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Designing Efficient Education and Tax Policies*

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Preliminary version. Comments welcome!

Abstract

We study education and income tax policies in a model with endogenous selection into college. Our framework is strongly influenced by the empirical college literature and incorporates heterogenous returns and tastes for college, earnings risk (implying uncertain returns to college) and potentially borrowing constraints. We (i) calculate revenue effects of various policy reforms starting from the current system and (ii) derive conditions for optimal education and tax policies with various degrees of sophistication: optimal college subsidies for given income taxes and vice versa, jointly optimal taxes and subsidies, and optimal education dependent taxes.

We estimate the relevant parameters of the model for quantitative analysis. We find that the endogeneity of the college choice has only a small impact on optimal taxes and increasing subsidies to their optimal level leads to large welfare gains. Finally, we find that for the current US policies, an increase in education subsidies is self-financing via higher tax revenue in the future; if we allow grants to condition on parental background, this effect gets even stronger and children with poor academic background should receive higher subsidies for pure efficiency reasons – efficient policies favor social mobility.

JEL-classification: H21, H23

Keywords: Optimal Taxation, College Enrollment, Tax Reforms

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1 Introduction

Income inequality has been rising in many industrialized countries over the last 30 years. This increase in income inequality can partly be explained by an increase in the college wage premium.\footnote{See Krueger, Perri, Pistaferri, and Violante (2010).} Education and tax policies both provide ways to deal with income inequality. Higher college subsidies can push more people into college and thereby reduce the number of people with low labor earnings. Redistributive tax policies can mitigate the consequences of income inequality by making the distribution of disposable income more equal. There is also an important interaction between tax and education policies. Public finance theory tells us that the presence of redistributive taxes alone provides a strong rationale for education subsidies as educational investment imposes a positive externality on the government budget. This is particularly relevant because empirical evidence suggests that college enrollment is very responsive to subsidies.\footnote{See the discussion of the empirical college literature in Section 2.2.}

Motivated by these observations, we set up a public finance framework with endogenous college enrollment to study education and tax policies. This framework is simple enough to allow for a complete analytical treatment. The framework is also sufficiently rich to be brought to the data and match and address key empirical phenomena. The main three questions we aim to answer are the following: (i) What does the endogeneity of the college enrollment decision theoretically and quantitatively imply for the welfare effects of tax reforms and – as a consequence – for the design of optimal nonlinear income taxes? (ii) What exactly determines an efficient college subsidy theoretically and what are the quantitative implications? (iii) To what extent is it welfare increasing to differentiate college subsidies (or grants, equivalently) across different groups of students (e.g., according to parental background)?

In short, we arrive at the following main conclusions. For (i), we find the impact of enrollment on optimal taxes to be theoretically ambiguous in its sign and quantitively small. Concerning (ii), we show that the number of infra-marginal and the number of marginal students (or equivalently, the semi-elasticity of enrollment) is key to understand the welfare implications of an increase in subsidies. Based on these theoretical considerations, we find that an increase in education subsidies in the US is potentially self-financing and the optimal subsidy is much larger than the current subsidy. Finally, for (iii), we find that increasing education subsidies for individuals with poor parental background (in terms of education) is much more cost-effective and thus provide a pure efficiency rationale for education policies that favor social mobility.

Our underlying formal framework explicitly incorporates behavioral responses of labor supply and post-secondary educational attainment with respect to policies. The effect of taxes on labor supply is theoretically well operationalized in the public finance literature and has been subject to a long and continuing line of investigation.\footnote{See Mirrlees (1971), Diamond (1998) and Saez (2001) for elaborations in a static framework and Golosov, Kocherlakota, and Tsyvinski (2003), Farhi and Werning (2013) as well as Golosov, Troshkin, and Tsyvinski (2013) for dynamic frameworks, respectively.} In addition, there is a strong and clear link between the empirical literature on labor supply elasticities and the theoretical public finance
literature. The college enrollment channel is, perhaps surprisingly, not as well operationalized in a public finance framework despite the potentially large implications for social welfare.\footnote{There is a large empirical literature on the responsiveness of college enrollment with respect to education policies, see the literature review in Section 2.2. This literature provides guidance for how to incorporate college enrollment into a public finance framework. Optimal tax papers addressing the endogeneity of education typically focus on human capital accumulation over the life-cycle or on educational investment in a more abstract sense. There is also no strong connection between these papers and the empirical college literature. Krueger and Ludwig (2013a,b) are an exception, see the literature review in Section 2.1.}

The way we incorporate the endogenous college decision into the workhorse optimal income tax problem is strongly guided by the empirical literature on the topic. In particular, we discipline our modeling choice such that the following three empirical features are addressed: (i) the returns to college are risky, (ii) individuals differ ex-ante with respect to their ability that determines the returns to college and (iii) individuals also differ with respect to tastes for college (henceforth, \textit{psychic costs}) that determine college entry via factors outside the budget constraint.\footnote{In future versions of this paper, we also intend to include heterogeneity in initial assets as a consequence of differences in parental wealth/income, college dropout as well as the decision between a two-year or a four-year college.}

Based on that framework, we theoretically and quantitatively elaborate the welfare and revenue consequences of policy reforms and characterize optimal policies with various degrees of sophistication: optimal college subsidies for given income taxes, optimal income taxes for given college subsidies, jointly optimal income taxes and college subsidies and optimal education dependent taxes.

\textbf{Theoretical Results:} We derive simple formulas for the revenue effects of college and tax policy reforms in the spirit of Golosov, Tsyvinski, and Werquin (2013). We show how the endogeneity of enrollment alters the ability of the government to raise revenue via higher taxes. Interestingly, the additional enrollment margin does not necessarily decrease the effectiveness of raising tax revenue. Due to income effects, higher taxes can theoretically increase enrollment.

In addition, we propose a simple and intuitive test for whether an increase in college subsidies is self-financing via higher tax revenue in the future. This test depends on the number of infra-marginal students, the number of marginal students and the average increase in expected tax payment per marginal student. Alternatively, it depends on the semi-elasticity of enrollment and the average increase in expected tax payment per marginal student.

Relatedly, we also derive a simple formula for the optimal college subsidy that holds in the presence of both, optimal and suboptimal income taxes. It depends on the elasticity of enrollment and the average increase in expected tax payment per marginal student, but also on the welfare weights the government assigns to students capturing the redistributive effect of subsidizing college.

For optimal income taxes, we show how the results from the static Mirrlees literature are extended by the endogeneity of college enrollment. This formula holds for optimal and suboptimal college subsidies. As for the revenue effect of tax reforms, the effect of the endogeneity of enrollment on marginal tax rates is unclear concerning its sign because a substitution and an income effect on college enrollment are at work. This is in contrast to the results obtained
in the Mirrlees model with an extensive labor supply margin by Saez (2002) and others, where the effect unambiguously reduces marginal tax rates.

Lastly, we consider the case where taxes are allowed to be education dependent. In this case, the effect of taxes on enrollment is no longer ambiguous yielding lower taxes for college graduates and higher taxes for high school graduates as compared to the case with exogenous enrollment.

Quantitative Results: We estimate the relevant parameters of our model using the NLSY, which includes data on earnings, college decisions, innate ability (measured by AFQT scores) and other factors that determine college entry. The model comes close to replicating established quasi-experimental evidence on the responsiveness of enrollment rates w.r.t increases education subsidies (Deming and Dynarski 2009).  

As our first result, we find that the endogeneity of college enrollment has only very small effects on the revenue gains from tax increases. Once, we allow tax reforms to be education dependent, however, the effect becomes quantitatively important.

Concerning education subsidies, we find that a $1 increase in grants leads to an increase in future tax revenue with a present value of $1.40. In fact, an increase in the yearly college subsidy by up to $5,108 Dollar per year is self-financing via higher tax revenue in the future.

We also look at the case, where subsidies are targeted and condition on parents’ education; we use this as a proxy for parental income in this version of the paper. For the group of individuals whose parents did not complete college, a $1 increase leads to a $1.46 increase in tax revenue. For the other group with an academic family background, the number is $1.20. For the first (latter) group an increase in the subsidy of up to $9,669 ($6,960) is self-financing under the current income tax.

We then explore fully optimal Utilitarian policies with different degrees of sophistication and the implied welfare gains. Our results can be summarized as follows:

1. Abolishing borrowing constraints and setting education subsidies optimally leads to large welfare gains.

2. Welfare gains from higher income taxes are also large.

3. The effect of the endogeneity of college enrollment is of second-order (first-order) importance for optimal education independent (dependent) marginal tax rates.

4. The U-shaped form of optimal rates is preserved (Diamond 1998).

5. Gains from education dependence of the tax code are significant.

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6 Naturally, this depends on our assumptions on borrowing constraints. Currently, we set the borrowing constraints such that individuals’ consumption during college cannot exceed $10,000 per year. In future versions, we want to include parental transfers into the model and address the issue of borrowing constraints in a more convincing way. If we do not impose no borrowing constraints at all as an extreme case, our model predicts enrollment responses that are roughly a third of the size.

7 Conducting a thought experiment and assuming away borrowing constraints shows that this result hinges on students being borrowing constrained: Once borrowing constraints are removed, an increase in college grants is not self-financing and a $1 increase in subsidies only leads to an increase in tax revenue of $0.32. In this case, the desirability of such a reform hinges on the distribution of welfare weights.
2 Related Literature

2.1 Public Finance

We derive our main theoretical results using intuitive tax perturbation methods, which go back to Piketty (1997) and Saez (2001) in the static context and have recently been generalized to dynamic settings by Golosov, Tsyvinski, and Werquin (2013). As the latter authors, we use the formulas to calculate revenue effects of policy reforms. As the former authors, we also use these formulas to calculate optimal policies. Our paper provides an extension of their results to an environment with endogenous college enrollment decisions.

Concerning the results for optimal income taxes, we show how our formula is an extended version of the well known Diamond (1998) formula. Since college enrollment is modeled as a binary choice, our formal approach is similar to other optimal tax papers with both, intensive and extensive margin: Scheuer (2013) considers the occupational choice margin (become an entrepreneur or not), Saez (2002) and Jacquet, Lehmann, and Van der Linden (2013) consider the labor force participation margin and Lehmann, Simula, and Trannoy (2013) consider migration. From an abstract point of view, our formulas are of course similar. A notable difference is the consideration of uncertainty and timing aspects in our model.

An important theoretical benchmark for a public finance treatment of education is the paper by Bovenberg and Jacobs (2005). They have shown that education subsidies and redistributive taxes are “siamese twins” in that the former can counteract the distortion on the education margin of the latter. Our theoretical framework differs in two distinctive ways: first, we model education as a discrete choice. In Bovenberg and Jacobs (2005), individuals make a continuous education choice and as a consequence, the government can provide the efficient subsidy for each type. In our framework, the government has to subsidize individuals along the extensive margin, making the trade-offs quite different. Second, we incorporate uncertainty into the model.

Bohacek and Kapicka (2008) extend the findings of Bovenberg and Jacobs (2005) to a dynamic setting and provide a quantitative exploration of the theory as well. More recently, Kapicka and Neira (2013), Findeisen and Sachs (2013) and Stantcheva (2013) consider the issue in the “New Dynamic Public Finance” (NPDF) tradition with endogenous policy instruments. Our paper differs in that we explicitly consider college as education decision and consider policy instruments with different degrees of sophistication as opposed to only focusing on the second-best.

