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Loss Aversion and Inefficient Renegotiation†

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Abstract: We propose a theory of inefficient renegotiation that is based on loss aversion. When two parties write a long-term contract that has to be renegotiated after the realization of the state of the world, they take the initial contract as a reference point to which they compare gains and losses of the renegotiated transaction. We show that loss aversion makes the renegotiated outcome sticky and materially inefficient. The theory has important implications for the optimal design of long-term contracts. First, it explains why parties often abstain from writing a beneficial long-term contract or why some contracts specify transactions that are never ex post efficient. Second, it shows under what conditions parties should rely on the allocation of ownership rights to protect relationship-specific investments rather than writing a specific performance contract. Third, it shows that employment contracts can be strictly optimal even if parties are free to renegotiate.

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1 Introduction

Renegotiation plays a crucial role in the theory of incomplete contracts. This theory, going back to Grossman and Hart (1986) and Hart and Moore (1990), starts out from the observation that long-term contracts have to be written before the contracting parties know the realization of the state of the world that is relevant for the specifics of their trading relationship. Writing a complete, state-contingent contract is often impossible, so the parties have to rely on renegotiation to adapt the contract to the realization of the state of the world. The standard paradigm assumes that renegotiation is always efficient. Once the parties observe the state of the world they will engage in Coasian bargaining and reach an efficient agreement on how to adapt the contract.

More recently, Hart and Moore (2008) and Hart (2009) have put this approach into question. They argue that the traditional approach is ill suited to studying the internal organization of firms. If renegotiation is always efficient “it is hard to see why authority, hierarchy, delegation, or indeed anything apart from asset ownership matters” (Hart and Moore, 2008, p. 3). Coase (1937) and Williamson (1985) argued long ago that the organization of transactions within firms and by markets can be understood only if we understand the inefficiencies of adapting contracts to changes of their environment, i.e., the inefficiencies of renegotiation.

In this paper we propose a new theory of inefficient renegotiation that is based on loss aversion, a fundamental concept in behavioral economics and psychology (Kahneman and Tversky, 1979; Tversky and Kahneman, 1991). There is ample experimental and field evidence showing that people evaluate outcomes not (only) in absolute terms but (also) relative to a reference point, and that losses (in comparison to this reference point) loom larger than gains of equal size. Already Tversky and Kahneman (1991, p. 1057) conjectured that “contracts define the reference levels for [...] bargaining; in the bargaining context the aversion to losses takes the form of an aversion to concessions”. Following this idea we assume that the contract to which the parties agreed ex ante defines the reference point in the renegotiation game.

The initial contract determines the parties’ payoffs when renegotiation breaks down. Suppose a buyer and seller agreed ex ante to trade some specification \(\bar{x}\) of a good at price \(\bar{p}\). After the realization of the state of the world they realize that it would be efficient to adjust the
specification of the good. However, the buyer feels a loss if the renegotiated price $p$ is greater than the initially agreed payment $\bar{p}$. Similarly, the seller feels a loss if her cost to produce the new specification $x$ is larger than her cost to produce the initially agreed specification $\bar{x}$. These losses loom larger than equally sized gains of consuming a better quality for the buyer and receiving a larger payment for the seller. A crucial feature of our model is that monetary losses due to a difference between the renegotiated price $p$ and the price $\bar{p}$ are evaluated separately from losses due to a lower valuation or a higher cost of $x$ as compared to $\bar{x}$. This decomposability assumption is common in the literature on reference points (Tversky and Kahneman, 1991; Kőszegi and Rabin, 2006).

First, we show that the conjecture of Tversky and Kahneman is correct. Due to decomposability loss aversion drives a wedge between the benefit of the buyer and the cost of the seller. This renders the renegotiation outcome materially inefficient, i.e., it does not maximize the material surplus (net of loss aversion) of the two parties. Furthermore, the kink in the utility function at the reference point may prevent renegotiation altogether. We show that if the realization of the state of the world is not too far from the “expected” state of the world on which the initial contract $(\bar{x}, \bar{p})$ was based, then the parties will not renegotiate and leave the initial contract in place. If the realized state of the world is sufficiently far away from the expected state, the parties will renegotiate. The terms of trade, however, are insufficiently adjusted. Thus, loss aversion makes the renegotiation outcome sticky and materially inefficient.\footnote{This effect is reminiscent of the assumption of “sticky prices” in macroeconomics. While the macroeconomic literature attributes price stickiness to exogenously given menu costs, sticky prices can arise endogenously in our model. That “sticky prices” can be explained by loss aversion is also shown for models with price setting firms by Heidhues and Kőszegi (2005, 2008).}

The friction due to loss aversion is quite different from other bargaining frictions, such as asymmetric information, the risk of bargaining breakdown or other transaction costs. The difference is that loss aversion arises because of the initial contract. The initial contract sets the reference point that causes the feelings of losses if the contract is renegotiated. In contrast, if the parties are asymmetrically informed about the realization of the state of the world, this information asymmetry will be there no matter whether there is an ex ante contract or not. If anything, the initial contract can be used to mitigate the informational problem by setting up a sophisticated mechanism that induces the parties to reveal their private information.
truthfully. Thus, with asymmetric information the initial contract can only reduce the cost of contracting, but it can never be harmful, while with loss aversion there is a cost of writing the initial contract that arises endogenously.\footnote{The same argument applies to the risk of bargaining breakdown and other transaction costs. If anything, the risk of bargaining breakdown or the transaction costs of renegotiation are reduced by the initial contract.}

Our theory of renegotiation has several interesting and important implications for contract theory. If the parties understand that a contract sets a reference point that triggers potentially unfavorable comparisons and that gives rise to disutility from loss aversion and to materially inefficient renegotiation outcomes, then they have an incentive to design contracts so as to minimize these frictions. A first implication of our model is that it may be optimal not to write a long-term contract ex ante but to rely on spot contracting ex post. If the parties write a long-term contract, this contract sets the reference point and it is costly to renegotiate away from it. If the parties do not write a long-term contract but negotiate the terms of trade after the realization of the state of the world, the parties may also have a reference point which we take to be their outside options.\footnote{Note that this is analogous to the case of contract renegotiation: if renegotiation fails the initial contract is executed, so the reference point is given by the outside options of the renegotiation game.}

The more competitive the spot market is, the closer are the outside options of the two trading parties to what they can achieve by trading with each other, and thus the lower are the potential losses. Hence, spot contracting outperforms a long-term contract if the spot market is highly competitive, while writing a long-term contract is likely to be optimal if there is little competition ex post. Furthermore, if the parties do write a long-term contract, it can be optimal to contract on a specification of the good that is never materially efficient ex post, but that minimizes the cost to renegotiate away from it.

Second, the theory offers a fresh view on the hold-up problem and the property rights theory. It shows under what circumstances the parties should rely on the allocation of asset ownership to protect their relationship-specific investments, and when they should rather write a long-term specific performance contract. A long-term specific performance contract outperforms the allocation of ownership rights to protect relationship-specific investments if there is little uncertainty, if the degree of asset specificity is high, and if the party that has to make a relationship specific investment is in a weak bargaining position.

Third, our theory offers a rationale for the existence of “employment” contracts. According
to Coase (1937) and Simon (1951) a key feature of an employment contract is that it fixes the price (the wage) and gives the buyer (the employer) authority to order the seller (the employee) which specification of the good (the service) to deliver. Simon (1951) compares an employment contract to a specific performance contract that fixes the specific service to be delivered. The advantage of the employment contract is that it is flexible. The disadvantage is that it leaves room for abuse because the employer can order the employee to deliver a service that benefits the employer but that is inefficiently costly to the employee. Which type of contract is optimal depends on whether the expected cost of rigidity or of abuse is more important. However, there are two problems with Simon’s argument. First, he ignores the possibility of renegotiation. If costless renegotiation is possible the parties will always reach the efficient outcome and the difference between the two contracts disappears. Second, Simon ignores the fact that an employment contract is an “at-will” contract: the employee can leave if he feels abused. We deal with both of these issues and show that the cost of renegotiation differs between a specific performance and an employment contract thereby confirming and extending Simon’s original insight. A loss averse employee will not leave the relationship if there is some (but not too much) abuse, so the employer has some discretion to exercise power over the employee. For high degrees of loss aversion (that preclude renegotiation), we confirm that an employment contract strictly outperforms a specific performance contract if the scope for inefficient abuse is small as compared to the cost of rigidity. In addition, we show that the employment contract is always optimal if the degree of loss aversion is small. In this case the buyer cannot exploit the seller because the seller would quit. For intermediate degrees of loss aversion the employment contract outperforms the specific performance contract if makes renegotiations less costly.

There is some recent experimental evidence that is consistent with our theory. Bartling and Schmidt (2012) conduct a laboratory experiment on (re)negotiation. They compare a situation in which a buyer and a seller renegotiate an initial contract to a situation in which they negotiate in the absence of an initial contract. In all other respects the two situations are completely identical. They find that with an initial contract prices are sticky and react much less to the realization of the state of the world as in the situation without an initial contract. This is exactly what our theory predicts for this experiment. Moreover, the experiment shows that the existence of the initial contract is causal for the stickiness of prices because the
material and strategic situation is exactly the same in both treatments.

Our paper is closely related and complementary to Hart and Moore (2008) who were the first to point out that contracts may serve as reference points. They assume that a contract determines parties’ feelings of entitlement if the contract was written under competitive conditions. The parties do not feel entitled to outcomes that are outside the contract, but each party feels entitled to the best possible outcome that is consistent with the contract. Thus, when interpreting the contract parties have mutually inconsistent expectations with a self-serving bias. When a party does not get what it feels entitled to, it feels aggrieved and shades in non-contractible ways. Shading reduces the payoff of the other party, but is costless for the shader, i.e., it is a form of costless punishment. Hart and Moore (2008) compare a rigid contract to a flexible contract. The benefit of flexibility is that the contract can be better adjusted to the realization of the state of the world, but the cost is that it leads to aggrievement and shading. This tradeoff gives rise to an optimal degree of flexibility. Hart (2009), Hart and Holmstrom (2010) and Hart (2013) use this approach to develop theories of asset ownership and firm boundaries. Contract renegotiation – which is at the heart of our paper – is beyond the scope of the aforementioned papers. The Hart-Moore approach is extended to allow for renegotiation by Halonen-Akatwijuka and Hart (2013), who show that it may be optimal to leave a contract deliberately incomplete.\(^4\) There are several important differences between the Hart-Moore approach and our approach. First, in Hart and Moore the ex post inefficiency is due to self-serving biases and aggrievement, while our approach is based on loss aversion, a well established and widely documented behavioral phenomenon. Second, Hart and Moore require a second stage of “shading” at which parties can punish each other free of cost. This is not necessary for our approach. Finally, in Halonen-Akatwijuka and Hart (2013) there is no material inefficiency in renegotiation (the only inefficiency is “shading”), while our model generates materially inefficient renegotiation outcomes (in addition to the feelings of losses).

The rest of the paper is organized as follows. The next section sets up the model. In Section 3 we take the initial contract as given and characterize the renegotiation outcome.

\(^4\)Fehr, Zehnder, and Hart (2009); Fehr, Hart, and Zehnder (2011a,b) run several experiments on the Hart-Moore model. They find support for the hypothesis that people shade more when the contract is more flexible if the contract was written under competitive conditions, but not if one party had monopoly power and could dictate the terms of the contract. Hoppe and Schmitz (2011) experimentally investigate the hold-up problem and also find some support for the Hart-Moore approach.
after the state of the world has materialized. In Section 4 we look at the implications for ex ante contracts. First, we show that it can be optimal not to write a long-term contract at all. Second, we consider a hold-up problem and show under what conditions the parties should rely on the allocation of ownership rights rather than on a specific performance contract to protect relationship specific investments. Third, we reconsider Simon’s problem of when to use an employment contract. Finally, we discuss the potential benefits of contract indexation. All proofs that are not outlined in the main text are relegated to the Appendix.

2 The Model

We consider two risk-neutral parties, a buyer B (he) and a seller S (she), who are engaged in a long-term relationship. The two parties can write a contract at date 0 that governs trade at date 3. The seller can deliver different specifications of a good $x \in X$, where $X$ is a compact space that can differ in multiple dimensions (quantity, quality, time and location of delivery, etc.). The buyer’s valuation $v = v(x, \theta)$ and the seller’s cost $c = c(x, \theta)$ depend on the specification $x$ of the good and on the realization of the state of world $\theta \in \Theta$. The exact shapes of the cost and valuation functions become commonly known at date 1, when the state of the world $\theta \in \Theta$ is realized. The state $\theta$ reflects exogenous uncertainty that is relevant for the optimal specification of the good to be traded. We assume that there is a unique materially efficient specification $x^*(\theta) \in X$ for each possible state of the world,

$$x^*(\theta) = \arg \max_{x \in X} \{v(x, \theta) - c(x, \theta)\}$$

(1)

that maximizes the material gains from trade.

At date 0, i.e. at the contracting stage, the two parties do not know the realization of the state of the world $\theta$, which is drawn from a compact space $\Theta$ according to a commonly known cumulative distribution distribution function $F(\theta)$. At date 1, i.e. before trade takes place, the state of the world is realized and observed by both parties. We assume that the realized state cannot be verified by a court or another third party. A court can verify only payments and which if any of the goods $x \in X$ is delivered. Thus, in this setting a contract cannot specify state contingent specifications and prices. However, the parties are free to renegotiate the terms of the contract after observing the state of nature. In this section we focus on
“specific performance contracts” \((\bar{x}, \bar{p})\) specifying one good to be delivered at a fixed price that can be enforced by each party. Other—more complex—forms of initial contracts are analyzed in Section 4, where we also discuss authority contracts, at-will contracts, and contracts on the allocation of ownership rights. The sequence of events is as follows:

\(t = 0\) Initial Contracting: The buyer and the seller negotiate the initial contract \((\bar{x}, \bar{p})\).

\(t = 1\) Realization of the State of the World: Nature draws \(\theta\) which is observed by \(B\) and \(S\). The contract in combination with the realized state determines the default options for both parties, \(U^B = v(\bar{x}, \theta) - \bar{p}\) and \(U^S = \bar{p} - c(\bar{x}, \theta)\).

\(t = 2\) Renegotiation: The buyer and the seller can renegotiate the initial contract to a new contract \((\hat{x}, \hat{p})\) that must be feasible and individually rational for both parties. If the parties do not agree upon a new contract, the initial contract \((\bar{x}, \bar{p})\) remains in place.

\(t = 3\) Trade: Trade is carried out according to the (renegotiated) contract.

\[
\begin{array}{cccc}
0 & 1 & 2 & 3 \\
\text{initial contract} & \text{state of the} & \text{renegotiation} & \text{trade} \\
(\bar{x}, \bar{p}) & \text{world } \theta & (\hat{x}, \hat{p}) & v(\hat{x}, \theta) - \hat{p} \\
\text{is realized} & & & \hat{p} - c(\hat{x}, \theta)
\end{array}
\]

Figure 1: Time structure

So far our model of renegotiation is completely standard. We now depart from the existing literature by assuming that the initial contract creates a reference point that determines how the parties evaluate the new contract. The parties compare the new contract \((\hat{x}, \hat{p})\) to what they would have received under the old contract in the realized state \(\theta\). This evaluation is distorted by loss aversion: The buyer feels a loss if the renegotiated price \(\hat{p}\) is greater than the initially agreed price \(\bar{p}\). Furthermore, he also feels a loss if his valuation for the renegotiated good \(\hat{x}\) is smaller than his valuation for the good \(\bar{x}\) given the realized state of nature. Similarly, the seller feels a loss if the renegotiated price \(\hat{p}\) falls short of the initially agreed price \(\bar{p}\) and if her cost for the renegotiated good \(\hat{x}\) is greater than her cost for the good \(\bar{x}\) in the realized
state $\theta$. Put differently, we posit that the default option—determined by the initial contract and the realized state of nature—shapes a reference point for the two parties.

The utility functions of the two parties at the renegotiation stage are given by

$$U^B(\hat{x}, \hat{p}|\theta) = v(\hat{x}, \theta) - \hat{p} - \lambda [\hat{p} - \bar{p}]^+ - \lambda [v(\hat{x}, \theta) - v(\hat{x}, \theta)]^+$$ (2)

$$U^S(\hat{x}, \hat{p}|\theta) = \hat{p} - c(\hat{x}, \theta) - \lambda [\bar{p} - \hat{p}]^+ - \lambda [c(\hat{x}, \theta) - c(\hat{x}, \theta)]^+$$ (3)

with $\lambda > 0$ and $[z]^+ \equiv \max\{z, 0\}$.

This specification follows Köszegi and Rabin (2006, 2007) in assuming that a party’s utility function has two additively separable components: standard outcome based utility and gain-loss utility. Furthermore, we assume that the gain-loss function satisfies decomposability as defined by Tversky and Kahneman (1991). Decomposability implies that a monetary loss due to a difference between $\hat{p}$ and $\bar{p}$ is evaluated separately from a loss due to a lower valuation or a higher cost. This assumption is now common in the literature on loss aversion and necessary for loss aversion to accommodate many well-known deviations from standard theory like the endowment effect (Thaler, 1980; Kahneman, Knetsch, and Thaler, 1990) or the status-quo bias (Samuelson and Zeckhauser, 1988). Moreover, we assume that the degree of loss aversion is the same across dimensions and across parties, i.e., we assume a universal gain-loss function (Köszegi and Rabin, 2006) and no buyer and seller specific values of $\lambda$. This assumption is merely imposed in order to reduce the number of parameters and has no qualitative impacts on our findings.\(^6\),\(^7\)

\(^5\)For simplicity we assume that losses in the two dimensions are additive and that the parameter of loss aversion, $\lambda$, is constant. How constant additive loss aversion can accommodate for many observed deviations from standard theory is explained by Tversky and Kahneman (1991). For more recent applications of constant additive loss aversion see e.g. Crawford and Meng (2011) and Ericson and Fuster (2011).

