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# Analyzing economic policies that affect supply and demand: a structural model of productivity, labor supply and rationing

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2014: Evidenzbasierte Wirtschaftspolitik - Session: Labour Demand, No. F08-V3

#### **Provided in Cooperation with:**

Verein für Socialpolitik / German Economic Association

Suggested Citation: Müller, Kai-Uwe (2014) : Analyzing economic policies that affect supply and demand: a structural model of productivity, labor supply and rationing, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2014: Evidenzbasierte Wirtschaftspolitik - Session: Labour Demand, No. F08-V3, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft, Kiel und Hamburg

This Version is available at: https://hdl.handle.net/10419/100471

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# Analyzing economic policies that affect supply and demand: a structural model of productivity, labor supply and rationing

Kai-Uwe Müller\*

#### Abstract

In this paper a labor supply model with demand side rationing is estimated to analyze the economic policies that directly affect incentives to work as well as labor costs. The framework is applied to evaluate the employment effects of a federal minimum wage in Germany and the impact of employervs. employee-oriented wage subsidies under a statutory minimum. We extend Laroque and Salanié (2002) by modeling the extensive and intensive margin of labor supply on the basis of desired working hours. While previous studies for Germany (Bargain et al., 2010) identify the rationing risk primarily from exogenous demand side factors, this paper structurally relates it to individual productivity which is determined in a jointly estimated wage/productivity equation. Unobserved individual factors are allowed to influence preferences and constraints. The variation needed to identify labor supply and demand is generated by the tax and transfer system and labor market regulations defining minimum standards of pay. Simulation exercises prove the value of the model for policy analysis. Differing adjustments at the extensive and intensive margin are revealed that are related to heterogeneous productivities.

KEYWORDS: household labor supply, rationing, minimum wage, wage subsidies, maximum simulated likelihood

JEL classification: J22, J23, J31, J38

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## 1 Introduction

After longstanding discussions the new government adopted a federal minimum wage for Germany. This far-reaching labor market reform implies a wage gain for workers bound by the minimum which increases net incomes and labor supply incentives. At the same time the rise in wage costs could lead to substitution away from (lowproductivity) labor and also induce output demand effects because of cost shifting to consumers. The effects of different types of wage subsidies for low wage employment would also change when a federal minimum wage is introduced. To analyze these issues empirically we will extend existing labor supply models that also determine the risk of being rationed by insufficient labor demand.

Involuntary unemployment is outside the scope of most microeconometric labor supply models (Blundell and Macurdy, 1999; Creedy and Kalb, 2006; Mroz, 1987). Estimated differences in labor supply would only equal changes in employment, if labor demand was perfectly elastic. Scenarios where an individual would like to work, but is not able to find a job, are not captured by these models. The severity of this limitation depends on the specific application as the bias in labor supply elasticities is a priori ambiguous (Ham, 1982; Bargain et al., 2010). The omission of labor demand constraints becomes particularly relevant when economic policies directly influence labor supply incentives and at the same time labor costs. A federal minimum wage is a textbook example for such a policy.

One approach to incorporate rationing into structural labor supply models is to combine them with labor demand elastities from computable general equilibrium models (CGE) (Peichl, 2009; Davies, 2009) or structural labor demand estimations (see Peichl and Siegloch, 2012 for an overview). We follow another strand of the literature where a rationing probability is directly integrated into a structural labor supply model (see, e.g., Laroque and Salanié, 2002). Extending previous papers for Germany that identify the rationing risk from exogenous labor demand factors (Bargain et al., 2010; Haan and Uhlendorff, 2013), we systematically relate an individual's productivity to the probability of not finding a job. This provides a structural interpretation of rationing and allows us to distinguish: (a) voluntary inactivity – the individual prefers not to work; (b) structural unemployment – the individual wants to work, but is constrained due to insufficient productivity; (c) cyclical/frictional unemployment – the individual prefers to work, her productivity is adequate, but she is constrained because of insufficient demand; (d) employment – the individual wants to work and finds a job. Estimating the wage/productivity, labor supply and rationing equations jointly helps to model the relationships between those processes caused by observed and unobserved characteristics. Identification of supply and demand is inter alia based on variation (conditional on individual productivity) generated by the tax and transfer system and labor market regulations that define minimum wage thresholds for employers (e.g. sectoral minimum wages, collective bargaining).

The following research questions will be addressed. How can a labor supply model with demand side constraints be formulated where the rationing risk depends among other things on individual productivity? Do labor supply elasticities change when rationing is taken into account? What are the consequences of a federal minimum wage for employment in Germany? Can different types of wage subsidies help to raise employment when a federal minimum wage is in place? The methodological contribution of the paper is to refine and extend existing labor supply models with demand constraints. Our model provides not only unbiased empirical labor supply elasticities for Germany. We are also able to analyze the consequences of a statutory minimum wage and related policies for labor supply and the rationing risk within a coherent framework.

The empirical analysis is focused on men. We find labor supply elasticities to be upward biased in the unconstrained model; participation and hours elasticities are significantly smaller in the model with rationing. Labor supply reactions to the implementation of a minimum wage which are estimated from the unconstrained model would be misleading. The labor supply model with constraints predicts negative participation effects which are larger in the East compared to the West. The loss in total working hours would be considerably smaller, though. While certain people lose their jobs, others benefit from higher wages and expand their hours of work. Although reductions in the volume of employment as a result of a federal minimum might be relatively moderate, jobs from low-productive people might be substituted by more productive workers. According to our simulations, employeeoriented subsidies would be ineffective in expanding employment when a federal minimum wage is in place. On the other hand, subsidies paid to employers and targeted at low-productive workers could nearly offset the negative effects of a federal minimum wage on participation.

The paper proceeds with a brief review of the related literature. In section 3 the econometric model is outlined consisting of a labor supply, wage/productivity, and rationing equation. These elements are then combined in a constrained labor supply model and identification is discussed. In section 4 the data sources and sample are characterized and descriptive statistics are provided. Section 5 presents the empirical results. After a brief discussion of the paramter estimates and labor supply elasticities, two policy simulations are presented: the introduction of a federal minimum wage as well as employee- and employer-oriented wage subsidies under a federal minimum wage. The final section concludes.

## 2 Literature

There are two basic approaches to take labor demand restrictions into account in the estimation of structural labor supply models. First, microeconometric labor supply estimation is combined with more or less disaggregated labor demand elasticities. These elasticities come either from CGE models (Bovenberg et al., 2000; Bourguignon et al., 2005; Arntz et al., 2008; Boeters and Feil, 2009).<sup>1</sup> Alternatively, demand elasticities are identified within a structural labor demand model (Creedy and Duncan, 2005; Haan and Steiner, 2005b; Peichl et al., 2010; Peichl and Siegloch, 2012). The advantage of this approach is that a wage adjustment process is simulated by iterating between the supply and demand modules. The price is heavy, though, as the highly aggregated demand elasticities affect all or many employees equally forgoing the main advantage of a microeconometric model.

Second, several studies explicitly incorporate involuntary unemployment into a structural labor supply model at the micro-level (Blundell et al., 1987; Bingley and Walker, 1997; Blundell et al., 2000; Hogan, 2004). Bargain et al. (2010) set up a discrete choice household labor supply model for Germany. They specify a latent rationing equation which is being identified by exogenous labor market conditions. The resulting double hurdle model is estimated in two independent steps. In addition to information on active search for a job and eligibility to the labor market,

<sup>&</sup>lt;sup>1</sup>Peichl and Schaefer (2009) and Peichl (2009) follow a similar approach.

Bargain et al. exploit information on desired hours for those willing to work. Haan and Uhlendorff (2013) extend this to an intertemporal model and analyze state dependence.

The rationing probability is not systematically related to individual productivity in the aforementioned papers. This contention seems questionable given that the risk of unemployment is often found to be related to productivity. Other labor supply models with demand side rationing are less restrictive in this regard. Institutional features of the labor market (e.g. a minimum wage) are exploited to identify the individual rationing probabilities also as a function of the wage. A wage and/or a productivity equation is jointly determined in those models. Meyer and Wise (1983a,b) took a first step towards this direction.

Laroque and Salanié (2002) estimate a static structural labor supply model for women in France. Their parsimonious model leaves out the intensive margin. It consists of jointly estimated wage and participation equations which are related by an unobserved random term. Laroque and Salanié take all features of the tax and transfer system into account. They are able to distinguish different types of non-employment: voluntary unemployment, classical unemployment (the minimum wage exceeds the estimated wage costs), and frictional or cyclical unemployment (as a residual category). The wage equation including an unobserved component is an argument of a highly non-linear tax function in the participation equation. Based on parametric assumptions Laroque and Salanié integrate out the random component of the likelihood numerically.

Nelissen et al. (2005) develop a more comprehensive approach. Latent productivity is the relevant quantity for labor demand. It is parameterized by observed individual characteristics, macro variables, and an unobserved random error term. The market wage is a function of productivity plus unemployment dynamics in certain segments of the labor market. Wages enter a tax function that determines net household incomes which together with leisure time are the main arguments in the utility function of a discrete choice labor supply model. Information on desired hours are exploited to identify preferences for work of the involuntarily unemployed. An equation for the latent minimum wage relevant to the employer is specified as a function of the legal minimum plus observed characteristics and a random error. This model allows distinguishing between voluntary unemployment, involuntary unemployment, and employment.

A comparable model is used by Euwals and van Soest (1999) to estimate the institutional constraints on desired working hours.<sup>2</sup> In the model by Aaberge et al. (1995) labor supply decisions are discrete choices among different packages of working hours, wage rates and other characteristics. Bloemen (2000) relates the labor supply model to the job search literature and considers different specifications for the distribution of job offers that put constraints on the labor supply decision.

We draw on elements from several of the aforementioned papers. The basic setup is similar to Bargain et al. (2010): we use the same information on observed labor market states, hours of work for employees, but also on involuntary unemployment and desired working hours. Exogenous variation in regional labor market conditions is exploited to identify the rationing risk. We also take up ideas from Nelissen et al. (2005) by giving the wage/productivity equation a structural interpretation and by relating the rationing risk also to the individual's productivity. The specification of the unobserved productivity components in the labor supply and rationing equation is inspired by Laroque and Salanié (2002). We also utilize variation in minimum wage thresholds created by institutional constraints on the wage setting for identification.

## 3 Econometric model

The empirical model consists of three equations. The first represents the individual's labor supply. We employ a discrete choice decision framework based on a household utility function with disposable income, leisure as well as (observed and unobserved) individual and household characteristics as arguments. Second, the model requires a wage/productivity equation for two reasons. It provides on the one side predicted market wages that are required to simulate household incomes for positive hours categories of non-employed individuals. Based on some structural assumptions it allows on the other side to predict an individual's productivity which is a crucial argument in the rationing equation of the model. This third equation represents

<sup>&</sup>lt;sup>2</sup>People might not only be rationed with respect to their participation decision, but also with respect to their preferred working time. The first paper on hours restrictions by Moffitt (1982) extends the Tobit model to account for institutional restrictions on part time work. In another paper van Soest et al. 1990 extend a Hausman (1980) type labor supply model with piecewise linear convex budget constraints to account for hours restrictions (see also Tummers and Woittiez, 1991).

the probability that an individual who decided to participate in the labor market is constrained by insufficient labor demand. The rationing risk depends ceteris paribus on an individual's productivity relative to some institutionally given minimum standard of pay (a minimum wage if applicable, collectively bargained wages, or the basic moral wage threshold as stated in the German Civil Code). Furthermore, a bunch of demand side indicators enter this equation as they also determine the rationing probability. We will subsequently describe the structure of the model and discuss the value of estimating the equations jointly.

#### 3.1 Labor supply decision

We assume that individuals maximize household utility by choosing among j = 1, 2, ..., m different labor market states. A discrete choice approach is conducive to modeling the joint labor supply decision of both spouses in couple households and to deal with non-linear budget sets (van Soest, 1995). To reduce the complexity of the model we take the spouse's decision as given (see Laroque and Salanié, 2002).<sup>3</sup> The utility of the individual *i*'s household for alternative *j* can be written as a function of leisure  $l_{ij}$ , household income  $y_{ij}$ , observed characteristics  $X_{ij}^{(LS)}$  and an unobserved component  $\epsilon_{ij}$ :

$$U_{ij} = U\left(l_{ij}, y_{ij}, X_{ij}^{(LS)}, \epsilon_{ij}\right) \tag{1}$$

Leisure  $l_{ij}$  for the inividual considered is given as  $l_{ij} = TE - h_{ij}$  with TE = 80being the total time endowment and  $h_{ij}$  the hours of work. The discrete labor supply model captures the extensive (zero vs. positive hours) and the intensive margin (different positive hours categories). The number of alternatives in the choice set fits the empirical distribution of hours. For women we consider six labor market states (non-work, marginal employment, low part time, high part time, full time, overtime). For men the set consists of four alternatives (non-work, marginal employment, full time, overtime).<sup>4</sup>

The household net income  $y_{ij}$  depends on the individual's labor income, other household income (including the spouse's labor income for couple households), taxes

<sup>&</sup>lt;sup>3</sup>The extension for couple households is straightforward:  $l_{ij}$  is replaced by  $lm_{ij}$  and  $lf_{ij}$  for men and women leading to a  $m_m \times m_f$  choice set. Yet, the model becomes more complicated as additional wage and rationing equations are needed for the spouse.

