\hat{x})$, and so the contractually stipulated repayment cannot be made. Such a repayment can therefore not be part of a feasible contract. Instead, feasibility requires that under non-verification, some rent is left to the entrepreneur with zero evasion (namely ϕ_{nv}), so that she always evades in such a way that fines still marginally matter to her. This ensures repayment to the investor is feasible even in case of a tax audit. In the following, we will only consider feasible contracts.

[7] Note that agreeing to a repayment larger than the statutory net income amounts to contracting on the gains of tax evasion, which is illegal and thus not enforceable in a court of law. $(1-\tau)x$ is therefore the legal maximum of contractible income.

3.2 The entrepreneur's report of $\hat{x}$ to the investor

In this section, we look at the entrepreneur's reporting behavior toward the investor in stage 1 of the game. Given a feasible contract (R, β) and the realization x, the entrepreneur makes a report, $\hat{x}$, to the investor. E will also take into account the implications of her reporting behavior on tax evasion choices as derived in the previous section. Let us distinguish two cases.

First, the case where $\beta(x) = 0$, i.e. contract (R, β) and realization x are such that a truthful report, $\hat{x} = x$ would not be verified by the investor. Second, the case where $\beta(x) = 1$, i.e. a truthful report would be verified. We consider the entrepreneur's reporting behavior in each of these cases and derive incentive compatibility constraints required for contracting.

3.2.1 Reporting $\hat{x}$ when $\beta(x) = 0$, i.e. a truthful report would not be verified by I

Consider the entrepreneur's possibilities for reporting to the investor if $\beta(x) = 0$. E can either report truthfully, $\hat{x} = x$, in which case, assuming feasibility (F_{nv}) holds, her expected utility would be

$$U_{nv}(e^*_{nv,(a)}, x, x) = (1 - \tau)x + \frac{1}{2}(1 - p\lambda)^2 \frac{\tau^2}{k} - R_{nv}(x)$$

Alternatively, the entrepreneur may choose to misrepresent true earnings and report some $\hat{x} \neq x$ with $\beta(\hat{x}) = 0$.[8] Then E's expected utility is given by

$$U_{nv}(e^*_{nv,(a)}, \hat{x}, x) = (1 - \tau)x + \frac{1}{2}(1 - p\lambda)^2 \frac{\tau^2}{k} - R_{nv}(\hat{x})$$

Truthful reporting when $\beta(x) = 0$ is therefore preferred by the entrepreneur if and only if

$$U_{nv}(e^*_{nv,(a)}, x, x) \geq U_{nv}(e^*_{nv,(a)}, \hat{x}, x) \qquad \forall x, \hat{x} \; with \; \beta(\hat{x}) = 0 \; , \; \beta(x) = 0 \, and \, \hat{x} \neq x$$

But comparing the two expected utility terms above, this implies

$$R_{nv}(x) \leq R_{nv}(\hat{x}) \qquad \forall x, \hat{x} \; with \; \beta(\hat{x}) = 0 \; , \; \beta(x) = 0 \, and \, \hat{x} \neq x$$

Put differently, the entrepreneur has an incentive to misreport true earnings if some other, non-verified report $\hat{x} \neq x$ induces a lower repayment. We therefore obtain the following

[8] Following the convention in the literature, we exclude the case of *a lie that induces verification*, i.e. reporting some $\hat{x} \neq x$ with $\beta(\hat{x}) = 1$. Such a lie would be found out and can be arbitrarily punished as part of the contract. This exclusion is without loss of generality, as any contract inducing such reporting can be rewritten to induce truthful reporting under verification. See for instance Bolton and Dewatripont (2005), or Freixas and Rochet (2008).

incentive-compatibility constraint familiar from the literature on costly state verification.

$$R_{nv}(\hat{x}) = D \qquad \forall\, \hat{x} \quad with \quad \beta(\hat{x}) = 0 \qquad (IC1)$$

where $D \in \mathbb{R}$ is a constant. Since under non-verification, a lie will not be detected by the investor, the repayment to I under non-verification cannot depend on the report. It has to be constant.

