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# Optimal Dynamic Carbon Taxation in a Life-Cycle Model with Distortionary Fiscal Policy

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## Abstract

We quantitatively characterize optimal carbon, capital, and labor income taxes in an economy-climate integrated assessment model that features overlapping generations and distortionary fiscal policy. First, we show that the optimal carbon tax significantly differs from the Pigouvian carbon levy in a first-best setting with overlapping generations in which fully rational households optimize over finite lifetimes. The key driving force behind this result is the life-cycle structure of the our model, in conjunction with endogenously chosen labor supply. We also show that the assumed labor supply elasticity is important for the size of deviation of the optimal carbon tax from the Pigouvian tax, but not the existence of the deviation from Pigouvian pricing. Second, interacting life-cycle household behavior with distortionary fiscal policy is shown to further drive a wedge between the second-best optimal carbon tax and a Pigouvian carbon levy.

**Keywords:** Optimal Carbon Taxation, Climate Policy, Capital Taxation, Overlapping Generations, Fiscal Distortions.

**JEL classification:** E13, H21, H24, Q54, C61

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# 1 Introduction

At what rate should the government tax greenhouse gas emissions such as carbon dioxide (CO<sub>2</sub>) in order to address the climate change externality? A central tenet of environmental economics—that can be traced back at least as far as [Pigou \(1938\)](#)—is that taxes are an effective mechanism for “getting the prices right” and can improve welfare outcomes by internalizing externalities. In a *first-best* setting in which pollution is the only market failure and there exist no other distortions in the economy, the optimal carbon tax is *Pigouvian* and equals the social cost of carbon—the value of marginal damages from another ton of greenhouse gas emissions.<sup>1</sup> The validity of this fundamental result in a *first-best* general equilibrium setting is confirmed by recent work by [Goloso et al. \(2014\)](#) who show that optimal dynamic fuel taxes follow the standard Pigou reasoning. An extensive literature (coined the “double-dividend” literature) has studied the interaction of environmental taxation and fiscal policy suggesting that in the presence of other (non-environmental) distortionary taxes the *second-best* optimal pollution tax equals marginal damages divided by the marginal cost of public funds ([Bovenberg and de Mooij, 1994](#); [Parry, 1995](#); [Bovenberg and Goulder, 1996](#)). The need to raise revenue through, at least partially, distortionary taxes (such as income, payroll, and sales taxes) to fund the levels of public spending observed in modern real-world economies, then implies that the optimal carbon tax is smaller than the *Pigouvian* carbon levy.

This paper contributes to the literature in two ways. First, we show that the optimal carbon tax differs from the *Pigouvian* carbon levy in a *first-best* setting with overlapping generations in which fully rational households optimize over finite lifetimes. The key driving force behind this result is the life-cycle structure of the our model, in conjunction with endogenously chosen labor supply. We also show that the assumed labor supply elasticity is important for the size of deviation of the optimal carbon tax from the *Pigouvian* tax, but not the existence of the deviation from *Pigouvian* pricing. Second, we interact life-cycle elements with distortionary fiscal policy and show that this further increases the deviation between the *second-best* optimal carbon tax and a *Pigouvian* carbon levy.

Our quantitative investigation of optimal carbon taxes and distortionary fiscal policy is carried out with a novel integrated assessment model (IAM) of the global economy that combines

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<sup>1</sup> Following [Goulder \(1995\)](#), [Bovenberg and de Mooij \(1994\)](#), and [Barrage \(2013\)](#), we define “Pigouvian” as marginal damages value by the households’ marginal utility of consumption evaluated at the optimal allocation. In contrast, studies such as [Cremer et al. \(1998\)](#) define it as marginal damages valued by the government’s marginal utility of income.

elements from a standard economy-climate IAMs—following closely the state-of-the-art representation of the carbon cycle and climate-economy feedbacks of the seminal DICE framework (Nordhaus, 2013)—with a large-scale Auerbach and Kotlikoff (1987) overlapping generations model that features distortionary capital and labor taxation. Emissions and output growth as well as the climate damage function are calibrated to follow closely the baseline of the DICE Nordhaus (2013) model. To overcome the difficulty of incorporating *second-best* features such as distortionary taxes in optimizing IAM models which are typically formulated as non-linear programs<sup>2</sup>, we formulate the problem of solving for the general equilibrium allocation of the underlying model that features a climate externality *and* fiscal distortions as a mixed complementarity problem (MCP).<sup>3</sup> The problem of finding optimal policies is then solved as a Mathematical Program with Equilibrium Constraints (MPEC) (Luo et al., 1996).<sup>4</sup>

Our two main results build on and are related to several branches of the literature. The main focus of this paper, namely to study the optimal design of carbon taxes both as an instrument to address climate change and as a part of fiscal policy is on the one hand related to the “double-dividend” literature (Bovenberg and de Mooij, 1994; Parry, 1995; Bovenberg and Goulder, 1996; and Babiker et al., 2003) and on the other hand to the literature on optimal taxation (Sandmo, 1975; Cremer et al., 1998). These papers, however, are characterized by two major limitations: first, they do not incorporate the environmental benefits of climate policy, and second, they tend to be predominantly focus on static setting, often highlighting the implications arising from distorted (static) labor markets. As such, they fail to ignore any intertemporal distortions and their effects on dynamic processes such as carbon and capital accumulation. In contrast, this paper investigates optimal carbon and fiscal taxation in a dynamic general equilibrium framework that also incorporates a fully endogenous link between the climate and economic system thereby incorporating the environmental benefits and damages of climate policy.

Similar to this paper, recent work by Barrage (2013) and Schmitt (2013) also addresses

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<sup>2</sup> See Böhringer et al. (2007) for a more detailed discussion of this aspect.

<sup>3</sup> Rutherford (1995) and Mathiesen (1985) have shown that a complementary-based approach is convenient, robust, and efficient. A characteristic of many economic models is that they can be cast as a complementary problem, i.e. given a function  $F: \mathbb{R}^n \rightarrow \mathbb{R}^n$ , find  $z \in \mathbb{R}^n$  such that  $F(z) \geq 0$ ,  $z \geq 0$ , and  $z^T F(z) = 0$ . The complementarity format embodies weak inequalities and complementary slackness, relevant features for models that contain bounds on specific variables, e.g. activity levels which cannot a priori be assumed to operate at positive intensity. Such features are not easily handled within an optimizing approach.