\(^6\)Most of the evidence regarding the size of $\lambda$ comes from experimental findings about the willingness to accept (WTA) and willingness to pay (WTP) ratio. The WTA is the amount a subject who received an item (typically a coffee mug) demands so that she is willing to sell it. The WTP is the amount a subject who has not received an item is willing to pay for it. Reviewing 45 studies on WTA-WTP ratios with a remarkable range of goods, Horowitz and McConnell (2002) report that the median (mean) ratio of average WTA and average WTP is 2.6 (7), which corresponds to $\lambda = 0.61$ ($\lambda = 1.6$) in our model. The classic investigation of the endowment effect by Kahneman, Knetsch, and Thaler (1990), compares the WTA of sellers with the amount of money that makes the so-called “choosers” indifferent between obtaining either the item or the money. The advantage of the classic approach is that “choosers” and sellers face precisely the same decision problem, whereas WTA-WTP ratios (slightly) above one can also be explained by income effects. Kahneman, Knetsch, and Thaler (1990) report estimates corresponding to $\lambda \approx 1.28$ in one experiment, while they reported estimates corresponding to $\lambda \approx 1.0$ for another experiment.

\(^7\)Loss averse behavior need not arise from the behavioral anomaly of loss aversion but may also be caused
3 Renegotiation

In this section we take the initial contract \((\bar{x}, \bar{p})\) as exogenously given and analyze the renegotiation game at date 2. We first characterize the renegotiation set, i.e., the set of specifications \(\hat{x}\) that are feasible and individually rational given the initial contract \((\bar{x}, \bar{p})\). Then, we impose some structure on how the parties renegotiate the initial contract and characterize the renegotiation outcome. We will show that the renegotiation outcome is sticky and materially inefficient: Parties often fail to renegotiate even if a materially more efficient contract is available, and if they do renegotiate they adjust the contract too little to the realization of the state of the world and do not agree ex post on trading the materially efficient \(x^*(\theta)\) that maximizes \(v(\cdot) - c(\cdot)\). Finally, we characterize the cost and the likelihood of renegotiation.

3.1 The Renegotiation Set

Suppose that an initial contract \((\bar{x}, \bar{p})\) is in place and that the state of the world \(\theta\) has materialized. Thus, if the initial contract is not renegotiated the parties will trade \(\bar{x}\) at price \(\bar{p}\) which yields the outside option utilities \(U^B = v(\bar{x}, \theta) - \bar{p}\) and \(U^S = \bar{p} - c(\bar{x}, \theta)\).

The buyer prefers a new contract \((\hat{x}, \hat{p})\) to the initial contract if and only if his utility under the new contract is greater than his utility from the initial contract. This is the case if and only if

\[
v(\hat{x}, \theta) - \hat{p} - \lambda[v(\bar{x}, \theta) - v(\hat{x}, \theta)]^+ - \lambda[\hat{p} - \bar{p}]^+ \geq v(\bar{x}, \theta) - \bar{p}
\]

\[
\iff v(\hat{x}, \theta) - v(\bar{x}, \theta) - \lambda[v(\bar{x}, \theta) - v(\hat{x}, \theta)]^+ \geq \hat{p} - \bar{p} + \lambda[\hat{p} - \bar{p}]^+.
\] \hfill (4)

The seller prefers the new contract \((\hat{x}, \hat{p})\) to the original contract if and only if

\[
\hat{p} - c(\hat{x}, \theta) - \lambda[c(\hat{x}, \theta) - c(\bar{x}, \theta)]^+ - \lambda[\hat{p} - \bar{p}]^+ \geq \bar{p} - c(\bar{x}, \theta)
\]

\[
\iff c(\hat{x}, \theta) - c(\bar{x}, \theta) + \lambda[c(\hat{x}, \theta) - c(\bar{x}, \theta)]^+ \leq \hat{p} - \bar{p} - \lambda[\hat{p} - \bar{p}]^+.
\] \hfill (5)

by organizational constraints. For example, consider a company A renegotiating a contract with some other company B. The renegotiation proposal affects two divisions of company A, say production and marketing. The division that has to bear the cost of the contract adjustment (production) may oppose it more strongly than it is supported by the division that benefits from it (marketing). The CEO of company A has to push through the renegotiation proposal in both divisions. Thus, even if the CEO himself is not loss averse, he may behave as if he was affected by loss aversion.
Contracts \((\hat{x}, \hat{p})\) satisfying (4) and (5) are called individually rational. The renegotiation set is the set of goods \(\hat{x}\) to which the parties could voluntarily renegotiate to, i.e., the set of \(\hat{x} \in X\) for which there exists a price \(\hat{p}\) such that \((\hat{x}, \hat{p})\) is individually rational for both parties.

We have to distinguish whether \(\hat{x}\) leads to higher or lower benefits for the buyer and higher or lower costs for the seller as compared to \(\bar{x}\). Obviously, if \(\hat{x}\) leads to higher costs and lower benefits than \(\bar{x}\), then there does not exist any price \(\hat{p}\) such that \((\hat{x}, \hat{p})\) is preferred by both parties to \((\bar{x}, \bar{p})\). This leaves us with three cases, covered in the following proposition.

**Proposition 1.** Consider an initial contract \((\bar{x}, \bar{p})\) and suppose that state \(\theta \in \Theta\) is realized. The renegotiation set, i.e., the set of all \(\hat{x} \in X\) to which the parties may voluntarily renegotiate to, is characterized as follows:

(i) If \(\hat{x} \in X\) yields (weakly) higher benefits for the buyer and (weakly) lower costs for the seller as compared to \(\bar{x}\), then it can always be reached by renegotiation.

(ii) If \(\hat{x} \in X\) yields higher benefits for the buyer but is more costly to produce for the seller as compared to \(\bar{x}\), then it can be reached by renegotiation if and only if

\[
v(\hat{x}, \theta) - v(\bar{x}, \theta) \geq (1 + \lambda)^2[c(\hat{x}, \theta) - c(\bar{x}, \theta)]. \tag{6}
\]

(iii) If \(\hat{x} \in X\) is less costly to produce for the seller but also less beneficial to the buyer as compared to \(\bar{x}\), then it can be reached by renegotiation if and only if

\[
c(\bar{x}, \theta) - c(\hat{x}, \theta) \geq (1 + \lambda)^2[v(\bar{x}, \theta) - v(\hat{x}, \theta)]. \tag{7}
\]

The intuition for this result is straightforward. Clearly, a good \(\hat{x}\) that is preferred to \(\bar{x}\) by both parties can always be reached by renegotiation by leaving the price unchanged. The interesting cases arise when there is a tradeoff, i.e., either the buyer or the seller suffers if the new good is implemented and the price is not adjusted. For instance, suppose that \(\hat{x}\) benefits the buyer but is more costly to produce for the seller. In order to compensate the seller, the buyer has to increase the price by at least \(\Delta_p = (1 + \lambda)[c(\hat{x}, \theta) - c(\bar{x}, \theta)]\). The buyer is willing to offer this price increase only if his valuation increases by at least \((1 + \lambda)\Delta_p\), i.e., if \(v(\hat{x}, \theta) - v(\bar{x}, \theta) \geq (1 + \lambda)\Delta_p\).\(^8\) Note that the renegotiation set becomes smaller as \(\lambda\)

\(^8\)If the two parties differ in their degree of loss aversion, so that party \(i\)'s degree is \(\lambda_i\) with \(i \in \{B, S\}\), in Proposition 1 the term \((1 + \lambda)^2\) in (6) and (7) needs to be replaced by the term \((1 + \lambda_S)(1 + \lambda_B)\).
increases. Note also that whether implementing $\hat{x}$ is individually rational for both parties ex post is independent of the initial price $p$. This is due to the assumed quasi-linear structure of preferences in combination with linear loss aversion. The renegotiation set for $\lambda = 0$ and $\lambda > 0$ is depicted in Figure 2: If the parties are not loss averse, all goods that are north-east of the straight line can be reached by renegotiation. If the parties are loss averse, the renegotiation set shrinks to the goods that are located north-east of the dashed lines.

### 3.2 The Renegotiation Outcome

So far we characterized the set of renegotiation outcomes that are feasible and individually rational. In order to characterize the renegotiation outcome that will actually obtain we have to be more specific about the bargaining game played at the renegotiation stage. In the following we employ the Generalized Nash Bargaining Solution (GNBS). The GNBS is the only bargaining solution that is Pareto efficient, invariant to equivalent utility representations and independent of irrelevant alternatives. Furthermore, it reflects the relative bargaining power of the two parties.\(^9\) The GNBS is the contract $(\hat{x}(\theta), \hat{p}(\theta))$ that maximizes the Generalized

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\(^9\)See Roth (1979) for a discussion of the Generalized Nash Bargaining Solution and other axiomatic models of bargaining. Binmore, Rubinstein, and Wolinsky (1986) derive the GNBS as a non-cooperative equilibrium of an alternating offer game between one seller and one buyer.
Nash Product (GNP), i.e.,

\[
(\hat{x}(\theta), \hat{p}(\theta)) \equiv \arg \max_{x,p} \left\{ (U^B(x,p|\theta) - U^B) - (U^S(x,p|\theta) - U^S)  \right\},
\]

where \(U^B\) and \(U^S\) are the outside option utilities of the buyer and the seller, respectively—i.e., the utilities they achieve if no agreement is reached and the initial contract is carried out.\(^{10}\)

The share of the surplus going to the buyer increases with \(\alpha\), a parameter that is commonly interpreted as a measure of the buyer’s relative bargaining skill/power.\(^{11}\)

Because of the very general structure of \(X\) which may be a discrete or multi-dimensional space, it is not possible to characterize \(\hat{x}(\theta)\) without imposing additional structure on the renegotiation problem. We will do this in the next subsections. However, for a given renegotiated \(\hat{x}(\theta)\) we can characterize the renegotiated price \(\hat{p}(\theta)\) in general.

**Proposition 2.** Let \(\Delta_v := [v(\hat{x}, \theta) - v(\bar{x}, \theta)]\) and \(\Delta_c := [c(\hat{x}, \theta) - c(\bar{x}, \theta)]\). The Generalized Nash Bargaining Solution implies that for a given \(\hat{x}(\theta)\) the renegotiated price \(\hat{p}(\theta)\) is given by

\[
\hat{p}(\theta) = \begin{cases} 
\bar{p} + (1 - \alpha) \frac{1+\lambda_1}{1+\lambda_2} \Delta_v + \alpha (1 + \lambda_2) \Delta_c & \text{if } (1 - \alpha) \frac{1+\lambda_1}{1+\lambda_2} \Delta_v + \alpha (1 + \lambda_2) \Delta_c \geq 0 \\
\bar{p} & \text{otherwise}
\end{cases}
\]

with

\[
\lambda_1 = \begin{cases} 
\lambda & \text{if } v(\bar{x}, \theta) - v(\hat{x}, \theta) > 0 \\
0 & \text{otherwise}
\end{cases}, \quad \lambda_2 = \begin{cases} 
\lambda & \text{if } c(\bar{x}, \theta) - c(\hat{x}, \theta) > 0 \\
0 & \text{otherwise}
\end{cases}
\]

To see the intuition for Proposition 2 note that for any given \(\hat{x}\) the Pareto frontier is linear with a kink at \((U_B(\hat{x}, \bar{p}), U_S(\hat{x}, \bar{p}))\). Hence, it is possible to transfer utility from one player to the other, but – due to loss aversion – not one to one and at different rates in different directions. Because of this kink the parties will not adjust the price if the absolute values of \(\Delta_v\) and \(\Delta_c\) are small and if both parties have some bargaining power. Consider now a case

\(^{10}\)In general, the GNBS need not be unique. In Subsection 3.3 we impose additional assumptions that guarantee uniqueness of the GNBS.

\(^{11}\)By assuming that renegotiation leads to the GNBS we take a reduced form approach that does not model the bargaining game explicitly. This approach assumes that the reference point of each party is fixed and unaffected by the offers and counteroffers made in the negotiation game. Even if this was not the case and if the parties incurred losses when updating the reference point, the accumulated losses until an agreement is reached should be similar to the losses the parties incur when implementing the GNBS directly. However, modeling the adjustment of the reference point in different bargaining games is beyond the scope of this paper.
where the price is adjusted. For concreteness suppose that \( \hat{x} \) is such that the buyer’s valuation and the seller’s cost go up as compared to \( \bar{x} \), so \( \Delta_\nu > 0 \) and \( \Delta_\epsilon > 0 \) which implies \( \lambda_1 = 0 \) and \( \lambda_2 = \lambda \). In this case the price must go up to compensate the seller for her higher cost. If the buyer has all the bargaining power \((\alpha = 1)\), the price increases by \((1 + \lambda)[v(\hat{x}, \theta) - c(\bar{x}, \theta)]\), just enough to compensate the seller for her increase in cost and her feeling of a loss because of this cost increase. If the seller has all the bargaining power \((\alpha = 0)\), the price increases by \( \frac{v(\hat{x}, \theta) - v(\bar{x}, \theta)}{1 + \lambda} \), so the price increase multiplied by \((1 + \lambda)\) just equals the increase of the buyer’s valuation because the buyer feels a loss due to the price increase.

It is interesting to note that the price adjustment \( \Delta_p := \hat{p} - \bar{p} \) is independent of the initially specified price \( \bar{p} \). The price \( \bar{p} \) defines the wealth position of the buyer and the seller from which renegotiation starts. Because the utility functions are quasi-linear there are no income effects and the price \( \bar{p} \) has no impact on the price adjustment. A second interesting observation is that the price adjustment \( \Delta_p := \hat{p} - \bar{p} \) is often decreasing in \( \lambda \). For example, if renegotiation takes place and both parties have the same bargaining power \((\alpha = 0.5)\), an increase in loss aversion reduces the price adjustment and makes prices more sticky.\(^{12}\)

### 3.3 The Stickiness of the Initial Contract

In this subsection we assume that the specification of good \( x \) is one-dimensional and can be changed continuously, i.e. \( X \equiv \mathbb{R}_0^+ \) and that the state of the world is drawn from a one-dimensional continuous space \( \Theta \subset \mathbb{R} \).

**Assumption 1.** For any state \( \theta \in \Theta \subset \mathbb{R} \) and any quantity \( x \in X \equiv \mathbb{R}_0^+ \) the buyer’s valuation and the seller’s cost function are twice continuously differentiable and satisfy the following (Inada) conditions: \( \forall x > 0 \)

\[
\begin{align*}
(a) \quad & v(0, \theta) = 0, \quad \frac{\partial v(x, \theta)}{\partial x} > 0, \quad \frac{\partial^2 v(x, \theta)}{\partial x^2} < 0, \quad \frac{\partial^2 v(x, \theta)}{\partial x \partial \theta} > 0,
\end{align*}
\]

\(^{12}\)Proposition 2 is consistent with the experimental evidence in Bartling and Schmidt (2012). They conduct a (re-)negotiation experiment in which the seller can make a take-it-or-leave-it renegotiation offer, so \( \alpha = 0 \), and the buyer always benefits from renegotiation, i.e., \( \Delta_\nu > 0 \). In this case Proposition 2 implies sticky prices. Bartling and Schmidt find that sellers often deliver the ex post efficient good without charging any markup if \( x^*(\theta) \) is less costly to produce than \( \hat{x} \). Moreover, they find that if the seller demands a higher price, which almost always happens if \( x^*(\theta) \) is more costly to produce, then the demanded markup is lower with an initial contract than in an equivalent situation without an initial contract. Note that we do not generally predict sticky prices, i.e. the price change may also be larger with loss aversion than without.
(b) \( c(0, \theta) = 0, \frac{\partial c(x, \theta)}{\partial x} > 0, \frac{\partial^2 c(x, \theta)}{\partial x^2} \geq 0, \frac{\partial^2 c(x, \theta)}{\partial x \partial \theta} \leq 0, \)

(c) \( \lim_{x \to 0} \frac{\partial v(x, \theta)}{\partial x} > \lim_{x \to 0} \frac{\partial c(x, \theta)}{\partial x} = 0, \lim_{x \to \infty} \frac{\partial v(x, \theta)}{\partial x} < \lim_{x \to \infty} \frac{\partial c(x, \theta)}{\partial x}. \)

Assumption 1 guarantees that there exists a unique materially efficient quantity \( x^*(\theta) > 0 \) that is fully characterized by the first-order condition. Furthermore, it implies that an increase in \( \theta \) increases marginal benefits and reduces marginal costs. Thus, the higher the state, the higher is the materially efficient quantity, i.e., \( x^*(\theta) \) is increasing in \( \theta \).

Suppose the parties start out from an initial contract \((\bar{x}, \bar{p})\), which implements the materially efficient good in state \( \bar{\theta} \), i.e. \( \bar{x} = x^*(\bar{\theta}) \). We have to distinguish two cases, i.e., whether the realized state is larger or smaller than \( \bar{\theta} \). In the former case the parties want to (weakly) increase \( x \), while in the latter case the parties want to (weakly) decrease \( x \). The following proposition fully characterizes the renegotiation outcome for both cases.