3.2.2 Reporting $\hat{x}$ when $\beta(x) = 1$, i.e. a truthful report would be verified by I

There are two possibilities the entrepreneur has when making her report to the investor. Either she can tell the truth and report $\hat{x} = x$, or she may lie and make a report that does not induce verification, i.e. report some $\hat{x} \neq x$ with $\beta(\hat{x}) = 0$.[9] Denoting, as above, her best-response evasion choices under these two options as e_v^* and e_{nv}^*, we compare E's resulting expected utilities and hence deduce E's reporting behavior.
If E reports $\hat{x} = x$, the report will be verified and so $e_v^* = 0$ will be chosen by the entrepreneur in stage 3 of the game. Her expected utility is then given by

$$U_v(e_v^*, x, x) = (1 - \tau)x - R_v(x, x)$$

If on the other hand, E chooses to misrepresent and report $\hat{x} \neq x$ with $\beta(\hat{x}) = 0$, she will subsequently choose $e_{nv,(a)}^* = (1 - p\lambda)\frac{\tau}{k}$. E's expected utility is then

$$U_{nv}(e_{nv,(a)}^*, \hat{x}, x) = (1 - \tau)x + \frac{1}{2}(1 - p\lambda)^2 \frac{\tau^2}{k} - R_{nv}(\hat{x})$$

Truthful reporting is therefore preferred if and only if

$$(1-\tau)x - R_v(x, x) \geq (1-\tau)x + \frac{1}{2}(1-p\lambda)^2 \frac{\tau^2}{k} - R_{nv}(\hat{x}) \qquad \forall\, x, \hat{x}\ with\ \beta(x) = 1\ and\ \beta(\hat{x}) = 0$$

The entrepreneur's payoff from truthful reporting on the left-hand side of the inequality must be at least E's expected utility from reporting a non-verified report. This is given on the right-hand side as the statutory net income $(1-\tau)x$ minus the repayment $R_{nv}(\hat{x})$, plus the expected gain from tax evasion, $\frac{1}{2}(1 - p\lambda)^2 \frac{\tau^2}{k}$. Rearranging this condition allows us to summarize the conditions for incentive compatibility in the following proposition.

[9] Again, without loss of generality, we exclude from our analysis the case of misreporting that induces verification, i.e. reporting some $\hat{x} \neq x$ with $\beta(\hat{x}) = 1$.

Proposition 2. [Incentive compatibility constraints on financial contracting]

A feasible contract (R, β) is incentive compatible if and only if

$$R_{nv}(\hat{x}) = D \qquad \forall\, \hat{x} \quad with \quad \beta(\hat{x}) = 0 \tag{$IC1$}$$

for some constant $D \in \mathbb{R}$ and

$$R_v(x, x) \leq R_{nv}(\hat{x}) - \phi_T \qquad \forall\, x, \hat{x}\ with\ \beta(x) = 1\ and\ \beta(\hat{x}) = 0 \tag{$IC2$}$$

where $\phi_T = \frac{1}{2}(1 - p\lambda)^2\, \frac{\tau^2}{k}$ is the entrepreneur's expected gain from tax evasion.

Proof. Follows from the arguments above. □

As will become evident in the subsequent section, the incentive compatibility constraints in Proposition 2 are key determinants of the shape of the optimal financial contract. Constraint $(IC1)$ requires that whenever a report is not verified by the investor, the repayment is constant. This constant payment under non-verification is familiar from the literature on costly state verification and is typically associated with the face-value payment of a debt-contract. $(IC1)$ thus induces a debt-like property of the optimal contract.

The constraint $(IC2)$ requires that the repayment under non-verification always exceeds the repayment under verification by at least the entrepreneur's expected gain from tax evasion, ϕ_T. This constraint is a variation of the incentive-compatibility constraint of Gale and Hellwig (1985). To prevent misreporting by the entrepreneur in this setting, however, it is not simply enough to stipulate a (weakly) higher repayment under non-verification as in Gale and Hellwig's (1985) standard debt contract. Instead, a strictly larger (by the amount ϕ_T) repayment is required because the entrepreneur has an additional incentive to lie and claim a non-verified project success so as to be able to engage in tax evasion.[10] The jump in the optimal contract thus induced by $(IC2)$ due to the entrepreneur's possibility to engage in tax evasion is a key novelty in the present paper.