<sup>&</sup>lt;sup>4</sup>In applications with German data from the SOEP, results have found to be quite robust with respect to the number of states (Steiner et al., 2012).

and contributions, transfers (unemployment assistance, social assistance, child or housing benefits). The microsimulation model STSM (see Steiner et al., 2012; subsection 4.3 below) is employed to simulate available household incomes  $y_{ij}$  for different choices of labor supply. A sizeable component of household ressources is labor income. Hourly wages are calculated on the basis of reported monthly gross wage earnings and working time for those being in work. For non-employed individuals hourly wages  $\hat{w}_i$  are predicted on the basis of parameters from a wage equation and then inserted into the labor supply equation. We assume throughout this paper hourly wages to be constant for different categories of working hours. In the unconstrained choice model the wage equation is estimated independently from labor supply (see sub-section 3.2 for a discussion). The available household income for alternative j can be written as:

$$y_{ij} = R\left[w_i \times (TE - l_{ij}), y_i^{(other)}, X_{ij}^{(LS)}\right]$$
(2)

R(.) is a highly nonlinear tax function (including benefits for entitled households) with gross labor income  $w_i * (TE - l_{ij})$ , other household income  $y_i^{(other)}$ , and observed characteristics  $X_{ij}^{(LS)}$  as arguments. Note that for couple households the spouse's gross income  $w_i^{(spouse)} * (TE - l_i^{(spouse)})$  is part of  $y_i^{(other)}$  but does not vary with j. When wages are taken to be exogenous  $y_{ij}$  can be simulated for each potential jand inserted into the labor supply equation. When the wage/productivity equation is jointly estimated in the extended model with labor demand constraints below, R(.) becomes part of the labor supply equation resulting in a more complicated likelihood (see sub-section 3.6 below). Since there are no savings in this model, income  $y_{ij}$  equals consumption  $c_{ij}$  and the budget constraint is:

$$C_i = y_{ij} \tag{3}$$

We approximate the utility function with a quadratic specification for household i and choice j:

$$V_{ij} = U(l_{ij}, y_{ij}, x_{ij}, \epsilon_{ij})$$

$$= \alpha_c + \alpha_y y_{ij} + \alpha_l l_{ij} + \alpha_{y^2} y_{ij}^2 + \alpha_{l^2} l_{ij}^2 + \alpha_{yl} l_{ij} y_{ij} + \epsilon_{ij}$$
(4)

Preference heterogeneity is introduced by a number of household- or individualspecific taste shifters  $X^{(LS)}$  with respect to labor supply (age, dummy for singles, dummies for small children, interaction of single and small kid dummies for lone parents, handicap, region). The matrix  $X^{(LS)}$  also includes a choice-specific dummy variable for the part time category of men that improves the model fit of the empirical hours distribution (van Soest, 1995). Otherwise the part time category would be over-predicted by the model. This variable represents the men's distaste of working a low number of hours. The parameters in the utility function are thus functions of  $X^{(LS)}$ :

$$\alpha_c = \alpha_{c0} + \alpha_{c1} X_1^{(LS)}$$
$$\alpha_l = \alpha_{l0} + \alpha_{l1} X_2^{(LS)}$$

Assuming that  $\epsilon_{ij}$  are independently Type I Extreme Value distributed the probability of choosing the alternative k can be shown to be (McFadden, 1974):

$$Pr_{ik} = Pr\left(V_{ik} > V_{ij}, \forall j = 0, \dots, m\right) = \frac{exp\{U(y_{ik}, l_{ik}, X_{ik})\}}{\sum_{j=0}^{m} exp\{U(y_{ij}, l_{ij}, X_{ij})\}}$$
(5)

The probability of choosing a given category results from the comparison of its utility with the utility of all alternatives. In expectation utility maximizing households will always choose the alternative which generates the highest utility. Assumptions about the independence and homoskedasticity of the error terms imply the independence of irrelevant alternatives (IIA) property. The ratio of probabilities for two given categories does not depend on other alternatives.<sup>5</sup>

In the labor supply model without demand side constraints the individuals' (households') preferences are assumed to be in line with the actual labor supply responses. Changes in labor supply are interpreted in terms of employment effects as demand is assumed to be perfectly elastic. In the following sub-sections we extend this choice model with a structural wage/productivity equation and a rationing equation leading to a labor supply model with demand side rationing.

#### **3.2** Wages and productivity

In the unconstrained labor supply model wages are considered to be exogenous conditional on observed covariates. Since hourly wages are needed for all individuals regardless of their labor market status, a (log) wage equation is estimated in order

<sup>&</sup>lt;sup>5</sup>Several alternatives have been suggested in the literature, among them generalized extreme value, discrete choice probit or random coefficient models (Train, 2009).

to predict potential wages for non-employed individuals. The predictions (including randomly drawn error terms to emulate the variance of observed wages) are used to simulate the disposable household incomes for each category. Usually, a Heckman type selection model is estimated to take selection into employment into account. In the unconstrained model a structural interpretation of the wage equation is hardly needed; it mainly serves to create consistent gross wage predictions for the decision model.

In the extended model with demand side rationing different states of nonemployment are distinguished systematically. Potential market wages for nonemployed individuals are needed here, too. In addition, we give different arguments in the wage equation a structural interpretation. The basic assumption is that observed market wages are a function of an individual's productivity  $p_i^*$ , i.e. more productive people earn higher wages. On the other hand market wages are influenced by the labor demand side denoted as  $ld_i^*$ . What a firm is willing or able to pay may depend on its productivity, its market power, or macro-economic shocks. Firms might also discriminate among different employees. The individual productivity and the labor demand (firm) wage components as such are not observable. We assume that both can be written as a function of observed covariates and an unobserved term:

$$p_i^* = X_i'^{(w_p)} \beta^{(w_p)} + e_i^{(w_p)}$$

$$ld_i^* = X_i'^{(w_{ld})} \beta^{(w_{ld})} + e_i^{(w_{ld})}$$
(6)

The observed wage rate paid consists of those two components which are assumed to be linked additively<sup>6</sup>:

$$\ln w_{i} = p_{i}^{*} + ld_{i}^{*}$$

$$= X_{i}^{\prime(w_{p})}\beta^{(w_{p})} + X_{i}^{\prime(w_{ld})}\beta^{(w_{ld})} + e_{i}^{(w_{p})} + e_{i}^{(w_{ld})}$$

$$= X_{i}^{\prime(w_{p})}\beta^{(w_{p})} + X_{i}^{\prime(w_{ld})}\beta^{(w_{ld})} + e_{i}$$
(7)

We cannot separately identify  $e_i^{(w_p)}$  and  $e_i^{(w_{ld})}$  as we rely on employee data in this paper.<sup>7</sup> Therefore we usually refer to the composite term of the wage/productivity

<sup>&</sup>lt;sup>6</sup>This implies that both elements are separable and do not reinforce each other. Alternative specifications that make both components depend on each other could be equally plausible. Yet, the error terms would also be related with the other component and could not be separated as easily.

<sup>&</sup>lt;sup>7</sup>If firm information were available, one could identify an unobserved firm-specific effect.

equation. Assumptions on these unobserved components, particularly on  $e_i^{(w_p)}$ , are crucial for the different specifications of the empirical model.

The matrix  $X_i^{\prime(w_p)}$  contains various variables that can be interpreted as determinants of an individual's productivity: age, school and vocational education, previous labor market experience (years in full time and in part time employment), and the depreciation of human capital due to unemployment and other work interruptions. These variables are related to the individual and do not depend on the firm.

The second category of variables that influence the market wage is primarily related to the firm, i.e. the labor demand side.  $X'_i^{(w_{ld})}$  thus contains sets of industry and firm size dummies to account for variation in pay along these dimensions. A set of dummies for the German federal states is included to mirror regional differences in wage scales. Moreover, a dummy variable for the German nationality controls for discrepancies in pay between natives and foreign nationals which may be due to discrimination. A further dummy for civil servants controls for the gap between public and private sector wages.

Since  $w_i$  is censored for non-working individuals, some form of selection correction has to be included in the wage equation when there are unobserved differences between working and non-working individuals that affect their (potential) wages conditional on  $X^{(w)}$ . When exclusion restrictions are available and under the assumption that the error terms in the wage and selection equation are distributed jointly normal, one can add the inverse Mills ratio from a first-step selection equation according to Heckman (1979) as a selection term. As indicated above this is the standard procedure for discrete choice labor supply models without rationing (van Soest, 1995; Creedy and Duncan, 2005; Steiner et al., 2012). We follow this procedure in the labor supply model with constraints when the equations are assumed to be independent and are estimated separately. Available exclusion restrictions are the degree of disability, marital status, the existence of children in the household, and other household income. An alternative specification is suggested below for the joint estimation of the extended model with rationing (see sub-section 3.5).

#### 3.3 Demand side rationing

The unconstrained labor supply model consists of only two different labor market states: inactivity and employment (with different hours of work). The zero hours category is chosen voluntarily as it maximizes the household's utility. This setup does not capture situations where an individual prefers to work – as this would increase his utility – but is constrained by the labor market. An individual might either not be productive enough to find a job at going market wage rates (structural unemployment), or labor market frictions could create a mismatch between supply and demand (frictional, cyclical unemployment).<sup>8</sup>

In the extended model with rationing three basic labor market states are distinguished: (voluntary) inactivity, (involuntary) unemployment, and employment. For inactive people actual  $(h^a)$  and desired  $(h^d)$  working hours are equal to zero, i.e.  $h^a = h^d = 0$ . Inactive individuals do not intend to work and are not looking for a job. Unemployed people, on the other hand, would like to work, i.e.  $0 = h^a < h^d$ , and are actively searching for a job. For employed people desired working time equals their observed hours  $(h^a = h^d)$ . In addition to observed working hours we thus exploit information about the desired working hours of non-employed people in the model with rationing. Individuals who are observed to work zero hours and are identified to be involuntarily unemployed are assigned to their preferred hours of work category in the labor supply estimation. Based on the assumption that employed individuals are not hours-constrained, i.e.  $h^a = h^d$  for all  $h^a > 0$ ,<sup>9</sup> the decision model of labor supply identifies the true preferences for work regardless of labor market rationing.

Besides the re-specification of the hours categories in the labor supply decision model, we introduce the risk of involuntary unemployment into the model. An equation is added that describes the probability of not finding a job when an individual seeks work. To be identified as involuntarily unemployed in our data set, the indi-

<sup>&</sup>lt;sup>8</sup>See Laroque and Salanié (2002) for a similar categorization. They distinguish classical unemployment (individual's productivity is below the statutory minimum wage) from frictional or cyclical unemployment. The latter types are included in a residual category which can be identified with their data.

<sup>&</sup>lt;sup>9</sup>In principle our model could be extended to capture not only employment constraints, but also restrictions for specific hours constraints. To keep the model simple in this regard we disregard hours constraints here. For models that consider hours constraints see the discussion in sub-section 2 above.

vidual must state that he or she is willing and able to work and is actively searching for a job without being in an employment relationship.<sup>10</sup> Note that for inactive individuals who are (currently) not searching for a job, we have no information on their rationing status. They might be rationed, if they decide to pursue employment. Inactive people thus cannot contribute to the identification of the rationing probability.

In our model the rationing risk depends on various factors. First, we assume that an individual's productivity  $p_i^*$  relative to some institutionally influenced minimum standard of pay  $mw_i$  determines the probability of being constrained. More productive persons have ceteris paribus a higher probability of finding a job. The determinants of productivity are detailed in sub-section 3.2 above. On the other hand this probability also depends on the wage level a firm has at the very least to pay for an employee. We contend that a firm only creates the job, if the employee's (perceived) productivity at least equals the wage costs. If productivity is below the institutionally defined lower wage cost threshold, the position will not be generated. This lower threshold  $mw_i$  is influenced by labor market regulations like a federal minimum wage, sectoral or regional minimum wages, collectively bargained wages where firms or employees are covered by those agreements, or the basic moral wage threshold.

Second, workers could be paid differently because of observed characteristics that are not directly related to their productivity. Therefore the wage equation includes variables that indicate potential reasons for wage discrimination. Third, the probability of rationing depends on labor demand factors that are independent from productivity. One can think of technological or structural change and also exogenous demand shocks that might vary over regions and also sectors or occupations. Since the unemployment risk of individual i is not directly observable, we specify a latent equation for the probability  $ue_i^*$  to be involuntarily unemployed:

$$ue_i^* = \beta^{(ue_p)}(p_i^* - \ln(mw_i)) + X_i^{\prime(ue_{dis})}\beta^{(ue_{dis})} + X_i^{\prime(ue_{ld})}\beta^{(ue_{ld})} + \eta_i$$
(8)

The first term  $(p_i^* - \ln(mw_i))$ , the difference between the employee's productivity  $p_i^*$  and the lower wage threshold  $mw_i$  taken in logs, depicts a measure of produc-

<sup>&</sup>lt;sup>10</sup>We apply the standard ILO definition: if the individual states that he or she has actively searched in the last 4 weeks and is ready to take up employment within 2 weeks without being currently employed we assign the state involuntary unemployment.

tivity in relation to minimum wage costs.<sup>11</sup> Productivity as such is unobserved and identified in eq. (7). Different assumptions on the determinants of productivity lead to alternative specifications of the model. If we suppose that a worker's productivity can be fully explained by observed characteristics and the unobserved component  $e_i^{(w_p)}$  to be purely random, we can estimate eq. (7) separately and put the expected value  $\hat{p}_i$  into the rationing equation. Should unobserved factors systematically influence productivity and not be included in the rationing equation, the estimates could suffer from omitted variable bias. In an alternative specification (see sub-section 3.5 below) the residual  $e_i$  together with  $\hat{p}_i$  from the wage/productivity equation represent individual productivity  $p_i^*$ .<sup>12</sup>

The variable  $mw_i$  approximates the minimum wage cost threshold that applies to the individual. Germany does not have a federal minimum wage, but several sectoral minima have been introduced over the last years. Therefore  $mw_i$  is set to the sectoral minimum wage for all covered employees. In all other cases we insert those wage rates for  $mw_i$  that are paid at the bottom of the hourly wage distribution respectively for men or women, in a certain region, for a certain age group and in a certain sector. These lower wage rates represent the minimum standard of pay that is influenced by labor market regulations like collective bargaining or the basic moral threshold according to the German Civil Code.<sup>13</sup> The lower wage thresholds are approximated by the 5th quantile of the observed distribution of hourly wages in cells defined by gender, region, age group and sector.<sup>14</sup> Since the SOEP does not include enough observations to adequately display the wage distribution for this level of disaggregation, we employ the German Structure of Earnings Survey (GSES, see sub-section 4.1 below) for this variable.<sup>15</sup> We use the latest available cross-section

<sup>&</sup>lt;sup>11</sup>The wage equation (7) where productivity is identified is estimated in logs. Therefore the variable approximating the lower wage threshold also enters in logs. As explained below, it holds by definition that  $mw_i > 0$ .