The work of Krueger and Ludwig (2013a, 2013b) is closely related. In a large scale macroeconomic overlapping generations model, they solve for the optimal linear labor income tax rate (with a lump sum rebate) and the optimal college subsidy. We view our analysis as complementary in that we provide analytical results and our policy space is richer whereas they consider a larger model in the macroeconomic tradition.

8Kapicka (2013) considers unobservable human capital taxation over the life cycle and finds that a force towards decreasing taxes over the life cycle to incentivize human capital investment. Best and Kleven (2013) make a similar case, connecting optimal tax formulas to empirically estimated career effects.
From a methodological point of view, our paper is related to the work of Blundell and Shephard (2012), who explore optimal income taxation in a structurally estimated model of labor supply. Whereas our empirical approach is less ambitious in that we set some parameters (labor supply elasticity, risk aversion, discount factor) in line with benchmark numbers from the literature instead of estimating them, our approach has the advantage of providing an analytical treatment of the policy problem making our results particularly transparent.

2.2 Empirical Papers

Several papers in labor and education economics have described college decisions of individuals and how they respond to changes in policies. Our analysis is strongly guided by this empirical literature. We discuss the most relevant papers in this subsection.

A considerable reduced form literature has investigated the responsiveness of college enrollment with respect to education policies. A benchmark result from quasi-experimental studies for the US is that an increase in yearly college grants of $1,000 leads to an increase in college enrollment by 3-5 percentage points (Kane 2006, Deming and Dynarski 2009).9 Concerning the impact of parental income/wealth on college enrollment, empirical evidence is less conclusive. See, e.g., Lovenheim (2011) and Hilger (2013) for two recent papers with differing results. Lovenheim (2011) uses variation in housing wealth with the PSID as data source, finding relatively big effects. Hilger (2013) exploits differences in the timing of parental layoffs using IRS micro data and finds small effects. Very recently, Manoli and Turner (2014) find relatively large cash-on-hands effects exploiting two kink points in the US federal income tax code.

Empirical evidence on the importance of borrowing constraints is also less conclusive. Belley and Lochner (2007) find that parental income and wealth have a stronger impact on the enrollment decision than 30 years ago. They set up an educational choice model and find that the model can explain this increasing importance only via borrowing constraints. Johnson (2013) considers a state of the art structural college model with three choices (no college, 2 years college or 4 years college). He finds that relaxing borrowing constraints only has a modest impact on enrollment and that an increase in tuition subsidies is needed in order to obtain a larger increase in college enrollment. Navarro (2011) reaches different conclusions in a semi-parametrically identified structural model. Lochner and Monge-Naranjo (2011) explicitly distinguish between private and public lending and endogenize the privat borrowing limit and can explain more empirical patterns. Winter (2013) uses a structural life-cycle model of college entry where children receive transfers from their altruistic parents and finds that a quarter of all households are financially constrained. This whole issue is comprehensively surveyed by Lochner and Monge-Naranjo (2012).

There is also strong empirical evidence that returns to college cannot be the only determinant of the college entry decision. Other variables that are considered in the literature as driving forces are parental education, ability and the geographical environment. Taken together these variables are often summarized by one variable labelled psychic costs or tastes for college, see

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9Performing such policy experiments within structural models, Johnson (2013) and Abbott, Gallipoli, Meghir, and Violante (2013) arrive at similar numbers. Using a semi-structural approach, Navarro (2011) arrives at lower numbers. His results suggest that abolishing tuition would increase enrollment by only 2 percentage points.
Abbott, Gallipoli, Meghir, and Violante (2013) provide a state of the art large scale college enrollment model with endogenous parental transfers and imperfect substitutability between college and high-school labor. Similar as Johnson (2013) they find that increasing loan limits would have no salient effect on enrollment. For grant increases they obtain similar numbers, however, when taking into account general equilibrium effects (i.e. that the college wage premium declines when graduation rates increase) the effects of grant increases on enrollment are 3-4 times smaller.

Finally, Johnson (2013) and Abbott, Gallipoli, Meghir, and Violante (2013) find that need based college grants are more cost-effective in increasing enrollment than merit-based grants because among low income individuals, the share of infra-marginal students is low.

3 The Formal Framework

We first consider a static version of the model. Whereas this version of the model neither takes time costs of college nor borrowing issues into account, it still describes the major trade-offs of the policy maker in a simple and transparent manner. In Section 6, we show how the analysis changes if time costs and borrowing constraints are considered as well.

This section is organized as follows: In Section 3.1, the economic environment is introduced. In Section 3.2, we describe individual behavior and the equilibrium given taxes. In Section 3.3, we derive analytical expressions for behavioral responses along the extensive college margin with respect to various policy changes. We then turn to the social planners objective in Section 3.4 and derive formulas for the welfare effects of policy reforms in Section 3.5. The latter serves as a basis to calculate revenue effects of various policy reforms in Section 4 and optimal policies in Section 5.

3.1 Individual Heterogeneity, Preferences and Choices

Short Verbal Summary of the Model. Individuals differ in innate ability and preferences for college. They decide whether to obtain a college degree or not. Obtaining a college degree matters because of (i) tuition fees, (ii) its direct effect on utility and, most importantly, (iii) because of the effect on the wage. The wage that an individual earns, however, is uncertain from an ex-ante point of view. Uncertainty materializes after the college decision is made and the distribution from which individuals draw their wage depends on their innate ability and their college decision.

Heterogeneity. Individuals differ among their innate ability $\theta \in \Theta := [\theta, \bar{\theta}]$ and their psychic costs or preferences for college $\chi \in \chi := [\chi, \bar{\chi}]$. Innate ability is distributed according to $cdf F(\theta)$ with density $f(\theta)$. Psychic costs are distributed according to $cdf K_{\theta}(\chi)$ conditional on innate ability; the respective $pdf$ is $k_{\theta}(\chi)$. After individuals make the decision to go to college or not, their wage $\omega \in \Omega := [\omega, \bar{\omega}]$ materializes. If they go to college – i.e. if they are college graduates – they draw their wage from a distribution with $cdf G^{co}(\omega|\theta)$ and density $g^{co}(\omega|\theta)$. 
Otherwise – if they are high-school graduates – they draw their wage from a distribution with cdf $G_{hs}(\omega|\theta)$ and density $g_{hs}(\omega|\theta)$. In the theoretical part of this paper, we do not make any assumptions on these wage distributions; note that they capture returns to innate ability, returns to college, a possible complementarity between ability and schooling as well as the degree of uncertainty. Lastly, note that $\theta, \chi$ and $\omega$ are an individual’s private information.

Preferences. We assume quasi-linear preferences of the form $U(c - v(y))$, where $c$ denotes consumption, $y$ denotes gross income, $U(\cdot)$ is a concave transformation and $v(\cdot)$ is a convex transformation. We make the quasi-linearity assumption for tractability. Further, we assume that individuals are expected utility maximizers. Denote by $c_{hs}(\omega)$ their consumption as a high school graduate with wage $\omega$ and $y_{hs}(\omega)$ their respective gross income. Expected utility of a high school graduate with innate ability $\theta$ is then given by

$$\int_{\Omega} U\left(c_{hs}(\omega) - v\left(\frac{y_{hs}(\omega)}{\omega}\right)\right) dG_{hs}(\omega|\theta).$$

Denote, equivalently by $c_{co}(\omega)$ and $y_{co}(\omega)$ consumption and income of a college graduate with wage $\omega$. In the following sub- oder superscripts $co$ and $hs$ always refer to college and high school graduates. Expected utility of a college graduate with innate ability $\theta$ is given by

$$\int_{\Omega} U\left(c_{co}(\omega) - v\left(\frac{y_{co}(\omega)}{\omega}\right)\right) dG_{co}(\omega|\theta) - \chi.$$

Psychic costs $\chi$ are directly subtracted from utility. Note that $\chi$ can also take negative values reflecting psychic benefits from going to college. $\chi$ should be interpreted as a one-dimensional aggregate of different factors that determine the college entry decision via channels outside the budget constraint. In the empirical section of this paper, we will be more clear about these factors.

Choices. In this static environment, individuals only make two choices: (i) going to college or not and (ii) how much to work given a realized wage $\omega$. Besides the psychic costs (or benefits) associated with going to college, individuals also have to pay tuition fees $C$. In the dynamic extension of the model in Section 6, individuals that go to college also make a borrowing decision.

3.2 Equilibrium Given Taxes

The tax functions that we consider are possibly education dependent. We denote by $T_{hs}(y)$ the tax schedule for high school graduates and by $T_{co}(y)$ the tax schedule for college graduates. Current real world policies in most countries are characterized by marginal tax rates that are independent from the education choice, i.e. $T_{y}^{hs}(y) = T_{y}^{hs}(y) \forall y$. Further, college education

10The empirical literature using micro data sets has typically found only small income effects on labor supply (see Gruber and Saez (2002) for the US or a recent paper by Kleven and Schultz (2012) using the universe of danish tax records). In macroeconomics, this class of preferences has shown to be very useful in matching business cycle moments (Greenwood, Hercowitz, and Huffman 1988, Mendoza and Yue 2012).
is subsidized in most countries which would be reflected by a lower lump sum tax for college graduates in our model: $T_{y}^{hs}(0) \geq T_{y}^{co}(0)$.

We now look at optimal individual behavior given taxes. We start with the decision of how much to work once the wage $\omega$ has materialized. Value functions are defined by:

$$
\forall \omega \in \Omega \text{ and } i = co, hs : V_{i}(\omega) = \max_{y} U \left( c - v \left( \frac{y}{\omega} \right) \right) \text{ s.t. } c \leq y - T_{i}(y) - 1_{i=co}C,
$$

where $C$ is college tuition. Denote by $y_{i}(\omega)$ the respective optimal gross income choice. Note that in general $V_{i}(\omega)$ and $y_{i}(\omega)$ are functions of the tax schedule $T$ as well. For ease of notation, we suppress the dependence of the endogenous variables on the tax schedule $T$ in the following.

Based on (1), we can also define the value function of going to college and going to high school.

$$
V_{co}(\theta) = \int_{\Omega} V_{co}(\omega) dG^{co}(\omega|\theta) \text{ and } V_{hs}(\theta) = \int_{\Omega} V_{hs}(\omega) dG^{hs}(\omega|\theta).
$$

We now turn to the college entry decision. Individuals go to college whenever

$$
V_{co}(\theta) - \chi \geq V_{hs}(\theta).
$$

From (3) it follows that – for a sufficiently large support of $\chi$ – there exists a unique threshold of $\chi$ for each ability level that divides individuals into college and high school graduates. Formally, this threshold is defined by

$$
\tilde{\chi}(\theta) = V_{co}(\theta) - V_{hs}(\theta)
$$

for each $\theta$.

Based on all the above considerations, we can now formally define an equilibrium given taxes in this simple economy:

\textbf{Definition 1.} An allocation $\{\tilde{\chi}(\theta)\}_{\theta \in \Theta}, \{c_{co}(\omega), c_{hs}(\omega), y_{co}(\omega), y_{hs}(\omega)\}_{\omega \in \Omega}$ is an equilibrium given taxes $T_{y}^{co}(y)$ and $T_{y}^{hs}(y)$ if

(i) $\forall \omega \in \Omega$ and for $i = hs, co$: $y_{i}(\omega)$ solves (1)

(ii) $\forall \omega \in \Omega$ and for $i = hs, co$: $c_{i}(\omega) \leq y_{i}(\omega) - T_{i}(y_{i}(\omega)) - 1_{i=co}C$

(iii) $\forall \theta \in \Theta$: $\tilde{\chi}(\theta)$ satisfies (3).