3.3 The optimal contract

We are now in a position to derive the optimal contract offered by the entrepreneur in the initial stage of the game. In addition to the feasibility constraints (F_v) and (F_{nv}) from Corollary 1 and the incentive-compatibility constraints $(IC1)$ and $(IC2)$ from Proposition 2, the optimal contract also has to satisfy the investor's participation constraint. That is, the

[10] Note that the expected gains from tax evasion, and hence the rigidity of $(IC2)$ and the discontinuity in any incentive-compatible financial contract, depend in expected ways on the parameters that are traditionally thought to govern tax evasion behavior, namely p, λ, τ, and k. As a consequence, tax enforcement policy directly influences financial contracting, making it more standard debt-like as audit probability and fines increase, and less so as the tax rate τ increases.

financial contract has to yield, in expectation, at least $\underline{v}$ to the investor, his reservation utility. Formally, we have

$$\int_{\underline{x}}^{\overline{x}} [\,(1-\beta(\hat{x}))R_{nv}(\hat{x}) + \beta(\hat{x})R_v(\hat{x},x)\,]dF(x) \geq \underline{v} \qquad (PC)$$

The optimal contract (R,β) therefore solves the following problem of maximizing the entrepreneur's expected utility subject to the constraints (F_v), (F_{nv}), $(IC1)$, $(IC2)$, and (PC).

$$\max_{R,\beta} \quad \{ \int_{\underline{x}}^{\overline{x}} [\,\beta[(1-\tau)x - R_v] + (1-\beta)[(1-\tau)x - R_{nv} + \phi_T]\,]\, dF(x) \,\}$$

subject to

$$R_v(\hat{x},x) \leq (1-\tau)x \qquad (F_v)$$

and

$$R_{nv}(\hat{x}) \leq (1-\tau)x - \phi_{nv} \qquad (F_{nv})$$

and

$$R_{nv}(\hat{x}) = D \; for\, some\; D \in \mathbb{R} \qquad \forall\, \hat{x} \quad with \quad \beta(\hat{x}) = 0 \qquad (IC1)$$

and

$$R_v(x,x) \leq R_{nv}(\hat{x}) - \phi_T \qquad \forall\, x,\hat{x}\; with\; \beta(x) = 1\; and\; \beta(\hat{x}) = 0 \qquad (IC2)$$

and

$$\int_{\underline{x}}^{\overline{x}} [\,(1-\beta)R_{nv} + \beta R_v\,]\, dF(x) \geq \underline{v} \qquad (PC)$$

where $\phi_T = \frac{1}{2}(1-p\lambda)^2\,\frac{\tau^2}{k}$ is the expected gain from tax evasion
and $\phi_{nv} = [\,(1-p)^2 - (1-p\lambda)^2\,]\,\frac{\tau^2}{2pk}$ is the feasibility constant derived above.

Before we derive the optimal contract (R,β) solving this problem, let us consider two aspects of the maximization problem that will provide intuition regarding its solution.
First, we may rewrite the objective function of the problem, the entrepreneur's expected

utility, as

$$\int_{\underline{x}}^{\overline{x}} (1-\tau)x\, dF(x) - \int_{\underline{x}}^{\overline{x}} [(1-\beta)R_{nv} + \beta R_v]\, dF(x) + \int_{\underline{x}}^{\overline{x}} (1-\beta)\phi_T\, dF(x)$$

The first term is just the expected statutory net income, on which the choice of contract has no influence.

The second term is the expected repayment to the investor, familiar from the participation constraint (PC). Since the investor's participation constraint binds at the optimum[11], however, the second term reduces to $\underline{v}$. This focuses the entrepreneur's contracting challenge on the third term, her expected gain from tax evasion, in a reasoning familiar from the literature on costly state verification.[12] Since evasion only happens under non-verification, (i.e. when $\beta(\hat{x}) = 0$, whereas the third term above becomes 0 if $\beta(\hat{x}) = 1$), the optimal contract will *maximize* the range of non-verification subject to satisfying the constraints. Equivalently, the range of realizations that are verified by the investor, thus leaving no room for a tax evasion gain, ought to be *minimized* by an optimal contract.