<sup>&</sup>lt;sup>12</sup>Ideally  $e_i^{(w_p)}$  would be added to  $\hat{p}_i$ . As indicated above, we cannot separate  $e_i^{(w_p)}$  from  $e_i^{(w_{ld})}$ , therefore the composite residual is added.

<sup>&</sup>lt;sup>13</sup>Article 138 of the German Civil Code states that effort and pay must not be in stark disproportion. According to consistent case-law this is fulfilled, if a wage is below two thirds of collectively bargained wages customary in the sector and region.

<sup>&</sup>lt;sup>14</sup>We utilize the regional differentiation available in the GSES data: north-western states (Schleswig-Holstein, Lower Saxony, Hamburg), western Germany (North Rhine-Westfalia), middle of Germany (Hesse, Rhineland-Palatinate, Saarland), southern states (Baden-Wurttemberg, Bavaria), and eastern Germany (Mecklenburg-Western Pommerania, Brandenburg, Berlin, Saxony-Anhalt, Thuringia, Saxony). The age groups consist of 18-30, 31-50, 51-65 years. Sectors are distinguished at the 2-digit level.

 $<sup>^{15}</sup>$ The 5th percentile is a somewhat arbitrary choice; there is a trade-off between approximating

for the year 2006 and forward-project the wages with constant growth rates to the year of analysis.

The variation in productivity-minimum wage ratio is thus generated by differences in individual productivity  $p_i^*$  as well as the lower wage thresholds  $mw_i$ . The distributions of estimated individual productivities, the assigned lower wage thresholds and the log ratio of both variables are documented in Figures 1, 2, and 3 in the Appendix. There is indeed sizeable variation in both components; the productivityminimum wage ratio is approximately normally distributed.

The matrix  $X_i^{\prime(ue_{dis})}$  includes individual and household characteristics that have an effect on the individual's rationing probability  $ue_i^*$  which does not run through individual productivity (as a component of the wage  $w_i$ ).<sup>16</sup> We include dummies for German nationality, single households, and children younger than three in the household. Those variables might indicate discrimination on the labor market that leads to a higher rationing probability than for the respective reference groups.

The variables denoted by  $X'_i^{(ue_{ld})}$  serve as proxies for the demand side of the labor market; again we assume that they neither directly affect the individual's labor supply decision, nor determine the rationing risk through productivity.<sup>17</sup> As the demand for certain skills and professions fluctuates over time, the set of occupation dummies included in  $X'_i^{(ue_{ld})}$  captures some of that heterogeneity.<sup>18</sup> Furthermore, a bunch of indicators varying at the regional level cover the economic situation of firms as well as the performance and structure of the local labor market.  $X'_i^{(ue_{ld})}$ includes the unemployment rate, the employment rate, the GDP/capita, and the share of part time employment. It depicts demand side shocks as well as cyclical fluctuations. These indicators vary at the level of NUTS 2 regions<sup>19</sup> (so-called 'Raumordnungsregionen') which is an administrative rank just below the federal states.

the lower wage treshold and being prone to measurement error for the lowest hourly wages.

<sup>&</sup>lt;sup>16</sup>I.e.  $X_i^{\prime(ue_{dis})}$  is not part of  $X_i^{(w)}$ ; see eq. (7) in sub-section 3.2.  $X_i^{\prime(ue_{dis})}$  is also not identical with the exclusion restrictions for the selection equation in the specifications with a selection correction.

<sup>&</sup>lt;sup>17</sup>As with  $X'_i^{(ue_{dis})}$  the variables in  $X'_i^{(ue_{ld})}$  are also neither contained in  $X^{(w)}_i$ , nor identical to the exclusion restriction in the selection equation of the two-step specification of the model.

<sup>&</sup>lt;sup>18</sup>For non-employed persons we insert the occupation that was carried out during the last employment spell. We add a residual category for cases where no occupation is observed; it amounts to less than 2 percent in the sample.

<sup>&</sup>lt;sup>19</sup>Regulation (EC) No 1059/2003 of the European Parliament and of the Council of 26 May 2003 on the establishment of a common classification of territorial units for statistics (NUTS).

Assuming  $\eta_i$  to be distributed standard normally we arrive at a probit specification for the rationing probability:

$$Pr(ue_{i} = 1) = \Phi \left\{ \beta^{(ue_{p})}(p_{i}^{*} - \ln(mw_{i})) + X_{i}^{\prime(ue_{dis})}\beta^{(ue_{dis})} + X_{i}^{\prime(ue_{ld})}\beta^{(ue_{ld})} \right\}$$

$$Pr(ue_{i} = 0) = \Phi \left\{ - \left( \beta^{(ue_{p})}(p_{i}^{*} - \ln(mw_{i})) + X_{i}^{\prime(ue_{dis})}\beta^{(ue_{dis})} + X_{i}^{\prime(ue_{ld})}\beta^{(ue_{ld})} \right) \right\}$$
(9)

Having described the separate elements of the model, we can now combine the labor supply decision with the rationing probability and write down the labor supply decision with demand side constraints.

#### 3.4 Labor supply with rationing

In the first specification we assume that conditional on the observed covariates the choice probability of the labor supply equation and the rationing probability are independent. The probabilities for the different labor market states in the labor supply model with constraints can be written as follows:

$$Pr_{i0}^{inact} = Pr\left(h^{a} = h^{d} = 0\right) = \frac{exp\{U(y_{i0}, l_{i0}, X_{i0})\}}{\sum_{j=0}^{m} exp\{U(y_{ij}, l_{ij}, X_{ij})\}}$$

$$Pr_{ik}^{ue} = Pr\left(h^{a} = 0, h^{d} > 0, ue_{i} = 1\right) = \frac{exp\{U(y_{ik}, l_{ik}, X_{ik})\}}{\sum_{j=0}^{m} exp\{U(y_{ij}, l_{ij}, X_{ij})\}} \Phi\left\{.\right\}$$
(10)
$$Pr_{ik}^{emp} = Pr\left(h^{a} = h^{d} > 0, ue_{i} = 0\right) = \frac{exp\{U(y_{ik}, l_{ik}, X_{ik})\}}{\sum_{j=0}^{m} exp\{U(y_{ij}, l_{ij}, X_{ij})\}} \Phi\left\{-(.)\right\}$$

The probability of being inactive  $Pr_{i0}^{inact}$  is only determined by the utility model for the labor supply decision. Individuals that are located in this category prefer not to work ( $h^d = 0$ ). For these individuals the 'reservation net ressources' exceed the household's utility for each category with positive working hours.<sup>20</sup> The likelihood of involuntary unemployment  $Pr_{ik}^{ue}$  results from choosing a category with positive working hours according to the household's utility maximization. This choice probability is multiplied by the risk of being rationed on the labor market  $\Phi$  {.} from eq. (9). For the probability of being employed  $Pr_{ik}^{emp}$  the individual again chooses positive working hours as this maximizes the household's utility. This choice probability is multiplied by the probability of being employed  $\Phi$  {-(.)}.<sup>21</sup> The specification cor-

<sup>&</sup>lt;sup>20</sup>The group might include people that are *discouraged* to search for a job. Those individuals actually prefer to work, but fail to find a job and therefore might state they do not want to work and do not search (any longer). Then, their labor supply preferences are mis-specified in the estimation, as individuals who actually want to work are rationed and therefore discouraged. This does not apply to the rationing equation, because those individuals do not contribute to identification. But it reduces efficiency because information on some rationed individuals is not exploited.

<sup>&</sup>lt;sup>21</sup>We mentioned above that we only consider one individual per household. An extended model for couples would have a combined choice set of all hours combinations multiplied by rationing probabilities for both spouses. We leave this extension for future research.

responds to Bargain et al. (2010) but extends their model by the productivity term in the rationing equation.

The independence assumption means that conditional on the variables in X household utility and the rationing probability are not correlated, i.e.  $Cov(\epsilon_{ij}, \eta_i) = 0$ . There are no unobservables that have an effect on the labor supply decision and simultaneously on the risk of rationing. In substantive terms this implies that unobserved individual characteristics are not systematically linked to an employee's productivity. Through the wage productivity is related to the household's utility and labor supply, but likewise a crucial explanatory variable in the rationing equation. When wages are exogenous for the labor supply decision and productivity is exogenous for labor market rationing, the wage/productivity equation (7), the rationing equation (9) and the labor supply equation (5) can be estimated separately. Expected values for productivity  $\hat{p}_i$  are inserted into the rationing equation and predicted wages  $\hat{w}_i$  for non-employed individuals enter into the labor supply decision model. The predicted rationing probability can simply be multiplied with  $Pr_{ik}^{emp}$  and  $Pr_{ik}^{ue}$  to arrive at the choice probabilities for the labor supply model with constraints.

#### 3.5 Joint estimation of the model

Thinking of personal traits like motivation, talent, creativity or ability the independence assumptions from the last sub-section seem overly restrictive. Such unobserved characteristics will influence an individual's productivity and thus be reflected in the wage. Therefore the individual rationing risk should be affected by unobservables through productivity.<sup>22</sup> The labor supply decision is likewise affected. Unobserved differences in productivity translate through the wage into net household income. In addition, these unobservables might be correlated with an individuals preferences for work. In sum, more able and more motivated people should therefore ceteris paribus have a higher labor market participation, earn higher wages and face a lower risk of involuntary unemployment. This means that the error terms of the different equations are correlated. This has to be taken into account; otherwise the

<sup>&</sup>lt;sup>22</sup>The unobserved traits could also be correlated with the individual's search intensity and/or effectiveness; through this channel they might also affect the rationing probability. These different channels cannot be identified separately in our framework.

model parameters cannot be estimated consistently.

The alternative specification borrowed from Laroque and Salanié (2002) who estimate a somewhat more simple framework<sup>23</sup> reflects these associations. The unobserved component  $e_i$  from the wage/productivity equation is incorporated into the labor supply decision and the rationing equation. It enters the utility function through  $y_{ij} = R \left[ w_i \times (TE - l_{ij}), y_i^{(other)}, X_{ij}^{(LS)} \right]$  from eq. (2) as  $e_i$  is part of the gross wage. Observed  $w_i$  for the employed are used as in eq. (10). For non-employed individuals we now not only include the predicted wage based on the observables and parameters from the wage equation  $X_i^{\prime(w_p)}\beta^{(w_p)} + X_i^{\prime(w_{ld})}\beta^{(w_{ld})}$ , but also the unobserved wage component  $e_i$ . This can only be done when the wage and labor supply equations are estimated jointly. Since the predicted wages are not estimated in a separate step before the maximization of the utility function, the complex tax and transfer function R(.) becomes part of the likelihood. Net household incomes are endogenously determined within the estimation procedure for non-employed people which complicates the maximization.

In addition to that, Laroque and Salanié (2002) include the error term from the wage equation into the choice probability for labor supply. It is added to the reservation net ressources in the non-employment alternative. Through a parameter  $\rho$  the unobserved component from the wage equation is flexibly linked to the labor supply decision. For our model this translates to the inclusion of  $e_i$  into the utility function from eq. (4)

$$V_{ij} = \alpha_c + \alpha_y y_{ij} + \alpha_l l_{ij} + \alpha_{y^2} y_{ij}^2 + \alpha_{l^2} l_{ij}^2 + \alpha_{yl} l_{ij} y_{ij} + \rho^{(LS)} e_i d_{i0} + \epsilon_{ij}$$
(11)

The term  $e_i$  is interacted with a choice-specific dummy for the non-working alternative leading to the interaction term  $e_i d_{ij}$  which is  $e_i$  for j = 0 and zero otherwise. Besides the translated effect through the income this specification allows for an additional direct effect of the unobservables on the extensive margin of labor supply. The underlying assumption is that unobserved personal traits influence the decision whether to work or not.<sup>24</sup> The choice of a certain number of working hours is only determined through different incomes (including the unobserved  $e_i$ ) and leisure times.

 $<sup>^{23}{\</sup>rm Laroque}$  and Salanié (2002) analyze only the participation decision and do not have a separate rationing decision.

<sup>&</sup>lt;sup>24</sup>Not including the interaction terms with the other hours categories also avoids the overparameterization of the model. For men a choice-specific part time dummy is already in the model.