To ease notation, we will now also define some cross sectional wage distributions. First, we define cross sectional distributions conditional on education:

$$
H^{co}(\omega) = \int_{\Theta} \int_{\omega} dG^{co}(\tilde{\omega}|\theta) K_{\theta}(\tilde{\chi}(\theta)) dF(\theta)
$$

and

$$
H^{hs}(\omega) = \int_{\Theta} \int_{\omega} dG^{hs}(\tilde{\omega}|\theta) (1 - K_{\theta}(\tilde{\chi}(\theta))) dF(\theta).
$$
Note that \( H^{co} \) and \( H^{hs} \) are not classical cumulated distribution functions because \( H^{co}(\bar{\omega} < 1 \) and \( H^{hs}(\bar{\omega} < 1 \). Finally, the overall wage distribution is given by

\[
H(\omega) = H^{co}(\omega) + H^{hs}(\omega).
\]

\( H \) is a classical cdf because we have \( H(\bar{\omega}) = 1 \). Note that these wage distributions are endogenous with respect to the tax system. For ease of notation we suppress dependence of these functions with respect to policies.

### 3.3 Responses Along the College Margin to Policy Changes

In this subsection, we elaborate how the individual college decision responds to changes in policies. We do this for two reasons. First, to better understand the mechanics of the model and second, because the responsiveness of the enrollment decision is crucial for policy design.

In general, individuals can respond to policies along two margins: the labor supply margin (intensive margin) and the education margin (extensive margin). As individual preferences satisfy quasi-linearity, labor supply will not respond to changes in the absolute value of taxes (and therefore not to changes in the lump sum elements of the tax schedules), but only to changes in marginal tax rates. These responses are standard considerations in economic theory and well understood. Therefore we refrain from analyzing labor supply responses in detail and solely focus on college enrollment responses in this subsection.

College enrollment is responsive to changes in the absolute value of taxes. In this subsection, we will define enrollment semi-elasticities with respect to taxes. We start with the case where taxes are education dependent and first we consider taxes for college graduates. We define the semi-elasticity of enrollment for ability level \( \theta \) with respect to the absolute value of the college tax \( T^{co}(y^{co}(\omega)) \) as:

\[
\xi^{co}(\theta, \omega) \equiv \frac{\partial \tilde{\chi}(\theta)}{\partial T^{co}(y^{co}(\omega))} k_\theta(\tilde{\chi}(\theta)) f(\theta) = \frac{-U'_{co}(w) g^{co}(w|\theta) k_\theta(\tilde{\chi}(\theta))}{K_\theta(\tilde{\chi}(\theta))},
\]

where \( U_{co}(\omega) \) is the marginal utility of income of a college graduate with wage \( w \). The numerator captures the negative of the mass of marginal individuals that would have gone to college in the absence of the increase of \( T^{co}(y^{co}(\omega)) \), but do not go because of this tax increase. To obtain a semi-elasticity, the mass of those marginal individuals of type \( \theta \) is divided by the share of college graduates of type \( \theta \). The equal sign follows from the fact that \( \frac{\partial \tilde{\chi}(\theta)}{\partial T^{co}(y^{co}(\omega))} = U'_{co}(\omega) g^{co}(\omega|\theta) \) which directly follows from differentiation of (3). Finally, note that we define semi-elasticities such that they are negative whenever less individuals obtain a college degree.

Next, we define the semi-elasticities of enrollment with respect to high school taxes: Increasing \( T^{hs}(y^{hs}(\omega)) \) induces a percentage increase in college enrollment for individuals of type \( \theta \) by

\[
\xi^{hs}(\theta, \omega) \equiv \frac{\partial \tilde{\chi}(\theta)}{\partial T^{hs}(y^{hs}(\omega))} k_\theta(\tilde{\chi}(\theta)) f(\theta) = \frac{U'_{hs}(\omega) g^{hs}(\omega|\theta) k_\theta(\tilde{\chi}(\theta))}{K_\theta(\tilde{\chi}(\theta))},
\]

9
where \( U'_{hs}(\omega) \) is the marginal utility of income of a high school graduate with wage \( \omega \) and the equal sign again follows from differentiating (3) respectively. An increase in high school taxes makes college more attractive and therefore increases enrollment; therefore \( \xi^{hs}(\theta, \omega) > 0 \).

Whereas the impact of education dependent taxes on enrollment is unambiguous, the response of enrollment with respect to an increase of education-independent taxes is in general ambiguous. In case that taxes are education independent, a marginal increase in \( T(y(\omega)) \) yields a percentage increase in enrollment for type \( \theta \)-individuals given by

\[
\xi^{ind}(\theta, \omega) = \xi^{co}(\theta, \omega) + \xi^{hs}(\theta, \omega),
\]

which can in general be positive or negative. An issue we now discuss in more detail.

**Do Higher Taxes Lead to Lower Enrollment?** An increase in the income tax has in general an ambiguous effect on college enrollment. To understand this better, let us think about an increase in the absolute tax payment for all individuals with \( \omega > \omega^* \), which can be thought of an increase of the marginal tax rate at income level \( y(\omega^*) \). Formally, we know that the overall impact on enrollment reads as

\[
\int_{\omega^*}^{\omega} \int_{\Theta} \left( \xi^{co}(\theta, \omega) + \xi^{hs}(\theta, \omega) \right) K_\theta(\bar{\chi}(\theta))dF(\theta)d\omega.
\]

To gain a better understanding, we substitute the semi-elasticities by its definitions, yielding:

\[
\int_{\omega^*}^{\omega} \int_{\Theta} \left( U'_{hs}(\omega)g^{hs}(\omega|\theta) - U'_{co}(\omega)g^{co}(\omega|\theta) \right) k_\theta(\bar{\chi}(\theta))dF(\theta)d\omega. \tag{4}
\]

On the one hand, higher income taxes make college graduation more attractive since the tax burden for high school graduates increases (captured by \( +U'_{hs}(\omega)g^{hs}(\omega|\theta) \) in the above formula) tending to make (4) positive. On the other hand, higher income taxes also affect me as a college graduate (captured by \( -U'_{co}(\omega)g^{co}(\omega|\theta) \) in the above formula) tending to make (4) negative. The question is whether individuals are affected more heavily by this tax increase (in expectation) as a college or as a high school graduate. To understand this, it is useful to distinguish between a price and an income effect.

1. **Price Effect:** This price effect could also be called a return effect. Whenever \( G^{co}(\omega^*|\theta) < G^{hs}(\omega^*|\theta) \) (college induces a first-order stochastic dominance shift in the wage distribution), an individual is more likely to be affected by the tax increase if it goes to college. By this reasoning a tax increase for individuals with \( \omega > \omega^* \) renders college less attractive.

2. **Income Effect:** Whereas one is more likely to be affected by the tax increase as a college graduate it is not clear whether one is going to be affected more severely. The “severeness” is measured by \( E_{\omega|\theta}(U'_{hs}|\omega > \omega^*) \) and \( E_{\omega|\theta}(U'_{co}|\omega > \omega^*) \). Also here, we have to distinguish two effects.

   a) **Tuition Effect:** Whenever the college subsidy is lower than tuition costs, college graduates are poorer than high school graduates for a given realization of \( \omega \). This
tuition effect implies that higher taxes lead to lower college enrollment and therefore works in the same direction as the price effect.

b) **Probability Effect:** Whereas college graduates are poorer conditional on \( \omega \), college graduates have a higher expected wage conditional on \( \omega > \omega^* \). Thus, the expected wage conditional on being affected should be lower for a high school graduate than for a college graduate. In contrast to the other effects, the probability effect implies that higher taxes lead to higher enrollment.

In general, we can conclude that the impact of taxes on enrollment is ambiguous. Assuming that the probability effect outweighs the tuition effect, the income effect and the price effect are of opposite sign. The former is more likely to outweigh the latter for low values of \( w^* \). For a subsidy \( \geq 100\% \), i.e. \( T^{co}(0) + C \leq T^{hs}(0) \), we can prove that an increase in lump-sum taxes makes college more attractive:

**Proposition 1.** Assume that taxes are education independent and college subsidies \( \geq 100\% \), i.e. \( T^{co}(0) + C \leq T^{hs}(0) \). Then an increase in the lump sum tax for all individuals unambiguously increases college enrollment.

**Proof.** A 100% subsidy, i.e. \( T^{co}(0) + C = T^{hs}(0) \), implies \( U'_{hs}(\omega) = U'_{co}(\omega) \equiv U'(\omega) \). Whenever, FOSD holds, we therefore have for each \( \theta \):

\[
\int_{\omega} U'(\omega) \left( g^{hs}(\omega|\theta) - g^{co}(\omega|\theta) \right) d\omega > 0
\]

since marginal utility is decreasing in \( \omega \). For a subsidy above 100%, i.e. \( T^{co}(0) + C < T^{hs}(0) \) we have \( U'_{hs}(\omega) > U'_{co}(\omega) \) and the argument goes through a fortiori. \( \square \)

### 3.4 Social Welfare and Policy Design

So far we have described the economy and individual behavior given taxes. In the following we turn to a normative assessment and consider a social planner that designs policies.

**Optimizing Planner versus Reform Planner.** We consider a social planner that designs \( T^{hs}(\cdot) \) and \( T^{co}(\cdot) \). We thereby do not only look at a planner that chooses these instruments optimally but also ask how the planner would value certain reforms of some given policies. In the following we will use the notions optimizing planner and reform planner to refer to these two considerations. Regardless of whether we think about an optimizing planner or a reform planner, the planner’s objective is assumed to be given by:

\[
\int_{\Theta} \int_{\tilde{\chi}} (V_{co}(\theta) - \chi) d\tilde{K}_{\theta}(\chi) d\tilde{F}(\theta) + \int_{\Theta} \int_{\tilde{\chi}} V_{hs}(\theta) d\tilde{K}_{\theta}(\chi) d\tilde{F}(\theta),
\]

where \( \tilde{\chi}(\theta) \) is defined by (3) and \( V_{co}(\theta) \) and \( V_{hs}(\theta) \) are defined by (2). \( \tilde{K}_{\theta}(\chi) \) and \( \tilde{F}(\theta) \) denote the cumulated Pareto weights. If, e.g., \( \tilde{K}_{\theta}(\chi) = K_{\theta}(\chi) \forall \theta, \chi \) and \( \tilde{F}(\theta) = F(\theta) \forall \theta \), the social planner is Utilitarian.
Taxes (whether chosen optimally or not) have to satisfy a government budget constraint:

\[
\int_{\Theta} \int_{\Omega} T^{co}(y_{co}(\omega))dG^{co}(\omega|\theta)K_{\theta}(\tilde{\chi}(\theta))dF(\theta) \\
+ \int_{\Theta} \int_{\Omega} T^{hs}(y_{hs}(\omega))dG^{hs}(\omega|\theta)(1 - K_{\theta}(\tilde{\chi}(\theta)))dF(\theta) \geq 0. 
\] (6)

Further, we denote as \(\rho\) the marginal value of public funds. It says by how much welfare increases if (6) is relaxed by a marginal dollar. If policies are chosen optimally, \(\rho\) will take the same value as the Lagrangian multiplier on (6).