<sup>4</sup> Solving for the optimal design of the policy instruments, changes the equilibrium model from a MCP to a bi-level optimization problem. The first level is a mathematical programming problem which maximizes the objective function subject to a number of constraints. The second-level problem is a MCP that characterizes the equilibrium conditions for the economic and climate models.

these two major shortcomings of the previous literature by employing a dynamic optimal tax framework that is linked to a DICE-type model of the climate and the carbon cycle. While these papers are similar in terms of the key questions they ask, our paper differs with respect to one important dimension: we investigate the optimal dynamic carbon taxation problem in a world which is characterized by overlapping generations (whereas their work assumes a standard infinitely lived agent). This difference is found to substantially alter the key results found in [Barrage \(2013\)](#) and [Schmitt \(2013\)](#). Whereas [Barrage \(2013\)](#) finds that if it is optimal to set capital taxes to zero, then optimal carbon taxation—in the absence of distortionary fiscal taxes—fully internalizes production damages from climate change at the *Pigouvian* rate”, we find—quite differently—that in an overlapping generations economy it is optimal to tax carbon below the *Pigouvian* rate even if fiscal taxes are non-distortionary, i.e. the sequence of exogenous government expenditures can be financed through (personalized) lump-sum taxes. Interpreting climate as an environmental capital ([Nordhaus, 2010](#); [Barrage, 2013](#)) allows us to relate this result to major insights established by the macroeconomics literature on optimal capital income taxation. While the seminal contributions by [Chamley \(1986\)](#) and [Judd \(1985\)](#) and subsequent literature (for example, [Atkeson et al. \(1999\)](#) [Chari and Kehoe \(1999\)](#)) have found that it is optimal to tax capital income in the long-run at a rate of zero, [Conesa et al. \(2009\)](#) find that endogenous labor supplied differentially over the life cycle is crucial in driving an optimal long-run tax on capital income that is significantly positive.<sup>5</sup>

From a methodological point of view, a limited number of papers has used dynamic general equilibrium models with overlapping generations model to study issues of environmental taxation ([Rasmussen, 2003](#); [Carbone et al., 2012](#); [Rausch, 2013](#)). These papers have investigated the economic and welfare impacts of achieving an exogenous emissions reduction target but do not consider the climate-economy linkage, i.e. they assume that environmental quality is constant. [Leach \(2009\)](#) is the only paper, to the best of our knowledge, that combines an integrated assessment model with an OLG approach. This latter paper is, however, focused on assessing the generational welfare impacts of Kyoto-type climate policies and does not consider the interaction with fiscal and optimal policies.

The structure of the paper is as follows. Section 2 characterizes the optimal and Pigouvian emission taxes in an overlapping generation framework. Section 3 introduces the numerical model used and describes the calibration procedure. Section 4 describes our computational

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<sup>5</sup> Using a realistically calibrated large-scale overlapping generations model [Conesa et al. \(2009\)](#) find that the optimal capital income tax rate for the United States economy is positive at 36 percent.

experiment, and present the key results. Section 5 concludes.

## 2 Model and Theory Results

We consider an economy of overlapping generations of identical consumers similar to the one studied by [Erosa and Gervais \(2002\)](#). Individuals gain utility out of commodity and leisure consumption. Commodities are produced using capital, labor, and carbon emissions as inputs. Emissions cause economic damage in future periods which we assume materialize in form of production losses only, i.e. we do not allow for the case in which climate damages directly affect private utility.<sup>6</sup> Throughout our analysis, we assume that an exogenous time path of government expenditures is given that has to be financed by using either age-dependent labor and capital income taxes, or carbon taxes.

### 2.1 Economy

Each individual lives for  $J + 1$  periods. We denote generations by their period of birth  $t \in \{-J, -J + 1, \dots, 0, 1, \dots\}$  where zero denotes the period when the policy decision takes places, i.e. the initial period. The age of individuals in the initial period is given by  $j_0(t) := \max(-t, 0)$ . Labor productivity at age  $j$  is given as  $z_j$ . The population share of age  $j$  individuals is time invariant and given as  $\mu_j = \frac{1}{J+1}$ . Labor endowment is normalized to one and the labor supply, consumption, and asset holdings of generation  $t$  in age  $j$  are denoted by  $l_{tj}$ ,  $c_{tj}$ , and  $a_{tj}$ , respectively. The net-of (labor income) tax wage rate is given as  $w_{tj} := (1 - \tau_{tj}^l) \hat{w}_{t+j}$  and  $w_{t+j}$  is the market clearing wage in period  $t + j$ . Similarly, the real net of (capital income) tax interest rate is given as  $r_{tj} := (1 - \tau_{tj}^r) \hat{r}_{t+j}$  where  $\hat{r}_{t+j}$  is the real interest in period  $t + j$ . Lifetime utility is assumed to be linear time-separable and the period-utility function  $U(c_{tj}, 1 - l_{jt})$  is increasing in consumption and leisure, strictly concave, and satisfies the standard Inada conditions. The

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<sup>6</sup> This is unarguably a major (and unrealistic) simplification. For example, recent work by [Barrage \(2013\)](#) suggests that optimal carbon taxes must value climate damages that affect production differently from those that affect utility directly. Because internalizing utility damages yields no production gain and creates efficiency costs due to (fiscal) tax interactions, she finds that the optimal carbon tax does not fully internalize utility damages, taxing them below the Pigouvian rate. Given the complexity of the problem at hand, we focus in this paper on production damages from climate only and leave for future research to investigate the implications of utility damages for optimal carbon and distortionary income taxation in an overlapping generations framework.

utility maximization problem of generation  $t$  is:

$$\max_{\{c_{tj}, l_{tj}\}_{j=j_0(t)}^J} \sum_{j=j_0(t)}^J \beta^{j-j_0(t)} U(c_{tj}, 1 - l_{tj}) \quad (1)$$

$$c_{tj} + a_{t(j+1)} \geq w_{tj} z_{tj} l_{tj} + (1 + r_{tj}) a_{tj} \quad \forall j \quad (2)$$

$$a_{tj_0(t)} \text{ given; } j_0(t) = 0 \Rightarrow a_{tj_0(t)} = 0 \quad (3)$$

where  $\beta$  denotes the private discount factor. Aggregated consumption and labor in period  $t$  are given as:

$$c_t := \sum_{j=0}^J \mu_j c_{(t-j)j} \quad L_t := \sum_{j=0}^J \mu_j z_j l_{(t-j)j}. \quad (4)$$

In each period a single commodity is produced using capital  $K_t$ , labor  $L_t$ , and emissions  $E_t$  as inputs. The production technology is assumed to exhibit constant returns to scale and is represented by a neoclassical production function  $y_t = f(K_t, L_t, E_t)$ . Emissions cause damages in future periods. These damages are represented by the damage function  $D_t := D_t(E_{t-1}, E_{t-2}, \dots)$  which determine the climate damage as fraction of total output in period  $t$ . The damage function is assumed to be strictly increasing in past emissions ( $\frac{\partial D_t}{\partial E_\tau} > 0; t > \tau$ ). Due to the constant-returns-to-scale assumption, capital and labor are paid in equilibrium with their respective net-of-damage marginal products, and the marginal product of emissions equals the emission tax  $\tau_t^E$ :

$$(1 - D_t) f_t^K = \hat{r}_t + \delta \quad (1 - D_t) f_t^L = \hat{w}_t \quad (1 - D_t) f_t^E = \tau_t^E \quad (5)$$

where  $\delta$  denotes the capital depreciation rate. Output net of damage can be used as an investment commodity, for final consumption, and as a public good  $g_t$ . Denoting the exogenous and constant capital depreciation rate by  $\delta$ , the feasibility or market clearing constraint in period  $t$  becomes:

$$(1 - D_t) y_t \geq K_{t+1} - (1 - \delta) K_t + c_t + g_t. \quad (6)$$

Public good consumption  $g_t$  is exogenous. Besides carbon taxes  $\tau_t^E$ , the government has access to age-dependent labor and capital income taxes,  $\tau_{tj}^l$  and  $\tau_{tj}^r$  to finance the exogenous sequence of  $g_t$ . As we do not allow for government debt, the public budget constraint for period  $t$  is given by:

$$\sum_{j=0}^J \mu_j \left[ \tau_{(t-j)j}^r \hat{r}_t + \tau_{(t-j)j}^l z_j \hat{w}_t \right] + t_t^E E_t = g_t. \quad (7)$$