Finally, the unobserved term influences the risk of rationing through the productivity variable. Now not only expected productivity  $\hat{p}_i$  enters the rationing probability as in eq. (10), but also its unobserved part  $e_i$ . Both components are determined in the wage/productivity equation which is estimated together with the labor supply and the rationing equation; the latter now becomes:

$$ue_{i}^{*} = \beta^{(ue_{p})} \left[ (X_{i}^{\prime(w_{p})}\beta^{(w_{p})} + e_{i}) - \ln(mw_{i}) \right] + X_{i}^{\prime(ue_{dis})}\beta^{(ue_{dis})} + X_{i}^{\prime(ue_{ld})}\beta^{(ue_{ld})} + \eta_{i} \quad (12)$$

Note that we have to assume the unobserved characteristics influencing productivity to be in some way known to employers. Thus they should be reflected in wages and influence the individual's rationing probability. Assuming  $e_i$  to be distributed standard normally we can re-write the likelihood contributions for the choice probabilities conditional on the unobserved component:<sup>25</sup>

$$Pr_{i0}^{inact} = \int \phi(e) \left[ \frac{exp\left\{ U\left[ R\left( y_{i0}^{other}, X_{i0}^{(LS)} \right), l_{i0}, X_{i0}^{(LS)}, e_{i} * d_{i0} \right] \right\}}{\sum_{j=0}^{m} exp\left\{ U\left[ R\left( exp(X_{i}^{\prime(w)})\beta^{(w)} + e_{i} + \sigma_{e}^{2}/2 \right) * (TE - l_{ij}), y_{ij}^{other}, X_{ij}^{(LS)} \right), l_{ij}, X_{ij}^{(LS)}, e_{i} * d_{ij} \right] \right\}} \right] de$$

$$Pr_{ik}^{ue} = \int \phi(e) \left[ \frac{exp\left\{ U\left[ R\left( exp(X_{i}^{\prime(w)})\beta^{(w)} + e_{i} + \sigma_{e}^{2}/2 \right) * (TE - l_{ik}), y_{ik}^{other}, X_{ik}^{(LS)} \right), l_{ij}, X_{ij}^{(LS)}, e_{i} * d_{ik} \right] \right\}}{\sum_{j=0}^{m} exp\left\{ U\left[ R\left( exp(X_{i}^{\prime(w)})\beta^{(w)} + e_{i} + \sigma_{e}^{2}/2 \right) * (TE - l_{ij}), y_{ij}^{other}, X_{ij}^{(LS)} \right), l_{ij}, X_{ij}^{(LS)}, e_{i} * d_{ik} \right] \right\}} \right] de$$

$$Pr_{ik}^{emp} = \int \phi(e) \left[ \frac{exp\left\{ U\left[ R\left( exp(X_{i}^{\prime(w)})\beta^{(w)} + e_{i} + \sigma_{e}^{2}/2 \right) * (TE - l_{ij}), y_{ij}^{other}, X_{ij}^{(LS)} \right), l_{ij}, X_{ij}^{(LS)}, e_{i} * d_{ij} \right] \right\}} \right] de$$

$$Pr_{ik}^{emp} = \int \phi(e) \left[ \frac{exp\left\{ U\left[ R\left( exp(X_{i}^{\prime(w)})\beta^{(w)} + e_{i} - \ln(mw_{i}) \right] + X_{i}^{\prime(ue)}\beta^{(ue)} \right\} \right] de}{\sum_{j=0}^{m} exp\left\{ U\left[ R\left( w_{i} * (TE - l_{ij}), y_{ij}^{other}, X_{ij}^{(LS)} \right), l_{ij}, X_{ij}^{(LS)}, \hat{e}_{i} * d_{ij} \right] \right\}} \right] de$$

$$Y_{ik}^{emp} = \int \phi(e) \left[ \frac{exp\left\{ U\left[ R\left( w_{i} * (TE - l_{ij}), y_{ij}^{other}, X_{ij}^{(LS)} \right), l_{ij}, X_{ij}^{(LS)}, \hat{e}_{i} * d_{ij} \right] \right\}}{\sum_{i=0}^{m} exp\left\{ U\left[ R\left( w_{i} * (TE - l_{ij}), y_{ij}^{other}, X_{ij}^{(LS)} \right), l_{ij}, X_{ij}^{(LS)}, \hat{e}_{i} * d_{ij} \right] \right\}}$$

$$Y_{ik}^{emp} = \int \phi(e) \left[ \frac{exp\left\{ U\left[ R\left( w_{i} * (TE - l_{ij}), y_{ij}^{other}, X_{ij}^{(LS)} \right), l_{ij}, X_{ij}^{(LS)}, \hat{e}_{i} * d_{ij} \right] \right\}}{\sum_{i=0}^{m} exp\left\{ U\left[ R\left( w_{i} * (TE - l_{ij}), y_{ij}^{other}, X_{ij}^{(LS)} \right), l_{ij}, X_{ij}^{(LS)}, \hat{e}_{i} * d_{ij} \right] \right\}}$$

$$Y_{ik}^{emp} = \int \phi(e) \left[ \frac{exp\left\{ U\left[ R\left( w_{i} * (TE - l_{ij}), y_{ij}^{other}, X_{ij}^{(LS)} \right), l_{ij}, X_{ij}^{(LS)}, \hat{e}_{i} * d_{ij} \right] \right\}} \left[ \frac{exp\left\{ \left( \left[ R\left( w_{i} * (TE - l_{ij}), y_{ij}^{other}, X_{ij}^{(LS)} \right), l_{ij}, X_{ij}^{(LS)}, \hat{e}_{i} * d_{ij} \right] \right\}} \right] \left[ \frac{exp\left\{ \left[ \frac{exp\left\{ \left[ R\left( w_{i} * (TE - l_{ij}), y_{ij}^{other}, X_{ij}^{(LS)}$$

For employed people we can compute the residuals  $\hat{e}_i$ . For the non-employed  $e_i$  has to be integrated out of the likelihood. Since this term involves the highly nonlinear function R(.), the integral has no closed-form solution and must be solved numerically. Therefore we have to rely on maximum simulated likelihood to estimate this model. The procedure boils down to repeatedly taking random draws from the standard normal distribution and average the estimation results over this simulated distribution of the error terms of non-employed individuals (Train, 2009). Haan and Uhlendorff (2013) propose a different specification for the unobserved heterogeneity. They assume a discrete distribution of unobserved terms in their labor supply and rationing equation in a two-factor loading model. Their approach requires less

<sup>&</sup>lt;sup>25</sup>We simplified the notation for readability and collect the different explanatory variables for the wage  $X'_i^{(w)}\beta^{(w)}$  and rationing  $X'_i^{(ue)}\beta^{(ue)}$  equation.

restrictive functional form assumptions and includes a more flexible and general variance-covariance matrix.<sup>26</sup>

#### 3.6 Likelihood and identification

The sample likelihood for the extended model with labor demand restrictions is given by:

$$L = \prod_{i=1}^{N_1} Pr_{i0}^{inact} \times \prod_{i=N_1+1}^{N_2} \prod_{k=1}^m (Pr_{ik}^{ue})^{\delta_{ik}} \times \prod_{i=N_2+1}^{N_3} \prod_{k=1}^m (Pr_{ik}^{emp})^{\delta_{ik}}$$
(14)

The individual choice probabilities are defined in eq. (13). The dummy  $\delta_{ik}$  is equal to one when individual *i* chooses alternative *k* and zero otherwise. Three different groups contribute to the likelihood:  $i = 1, \ldots, N_1$  individuals who are voluntarily inactive;  $i = N_1 + 1, \ldots, N_2$  individuals who are involuntarily unemployed; and  $i = N_2 + 1, \ldots, N_3$  employed individuals.

Some remarks are due on identification. The basic problem in the labor supply model with rationing is to separately identify labor supply decisions and labor demand constraints. Bargain et al. (2010) like Haan and Uhlendorff (2013) argue that demand side rationing is identified from regional labor market conditions which are exogenous to the individual and influence the rationing probability. These factors are assumed not to have a direct effect on the labor supply.<sup>27</sup> The unemployment rate is the most important indicator. We also use this type of variation here.

Since we also want to make rationing dependent on the individual's productivity, identification is even thornier here because of the direct link of productivity to disposable income in the utility function for labor supply (going through the wage). There are several points to make where the necessary variation comes from. First, we do not identify the mere effect of an individual's productivity, but of productivity in relation to some minimum wage threshold inherent in German labor market institutions. We exploit sectoral minima – where applicable – or use the bottom of the distribution of observed wages in segments of the labor market defined by gender, age, region and sector. Figure 2 documents the sizeable variance in this variable as

 $<sup>^{26}</sup>$ This specification is more demanding in our context as we have three related equations. We leave a more general specification for future research.

<sup>&</sup>lt;sup>27</sup>A possible channel for that relationship could go through the (potential) wage. If wages were on average lower in regions with a more tense labor market, this could also lead to lower labor supply. Different labor market outlooks might also en- or discourage worker to search for jobs.

firms are allowed to pay lower wages in certain segments of the labor market than in others. Given individual productivity in the enumerator there is additional variation in the denominator of this variable. Second, we use the wage (in the tax function) of the labor supply model. Besides individual productivity firm-side variables provide additional variation in labor income. Third, we exploit variation generated by the tax system for labor supply. Given someone's productivity and gross wage, the disposable income varies with the household context (marital status, labor supply of the spouse, children, further income); such differences are not relevant for the firm's labor costs and thus the rationing probability.<sup>28</sup>

Fourth, we use observed covariates as exclusion restrictions in different equations of our model. The identification of individual productivity given the other covariates in the rationing equation hinges on valid exclusion restrictions in the wage/productivity equation that are not part of the rationing model. We use indicators derived from the individual labor market history like the depreciation of human capital (due to previous unemployment spells), tenure, previous incidence of full/part time employment, and qualification variables.<sup>29</sup> As explained the denominator in the productivity/minimum wage threshold variable provides additional variation. Regional labor market indicators were already mentioned as exclusion restrictions for labor demand in the rationing equation. We add a set of occupation dummies to approximate differences in the demand for and mismatch of skills. Occupational choices made earlier in life might influence the rationing probability today without having a direct link to the labor supply decision.

#### 4 Data

#### 4.1 Data sources

The simulation of wage effects, the microsimulation, and the labor supply estimations are based on data from the German Socio-Economic Panel (SOEP). The SOEP is a representative sample of households living in Germany with detailed information

<sup>&</sup>lt;sup>28</sup>Like in the unconstrained labor supply model we assume productivity and wages to be constant for different working time categories. Making wages dependent on working hours would complicate identification.

<sup>&</sup>lt;sup>29</sup>One could argue that some of these factors influence rationing not only through productivity, but to also through different mechanisms, e.g. discriminiation.

on household incomes, working hours and the household structure (Wagner et al., 2007). We use the wave for the year 2009. Since the STSM is based on retrospective information on income components for the simulation of net household incomes for a given year, wages and incomes computed on basis of the SOEP wave from 2009 refer to 2008. Because our analysis refers to the year 2010, we extrapolate incomes on the basis of realized average growth rates for 2009 and 2010.<sup>30</sup> The tax-benefit system includes all changes in regulations up to the year 2010.

For the regional labor demand variables we resort to data that is collected and edited jointly by the German Statistical Office with the Federal Institute for Research on Building, Urban Affairs and Spatial Development within the Federal Office for Building and Regional Planning. The dataset "Indicators and Maps on the Spatial Development" ("Indikatoren und Karten zur Raumentwicklung", INKAR, see Helmcke, 2008) allows longitudinal comparisons at different regional levels for Germany. The regional classification conforms to the Nomenclature of territorial units for statistics (NUTS, see European Commission, 2003). The information used here is aggregated at the NUTS 2 level which consists mainly of administrative districts in Germany at which regional policies are planned and implemented. We utilize indicators for employment, unemployment and economic performance. The INKAR and the SOEP data are merged at the regional NUTS 2 level.

The information for the denominator of the variable that relates individual productivity and existing minimum standards for pay comes from the 2006 wave of the German Structure of Earnings Survey (GSES). This is a linked employer-employee data set provided by the German Federal Statistical Office (Hafner, 2006; Statistisches Bundesamt, 2009). The large sample size (about 1 million observations in total) allows to precisely assess the lower quantiles of the wage distribution for sub-groups of employees. The data set contains cardinal information on working hours. The hourly wage measure is more reliable compared to the SOEP, since the information comes directly from the firm and is based on the employment contract. Measurement errors due to incomplete memory of the respondent, discrepancies between

<sup>&</sup>lt;sup>30</sup>Most interviews in the SOEP refer to the first quarter of the year. We assume that incomes will increase with the annual growth rate in that year. Average annual growth rates are derived from the following indices for the years 2009 and 2010: 1.004, 1.02 for consumer prices; 1.019, 1.007 for wages; 0.99, 1.05 for income from profits (source: national accounts; BMWi (2010); own calculations).

reported working hours and wage income are therefore less of a problem (Müller, 2009).

#### 4.2 Sample and descriptive statistics

We analyze the labor supply with rationing for men living in single and couple households. Following the chauvinist framework we take the labor supply of the partner in couple households as given.<sup>31</sup> We restrict the sample to men aged 18-65 as we do neither model the educational choices nor retiring decisions. Therefore we exclude households where the individual under consideration is retired, disabled, on maternity/paternity leave, or in fulltime education. This results in an estimation sample of 3,858 observations. We estimate the model and run all policy simulations on data referring to the year 2010.

Descriptive statistics are given in Tables 5 through 8 in the Appendix. For the *productivity variables* in the *wage equation* some differences between working and non-working individuals can be noticed. Currently employed have on average more years of schooling, more often a higher school certificate or a tertiary education, more working experience and a lower depreciation of human capital (Table 5). These discrepancies are more pronounced for rationed individuals whereas voluntarily unemployed are more similar to employed people. There are thus marked differences in observable characteristics that determine an individual's productivity. Regarding the *demand side variables* the dummies for firm size and sector cannot be observed for non-employed individuals; mean values of orthogonalized dummy variables are used in those cases (Table 6).