**Social Welfare Weights** In the remainder of the paper, we often make use of welfare weights that are endogenous with respect to policies. We define the welfare weight of an individual of type \((\theta, \chi, \omega)\) with \(\chi < \tilde{\chi}(\theta)\):

\[
W(\theta, \chi, \omega) = \frac{1}{\rho}U'(c_{co}(\omega) - v\left(\frac{y_{co}(\omega)}{\omega}\right)\tilde{k}_{\theta}(\chi)\tilde{f}(\theta)}{h^{co}(\omega)}. 
\] (7)

Such a weight is endogenous because it depends on the marginal utility of consumption and therefore on policies. It can be read as follows: marginally increasing consumption for individuals of type \((\theta, \chi, \omega)\) increases welfare in units of public funds by \(W(\theta, \chi, \omega)\times h^{co}(\omega)\tilde{k}_{\theta}(\chi)\tilde{f}(\theta)\), additional dollars of public funds. We now also define two aggregated welfare weights. The welfare weights of all individuals with wage \(\omega\) and with/without a college degree are given by:

\[
W^{co}(\omega) = \frac{1}{\rho}U'(c_{co}(\omega) - v\left(\frac{y_{co}(\omega)}{\omega}\right)\int_{\Theta} \int_{\tilde{\chi}(\theta)} g^{co}(\omega|\theta)\tilde{k}_{\theta}(\chi)\tilde{f}(\theta)}{h^{co}(\omega)}
\] and

\[
W^{hs}(\omega) = \frac{1}{\rho}U'(c_{hs}(\omega) - v\left(\frac{y_{hs}(\omega)}{\omega}\right)\int_{\Theta} \int_{\tilde{\chi}(\theta)} g^{hs}(\omega|\theta)\tilde{k}_{\theta}(\chi)\tilde{f}(\theta)}{h^{hs}(\omega)}. 
\] (8)

Based on that, the social marginal welfare weight of all individuals with wage \(\omega\) is given by \(W(\omega) = W^{co}(\omega) + W^{hs}(\omega)\).

**Restrictions on Policy Instruments.** Whether we consider the optimizing planner or the reform planner, we will in general consider different scenarios for the sophistication of policy instruments. The most general scenario is where both \(T^{co}(\cdot)\) and \(T^{hs}(\cdot)\) are arbitrarily nonlinear functions. We refer to this case as *education-dependent taxes*. Another scenario – more in line with real world policies – is the one, where \(T^{hs}(y) = T^{co}(y) \forall y\). In this case marginal tax rates are independent of the education decision, and we therefore call this case *education-independent taxes*. For the case of education-independent taxes, we can also distinguish two subcases: one where college entry is potentially subsidized (or taxed). In this case, we can

---

11 For \(\chi > \tilde{\chi}(\theta)\), we only would have to replace subscript \(co\) by \(hs\).
have $T^{hs}(0) \neq T^{cc}(0)$. In the other subcase, college cannot be taxed or subsidized and we have $T^{hs}(0) = T^{cc}(0)$.

3.5 Welfare Effects of Tax and Education Policy Reforms

Based on the previous subsection, we now define the welfare effects of tax reforms in the spirit of Piketty (1997), Saez (2001) and Golosov, Tsyvinski, and Werquin (2013). These formulas will then be used to calculate revenue effects from perturbing some given policies in Section 4 as in Golosov, Tsyvinski, and Werquin (2013) and to derive optimal policy formulas in Section 5.

We therefore look at two kinds of reforms: first, we look at small changes in marginal tax rates at a certain income level as illustrated in Figure 1. Secondly, we look at welfare effects of changes in the lump sum elements of the tax function. We will also distinguish the cases where taxes are constrained to be independent of education (Section 3.5.1) and where the planner can condition taxes on the educational choice (Section 3.5.2).

3.5.1 Education Independent Taxes

We start with perturbations of the marginal tax rates as illustrated in Figure 1. The marginal tax rate is slightly increased by $\Delta T_y$ in an interval of infinitesimal length $\Delta y$ around income $y(\omega^*)$. Such a perturbation has three effects on social welfare. First, it has a mechanical effect on welfare since $\Delta y \times \Delta T_y$ dollars are taken from individuals with $\omega > \omega^*$. For simplicity but w.l.o.g., we will assume that $\Delta y \times \Delta T_y = 1$ in the following. Second, the perturbation will have an effect on labor supply for individuals within this small income interval around $y(\omega^*)$ as their marginal returns of supplying labor have changed. Whereas this has no first-order effect on welfare via individual utilities because of the envelope theorem, it will affect welfare via the implied reduction in public funds. Third, this tax reform will change enrollment behavior of individuals that are just indifferent between going to college or not. Again, this will only affect welfare via the implied change in public funds.
Mechanical Effect. As described, the tax reform increases the absolute tax liability of individuals with $\omega > \omega^*$ by one dollar. This will increase tax revenue by $1 - H(\omega^*)$. However, a benevolent government will also value the utility loss of these individuals. We denote the effect on welfare of taking one dollar from individuals with wage $\omega > \omega^*$ and with/without a college degree by
\[ \forall i = hs, co : M_i(\omega^*) \equiv \int_{\omega^*}^{\omega} h_i(\omega) \left( 1 - W_i(\omega) \right) d\omega, \]
where the welfare weight $W_i(\omega)$ is defined by (7) and (8) respectively. As both education groups are affected by this tax reform, it is useful to define an education independent mechanical effect:
\[ M(\omega^*) = M^{hs}(\omega^*) + M^{co}(\omega^*). \]

Labor Supply Effect. As the mechanical effect, the labor supply effect is rather standard in the literature. Define as $\epsilon_i(\omega^*)$ the elasticity of income with respect to $1 - T_i y$ for individuals with wage $\omega^*$ and education $i$. Again, this elasticity is actually also a function of the tax schedule $T_i$ and we suppress the dependence for simplicity. Slightly increasing the marginal tax rate $T_i(y_i(\omega^*))$ in the small interval around $y_i(\omega^*)$ therefore causes individuals with income in that interval and educational choice $i$ to change their income by
\[ \epsilon_i(\omega^*) \frac{y_i(\omega^*)}{1 - T_i(y_i(\omega^*))}. \]
As first shown by Piketty (1997), some algebraic steps reveal that the implied change in tax revenue is given by
\[ LS^i(\omega^*) \equiv -\frac{\epsilon_i(\omega^*)}{1 + \epsilon_i(\omega^*)} \frac{T_i^2(y_i(\omega^*))}{1 - T_i^2(y_i(\omega^*))} h_i(\omega^*) \omega^*, \]
where we call $LS^i(\omega^*)$ the labor supply effect. Since we look here look at the case of education-independent taxes, we need the impact on public funds of an increase in $T_i^2$. It is given by:
\[ LS(w^*) = LS^{co}(w^*) + LS^{hs}(w^*). \]

Enrollment Effect. The tax reform yields to an increase of the absolute tax payment of one dollar for each wage realization $\omega > \omega^*$ – independent from the education choice. Based on the definitions of the enrollment semi-elasticities in Section 3.3, we know that this reform has the following impact on enrollment:
\[ \int_{\omega} \int_{\omega^*} \left( \xi^{co}(\theta, \omega) + \xi^{hs}(\theta, \omega) \right) K_{\theta}(\tilde{\chi}(\theta)) d\omega dF(\theta). \]
This has no first-order effects on welfare because these marginal individuals have just been indifferent between obtaining a college degree or not. However, they will pay a different amount of taxes. The expected tax differential of an individual of type $\theta$ is given by:
\[ \Delta T(\theta) = \int_{\Omega} T^{co}(y_{co}(\omega))dG^{co}(\omega|\theta) - \int_{\Omega} T^{hs}(y_{hs}(\omega))dG^{hs}(\omega|\theta). \]
Thus, the impact on public funds of this change in enrollment is given by

\[ E(\omega^*) = \int_\Theta \Delta T(\theta) \int_{\omega^*} (\xi^{co}(\theta, \omega) + \xi^{hs}(\theta, \omega)) K_\theta(\bar{\chi}(\theta)) d\omega dF(\theta). \]  

(10)

In general, the sign of (10) is unclear. First, we do not know whether higher taxes really decrease enrollment as discussed in Section 3.3. Second, we do not know the sign of (9) in general because this depends on policies and returns to college.

For education-dependent taxes, we can equivalently define:

\[ E^{co}(\omega^*) = \int_\Theta \Delta T(\theta) \int_{\omega^*} \xi^{co}(\theta, \omega) K_\theta(\bar{\chi}(\theta)) d\omega dF(\theta). \]

and

\[ E^{hs}(\omega^*) = \int_\Theta \Delta T(\theta) \int_{\omega^*} \xi^{hs}(\theta, \omega) K_\theta(\bar{\chi}(\theta)) d\omega dF(\theta). \]

For \( \Delta T(\theta) > 0 \), we can make clearer statements about \( E^{co}(\omega^*) \) and \( E^{hs}(\omega^*) \) than on \( E(\omega^*) \); namely we then have \( E^{co}(\omega^*) < 0 \) and \( E^{hs}(\omega^*) > 0 \).

**Overall Welfare Effects.** Based on these definitions, we can now state the impact of an increase in the marginal tax rate at income level \( y(w^*) \) on welfare. This term will be very useful to obtain formulas for normalized revenue gains in Section 4 and to obtain expressions for optimal marginal income tax rates in the presence of endogenous college enrollment. The welfare effect of increasing \( T_y(y(\omega)) \) is given by:

\[ \Gamma(w^*) = M(w^*) + E(w^*) + LS(w^*). \]  

(11)

Using similar arguments, we can now also define the welfare effect of an increase in the lump sum element. It is given by:

\[ \Gamma(w) = M(\omega) + E(\omega). \]  

(12)

Here the labor supply effect drops out because an increase in the lump sum element has no effect on labor supply if preferences satisfy quasi-linearity.

Finally, let us define the welfare effect of a marginal increase in the college subsidy (i.e. a decrease in \( T^{co}(0) \)):

\[ \Gamma^S \equiv -\Gamma^{co}(w) = -M^{co}(\omega) + E^{co}(\omega). \]  

(13)

### 3.5.2 Education Dependent Taxes

The welfare effect of increasing the marginal education dependent tax rate reads as

\[ \forall i = hs, co : \Gamma^i(\omega^*) = M^i(\omega^*) + E^i(\omega^*) + LS^i(\omega^*). \]

Since \( E^{co}(\omega^*) > 0 \) (\( E^{hs}(\omega^*) < 0 \)) whenever \( \Delta T(\theta) > 0 \), one can directly see that the endogeneity of college enrollment decreases (increases) the welfare gains from raising marginal tax rates for
college graduates (high school graduates). The same is true for the lump sum element, where the effect on welfare reads as

\[ \forall i = hs, co : \Gamma^i(\omega) = M^i(\omega) + E^i(\omega), \]

where for \( i = co \), an increase in the lump sum element is equivalent to a decrease in the college subsidy, see equation (13).

4 Revenue Effects of Policy Reforms

Following Golosov, Tsyvinski, and Werquin (2013), we now ask how effective policy changes (starting from the current system) are in generating revenue. Within the static modeling framework, current policies are well approximated by \( T_{hs}(y) = T_{co}(y) \) \( \forall y \) and \( T^{hs}(0) > T^{co}(0) \).

The former reflects education independent taxes and the latter reflects subsidies to college in the form of student grants, publicly financed colleges and subsidized loans.\(^{12}\)

To rewrite the terms for the welfare gains of policy reforms into revenue gains, it suffices to set all welfare weights in the welfare gain formulas to zero, i.e. \( W(\theta, \chi, \omega) = 0 \) \( \forall \theta, \chi, \omega \).

Thus, let \( \Gamma_{R}^{co}(\omega^*), \Gamma_{R}^{hs}(\omega^*) \) and \( \Gamma_{R}(\omega^*) \) be the revenue gain counterparts to \( \Gamma^{co}(\omega^*), \Gamma^{hs}(\omega^*) \) and \( \Gamma(\omega^*) \). To get the overall revenue gains from changing current policies, one only would have to multiply the terms by population size. In the spirit of Golosov, Tsyvinski and Werquin (2013), we rather focus on normalized revenue gains that measure how effective different policy reforms are in increasing revenue.