## 2.2 Ramsey Optimal Tax Problem

The government chooses the sequence of income and emissions taxes to finance a given, exogenous path of government expenditures  $\{g_t\}_{t=-J}^{\infty}$  in order to maximize the following Bergson-Samuelson functional:

$$W := \sum_{t=-J}^{\infty} \gamma^t U_t \quad (8)$$

where  $\gamma$  denotes the social discount factor and  $U_t$  the indirect utility function of the generation born in period  $t$ . Denoting the Lagrangian multiplier on the implementability constraint for generation  $t$  by  $\phi_t$ , we define the modified welfare function for generation  $t$  as:<sup>7</sup>

$$\tilde{W}_t := \sum_{j=j_0(t)}^J \beta^{j-j_0(t)} \left[ U(c_{tj}, 1 - l_{jt}) + \phi_t \left( U_{tj}^c c_{tj} + U_{tj}^l l_{tj} \right) \right] + \phi_t A_{tj_0(t)}. \quad (9)$$

$U_{tj}^c := \frac{\partial U}{\partial c_{tj}}$  and  $U_{tj}^l := \frac{\partial U}{\partial l_{tj}}$  denote the marginal utility of commodity and leisure consumption and  $A_{tj_0(t)} = U_{tj_0(t)}^c (1 + t_{tj_0(t)}) a_{tj_0(t)}$  the valuation of initial asset holdings. The optimal taxation problem in primal form maximizes the discounted sum of the modified welfare function under the feasibility constraint:<sup>8</sup>

$$\max_{\{c_{tj}, l_{tj}\}_{j=j_0(t)}^J, K_{t+J+1}, E_{t+J}} \sum_{t=-J}^{\infty} \gamma^t \tilde{W}_t \quad (10)$$

$$\text{s.t.} \quad (1 - D_t) y_t \geq K_{t+1} - (1 - \delta) K_t + c_t + g_t. \quad (11)$$

## 2.3 Pigouvian and Optimal Taxes

The optimal tax problem given in (10) and (11) is similar to problem studied in [Erosa and Gervais \(2001\)](#). Besides neglecting population growth, it differs from their approach by including carbon emissions and production damages that depend on past emissions. Let  $H_{tj}^c := \frac{\frac{\partial^2 U}{\partial c_{jt}^2} c_{jt} + \frac{\partial^2 U}{\partial l_{jt} \partial c_{jt}} l_{jt}}{U_{tj}^c}$  and  $H_{tj}^l := \frac{\frac{\partial^2 U}{\partial l_{jt}^2} l_{jt} + \frac{\partial^2 U}{\partial c_{jt} \partial l_{jt}} c_{jt}}{U_{tj}^l}$  denote the general equilibrium elasticities for consumption and leisure, respectively (see, for example, [Atkeson et al., 1999](#)). The main result of [Erosa and Gervais \(2001\)](#) carries over to our model with emissions taxes and production damages from climate.

**Proposition 1** *At each date, the optimal tax rate on labor income is different from zero unless*

<sup>7</sup> Appendix A derives the implementability constraint.

<sup>8</sup> Walras' Law implies that the public budget restriction holds which is omitted here.



$H_{tj}^l = H_{tj}^c$  and the optimal capital income tax is different from zero unless  $H_{tj}^c = H_{t(j+1)}^c$ .

**Proof.** See Appendix B. ■

As labor productivity typically varies over the life cycle of a household, Proposition 1 suggests that optimal capital income taxes are likely to be positive in an OLG framework. Erosa and Gervais (2001) show that in the absence of productivity variations over the life cycle the optimal capital becomes zero if the private and social discount factors coincide. This is in contrast to the seminal zero-capital income tax result for a Ramsey economy with an infinitely-lived representative consumer in which  $H_{tj}^c$  converges to a constant in the steady state (Judd, 1985; Chamley, 1986).

Equation (12) below defines the Pigouvian tax for our overlapping generations framework in terms of the marginal welfare impact. Intuitively, the Pigouvian evaluates the marginal damage of period  $t$  emissions in all future periods. The expression in the de-/nomination represents the marginal utility of an additional unit of consumption for all individuals alive in a given time period. Consequently, the fraction evaluates the marginal utility loss of future individuals caused by one unit of emissions today (nominator) against the marginal utility gained by individuals consuming in period  $t$ .

**Definition 1** *The Pigouvian emission tax is the present value of future marginal damages evaluated at the government marginal welfare of consumption at the optimal allocation:*

$$\begin{aligned} \tau_t^{Pigou} &:= \sum_{\tau=t+1}^{\infty} \frac{\sum_{j=j_0(\tau)}^J \frac{\partial W}{\partial c_{(\tau-j)j}}}{\sum_{j=j_0(t)}^J \frac{\partial W}{\partial c_{(t-j)j}}} \frac{\partial D_{\tau}}{E_t} y_{\tau} \\ &= \sum_{\tau=t+1}^{\infty} \gamma^{\tau-t} \frac{\sum_{j=j_0(\tau)}^J \left(\frac{\beta}{\gamma}\right)^j U_{(\tau-j)j}^c}{\sum_{j=j_0(t)}^J \left(\frac{\beta}{\gamma}\right)^j U_{(t-j)j}^c} \frac{\partial D_{\tau}}{\partial E_t} y_{\tau}. \end{aligned} \quad (12)$$

We next derive the optimal emissions tax which is given by the discounted sum of future damages. For optimality, it is important to note that the discount factor has to be equal to the real interest rate between two periods  $\hat{r}_k$ .

**Proposition 2** *At each date, the optimal emission tax is the sum of future marginal damages dis-*

counted with the real interest rate:

$$\tau_t^E = \sum_{\tau=t+1}^{\infty} \left( \prod_{k=0}^{\tau} \frac{1}{1 + \hat{r}_k} \right) \frac{\partial D_{\tau}}{\partial E_t} y_{\tau} \quad (13)$$

$$= \sum_{\tau=t+1}^{\infty} \frac{\sum_{j=0}^J \left(\frac{\beta}{\gamma}\right)^j U_{(\tau-j)j}^c \left[1 + \phi_t \left(1 + H_{(\tau-j)j}^c\right)\right]}{\sum_{j=0}^J \left(\frac{\beta}{\gamma}\right)^j U_{(t-j)j}^c \left[1 + \phi_t \left(1 + H_{(t-j)j}^c\right)\right]} \frac{\partial D_{\tau}}{\partial E_t} y_{\tau} \quad (14)$$

**Proof.** See Appendix C. ■

We are now in a position to compare the optimal emissions tax with the Pigouvian tax. Comparing (14) with (12) shows that the two are likely to deviate if the implementability constraint is binding, i.e. if  $\phi_t$  becomes positive. Intuitively, the deviation of the optimal emissions from the Pigouvian tax rate is caused by the fact that the latter takes into account the marginal welfare change whereas the first takes into account the modified welfare function (9) which reflects distortionary (income) taxation.