The explanatory variables in the rationing equation should be compared between the working individuals and those who are rationed, since those two groups contribute to the identification of the rationing probability. The assigned lower wage threshold is on average a bit higher for employed in comparison to non-employed people (Table 7). It varies between  $3.20 \in$  and  $16 \in$ /hour. The share of singles is higher whereas that of German nationals is lower among the rationed individuals. There are also substantial differences in the occupational composition of both

<sup>&</sup>lt;sup>31</sup>An obvious extension of the framework of this paper would be a household labor supply model for couple with both spouses being flexible. This yields a more complicated likelihood as it includes additional wage and rationing equations for the spouse that have to be estimated simultaneously. We leave this extension for future research.

groups. Finally, the labor demand indicators show that the incidence of unemployment and the economic clout is lower in regions where the rationed individuals live. The characteristics of the voluntarily unemployed people lie in between the employed and rationed individuals, although they more closely resemble the properties of the latter group.

The descriptive statistics for observed taste shifters in the *labor supply* decision model are reported for the observed choice category, but cannot be directly interpreted, since the variables are interacted with income and leisure (Table 8). Looking at the observed distribution of working hours we observe 11% of the sample to be voluntarily inactive in the unconstrained case (Table 1). Only 2% of the sample work in marginal or part time employment whereas nearly 50% of the sample work in the full time category and almost 40% work more than 40 hours. We see that 5% of men in our sample are identified to being rationed on the labor market. Accordingly the share of volunatarily unemployed is reduced by 5 percentage points and the share in the (preferred) full time category increases to 54%.

Alternative	Hours	Men Unconstr. Share	Constr. Share
Inactivity	0.0	0.12	0.07
v	0.0	0	0.0.
Small part time Medium part time	12.5	0.02	0.02
Large part time			
Full time	36.5	0.49	0.54
Overtime	46.5	0.37	0.37
Share rationed			0.05
Observations			3,858

Table 1: Unconditional hours distribution

Notes: Unconstr.=unconstrained model, Constr.=constrained model.

Source: Own calculations based on SOEP, wave 2009.

The share of rationed individuals is clearly smaller than aggregate numbers on unemployment for Germany at that time. Except for conceptual differences between registered unemployment and the ILO definition applied here, the discrepancy may also be related to the measurement errors mentioned above. There might be discouraged workers who claim to be voluntarily unemployed.

#### 4.3 Microsimulation model

In order to translate individual gross earnings into disposable incomes at the household level we use the tax-benefit microsimulation model STSM (Steiner et al., 2012; Creedy and Duncan, 2002). The SOEP provides the necessary information on the composition of the households, incomes from various sources (labor, capital, transfers, etc.), working hours, and socio-demographic characteristics. A tax-transfer calculator simulates net household incomes based on gross hourly wages of the employed persons in the household for different working hours categories.

The STSM contains the main features of the German tax and transfer system. The income tax is computed by applying the income tax formula to the individual incomes of unmarried spouses; for married spouses, income is taxed jointly based on an income splitting factor of 2. Employees' social security contributions and the income tax are deducted from gross household income and social transfers are added to get net household income. Social transfers include child allowances, childrearing benefits, educational allowances for students and apprentices, unemployment compensation, the housing allowance, and social assistance. The model accounts for nonlinearities and interactions within the German tax-benefit system, in particular means-tested income-support schemes, exemptions of very low earnings from social security contributions, and the joint income taxation of married couples imposing relatively high marginal tax rates on secondary earners.

### 5 Empirical results

#### 5.1 Parameter estimates

Although the parameters of single variables are not of primary interest, their estimates can be used to check the theoretical consistency of our model. With regard to the *wage equation* we see that among the productivity variables age, years of schooling and holding a higher school certificate are significantly related to higher wages (Table 9 in the Appendix). The depreciation of human capital has a stronly negative association with the earned wage. Variables measuring the experience on the labor market and the tenure with a firm are also significantly related to the wage. The demand side variables show that there is a wage premium for German employees. Civil servants earn on average less than all other employees. People working in small firms have lower hourly wages compared to those in larger firms. There are significant differences in pay between sectors. Moreover, there is a substantial difference in average wages between West and East German federal states.

In the rationing equation we see that the variable relating an individual's productivity to institutional wage constrains at the lower end of the distribution is strongly and significantly related to the risk of rationing (Table 10 in the Appendix). A higher productivity reduces the rationing probability, whereas the effect of institutional constraints at the lower end of the wage distribution goes in the opposite direction. Among covariates indicating individual constraints or discrimination on the labor market only the effect for singles is statistically significant. The demand side variables show that the rationing risk varies significantly among different occupations. The regional unemployment rate as an indicator for the tightness of the local labor market is strongly and positively related to the risk of being involuntarily unemployed. This holds – although to a much lesser degree – for the employment rate. Conditional on the other covariates neither the regional BIP per capita, nor the share of part time employment is significantly related to the rationing risk.

In the unconstrained as well as the constrained *labor supply* models the great majority of households fulfills monotonicity and concavity of the utility function with respect to the choice variables (Table 11 in the Appendix). Utility increases with the disposable household income for virtually all households which is the minimum requirement for the consistency of the policy simulations below. According to the derivatives with respect to leisure, for a limited proportion of the population positive monotonicity in leisure is not respected. The marginal utility of income and leisure varies with individual- and household-specific variables. Except for the age variables taste-shifters for the consumption of household income do not exhibit significant observable heterogeneity in preferences among. The preference for leisure is higher for singles, households with children and people with a handicap. The age variables display a significant nonlinear pattern with respect to the preferences for leisure. The choice-specific dummy variable which – specified as costs of part-time - has the expected positive sign and is statistically significant. Given the other covariates men indeed seem to dislike working reduced hours. The relationship between the unobserved productivity component and inactivity is negative, but not statistically different from zero. The model fit is adequate and comparable to previous estimations with similar data. The category of non-working individuals is a bit underpredicted.

#### 5.2 Elasticities

Wage elasticities cannot be derived directly from the parameter estimates, but have to be simulated numerically. The state probabilities are first simulated for disposable incomes in the status quo. Then wages (including the shadow wages for currently non-employed individuals) are increased by one percent, new disposable incomes are simulated and the state probabilities are predicted. The average difference between both scenarios yields participation and hours elasticities which are reported separately for West and East Germany. For the constrained model the labor supply elasticities measure the behavioral response of a worker to an (exogenous) wage increase in terms of desired hours irrespective of the rationing.

For the unconstrained model participation elasticities<sup>32</sup> for East and West Germany are about 0.15 which means that raising gross wages by 1% will increase the participation rate by about 0.15 percentage points (Table 2). The hours elasticity is 0.22 in West and 0.26 in East Germany. These values are in the ballpark of the empirical literature (Blundell and Macurdy, 1999) and confirm previous findings with a similar model and the same data set for previous years (Haan and Steiner, 2005a; Steiner and Wrohlich, 2005; Bargain et al., 2010).

		Men	L	
	E	ast	W	est
Percentage point change participation rate				
Unconstrained model	0.16	[0.13, 0.19]	0.14	[0.12, 0.16]
Constrained model	0.08	[0.06, 0.10]	0.08	[0.07, 0.09]
Constrained model with heterogeneity	0.05	[0.03, 0.07]	0.05	[0.03, 0.07]
Percent change hours				
Unconstrained model	0.26	[0.20, 0.31]	0.22	[0.19, 0.26]
Constrained model	0.16	[0.13, 0.19]	0.17	[0.14, 0.19]
Constrained model with heterogeneity	0.13	[0.09, 0.16]	0.14	[0.11, 0.17]

Table 2:	Labor	supply	elasticities
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*Notes:* PP change part. rate=change of participation rate in percentage points, Pct. change hours=change in working hours in percent, Unconstrained=unconstrained model, Constrained=constrained model, Constrained with heterogeneity=constrained model with unobserved heterogeneity (joint estimation), Bootstrapped 95%-confidence bands in parentheses.

Source: Own calculations based on SOEP, wave 2009; INKAR, wave 2010.

As discussed at the outset the omission of demand side constraints in labor supply estimation will bias elasticities for several reasons. Participation bias results

 $<sup>^{32}</sup>$ In technical terms this is not an elasticity, but the participtation change in percentage points.

from falsely depicting rationed individuals as voluntarily inactive. Preference bias follows from overstating the value of leisure in the unconstrained model by characterizing involuntarily unemployed as inactive. There might also be specification bias in the unconstrained model. The first two biases go into opposite directions and the third one is a priori indeterminate. The simulated participation elasticities show that an upward participation bias indeed exists. The participation elasticities for the model with demand side constraints are reduced by half to 0.08 for West and East Germany compared to the unconstrained model. They are further diminished to 0.05 when unobserved heterogeneity is taken into account (Table 2). The differences in the hours elasticities are smaller; the model with rationing yields elasticities of 0.17 (0.16) without and 0.14 (0.13) with unobserved heterogeneity for West (East) Germany. The participation bias dominates dominates overall, since the hours elasticities (incorporating the extensive and intensive margin) are smaller in the constrained models. Yet, the participation bias also seems to play a role because the differences between the unconstrained and constrained models are smaller than for the participation elasticities.

#### 5.3 Policy simulations

Based on the parameters of our structural model we carry out several policy simulations. Sub-section 5.3.1 considers the implementation of a federal minimum wage of  $7.50 \in$  in Germany. Section 5.3.2 compares the consequences of employee- and employer-oriented wage subsidies for employment when a federal minimum wage is already in place.

#### 5.3.1 Employment effects of a federal minimum wage

In this sub-section we simulate the employment effects a federal minimum wage of  $7.50 \in$  would have induced, had it been introduced in 2010. We compare this counterfactual scenario with the observed situation in 2010 without a federal, but several sectoral minima. We follow Müller and Steiner (2010) and simulate a counterfactual wage distribution under a federal minimum wage by setting all hourly wages below the minimum to  $7.50 \in$ . This means that all employees that previously earned sub-minimum wages are assumed to be paid exactly the minimum wage. Moreover, spillover effects are ruled out meaning that the wage distribution above the minimum wage would by assumption not be affected.<sup>33</sup>

How is the introduction of a federal minimum wage reflected in the different specifications of the structural labor supply model? In both versions of the model hourly wages become  $w_i^{MW} = max(w_i, 7.50)$ ; this holds for observed wages of employed people and also for the predicted wage rates  $\hat{w}_i^{MW}$  of non-working individuals. Through the tax and transfer function R(.) the disposable household income  $y_{ij}^{MW}$ will also be adjusted with marginal tax rates on the additional labor income depending on the individual's working hours and the household context. The amount of additional household income determines the labor supply incentives of this reform for a specific individual. The increase in net household income is markedly smaller than the nominal changes in gross labor earnings as marginal tax rates can be very high when, e.g., welfare transfers are substituted (Müller and Steiner, 2009).

On the other hand, the institutionally fixed lower bound for wages puts a strain on companies' wage costs. Firms may not be willing to employ a person whose productivity does not match the wage costs. When an individual is bound by the wage threshold, his rationing risk is also affected. This side of the labor market is not reflected in the unconstrained labor supply model where it is assumed that the additional labor supply induced by the minimum wage policy equals the rise in realized employment. In the extended model with demand side rationing such wage rigidities are captured in the equation for the latent risk of involuntary unemployment. An individual's productivity  $p_i^*$  is related to lower wage thresholds  $mw_i$  determined by, e.g., collective bargaining agreements or sectoral minima, that apply to the respective person. In the scenario with a federal minimum this variable becomes  $mw_i^{MW} = 7.5 \ \forall \ mw_i < 7.5$ . For all individuals with thresholds below the statutory minimum wage,  $m w_i^{MW}$  is set to this level. According to the estimates (Table 10 in the Appendix) this will increase the rationing probability. The total employment effect will thus depend on the increase in labor supply incentives and the adjustment of labor demand estimated in the rationing equation.

Our model does not feature equilibrium wage effects. When supply and demand shift after a legal minimum wage is introduced and the equilibrium wage distribution adjusts, these 'second round' wage changes are not captured in our policy

 $<sup>^{33}</sup>$ The gross hourly wage is calculated by dividing reported earnings in the month before the interview by the number of hours worked in that month; paid overtime hours are included in this measure. See Müller and Steiner (2010) for a detailed discussion.

simulation. We rely on the nominal changes in wages and labor costs. Based on these assumptions we calculate the expected change in labor supply – as well as adjusted rationing probabilities in the constrained model – under the observed and the counterfactual distributions in wages and wage constraints.

The results from the unconstrained model show that a federal minimum wage of  $7.50 \in$  would increase labor supply (Table 3). The magnitude is limited due to high marginal tax rates of minimum wage earners ((Müller and Steiner, 2010)). The participation rate would increase by 0.2 (0.8) percentage points in West (East) Germany which amounts to about 23,000 (22,000) additional male employees represented by our sample. Working hours would on average increase by about 0.4% in the West and 1.6% in the East corresponding to about 23,000 and 22,000 full time equivalents in total.

The unconstrained model disregards the increase in labor costs induced by a federal minimum wage. The model with demand side rationing entails this margin in the calculation of the expected employment change due to a statutory minimum wage. In addition to the labor supply increases, the individual rationing risk is also affected. The rationing probability increases by 0.6 percentage points in West Germany and by 2.5-2.8 percentage points in the East. Since nominal wage increases because of the minimum are markedly higher, firms in East Germany are hit harder by this reform and would decrease their labor demand more strongly. Note that all those changes refer to the rationing probability of the whole sample. The effects are smaller for people that are active on the labor market, i.e. employees and individuals looking for work. Currently inactive persons face on average a higher risk of not finding job, as their productivity is estimated to be lower compared to people attached to the labor market (Table 3).