4.1 Increasing Lump Sum Taxes

We first look at the normalized revenue gain of an increase in the lump-sum element of the tax function:

\[ \gamma_{R}(\omega) = \frac{\Gamma_{R}(\omega)}{1 - H(\omega)} = 1 + E(\omega). \]

For clarification, note that \( \frac{M(\omega)}{1 - H(\omega)} = 1 \) if all welfare weights are set to zero. The above term is likely to be larger than one because this lump-sum increase causes no labor supply responses and possibly leads to larger college enrollment as argued in Section 3.3. Stated differently, for each dollar that is mechanically raised through the lump sum increases, the government gets some additional cents because of the implied increase in college graduates.

4.2 Increasing Education Subsidies

We now ask the following question: \textit{What are the fiscal costs of increasing grants by one dollar?}\n
The following formula gives the respective number:

\[ \gamma^S \equiv \gamma^co_{R}(\omega) = 1 + \frac{E^{co}(\omega)}{H^{co}(\omega)}, \]  

\(^{12}\)The latter will be modeled distinctively in the dynamic version of the model in future versions of the paper.
If this number is 0.25, it says that the government gets 75 Cents of each invested dollar back via higher tax revenue. Thus, if this number is negative, it implies that higher grants can even be self-financing. In this case, one dollar invested into the education of young individuals will yield more than one additional dollar of expected tax revenue. In such a case, an increase in the education subsidy is a Pareto improvement.

We can rewrite (14) as:

$$\gamma^S = 1 - \frac{\int_\theta \Delta e(\theta) \Delta T(\theta) d\theta}{H^{co}(\omega)},$$

where $\Delta e(\theta)$ is the number of marginal students of type $\theta$. Further, note that $H^{co}(\omega)$ is the college graduation rate in the economy and therefore the number of infra-marginal students. We now further simplify this formula by defining $\Delta \tau = \int_\theta \Delta e(\theta) d\theta$ as the overall number of marginal students and $\Delta \bar{T} = \int_\theta \frac{\Delta e(\theta) \Delta T(\theta) d\theta}{\Delta \tau}$ as the average tax differential for a marginal student:

$$\gamma^S = 1 - \frac{\Delta \tau \Delta \bar{T}}{H^{co}(\omega)}. \tag{15}$$

This formula intuitively describes which economic forces determine $\gamma^S$. The larger the returns to college (for marginal students) are, the larger is $\Delta \bar{T}$ and therefore the more desirable the reform becomes ceteris paribus. Additionally, the larger the share of marginal students $\Delta \tau$ over infra-marginal students $H^{co}(\omega)$ is, the more desirable such a reform becomes. Note, that as $\Delta \tau / H^{co}(\omega)$ can be interpreted as the semi-elasticity of enrollment, this formula basically only depends on two empirical measures: (i) the semi-elasticity of enrollment and (ii) the average increase in tax payment of the marginal students. These ideas reveal that (15) is a sufficient-statistics formula/test. Different models with different assumptions about important primitives like borrowing constraints of students, risk-aversion or the returns for marginal students, will be encompassed by the local behavioral response.\(^{13}\)

**Graphical Illustration.** Figure 2 intuitively illustrates the fiscal trade-off of increasing the college subsidy. For the ease of exposition, assume that there is only one innate ability level. The bell shaped curves reflect the density of the psychic cost distribution. We look at two scenarios: a) the enrollment share is low and b) the enrollment share is high. The left vertical black line gives the psychic costs threshold before a potential grant increase, respectively. The vertical line to the right in each graph gives the psychic cost threshold after a grant increase of say $1,000. The green area reflects the number of infra-marginal students. The yellow area represents the number of marginal students. Assuming that the tax differential $\Delta \bar{T}$ is the same in both cases, one can directly see that increasing the grant is a better investment in future tax revenue in the low enrollment case as compared to the high enrollment case.

**Tagging.** These considerations also indicate that the trade-off for increasing grants for different groups can be quite distinct. Especially the ratio of marginal to infra-marginal students can differ drastically among different groups. Thus, even though higher grants might not be

\(^{13}\)Lawson (2014) has derived a very similar sufficient-statistics argument in a homogeneous-agent framework.
entirely self-financing in general, they might be self-financing for certain subgroups where this ratio of marginal to infra-marginal students is large. In the latter case, the introduction of tagging (i.e. increasing subsidies only for a subgroup of the population with a certain “tag”) can be Pareto improving.

4.3 Increasing Marginal Tax Rates for Both Education Groups

We now look at the effectiveness of raising tax revenue via increases of marginal tax rates. Therefore, based on Golosov, Tsyvinski and Werquin (2013), we define the following measure

\[
\gamma_R(\omega^*) = \frac{\Gamma_R(\omega^*)}{1 - H(\omega^*)} = 1 + \frac{LS(\omega^*)}{1 - H(\omega^*)} + \frac{E(\omega^*)}{1 - H(\omega^*)}.
\]

This measure can be read as follows: For each dollar the government mechanically raises by increasing \( T(y(w^*)) \), it keeps \( \gamma_R(w^*) \). In the absence of the college margin, this number would always be smaller than one because rising tax revenue mechanically for individuals \( \omega > \omega^* \) necessarily implies a downward distortion of labor supply for individuals with \( w = w^* \). The presence of endogenous college enrollment should typically exacerbate this effect and make \( \gamma_R(\omega^*) \) smaller. However, as argued in Section 3.3, the implied enrollment responses do not

\[14\] The whole analysis can certainly also be used to ask the opposite question: “How to effectively decrease the tax burden?” In times of large fiscal deficits, we consider the other approach as more informative for policy.
necessarily imply a fiscal loss for the government as (i) higher taxes for \( \omega > \omega^* \) can theoretically increase college enrollment and (ii) it is in general unclear whether one more college graduate implies a fiscal gain or loss for the government. Thus, \( \gamma_R(\omega^*) \) could even be larger than one.

For a calibrated version of the model and a quantitative approximation of the current policies, one can obtain the numerical values \( \gamma_R(\omega^*) \forall \omega^* \in \Omega \). These values are very informative for how tax revenue could be increased most effectively.

### 4.4 Increasing Marginal Tax Rates only for One Education Group

Allowing taxes to be education dependent, we can define the analogous measures as before:

\[
\gamma_R^{co}(\omega^*) = \frac{\Gamma_R^{co}(\omega^*)}{H^{co}(\omega) - H^{co}(\omega^*)} = 1 + \frac{LS^{co}(\omega^*)}{H^{co}(\omega) - H^{co}(\omega^*)} + \frac{E^{co}(\omega^*)}{H^{co}(\omega) - H^{co}(\omega^*)},
\]

\[
\gamma_R^{hs}(\omega^*) = \frac{\Gamma_R^{hs}(\omega^*)}{H^{hs}(\omega) - H^{hs}(\omega^*)} = 1 + \frac{LS^{hs}(\omega^*)}{H^{hs}(\omega) - H^{hs}(\omega^*)} + \frac{E^{hs}(\omega^*)}{H^{hs}(\omega) - H^{hs}(\omega^*)}.
\]

The first main difference to the education independent case is that each expression is normalized by an education dependent share of individuals with \( w > w^* \). The second main difference is that the revenue effects from changed college enrollment now have an unambiguous sign if \( \Delta T(\theta) > 0 \forall \theta \), as we discussed in Section 3.3 in detail.

Following Golosov, Tsyvinski, and Werquin (2013), we now aim at formalizing the gains. For this need, we define the education dependent hazard ratios as \( \lambda^{co}(\omega) = \frac{h^{co}(\omega)}{H^{co}(\omega) - H^{co}(\omega^*)} \) and \( \lambda^{hs}(\omega) = \frac{h^{hs}(\omega)}{H^{hs}(\omega) - H^{hs}(\omega^*)} \), and for the aggregated wage distribution as \( \lambda(\omega) = \frac{h(\omega)}{1 - H(\omega)} \).

Using this simplified notation, one can show that the following relationship between the revenue effects of the education independent reform and the education dependent case holds.

**Proposition 2.** The revenue effect of a marginal increase of \( T^{co}(y^{co}(\omega^*)) \) can be written as

\[
\gamma_y^{co}(\omega^*) = \frac{\lambda^{co}(\omega^*)}{\lambda(\omega^*)} \gamma(\omega^*) + \left( 1 - \frac{\lambda^{co}(\omega^*)}{\lambda(\omega^*)} \right) \frac{E^{co}(\omega^*)}{H^{co}(\omega) - H^{co}(\omega^*)} - \frac{\lambda^{co}(\omega^*)}{\lambda(\omega^*)} \frac{E(\omega^*)}{1 - H(\omega^*)}.
\]

To gain more intuition, consider the case where education independent taxes are set optimally (i.e. such that revenue is maximized), so \( \gamma(\omega^*) = 0 \). Starting from there, the government introduces education dependence at some income level, by increasing the marginal tax rate for college graduates. This can be, for example, implemented with an income-contingent student loan system (Findiesen and Sachs 2013). The revenue gain from this reform can be shown to be:

\[
\left( 1 - \frac{\lambda^{co}(\omega^*)}{\lambda(\omega^*)} \right) + \left( 1 - \frac{h^{co}(\omega^*)}{h(\omega^*)} \right) \frac{E^{co}(\omega^*)}{H^{co}(\omega) - H^{co}(\omega^*)} - \frac{\lambda^{co}(\omega^*)}{\lambda(\omega^*)} \frac{E^{hs}(\omega^*)}{1 - H(\omega^*)}.
\]

In particular, under reasonable assumptions on the returns to college we have \( \frac{\lambda^{co}(\omega^*)}{\lambda(\omega^*)} < 1 \) and the first hazard effect is positive. The intuition is that tax rates on college graduates tend to be less
distorting, whenever $\frac{\lambda^\text{co}(\omega^*)}{\lambda(\omega^*)} < 1$. The second effect counteracts the first effect because taxing college graduates higher than high school graduates has adverse effects on college enrollment. In the quantitative section, we discuss in detail which effect dominates at which part of the income distribution. Note that these formulas are closely related to the results in Golosov, Tsyvinski, and Werquin (2013), who consider the welfare gains from age-dependence, history-dependence and joint taxation of capital and labor income in the tax code.

5 Optimal Education and Tax Policies

5.1 Optimal Education Subsidies

Formally, the college subsidy is defined by: $S = T_{ha}(0) - T_{co}(0)$. The optimal subsidy can be obtained from setting $\Gamma_S$ (as defined in (13)) equal to zero, which yields the following proposition.

Proposition 3. The optimal education subsidy $S$ satisfies:

$$S = \int_{\theta} \eta^\text{co}(\theta) K_{ha}(\bar{\chi}(\theta)) \Delta T(\theta) dF(\theta)$$

$$H^\text{co}(\bar{\omega}) - \int_{\omega} W^\text{co}(\omega) d\omega,$$

where $\eta^\text{co}(\theta)$ is the elasticity of college graduation w.r.t. to the education subsidy for group $\theta$. The intuition is very similar to the intuition behind the revenue gains from increasing college subsidies outlined in Section 4.2: The numerator captures the fiscal externality, i.e. the gains from subsidizing college. The stronger individuals respond to subsidies (captured by the elasticity $\eta(\theta)$), the larger subsidies should be. In addition, the higher taxes are and the higher the returns to college are, the larger is $\Delta T(\theta)$ and the larger subsidies should be.