Second, notice that (F_v) and ($IC2$) may be combined into the following constraint on repayment under verification, R_v,

$$R_v \leq min\,\{\,(1-\tau)x\,,\, R_{nv} - \phi_T\,\}$$

So repayment under investor verification needs to be weakly smaller than the statutory maximum contractible income, $(1-\tau)x$, but also than repayment under non-verification, R_{nv}, less the expected gain from tax evasion ϕ_T. This points at the intuitive discontinuity discussed in the previous section. If repayment under verification, R_v, gets "too close" (closer than ϕ_T) to repayment under non-verification, R_{nv}, the entrepreneur would prefer to misrepresent her true earnings and promise the slightly higher repayment to be able to take a chance at tax evasion. To be incentive-compatible, a contract therefore needs to demand from E a repayment under non-verification that exceeds the repayment under verification by at least the expected gain from evading taxes. This combined constraint on R_v is responsible for the characteristic shape of the optimal contract that we derive in the following proposition.

[11] To see why the participation constraint (PC) is binding at the optimum, consider towards a contradiction a non-binding (PC) at the optimum. Then R_v could be lowered, thereby strictly increasing the entrepreneur's profits while relaxing all constraints it affects and still satisfying (PC). So the original situation cannot have been an optimum.

[12] See, for instance, Tirole (2005), Ch.3

Proposition 3. [Optimal contract]

a) Every contract satisfying the constraints (F_v), (F_{nv}), $(IC1)$, $(IC2)$, and (PC) is weakly dominated by a contract of the following form, denoted by χ,

$$R(x,\hat{x}) = \begin{cases} D & if \quad \beta(\hat{x}) = 0 \\ \\ min\{\,(1-\tau)x\,,\, D-\phi_T\,\} & if \quad \beta(\hat{x}) = 1 \end{cases}$$

where

$$\beta(\hat{x}) = 0 \quad iff \quad \hat{x} \in B_{nv} = \{x \mid x \geq \tilde{x}\}$$

and

$$\beta(\hat{x}) = 1 \quad iff \quad \hat{x} \in B_v = \{x \mid x < \tilde{x}\}$$

and $\tilde{x}$ is such that

$$D = (1-\tau)\tilde{x} - \phi_{nv}$$

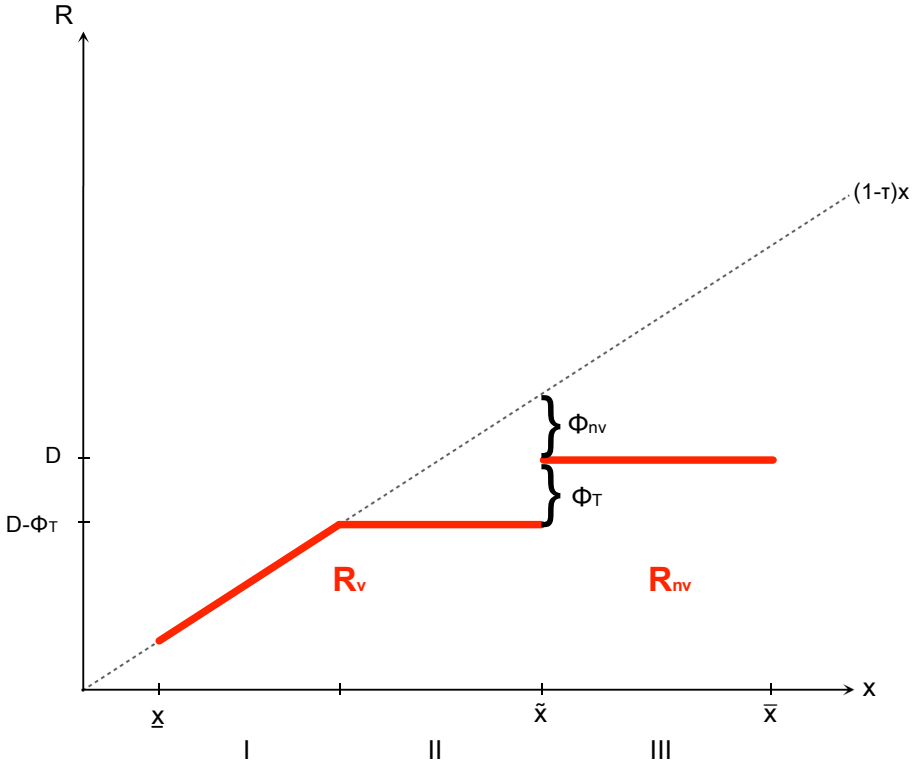
where $\phi_T = \frac{1}{2}(1-p\lambda)^2\,\frac{\tau^2}{k}$ and $\phi_{nv} = [\,(1-p)^2 - (1-p\lambda)^2\,]\,\frac{\tau^2}{2pk}$.

b) Contracts of the form χ are uniquely optimal.