**Proposition 3.** Suppose that Assumption 1 holds. Consider any initial contract \((\bar{x}, \bar{p})\) with \( \bar{x} > 0 \) and any realized state of the world \( \theta \in \Theta \). The GNBS implies that the parties will renegotiate to

\[
(\hat{x}(\theta), \hat{p}(\theta)) = \begin{cases} 
(\hat{x}^L(\theta), \hat{p}^L(\theta)) & \text{if } \theta < \theta^L \\
(\bar{x}, \bar{p}) & \text{if } \theta^L \leq \theta \leq \theta^H \\
(\hat{x}^H(\theta), \hat{p}^H(\theta)) & \text{if } \theta^H < \theta
\end{cases}
\]  

(10)

where \( \hat{x}^i \) and \( \hat{p}^i \), \( i \in \{L, H\} \) are given by:

\[
\frac{\partial v(\hat{x}^L(\theta), \theta)}{\partial x} = \frac{1}{(1 + \lambda)^2} \frac{\partial c(\hat{x}^L(\theta), \theta)}{\partial x} 
\]  

(11)

\[
\frac{\partial v(\hat{x}^H(\theta), \theta)}{\partial x} = (1 + \lambda)^2 \frac{\partial c(\hat{x}^H(\theta), \theta)}{\partial x} 
\]  

(12)

\[
\hat{p}^L(\theta) = \bar{p} + (1 - \alpha)(1 + \lambda) \left[ v(\hat{x}^L(\theta), \theta) - v(\bar{x}, \theta) \right] + \frac{\alpha}{1 + \lambda} \left[ c(\hat{x}^L(\theta), \theta) - c(\bar{x}, \theta) \right] 
\]  

(13)

\[
\hat{p}^H(\theta) = \bar{p} + \frac{1 - \alpha}{1 + \lambda} \left[ v(\hat{x}^H(\theta), \theta) - v(\bar{x}, \theta) \right] + \alpha(1 + \lambda) \left[ c(\hat{x}^H(\theta), \theta) - c(\bar{x}, \theta) \right] 
\]  

(14)

and \( \theta^L \) and \( \theta^H \) are the unique solutions to \( \hat{x}^L(\theta^L) = \bar{x} \) and \( \hat{x}^H(\theta^H) = \bar{x} \) if these solutions exist; otherwise, \( \theta_L \) and \( \theta_H \) coincide with \( \inf\{\Theta\} \) and \( \sup\{\Theta\} \), respectively.

Loss aversion causes a kink in the utility functions of the buyer and the seller at \( x = \bar{x} \) which leads to the existence of a range of states of the world \([\theta^L, \theta^H]\) around state \( \bar{\theta} \) in which the
parties prefer to stick to the initial contract, even though this is inefficient in the absence of loss aversion. This range depends on the initially specified good but not on the initially specified price. If a state materializes that is far enough away from $\bar{\theta}$, the parties will renegotiate, but the contract is sticky. The quantity change always falls short of the quantity change that would be necessary to achieve the materially efficient $x^*(\theta)$.

If the parties do renegotiate they choose $\hat{x}$ so as to push out the Pareto frontier as far as possible and then split the surplus by adjusting the price. Thus, as in the Coase theorem, the renegotiated $\hat{x}$ is independent of the relative bargaining power ($\alpha$) of the parties. However, in contrast to the Coase theorem transferring utility is costly because of loss aversion. As in Proposition 2 the relative bargaining power determines how the additional achievable surplus is split between the two parties by adjusting the price.

Figure 3 illustrates the renegotiation outcome for a simple example with $v(x, \theta) = \theta x$.

---

$1^{13}$If $x$ can be changed only in discrete steps or if costs and benefits are linear, the renegotiated quantity may coincide with the materially efficient quantity.
\(c(x, \theta) = \frac{1}{2} x^2\), and \(X = \Theta = [0, 10]\). In this example the ex post efficient quantity is \(x^*(\theta) = \theta\). The initial contract has \(\bar{x} = 1\) which implies \(\bar{\theta} = 1\). The dashed lines in Figure 3 show the renegotiated quantities \(\hat{x}(\theta)\) for \(\lambda = 1\) and \(\lambda = 0.1\). Many experimental studies found that losses are valued about twice as much as equally sized gains, which corresponds to \(\lambda = 1\).\(^{14}\) If \(\lambda = 1\) (short-dashed line), there is very little renegotiation. Only in extreme states of the world (\(\theta < 0.25\) and \(\theta > 4\)) do the parties renegotiate. On the other hand, the experimental evidence also suggests that experienced “traders” (i.e., people who frequently trade goods not to own them but in order to make money) are much less attached to the goods they trade and suffer much less from loss aversion.\(^{15}\) But even if \(\lambda = 0.1\) (long-dashed line) there is a significant effect. There is no renegotiation for \(\theta \in [0.87, 1.21]\). If there is renegotiation the renegotiated quantity is sticky and does not fully adjust to \(x^*(\theta)\). In this example the relative distortion, 

\[
\frac{\left| x^*(\theta) - \hat{x}(\theta) \right|}{x^*(\theta)},
\]

increases when \(\theta\) moves away from \(\bar{\theta}\) until it reaches \(\theta^H\) (\(\theta^L\), respectively). From there on the relative distortion is constant. Finally, if only one party (say the buyer) suffers from loss aversion (\(\lambda_B = 1\)) while the other party is a very experienced trader (\(\lambda_S = 0\)) we get the dotted intermediate curve with no renegotiation for \(\theta \in [0.5, 2]\). Thus, it is sufficient if one party is loss averse to have an economically significant effect.

### 3.4 The Cost and Likelihood of Renegotiation

By employing the Generalized Nash Bargaining Solution we implicitly assume that the parties will always come to an ex post efficient agreement in utility terms. However, from an ex ante perspective there is a cost to writing a specific performance contract that is later renegotiated. Renegotiation may yield an outcome that is materialy inefficient, i.e., it does not maximize the material social surplus \(S(x, \theta) = v(x, \theta) - c(x, \theta)\). Furthermore it may give rise to feelings of losses. From an ex ante perspective both of this is inefficient.

\(^{14}\)E.g., Kahneman, Knetsch, and Thaler (1990) report estimates corresponding to \(\lambda\) slightly above 1. See also Footnote 6 for additional evidence on the size of \(\lambda\).

\(^{15}\)Evidence that market experience can eliminate the endowment effect caused by loss aversion is provided by List (2004, 2011). Horowitz and McConnell (2002), however, point out in their review of 45 endowment experiments, that the evidence that the endowment effect is reduced by subjects’ familiarity with the experiment is weak. One explanation that has been put forth in the literature in order to explain the different behavior of “traders” and “non-traders” is that traders expect to sell their items while non-traders expect to keep them. People who expect not to keep an item are less attached to that item and in turn suffer less from loss aversion when loosing it (Kószegi and Rabin, 2006).
The social surplus of a specific performance contract \((\bar{x}, \bar{p})\) that is renegotiated to \((\hat{x}(\theta), \hat{p}(\theta))\) is given by

\[
S(\theta \mid \lambda, \bar{x}, \bar{p}) = v(\hat{x}(\theta), \theta) - c(\hat{x}(\theta), \theta) - \lambda [v(\bar{x}, \theta) - v(\hat{x}(\theta), \theta)]^+ - \lambda [c(\hat{x}(\theta), \theta) - c(\bar{x}, \theta)]^+ - \lambda |\hat{p}(\theta) - \bar{p}|
\]  

We define the efficiency loss of a specific performance contract with renegotiation as the expected difference between the materially efficient social surplus, \(S^*(\theta) := \max_x \{v(x, \theta) - c(x, \theta)\}\), and the social surplus that the parties actually achieve through renegotiation:

\[
L(\lambda, \bar{x}, \bar{p}, \alpha) = E_{\theta} [S^*(\theta) - S(\theta \mid \lambda, \bar{x}, \bar{p})]
\]

**Proposition 4.** Suppose Assumption 1 holds. The efficiency loss of a specific performance contract with renegotiation \(L(\lambda, \bar{x}, \bar{p}, \alpha)\) is independent of \(\bar{p}\) and increasing in \(\lambda\). It is strictly increasing in \(\lambda\) at \(\lambda = 0\).

As we have seen before, the initial price \(\bar{p}\) does not affect the renegotiated good \(\hat{x}\) nor the price adjustment \(|\hat{p} - \bar{p}|\). Thus, the efficiency loss of a specific performance contract is also independent of \(\bar{p}\). An increase in the degree of loss aversion increases the efficiency loss because of two effects. First, keeping the good \(\hat{x}\) fixed, increasing \(\lambda\) increases the inefficiency because the disutility associated with a given loss increases. Second, the renegotiated good \(\hat{x}\) also depends on \(\lambda\). If \(\lambda\) increases, \(\hat{x}\) reacts less strongly to changes of \(\theta\) which reduces the material surplus \(v(\cdot) - c(\cdot)\) achieved ex post.

We now turn to the likelihood that a given contract \((\bar{x}, \bar{p})\) is in fact renegotiated. From Proposition 3 it seems intuitive that renegotiation is more likely if the environment is more uncertain. In a more uncertain environment, it turns out more often that the initially contracted specification \(\bar{x}\) is far from optimal and thus will not be delivered ex post, even though the parties are loss averse and dislike renegotiation. In order to formalize this intuition, assume that \(\theta\) is distributed according to some cumulative distribution function \(F(\theta)\). The initial contract will be renegotiated for \(\theta < \theta^L\) and \(\theta > \theta^H\), where \(\theta^L\) and \(\theta^H\) are characterized by Proposition 3. Note that \(\theta^L\) and \(\theta^H\) are independent of the cumulative distribution function \(F(\cdot)\). We denote the ex-ante probability of renegotiation by \(\rho(F) = F(\theta^L) + 1 - F(\theta^H)\) which depends on the distribution function and the initial contract. The following result shows that our conjecture is correct.
Corollary 1. Suppose Assumption 1 holds. If $F_1(\theta)$ crosses $F_2(\theta)$ once from below at $\hat{\theta} \in (\theta^L, \theta^H) \subset \Theta$, then the initial contract $(\bar{x}, \bar{p})$ is more likely to be renegotiated if $\theta$ is drawn from $F_2$ than from $F_1$, i.e., $\rho(F_1) < \rho(F_2)$.

The condition that $F_1(\theta)$ crosses $F_2(\theta)$ once from below at $\hat{\theta} \in (\theta^L, \theta^H)$ means that $F_2(\theta)$ has more “weight in the tails” than $F_1(\theta)$ and is “more risky” in this sense. If $F_1$ and $F_2$ have the same mean, $F_2$ is a mean preserving spread of $F_1$.

Another direct implication of Proposition 3 is that renegotiation becomes less likely the higher $\lambda$. An increase of $\lambda$ shifts $\theta^L$ to the left and and $\theta^H$ to the right and thereby reduces the set of states of the world in which renegotiation takes place.

Corollary 2. Suppose Assumption 1 holds and that $(\theta^L, \theta^H) \subset \Theta$. The probability that the initial contract is renegotiated is strictly decreasing in $\lambda$.

3.5 Alternative Specifications of the Reference Point

Our model is based on the assumption that the reference point is what the contract stipulates given the realized state of the world $\theta$, i.e. trading $\bar{x}$ at price $\bar{p}$ which gives rise to value $v(\bar{x}, \theta)$ and cost $c(\bar{x}, \theta)$. We believe that this is a highly natural and plausible specification. After all, the parties negotiated the contract, they both agreed to it, and when it comes to the renegotiation stage in state $\theta$ the contract determines the rights and obligations of both parties (and thus the threatpoint payoffs) if renegotiation fails.

An alternative specification is that the buyer and the seller form a reference point before the realization of the state of the world, i.e. shortly after the initial contract has been signed. In this case, the buyer and the seller compare the renegotiation proposal to the ex ante expected value, $E_\theta[v(\bar{x}, \theta)]$, and the expected cost $E_\theta[c(\bar{x}, \theta)]$, respectively. The analysis of the renegotiation game is very similar and gives rise to same frictions. In particular, it is still

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16If $F_1$ and $F_2$ have the same mean this definition of “more risky” implies Second Order Stochastic Dominance (SOSD). However, not every mean preserving spread of $F_1$ yields a distribution that is “more risky” according to the definition given above. It is possible to construct $F_2$ by adding a mean preserving spread to $F_1$ in such a way that $F_2$ has less weight in the tails than $F_1$ (see Levy (1992, p. 563) for an example). In this case, the likelihood of renegotiation is smaller under $F_2$ than under $F_1$. Thus, SOSD is not sufficient for Corollary 1. In fact, Corollary 1 holds as long as the following local properties of the distribution are satisfied: $F_2(\theta^L) > F_1(\theta^L)$ and $F_2(\theta^H) < F_1(\theta^H)$.
the case that the good \( x \) specified in the contract determines the reference point and that the parties incur losses in renegotiation that distort the renegotiation outcome.

Going one step further Köszegi and Rabin (2006) assume that parties do not form a point prediction ex ante but rather look at the full distribution of ex post outcomes. Furthermore, they assume that parties rationally expect that the contract will be renegotiated and take the renegotiation outcomes into account. With this form of expectation-based loss aversion the contract shapes the reference point only indirectly by affecting the parties expectations about the feasible ex post outcomes. Nevertheless, the initial contract trades off maximizing material efficiency and minimizing expected losses, so the same basic tradeoff arises. However, the analysis of expectation-based loss aversion is far more complicated.\(^{17}\)

Finally, it might be argued that the parties form rational expectations about the renegotiation outcome and that the reference point is the expected outcome given the realized state of the world. In this case there exists an equilibrium in which the first best is implemented: The parties expect \( x^*(\theta) \) to be traded in state \( \theta \) for all \( \theta \in \Theta \), and they do not incur any losses because this is exactly what happens. However, this assumption describes perfectly rational behavior in a world in which the parties can manage their reference points so as to avoid any loss aversion. It is inconsistent with the large body of evidence showing that loss aversion affects economic behavior.

### 4 Implications for Ex Ante Contracts

In this section we want to compare long-term specific performance contracts to other contractual arrangements such as spot contracting, the allocation of ownership rights, and authority contracts. For this we have to know how the reference point of the parties and the feelings of losses are affected by these more general contracts (or if no ex ante contract was written at all). In the following we extend the logic of Section 3 to more general contracts.

With a specific performance contract the reference point in renegotiation is the outcome

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\(^{17}\)Note that the parties experience losses even if they form rational expectations. With expectation-based loss aversion a party compares the rationally expected outcome in state \( \theta \) to all outcomes that would have obtained if some other state \( \theta' \neq \theta \) had materialized. See Herweg, Karle, and Müller (2013) for an application and detailed discussion of this approach to an incomplete contracts problem.
prescribed by the contract, i.e. what would happen if renegotiation fails. Analogously, we posit that if the parties do not write a long-term specific performance contract but wait until the state of the world materializes, their reference point is the outcome that would obtain if spot contracting failed and each party had to choose her next best outside option. Similarly, if the parties write a contract that is different from a specific performance contract, the reference point in the renegotiation game after the realization of the state of the world is the outside option induced by the contract that each party would get if renegotiation failed.

How strong are these reference points? The reference point that is induced by a specific performance contract presumably is stronger than the reference point that obtains if no contract had been written. After all, if the parties wrote a contract, they spent time and effort discussing and negotiating it, so the contract will loom prominently on their mind at the renegotiation stage. Thus, the feelings of losses (the degree of loss aversion $\lambda$) should be larger if the parties renegotiate a specific performance contract than if they negotiate from scratch.

What if the parties came to a contractual agreement that is not a specific performance contract (e.g. a contract on the allocation of ownership rights or on authority)? Here the reference point could be weaker or stronger than the reference point given by a specific performance contract depending on how prominent these other contracts are on the minds of the contracting parties. For simplicity we ignore the possibility of contract dependent degrees of loss aversion in the following and use the same degree of loss aversion $\lambda$ for all contracts. How strong reference points – induced by different contracts – are, is an important empirical question. From a theoretical perspective the extension of the model to contract dependent degrees of loss aversion is straightforward.

A related question is whether the subjects suffer from loss aversion at date 0 when writing an initial long-term contract. In Section 3 we took the initial contract as exogenously given and focused on the renegotiation game. If the performance of different contractual arrangements are to be compared it could be argued that not only ex post losses at the renegotiation stage but also ex ante losses from negotiating the initial contract should be taken into account. However, the focus of this paper is on the losses and the frictions occurring at the renegotiation stage. Therefore, we ignore reference points at date 0 and assume that there are no feelings of losses
when the initial contract is written. In fact, there are good economic reasons that are well in line with our general approach for why feelings of losses at stage 0 are of second order importance: Suppose that at date 0 the parties compare the proposed initial contract to their next best alternatives. These alternatives depend on the degree of competition on the date 0 market. It is natural to assume that there is more competition at date 0 than at date 2, for instance, because the buyer and the seller have more time to look for alternative trading partners. Suppose that there is perfect competition at date 0, while there may be less than perfect competition at date 2. Thus, if a buyer and a seller consider writing a contract on $\bar{x}$ at the competitive price $\bar{p}$ at date 0, the buyer (seller) knows that there are many other sellers (buyers) on the market willing to agree to the same contractual terms. Hence, the next best alternative to $(\bar{x}, \bar{p})$ is $(\bar{x}, \bar{p})$. Because the reference point is equal to the contract there are no feelings of losses when $(\bar{x}, \bar{p})$ is agreed upon.

4.1 The Costs and Benefits of Long-term Contracts

Instead of writing a long-term specific performance contract at date 0 that has to be renegotiated the parties could also wait until the state of the world has materialized and then write a contract on the spot at date 2. The long-term contract is costly because of costly renegotiation, but the spot contract may also be costly depending on the reference point that governs spot contracting at date 2 and the strength of this reference point, i.e. the size of $\lambda^{SC} \leq \lambda$. The following simple example illustrates the costs and benefits of writing a long-term contract. It also shows that it can be optimal to write a long-term contract on a good that is never materially efficient ex post.

Suppose that there are two states $\Theta = \{\theta_1, \theta_2\}$ and two specifications of the good $X = \{x_1, x_2\}$. The state-contingent payoffs are given in Figure 1. Good $x_1$ is materially efficient in state $\theta_1$ and good $x_2$ in state $\theta_2$. Without loss of generality, we assume that state $\theta_1$ materializes with probability $\pi \geq \frac{1}{2}$. Moreover, let $v^* > c^* > 0$.