The denominator reflects the cost of increasing college subsidies. Ceteris paribus, higher college graduation rates $H^\text{co}(\bar{\omega})$ are associated with a lower subsidy, as all graduates receive the subsidy. But this cost term is corrected by $\int_{\bar{\omega}} \hat{h}^\text{co}(\omega) d\omega$ which measures the welfare gain of the increased utility of these infra-marginal students.

Note also that this formula for the optimal education subsidy does not only apply for the optimal income tax but also for any given suboptimal income tax function.

Optimal Subsidies and the College-Wage Premium. Interestingly, the formula transparently indicates, why a rise in returns to education/the college wage premium can have ambiguous effects on the optimal subsidies. On the one hand, as said above, the government has now a higher incentive to subsidize college education because the fiscal externality will be larger. On the other hand, without offsetting changes in the tax code, inequality between college graduates and non-graduates will increase, which will tend to decrease $\int_{\bar{\omega}} \hat{h}^\text{co}(\omega) d\omega$. The intuition here is, that most college graduates will end up relatively well-off, reducing their marginal utility and the desirability of transfers to this groups.
5.2 Optimal Education Independent Taxes

The formulas for the welfare gains (11) and (12) can also be used to calculate optimal marginal tax rates for some given set of welfare weights. As a tax reform should have no first-order effect on welfare in case of an optimal tax system, optimality requires
\[ \Gamma_{\text{ind}}(\omega^*) = 0 \quad \forall \omega \in \Omega. \]

Some manipulations of this optimality condition then yield the following proposition:

**Proposition 4.** In the static model, optimal education independent marginal tax rates satisfy
\[
\frac{T_y(\omega^*)}{1 - T_y(\omega^*)} = \left( 1 + \frac{1}{\varepsilon(\omega^*)} \right) \frac{1 - H(\omega^*)}{h(\omega^*) \omega^*} (M(\omega^*) + E(\omega^*))
\]

*Proof.* Rearranging \( \Gamma_{\text{ind}}(\omega^*) = 0 \) yields the result. \( \square \)

This formula for the optimal marginal tax rate is related to the formulas of Saez (2002) and Jacquet, Lehmann and van der Linden (2013) where the extensive margin is due to labor market participation.\(^{15}\) In these papers, the extensive margin is an unambiguous force towards lower marginal tax rates whenever workers pay more taxes than non-workers. In contrast, the endogeneity of college enrollment does not necessarily lead to lower marginal tax rates as the additional term is ambiguous in its sign. First, we do not know the sign of \( \Delta T(\theta) \) in general. Second, we do not know whether higher taxes for individuals with \( \omega > \omega^* \) indeed lead to lower college enrollment because of possibly counteracting income and substitution effects as outlined in Section 3.3. Whether and to what extent the endogeneity of college enrollment leads to lower optimal marginal tax rates is a quantitative question we aim to answer in Section 8.

5.3 Education Dependent Taxes

Given that the government can observe the college choice, a natural question is to what extent making taxes education dependent can improve welfare and how taxes should be set in that case. The latter question is addressed in the following proposition:

**Proposition 5.** In the static model, optimal education dependent marginal tax rates satisfy
\[
\frac{T_y^{\text{co}}(\omega^*)}{1 - T_y^{\text{co}}(\omega^*)} = \left( 1 + \frac{1}{\varepsilon^{\text{co}}(\omega^*)} \right) \frac{1 - H^{\text{co}}(\omega^*)}{h^{\text{co}}(\omega^*) \omega^*} (M^{\text{co}}(\omega^*) + E^{\text{co}}(\omega^*)).
\]

and
\[
\frac{T_y^{\text{hs}}(\omega^*)}{1 - T_y^{\text{hs}}(\omega^*)} = \left( 1 + \frac{1}{\varepsilon^{\text{hs}}(\omega^*)} \right) \frac{1 - H^{\text{hs}}(\omega^*)}{h^{\text{hs}}(\omega^*) \omega^*} (M^{\text{hs}}(\omega^*) + E^{\text{hs}}(\omega^*)).
\]

*Proof.* \( \Gamma^{\text{co}}(\omega^*) = 0 \) and \( \Gamma^{\text{hs}}(\omega^*) = 0 \) yield the result. \( \square \)

These formulas are very similar to the education independent tax formula in Proposition 4. A difference is that all the terms now carry the superscript \( \text{co} \) or \( \text{hs} \). It is well known\(^{15}\) See also Lehmann, Simula, and Trannoy (2013) for a related formula in an optimal tax setting with migration.

15See also Lehmann, Simula, and Trannoy (2013) for a related formula in an optimal tax setting with migration.
from the tagging literature\textsuperscript{16} that conditioning taxes on characteristics other than income is a powerful tool to tailor marginal tax rates to different subgroups. Importantly, education is not an exogenous tag here. Instead it is an endogenous choice. The planner therefore has to take into account the endogeneity of enrollment similarly as in Proposition 4. Whereas for education independent taxes, the effect of higher taxes on enrollment is theoretically ambiguous, this is not the case with education dependent taxes. Thus, for $\Delta \mathcal{T}(\theta) > 0$, endogeneity of enrollment is a clear force towards lower college taxes and higher high school taxes (as compared to the case with exogenous enrollment, i.e. classical tagging). Note that these formulas are related to Scheuer (2013), who considers the differential tax treatment of entrepreneurs and workers.

5.4 Self-Confirming Policy Equilibria

We also study tax policies of a naive planner that does not take into account the endogeneity of the enrollment decision. In order to coherently elaborate this case, we look at self-confirming policy equilibria (SCPE) as defined by Rothschild and Scheuer (2011, 2013). If a government does not take into account the endogeneity of the enrollment decision, it takes the wage distribution as exogenous. A SCPE is defined as a situation where the wage distribution that endogenously emerges given the government policies is the same wage distribution that renders these government policies optimal for a naive government that takes this wage distribution as exogenous.

5.4.1 Education Independent Taxes

First, we characterize education independent taxes in the following proposition.

**Proposition 6.** In the static model, education independent marginal tax rates in a SCPE are given by

$$\frac{T_y(\omega^*)}{1 - T_y(\omega^*)} = \left(1 + \frac{1}{\varepsilon(\omega^*)}\right) \frac{1 - H(\omega^*)}{h(\omega^*)w^*} M(\omega^*)$$

(16)

The optimal education subsidy in a SCPE is set s.t.:

$$M^{co}(\omega) = 0.$$

The naive government ignores any effects of taxes on enrollment decisions. Consequently, the tax formula collapses to the standard one (Diamond 1998). Equivalently, ignoring all behavioral responses caused by education policies, the subsidy is just set to equate the mechanical benefit (raise of welfare for college graduates) to the mechanical costs (fiscal costs of the subsidy).

5.4.2 Education Dependent Taxes

Next, we discuss education dependent taxes in the SCPE.

\textsuperscript{16}This literature goes back to Akerlof (1978). For more recent contributions addressing this topic, see Mankiw and Weinzierl (2010), Cremer, Firouz, and Lozachmeur (2010) and Weinzierl (2013).
Proposition 7. In the static model, education dependent marginal tax rates in a SCPE satisfy

\[
\frac{T_y^{co}(\omega^*)}{1 - T_y^{co}(\omega^*)} = \left(1 + \frac{1}{\varepsilon^{co}(\omega^*)}\right) \frac{1 - H^{co}(\omega^*)}{F^{co}(\omega^*)} M^{co}(\omega^*).
\]

and

\[
\frac{T_y^{hs}(\omega^*)}{1 - T_y^{hs}(\omega^*)} = \left(1 + \frac{1}{\varepsilon^{hs}(\omega^*)}\right) \frac{1 - H^{hs}(\omega^*)}{F^{hs}(\omega^*)} M^{hs}(\omega^*).
\]

The education subsidy in a SCPE is set s.t.:

\[M^{co}(\omega) = 0.\]

Education is basically used as a pure tag for taxes. The subsidy follows the same intuition as in the education independent case.

6 A Simple Dynamic Extension of the Model

We now introduce time into the model. Figure 3 illustrates this. We still model the labor market as one period. The length of this period, however, is shorter for college graduates as compared to high school graduates. Thus going to college now has the additional cost of foregone earnings. In addition, borrowing now also becomes an issue for individuals to cover tuition and to finance consumption during college.

In general, the borrowing choice for college graduates implies additional room for policy. First, borrowing could potentially be subsidized, even in a non-linear way. Subsidizing loans for college students is a way to make college more attractive especially for those individuals with good earnings prospectives that want to borrow a lot. Subsidizing loans is thus a way to target government money to high ability students. Secondly, the government could potentially also condition labor earnings taxes on the borrowing decision or provide public loans that come
with income-contingent repayment. In future versions of this paper, we intend to address all these additional policy instruments. In this version, we abstract from these considerations but nevertheless shortly introduce the dynamic version of the model as the timing considerations are important for the calibration and the interpretation of the numerical results.

The model is almost equivalent to the static version. Again, after individuals have drawn their wage, they make a labor leisure decision. Individuals live for \( T \) years. High school graduates thus work for \( T \) years, whereas college graduates work only \( T - T_e \) years because they spend \( T_e \) years at college. Before stating the problem of the individuals, let us define the following discount factors:

\[ \beta_{hs} \equiv \sum_{t=1}^{T} \beta^{t-1} \] captures the discounted length of the working period for a high school graduate; if per period utility for a high school graduate is \( X \), then the (discounted) lifetime utility is \( \beta_{hs} X \). Equivalently, define \( \beta_{co2} \equiv \sum_{t=T_e+1}^{T} \beta^{t-1} \) and \( \beta_{co1} \equiv \sum_{t=1}^{T_e} \beta^{t-1} \).

For a high school graduate of type \( \theta \), the value function is defined by

\[
V_{hs}(\theta) = \max_{y_{hs}(\omega)} \int_{\omega} \beta_{hs} U \left( y_{hs}(\omega) - T_{hs}(y_{hs}(\omega)) - v \left( \frac{y_{hs}(\omega)}{\omega} \right) \right) dG_{hs}(\omega|\theta),
\]

where \( \beta \equiv \sum_{t=1}^{T} \beta^{t-1} \). The problem of a high school graduate is essentially unchanged in comparison to static model.

For the college graduate, the problem gets slightly more involved:

\[
V_{co}(\theta) = \max_{c_e, c_{co}(\omega), y_{co}(\omega)} \beta_{co1} (U (c_e)) + \beta_{co2} \int_{\Omega} U \left( c_{co}(\omega) - v \left( \frac{y_{co}(\omega)}{\omega} \right) \right) dG_{co}(\omega|\theta)
\]

subject to

\[
\forall \omega: \beta_{co2} c_{co}(\omega) \leq \beta_{co2} [y_{co}(\omega) - T_{co}(y_{co}(\omega))] + \beta_{co1} s
\]

and

\[
c_e = -s + \mathcal{G} - C,
\]

where \( c_e \) captures consumption during education, \(-s\) is the amount individuals borrow in period during education. \( \mathcal{G} \) is a grant that college students receive.\(^{17} \) As stated above, in future versions of the paper we will allow for subsidized loans which could be captured by making \( \mathcal{G} \) a function of \( s \). We could also consider history dependent taxes or income-contingent repayment of loans by having \( T_{co}(y_{co}(\theta, \omega), s(\theta)) \); as soon as taxes condition on savings, labor supply will also be a function of \( \theta \).