Equation (14) shows that there is direct relationship between the optimal capital income tax and the optimal tax on carbon emissions:

**Proposition 3** *Along the steady state path, if the optimal capital income tax is zero, then the optimal emission tax equals the Pigouvian tax.*

**Proof.** In a steady state, the general equilibrium elasticity of consumption is constant across generations:  $H_{tj}^c = H_{(t-1)j}^c$ . From Proposition 1, we have that a zero capital income tax implies that the general equilibrium elasticity of consumption is constant across the life cycle, that is  $H_{tj}^c = H_{t(j+1)}^c$ . This implies that terms in squared brackets in equation (14) are constant and equal in all periods, and hence the optimal emission tax is equal to the Pigouvian emissions tax (12). ■

### 3 Functional Forms and Calibration

In order to carry out the numerical investigation of the optimal tax code, we first choose a model parametrization, which we now describe. Table 1 summarizes our choices for all parameters.

#### 3.1 Demographics, Preferences, and Labor Productivity

In our model, households are born at age 20 (model age 1) and they die with probability 1 at model age 51 (age 70 in the real world). Households have time-separable preferences over

consumption and leisure and discount the future with factor  $\beta$ . We consider the following constant-elasticity-of-substitution (CES) period utility function:

$$UT_{tj}(c_{tj}, l) = (\phi c^{\sigma_{UT}} + (1 - \phi)l^{\sigma_{UT}})^{1/\sigma_{UT}},$$

where  $\phi$  is a share parameter determining the relative importance of consumption, and  $\sigma_{UT}$  determines the elasticity of substitution between consumption and leisure. These period utility functions,  $UT_{tj}$ , are aggregated to lifetime consumption of generation  $t$ ,  $U_t$ , with the intertemporal substitution elasticity  $\sigma_U$ .<sup>9</sup>

To describe the evolution of labor productivity over the life cycle, we use an age-related productivity profile according to:

$$\epsilon_j = \exp(\lambda_0 + \lambda_1(21 + j) + \lambda_2(j + 21)^2 + \lambda_3(j + 21)^3),$$

where the parameters of this function are selected to minimize the difference from the profile arising by taking the average of multiple income groups as discussed in [Altig et al. \(2001\)](#). The coefficients used are:  $\lambda_0 = 1.0785$ ,  $\lambda_1 = 0.0936$ ,  $\lambda_2 = -0.0015$ , and  $\lambda_3 = 7 \times 10^{-6}$ .

### 3.2 Production Technology

The production technology  $y_t = f_t(K_t, L_t, E_t)$  (gross of climate damages) is given by the following nested CES function:

$$y_t = [\theta_{VA}(VA_t)^{\sigma_Y} + (1 - \theta_{VA})E_t^{\sigma_Y}]^{1/\sigma_Y},$$

where  $VA$  and  $E$  are themselves CES composites. [Figure 1](#) depicts the nested CES function used to characterize production. At the lower level, capital and labor produce an energy commodity which is combined in a Leontief manner (with elasticity of substitution equal to zero) with carbon dioxide emissions. This carbon inclusive energy bundle is combined with a value added aggregate of capital and labor with the substitution elasticity  $\sigma_y$  to produce the final commodity. Note that unlike in the DICE2013 model, which includes an explicit abatement cost function that reduces output available for final (private and public) consumption and investment purposes, we represent abatement costs implicitly through the nested CES structure. Abatement costs

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<sup>9</sup> [Lau et al. \(2002\)](#) show that the specification of utility using a nested CES function is consistent with linear time-separable specification as used in the section above.

are thus modelled through endogenous substitution between inputs at the top-nest of the CES structure.

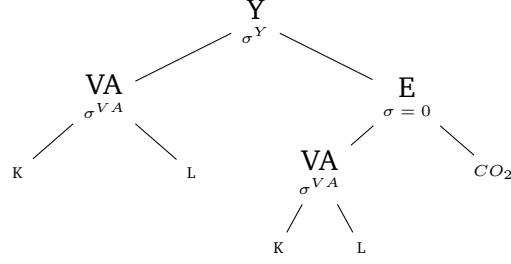


Figure 1: Nested CES structure for production of output (gross of climate damages)

### 3.3 Climate and Carbon Cycle

The link between emissions and economic damages is adopted from the DICE2013 model (Nordhaus, 2013). Total emissions—the sum of industrial ( $e_t^{ind}$ ) and exogenous emission from land use changes ( $e_t^{tree}$ )—enter the atmosphere layer of the carbon cycle. The carbon cycle is represented by a three-layer model which includes the atmosphere ( $a$ ), upper ( $u$ ) and lower ( $l$ ) ocean. Transfer of carbon between different layers is characterized by the following matrix of transfer coefficients  $\phi$ :

$$\begin{pmatrix} M_{a,t} \\ M_{u,t} \\ M_{l,t} \end{pmatrix} = \begin{pmatrix} \phi_{a,a} & \phi_{u,a}^M & 0 \\ \phi_{a,u}^M & \phi_{u,u}^M & \phi_{l,u}^M \\ 0 & \phi_{u,l}^M & \phi_{l,l}^M \end{pmatrix} \begin{pmatrix} M_{a,t-1} \\ M_{u,t-1} \\ M_{l,t-1} \end{pmatrix} + \begin{pmatrix} E_t + e_t^{tree} \\ 0 \\ 0 \end{pmatrix}. \quad (15)$$

Radiative forcing  $F_t$  depends on the change of atmospheric  $\text{CO}_2$  concentration relative to its pre-industrial level  $\overline{M_{a,1750}}$  as well as on some exogenous forcing  $f_t^{ex}$ :

$$F_t = \eta \frac{M_{a,t}}{\overline{M_{a,1750}}} / \ln(2) + f_t^{ex}. \quad (16)$$

Depending on the forcing, *DICE2013* determines atmospheric ( $T_{a,t}$ ) and upper ocean ( $T_{u,t}$ ) temperature increase:

$$T_{a,t} = T_{a,t-1} + \zeta_1 [F_t - \zeta_2 T_{a,t-1} - \zeta_3 (T_{a,t-1} - T_{u,t-1})] \quad (17)$$

$$T_{u,t} = T_{u,t-1} + \zeta_4 (T_{a,t-1} - T_{u,t-1}). \quad (18)$$

The damage  $D_t$  from climate change depends on the atmospheric temperature increase  $T_{a,t}$ :

$$D_t = \chi_1 T_{a,t} + \chi_2 T_{a,t}^{\chi_3}. \quad (19)$$

Our parametrization of the climate and carbon cycle is identical to the 2013 version of the DICE model Nordhaus (2013).