Proof. See appendix A.2. □

The optimal contract minimizes verification by charging the maximum possible repayment inside the verification region. Crucially, this does not lead to a standard debt contract in this setting, which is neither feasible nor incentive-compatible when entrepreneurs can evade taxes. Instead, while sharing some characteristics with standard debt, our optimal contract differs from it in important respects, as the following figure illustrates.

Figure 2 - Shape of the optimal contract



First, if a report is not verified (range III in Figure 2), the optimal contract stipulates a constant repayment of size $D = (1-\tau)\tilde{x} - \phi_{nv}$ to the investor. This constant face-value, upon repayment of which no verification takes place, is a debt-like characteristic of our optimal contract. However, as we argued in section 3.1.2, the repayment $R_{nv} = D$ cannot charge up to the limited liability $(1-\tau)x$ (dotted line) anywhere in the non-verification range III. Instead, the repayment has to always be ϕ_{nv} below this level. This is required for a feasible contract because otherwise the entrepreneur could profitably use her limited liability protection when evading taxes, making the repayment unfeasible in case a tax audit occurs.

Second, consider region I, where verification takes place, i.e. $\beta(\hat{x}) = 1$, and the repayment is the entire contractible income, $R_v = (1-\tau)x$, leaving no rent to the entrepreneur. Here, as in a standard debt-contract, the repayment charges up to the agent's limited liability.

Third, consider region II, where verification takes place but the repayment is constant at $R_v = D - \phi_T$. This is a hybrid region combining elements of debt (constant repayment) and equity (leaving some rent to the entrepreneur under verification). As argued in section 3.2.2, ϕ_T is the entrepreneur's expected gain from tax evasion. To induce incentive compatibility on part of the entrepreneur, the repayment under verification, R_v, (where no tax evasion is possible) has to always be at least ϕ_T below the repayment under non-verification, R_{nv}, where evasion is possible. Otherwise, the entrepreneur would simply misrepresent her true earnings

as being in the non-verification region III of the contract and take a chance at tax evasion, which, in expectation, would then be profitable for her.

4 Conclusion

This paper analyzes a model of costly state verification with tax evading entrepreneurs. We posit an informational externality between investor verification and tax auditing. When investors choose to verify the true state of the world, this delivers information to the government, making tax evasion less profitable. The optimal financial contract therefore minimizes investor verification to provide the entrepreneur with a maximum of tax evasion possibilities. Yet the contract achieving this is not a standard debt contract, as the original work on costly state verification by Townsend (1979) and Gale and Hellwig (1985) might suggest. Instead, we find an optimal contract which is debt-like only for very low and very high realizations, and combines elements of debt and equity in an intermediate range. This is because tax evasion represents a special challenge for financial contracting. Since it is illegal, the gains from evasion cannot be contracted on. But if tax evasion is detected, fines may reduce the entrepreneur's ability to repay the investor. Except for very low realizations, the optimal contract in this setting always leaves the entrepreneur with a positive rent to prevent her from abusing her limited liability protection for excessive tax evasion activities. This makes the optimal contract less efficient at minimizing investor verification than a standard debt contract, which is neither feasible nor incentive compatible when borrowers may evade taxes.

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Appendix

A.1 Proof of Proposition 1

As argued in section 3.1.1 above, $e_v^* = 0$ follows immediately from the public revelation of x in case $\beta(\hat{x}) = 1$.

Let us now focus on the entrepreneur's evasion choice under non-verification, e_{nv}^*. So suppose $\beta(\hat{x}) = 0$.