Taking the change in rationing probabilities into account, the net effect on participation becomes negative. Depending on the specification participation rates would decrease by about 0.4-0.5 (2.0-2.4) percentage points which in total equals about 51-58,000 (53-63,000) employees for West (East) Germany represented in our estimation sample. Interestingly, the change in the volume of employment is significantly smaller. Measured in full time equivalents employment would be reduced by about 20,000 (23-29,000) in West (East) Germany. The total employment effects were not captured in previous papers as these either focused on supply or demand. Jobs that

MW=7.50€	
loyment effects:	
ble 3: Empl	
Tab	

						Men	en					
				$\mathbf{East}$						West		
	Unco	Unconstrained	C	Constrained	C	Constr. Het.	Uncc	Unconstrained	Co	Constrained	Col	Constr. Het.
Employment Participation												
Pp. change participation rate	0.80	[0.53, 1.08]	-1.95	[-2.66, -1.25]	-2.42	[-3.35, -1.64]	0.19	[0.11, 0.27]	-0.41	[-0.62, -0.21]	-0.48	[-0.76, -0.25]
Change employment in 1000 Working hours	22	[15,28]	-53	[-75,-33]	-63	[-91, -41]	23	[14, 33]	-51	[-78,-25]	-58	[-94,-31]
Change hours	0.37	[0.25, 0.48]	-0.38	[-0.67, -0.08]	-0.47	[-0.86, -0.09]	0.09	[0.05, 0.12]	-0.07	[-0.15, 0.01]	-0.07	[-0.16, 0.03]
Pct. change hours	1.62	[0.96, 2.28]	-1.21	[-2.52, 0.10]	-1.66	[-3.25, -0.24]	0.37	[0.19, 0.54]	0.02	[-0.36, 0.40]	0.04	[-0.40, 0.56]
Change hours 1000 fte.	22	[15,29]	-23	[-40, -5]	-29	[-56,-5]	23	[14, 33]	-19	[-41,4]	-18	[-46,10]
Rationing												
$Whole \ population$												
Rationing probability SQ			12.48	[9.68, 15.27]	11.08	[8.53, 13.92]			7.33	[6.05, 8.62]	7.32	[6.18, 8.49]
Rationing probability MW			14.96	[11.76, 18.15]	13.85	[10.66, 17.32]			7.89	[6.52, 9.25]	7.89	[6.49, 9.09]
Pp. change rationing probability			2.48	[2.08, 2.87]	2.76	[2.13, 3.39]			0.55	[0.48, 0.63]	0.57	[0.31, 0.60]
Active on labor market												
Rationing probability SQ			9.80	[7.69, 11.92]	8.09	[6.12, 10.30]			5.54	[4.55, 6.53]	5.15	[4.17, 5.97]
Rationing probability MW			11.91	[9.30, 14.52]	10.34	[8.06, 13.36]			6.02	[4.95, 7.08]	5.62	[4.66, 6.55]
Pp. change rationing probability			2.10	[1.61, 2.59]	2.25	[1.93, 3.06]			0.47	[0.40, 0.55]	0.47	[0.49, 0.58]

dnc ., , erty (JOI 2020 pp.=percentage points, pct.=percent, Bootstrapped 95%-confidence bands in parentheses. Source: Own calculations based on SOEP, wave 2009; INKAR, wave 2010.

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would be lost following the introduction of a minimum wage do often not involve full time contracts. On the other hand people are to some degree incentivized by the minimum wage to extend their working hours.

The comparison of the different model specifications shows that a labor supply model with demand side constraints is suited for policy simulations in cases where not only labor supply incentives are affected, but labor costs are directly influenced. Under these circumstances a pure labor supply model will not provide policy relevant results as estimated labor supply adjustments are not informative about employment changes. The extended model is also able to shed light on different margins of employment. According to the estimates, the total number of employed would be reduced, but this loss would be partially compensated by increased working hours of people remaining employed. While people with low individual productivity are in danger of losing their job, other, more productive persons who earned low hourly wages without the minimum will benefit from such a reform. Such substitution mechanisms have been found to be relevant (Ahn et al., 2011).

#### 5.3.2 Minimum wage and wage subsidies

In the second policy simulation we analyze different wage subsidies in the presence of a federal minimum wage. In a partial model of the labor market with fully flexible wages the incidence of wage subsidies does not depend on whether they are paid to employers or employees, but rather on the labor demand and supply elasticities. If a subsidy is paid to employees, more people will be prepared to work for lower nominal wages because these are topped up by the subsidy resulting in higher effective wages. Likewise already employed people might extend their working hours for given nominal wages. The expansion of labor supply leads to lower equilibrium wages and more employment. If the subsidy is directed to employers, they will agree to employ also less productive workers at given nominal wages as they are compensated for the difference in wage costs and productivity. They would agree to higher wages for already employed people when the state covered the increase in wage costs. This would also result in higher employment levels.

This reasoning does not hold in a scenario where the wage adjustment mechanism is limited by a binding lower wage threshold. Employee-oriented wage subsidies are no longer effective: while employees would accept lower nominal gross wages, these must not be paid because of the minimum wage. Therefore an equilibrium with lower wages and higher employment levels cannot be reached. Since a minimum wage only impedes downward wage adjustments below the threshold, the opposite mechanism still works. An employer-oriented wage subsidy reduces wage costs. Subsidized firms are therefore prepared to employ people at higher nominal wages as they are compensated by the subsidy. The higher wage in turn incentivizes people to supply more labor boosting employment.

In this sub-section we simulate wage subsidies with a statutory minimum wage in place. We compare a situation where a federal minimum wage of  $7.50 \notin$ /hour exists without subsidies with the same minimum wage scenario including employeeor employer-oriented wage subsidies. The subsidies are targeted at low *hourly* gross wages, between the minimum wage threshold of  $7.50 \notin$  and  $10.00 \notin$ /hour. They benefit low productive employees, not low labor earnings per se. We calibrate the subsidy in a way that its total volume does not exceed 100 million  $\notin$ /month which is about half of the amount Müller and Steiner (2011) calculated to be available for re-distribution from a minimum of  $7.50 \notin$  after labor demand adjustments. The lowest wages, i.e. those at the nominal minimum, receive the largest subsidy which – according to our calibration – amounts to  $0.18 \notin$  per  $1 \notin$  earned. After that the subsidy is linearily phased out up to a wage of  $10.00 \notin$ /hour. We assume the economic and the de jure incidence to coincide. Employees and employers benefit fully from their respective subsidy without any adjustments of gross wages.<sup>34</sup>

How are the wage subsidies implemented in our framework? With the unconstrained model we are only able to analyze employee-oriented wage subsidies. In this framework the subsidy induces a proportional rise in the hourly wage  $w_i^{MW}$ of individuals benefitting from the subsidy. The (potential) increase in household income that is simulated for this scenario incentivizes individuals to supply additional labor. In this model this additional supply equals realized employment. We calculate the change in expected participation and working hours in comparison to the situation with a federal minimum wage but no subsidies. Given our assumptions on the incidence, wage subsidies targeted at employers will not lead to any changes in the (potential) wages of employees. That the subsidy diminishes the cost of labor

<sup>&</sup>lt;sup>34</sup>This obviously is a simplifying assumption. The literature has provided mixed results on the incidence of labor taxes. Recent evidence shows that nominal and economic incidence might be closer than previously thought (Neumann, 2013).

for jobs paying the minimum wage and that demand for those jobs will increase is not captured by this model.

In the model with demand side constraints wage subsidies for employees and employers can be investigated. Subsidies for employees increase the (potential) wage for covered individuals and – through the change in disposable incomes – labor supply incentives. Labor costs for firms are not affected, though. Wages below the minimum wage threshold must not be paid. Therefore the rationing probability for people receiving the subsidy does not change. Whether the estimated employment effects are smaller or larger compared to the unconstrained model depends on the relative importance of the participation, preference and specification biases. An employer-oriented subsidy, however, is directly targeted at the wage costs for low wage earners. In the constrained model this is reflected in a proportional reduction of the institutionally fixed wage costs  $mw_i$  for low wage earners (up to  $10 \in$ /hour with a declining rate) which in turn increases the labor demand and reduces the rationing risk for given productivities. Although the (potential) wages  $w_i^{MW}$  of employees do not change in this scenario, the labor supply incentives induced by the minimum wage may be realized as the related increase in wage costs is compensated by the subsidy and labor demand rises.

Looking first at the results for the *employee-oriented subsidy*, the unconstrained model predicts a moderate increase of labor supply. The participation rate rises by 0.2 in West and 0.9 percentage points in East Germany which amounts to 24,000 and 23,000 employees in total (Table 4). The change of total hours in full time equivalents roughly equals the extensive margin. In the extended model with rationing<sup>35</sup> the changes in participation lead to smaller effects. The participation rates for West (East) Germany would increase by merely 0.1 (0.3) percentage points which equals 13,000 (6,000) employees in total. The unconstrained model thus overestimates the consequences of employee-oriented subsidies on the extensive margin reflecting the participation bias in the elasticities (Table 2). It also ignores that those people who decide to start working because of the subsidy could be rationed.

The model with demand side rationing estimates larger hours gains for West Germany, East German men react less to the subsidiy in comparison to the un-

 $<sup>^{35}\</sup>mathrm{We}$  will refer to the constrained model with unobserved heterogeneity as this is the preferred specification.

	Unco	Unconstrained	Col	Constrained	Col	Constr. Het.	Unco	Unconstrained	Con	Constrained	Con	Constr. Het.
Employee-oriented subsidies Employment Participation												
Pp. change participation rate Change employment in 1000	0.86 23	[0.61, 1.12] [17, 29]	0.43 12	[0.29, 0.56] $[8, 15]$	0.25 6	[0.12, 0.40] $[3, 10]$	$\begin{array}{c} 0.19\\ 24\end{array}$	[0.13, 0.26] $[16, 32]$	$0.15 \\ 19$	[0.10, 0.20] $[13, 25]$	$0.10 \\ 13$	[0.05, 0.17] [6, 21]
Change hours	0.40	[0.29, 0.52]	0.32	[0.21, 0.42]	0.25	[0.14, 0.39]	0.09	[0.06, 0.12]	0.11	[0.08, 0.15]	0.11	[0.06, 0.16]
Pct. change hours Change hours 1000 fte.	$1.56 \\ 24$	[1.02, 2.10] $[17, 32]$	$1.28 \\ 19$	[0.88, 1.69] $[12, 26]$	1.06 $16$	[0.63, 1.53] $[8, 24]$	0.33 25	[0.21, 0.45] $[16, 33]$	$0.52 \\ 31$	[0.31, 0.73] $[21, 41]$	0.45 30	[0.24, 0.72] $[18, 45]$
<b>Rationing</b> Whole vovulation												
Rationing prob. MW			14.96	$\begin{bmatrix} 11.76, 18.15 \end{bmatrix}$	13.85	[10.66, 17.32]			7.89	[6.52, 9.25]	7.89	[6.49, 9.09]
Kationing prob. MW & subsidy Pp. change rationing probability			$14.96 \\ 0.00$	[11.76, 18.15] $[0.00, 0.00]$	$13.85 \\ 0.00$	[10.66, 17.32] $[0.00, 0.00]$			7.89 0.00	[6.52, 9.25] $[0.00, 0.00]$	7.89 0.00	[6.49, 9.09] [0.00, 0.00]
Employer-oriented subsidies Employment												
Participation												
Pp. change participation rate	0.00	[0.00, 0.00]	2.25	[1.75, 2.76]	2.63	[1.71, 3.69]	0.00	[0.00, 0.00]	0.70	[0.53, 0.86]	0.94	[0.64, 1.31]
Change employment in 1000 Working hours	0	[0,0]	10	[49,72]	20	[40,93]	0	[0,0]	2.8	[68,1U5]	611	[80,158]
Change hours	0.00	[0.00, 0.00]	0.77	[0.61, 0.93]	0.68	[0.40, 0.98]	0.00	[0.00, 0.00]	0.23	[0.17, 0.28]	0.21	[0.14, 0.29]
Pct. change hours	0.00	[0.00, 0.00]	4.03	[2.61, 5.45]	5.04	[2.55, 9.08]	0.00	[0.00, 0.00]	1.28	[0.81, 1.76]	2.06	[0.99, 3.39]
Change hours 1000 fte.	0	[0,0]	46	[36,57]	42	[26,60]	0	[0,0]	62	[48, 77]	60	[40, 80]
Rationing												
Whole population									0 0 1		0 0 1	
Rationing prob. MW			14.96	[11.76, 18.15]	13.85	[10.66, 17.32]			7.89	[6.52, 9.25]	7.89	[6.49, 9.09]
Rationing prob. MW & subsidy			12.99	[10.13, 15.84]	11.06	[7.46, 14.71]			7.24	[5.97, 8.50]	6.83	[5.52, 7.96]
Pp. change rationing probability			-1.97	[-1.63, -2.31]	-2.79	[-3.21, -2.61]			-0.65	[-0.55, -0.75]	-1.06	[-0.97, -1.13]

**Table 4:** Employment effects:  $MW=7.50 \in$ 

pp.=percentage points, pct.=percent, Bootstrapped 95%-confidence bands in parentheses. Source: Own calculations based on SOEP, wave 2009; INKAR, wave 2010.

constrained model. Already employed individuals would thus extend their working hours.<sup>36</sup> For those employees in West Germany who earn low wages and are covered by the subsidy, the preference bias seems to dominate the participation bias.