**Long-term contract.** If the parties write a long-term specific performance contract, the contract optimally specifies $\bar{x} = x_1$. Thus, in the more likely state $\theta_1$ there is no need for renegotiation. In state $\theta_2$, however, the contract is inefficient. By Proposition 1 the parties
$v^* \geq (1 + \lambda)^2 c^* \iff \lambda \leq \sqrt{\frac{v^*}{c^*}} - 1 = \bar{\lambda}$

(17)

Thus, if $\lambda \leq \bar{\lambda}$ the parties will renegotiate to $\hat{x} = x_2$ and, by Proposition 2,

$$\hat{p} = \bar{p} + (1 - \alpha) \frac{1}{1 + \lambda} v^* + \alpha(1 + \lambda)c^*,$$

(18)

giving rise to final payoffs:

$$U_B(\theta_2) = v^* - \bar{p} - (1 + \lambda) \left[ (1 - \alpha) \frac{1}{1 + \lambda} v^* + \alpha(1 + \lambda)c^* \right],$$

$$U_S(\theta_2) = \bar{p} + (1 - \alpha) \frac{1}{1 + \lambda} v^* + \alpha(1 + \lambda)c^* - (1 + \lambda)c^*. 

(19)

If $\lambda > \bar{\lambda}$ there is no renegotiation and $x_1$ is traded at price $\bar{p}$. In this case payoffs are $U_B(\theta_2) = -\bar{p}$ and $U_S(\theta_2) = \bar{p}$. Thus, the ex ante expected social surplus of a long-term contract is given by

$$S_{LTC}(x_1, \bar{p}) = \begin{cases} v^* - c^* - (1 - \pi)\lambda \left\{ \frac{1 - \alpha}{1 + \lambda} v^* + [1 + \alpha(1 + \lambda)]c^* \right\} & \text{if } \lambda \leq \bar{\lambda} \\ \pi(v^* - c^*) & \text{if } \lambda > \bar{\lambda}. \end{cases} 

(20)

**Spot contract.** Suppose now that the parties wait until the state has materialized and then negotiate a spot contract on $x^*(\theta)$. The size of the losses they experience in doing so depends on the competitiveness of the market at date 2. Suppose that if the buyer and the seller do not come to an agreement on $x^*(\theta)$ they can trade some generic good $x$ with another trading partner at price $p$. This generic good is less customized to the needs of the buyer and therefore
generates a lower surplus: In both states of the world the buyer's valuation for $x$ is $\beta v^*$ and the seller's cost of producing it is $\beta c^*$, with $0 \leq \beta \leq 1$, so the surplus is $\beta(v^* - c^*)$. We interpret $\beta$ as the degree of competition on the market at date 2. The smaller $\beta$, the less attractive are the competing alternatives on the market and the more are the two parties locked into each other. Because trading $x$ at price $\tilde{p}$ is the next best alternative, the reference point for the buyer and the seller is $(\beta v^*, \tilde{p})$ and $(\beta c^*, \tilde{p})$, respectively.

By Proposition 1 the parties write a spot contract at date 2 if and only if

$$(1 - \beta)v^* \geq (1 + \lambda^{SC})(1 - \beta)c^* \iff \lambda^{SC} \leq \sqrt{\frac{v^*}{c^*}} - 1 = \tilde{\lambda}$$

(21)

Thus, if $\lambda^{SC} \leq \tilde{\lambda}$ the parties write a spot contract on $\hat{x} = x^*(\theta)$ at price

$$\hat{p} = p + (1 - \alpha)\frac{1}{1 + \lambda^{SC}(1 - \beta)v^* + \alpha(1 + \lambda^{SC})(1 - \beta)c^*}.$$  

(22)

If $\lambda^{SC} > \tilde{\lambda}$ the parties do not come to an agreement and trade good $\bar{x}$ on the market. Hence, the ex ante expected social surplus of a spot contract is given by

$$S_{SC} = \begin{cases} 
    v^* - c^* - (1 - \beta)\lambda^{SC} \left\{ \frac{1 - \alpha}{1 + \lambda^{SC}}v^* + [1 + \alpha(1 + \lambda^{SC})]c^* \right\} & \text{if } \lambda^{SC} \leq \tilde{\lambda} \\
    \beta(v^* - c^*) & \text{if } \lambda^{SC} > \tilde{\lambda}.
\end{cases}$$

(23)

**Proposition 5.** A spot contract outperforms a long-term specific performance contract if the degree of competition on the spot market is sufficiently high as compared to the probability that the long-term contract does not have to be renegotiated, i.e. if $\beta \geq \pi$. If $\beta < \pi$, there exists a critical threshold $\hat{\lambda}(\lambda) < \tilde{\lambda}$ such that spot contracting is optimal if and only if $\lambda^{SC} \leq \hat{\lambda}(\lambda)$.

To illustrate this result consider the following extreme cases: If there is perfect competition on the spot market ($\beta = 1$), it is clearly optimal to wait until date 2. In this case the spot contract is always materially efficient and causes no feelings of losses, while a long-term contract requires costly renegotiation with positive probability. Similarly, if $\lambda^{SC}$ is equal to 0, the spot contract is also materially efficient and there are no feelings of losses. However, if $\lambda^{SC}$ is close to $\lambda$ and if there is little competition on the date 2 market ($\beta$ is small), a long-term contract can be superior, because it replaces the reference point associated with $(x, \tilde{p})$ with the more efficient reference point associated with $(x_1, \hat{p})$.  

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Contracts that are never ex post efficient. A slightly modified version of this example can be used to show that it can be optimal to write a long-term contract on a good that is never materially efficient ex post. To see this suppose that the parties have to write a long-term contract (e.g. because a spot market does not exist). However, in addition to the goods \( x_1 \) and \( x_2 \) there is a third good \( \bar{x} \) that yields a benefit of \( \beta v^* \) for the buyer and costs the seller \( \beta c^* \) to produce in both states of the world. Thus, \( \bar{x} \) is a “compromise good” that is never materially efficient, but yields some moderate surplus \( \beta(v^* - c^*) \) in both states of the world.

Suppose that the parties write a long-term contract on \( \bar{x} \) at price \( \bar{p} \). After the realization of the state of the world they consider renegotiating this contract. The analysis of the renegotiation game is exactly the same as the analysis of spot contracting above, but now the same degree of loss aversion \( \lambda \) applies to all long-term contracts. If \( \lambda \leq \bar{\lambda} \) the parties will renegotiate to the efficient good \( x^*(\theta) \), if \( \lambda > \bar{\lambda} \) renegotiation fails and \( \bar{x} \) is traded. The social surplus of this contract is given by (23). Thus, it is optimal for the parties to contract on \( \bar{x} \) rather than on \( x_1 \) if and only if \( \beta \geq \pi \), even though the parties know that \( \bar{x} \) is never ex post efficient. For low degrees of loss aversion (\( \lambda \leq \bar{\lambda} \)) the “compromise good” \( \bar{x} \) makes renegotiation less painful because smaller adjustments in prices and costs are needed to get to the materially efficient good. For high degrees of loss aversion (\( \lambda > \bar{\lambda} \)) renegotiation costs are prohibitive. In this case good \( \bar{x} \) is an attractive compromise that yields an intermediate surplus in both states of the world which is preferable to getting the full surplus in one state and nothing in the other state. Thus, even though the compromise good is never materially efficient it minimizes renegotiation costs.

4.2 Asset Ownership, Long-term Contracts, and the Hold-up Problem

In this subsection we discuss the implications of loss aversion for the hold-up problem and the protection of relationship specific investments. We show under what conditions the parties want to rely on the allocation of ownership rights rather than on a long-term specific performance contract to mitigate the hold-up problem.

Suppose that the buyer can make a non-contractible relationship specific investment that increases his valuation of the good. If a complete, state-contingent contract could be written,
there would be no problem to induce the buyer to invest efficiently. However, in a complex environment with many different states of the world it is often impossible to write a long-term contract that specifies the rights and obligations of both parties in all conceivable contingencies. In this case the hold-up problem arises which may induce the buyer to choose an inefficient investment level.

The literature on incomplete contracts discusses two distinct ways of how to deal with the hold-up problem. The property rights approach (Grossman and Hart, 1986; Hart and Moore, 1990) assumes that it is impossible to write any long-term contract on trade. The only contracts that can be written to protect relationship-specific investments are contracts on the allocation of ownership rights. If a party owns an asset that is required for production this party has a stronger bargaining position when the terms of trade are negotiated because it can threaten to take the asset and trade with some other party. Therefore the owner of the asset will get a larger share of the surplus which increases his or her investment incentives. Hence, the allocation of ownership rights can be used to mitigate the hold-up problem.

A second approach going back to Hart and Moore (1988) allows for a long-term contract on trade, but the contract cannot be state-contingent and is therefore likely to be suboptimal after the realization of the state of the world in which case the parties have to renegotiate it. Several papers including Aghion, Dewatripont, and Rey (1994), Nöldeke and Schmidt (1995), and Edlin and Reichelstein (1996) show that simple contracts cum renegotiation can be used to implement the first best under fairly general conditions. However, if the renegotiation outcome is inefficient because the parties suffer from loss aversion, first best efficiency cannot be achieved any more. In this case it is an interesting and important question under what circumstances the parties should rely on the allocation of ownership rights rather than on a long-term contract to protect their relationship specific investments.

It is important to note that writing a specific performance contract and allocating ownership rights on assets are mutually exclusive instruments to encourage relationship-specific investments. Ownership of an asset improves the bargaining position of the owner only if he can threaten to trade with some outside party and take the asset with him. A specific performance contract precludes this possibility. With a specific performance contract each party can insist that good $\bar{x}$ is traded at price $\bar{p}$. Thus, the parties have to take a decision: Either
they write a specific performance contract or they rely on the allocation of ownership rights to protect their investments.

Of course, if the parties allocate ownership rights this will also create a reference point that affects the ex post negotiation game. The allocation of ownership rights determines the outside options of the two parties. As discussed at the beginning of this section we assume that in this case the reference point is determined by what happens if the negotiation fails, i.e. if the two parties do not come to an agreement and get their outside options.

To illustrate the trade-off between writing a long-term specific performance contract and allocating ownership rights we consider the following simple model. At date 0 the two parties can write a contract. At date \( t = 1 \) the buyer can make a relationship-specific investment \( I \in \mathbb{R}_0^+ \) that increases his benefit from trade at cost \( \psi(I) = (1/2)I^2 \). The investment is beneficial only if the buyer has access to an asset \( A \). At date 1 the state of the world, \( \theta \in \Theta \), materializes that affects the valuation of the buyer, \( v(x, \theta, I) \), and the cost of the seller, \( c(x, \theta) \). At date 2 parties can (re)negotiate the specification of the good \( x \in X \) and the price \( p \in \mathbb{R} \). Finally, at date 3 trade takes place and payoffs are realized.

Suppose that there are \( n \geq 2 \) states of the world, \( \theta \in \{ \theta_1, \ldots, \theta_n \} \equiv \Theta \), and \( n \) relevant specifications of the good that can be produced at stage 3, \( x \in \{ x_1, \ldots, n_n \} \equiv X \). Good \( x_i \) is materially efficient ex post if and only if state \( \theta_i \) materializes, i.e.

\[
x_i = x^*(\theta_i) = \arg \max_x \{ v(x, I, \theta_i) - c(x, \theta_i) \}.
\] (24)

Let \( \text{Prob}(\theta = \theta_i) = \pi_i \) with \( \sum_{i=1}^n \pi_i = 1 \), and assume w.l.o.g. that \( \pi_1 \geq \pi_i \) for all \( i \geq 2 \). For simplicity, we assume that only two configurations of costs and benefits can arise ex post depending on whether or not the efficient good is traded. Given \( \theta_i \) the buyer’s and the seller’s ex-post utility is

\[
U^B = \begin{cases} 
v^* + I - p - \frac{1}{2}I^2 & \text{if } x = x_i \\
v - p - \frac{1}{2}I^2 & \text{if } x \neq x_i
\end{cases}
\] (25)

and

\[
U^S = \begin{cases} 
p - c^* & \text{if } x = x_i \\
p - \bar{c} & \text{if } x \neq x_i
\end{cases}
\] (26)

respectively, with \( v^* - c^* > v - \bar{c} > 0 \) and \( c^* > \bar{c} > 0 \). Note that the investment pays off only if the efficient good is traded. In the benchmark case without frictions caused by contracting,
the parties trade $x_i$ in state $\theta_i$ and the buyer invests

$$I^* = \arg \max_{I} \{v^* - c^* + I - \frac{1}{2}I^2\} = 1.$$  \hfill (27)

**Long-term specific performance contracts.** Suppose that the parties write a specific performance contract $(\bar{x}, \bar{p})$ at stage 0. Obviously, it is optimal to specify $\bar{x} = x_1$ in the contract because good $x_1$ is most likely to be materially efficient ex post. If state $\theta = \theta_1$ is realized, the specific performance contract is materially efficient and will be executed. If some other state $\theta \neq \theta_1$ is realized trading $x_1$ is materially inefficient. In this case, the contract will be renegotiated if and only if the parties are not too loss averse, i.e. if and only if

$$\lambda \leq \sqrt{\frac{v^* - \bar{v} + \bar{v}}{c^* - \bar{c}}} - 1 \equiv \bar{\lambda}(I).$$  \hfill (28)

If $\lambda > \bar{\lambda}(I)$ the parties do not renegotiate and trade good $x_1$ in all states even though good $x_1$ is materially inefficient in all states but state $\theta_1$. The crucial difference to the previous analysis is that the critical $\lambda$-threshold depends on the buyer’s investment. The buyer’s investment $I^C$ maximizes his ex ante expected utility, which depends on whether or not renegotiation takes place ex post for states $\theta \neq \theta_1$. The following result shows that there is a unique critical degree of the parties’ loss aversion and a unique optimal investment level.

**Lemma 1.** If the parties write a long-term specific performance contract there exists a unique cutoff $\lambda = \sqrt{\frac{v^* - \bar{v} + \bar{v}}{c^* - \bar{c}}} - 1 > 0$ such that the contract will be renegotiated if and only if $\lambda \leq \bar{\lambda}$. The buyer’s investment is given by

$$I^C = \begin{cases} 
I^{CR} = \pi_1 + (1 - \pi_1)\alpha & \text{if } \lambda \leq \bar{\lambda} \\
I^{CNR} = \pi_1 & \text{if } \lambda > \bar{\lambda}.
\end{cases}$$

The expected surplus generated by the contract is given by

$$ES^C = \begin{cases} 
v^* - c^* + I^C - \frac{1}{2}(I^C)^2 - \lambda(1 - \pi_1) \left\{ \frac{1}{1+\lambda}(v^* + I^C - \bar{v}) + [\alpha(1+\lambda) + 1](c^* - \bar{c}) \right\} & \text{if } \lambda \leq \bar{\lambda} \\
\pi_1[v^* - c^* + I^C] + (1 - \pi_1)(\bar{v} - \bar{c}) - \frac{1}{2}(I^C)^2 & \text{if } \lambda > \bar{\lambda}.
\end{cases}$$

Note that $I^C$ is always smaller than the first-best investment level $I^* = 1$. The investment $I^C$ increases with $\pi_1$, the probability that the contract is ex post efficient. The degree of loss aversion determines whether an inefficient contract will be renegotiated or not. If there is
renegotiation the buyer will invest more and the expected surplus is higher. If \( \alpha = 1 \) (the buyer has all the bargaining power) he invests efficiently. The smaller \( \alpha \), the more severe is the hold-up problem. Loss aversion has two effects on the investment that go in opposite directions if \( \lambda \leq \bar{\lambda} \). On the one hand, the increase in the renegotiation price caused by a higher investment is lower the more loss averse the parties are, i.e. loss aversion mitigates the holdup problem. On the other hand, a higher degree of loss aversion reduces the buyer’s expected payoff and thereby discourages investment. These two effects just cancel out so that \( I_C \) is independent of \( \lambda \). Total surplus, however, decreases as \( \lambda \) goes up.

**Asset ownership and spot contracting.** Suppose now that the parties allocate ownership rights at date 0 and contract on trade only at date 2, after the state of the world has materialized. If they do not come to an agreement the next best alternative for the buyer is to leave the relationship and trade \( x^*(\theta) \) with another seller at some price \( p \) and get

\[
U^B = \begin{cases} 
 v^* + \beta I - p - \frac{1}{2} I^2 & \text{if the buyer owns } A \\
 v^* - p - \frac{1}{2} I^2 & \text{if the seller owns } A 
\end{cases}
\]

where \( \beta \in [0, 1] \) measures the specificity of the buyer’s investment. The smaller \( \beta \) the lower is the value of the investment if the buyer trades with a different seller, so the more relationship specific is his investment. For the seller the next best alternative is to walk away, too, and trade \( x^*(\theta) \) with another buyer at price \( p \). If she walks away, she gets

\[
U^S = p - c^* ,
\]

independent of whether the buyer or the seller owns \( A \).

These utilities from walking away determine the parties’ reference points in the negotiation game. Clearly, it is optimal to give ownership of \( A \) to the buyer in order to maximize his investment incentives.

Note that trade with each other is an outcome that is unanimously preferred to trade with an outsider (because the buyer’s valuation is higher and the seller’s cost remains the same). Thus, by Propositions 1 and 2, the buyer and the seller will always agree to trade \( x = x^*(\theta) \) at price

\[
p = p + \frac{1 - \alpha}{1 + \lambda} (v^* + I - v^* - \beta I) + \alpha (1 + \lambda) (c^* - c^*) = p + \frac{1 - \alpha}{1 + \lambda} (1 - \beta) I .
\]
Loss aversion has two effects on the buyer’s utility that go in opposite directions. On the one hand, an increase in $\lambda$ reduces the price that the buyer has to pay and thereby it mitigates the holdup. On the other hand, loss aversion reduces the buyer’s utility. These two effects cancel each other out so that the buyer’s utility, which is given by

$$U^B = v^* + I - p - (1 - \alpha)(1 - \beta)I - \frac{1}{2}I^2,$$  \hspace{1cm} (32)

does not depend on $\lambda$. The seller’s utility, however, is decreasing with $\lambda$ because the renegotiated price decreases as $\lambda$ goes up.