The first order conditions for a maximum are given by

$$(1 - p\lambda)\tau - c_e(e_{nv,(a)}^*) = 0 \qquad if \qquad (a)$$

and

$$(1 - p)\tau - c_e(e_{nv,(b)}^*) = 0 \qquad if \qquad (b)$$

The assumptions on $c(e)$ imply that the second order conditions for a maximum are satisfied. From the first order conditions for a maximum, and the fact that $c_e(e) = ke$, we obtain

$$e_{nv,(a)}^* = (1 - p\lambda)\frac{\tau}{k}$$

and

$$e_{nv,(b)}^* = (1 - p)\frac{\tau}{k}$$

The entrepreneur now chooses $e_{nv}^* = e_{nv,(a)}^*$ if and only if

$$U_{nv}(e_{nv,(a)}^*, \hat{x}, x) \geq U_{nv}(e_{nv,(b)}^*, \hat{x}, x)$$

or equivalently

$$(1-\tau)x + (1-p\lambda)\tau e_{nv,(a)}^* - R_{nv}(\hat{x}) - c(e_{nv,(a)}^*) \geq (1-p)[(1-\tau)x + \tau e_{nv,(b)}^* - R_{nv}(\hat{x})] - c(e_{nv,(b)}^*)$$

Using the expressions for $e_{nv,(a)}^*$ and $e_{nv,(b)}^*$ above and the fact that $c(e) = \frac{k}{2}e^2$, this inequality becomes

$$(1 - \tau)x + (1 - p\lambda)^2\frac{\tau^2}{2k} - R_{nv}(\hat{x}) \geq (1 - p)[(1 - \tau)x - R_{nv}(\hat{x})] + (1 - p)^2\frac{\tau^2}{2k}$$

And rearranging, we obtain

$$R_{nv}(\hat{x}) \leq (1 - \tau)x - [(1 - p)^2 - (1 - p\lambda)^2]\frac{\tau^2}{2kp} \equiv (1 - \tau)x - \phi_{nv}$$

as required. □

A.2 Proof of Proposition 3

a)

Consider any arbitrary contract $C : (R, \beta)$ satisfying (F_v), (F_{nv}), $(IC1)$, $(IC2)$, and (PC).
Since C satisfies $(IC1)$, repayment under non-verification is constant and we denote this constant by D.
Now construct a contract of the form χ, denoted $C^\chi : (R^\chi, \beta^\chi)$ with the same constant repayment D under non-verification as the original contract C.
We have thus

$$R^\chi(x, \hat{x}) = \begin{cases} D & if \quad \beta^\chi(\hat{x}) = 0 \\ \\ min\{\,(1-\tau)x\,,\, D - \phi_T\,\} & if \quad \beta^\chi(\hat{x}) = 1 \end{cases}$$

where

$$B^\chi_{nv} = \{x \mid \beta^\chi(x) = 0\} = \{x \mid x \geq \tilde{x}\}$$

and

$$B^\chi_{v} = \{x \mid \beta^\chi(x) = 1\} = \{x \mid x < \tilde{x}\}$$

and $\tilde{x}$ is such that

$$D = (1-\tau)\tilde{x} - \phi_{nv}$$

By construction, the new contract satisfies (F_v), (F_{nv}), $(IC1)$, and $(IC2)$.
Denote by $B_v = \{x \mid \beta(x) = 1\}$ the verification set under the original contract, and by $B_{nv} = \{x \mid \beta(x) = 0\}$ its complement, the non-verification set.
First, we show that $B^\chi_v \subseteq B_v$, meaning the verification region is weakly smaller under the new contract.
So suppose $x \in B^\chi_v$. Then in particular, $x < \tilde{x}$. We need to show that this implies $x \in B_v$.
Toward a contradiction, suppose this were not the case, i.e. $x \in B_{nv}$.
Then since x is not verified under the old contract, the repayment under the old contract is $R_{nv}(x) = D = (1-\tau)\tilde{x} - \phi_{nv}$.
However, since $x < \tilde{x}$, we have

$$R_{nv}(x) = D = (1-\tau)\tilde{x} - \phi_{nv} > (1-\tau)x - \phi_{nv}$$

This is a violation of (F_{nv}), contradicting our assumption that the original contract satisfies (F_{nv}). Therefore, it cannot be true that $x \in B_{nv}$ and so we must have $x \in B_v$.
Thus we have shown that $B^\chi_v \subseteq B_v$. The verification set under the new contract C^χ is weakly smaller than under the original contract C.