As in the static model we can define a threshold value for the psychic costs:

\[
\tilde{\chi}(\theta) = V_{co}(\theta) - V_{hs}(\theta)
\]

\(^{17}\)Note that \( \mathcal{G} \) is redundant for perfect credit markets as these subsidies can also be provided via different lump sum elements in \( T_{hs} \) and \( T_{co} \).
for each \( \theta \). In addition, we can also define the cross-sectional wage distributions in a similar manner. Now they are adjusted for the time, which then reflects the fact that college graduates spend less years on the labor market.

\[
H^{co}(\omega) = \beta^{co2} \int_{\theta} \int_{\omega} dG^{co}(\tilde{\omega}|\theta) K(\tilde{\chi}|\theta) dF(\theta)
\]

and

\[
H^{hs}(\omega) = \beta^{hs} \int_{\theta} \int_{\omega} dG^{hs}(\tilde{\omega}|\theta) (1 - K(\tilde{\chi}|\theta)) dF(\theta).
\]

The overall wage distribution is the given by

\[
H(\omega) = H^{co}(\omega) + H^{hs}(\omega).
\]

All results from Section 3 remain essentially unchanged in this version of the paper. The formulas in some cases only need to be adjusted by \( \beta^{hs} \), \( \beta^{co1} \) and/or \( \beta^{co2} \) respectively.

7 Paremetrization & Estimation of the Model

In general, we use two data sets: the NLSY79 and the NLSY97. The big advantage of these data sets is that they contain the Armed Forced Qualification Test Score (AFQT-score) for most individuals. This test score is a very good signal for innate ability. Cunha, Karahan, and Soares (2011), e.g., show that it is the most precise signal for innate ability among comparable scores in other data sets. We use the method of Altonji, Bharadwaj, and Lange (2011) to make the AFQT-scores comparable between the two samples.

To get the wage distributions, we use the NLSY79 data as this data set contains more information about labor market outcomes. To estimate the psychic costs distribution, we use the NLSY97 data as more information is provided on students. Combining both data sets has proven to be a fruitful way in the literature to overcome the limitations of each individual data set, see Johnson (2013) and Abbott, Gallipoli, Meghir, and Violante (2013).

The key primitives of the model are the distribution of psychic costs and the conditional wage distributions. We describe the respective estimation in Section 7.3 and 7.2 respectively. Before that, we present other assumptions about parameters and current policies in Section 7.1

7.1 Data and Parameter Assumptions

Concerning the utility function, we assume the following form

\[
U = \frac{(C - \frac{p^{1+\varepsilon}}{1+\varepsilon})^{1-\gamma}}{1-\gamma},
\]

where we set \( \varepsilon = 0.5 \) in the benchmark case and \( \gamma = 2 \). We assume that college takes 4 years (i.e. \( T_c = 4 \)) and assume that individuals spend 44 or 48 years on the labor market depending on whether they went to college before. We set the risk free interest to 4%, i.e. \( R = 1.04 \) and assume that individuals discount factor is \( \beta = \frac{1}{R} \).
Concerning current policies, we assume that the labor income tax rate in the US is flat at a value of 30%. We set average yearly tuition costs to $6,820 and assume that each student receives a grant of $2,753 (Abbott, Gallipoli, Meghir, and Violante 2013).

We set the value of exogenous government spending to 15% of the GDP and set the lump sum element of the tax schedule such that the government budget is balanced (implying roughly $7,000 of transfers per year).

Finally, we make the assumption that individuals are borrowing constrained. Though there is no definite evidence on the importance and magnitude of borrowing constraints, Lochner and Monge-Naranjo (2012) conclude that they became more important in the recent time. In the current version of this paper, we set them to $14,000 per year of college, implying a maximum of yearly consumption of roughly $10,000 per year of college.

Given all these assumptions about parameters, we proceed in three steps:

1. We estimate $G^{co}(w|\theta)$ and $G^{hs}(w|\theta)$ in Section 7.2.
2. Based on that, we can calculate $V_{hs}$ and $V_{co}$ for each individual.
3. Based on the value function, we estimate the distribution of psychic costs in Section 7.3.

### 7.2 Wage Distributions

As a first step, we estimate $G^{co}(w|\theta)$ and $G^{hs}(w|\theta)$. We adopt the approach of Abbott, Gallipoli, Meghir, and Violante (2013) to estimate the relationship between innate ability, education and labor market outcomes. For each individual in our final sample of individuals from the NLSY79, we calculate average annual (gross) incomes for the age bracket between 35 and 40. For both education levels, graduating from college or not – similar as in Abbott, Gallipoli, Meghir, and Violante (2013) – we run regressions of the form:

$$\ln y_i = \alpha + \beta \ln(\theta_i) + \epsilon_i,$$

where $\theta_i$ corresponds to an individual’s AFQT-score. The reduced-form results imply a familiar complementarity between early or innate ability (AFQT) and formal college education. The gradient for college implies that a 10% increase in test score implies a 16.8% increase in earnings, whereas the effect for non-graduates is 10.4%.

We assume that incomes are distributed log-normally, except for a right-side Pareto tail. The above procedure gives us the mean of log incomes as function of an individual’s AFQT-score and education level. To determine the second moment of the log-normal part of the income distributions across education and innate ability levels, we use the sample variances of the error terms from (18) for each education level.\(^{18}\) We then append Pareto tails to each income distribution, starting at incomes of $150,000. We set shape parameter $a$ of the Pareto distribution to 2 for all income distributions.\(^{19}\) Figure 4 shows the expected income as a

\(^{18}\)In unreported results, we also allow the variance to be a quadratic function of innate ability. Since the coefficients in this auxiliary regressions are not significant, we stick to the case where the variance is just a function of education.

\(^{19}\)The rationale for this choice is that we estimated a separately for each education group, obtaining estimates very close to 3 in both cases. Additionally, we tested for any systematic patterns for a direct relationship between
function of the AFQT (in percentiles) for both education levels and clearly highlights the complementarity between innate ability and education.

Figure 4: Expected Income

The last step consists of calibrating the respective skill/wage distribution from the income distributions by using the an approximation of the US tax system and exploiting the first-order condition of individuals as pioneered by Saez (2001) and done in many papers after that. Appendix A.1.1 provides more details on the whole procedure.

As was argued in the theoretical section, the returns to education play an important role for the fiscal effects of an increase in college enrollment. Figure 5 plots the expected increase in tax revenue (in present value terms) as a function of the AFQT-score. As can be seen, this return is negative for small IQ levels reflecting the small returns to college. However, this number gets positive already for the 0.2 percentile and for the highest ability individuals the fiscal gain gets close to $300,000 in present value terms.

7.3 Psychic Costs

The distribution of psychic costs is key for the responsiveness of enrollment with respect to policies. In line with the literature (see Cunha, Heckman, and Navarro 2005), we assume that these psychic costs are determined by parental education, innate ability and whether individuals live in an urban area when they graduate from high school.

We thus estimate the following probit model

\[
P(College_i = 1) = \text{Prob}(Y_i^* > 0)
\]

\(a\) and \(\theta\) without finding any robust or significant patterns. The next consideration is that the well-known problem that high incomes are severely under-sampled in survey data as the NLSY or the PSID. We choose 2, as the evidence from tax returns data, for example Atkinson, Piketty, and Saez (2011), suggests. As is standard, \(a\) is defined such that \(y_m / y = a / (a - 1)\), where \(y_m = E[y_i | y_i > y]\) is defined as average earnings among those with earnings above \(y\).
where

$$Y_i^* = V^i_{co} - V^i_{hs} + \beta_1 + \beta_2 Urban_i + \beta_3 \cdot AFQT_i + \beta_4 S^{'father}_i + \beta_5 S^{'mother}_i + \varepsilon_i$$

and where $\varepsilon_i \sim N(0, \sigma)$. We restrict the coefficient on the difference in the value function to be one as utility is our unit of measurement. For the power of the estimation, however, this is no restriction as a binary choice model always leaves on degree of freedom for the parameters. As expected all variables have a positive and significant impact on the college choice.

Based on these estimations, we can calculate the estimated psychic costs for each individual:

$$\hat{\kappa}_i = \hat{\beta}_1 + \hat{\beta}_2 Urban_i + \hat{\beta}_3 AFQT_i + \hat{\beta}_4 S^{'father}_i + \hat{\beta}_5 S^{'mother}_i + \varepsilon_i.$$  

We draw 100 values for each $\varepsilon_i$ and then fit a normal distribution of $\kappa$ for each level of innate ability. Then, we are equipped with distributions of psychic costs conditional on innate ability as measured by the AFQT-score.

### 7.4 Performance of the Model

An important criterion for the credibility of optimal policy simulations is of course how well the model performs in replicating empirical facts given current policies. We start by comparing the enrollment share as a function of the AFQT score. Figure 6(a) shows that the model does well in replicating the enrollment shares, which is no surprise as the enrollment share was targeted in the estimation.

Of interest is how well the model does in replicating quasi-experimental evidence. To test the model, we thus increased the grant by $1,000 Dollar from the calibration value of $2,753: enrollment increases by 2.23 percentage points, see Figure 6(b) to see this increase as a function of AFQT. Thus, we come close to the benchmark number from Deming and Dynarski (2009), who report an increase in enrollment by 3-5 percentage points. In fact, we are comfortable with having slightly lower numbers as in our model there is no difference between college enrollment and college completion. Finally, the college wage premium is 75%, i.e. on average the income
of college graduates is 75% higher. As our earnings data are for the mid 90s, this is in line with empirical evidence, see Acemoglu and Autor (2011).

8 Simulation of Policies

This section is organized as follows: In Section 8.1, we look at revenue gains from partial reforms of college subsidies and income taxes. In Section 8.2, we elaborate optimal policies with various degrees of sophistication and compare the implied welfare gains.

8.1 Partial Reforms

8.1.1 Increasing Education Subsidies

We first ask the following question: For one dollar of investment in education grants, how much dollars in present value terms does the government get back via higher tax revenue? To obtain this number, we simply have to evaluate (15) in a version that is slightly adjusted for period length. To obtain this number, we decompose (15) into three parts: the share of infra-marginal students, the share of marginal students and the average increase in tax payment per marginal student. Benchmark enrollment – reflecting infra-marginal students – is at 29.26%. The share of marginal students due to a $1 Dollar increase in grants is 0.00242%\(^{20}\). The average fiscal gain per marginal student for the government in present value terms is $63,957. The marginal costs of \(\sum_{t=0}^{3} \frac{1}{104} \times 0.2923 \times $1 \approx $1.105\) are thus clearly outweighed by the marginal gains 0.0000242 \* $63,957 \approx $1.542. Thus, for each dollar invested, the government obtains $1.40. Investing more tax revenue into college subsidies would thus be a profitable investment for society. Given that one marginal dollar of education subsidies is self-financing, the following question naturally arises: Up to which amount is an increase in college subsidies self-financing? According to the current parametrization, our simulations reveal that an increase in grants from $2,753 per year up to $7,861 is self-financing.

\(^{20}\)Multiplying this number by 1,000 gives us the number from grant increase experiment in Section 7.4.
**Targeting Subsidies:** The profitability of such investments can differ substantially across groups. In reality, students from poor families often receive larger grants. We now show that such policies do not only favor social mobility but that investment in the education of poor children has a higher return. At this point, we do not model parental income or wealth and thus address this issue rather indirectly. We let college grants be different for children where both parents have no bachelor degree. It turns out that the amount of additional tax revenue for each dollar invested in grants is $1.46 for the children without academic background and $1.20 for children with academic background. Children with uneducated parents are thus the better investment opportunity for the society. Note that this is the case even though the average marginal student from the other group yields more tax revenue because of higher innate ability on average.\(^21\) Thus, it is the share of marginal over infra-marginal students that mainly drives the results.\(^22\) We also ask the question to what extent an increase in college subsidies is self-financing for these two groups: we calculate the numbers $9,669 for children with and $6,960 for children without academic background. In future versions of the paper, we plan to make grants contingent on parental income and/or wealth. As parental education is correlated with parental wealth and income which is correlated with parental transfers, we conjecture that our result gets strengthened. Individuals with low income background should typically react stronger to financial incentives as borrowing constraints are more likely to be binding.