**Lemma 2.** If the parties rely on asset ownership, they optimally allocate ownership of the asset to the buyer. In equilibrium the parties always agree to trade with each other at date 2. The buyer’s investment is given by

$$I^A = \beta + \alpha(1 - \beta) < 1.$$  

The surplus generated by giving asset ownership to the buyer is given by

$$S^A = v^* - c^* + I^A - \frac{1}{2}(I^A)^2 - \frac{\lambda}{1 + \lambda}(1 - \alpha)(1 - \beta)I^A.$$  

The higher the buyer’s bargaining power and the less relationship specific the investment, the more is the buyer going to invest. Thus, the investment and the surplus are increasing in $\alpha$ and $\beta$. The degree of loss aversion $\lambda$ does not affect the optimal investment level but it reduces the surplus.

**Specific Performance Contract vs. Allocation of Ownership Rights** The comparison of the two types of contracts is now straightforward.

**Proposition 6.** Relying on the allocation of ownership rights outperforms a long-term specific performance contract if and only if $D(\alpha, \beta, \pi_1, \lambda) := S^A - E^{SC} \geq 0$, where $E^{SC}$ and $S^A$ are given in Lemma 1 and Lemma 2, respectively.

(i) $D(\cdot)$ is strictly increasing in $\beta$ and strictly decreasing in $\pi_1$, i.e., allocating ownership rights is more likely to outperform a long-term specific performance contract the smaller the degree of asset specificity (the larger $\beta$) and the more uncertain the environment (the smaller $\pi_1$).
The allocation of ownership rights is optimal if the buyer’s bargaining power is sufficiently high, i.e. \( \lim_{\alpha \to 1} D > 0 \).

The intuition for this result is as follows. Note first that in the benchmark case without loss aversion (\( \lambda = 0 \)) the contractual arrangement that leads to higher investments generates the higher expected surplus. In this case, specific performance leads to higher investments and outperforms asset ownership if \( \pi_1 > \beta \), i.e., if there is little uncertainty in the environment and if asset specificity is high. The uncertainty of the environment and asset specificity also play a crucial role if the parties are loss averse, i.e., if \( \lambda > 0 \). An increase of \( \pi_1 \) – a less uncertain environment – does not affect the performance of ownership rights, but it improves the performance of a specific performance contract in two ways. First, as in the case without loss aversion, increasing \( \pi_1 \) increases the buyer’s investment. Secondly, with a more certain environment the probability of costly renegotiation is reduced. If, on the other hand, the asset specificity of the investment decreases (\( \beta \) increases) this improves the performance of ownership rights for two reasons. First, the buyer’s investment increases and moves closer to the efficient level \( I^* \). Second, the reference point gets closer to the bargaining outcome, so the losses felt ex post are reduced. In the limit, as \( \beta \to 1 \), the first best is achieved by using ownership rights while the social surplus generated by a specific performance contract is unaffected. Finally, and most interestingly, the relative bargaining power affects the performance of the contractual arrangements if the parties are loss averse but not if the parties are loss neutral. If the buyer’s bargaining power \( \alpha \) goes to 1, the performance of ownership rights is improved. The buyer’s investment goes to the efficient investment level and his feelings of losses disappear because he has to pay less to the seller. With a specific performance contract the buyer will also invest more if \( \alpha \) goes up, but only if \( \lambda \leq \bar{\lambda} \). Furthermore, the feelings of losses do not disappear in the renegotiation game because the buyer has to compensate the seller for the cost increase \( c^* - \bar{c} \).

4.3 Authority Contracts and the Employment Relation

Instead of writing an ex-ante contract that specifies a particular good \( \bar{x} \) to be traded, the parties could also write an “authority contract” that gives one party the right to choose \( x \) out of some admissible set \( A \subseteq X \). For example, the buyer could have the right to “order” the
seller to deliver any good or service $x \in A$. According to Simon (1951) this is the nature of
the employment relation. An employment contract does not specify a specific service to be
delivered by the employee (the seller), it rather gives the employer (the buyer) the right to
order the employee which service to provide (within the limits specified by the employment
contract). Simon compares an authority contract to a specific performance contract and
argues that there is a tradeoff. The authority contract has the advantage of flexibility, i.e.,
the employer can easily adjust the service to be provided to the realization of the state of the
world. However, the authority contract is also prone to abuse. The employer has an incentive
to choose $\hat{x}(\theta) = \arg \max_{x \in A} v(x, \theta)$ which maximizes his own utility rather than the materially
efficient service $x^*(\theta) = \arg \max_{x \in A} [v(x, \theta) - c(x, \theta)]$. The employee anticipates this and has
to be compensated ex ante for her expected cost $E_\theta [c(\hat{x}(\theta), \theta)]$. Thus, the efficiency loss will
be borne by the employer. A specific performance contract, on the other hand, leaves no scope
for abuse. But, this advantage comes at the cost of rigidity. The employee will provide $\bar{x}$ in
all states of the world. Hence, according to Simon, whether an authority contract or a specific
performance contract is optimal depends on whether the cost of abuse exceeds the cost of
rigidity.

A crucial problem with Simon’s argument is that the specific performance contract need
not be rigid because the parties are free to renegotiate. If the parties write a contract $(\bar{x}, \bar{p})$
they can later renegotiate it to $(x^*(\theta), \hat{p})$. The specific performance contract protects the
employee against abuse (she must always get at least $\bar{p} - c(\bar{x}, \theta)$), while renegotiation makes
the contract flexible. With a specific performance contract the employer has to “bribe” the
employee to provide $x^*(\theta)$ rather than $\bar{x}$. The authority contract can also be renegotiated to
prevent that the buyer’s preferred good, i.e. the inefficient good $\hat{x}(\theta)$, is implemented ex post.
With an authority contract the employee has to “bribe” the employer to choose $x^*(\theta)$ rather
than $\hat{x}(\theta)$. If renegotiation is costless the final outcome will always be materially efficient and
the expected payments will be the same under both contracts. If renegotiation is imperfect
due to loss aversion, however, the two contracts are no longer equivalent.

A second problem with Simon’s argument is the assumption that the employee has to
carry out the order of the employer. In most legislations the employment contract is “at will”,
i.e. the employee can always refuse to comply and quit. This limits the flexibility and the
scope for exploitation.

In this section we address both of these problems. We compare a specific performance contract to an at-will authority contract and we allow for renegotiation. With the authority contract the buyer can order the seller which specification of the good to produce. However, the authority contract is at-will, i.e. the seller can always quit. In this case she does not have to incur any costs, but she also forgoes the agreed upon price. We show that loss aversion affects an authority contract differently than a specific performance contract. In particular, loss aversion makes it easier for the buyer to “exploit” the seller. Furthermore, with an at-will contract the initial price $\bar{p}$ affects the ex post outcome.

Suppose that $X \subset \mathbb{R}^N$ and $\Theta \subset \mathbb{R}^S$ are some continuous subsets of Euclidean spaces and that $\theta$ is drawn by nature according to the density function $f(\theta)$ out of set $\Theta$. Let $x^*(\theta) : \Theta \to X$ be a bijective function, i.e., for any $x \in X$ there exists one and only one $\theta \in \Theta$ in which $x$ is efficient. Similarly, let $\tilde{x}(\theta) : \Theta \to X$ be also a bijective function, i.e., for any $x \in X$ there is exactly one $\theta \in \Theta$ in which $x$ is profit maximizing for the buyer. Furthermore, we assume that $x^*(\theta) \neq \tilde{x}(\theta)$ for all $\theta \in \Theta$. These assumptions imply that without renegotiation the specific performance contract and the authority contract implement the efficient outcome with probability zero.\(^{18}\)

**Assumption 2.** For all $\theta \in \Theta$ we have that $v(x^*(\theta), \theta) = v^*$, $c(x^*(\theta), \theta) = c^*$, $v(\tilde{x}(\theta), \theta) = \tilde{v}$, $c(\tilde{x}(\theta), \theta) = \tilde{c}$, $v(x, \theta) = v$, and $c(x, \theta) = c$ for all $x \in X \setminus \{x^*(\theta), \tilde{x}(\theta)\}$. Furthermore, $\tilde{v} > v^* > v$, $\tilde{c} > c^* > c$, $v^* - c^* > \tilde{v} - \tilde{c} > 0$, and $v^* - c^* > v - c > 0$.

Assumption 2 simplifies the problem considerably by assuming that there are only three different outcomes, $(v^*, c^*)$, $(\tilde{v}, \tilde{c})$ and $(\bar{v}, \bar{c})$, and two relevant services in each state of the world at the renegotiation stage. The relevant services are the materially efficient service $x^*(\theta)$ and the service $\tilde{x}(\theta)$ that maximizes the buyer’s benefit.

**Specific performance contract.** Given the continuous state space a specific performance contract prescribes the efficient outcome with probability 0. Thus, if $\lambda$ is sufficiently small,

\(^{18}\)These assumptions are useful to avoid cumbersome case distinctions, but they are not crucial for any of the following results. It is straightforward to set up a similar model with a discrete state space and without the bijectivity assumptions.
the contract will be renegotiated, otherwise the parties are stuck with \((v, c)\). The analysis of the specific performance contract is straightforward and summarized in the following lemma.

**Lemma 3.** If the parties write a specific performance contract, the contract will be renegotiated if and only if \(\lambda \leq \sqrt{\frac{v^* - v}{c^* - c}} - 1 \equiv \bar{\lambda}^S\). The total surplus that is generated by this contract is given by

\[
S_{SPC}(\lambda) = \begin{cases} 
    v^* - c^* - \lambda(1 + \alpha(1 + \lambda))[c^* - c] - \frac{\lambda(1 - \alpha)}{1 + \alpha}[v^* - v] & \text{if } \lambda \leq \bar{\lambda}^S \\
    v - c & \text{if } \lambda > \bar{\lambda}^S
\end{cases}
\]

**Authority contract.** The authority contract is an at-will contract, i.e. the seller is free to quit if she does not want to carry out the buyer’s order. In this case the seller incurs no cost, but she does not get \(\bar{p}\) either. To make things interesting we assume that \(\bar{p}\) is such that \(c^* \leq \bar{p} < \bar{c}\), i.e., in the absence of loss aversion the seller is willing to deliver the efficient service \(x^*(\theta)\) but not the exploitative service \(\tilde{x}(\theta)\) that maximizes the buyer’s utility.\(^{19}\)

What is the reference point induced by an authority contract? The contract says that the buyer decides which service the seller has to deliver subject to the constraint that the seller may quit. Hence the reference point in the renegotiation game is the outcome that would obtain if renegotiation breaks down and the buyer exercised his authority. In particular, if it is optimal for the buyer to choose \(\tilde{x}(\theta)\) and if this does not induce the seller to quit, then the seller expects this to happen and does not feel a loss. If the seller prefers to quit when the buyer demands \(\tilde{x}(\theta)\), then demanding \(\tilde{x}(\theta)\) is not optimal for the buyer. In this case, the buyer prefers to demand \(x^*(\theta)\) and the seller’s reference point is \((c^*, \bar{p})\). Note that whether the seller prefers to quit depends on her reference point. If the reference point is \((\bar{c}, \bar{p})\), she is less likely to quit than for other reference points, such as \((c^*, \bar{p})\) and \((0, 0)\). In other words, there are multiple “equilibria”. We assume that the seller complies as long as complying constitutes an equilibrium in the following sense: the reference point is \((\bar{c}, \bar{p})\) as long as – given this reference point – the seller prefers to deliver \(\tilde{x}(\theta)\) at price \(\bar{p}\) to quitting the relationship.\(^{20}\)

\(^{19}\)The initial price \(\bar{p}\) is part of the ex ante contract. The parties will choose \(\bar{p}\) according to their initial bargaining powers. We take \(\bar{p}\), here, as given. Note, however, that for \(\bar{p} < c^*\) the seller may prefer to quit even when the buyer demands the materially efficient service. If this is the case, a high price mark-up is necessary in order to implement \(x^*(\theta)\) via renegotiation. Therefore, initial prices \(\bar{p} < c^*\) can hardly be part of an optimal authority contract. If \(\bar{p} \geq \bar{c}\), then one of the three cases we identify does not exist. Otherwise, the analysis and the results remain unchanged.

\(^{20}\)The equilibrium selection has no qualitative effects on our results. The equilibrium selection only affects
Note that if the buyer requests the efficient service, the seller always complies because her production cost $c^*$ is smaller than the price $\bar{p}$ – provided that her reference point is $(\hat{c}, \bar{p})$ or $(c^*, \bar{p})$. What if the buyer requests the exploitative service? If the seller complies she gets $\bar{p} - \hat{c} < 0$. If she quits, she saves the production cost $\hat{c}$ but she loses the price $\bar{p}$. Because losing $\bar{p}$ is considered a loss, her utility from quitting is $-\lambda \bar{p}$. Thus, the seller complies and produces $\tilde{x}(\theta)$ if and only if

$$\bar{p} - \hat{c} \geq -\lambda \bar{p} \iff \lambda \geq \frac{\hat{c} - \bar{p}}{\bar{p}} \equiv \lambda_1^A.$$  

(33)

If $\lambda < \lambda_1^A$ the seller would quit if the buyer demanded $\tilde{x}(\theta)$. Thus, the buyer is better off requesting $x^*(\theta)$ which is always accepted by the seller, and the surplus is given by $S^A = v^* - c^*$.

Suppose now that $\lambda \geq \lambda_1^A$. In this case the seller does not quit if requested to deliver $\tilde{x}(\theta)$ and the buyer will request the exploitative service. However $\tilde{x}(\theta)$ is materially inefficient. The parties will renegotiate to $\hat{x} = x^*(\theta)$ if and only if $\lambda \leq \sqrt{\frac{\hat{c} - c^*}{\hat{c} - v^*}} - 1 \equiv \lambda_2^A$.

The following Lemma summarizes the analysis of the authority contract.

**Lemma 4.** If the parties write an authority contract the buyer requests the efficient service if $\lambda \leq \lambda_1^A$. If $\lambda_1^A < \lambda \leq \max\{\lambda_1^A, \lambda_2^A\}$ the buyer requests the exploitative service $\tilde{x}$, but the parties renegotiate to the efficient service $x^*(\theta)$. If $\max\{\lambda_1^A, \lambda_2^A\} < \lambda$ there is no renegotiation and the seller delivers the exploitative service $\tilde{x}(\theta)$. The surplus generated by this contract is given by

$$S^{AC}(\lambda) = \begin{cases} v^* - c^* & \text{if } \lambda < \lambda_1^A \\ v^* - c^* - \lambda[1 + (1 - \alpha)(1 + \lambda)](\hat{v} - v^*) - \frac{1}{1 + \lambda} \alpha(\hat{c} - c^*) & \text{if } \lambda_1^A \leq \lambda \leq \max\{\lambda_1^A, \lambda_2^A\} \\ \hat{v} - \hat{c} & \text{if } \max\{\lambda_1^A, \lambda_2^A\} \leq \lambda \end{cases}$$

Note that a higher degree of loss aversion promotes the exploitation of the seller in an authority contract because it makes her more reluctant to quit. The generated surplus $S^{AC}$ is continuous in $\lambda$ except for the point $\lambda_1^A$, where it has a downward discontinuity.

**Comparison of authority and specific performance contracts.** It is now straightforward to compare the two contracts.

the $\lambda$-thresholds discussed below.
Proposition 7. An authority contract outperforms a specific performance contract if and only if $D(\lambda) := S^{AC}(\lambda) - S^{SPC}(\lambda) > 0$, where $S^{SPC}(\lambda)$ and $S^{AC}(\lambda)$ are given in Lemma 3 and Lemma 4, respectively.

(i) If the parties are loss neutral, $\lambda = 0$, then both contractual arrangements perform equally well, $D(0) = 0$.

(ii) If $\lambda$ is small, the authority contract is strictly optimal, i.e. if $0 < \lambda < \bar{\lambda}_1^A$, then $D(\cdot) > 0$.

(iii) If $\lambda$ is large, so that there is no renegotiation of either contract, the authority contract outperforms the specific performance contract if and only if the efficiency loss due to abuse is smaller than the efficiency loss due to rigidity, i.e. if $\lambda \geq \max\{\bar{\lambda}_1^A, \bar{\lambda}_2^A, \bar{\lambda}_S\}$, then $D(\cdot) > 0 \iff \hat{v} - \hat{c} > v - c$.

(iv) If $\lambda$ is intermediate, the authority contract is more likely to be optimal the less costly (in terms of experienced losses due to loss aversion) it is to move from $(\hat{v}, \hat{c})$ to $(v^*, c^*)$, i.e. the smaller $\hat{v} - v^*$ and $\hat{c} - c^*$, and the more costly it is to move from $(\hat{v}, \hat{c})$ to $(v^*, c^*)$, i.e. the larger $v^* - \hat{v}$ and $c^* - \hat{c}$. Formally, if $\max\{\bar{\lambda}_1^A, \bar{\lambda}_2^A\} \leq \lambda < \max\{\bar{\lambda}_1^A, \bar{\lambda}_2^A, \bar{\lambda}_S\}$:

$$\frac{\partial D}{\partial (\hat{c} - c^*)} < 0, \quad \frac{\partial D}{\partial (v^* - \hat{v})} > 0, \quad \frac{\partial D}{\partial (\hat{v} - v^*)} < 0, \quad \frac{\partial D}{\partial (c^* - \hat{c})} > 0,$$

This result confirms and extends the original insights of Simon. If $\lambda > \max\{\bar{\lambda}_1^A, \bar{\lambda}_2^A, \bar{\lambda}_S\}$ so that neither the authority contract nor the specific performance contract is renegotiated the comparison boils down to Simon’s comparison of whether $v - c$ is greater than $\hat{v} - \hat{c}$, i.e., whether rigidity or abuse is more efficient. If $\lambda$ is small ($\lambda \leq \bar{\lambda}_1^A$), the authority contract implements the first best, because the seller can credibly threaten to quit if the exploitative good is requested. The specific performance contract does less well because it requires costly renegotiation. For intermediate values of $\lambda$ the crucial question is by how much costs and benefits have to be shifted to reach efficiency. To see this compare two situations, one in which a specific performance contract (without renegotiation) yields $(\hat{v}, \hat{c})$ and one where it yields $(\hat{v} - \Delta, \hat{c} - \Delta)$ with $\Delta > 0$, while the materially efficient good always yields $(v^*, c^*)$. Even though the specific performance contracts are equally inefficient in the two situations, it is less costly to renegotiate in the first situation than in the second. This is because $v^* - \hat{v} < v^* - (\hat{v} - \Delta)$ and $c^* - \hat{c} < c^* - (\hat{c} - \Delta)$, i.e., loss aversion kicks in more strongly in the second case.
4.4 Price Indexation

Suppose that there is a verifiable signal $\sigma$ that is correlated with the state of the world $\theta$. Is it possible to improve efficiency by making the payment in the initial contract conditional on this signal? One of the two main results in Hart (2009) is that indexation can be very useful. By making the price $\bar{p}$ conditional on $\sigma$ it becomes more likely that $c(\cdot, \theta) < \bar{p}(\sigma) < v(\cdot, \theta)$, so that parties are willing to trade voluntarily and costly renegotiation can be avoided. Perhaps surprisingly, this is not the case in our set-up. To show this we return to the modeling assumptions of Section 3.3 where $x^*$ is a continuous function of $\theta$.