### 3.4 Computational Strategy

We formulate the equilibrium of the integrated economy-climate model with overlapping generations as mixed complementarity problem (MCP) following the seminal work by Mathiesen (1985) and Rutherford (1995).<sup>10</sup> Using a MCP approach it is straightforward to incorporate second-best effects from distortionary taxes into general equilibrium. Importantly, this allows us to overcome a major disadvantage of optimizing IAMs that are traditionally formulated as a nonlinear program (see, for example, Nordhaus, 2013): “optimal” policies emerging from IAMs formulated as a non-linear program are only optimal in a first-best, undistorted economy (see, for example, Böhringer et al. (2007) for further discussion of this issue). Building on the MCP formulation of the equilibrium problem of the underlying climate-economy model, we then solve the dynamic Ramsey (optimal) tax problem as a Mathematical Programs with Equilibrium Constraints (MPEC) (Luo et al., 1996).<sup>11</sup> The MCP model is solved using the General Algebraic Modeling System (GAMS) (GAMS Development Corporation, 2013) and the PATH solver (Dirkse and Ferris, 1995). The MPEC problem is solved using GAMS/NLPEC. NLPEC reformulates the MPEC problem as an non-linear program which is then solved using CONOPT (Drud, 1992).

<sup>10</sup> Rutherford (1995) and Mathiesen (1985) have shown that a complementary-based approach is convenient, robust, and efficient. A characteristic of many economic models is that they can be cast as a complementary problem, i.e. given a function  $F: \mathbb{R}^n \rightarrow \mathbb{R}^n$ , find  $z \in \mathbb{R}^n$  such that  $F(z) \geq 0$ ,  $z \geq 0$ , and  $z^T F(z) = 0$ . The complementarity format embodies weak inequalities and complementary slackness, relevant features for models that contain bounds on specific variables, e.g. activity levels which cannot a priori be assumed to operate at positive intensity. Such features are not easily handled within an optimizing approach. The equilibrium of the integrated climate-economy model can be characterized by three classes of equilibrium conditions: (i) zero-profit conditions for constant returns activities, and (ii) market clearing conditions for goods and factors. The decision variables are two vectors: (i) activity levels of constant returns production, and (ii) prices for goods and factors. In equilibrium each of these variables is linked to one inequality condition: (i) an activity level to a zero profit condition, and (ii) a price to a market clearance condition. In a model with multiple agents, we must add an additional class of income balances that relate factor income to expenditure of agents (with associated income variables).

<sup>11</sup> These kind of problems are usually hard to solve due to non-convex and non-smooth characteristics. Ban et al. (2006) developed a rigorous solution approach for mathematical programs with complementarity constraints (MPCCs), which convert the bi-level formulation to a single-level NLP and use an iterative algorithm based on a relaxation scheme to solve this problem. We use Bussieck and Meeraus (2004) and pre-customized solvers to solve the MCP and MPEC problem in an efficient way.

In line with our numerical solution method, we can formulate the equilibrium of the integrated economy-climate model as follows:<sup>12</sup>

**Definition 2 (Competitive equilibrium)** *Given an exogenous sequence of government expenditure  $\{g_t\}_{t=0}^{\infty}$ , initial conditions for asset holdings (claims on the global capital stock), and carbon concentrations  $(M_{a,0}, M_{u,0}, M_{l,0})$ , a competitive equilibrium is a sequence of non-negative prices and activity levels and set of policies  $\{\tau_t^R, \tau_t^L, P_t^{CO_2}\}_{t=0}^{\infty}$  such that: (i) zero-profit conditions (D.4)–(D.9) are satisfied, (ii) market clearing conditions (D.10)–(D.17) hold, (iii) the government budget constraint (D.19) is satisfied, and (iv) equations (D.20)–(D.24) defining the climate and carbon cycle are satisfied.*

**Definition 3 (Ramsey equilibrium)** *A Ramsey equilibrium is the competitive equilibrium with the highest social welfare  $W := \sum_{t=-J}^{\infty} \gamma^t U_t$ .*

### 3.5 Calibration

The model is calibrated to (largely) follow the baseline path of the DICE2013 model. Our baseline calibration deviates somewhat from DICE2013 as we incorporate government spending, distortionary income taxation, and an OLG household structure. More specifically, we match output growth in the baseline by adjusting the effective augmentation rate for labor which comprises population growth and Harrod-neutral technological progress. All parameters for the climate and carbon cycle equations, the damage function, and the emissions coefficients are identical to DICE2013 (see Nordhaus, 2013 for a detailed overview). The level of total global output is taken from DICE2013 and the production function is calibrated using value shares given in Table 1. Given total output in the base year, we derive aggregated consumption and labor demand given the government expenditure as a share of total production. We then use the calibration procedure described by Rasmussen and Rutherford (2004) together with an exogenously set leisure consumption share to calibrate parameters of utility functions of the different generations. The household calibration procedure endogenously determines the private discount rate (which is uniform across all generations). For our central simulations we assume that social discount factor  $\gamma$  equals the private discount rate. The model is solved with ten year time steps.

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<sup>12</sup> Appendix D provide a complete algebraic description of the model detailing the equilibrium conditions used in the definition below.

Table 1: Parameter values for central case

Parameter	Description	Value
$\theta_{KVA}$	Capital value share in value-added	.30
$\theta_{KE}$	Capital value share in energy production	.50
$\theta_{EY}$	Energy value share in production	.10
$\phi$	Leisure consumption share	.80
$\sigma^U$	Inter-temporal elasticity of substitution	0.50
$\sigma^{UT}$	Substitution elasticity leisure vs. consumption	.80
$\sigma^{VA}$	Substitution elasticity capital vs. labor	1.00
$\sigma_Y$	Substitution elasticity valued-added vs. energy	1.50
$\rho$	Period utility discount rate	<i>Calibrated</i>
$\bar{r}$	Annual interest rate	0.04
$\delta$	Annual capital depreciation rate	0.07
$\Psi$	Government expenditure as share of total production	.18

## 4 Quantitative Results

Our quantitative analysis further investigates the problem of combined optimal fiscal and climate policy, i.e. it combines the traditional problem of optimal income taxation with the problem of optimally controlling carbon emissions. Throughout our entire analysis, and in the spirit of the classical (dynamic) Ramsey taxation problem, we assume that an exogenous sequence of government expenditure ( $\{G_t\}_{t=0}^{\infty}$ ) has to be financed by raising revenue through a combination of non-negative capital income, labor income, and carbon taxes. Given  $\{G_t\}_{t=0}^{\infty}$ , which is assumed to be identical throughout, we solve for the optimal time paths of income and carbon taxes. More specifically, our analysis is structured around two sets of scenarios that are designed to explore the interplay of jointly optimized fiscal and carbon pricing policy:

- BAU (“business-as-usual”): Capital and labor income taxes are assumed to remain fixed at baseline levels and there are no carbon taxes. Income taxes are sufficient to fully finance a given path of exogenous government expenditures.
- INCTAX (“income tax reform”): Capital and labor income taxes are optimized but there are no carbon taxes. This scenario measures the efficiency gains from conventional tax reform as considered by the literature on optimal capital income taxes.
- FULLOpt (fully optimized income and carbon tax): Optimization over capital and labor income taxes as well as carbon taxes.
- GREENKTaxRecycl (“green tax reform with optimal carbon and capital income taxes”): Optimized carbon and capital income taxes where the labor income tax rates remain fixed at BAU levels. This scenario may also be viewed as a tax reform which recycles the receipts

from a carbon tax through adjusting capital income tax rates.