The situation is different for an *employer-oriented wage subsidy*. Such a policy is beyond the scope of the unconstrained labor supply model. In the model with demand side restrictions an employer-oriented subsidy changes the labor supply predictions in a scenario with a statutory minimum wage. The subsidy reduces the labor costs of low-productive workers who earn low wages. This significantly reduces the predicted rationing probability for those individuals. The rationing risk would decrease by 1.1 (2.8) percentage points in West (East) Germany (Table 4). The share of low wage earners is larger in East than in West Germany.

This substantial reduction of the rationing risk leads to a considerable surge in employment at the extensive and intensive margin. With an employer-oriented subsidy the participation rate would rise by 0.9 (2.6) percentage points in West (East) Germany which amounts to 115,000 (68,000) employees. The employment losses induced by the minimum wage would be virtually compensated. The effect on total working hours is also substantial amounting to 60,000 (42,000) full time equalivalents in West (East) Germany. Most of the jobs created by an employeroriented wage subsidy would thus be some form of part time arrangement.

Several conclusions can be drawn from these findings. First, the model with demand side constraints improves the applicability of labor supply models for the evaluation of wage subsidies under a federal minimum wage. An unconstrained model provides very limited evidence at best; interpreting its labor supply predictions as employment effects can lead to misleading policy conclusions. Second, it shows that the type of wage subsidy makes a fundamental difference when a statutory minimum wage is in place. It confirms that employer-oriented subsidies are more effective in increasing employment under those circumstances. Third, in economic policy terms the findings show the potential of wage subsidies for employers that are related to low hourly wages, i.e. predominantly low-productivity jobs, when they flank a federal minimum wage. They could serve as a complentary policy tool during the implementation of a general minimum wage to protect low-productive

<sup>&</sup>lt;sup>36</sup>As discussed above our model does not include hours constraints. When individuals find a job, i.e. are not constrained by labor demand, we assume that they can freely choose their number of hours worked.

employees from adverse consequences. However, this is a stylized simulation in order to illustrate the outcomes of different subsidies in various model specifications. Further questions, e.g. alternative assumptions on the incidence of subsidies, equilibrium effects on wages, or deadweight effects, go beyond the scope of this paper.

## 6 Conclusions

We developed a discrete choice labor supply model with labor demand constraints. The framework extends previous approaches as it models the intensive margin and identifies the rationing risk not only from exogenous labor demand shocks, but also from individual productivity. Institutional variation in minimum wage thresholds is used for identification in the rationing equation. We also include unobserved individual effects that – through productivity – influence household income and labor supply as well as the rationing probability. The model is estimated with simulated maximum likelihood.

The extensions of the model have been shown to matter. We demonstrated the elasticities of the unconstrained labor supply model to be biased. Participation elasticities are upward biased in the unconstrained model which therefore overestimates labor supply reactions at the extensive margin. Hours elasticities (including the extensive and intensive margin) are also upward biased in the unconstrained model. Leaving aside potential specification bias, the participation bias in the unconstrained model apparently dominates.

The discrepancies in elasticies are in the policy simulations. Predictions from the unconstrained labor supply model are further biased as the interventions also affect labor costs. An example is the introduction of a federal minimum wage. Contrary to the unconstrained estimation, the labor supply model with demand side constraints predicts universally negative participation effects which are particularly large in East Germany. The reduction in total working hours is estimated to be considerably smaller, though. The decline in the volume of employment as a result of a federal minimum could therefore be relatively moderate. On the other hand, this masks that employees, especially those with low productivities, run the risk of losing their jobs which might be substituted by more productive labor. This sheds light on the dynamics a minimum wage might induce on the labor market, even if the employment level is not or only moderately reduced.

Finally, we demonstrate that various types of wages subsidies yield different effects in a scenario with a statutory minimum wage. Employee-oriented subsidies would be largely ineffective in expanding employment. On the other hand, subsidies paid to employers and targeted at low-productive workers could on aggregate virtually offset the negative effects of a federal minimum wage on participation. Although they cannot not be perfectly targeted at the most disadvantaged individuals and fully reverse the detrimental effects of a minimum wage, employer-oriented subsidies could nevertheless be an effective tool to absorb the shock on labor costs induced by the minimum for employees with low productivities.

The model of this paper suffers from several limitations. The crucial problem is to identify an individual's productivity as a function of observed and unobserved characteristics and separate it from other determinants of the market wage. Ultimately, this boils down to distinguishing between labor supply and demand. Ideally one would would have information about firms to get variation that is exogenous to the individual and household. A next step could be to exploit the longitudinal dimension of our data. One could, e.g., try to identify a time-invariant individual component of productivity. Similarly the denominator in the productivity/minimum wage threshold-ratio can be questioned. Using lower quantiles of the wage distribution for different labor market segments where no sectoral minimum wage is in place, might not be an optimal solution because the observed market wages are a function of supply, demand and institutional constraints. Again, having firm information would also provide better instruments for lower wage constraints on the labor demand side.

Another limitation of the framework is that it lacks an equilibrium wage adjustment (à la Haan and Steiner, 2005b or Peichl and Siegloch, 2012). Although labor supply and demand changes are analyzed in a common model, there is no feedback mechanism of either supply or demand to the wage equation. Therefore we can only consider the nominal wage changes of a minimum wage or wage subsidies in the policy analysis and leave out equilibrium wage adjustments after supply and demand have adapted. An obvious extension would be to include the unemployment rate in the wage equation. This would in principle make the simulation of wage adjustments possible as a reaction to demand and supply shifts. At the same time some of the assumptions would have to be modified. The regional unemployment rate is assumed to be exogenous for the rationing risk in the current specification. The fact that this equilibrium mechanism is not modeled, should be taken into account for the interpretation of the results.

Labor supply is estimated conditional on the spouse's decision in couple households. The obvious extension would be to specify a household labor supply model for both spouses. This complicates the likelihood considerably, though, as additional rationing and wage/productivity equations have to be included for the spouse. We currently use a static model. Since labor supply decisions and the probability of being rationed depend on the individual's labor market history, specifying a dynamic model like Haan and Uhlendorff (2013) would be a reasonable extension. Lastly, one could think about the treatment of unobserved hetreogeneity. Exploiting the longitudinal dimension of the data set will help identification when individuals switch between labor market states. Less restrictive assumptions on the functional form of the unobserved components (see Haan and Uhlendorff, 2013) will probably lead to more complicated specifications. It would still be worthwhile to test the robustness of the results in this regard. In spite of these issues the labor supply model with demand side constraints has proven to be a promising route for the integration of the labor demand side into microeconometric labor supply models.

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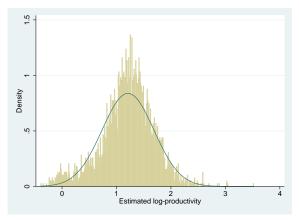
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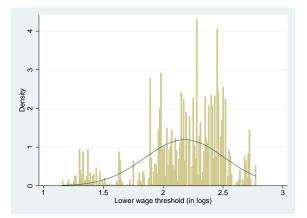
## Appendix

Figure 1: Estimated log productivity



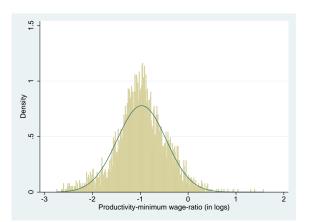
*Notes:* Estimated prouctivities from constrained model with constraints, joint estimation. *Source:* Own calculations based on SOEP, wave 2009.

Figure 2: Minimum wage thresholds (in logs)



*Notes:* Sectoral minimum wages, if applicable, observed lower wage thresholds otherwise. *Source:* Own calculations based on GSES, wave 2006, WSI minimum wage data base.

Figure 3: Productivity-minimum wage-ratio (in logs)



Source: Own calculations based on SOEP, wave 2009, GSES, wave 2006, WSI minimum wage data base.

	All		Working	20	Non-working	king	Non-working, rationed	, rationed	Non-work., vol. uenem	l. uenem.
	mean	[s.d.]	mean	[s.d.]	mean	[s.d.]	mean	[s.d.]	mean	[s.d.]
Log hourly wage	2.89	[0.45]	2.89	[0.45]						
Age	45.78	[9.88]	45.45	[9.58]	48.36	[11.67]	45.05	[10.99]	51.24	[11.51]
Years of schooling	13.79	[2.41]	13.90	[2.40]	12.90	[2.31]	12.67	[2.23]	13.09	[2.37]
Primary/secondary school certificate	0.03	,	0.03	,	0.04		0.05	,	0.04	
Higher school certificate	0.10		0.10		0.07		0.05		0.09	
Vocational education	0.58		0.57		0.61		0.63		0.60	
Tertiary education	0.24		0.25		0.14		0.11		0.16	
Years of experience	21.83	[10.42]	21.91	[10.19]	21.15	[12.05]	18.25	[10.60]	23.67	[12.67]
Years of $experience^2/100$	5.85	[4.64]	5.84	[4.57]	5.92	[5.21]	4.45	[4.00]	7.20	[5.78]
Tenure	12.25	[11.00]	13.68	[10.86]	1.00	[1.24]	2.15	[0.91]	0.00	0.00]
$\mathrm{Tenure}^2/100$	2.73	3.88	3.05	[4.00]	0.22	[0.28]	0.48	[0.20]	0.00	0.00]
Depreciation of human capital	0.45	[0.93]	0.23	[0.53]	2.18	[1.43]	2.08	[1.35]	2.26	[1.49]
Years of full time experience	21.27	[10.64]	21.39	[10.42]	20.38	[12.21]	17.45	[10.75]	22.93	[12.83]
Years of full time experience $^2/100$	5.66	[4.63]	5.66	[4.56]	5.64	[5.15]	4.19	[3.96]	6.90	[5.72]
Observations	3,858		3,424		434		202		232	

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	All mean	[ b s]	Working	الع ا	Non-working mean [s d ]	Non-working, rationed mean [s d ]	Non-work., vol. uenem. mean
		[		[]			
German nationality	0.94		0.94		0.88	0.88	0.89
Civil servant	0.27		0.30		0.00	0.00	0.00
Firm size: 1-4	-0.02		-0.02		0.00	0.00	0.00
Firm size: 5-19	-0.02		-0.02		0.00	0.00	0.00
Firm size: 20-199	0.00		0.00		0.00	0.00	0.00
Industry: Engineering, electronics	-0.04		-0.05		0.00	0.00	0.00
Industry: Mining and energy	0.00		0.00		0.00	0.00	0.00
Industry: Chemical industry, wood, paper	-0.02		-0.02		0.00	0.00	0.00
Industry: Clay, stones, earthes, construction	-0.02		-0.02		0.00	0.00	0.00
Industry: Iron, steel, heavy industry	-0.01		-0.01		0.00	0.00	0.00
Industry: Clothes	0.00		0.00		0.00	0.00	0.00
Industry: Wholesale trade	-0.02		-0.03		0.00	0.00	0.00
Industry: Train, post, communication	-0.01		-0.01		0.00	0.00	0.00
Industry: Public services	-0.02		-0.02		0.00	0.00	0.00
Industry: Private services	-0.01		-0.02		0.00	0.00	0.00
Other industries	-0.02		-0.03		0.00	0.00	0.00
Schleswig-Holstein and Hamburg	0.04		0.04		0.04	0.03	0.04
Lower Saxony and Bremen	0.10		0.10		0.09	0.11	0.07
North Rhine-Westphalia	0.20		0.21		0.17	0.15	0.19
Hesse	0.07		0.07		0.05	0.04	0.06
Rhineland-Palatinate and Saarland	0.06		0.06		0.06	0.07	0.06
Baden-Wurttemberg	0.12		0.13		0.07	0.07	0.07
Bavaria	0.15		0.15		0.13	0.08	0.16
Mecklenburg-Western Pomerania	0.02		0.02		0.03	0.03	0.02
Brandenburg	0.04		0.04		0.08	0.09	0.07
Saxony-Anhalt	0.04		0.04		0.07	0.08	0.06
Thuringia	0.04		0.04		0.06	0.06	0.06
Saxony	0.08		0.07		0.10	0.11	0.09
Observations	3,858		3,424		434	202	232

 Table 6: Descriptive statistics: wage equation – labor demand variables

	All		Working	50	Non-working	ting	Non-working, rationed	rationed	Non-work., vol. uenem	uenem.
	mean	[s.d.]	mean	[s.d.]	mean	[s.d.]	mean	[s.d.]	mean	[s.d.]
Rationing dummy	0.05		0.00		0.47		1.00		0.00	
Lower wage threshold	9.32	[2.73]	9.41	[2.73]	8.66	[2.62]	8.25	[2.73]	9.02	[2.47]
Dummy single	0.21		0.20		0.31		0.34		0.28	1
Dummy German nationality	0.94		0.94		0.88		0.88		0.89	
Dummy children $< 3$ years	0.08		0.08		0.09		0.09		0.09	
Occ.: Armed forces	0.01		0.01		0.00		0.00		0.00	
Occ.: Managers	0.07		0.08		0.01		0.00		0.01	
Occ.: Professionals	0.19		0.21		0.05		0.04		0.06	
Occ.: Technicians	0.17		0.19		0.07		0.06		0.09	
Occ.: Clerical support workers	0.07		0.07		0.05		0.05		0.04	
Occ.: Service & sales workers	0.05		0.05		0.03		0.03		0.03	
Occ.: Agricultural, forestry	0.01		0.01		0.03		0.02		0.03	
Occ.: Craft & related trades	0.23		0.21		0.39		0.40		0.38	
Occ.: Plant & machine operators	0.11		0.12		0.09		0.10		0.07	
Regional unemployment rate	8.62	[3.80]	8.48	[3.75]	9.71	[4.00]	10.15	[4.00]	9.31	[3.97]
Regional employment rate	55.46	[3.55]	55.53	[3.57]	54.97	[3.30]	54.77	[3.28]	55.15	[3.30]
Regional part time share	18.14	[1.67]	18.14	[1.68]	18.12	[1.53]	18.29	[1.57]	17.98	[1.49]
Regional BIP	29.01	[7.32]	29.17	[7.31]	27.69	[7.30]	27.01	[7.01]	28.28	[7.50]
Observations	3,858		3,424		434		202		232	
Notes: All=whole estimation sample, Working=all employed individuals, Non-working=all non-working individuals, Non-working, rationed=involuntarily unemployed individuals, Non-work. vol. unemployed=voluntarily unemployed individuals, Occ.=occupation. Source: Own calculations based on SOEP, wave 2009; INKAR, wave 2010.	l employed indi ed individuals, 009; INKAR, w	ividuals, Non-work , Occ.=occupation /ave 2010.	-working=all ation.	non-working	individuals, l	Von-workin	g, rationed=inv	oluntarily	unemployed indi	viduals,