**Proposition 8.** Suppose that Assumption 1 holds and that there exists a verifiable signal $\sigma$ that is correlated with the state of the world $\theta$. Making the initially agreed upon price $\bar{p}$ a function of $\sigma$ has no effect on the renegotiation outcome and on the efficiency of the initial contract.

The intuition for this result is simple. In our model the only role of the initial price $\bar{p}$ is to share the available surplus ex ante. Renegotiation is only about $\hat{x}$ and the markup $\hat{p} - \bar{p}$ in which $\bar{p}$ cancels out. Once the state of the world has materialized and some $\bar{p}(\sigma)$ is in place, this $\bar{p}(\sigma)$ defines the reference point. Only deviations from the reference point matter, but not the reference point itself.

The striking difference to Hart (2009) is due to the fact that Hart considers at-will contracts in which each party can freely walk away from the contract while we consider specific performance contracts in which preventing the parties to leave is not an issue. Of course, if it was possible to make the specification of the good contingent on $\sigma$ this would improve efficiency if it reduces $\bar{x}(\sigma) - x^*(\theta)$ as compared to $\bar{x} - x^*(\theta)$ and thereby the welfare loss due to renegotiation in expectation. This can be (partially) achieved by a contract that makes the price per unit of output conditional on an index (say inflation, the exchange rate, or the price of oil) and gives the buyer the right to choose the quantity of trade ex post. Such a contract is similar to the employment contract because one party can tell the other party what to deliver, but it is more complex because the total payment depends on the quantity chosen by the buyer.

The following example shows that such a contract with price indexation can be very
beneficial and may implement the first best if the signal is sufficiently informative. Let \( x \in \mathbb{R}^+ \) denote the quantity of trade and let \( \theta = (\sigma, \tau) \) be a two-dimensional state of the world, where \( \sigma \) is publicly observable and verifiable. Assume that \( c(x, \theta) = c(\sigma) \cdot x \). In this case a contract with price indexation that gives the buyer the right to decide on the quantity \( x \) is very useful. If the contract stipulates that the price per unit is \( w(\sigma) = c(\sigma) \), i.e \( p = w(\sigma) \cdot x \), then the buyer will choose

\[
\hat{x} = \arg \max_x \{v(x, \theta) - w(\sigma) \cdot x\} = \arg \max_x \{v(x, \theta) - c(\sigma) \cdot x\} = x^*(\theta) .
\]

Thus, this contract implements the first best without renegotiation. Note, however, that for price indexation to be beneficial it is crucial that it not only affects the total payment but also implicitly the quantity (or specification) \( x \) that is traded if the initial contract is not renegotiated.

5 Conclusions

This paper explores the implications of one important behavioral phenomenon, loss aversion, for optimal (incomplete) contracting and renegotiation. It shows that loss aversion makes the initial contract sticky and prevents parties to adjust the contract to the materially efficient allocation. This inefficiency of renegotiation has important implications for the optimal design of contracts. In particular, it can explain why people often abstain from writing (beneficial) long-term contracts or why they write long-term contracts that are obviously inefficient, it can explain under what conditions the allocation of ownership rights should be used to promote investment incentives rather than specific performance contracts, and it predicts under what conditions employment contracts strictly outperform specific performance contracts. Moreover, the model we propose is simple and tractable and thus can easily be applied to other contracting problems as well.

We assume that the contracting parties are sophisticated in that they are aware of their loss aversion when they write the initial contract. Nevertheless, they continue to weigh gains

\footnote{Here, we stick to our assumption that the buyer feels a loss if total payment after renegotiation is higher than total payment under the initial contract, which depends on the verifiable signal and the buyer’s action. With the contract specifying a unit price, it might also be sensible to assume that the buyer feels a loss if the renegotiated unit price is higher than the initially specified unit price. In the case with indexation this would not change the example, because the initial contract is not renegotiated if the contract is properly indexed.}
and losses differently.\textsuperscript{22} It would be interesting to extend our model to the case of contracting parties who are less sophisticated and do not anticipate that loss aversion will distort renegotiation in the future. If the parties are “naïve” and believe that all future renegotiations will be materially efficient then they will write contracts that are suboptimal in an additional respect.

Finally, it would be interesting to study the interaction of loss aversion with other behavioral biases such as concerns for fairness, self-serving biases, and overconfidence that may effect or create additional reference points. The interaction of these effects and their impact on contracting is a fascinating topic for future research.

\textsuperscript{22}The behavior of the contracting parties in our model is akin to the behavior of a house owner who is reluctant to sell his house at a price that is below the price he bought it for, even though he understands that the historic price at which he bought is bygone and should not affect his decision to sell. Empirically investigating the Boston condominium market in the 1980’s, Genesove and Mayer (2001) provide evidence that the original purchase price has indeed a significant effect on seller behavior in line with nominal loss aversion. Moreover, they show that not only owner-occupants but also professional investors behave in a loss averse fashion.
A Appendix

Proof of Proposition 1. In case (i) we have \( v(\hat{x}, \theta) \geq v(\bar{x}, \theta) \) and \( c(\hat{x}, \theta) \leq c(\bar{x}, \theta) \). In this case, there always exists a set of prices \( \hat{p} \) such that both parties prefer \((\hat{x}, \hat{p})\) to \((\bar{x}, \bar{p})\). In particular \( \hat{p} = \bar{p} \) is an element of this set.

In case (ii) it holds that \( v(\hat{x}, \theta) > v(\bar{x}, \theta) \) and \( c(\hat{x}, \theta) \geq c(\bar{x}, \theta) \). The buyer is willing to accept an increase in price if and only if
\[
\hat{p} \leq \bar{p} + \frac{v(\hat{x}, \theta) - v(\bar{x}, \theta)}{1 + \lambda}. \tag{A.1}
\]
The seller is willing to incur the higher production cost if and only if she is compensated by a higher price \( \hat{p} \) where
\[
\hat{p} \geq \bar{p} + (1 + \lambda)[c(\hat{x}, \theta) - c(\bar{x}, \theta)]. \tag{A.2}
\]
Combining the two inequalities above reveals that there exists a price \( p > \bar{p} \) for \( \hat{x} \) that is acceptable to both parties if and only if (6) holds.

The proof of case (iii) proceeds by similar steps as the proof of case (ii).

Proof of Proposition 2. The generalized Nash product can be written as follows
\[
GNP(p) = [v(\hat{x}, \theta) - p - \lambda_1[v(\bar{x}, \theta) - v(\hat{x}, \theta)] - \lambda_3[p - \bar{p}] - v(\bar{x}, \theta) + \bar{p}]^\alpha \\
\times [p - c(\hat{x}, \theta) - \lambda_4[p - \bar{p}] - \lambda_2[c(\hat{x}, \theta) - c(\bar{x}, \theta)] - \bar{p} + c(\bar{x}, \theta)]^{1 - \alpha} \tag{A.3}
\]
where
\[
\lambda_3 = \begin{cases} 
\lambda & \text{if } p - \bar{p} > 0 \\
0 & \text{otherwise}
\end{cases} \quad \lambda_4 = \begin{cases} 
\lambda & \text{if } \bar{p} - p > 0 \\
0 & \text{otherwise}
\end{cases}
\]
Note that \( GNP(p) \) is strictly concave and differentiable for all \( p \) but \( p = \bar{p} \). Because we consider a given \( \hat{x}(\theta) \) it is clear whether or not \( \lambda_1 = 0 \) and/or \( \lambda_2 = 0 \). For \( \Delta_v \geq 0 \) and \( \Delta_c \geq 0 \) only prices \( p \geq \bar{p} \) can lead to \( U^S(\hat{x}(\theta), p|\theta) \geq U^S \). Thus, in this case \( GNP(p) \) is differentiable for all prices in the relevant range. Moreover, for \( \Delta_v \leq 0 \) and \( \Delta_c \leq 0 \) only prices \( p \leq \bar{p} \) can lead to \( U^B(\hat{x}(\theta), p|\theta) \geq U^B \), and thus \( GNP(p) \) is differentiable for all prices in the relevant range. Only for \( \Delta_v > 0 \) and \( \Delta_c < 0 \) we need to consider prices \( p \) that are higher as well as
lower than \( \tilde{p} \). (For \( \Delta_v \leq 0 \) and \( \Delta_c \geq 0 \) with at least one inequality being strict renegotiation does not take place.)

Differentiating the generalized Nash product with respect to \( p \) yields the following first-order condition

\[
\frac{\partial GNP(p)}{\partial p} = 0 \iff \alpha(1 + \lambda_3)[U^B(\hat{x}, p | \theta) - \bar{U}^B]^{-\alpha} + (1 - \alpha)(1 + \lambda_4)[U^S(\hat{x}, p | \theta) - \bar{U}^S]^{-\alpha} = 0. \tag{A.4}
\]

Inserting (2) and (3) for the utilities in (A.4) yields:

\[
\alpha(1 + \lambda_3)\left[(1 + \lambda_4)[p - \bar{p}] - (1 + \lambda_3)[c(\hat{x}, \theta) - c(\bar{x}, \theta)]\right] = (1 - \alpha)(1 + \lambda_4)\left[(1 + \lambda_4)[v(\hat{x}, \theta) - v(\bar{x}, \theta)] - (1 + \lambda_3)[p - \bar{p}]\right]. \tag{A.5}
\]

Solving for \( p \) yields the expressions for \( \hat{p}(\theta) \), given by equation (9) for the cases \((1 - \alpha)\frac{1 + \lambda_4}{1 + \lambda_3} \Delta_v + \alpha(1 + \lambda_2) \Delta_c \geq 0 \) and \((1 - \alpha)(1 + \lambda_1) \Delta_v + \alpha \frac{1 + \lambda_2}{1 + \lambda_1} \Delta_c \leq 0 \). Note that the two price formulas coincide for \( \Delta_v = 0 \) and \( \Delta_c = 0 \) the unique case where both conditions are satisfied with equality. Recall that it is impossible that \( \Delta_v \leq 0 \) and \( \Delta_c \geq 0 \) because in this case no renegotiation takes place. Therefore, \((1 - \alpha)\frac{1 + \lambda_4}{1 + \lambda_3} \Delta_v + \alpha(1 + \lambda_2) \Delta_c > 0 \) implies that \((1 - \alpha)(1 + \lambda_1) \Delta_v + \alpha \frac{1 + \lambda_2}{1 + \lambda_1} \Delta_c > 0 \) and \((1 - \alpha)(1 + \lambda_1) \Delta_v + \alpha \frac{1 + \lambda_2}{1 + \lambda_1} \Delta_c < 0 \) implies that \((1 - \alpha)\frac{1 + \lambda_4}{1 + \lambda_3} \Delta_v + \alpha(1 + \lambda_2) \Delta_c < 0 \). Hence, the two cases are disjunct.

It remains to analyze the case where \((1 - \alpha)\frac{1 + \lambda_4}{1 + \lambda_3} \Delta_v + \alpha(1 + \lambda_2) \Delta_c < 0 < (1 - \alpha)(1 + \lambda_1) \Delta_v + \alpha \frac{1 + \lambda_2}{1 + \lambda_1} \Delta_c \). This case can occur only if \( \Delta_v > 0 \) and \( \Delta_c < 0 \), i.e., if \( \hat{x}(\theta) \) is unambiguously better than \( \bar{x} \). By the concavity of \( GNP(p) \) it can readily be shown that

\[
\left. \frac{\partial GNP(p)}{\partial p} \right|_{p<\bar{p}} > 0 \quad \text{and} \quad \left. \frac{\partial GNP(p)}{\partial p} \right|_{p>\bar{p}} < 0. \tag{A.6}
\]

Thus, in this case the renegotiated price is \( \bar{p} \), which completes the proof. \( \square \)

**Proof of Proposition 3.** The proof is decomposed into two steps. First, we analyze the case \( \theta > \bar{\theta} \) and thereafter the case \( \theta < \bar{\theta} \). The threshold state \( \bar{\theta} \) is implicitly defined by \( \partial v(\hat{x}, \bar{\theta})/\partial x = \partial c(\bar{x}, \bar{\theta})/\partial x \) if a solution exists—i.e., the resulting \( \bar{\theta} \in \Theta \). If a solution does not exist, then
\( \hat{\theta} \in \{ \sup \{ \Theta \}, \inf \{ \Theta \} \} \). Precisely, if for all \( \theta \in \Theta \) (i) \( \partial v(\cdot)/\partial x > \partial c(\cdot)/\partial x \), then \( \hat{\theta} = \inf \{ \Theta \} \) (ii) \( \partial v(\cdot)/\partial x < \partial c(\cdot)/\partial x \), then \( \hat{\theta} = \sup \{ \Theta \} \).

**Case 1:** Suppose \( \theta > \hat{\theta} \). First, observe that the parties will never agree upon implementing a good \( x < \bar{x} \) ex post. For \( x < \bar{x} \) the buyer feels a loss in the good dimension and thus demands a price reduction. The necessary price reduction making the buyer accepting the contract is higher than the seller’s reduction in costs, because \( \theta > \hat{\theta} \). Hence, the parties either renegotiate to a \( x > \bar{x} \) or agree on performing the initially specified service \( \bar{x} \).

If the parties agree on a \( x > \bar{x} \), then the price has to increase, because the seller incurs higher costs for the new good, i.e., \( c(x, \theta) > c(\bar{x}, \theta) \). This implies that if renegotiation is successful, i.e. \( x > \bar{x} \), then the buyer feels a loss in the money dimension and the seller feels a loss in the good dimension. The \( GNP(x, p) \) in this case is given by:

\[
GNP(x, p) = \left\{ v(x, \theta) - v(\bar{x}, \theta) - (\lambda + 1)(p - \bar{p}) \right\}^\alpha \times \left\{ p - \bar{p} - (\lambda + 1)[c(x, \theta) - c(\bar{x}, \theta)] \right\}^{1-\alpha} \tag{A.7}
\]

If there is an interior solution, then the interior solution is characterized by the following first-order conditions:

\[
\frac{\partial GNP}{\partial p} = 0 \iff -\alpha \left[ U^B - U^B \right]^{\alpha - 1} \left[ U^S - U^S \right]^{1-\alpha} (\lambda + 1) + \left[ U^B - U^B \right]^\alpha \left[ U^S - U^S \right]^{-\alpha} (1 - \alpha) = 0, \tag{A.8}
\]

and

\[
\frac{\partial GNP}{\partial x} = 0 \iff \alpha \left[ U^B - U^B \right]^{\alpha - 1} \left[ U^S - U^S \right]^{1-\alpha} \frac{\partial v(x, \theta)}{\partial x} + \left[ U^B - U^B \right]^\alpha \left[ U^S - U^S \right]^{-\alpha} (1 - \alpha)(\lambda + 1) \frac{\partial c(x, \theta)}{\partial x} = 0. \tag{A.9}
\]

Rearranging (A.8) yields

\[
\frac{U^B - U^B}{U^S - U^S} = (\lambda + 1) \frac{\alpha}{1 + \alpha} \tag{A.10}
\]

Similarly, (A.9) can be written as

\[
\frac{U^B - U^B}{U^S - U^S} = \frac{1}{\lambda + 1} \frac{\alpha}{1 + \alpha} \frac{\partial v(x, \theta)/\partial x}{\partial c(x, \theta)/\partial x}. \tag{A.11}
\]
The first-order conditions (A.10) and (A.11) together imply that
\[ R(x, \theta) \equiv \frac{\partial v(x, \theta)}{\partial x} \frac{\partial c(x, \theta)}{\partial x} = (\lambda + 1)^2. \] (A.12)

Note that \( R(x, \theta) \) is strictly decreasing in \( x \) by Assumption 1. Moreover, \( \lim_{x \to \infty} R(x, \theta) < 1 \). Hence, if there are \( x > \bar{x} \) such that \( R(x, \theta) > (1 + \lambda)^2 \) then there is a unique \( x \) at which \( R(x, \theta) = (1 + \lambda)^2 \). We denote this solution by \( \hat{x}^H(\theta) \). If \( R(x, \theta) \leq (1 + \lambda)^2 \) for all \( x > \bar{x} \) then it does not pay off for the parties to renegotiate the original contract, because this would lead to losses for the parties that are higher than the net benefit in intrinsic utilities \( v - c \).

For which realizations of the state \( \theta \) does the optimality condition (A.12) characterize a \( \hat{x}^H > \bar{x} \)? Put differently, when do goods \( x > \bar{x} \) exist such that \( R(x, \theta) > (1 + \lambda)^2 \). This is the case if the realized state is sufficiently high, i.e., if \( \theta > \hat{\theta} \), with \( \hat{\theta} \) being implicitly defined by \( R(\bar{x}, \hat{\theta}) = (\lambda + 1)^2 \). Note that \( \hat{x}^H(\hat{\theta}) = \bar{x} \) by definition.