- GREENLTaxRecycl (“green tax reform with optimal carbon and labor income taxes”): Optimized carbon and labor income taxes where the capital income tax rates remain fixed at BAU levels. This scenario may also be viewed as a tax reform which recycles the receipts from a carbon tax through adjusting labor income tax rates.
- FIRSTBEST (“optimal carbon taxes and non-distortionary income taxation”): Optimal carbon taxes and non-distortionary lump-sum taxes.
- A second set of scenarios indicated with “NOLC” comprises the scenarios stated above but switches off life-cycle elements by assuming (i) exogenous labor supply and (ii) a flat labor productivity profile over the life cycle.

## **5 Conclusions**



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## Appendix

### A Derivation of the Implementability Constraint

Denoting the Lagrangian multiplier on the budget constraint (2) as  $\lambda_{tj}$ , the first order conditions for utility maximization problem of an individual born in period  $t$  are given as:<sup>13</sup>

$$\beta^{j-j_0(t)} U_{tj}^c = \lambda_{tj} \quad (\text{A.1})$$

$$\beta^{j-j_0(t)} U_{tj}^l = -\lambda_{tj} z_j w_{tj} \quad (\text{A.2})$$

$$(1 + r_{t(j+1)}) \lambda_{t(j+1)} = \lambda_{tj} \quad (\text{A.3})$$

Combining the conditions implies:

$$\prod_{k=1}^j \frac{1}{1 + r_{tk}} = \beta^{j-j_0(t)} \frac{U_{tj}^c}{U_{tj_0(t)}^c} \quad (\text{A.4})$$

$$-z_j w_{tj} = \frac{U_{tj}^l}{U_{tj}^c} \quad (\text{A.5})$$

The intertemporal budget constraint results from (2) as:

$$(1 + r_{tj_0(t)}) a_{tj_0(t)} = \sum_{j=j_0(t)}^J \left[ \prod_{k=1}^j \frac{1}{1 + r_{tk}} (c_{tj} - w_{tj} z_j l_{tj}) \right] \quad (\text{A.6})$$

Substituting from (A.4) and (A.5) yields the implementability constraint:

$$\sum_{j=j_0(t)}^J \beta^{j-j_0(t)} (U_{tj}^c c_{tj} + U_{tj}^l l_{tj}) = A_{tj_0(t)} = U_{tj_0(t)}^c (1 + r_{tj_0(t)}) a_{tj_0(t)} \quad (\text{A.7})$$

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<sup>13</sup> We formulate the conditions as equalities, i.e. assume an inner solution. For consumption an inner solution is ensured by the assumptions on the form of the utility function. However, labor supply can become zero in life cycle model, i.e. marginal utility of leisure becomes strictly smaller than the Lagrangian multiplier.

## B Proof of Proposition 1

By definition of the modified welfare function (9) its derivatives in period  $t$  are given as:

$$\frac{\partial \tilde{W}_t}{\partial c_{tj}} = \beta^{j-j_0(t)} U_{tj}^c \left( 1 + \phi_t \left( 1 + \frac{\frac{\partial^2 U}{\partial c_{jt}^2} c_{jt} + \frac{\partial^2 U}{\partial l_{jt} \partial c_{jt}} l_{jt}}{U_{tj}^c} \right) \right) = \beta^{j-j_0(t)} U_{tj}^c (1 + \phi_t (1 + H_{tj}^c)) \quad (\text{B.1})$$

$$\frac{\partial \tilde{W}_t}{\partial l_{tj}} = \beta^{j-j_0(t)} U_{tj}^c \left( 1 + \phi_t \left( 1 + \frac{\frac{\partial^2 U}{\partial l_{jt}^2} l_{jt} + \frac{\partial^2 U}{\partial c_{jt} \partial l_{jt}} c_{jt}}{U_{tj}^l} \right) \right) = \beta^{j-j_0(t)} U_{tj}^c (1 + \phi_t (1 + H_{tj}^l)) \quad (\text{B.2})$$

Using the first order conditions for production (5) and denoting the Lagrangian multiplier on the market clearing constraint ((11)) as  $\kappa_t$ , the first order condition for the welfare maximization problem become:

$$\kappa_t = \gamma [1 + \hat{r}_{t+1}] \kappa_{t+1} \quad (\text{B.3})$$

$$\gamma^j \kappa_{t+j} \mu_j = \frac{\partial \tilde{W}_t}{\partial c_{tj}} \quad (\text{B.4})$$

$$\gamma^j \kappa_{t+j} \mu_j z_j \hat{w}_{t+j} = -\frac{\partial \tilde{W}_t}{\partial l_{tj}} \quad (\text{B.5})$$

$$\tau_t^E = \sum_{\tau=t+1}^{\infty} \gamma^{\tau-t} \frac{\kappa_{\tau}}{\kappa_t} \frac{\partial D_{\tau}}{\partial E_t} y_{\tau} \quad (\text{B.6})$$

Equation (B.3) to ((B.5)) together with the partial derivatives (B.1) and (B.2) imply:

$$-\frac{\frac{\partial \tilde{W}_t}{\partial l_{tj}}}{\frac{\partial \tilde{W}_t}{\partial c_{tj}}} = -\frac{U_{tj}^l (1 + \phi_t (1 + H_{tj}^l))}{U_{tj}^c (1 + \phi_t (1 + H_{tj}^c))} = z_j \hat{w}_{t+j} \quad (\text{B.7})$$

$$\frac{\frac{\partial \tilde{W}_t}{\partial c_{tj}}}{\frac{\partial \tilde{W}_t}{\partial c_{t(j+1)}}} = \frac{U_{tj}^c (1 + \phi_t (1 + H_{tj}^c))}{\beta U_{t(j+1)}^c (1 + \phi_t (1 + H_{t(j+1)}^c))} = 1 + \hat{r}_{t+j+1} \quad (\text{B.8})$$

Substituting the marginal rates of substitution from (A.4) and (A.5) and using the definition of after tax factor returns proves proposition 1:

$$\tau_{tj}^l = \frac{\phi_t (H_{tj}^l - H_{tj}^c)}{1 + \phi_t (1 + H_{tj}^l)} \quad (\text{B.9})$$

$$\frac{1 + \hat{r}_{t+j+1}}{1 + (1 - \tau_{tj}) \hat{r}_{t+j+1}} = \frac{(1 + \phi_t (1 + H_{tj}^c))}{(1 + \phi_t (1 + H_{t(j+1)}^c))} \quad (\text{B.10})$$

## C Proof of Proposition 2

Repeated substitution of the first order condition for capital accumulation (B.3) the optimal emission tax equation (B.6) yields:

$$\tau_t^E = \sum_{\tau=t+1}^{\infty} \left( \prod_{k=0}^{\tau} \frac{1}{1 + \hat{r}_k} \right) \frac{\partial D_{\tau}}{\partial E_t} y_{\tau} \quad (\text{C.1})$$

The alternative expression for the emission tax results by summing the first order condition for consumption (B.4) over all generation which shows:<sup>14</sup>

$$\kappa_t = \sum_{j=0}^J \left( \frac{\beta}{\gamma} \right)^j U_{(t-j)j}^c \left[ 1 + \phi_t \left( 1 + H_{(t-j)j}^c \right) \right] \quad (\text{C.2})$$