**The Role Borrowing Constraints:** Our strong case for higher education subsidies hinges on the fact that individuals are borrowing constrained. Removing borrowing constraints might be an even more efficient way for the government to increase college enrollment and future tax revenue. In future versions of this paper, we will also incorporate parental transfers and address the problem of borrowing constraints in a more convincing way. Note, however, that our results on the profitability of a grant increase starting from current policies does not rely on this assumption on borrowing constraints, as for a local change in policies the enrollment response is a sufficient statistic.

8.1.2 Increasing Marginal Tax Rates

In the spirit of Golosov, Tsyvinski, and Werquin (2013), we present the revenue effects from tax reforms. The pictures are understood in the following way: how much tax revenue is raised if marginal tax rates at a given income level are raised such that they increase tax revenue by 1$ \textit{in the absence of behavioral responses} on the labor supply and education margin. So for each income level we are comparing the strength of the mechanical tax revenue effects relative to behavioral responses.

Figure 7(a) shows the normalized revenue gains from increasing education independent labor income taxes. They are positive and U-shaped. Increasing marginal tax rates clearly raises revenue for the government. The U-shaped pattern is primarily a consequence of the hazard

\(^{21}\) The average fiscal gains is $74,802 for children with and $57,447 for children without academic background. The reason is that innate ability is positively correlated with parental education.

\(^{22}\) Johnson (2013) and Abbott, Gallipoli, Meghir, and Violante (2013) also conclude that increasing enrollment among low income individuals is cheaper because the number of infra-marginal students smaller.
rate of the income distribution. Figure 7(b) shows the decomposition into the behavioral effects. The shape of the revenue gain is mainly determined by the (normalized) labor supply effect.

Figures 8(a) and 8(b) show the gain from education dependent increase in tax rates depending on education. These reforms, hence, introduce a small education dependence into the tax code. Whereas the enrollment effect was of minor importance in the education independent case, it is of major importance for the education dependent case. This becomes even more salient in Figures 9(a) and 9(b). In Figure 9(a), the education dependent revenue gains are displayed under the assumption that the college decision is exogenous. Because the income distribution of the college individuals is shifted to the right, revenue gains for college graduates are higher by the standard hazard rate interpretation. If, however, the endogeneity of enrollment is taken into account, the result reverses for low and intermediate incomes.
8.2 Optimal Policies

In the following subsections we look at optimal Utilitarian policies with various degrees of sophistication: optimal college subsidies in the presence of the current tax system (Section 8.2.2), optimal income taxes given current education subsidies (Section 8.2.3), optimal income tax and education policies (Section 8.2.4) and optimal education dependent taxes (Section 8.2.5). In Section 8.2.6, we show the welfare gains implied by each degree of sophistication. As optimal policies naturally imply an abolition of borrowing constraints, we also look at the case of current policies and no borrowing constraints as a first benchmark in Section 8.2.1.

8.2.1 Removing Borrowing Constraints

As a first scenario, we remove borrowing constraints and only adjust the lump sum element of the income tax such that the budget remains balanced (which implies a transfer of $7,259). As Figure 10(a) reveals, the removal of borrowing constraints leads to a significant increase of enrollment, especially for individuals with high innate ability. Intuitively, because of a high expected income, these individuals want to borrow a lot and for them, borrowing constraints have been particularly severe. The overall increase in enrollment is 5 percentage points; a number twice as high as found by Johnson (2013) and half the number found by Navarro (2011). The latter, however, does not only consider an abolition of borrowing constraints but also an abolition of risk, so that the numbers are of course not directly comparable. It seems natural that we obtain a higher number as Johnson (2013) as in our case without parental transfer basically all individuals are affected by credit constraints.

8.2.2 Optimal College Subsidies

Next, we look at the following scenario: the government cannot change marginal tax rates, however, it can adjust the lump sum element of the tax function and the college subsidies. In this case it slightly decreases the lump-sum element to $6,980 and increases grants to $9,020.
The implied increase in enrollment is shown in Figure 10(b). The increase of the subsidy increases enrollment across all ability levels. The overall increase in enrollment due to the optimal subsidies is 5 percentage points.

8.2.3 Optimal Income Taxes

We now consider the case where the government can set income taxes optimally given education subsidies as in the calibration. In Figure 11(a) optimal marginal tax rates are plotted as a function of yearly gross income. The shape of marginal tax rates is U-shaped as is typically the case with exogenous education (Diamond 1998, Saez 2001). In order to illustrate the impact of the endogeneity of college enrollment, we also plot marginal tax rates according to the standard optimal tax formula. We therefore take the optimal allocation as given and plug in the values into the RHS of (16). For low incomes and medium income levels marginal tax rates are slightly lower. For top income tax rates, the endogeneity of enrollment plays no significant role.

In Figure 11(b), we compare college enrollment to the case of current policies with and without borrowing constraints. The increase in taxes from current flat taxes to the optimal tax schedule decreases enrollment as we do not allow for higher college subsidies in this scenario. We address exactly this issue in the next subsection.

8.2.4 Optimal Income Taxes and College Subsidies

In this scenario, we allow the government also to set college subsidies optimally. The government sets optimal education subsidies to $16,289. Optimal marginal tax rates are displayed in Figure 12(a). They are slightly higher as compared to the previous subsection. This result is intuitive as higher education subsidies can counteract tax distortions and therefore allow for more redistribution and social insurance via taxes.

\footnote{An alternative would be to compare the optimum to the SCPE equilibrium. We illustrate this case in Appendix A.2.1; results are very similar.}
In this scenario, the endogeneity of enrollment has an even smaller impact on optimal marginal tax rates except for very low incomes. Intuitively, the fiscal gain of an additional college graduate is lower as college is subsidized at a higher rate in this case.  

Lastly, as Figure 12(b) reveals, optimal education subsidies boost college enrollment across all AFQT levels far above the numbers in the previous subsection.

### 8.2.5 Optimal Education Dependent Taxes

Finally, we look at the case of education dependent taxes. The NDPF-literature has shown that potential gains from such sophistications in the tax code are sizable (Golosov, Troshkin, and Tsyvinski 2013). In a setting with endogenous college choice that is more stylized than in the present paper, Findeisen and Sachs (2013) have analyzed student loans with income-contingent repayment and found significant welfare gains. Such income contingent repayment also implies education dependence in effective marginal tax rates.

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24The alternative comparison – the SCPE is illustrated in Appendix A.2.2. Results are very similar.
Figure 13(a) illustrates marginal tax rates as a function of gross income. Up to income levels of $130,000 marginal tax rates are higher for college graduates. For higher incomes, the result slightly reverses before marginal tax rates start to converge as the enrollment effect approaches zero. As a natural comparison, we also plot the optimal education independent marginal tax rates from the previous subsection in this graph. As intuition would suggest, they lie between the education dependent taxes. The optimal education subsidy is $18,577. In Appendix A.2.3, we also look at SCPE.

![Marginal Tax Rates](image1)

![Enrollment](image2)

Figure 13: Optimal Education Dependent Taxes

### 8.2.6 Welfare Gains

A question that naturally arises is how large the welfare gains from the different policies are. Figure 14 summarizes these numbers. The benchmark for comparison are current policies and the presence borrowing constraints as in the calibration. The following four lessons can be learned from this analysis:

1. Gains from removing borrowing constraints are large.

2. Gains from education subsidies are sizable and are larger if redistribution is at its optimal level.

3. Gains from redistribution are large.

4. Gains from education dependence in the tax code are smaller than the other gains but still significant ($\approx 0.26\%$ of consumption).

Note that these results are very similar to our analysis in Findeisen and Sachs (2013).
9 Conclusion

The goal of the paper is to build a transparent framework to explore the optimal design of college subsidies and income taxation. One of the main innovations is the close connection of the model to the empirical college literature and its successfulness in matching quasi-experimental evidence on behavioral effects along the college margin.

We obtain several analytical and quantitative results for the revenue effects of education and tax policy reforms and optimal education and tax policies. The three main results are that (i) increasing college subsidies is potentially self-financing via higher tax revenue in the future, (ii) targeting different subgroups (e.g. based on parental background) can substantially increase the power of college subsidies and yield Pareto improvements, (iii) the impact of endogenous college enrollment on optimal income taxes is rather small.
Figure 15: SCPE with exogenous college subsidies

A Appendix

A.1 Details on Calibration

A.1.1 Income and Skills

The sample selection from the NLSY79 is based on Johnson (2013), basically keeping all individuals, where crucial information is not missing. Our definition of college corresponds to the graduation from a postsecondary institution with a bachelor degree. In contrast to Johnson (2013), we also keep females in the sample. After running the regression as in (18), we eliminate all individuals from the sample with extreme realizations for \( \epsilon_i \). We keep all individuals who have realizations of \( \epsilon_i \) in between the 2th and the 98th percentile for each education level. We do so, to eliminate the influence of extreme outliers for our estimates of the second-moment of the log-normal distributions. The original sample consists of 1673 individuals and after the selection we are left with 1611 individuals.

For college, we obtain estimates of: \( \alpha^e = 1.993087 \) (standard error = 1.41) and \( \beta^e = 1.680102(0.27) \). For high-school: \( \alpha^e = 4.971929 \) (0.59) and \( \beta^e = 1.044688(0.12) \).

As described in the main text, the second-moments of the log-normal parts are education dependent, so that the up until 150k, \( \ln y \) is normal with standard deviation \( \sigma^e \). We directly take the estimates for \( \sigma^e \) from the distribution of residuals from (18). The values are 0.65 for college and 0.68 for high-school.

A.2 Numerical Illustration of SCPE

A.2.1 SCPE with Exogenous College Subsidies

Figure 15(a) compares optimal marginal tax rates with those marginal tax rates of an SCPE. As the endogeneity of enrollment is ignored in an SCPE, marginal tax rates are slightly higher. The change in enrollment (Figure 15(b)), however, is minuscule. The reason is that education subsidies are in both cases constraint to the value in the calibration.
A.2.2 SCPE with Endogenous College Subsidies

Figure 16(a) compares optimal marginal tax rates with the marginal tax rates in the SCPE. For low incomes, marginal tax rates are lower in the SCPE, for higher incomes there is basically no difference. Here it is harder to make out the impact of the enrollment effect because the overall wage distributions also differ since enrollment (Figure 16(b)) is much higher in the Utilitarian optimum, where college subsidies are much higher.

A.2.3 SCPE with Education Dependent Taxes

Figure 17(a) compares optimal education dependent marginal tax rates with the education dependent marginal tax rates in the SCPE. In the SCPE, a pure tagging logic applies and the government sets higher marginal tax rates for college graduates because the hazard rates differs. In the Utilitarian optimum, this gap gets smaller because the endogeneity of enrollment is a force towards higher high school taxes and lower college taxes.
Figure 17: SCPE with Education Dependent Taxes
References


CUNHA, F., F. KARAHAN, AND I. SOARES (2011): “Returns to skills and the college premium,” Journal of Money, Credit and Banking, 43(s1), 39–86.