We conclude the first step by noting that the parties are indeed better off when \( \hat{x}^H(\theta) > \bar{x} \) is implemented ex post for \( \theta > \hat{\theta} \). By Proposition 1 \( x \) can be implemented ex post iff
\[
\begin{align*}
v(x, \theta) - v(\bar{x}, \theta) &\geq (1 + \lambda)^2[c(x, \theta) - c(\bar{x}, \theta)] \\
\implies \int_{\bar{x}}^{x} \frac{\partial v(z, \theta)}{\partial x} dz &\geq (1 + \lambda)^2 \int_{\bar{x}}^{x} \frac{\partial c(z, \theta)}{\partial x} dz.
\end{align*}
\] (A.14)
The above condition is satisfied for \( x = \hat{x}^H \) by Assumption 1. Hence, there are prices \( p \geq \bar{p} \) such that both parties prefer the new contract with good \( \hat{x}^H \) to the initial contract.

Case 2: Suppose \( \theta < \hat{\theta} \). This case can be proved by similar reasonings as used in the proof of case 1. We outline only the few differences. Obviously, if renegotiation is successful, then the parties agree upon a good \( x < \bar{x} \) and a price \( p < \bar{p} \). The GNP is given by
\[
\text{GNP}(x, p) = \left\{ \bar{p} - p - (\lambda + 1)[v(x, \theta) - v(x, \theta)] \right\}^\alpha \times \left\{ c(\bar{x}, \theta) - c(x, \theta) - (\lambda + 1)(\bar{p} - p) \right\}^{1-\alpha}
\] (A.15)

From the two first-order conditions we obtain the following optimality condition which is independent of the price,
\[
R(x, \theta) = \frac{1}{(1 + \lambda)^2}.
\] (A.16)
If \( R(x, \theta) > 1/(1 + \lambda)^2 \) for all \( x \in [0, \bar{x}] \), then the parties carry out the initial service \( \bar{x} \). Note that \( \lim_{x \to 0} R(x, \theta) = \infty \). Thus, if there are \( x < \bar{x} \) such that \( R(x, \theta) < 1/(1 + \lambda)^2 \), then there
is a unique \( x \in (0, \bar{x}) \) at which \( R(x, \theta) = 1/(1 + \lambda)^2 \). We denote this solution by \( \hat{x}^L(\theta) \). The solution is indeed lower than the initially specified good (\( \hat{x}^L < \bar{x} \)) if \( \theta \) is sufficiently low, i.e. if \( \theta < \theta^L \), implicitly defined by \( R(\bar{x}, \theta^L) = 1/(1 + \lambda)^2 \). Noting that \( \hat{x}^L(\theta^L) = \bar{x} \) by definition completes the second step.

Obviously, for \( \theta = \hat{\theta} \) the parties cannot benefit from renegotiating the initial contract, which completes the proof. \( \square \)

**Proof of Proposition 4.** By Proposition 2 the renegotiated price \( \hat{p} \) is the initial price \( \bar{p} \) corrected by a term that depends only on \( \lambda, \bar{x}, \alpha, \) and \( \theta \). For \( L(\lambda, \bar{x}, \bar{p}, \alpha) \) only \( |\hat{p} - \bar{p}| \) plays a role. But in this term \( \bar{p} \) cancels out. Thus, \( L(\cdot) \) is independent of \( \bar{p} \).

Showing that \( L(\lambda, \bar{x}, \bar{p}, \alpha) \) is increasing in \( \lambda \) is equivalent to showing that for any initial contract \( (\bar{x}, \bar{p}) \) the surplus from renegotiation \( \Delta S = \Delta U^B + \Delta U^S \) is decreasing with \( \lambda \). Two cases have to be distinguished:

1. Suppose that \( \theta > \bar{\theta} \). In this case \( \hat{x} \geq \bar{x} \) and \( \hat{p} \geq \bar{p} \). Thus

\[
\Delta S = v(\hat{x}, \theta) - \hat{p} - \lambda[\hat{p} - \bar{p}] - v(\bar{x}, \theta) + \bar{p} + \hat{p} - c(\hat{x}, \theta) - \lambda[c(\hat{x}, \theta) - c(\bar{x}, \theta)] - \bar{p} + c(\bar{x}, \theta)
\]

\[
= [v(\hat{x}, \theta) - v(\bar{x}, \theta)] - (1 + \lambda)c(\hat{x}, \theta) - c(\bar{x}, \theta)] - \lambda[\hat{p} - \bar{p}]
\]

\[
= \Delta v - (1 + \lambda)\Delta c - \lambda[\hat{p} + \frac{1 - \alpha}{1 + \lambda}\Delta v + \alpha(1 + \lambda)\Delta c - \bar{p}]
\]

\[
= \frac{1 + \alpha\lambda}{1 + \lambda}[\Delta v - (1 + \lambda)^2\Delta c]
\]

Differentiating with respect to \( \lambda \) we get:

\[
\frac{\partial \Delta S}{\partial \lambda} = \frac{\alpha(1 + \lambda) - (1 + \alpha\lambda)}{(1 + \lambda)^2} [\Delta v - (1 + \lambda)^2\Delta c]
\]

\[
+ \frac{[\partial v \partial \hat{x}]}{\partial x \partial \lambda} - 2(1 + \lambda)\Delta c - (1 + \lambda)^2 \frac{\partial c \partial \hat{x}}{\partial x \partial \lambda} \frac{1 + \alpha\lambda}{1 + \lambda}
\]

\[
= -\frac{1 - \alpha}{(1 + \lambda)^2} \begin{cases} \Delta v - (1 + \lambda)^2\Delta c \geq 0 \\ < 0 \end{cases}
\]

\[
+ \left[ \frac{\partial v}{\partial x} - (1 + \lambda)^2 \frac{\partial c}{\partial x} \right] \frac{\partial \hat{x}}{\partial \lambda} - 2(1 + \lambda)\Delta c \right] \frac{1 + \alpha\lambda}{1 + \lambda} \leq 0.
\]

(A.18)
To see that \( \frac{\partial \Delta S}{\partial \lambda} < 0 \) at \( \lambda = 0 \), note that by Assumption 1 \( \Delta_v = 0 \) if and only if \( \Delta_c = 0 \). Hence, \( \frac{\partial \Delta S}{\partial \lambda} = 0 \) if and only if \( \Delta_v = \Delta_c = 0 \). Moreover, Proposition 3 implies that for every \( \theta > \bar{\theta} \) there exists a \( \lambda \) sufficiently close to zero such that for all \( \lambda < \bar{\lambda} \) we have \( \Delta_v > 0 \), and the strict inequality holds – i.e., for \( \lambda \) sufficiently close to 0 renegotiation to a larger \( x \) takes place if \( \theta > \bar{\theta} \), which implies that \( \Delta_v > 0 \) and \( \Delta_c > 0 \).

2. Suppose now that \( \theta < \bar{\theta} \). In this case \( \hat{x} \leq \bar{x} \) and \( \hat{p} \leq \bar{p} \). Thus

\[
\Delta S = v(\hat{x}, \theta) - \hat{p} - \lambda [\hat{v}(\hat{x}, \theta) - v(\hat{x}, \theta)] - \hat{p} + \hat{c}(\hat{x}, \theta) - \lambda [\hat{p} - p] - \hat{p} + c(\bar{x}, \theta)
\]

\[
= -(1 + \lambda) [v(\hat{x}, \theta) - v(\bar{x}, \theta)] + [\hat{c}(\bar{x}, \theta), -c(\hat{x}, \theta)] - \lambda [\hat{p} - p]
\]

\[
= \Delta_c - (1 + \lambda) \Delta_v - \lambda \left[ \hat{p} - \bar{p} + (1 - \alpha)(1 + \lambda) \Delta_v + \frac{\alpha}{1 + \lambda} \Delta_c \right]
\]

\[
= \frac{1 + \lambda - \alpha \lambda}{1 + \lambda} [\Delta_c - (1 + \lambda)^2 \Delta_v]. \quad (A.19)
\]

Differentiating with respect to \( \lambda \) we get:

\[
\frac{\partial \Delta S}{\partial \lambda} = \frac{(1 - \alpha)(1 + \lambda) - (1 + \lambda - \alpha \lambda)}{(1 + \lambda)^2} \left[ \Delta_c - (1 + \lambda)^2 \Delta_v \right]
\]

\[
+ \left[ \frac{\partial c}{\partial x} \frac{\partial \hat{x}}{\partial \lambda} - 2(1 + \lambda) \Delta_v - (1 + \lambda)^2 \frac{\partial v}{\partial x} \frac{\partial \hat{x}}{\partial \lambda} \right] \frac{1 + \lambda - \alpha \lambda}{1 + \lambda}
\]

\[
= - \left[ \frac{\partial c}{\partial x} - (1 + \lambda)^2 \frac{\partial \hat{x}}{\partial x} \frac{\partial \hat{x}}{\partial \lambda} \right] \frac{\partial \hat{x}}{\partial \lambda} \frac{\partial v}{\partial x} \frac{\partial \hat{x}}{\partial \lambda} \frac{2(1 + \lambda) \Delta_v}{\Delta_c - (1 + \lambda)^2 \Delta_v} \frac{1 + \lambda - \alpha \lambda}{1 + \lambda} \leq 0. \quad (A.20)
\]

Showing that \( \frac{\partial \Delta S}{\partial \lambda} < 0 \) at \( \lambda = 0 \) follows the same lines as in the case where \( \theta > \bar{\theta} \).

\[\boxdot\]

**Proof of Corollary 1.** First, note that the bounds \( \theta^L \) and \( \theta^H \) do not depend on the distribution. Moreover, for \( \theta < \bar{\theta} \) we have \( F_1(\theta) < F_2(\theta) \) and for \( \theta > \bar{\theta} \) it holds that \( F_2(\theta) < F_1(\theta) \). From the definition of \( \rho(F) \) it follows immediately that \( \rho(F_1) < \rho(F_2) \).

\[\boxdot\]

**Proof of Corollary 2.** We have to show that \( \rho(F, \lambda) = F(\theta^L(\lambda)) + 1 - F(\theta^H(\lambda)) \) is decreasing in \( \lambda \). We first show that \( \frac{\partial \rho^L(\lambda)}{\partial \lambda} < 0 \). By Proposition 3 \( \theta^L \) is implicitly defined by \( \hat{x}^L(\theta^L) - \bar{x} = 0 \).
By the implicit function theorem
\[
\frac{\partial \theta}{\partial \lambda} = -\frac{\partial L}{\partial \hat{x}^L / \partial \theta}.
\] (A.21)
\[\hat{x}^L(\theta, \lambda)\] is implicitly defined by
\[
\frac{\partial v(\hat{x}^L, \theta)}{\partial x} - \frac{1}{1+\lambda} \frac{\partial c(\hat{x}^L, \theta)}{\partial x} = 0.
\] Using the implicit function theorem twice again we get
\[
\frac{\partial \hat{x}^L}{\partial \theta} = -\frac{\partial^2 v}{\partial x^2} - \frac{1}{1+\lambda} \frac{\partial^2 c}{\partial x^2} > 0,
\] (A.22)
\[
\frac{\partial \hat{x}^L}{\partial \lambda} = -\frac{2(1+\lambda)^{-3} \frac{\partial v}{\partial x}}{1+\lambda} \frac{\partial^2 c}{\partial x^2} > 0.
\] (A.23)
In both equations the numerator is positive by Assumption 1, while the denominators are negative by Assumption 1. Thus, \(\frac{\partial \theta}{\partial \lambda} < 0\). By the same line of argument it is straightforward to show that \(\frac{\partial \theta}{\partial \lambda} > 0\).
Hence, we get
\[
\frac{\partial \rho}{\partial \lambda} = \frac{\partial F}{\partial \theta} \frac{\partial \theta^L}{\partial \lambda} > 0.
\] (A.24)

Proof of Proposition 5. The proof distinguishes three cases.

Case (i) \(\lambda^{SC} \leq \lambda \leq \lambda\): In this case, \(S^{SC} \geq S^{LTC}\) if and only if
\[
(1 - \pi)\lambda \left\{ \frac{1 - \alpha}{1 + \lambda} v^* + [1 + \alpha(1 + \lambda)]c^* \right\} \geq (1 - \beta)\lambda^{SC} \left\{ \frac{1 - \alpha}{1 + \lambda^{SC}} v^* + [1 + \alpha(1 + \lambda^{SC})]c^* \right\}
\] (A.25)
Inequality (A.25) is obviously satisfied if \(\pi \leq \beta\), because \(\lambda \geq \lambda^{SC}\). For \(\pi > \beta\), note that the left-hand side of (A.25) is strictly positive. The right-hand side approaches zero for \(\lambda^{SC} \to 0\). Moreover, the right-hand side is strictly increasing in \(\lambda^{SC}\) and the inequality is violated for \(\lambda^{SC} = \lambda\). Thus, there exists a threshold \(\tilde{\lambda}_1\) so that (A.25) holds if and only if \(\lambda^{SC} \leq \tilde{\lambda}_1 < \lambda\).

Case (ii) \(\lambda^{SC} \leq \tilde{\lambda} < \lambda\): In case (ii), we have \(S^{SC} \geq S^{LTC}\) if and only if
\[
(1 - \pi)(v^* - c^*) \geq (1 - \beta)\lambda^{SC} \left\{ \frac{1 - \alpha}{1 + \lambda^{SC}} v^* + [1 + \alpha(1 + \lambda^{SC})]c^* \right\}
\] (A.26)

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Again, the right-hand side is strictly increasing in \( \lambda^{SC} \) and approaches zero for \( \lambda^{SC} \to 0 \). For \( \lambda^{SC} = \bar{\lambda} \), inequality (A.26) simplifies to

\[
(1 - \pi)(v^* - c^*) \geq (1 - \beta)(v^* - c^*). \tag{A.27}
\]

Thus, if \( \pi \leq \beta \), then (A.26) holds for all \( \lambda^{SC} \in [0, \bar{\lambda}] \). For \( \pi > \beta \) inequality (A.26) is satisfied if and only if \( \lambda^{SC} \) is sufficiently small, i.e., if and only if \( \lambda^{SC} \leq \hat{\lambda}_2 < \bar{\lambda} \), where \( \hat{\lambda}_2 \) is defined as the \( \lambda^{SC} \) for which (A.26) holds with equality.

\textit{Case (iii) } \( \bar{\lambda} < \lambda^{SC} \leq \lambda \): Now, \( S^{SC} \geq S^{LTC} \) if and only if \( \beta \geq \pi \) follows directly from a comparison of equations (20) and (23).

In order to complete the proof, define \( \hat{\lambda}(\lambda) \) as follows: For \( \lambda^{SC} \leq \bar{\lambda} \), let \( \hat{\lambda}(\lambda) = \hat{\lambda}_1 \) if \( \lambda \leq \bar{\lambda} \) and \( \hat{\lambda}(\lambda) = \hat{\lambda}_2 \) if \( \lambda > \bar{\lambda} \). For \( \lambda^{SC} > \bar{\lambda} \), if \( \beta \leq \pi \) we have \( S^{LTC} \geq S^{SC} \) for all \( \lambda \geq \lambda^{SC} \). For \( \lambda^{SC} \geq \bar{\lambda} \) let \( \hat{\lambda}(\lambda) = 0 \), so that \( \lambda^{SC} \) is always greater than \( \hat{\lambda}(\cdot) \).

\textit{Proof of Lemma 1.} First, we solve for the buyer’s optimal investment for the two cases; renegotiation and no renegotiation. Thereafter, we show that renegotiation takes place if and only if \( \lambda \leq \bar{\lambda} \). Finally, we derive the expected surplus generated by the contract.

\textit{Step 1:} If \( \lambda \leq \bar{\lambda}(I) \) and \( \theta \neq \theta_1 \), the parties renegotiate to the materially efficient good \( \hat{x} = x^*(\theta) \) at price

\[
\hat{p} = \bar{p} + \frac{1 - \alpha}{1 + \lambda}(v^* + I + \psi) + \alpha(1 + \lambda)(c^* - \epsilon). \tag{A.28}
\]

Thus, the buyer’s expected utility is given by

\[
EU^B = v^* + I - \frac{1}{2}I^2 - \bar{p} - (1 - \pi_1) \left[ (1 - \alpha)(v^* + I - \psi) + \alpha(1 + \lambda)^2(c^* - \epsilon) \right], \tag{A.29}
\]

which is maximized at investment

\[
I^{CR} = \pi_1 + \alpha(1 - \pi_1) < 1. \tag{A.30}
\]

If \( \lambda > \bar{\lambda}(I) \), the parties do not renegotiate and trade the good \( x_1 \) at price \( \bar{p} \) in all states. The buyer’s expected utility is given by

\[
EU^B = \pi_1[v^* + I] + (1 - \pi_1)\psi - \bar{p} - \frac{1}{2}I^2. \tag{A.31}
\]

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In this case, the optimal investment is

\[ I^{CNR} = \pi_1 < 1. \]  \hspace{1cm} (A.32)

**Step 2:** Because \( I^{CNR} = \pi_1 < \pi_1 + \alpha(1 - \pi_1) = I^{CR} \) we have \( \lambda(I^{CNR}) < \lambda(I^{CR}) \). Thus, if \( \lambda \leq \lambda(I^{CNR}) \) the buyer anticipates that there will be renegotiation if \( \theta \neq \theta_1 \), so he invests \( I^{CR} \). Similarly, if \( \lambda > \lambda(I^{CR}) \) the buyer anticipates that there will be no renegotiation if \( \theta \neq \theta_1 \), so he invests \( I^{CNR} \). If, however, \( \lambda(I^{CNR}) < \lambda \leq \lambda(I^{CR}) \) there are two candidates for the optimal strategy of the buyer. He may invest \( I^{CR} \), which is the optimal investment given that with \( I^{CR} \) there will be renegotiation if \( \theta \neq \theta_1 \), or he may invest \( I^{CNR} \) which is the optimal investment given that with \( I^{CNR} \) there will be no renegotiation if \( \theta \neq \theta_1 \). If he chooses the first strategy and invests \( I^{CR} \), his expected utility is

\[
EU^B(I^{CR}, \lambda) = v^* + I^{CR} - \frac{1}{2}(I^{CR})^2 - \bar{p} - (1 - \pi_1) [(1 - \alpha)(v^* + I^{CR} - v) + \alpha(1 + \lambda)^2(c^* - c)]
\]  \hspace{1cm} (A.33)

If he follows the second strategy his expected utility is

\[
EU^B(I^{CN}, \lambda) = \pi_1(v^* + I^{CNR}) + (1 - \pi_1)v - \frac{1}{2}(I^{CNR})^2 - \bar{p} - (1 - \pi_1) [v^* + I^{CNR} - v]
\]  \hspace{1cm} (A.34)

The buyer prefers the first strategy over the second strategy if and only if

\[
I^{CR} - \frac{1}{2}(I^{CR})^2 - I^{CNR} + \frac{1}{2}(I^{CNR})^2 \geq (1 - \pi_1) [(1 - \alpha)(v^* + I^{CR} - v) - (v^* + I^{CNR} - v) + \alpha(1 + \lambda)^2(c^* - c)]
\]  \hspace{1cm} (A.35)

Inserting (A.30) and (A.32) into the above inequality and solving for \( \lambda \) yields

\[
\lambda \leq \sqrt{\frac{v^* - v + \pi_1 + \alpha(1 - \pi_1)}{c^* - c}} - 1 \equiv \bar{\lambda}
\]  \hspace{1cm} (A.36)

Note that \( \bar{\lambda}(I^{CNR}) < \bar{\lambda} < \bar{\lambda}(I^{CR}) \). Thus if \( \lambda \leq \bar{\lambda} \) the buyer prefers to invest \( I^{CR} \) and there will be renegotiation if \( \theta \neq \theta_1 \), while if \( \lambda > \bar{\lambda} \) the buyer invests \( I^{CNR} \) and there will be no renegotiation if \( \theta \neq \theta_1 \).