By substitution (B.6) reveals the alternative representation:

$$\tau_t^E = \sum_{\tau=t+1}^{\infty} \frac{\sum_{j=0}^J \left( \frac{\beta}{\gamma} \right)^j U_{(\tau-j)j}^c \left[ 1 + \phi_t \left( 1 + H_{(\tau-j)j}^c \right) \right]}{\sum_{j=0}^J \left( \frac{\beta}{\gamma} \right)^j U_{(t-j)j}^c \left[ 1 + \phi_t \left( 1 + H_{(t-j)j}^c \right) \right]} \frac{\partial D_{\tau}}{\partial E_t} y_{\tau} \quad (\text{C.3})$$

## D Model Description

This appendix provides a complete algebraic description of the numerical model used for the simulations in Section 4 of the paper. The framework has been developed to analyze optimal carbon taxes in an integrated economy-climate model that features overlapping generations and distortionary income taxation. As outlined in the main text above, we pose the model as a mixed complementarity problem (MCP). In this framework two types of conditions characterize the competitive equilibrium: zero-profit conditions determine the activity levels and market clearing the price levels. In order to determine optimal tax rates we use an Mathematical Program under Equilibrium Constraint (MPEC) approach which maximizes the welfare function and the public budget constraint and the equilibrium conditions of the CGE model. We formulate the problem using the General Algebraic Modeling System (GAMS) and the PATH as well as the NLPEC solver (Dirkse and Ferris, 1995) to solve for non-negative prices and quantities.

Below we first describe the functional forms used to characterized production and utility and the define the conditions for the competitive equilibrium in terms of zero-profit and market clearing restrictions as well as the climate equations. To enhance readability, all sets, parameters,

<sup>14</sup> For the ease of notation we look at periods  $t > J$ , i.e. at cases in which  $j_0(t) = 0$ .

and variables are defined subsequently to the model statement in Tables 2 and 3.

## D.1 Definitions of Functions

Production of output (gross of climate damages) is described by a nested CES function that combines energy with a value added capital-labor nest. Production of the energy commodity uses capital, labor, and emissions as inputs. The unit cost functions for the two level CES structure are given by:

$$c_t^Y = \left\{ \theta_Y^E (p_t^E)^{1-\sigma_Y} + \theta_Y^{KL} \left[ \theta_Y^K (P_t^R)^{1-\sigma_{VA}} + \theta_Y^L (P_t^L)^{1-\sigma_{VA}} \right]^{\frac{1-\sigma_Y}{1-\sigma_{VA}}} \right\}^{\frac{1}{1-\sigma_Y}} \quad (\text{D.1})$$

$$c_t^E = \left[ \theta_E^K (P_t^R)^{1-\sigma_E} + \theta_E^L (P_t^L)^{1-\sigma_E} \right]^{\frac{1}{1-\sigma_E}} + \phi_t P_t^{CO_2}. \quad (\text{D.2})$$

Utility functions of OLG households are also represented through nested CES functions. A CES structure for lifetime utility combines intra-period utilities which combine (material) consumption and leisure as inputs.<sup>15</sup> The unit expenditure function for generation  $t$  becomes:

$$c_t^C = \left\{ \sum_{j=j_0(t)}^J \theta_t^C \left[ \theta_L^C (P_{tj}^T)^{1-\sigma_{CL}} + \theta_C^C (P_{t+j}^Y)^{1-\sigma_{CL}} \right]^{\frac{1-\sigma_C}{1-\sigma_{CL}}} \right\}^{\frac{1}{1-\sigma_C}}. \quad (\text{D.3})$$

## D.2 Model Statement

Zero-profit conditions are given as:<sup>16</sup>

$$\frac{c_t^Y}{1+D_t} \geq P_t^Y \quad \perp \quad Y_t \geq 0 \quad \forall t \quad (\text{D.4})$$

$$c_t^E \geq P_t^E \quad \perp \quad E_t \geq 0 \quad \forall t \quad (\text{D.5})$$

$$P_t^K \geq (P_t^R - \delta P_{t+1}^K) (1 - \tau_t^R) + P_{t+1}^K \quad \perp \quad K_t \geq 0 \quad \forall t \quad (\text{D.6})$$

$$P_t^Y \geq P_{t+1}^K \quad \perp \quad I_t \geq 0 \quad \forall t \quad (\text{D.7})$$

$$c_t^C \geq P_t^U \quad \perp \quad U_t \geq 0 \quad \forall t \quad (\text{D.8})$$

$$P_{tj}^T \geq P_{t+j}^L \mu_j (1 + \tau_{t+j}^L) \quad \perp \quad L_{tj} \geq 0 \quad \forall t, j. \quad (\text{D.9})$$

<sup>15</sup> This CES representation of utility is equivalent to the functional form in the theoretical analysis (Section 2) as the CES function over time is a monotonic transformation of the additive separable function (see [Lau et al., 2002](#) for more detail).

<sup>16</sup> We use the “ $\perp$ ” operator to indicate complementarity between inequalities and variables (prices and quantities)

Market clearing conditions (using Shephard's and Hotelling's Lemma) are given by:

$$\frac{Y_t}{(1 + D_t)} \geq \sum_{j=0}^J z_t \frac{\partial c_{t-j}^C}{\partial p_t^Y} + I_t + g_t \quad \perp \quad p_t^Y \geq 0 \quad \forall t \quad (\text{D.10})$$

$$E_t \geq \frac{\partial c_t^Y}{\partial p_t^E} \quad \perp \quad p_t^E \geq 0 \quad \forall t \quad (\text{D.11})$$

$$\sum_{j=0}^J z_{t-j} \mu_j L_{(t-j)j} \geq \frac{\partial c_t^Y}{\partial p_t^L} + \frac{\partial c_t^E}{\partial p_t^L} \quad \perp \quad p_t^L \geq 0 \quad \forall t \quad (\text{D.12})$$

$$K_t \geq \frac{\partial c_t^Y}{\partial p_t^R} + \frac{\partial c_t^E}{\partial p_t^R} \quad \perp \quad p_t^R \geq 0 \quad \forall t \quad (\text{D.13})$$

$$\overline{T}_{tj} \geq L_{(t-j)j} + \frac{\partial c_t^C}{\partial p_{tj}^T} \quad \perp \quad P_{tj}^T \geq 0 \quad \forall t \quad (\text{D.14})$$

$$U_t * P U_t \geq INC_t \quad \perp \quad P_{tj}^T \geq 0 \quad \forall t \quad (\text{D.15})$$

$$(1 - A_t) \overline{CO2}_t \geq \frac{\partial c_t^E}{\partial P_t^{CO_2}} \quad \perp \quad P_t^{CO_2} \geq 0 \quad \forall t \quad (\text{D.16})$$

$$INC_t = \sum_{j=j_0(t)}^J P_{tj}^T \overline{T}_{tj} \quad \perp \quad INC_t \text{ free} \quad \forall t. \quad (\text{D.17})$$