Note that this result implies $B_{nv} \subseteq B_{nv}^{\chi}$, that is, the non-verification set under the new contract is weakly larger than under the original contract C.
Now compare the repayment to the investor under the two contracts.
If $x \in B_{nv}$, then $x \in B_{nv}^{\chi}$ and so the repayment is the same in both contracts, $R_{nv}(x) = R_{nv}^{\chi}(x) = D$.
If $x \in B_v$, there are two cases, since $B_v^{\chi} \subseteq B_v$. Either x induces verification also under the new contract, i.e. $x \in B_v^{\chi}$. Or x does not induce verification under the new contract, i.e. $x \in B_{nv}^{\chi}$.
Consider first the case where $x \in B_v^{\chi}$. Then the repayment under the new contract C^{χ} is $R_v^{\chi}(x, x) = min\{(1-\tau)x\,,\, D - \phi_T\}$. Under the old contract, which satisfies (F_v) and $(IC2)$, the repayment satisfies $R_v(x, x) \leq min\{(1-\tau)x\,,\, D - \phi_T\}$. So the repayment to the investor weakly increases under the new contract.
Now consider the second case, i.e. $x \in B_{nv}^{\chi}$. The new repayment is $R_{nv}^{\chi}(x) = D$, which is larger than the repayment under the old contract for such x, which satisfies $R_v(x, x) \leq min\{(1-\tau)x\,,\, D - \phi_T\} < D$. The repayment to the investor therefore increases in this case.
The new contract C^{χ} thus increases the expected repayment to the investor, making the participation constraint (PC) slack.
It also increases, however, the overall surplus for the contracting parties, since the non-verification region B_{nv}^{χ} weakly increases under the new contract. This means the expected tax evasion gain is larger under the new contract C^{χ}:

$$\int_{\underline{x}}^{\overline{x}} (1 - \beta^{\chi})\phi_T \, dF(x) \geq \int_{\underline{x}}^{\overline{x}} (1 - \beta)\phi_T \, dF(x)$$

But then we can decrease the threshold $\tilde{x}$ and payments R until the investor's participation constraint is binding again. This makes the entrepreneur strictly better off, since the overall surplus of the contract increases due to a smaller verification region but the expected repayment to I stays the same at $\underline{v}$, meaning the entire gain from less verification accrues to the entrepreneur through a higher expected tax evasion gain. $\square$

b)

So suppose the two contracts $C : (R, \beta)$ and $C^{\chi} : (R^{\chi}, \beta^{\chi})$ are both optimal contracts, where C^{χ} is a contract of the form χ derived from C as part a) of the proposition. Since they are both optimal, this means they maximize the entrepreneur's expected utility subject to the constraints (F_v), (F_{nv}), $(IC1)$, $(IC2)$, and (PC). But this implies

$$\int_{\underline{x}}^{\overline{x}} (1 - \beta^{\chi})\phi_T \, dF(x) = \int_{\underline{x}}^{\overline{x}} (1 - \beta)\phi_T \, dF(x)$$

and so

$$\int_{\underline{x}}^{\overline{x}} (\beta^{\chi} - \beta)\phi_T \, dF(x) = 0$$

Since $B_v^{\chi} \subseteq B_v$, we have $\beta^{\chi}(x) \leq \beta(x)$ for all $x \in [\underline{x}, \overline{x}]$. But then the above equality can only hold if

$$\beta^{\chi}(x) = \beta(x) \qquad \forall x$$

We will show that this implies the contracts have to be the same.
Consider first the case where $\beta^{\chi}(x) = \beta(x) = 0$. Then the repayment under both contracts is the same, $R_{nv}(x) = R_{nv}^{\chi}(x) = D$.
Now consider the other case, where $\beta^{\chi}(x) = \beta(x) = 1$. Under the new contract, the repayment is $R_v^{\chi}(x, x) = min\{(1-\tau)x \, , \, D - \phi_T\}$. But under the original contract, since it satisfies (F_v) and $(IC2)$, the repayment satisfies $R_v(x, x) \leq min\{(1-\tau)x \, , \, D - \phi_T\}$. If, however, there exists an x with $\beta^{\chi}(x) = \beta(x) = 1$ and $R_v(x, x) < min\{(1-\tau)x \, , \, D - \phi_T\}$, this implies a strictly higher payoff under the original contract than under the new contract C^{χ}. This contradicts our assumption that both contracts are optimal. So we must have $R_v(x, x) = R_v^{\chi}(x, x) = min\{(1-\tau)x \, , \, D - \phi_T\}$ for all x with $\beta(x) = \beta^{\chi}(x) = 1$.
This proves that both contracts are exactly equal and of the form χ, and so any optimal contract is necessarily of the form χ, as required. $\square$