**Step 3:** For \( \lambda \leq \bar{\lambda} \) the buyer invests \( I^{CR} \) and his expected utility is given by (A.33). The seller’s expected utility amounts to

\[
EU^S = \bar{p} - c^* + (1 - \pi_1) \left[ \frac{1 - \alpha}{1 + \lambda} (v^* + I^{CR} - v) + \alpha(1 + \lambda)(c^* - c) - \lambda(c^* - c) \right].
\]  \hspace{1cm} (A.37)
Thus, total expected social surplus is

\[ ES^{CR} = v^* - c^* + I^{CR} - \frac{1}{2} (I^{CR})^2 - \lambda (1 - \pi_1) \left[ \frac{1 - \alpha}{1 + \lambda} (v^* + I^{CR} - \bar{v}) + (\alpha (1 + \lambda) + 1) (c^* - \bar{c}) \right] \].

(A.38)

For \( \lambda > \bar{\lambda} \) the buyer chooses \( I^{CNR} \) and his expected utility is given by (A.34). The expected utility of the seller is

\[ EU^S = \bar{p} - \pi_1 c^* - (1 - \pi_1) \bar{c}, \]

(A.39)

and thus expected social surplus is given by

\[ ES^{CN} = \pi_1 [v^* - c^* + I^{CNR}] + (1 - \pi_1) (v - c) - \frac{1}{2} (I^{CNR})^2. \]

(A.40)

Combining steps 1–3 completes the proof.

\( \square \)

**Proof of Lemma 2.** Obviously, if the parties rely on asset ownership, the buyer optimally owns \( A \). In this case, the buyer’s utility is given by

\[ U^B = v^* + I - p - \frac{1 - \alpha}{1 + \lambda} (1 - \beta) I - \lambda \frac{(1 - \alpha)}{1 + \lambda} (1 - \beta) I - \frac{1}{2} I^2 \]

\[ = v^* + I - p - (1 - \alpha)(1 - \beta) I - \frac{1}{2} I^2, \]

(A.41)

which is maximized at investment \( I^A = \beta + (1 - \beta) \alpha \). The seller’s utility is

\[ U^S = \bar{p} + \frac{(1 - \alpha)(1 - \beta)}{1 + \lambda} I^A - c^*. \]

(A.42)

Thus, the surplus generated by allocating asset ownership is given by

\[ S^A = v^* - c^* + I^A - \frac{1}{2} (I^A)^2 - \frac{\lambda}{1 + \lambda} (1 - \alpha)(1 - \beta) I^A \]

(A.43)

\( \square \)

**Proof of Proposition 6.** From Lemmas 1 and 2 it follows immediately that ownership rights outperform specific performance contracts iff \( D = S^A - ES^C \geq 0 \). We prove the statements of the proposition separately for the cases \( \lambda \leq \bar{\lambda} \) and \( \lambda > \bar{\lambda} \).
Case I ($\lambda \leq \bar{\lambda}$): In this case, the difference in expected social surplus, $D_I = S^A - ES^C$, is given by

$$D_I = I^A - \frac{1}{2}(I^A)^2 - \left[I^{CR} - \frac{1}{2}(I^{CR})^2 \right] - \frac{\lambda}{1 + \lambda} (1 - \alpha)(1 - \beta)I^A$$

$$+ \frac{\lambda}{1 + \lambda} (1 - \alpha)(1 - \beta)I^{CR} + \lambda (1 - \pi_1) \left\{ \frac{1 - \alpha}{1 + \lambda} (v^* - v) + [\alpha (1 + \lambda) + 1](c^* - \bar{c}) \right\}.$$ \hspace{1cm} (A.44)

Note that $D_I$ is continuous in $\alpha$. Moreover, for $\alpha \to 1$, we have $I^A \to 1$ and $I^{CR} \to 1$. Thus, $\lim_{\alpha \to 1} D_I > 0$.

Next, we take the partial derivative of $D_I$ with respect to $\beta$:

$$\frac{\partial D_I}{\partial \beta} = (1 - \alpha)(1 - I^A) \left( \frac{1}{1 + \lambda} \right) + \frac{\lambda}{1 + \lambda} (1 - \alpha)(\alpha + \beta - \alpha \beta) > 0.$$ \hspace{1cm} (A.45)

Taking the partial derivative of $D_I$ with respect to $\pi_1$ yields

$$\frac{\partial D_I}{\partial \pi_1} = -(1 - \alpha) + (1 - \alpha)I^{CR} - \frac{\lambda}{1 + \lambda} (1 - \alpha)I^{CR} + \frac{\lambda}{1 + \lambda} (1 - \alpha)^2(1 - \pi_1)$$

$$- \lambda \left\{ \frac{1 - \alpha}{1 + \lambda} (v^* - v) + [\alpha (1 + \lambda) + 1](c^* - \bar{c}) \right\}.$$ \hspace{1cm} (A.46)

Rearranging the above equation leads to

$$\frac{\partial D_I}{\partial \pi_1} = -(1 - \alpha)(1 - I^{CR}) \frac{1}{1 + \lambda} - (1 - \alpha)(\alpha + \pi_1 - \alpha \pi_1) \frac{\lambda}{1 + \lambda}$$

$$- \lambda \left\{ \frac{1 - \alpha}{1 + \lambda} (v^* - v) + [\alpha (1 + \lambda) + 1](c^* - \bar{c}) \right\} < 0.$$ \hspace{1cm} (A.47)

Case II ($\lambda > \bar{\lambda}$): In this case, the difference in expected social surplus, $D_{II} = S^A - ES^C$, is given by

$$D_{II} = (v^* - c^*)(1 - \pi_1) + I^A - \frac{1}{2}(I^A)^2 - \frac{\lambda}{1 + \lambda} (1 - \alpha)(1 - \beta)I^A$$

$$- \pi_1 I^{CNR} - (1 - \pi_1)(v - \bar{c}) + \frac{1}{2}(I^{CNR})^2.$$ \hspace{1cm} (A.48)

First, note that $D_{II}$ is continuous in $\alpha$ and that

$$\lim_{\alpha \to 1} D_{II} = (1 - \pi_1) [(v^* - c^*) - (v - \bar{c})] + \frac{1}{2} (1 - \pi_1^2) > 0.$$ \hspace{1cm} (A.49)
Taking the partial derivative of $D_{II}$ with respect to $\beta$ give us the following expression:

$$
\frac{\partial D_{II}}{\partial \beta} = (1 - \alpha)(1 - I^A) \frac{1}{1 + \lambda} + \frac{\lambda}{1 + \lambda}(1 - \alpha)(\alpha + \beta - \alpha \beta) > 0.
$$

(A.50)

The partial derivative of $D_{II}$ with respect to $\pi_1$ is

$$
\frac{\partial D_{II}}{\partial \pi_1} = -[(v^* - c^*) - (\bar{v} - \bar{c})] - \pi_1 < 0.
$$

(A.51)

Combining case I and case II establishes that $\lim_{\alpha \to 1} D > 0$ and that $D(\cdot | \alpha, \pi_1, \lambda)$ is strictly increasing in $\beta$. The result that $D(\cdot | \alpha, \beta, \lambda)$ is strictly decreasing in $\pi_1$ does not follow directly from cases I and II, because the threshold $\bar{\lambda}$ is a function of $\pi_1$ (but not of $\beta$). That is, there can exist a critical $\bar{\pi}_1(\lambda)$ implicitly defined by $\lambda \equiv \bar{\lambda}(\bar{\pi}_1, \cdot)$. Crucially, $D$ is not continuous at $\bar{\pi}_1$, i.e., $D_I$ and $D_{II}$ are not equal at $\lambda = \bar{\lambda}$. Note that $\bar{\lambda}$ is increasing in $\pi_1$. Thus, for small $\pi_1$ we can have $\pi_1 < \bar{\pi}_1(\lambda)$, which implies that $\lambda > \bar{\lambda}$, while for high $\pi_1$ we have $\pi_1 \geq \bar{\pi}_1(\lambda)$, which implies that $\lambda \leq \bar{\lambda}$. In other words, $D(\cdot)$ as a function of $\pi_1$ is given by:

$$
D(\pi_1 | \alpha, \beta, \lambda) = \begin{cases} 
D_{II}(\cdot) & \text{if } \pi_1 < \bar{\pi}_1; \\
D_I(\cdot) & \text{if } \pi_1 \geq \bar{\pi}_1.
\end{cases}
$$

(A.52)

Next, we show that $D(\cdot)$ has a downward “jump” at $\pi_1 = \bar{\pi}_1$, which establishes the desired result. First, note that the difference between $D_I$ and $D_{II}$ is caused by the expected social surplus generated with a specific performance contract. Second, for $\pi_1 = \bar{\pi}_1$ we have $\lambda = \bar{\lambda}$. By the definition of $\bar{\lambda}$, the buyer is indifferent between his two strategies at this point. Hence, the discontinuity in expected social surplus of the specific performance contract is due to the seller’s expected utility. The seller prefers that renegotiation takes place to no renegotiation if and only if

$$
\frac{1 - \alpha}{1 + \lambda}(v^* - v + I^{CR}) + \alpha(1 + \lambda)(c^* - \bar{c}) - \lambda(c^* - \bar{c}) > c^* - \bar{c}.
$$

(A.53)

Thus, when (A.53) holds at $\lambda = \bar{\lambda}$, then $D(\pi_1 | \cdot)$ has a downward discontinuity at $\bar{\pi}_1$. Rearranging inequality (A.53) yields

$$
\lambda < \sqrt{\frac{v^* - v + I^{CR}}{c^* - \bar{c}}} - 1 = \bar{\lambda}(I^{CR}).
$$

(A.54)

Finally, noting that $\bar{\lambda} < \bar{\lambda}(I^{CR})$ completes the proof.

\[\square\]
Proof of Lemma 3. Suppose the parties have written a specific performance contract \((\bar{x}, \bar{p})\).

With probability one the realized state of the world is such that \(x^*(\theta) \neq \bar{x}\). Thus, by Proposition 2 there is scope for renegotiation if and only if there exists a \(p\) such that

\[
\frac{v^* - \bar{v}}{1 + \lambda} \geq p - \bar{p} \geq (1 + \lambda)(c^* - \bar{c}).
\]  

(A.55)

Such a price \(p\) exists if and only if

\[
\lambda \leq \sqrt{\frac{v^* - \bar{v}}{c^* - \bar{c}}} - 1 \equiv \lambda^S.
\]  

(A.56)

Hence, if \(\lambda < \lambda^S\), the parties renegotiate and trade the service \(x^*(\theta)\) at price

\[
\hat{p}^S = \bar{p} + \frac{1 - \alpha}{1 + \lambda} [v^* - \bar{v}] + \alpha (1 + \lambda) [c^* - \bar{c}].
\]  

(A.57)

In this case the buyer’s utility is \(U^B = v^* - \hat{p}^S - \lambda [\hat{p}^S - \bar{p}]\) while the seller’s utility is \(U^S = \hat{p}^S - c^* - \lambda [c^* - \bar{c}]\). If \(\lambda > \lambda^S\), there is no renegotiation and payoffs are \(U^B = v - p\) and \(U^S = p - \bar{c}\). Thus, the total surplus generated by a specific performance contract is given by

\[
S_{SPC} = \begin{cases} 
  v^* - c^* - \lambda (1 + \alpha (1 + \lambda)) [c^* - \bar{c}] - \frac{\lambda(1-\alpha)}{1+\lambda} [v^* - \bar{v}] & \text{if } \lambda \leq \lambda^S \\
  v - c & \text{if } \lambda > \lambda^S.
\end{cases}
\]  

(A.58)

Proof of Lemma 4. As shown in the main text – given our equilibrium selection – the seller complies and produces \(\tilde{x}(\theta)\) if and only if \(\lambda \geq \lambda^A_1\). If this is the case, then it is obviously optimal for the buyer to demand \(\tilde{x}(\theta)\) if renegotiation does not take place. For \(\lambda < \lambda^A_1\), the seller prefers to quit instead of producing \(\tilde{x}(\theta)\). In this case it is optimal for the buyer to demand good \(x^*(\theta)\). This is anticipated by the seller and therefore her reference point in this case is \((c^*, \bar{p})\). Note that for this reference point it is never optimal for the seller to quit, since \(\bar{p} \geq c^*\). Thus, the seller’s reference point is consistent with what the buyer demands if renegotiation fails. The social surplus generated is \(S_{AC} = v^* - c^*\) if \(\lambda < \lambda^A_1\).

For \(\lambda \geq \lambda^A_1\), the buyer demands \(\tilde{x}(\theta)\) if renegotiation fails in state \(\theta\). There is scope for voluntary renegotiation – given the threat point induced by the buyer’s behavior – if and only if there is a price \(p\) so that

\[
(1 + \lambda)(v - v^+) \leq p - \bar{p} \leq \frac{\bar{c} - c^*}{1 + \lambda},
\]  

(A.59)
which is the case if and only if \( \lambda \leq \bar{\lambda}_2^A \). As tie-breaking rule, we assume here that renegotiation does not take place if \( \lambda = \bar{\lambda}_2^A \), i.e., if the parties are indifferent between renegotiation and no renegotiation (has no impact on the results).

Hence, if \( \lambda \geq \max\{\bar{\lambda}_1^A, \bar{\lambda}_2^A\} \), good \( \tilde{x}(\theta) \) is traded. The buyer’s utility is \( U^B = \tilde{v} - \tilde{p} \) and the seller’s utility is \( U^S = \tilde{p} - \tilde{c} \). Thus, the social surplus amounts to \( S^{AC} = \tilde{v} - \tilde{c} \).

If \( \bar{\lambda}_1^A \leq \bar{\lambda}_2^A \), then there are values of \( \lambda \) so that \( \bar{\lambda}_1^A \leq \lambda \leq \bar{\lambda}_2^A \). In these cases the parties implement \( x^*(\theta) \) at price

\[
\hat{p}^A = \bar{p} - (1 - \alpha)(1 + \lambda)(\tilde{v} - v^*) - \frac{\alpha}{1 + \lambda}(\tilde{c} - c^*)
\]  

(A.60)

ex post through renegotiation. The buyer’s and the seller’s utility is

\[
U^B = v^* - \hat{p}^A - \lambda(\tilde{v} - v^*)
\]

and

\[
U^S = \hat{p}^A - c^* - \lambda(\bar{p} - \hat{p}^A) .
\]

Hence, the social surplus is given by

\[
S^{AC} = v^* - c^* - \lambda[1 + (1 - \alpha)(1 + \lambda)] - \frac{\lambda}{1 + \lambda}\alpha(\tilde{c} - c^*) .
\]  

(A.61)

Proof of Proposition 7. We prove the statements (i) – (iv) in turn.

(i) If \( \lambda = 0 \), we have \( S^{AC} = S^{SPC} = v^* - c^* \).

(ii) If \( 0 < \lambda < \bar{\lambda}_1^A \), comparing \( S^{AC} \) and \( S^{SPC} \) directly reveals that \( D > 0 \).

(iii) If \( \lambda \geq \max\{\bar{\lambda}_1^S, \bar{\lambda}_1^A, \bar{\lambda}_2^A\} \), then from Lemmas 3 and 4 it follows that \( D = (\tilde{v} - \tilde{c}) - (v - c) \).

(iv) If \( \max\{\bar{\lambda}_1^S, \bar{\lambda}_1^A\} \leq \lambda < \max\{\bar{\lambda}_1^A, \bar{\lambda}_2^A\} \), then it holds that

\[
D = \lambda[1 + \alpha(1 + \lambda)](c^* - c) + \frac{\lambda}{1 + \lambda}(1 - \alpha)(v^* - v) - \lambda[1 + (1 - \alpha)(1 + \lambda)](\tilde{v} - v^*) + \frac{\lambda}{1 + \lambda}\alpha(\tilde{c} - c^*) .
\]  

(A.62)

The signs of the partial derivatives follow directly from the above expression.
Proof of Proposition 8. By Proposition 1 the renegotiation set is independent of $\bar{p}$ and by Proposition 3 the renegotiation outcome and the renegotiation markup $\hat{p} - \bar{p}$ is also independent of $\bar{p}$. Thus, no matter which $\hat{p}(\sigma)$ is in place at the renegotiation stage, the renegotiation outcome is always the same.
References