The government objective function and revenue restriction are given as:

$$W = \sum_t \gamma^t U_t \quad (\text{D.18})$$

$$\sum_{j=j_0(t)}^J \tau_t^L P_t^L L_{(t-j)j} + \tau_t^R (P_t^R - \delta P_{t+1}^R) + A_t \overline{CO2}_t P_t^{CO_2} = \frac{g_t}{P_t^Y} \quad \forall t. \quad (\text{D.19})$$

The climate equations link carbon emissions to economic damages in the future periods and are adopted from the DICE model:

$$C_{rt} = \sum_{\bar{r}} \epsilon_{\bar{r}r} C_{\bar{r}(t-1)} + \left[ \frac{\partial c_{t-1}^E}{\partial P_{t-1}^{CO_2}} \right]_{\text{if } r = \text{Atmosphere}} \quad \forall t \quad (\text{D.20})$$

$$F_t = f^{CO_2} \frac{\ln(C_{\text{Atmosphere},t})}{588} + \bar{f}_t \quad \forall t \quad (\text{D.21})$$

$$T_t^{\text{Atmosphere}} = T_{t-1}^{\text{Atmosphere}} + \omega_1 \left[ F_t - \omega_2 T_{t-1}^{\text{Atmosphere}} - \omega_3 \left( T_{t-1}^{\text{Atmosphere}} - T_{t-1}^{\text{Ocean}} \right) \right] \quad \forall t \quad (\text{D.22})$$

$$T_t^{\text{Ocean}} = T_{t-1}^{\text{Ocean}} + \omega_4 \left( T_{t-1}^{\text{Ocean}} - T_{t-1}^{\text{Ocean}} \right) \quad \forall t \quad (\text{D.23})$$

$$D_t = \delta_1 T_t^{\text{Atmosphere}} + \delta_2 \left( T_t^{\text{Atmosphere}} \right)^{\delta_3} \quad \forall t \quad (\text{D.24})$$



### D.3 Terminal Conditions

To approximate the underlying infinite horizon economy by a finite-dimensional complementarity problem, we choose a “state variable targetting” approach as proposed by [Lau et al. \(2002\)](#). The infinite horizon economy can be decomposed into two distinct problems where one runs from  $0, \dots, T$  and the other runs from  $T + 1, \dots, \infty$ , where  $T$  denotes the last period of the numerical model.<sup>17</sup> Both subproblems are linked through the post-terminal capital stock in period  $T + 1$ . The level of post-terminal capital is computed endogenously by requiring that investment grows at the same rate as output (or any other “stable” quantity in the model):

$$I_{r,T}/I_{r,T-1} = 1 + \gamma. \quad (\text{D.25})$$

To compute a transition path to a new steady state of an infinite horizon economy, it is necessary to account for the special characteristics of generations alive in the post-terminal years (indexed by  $\hat{g}$ ). We adopt the approach described in [Rasmussen and Rutherford \(2004\)](#) and impose two additional constraints on the model. Whereas assets held at the start of the initial period are exogenous, a shock to the model may change the demand and supply for savings at a given interest rate and consequently the profile of asset holdings and the trade deficit in the new steady state. Assets held in year  $T$  are therefore computed as endogenous variables chosen to ensure that the model is on a steady-state growth in  $T$ . This implies that the percentage change in welfare, as measured by the equivalent variation ( $ev_{\hat{g}}$ ) of each of the generations living beyond the terminal period are of equal magnitude:

$$ev_{\hat{g}} = ev_{\hat{g}-1} \quad \text{for } T - N < \hat{g} \leq T. \quad (\text{D.26})$$

The second constraint ensures that consumption profiles of households living beyond  $T$  are held at the steady-state level. This requires that given the post-terminal consumption demands by these generations, the price path for consumption goods declines with the interest rate consistent with a steady-state projection of the price of consumption in period  $T$ .

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<sup>17</sup> Note that this method for approximating the infinite horizon relies on the assumption of time-separable utility functions.

Table 2: Sets and parameters

Symbol	Description
<i>Sets</i>	
$t$	Time periods
$j$	Age
$j_0(t)$	Initial age of generation $t$
$r$	Carbon reservoir in climate model (atmosphere, upper, and lower ocean)
<i>Parameters</i>	
$\theta_Y^E$	Share parameter energy in gross production
$\theta_Y^K$	Share parameter capital in gross production
$\theta_Y^L$	Share parameter labor in gross production
$\theta_E^K$	Share parameter capital in energy production
$\theta_E^L$	Share parameter labor in energy production
$\theta_t^C$	Share parameter period $t$ consumption in utility
$\theta_C^C$	Share parameter commodity consumption in utility
$\theta_C^L$	Share parameter leisure in utility
$\mu_j$	Labor productivity in age $j$
$\delta$	Capital depreciation rate
$\sigma_Y$	Substitution elasticity production top level
$\sigma_E$	Substitution elasticity energy production
$\sigma_{VA}$	Substitution elasticity production value-added nest
$\sigma_{CL}$	Substitution elasticity consumption versus leisure
$\sigma_C$	Intertemporal substitution elasticity
$\phi_t$	Carbon coefficient, i.e. carbon-output ratio for energy production
$\epsilon_{\tilde{r}r}$	Carbon transfer coefficient from reservoir $\tilde{r}$ to reservoir $r$
$\bar{f}_t$	Exogenous radiative forcing in period $t$
$\omega_1, \omega_2, \omega_3, \omega_4$	Parameters temperature equations
$\delta_1, \delta_2, \delta_3$	Parameters damage function
$z_t$	Size of generation $t$
$\bar{T}_{tj}$	Time endowment of generation $t$ in age $j$
$\overline{CO2}_t$	Baseline emissions in period $t$

Table 3: Price and quantity variables

Symbol	Description
<i>Prices</i>	
$P_t^r$	Capital rental rate in period $t$
$P_t^L$	Wage in period $t$
$P_t^E$	Energy price in period $t$
$P_{tj}^T$	Time price for generation $t$ in age $j$
$P_t^Y$	Output price in period $t$
$c_t^Y$	Unit cost function gross output in period $t$
$c_t^E$	Unit cost function energy production in period $t$
$P_t^K$	Capital price in period $t$
$P_t^U$	Expenditure index generation $t$
$c_t^C$	Unit expenditure function generation $t$
<i>Quantities</i>	
$Y_t$	Production in period $t$
$E_t$	Energy production in period $t$
$K_t$	Capital stock in period $t$
$D_t$	Damage as fraction of output in period $t$
$I_t$	Investment in period $t$
$U_t$	Expenditure generation $t$
$L_t$	Labor supply generation $t$ in age $j$
$INC_t$	Lifetime income generation $t$
$g_t$	Government demand in period $t$
$A_t$	Abatement fraction in period $t$
$C_{rt}$	Carbon concentration in reservoir $r$ in period $t$
$F_t$	Radiative forcing in period $t$
$f^{CO_2}$	Radiative forcing of doubling CO <sub>2</sub> concentration compared to pre-industrial times
$T_t^{Atmosphere}$	Atmospheric temperature in period $t$
$T_t^{Ocean}$	Upper ocean temperature in period $t$
<i>Tax policy variables</i>	
$\tau_t^L$	Labor income tax rate in period $t$
$\tau_t^R$	Capital income tax rate in period $t$
$P_t^{CO_2}$	Carbon price in period $t$