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	All		Working	1g	Non-working	king	Non-working, rationed	c, rationed	Non-work., vol. uenem.	ol. uenem.
	mean	[s.d.]	mean	[s.d.]	mean	[s.d.]	mean	[s.d.]	mean	[s.d.]
Consumption										
Age	368.99	[86.48]	371.00	[85.39]	353.13	[93.26]	334.26	[84.63]	369.57	[97.40]
Age squared	176.93	[73.51]	176.49	[72.78]	180.39	[79.08]	159.41	[70.28]	198.66	[81.87]
Leisure	29.88	[2.21]	29.87	[1.89]	29.98	[3.94]	28.08	[2.07]	31.63	[4.41]
Constant	8.06	[0.56]	8.14	[0.50]	7.46	[0.66]	7.66	[0.47]	7.28	[0.75]
Consumption squared	65.10	[9.25]	66.49	[8.18]	54.15	[9.93]	55.36	[8.17]	53.10	[11.15]
Leisure										
Age	170.61	[39.81]	166.96	[36.09]	199.39	[53.73]	170.49	[41.58]	224.54	[50.42]
Age squared	81.88	[34.70]	79.27	[31.98]	102.46	[46.49]	81.36	[35.68]	120.84	[47.07]
East	0.91	[1.61]	0.84	[1.55]	1.45	[1.96]	1.52	[1.86]	1.40	[2.05]
German	3.48	[0.93]	3.46	[0.86]	3.62	[1.35]	3.32	[1.25]	3.89	[1.39]
Handicapped	0.08	[0.56]	0.07	[0.49]	0.22	[0.95]	0.11	[0.64]	0.32	[1.14]
Child $\leq = 3$ years	0.31	[1.04]	0.31	[1.01]	0.39	[1.21]	0.36	[1.11]	0.42	[1.29]
Child 3-6 years	0.37	[1.11]	0.37	[1.11]	0.32	[1.09]	0.37	[1.13]	0.26	[1.05]
Single	0.78	[1.53]	0.72	[1.46]	1.25	[1.89]	1.27	[1.79]	1.23	[1.97]
Single x child $< 3$ years	0.02	[0.26]	0.01	[0.23]	0.05	[0.43]	0.06	[0.46]	0.04	[0.41]
Constant	3.72	[0.22]	3.67	[0.15]	4.10	[0.30]	3.78	0.00]	4.38	0.00]
Leisure squared	13.90	[1.73]	13.51	[1.14]	16.93	[2.44]	14.32	[00.0]	19.20	[0.00]
Observations	3,877		3,443		434		202		232	
	rking=all emple memployed ind , wave 2009; IN	yed individu ividuals. [KAR, wave 2	als, Non-work 2010.	ing=all non-	working indiv	riduals, Non	-working, ratic	oned=involu	intarily unemple	oyed individua

Table 8: Descriptive statistics: labor supply equation

Table 9:	Estimation	results:	wage	equation

	Uncon.	Model	Con. M	lodel	Con. Mod	el Het.
	coeff.	[s.e.]	coeff.	[s.e.]	coeff.	[s.e
Productivity variables						
Age	$0.014^{***}$	[0.003]	$0.007^{**}$	[0.003]	$0.009^{***}$	[0.002
Years of schooling	$0.054^{***}$	[0.011]	$0.061^{***}$	[0.011]	$0.060^{***}$	0.011
Primary/secondary school cert.	-0.001	[0.043]	0.009	[0.042]	0.006	0.043
Higher school certificate	0.117**	[0.054]	$0.114^{**}$	[0.052]	0.110**	0.05
Vocational education	-0.079	[0.052]	-0.099*	[0.051]	-0.102*	0.05
Tertiary education	0.034	[0.098]	0.006	[0.096]	0.007	0.10
Years of experience	-0.021**	[0.010]	-0.013	[0.009]	-0.012	[0.00
Years of experience $^2/100$	0.007	[0.021]	-0.002	[0.020]	-0.009	0.02
Tenure	0.010***	[0.002]	$0.014^{***}$	[0.002]	$0.017^{***}$	0.00
$Tenure^2/100$	-0.009*	[0.005]	-0.018***	0.005	-0.022***	0.00
Depreciation of human capital	-0.177***	[0.012]	-0.249***	[0.011]	-0.280***	0.01
Years full time experience	0.026***	[0.008]	0.025***	[0.008]	0.023***	0.00
Years full time experience $^{2}/100$	-0.040**	[0.020]	-0.038**	0.019	-0.032	0.02
Labor demand variables		· - J				
German nationality	0.072***	[0.025]	$0.078^{***}$	[0.025]	$0.080^{***}$	[0.02]
Civil servant	-0.028***	[0.007]	-0.028***	[0.007]	-0.029***	0.00
Firm size: 1-4	-0.186***	[0.025]	-0.173***	[0.025]	-0.173***	[0.02
Firm size: 5-19	-0.063***	[0.006]	-0.054***	[0.006]	-0.052***	0.00
Firm size: 20-199	$0.025^{**}$	[0.010]	$0.019^{*}$	[0.010]	0.017	0.01
Industry: Engineering, electronics	$0.037^{***}$	[0.012]	$0.031^{***}$	[0.012]	$0.029^{**}$	0.01
Industry: Mining and energy	$0.091^{**}$	[0.037]	0.088**	0.037	0.082**	0.03
Industry: Chemical, wood, paper	0.023	[0.021]	0.012	[0.021]	0.008	0.02
Industry: Clay, stones, construction	0.025	[0.018]	$0.039^{**}$	[0.018]	$0.034^{*}$	0.01
Industry: Iron, steel, heavy industry	0.021	[0.020]	0.010	[0.020]	0.007	0.02
Industry: Clothes	0.095	[0.073]	0.096	[0.074]	0.104	0.07
Industry: Wholesale trade	-0.063***	0.018	-0.072***	[0.018]	-0.069***	0.01
Industry: Train, post, communic.	-0.102***	[0.020]	-0.091***	[0.020]	-0.083***	0.02
Industry: Public services	-0.011	[0.011]	-0.009	[0.012]	-0.008	0.01
Industry: Private services	$0.076^{***}$	0.018	0.080***	0.018	$0.079^{***}$	0.01
Industry: Others	-0.028	[0.021]	-0.020	[0.021]	-0.014	0.02
Schleswig-Holstein and Hamburg	0.124***	[0.040]	0.135***	[0.040]	0.139***	[0.04
Lower Saxony and Bremen	$0.109^{***}$	[0.035]	$0.103^{***}$	[0.035]	0.110***	0.03
North Rhine-Westphalia	0.120***	[0.033]	0.121***	[0.032]	0.122***	[0.03
Hesse	0.155***	[0.036]	0.152***	[0.036]	0.155***	[0.03]
Rhineland-Palatinate and Saarland	0.154***	[0.038]	0.136***	[0.038]	0.147***	[0.03
Baden-Württemberg	0.180***	[0.034]	0.173***	[0.034]	0.179***	[0.034
Bavaria	0.137***	[0.033]	0.128***	[0.033]	0.133***	0.03
Mecklenburg-Western Pomerania	-0.065	[0.046]	-0.043	[0.047]	-0.037	[0.04]
Brandenburg	-0.043	[0.041]	-0.056	[0.041]	-0.046	0.04
Saxony-Anhalt	-0.108***	[0.040]	-0.117***	[0.041]	-0.118***	0.04
Thuringia	-0.166***	[0.040]	-0.181***	[0.040]	-0.172***	[0.04]
Saxony	-0.149***	[0.036]	-0.153***	[0.036]	-0.145***	0.03
Mills ratio	-0.081	[0.060]	0.061	[0.053]	-	
Constant	0.311***	[0.004]	1.505***	[0.110]	1.445***	[0.11]
Observations	3,443		3,858		3,858	
Log-likelihood	-860		-4,763		-4,718	

*Notes:* Uncon. Model=unconstrained model, Con. Model=constrained model, Con. Model Het.=constrained model with unobserved heterogeneity (joint estimation), coeff.=regression coefficient, s.e.=standard error. *Source:* Own calculations based on SOEP, wave 2009; INKAR, wave 2010.

Table 10:	Estimation	results:	rationing	equation

	Uncon. M	Model	Con. M	odel	Con. Mod	el Het.
	coeff.	[s.e.]	coeff.	[s.e.]	coeff.	[s.e.
Productivity-minimum wage	-1.251***	[0.121]	-1.405***	[0.107]	-2.062***	[0.171
Discrimination variables						
Dummy single	$0.401^{***}$	[0.090]	$0.389^{***}$	[0.094]	$0.408^{***}$	[0.110]
Dummy German nationality	-0.267*	[0.138]	-0.217	[0.144]	-0.250	[0.172]
Dummy children $< 3$ years	0.156	[0.136]	0.192	[0.140]	0.261	0.160
Labor demand variables						
Occ.: Armed forces	-0.534	[0.520]	-0.401	[0.541]	-0.207	[0.645]
Occ.: Managers	$-1.584^{***}$	[0.350]	-1.481***	[0.360]	-1.010**	[0.424]
Occ.: Professionals	-1.125***	[0.175]	-1.031***	[0.181]	-0.690***	[0.222]
Occ.: Technicians	$-1.103^{***}$	[0.158]	-1.013***	[0.163]	-0.749***	[0.194]
Occ.: Clerical support workers	-0.789***	[0.178]	-0.713***	[0.184]	$-0.621^{***}$	0.216
Occ.: Service & sales workers	-0.828***	[0.210]	-0.766***	[0.219]	-0.768***	0.258
Occ.: Agricultural, forestry	-0.100	[0.320]	-0.111	[0.336]	0.101	[0.396]
Occ.: Craft & related trades	$-0.543^{***}$	[0.114]	-0.500***	[0.118]	-0.360***	[0.130]
Occ.: Plant & machine operators	-0.888***	[0.146]	-0.852***	[0.153]	-1.012***	[0.18]
Regional unemployment rate	$0.101^{***}$	[0.021]	$0.102^{***}$	[0.021]	$0.126^{***}$	0.026
Regional employment rate	0.040**	[0.020]	$0.047^{**}$	[0.021]	$0.064^{**}$	[0.026]
Regional part time share	0.049	[0.030]	0.051	[0.031]	0.046	[0.03]
Regional BIP	0.006	[0.008]	0.006	0.008	0.011	0.01
Constant	-4.443**	[1.744]	$-7.139^{***}$	[1.856]	$-9.445^{***}$	[2.272
Observations	3,625		3,858			
Log-likelihood	-628		-4,739			

*Notes:* Uncon. Model=unconstrained model, Con. Model=constrained model, Con. Model Het.=constrained model with unobserved heterogeneity (joint estimation), coeff.=regression coefficient, s.e.=standard error, Occ.=occupation.

Source: Own calculations based on SOEP, wave 2009; INKAR, wave 2010.

	Uncon. N	/Iodel	Con. M	odel	Con. Mode	el Het.
	coeff.	[s.e.]	coeff.	[s.e.]	coeff.	[s.e.
Consumption						
Linear term	-0.300	[2.938]	$-27.186^{***}$	[4.082]	5.456	[3.968]
Quadratic term	$0.289^{***}$	[0.072]	$1.568^{***}$	[0.158]	$0.618^{***}$	[0.113]
x Age	0.120	[0.110]	0.176	[0.151]	-0.250	[0.159]
x Age squared	-0.138	[0.119]	-0.186	[0.164]	$0.302^{*}$	[0.175]
x Single	0.294	[0.400]	0.864	[0.546]	0.793	[0.593]
x Leisure	-1.122***	[0.225]	$0.338^{***}$	[0.037]	$-1.501^{***}$	[0.273]
Leisure						
Linear term	$49.388^{***}$	[4.213]	$51.678^{***}$	[3.001]	$90.089^{***}$	[5.217]
Quadratic term	-4.641***	[0.291]	-5.868***	[0.305]	-9.112***	[0.434]
x Age	-0.210**	[0.101]	-0.421***	[0.080]	-0.399***	[0.101]
x Age squared	$0.301^{***}$	[0.111]	$0.547^{***}$	[0.088]	$0.546^{***}$	[0.111]
x East	0.111	[0.153]	0.163	[0.187]	$-0.788^{***}$	[0.221]
x German	-1.004***	[0.235]	-1.475***	[0.306]	-0.523	[0.337]
x Handicapped	$1.450^{***}$	[0.374]	$1.584^{***}$	[0.457]	$1.897^{***}$	[0.538]
x Child $\leq = 3$ years	0.185	[0.268]	$1.201^{***}$	[0.351]	0.050	[0.363]
x Child 3-6 years	$0.512^{**}$	[0.237]	$0.579^{**}$	[0.295]	$0.716^{**}$	[0.325]
x Single	0.664	[0.429]	$1.812^{***}$	[0.290]	0.589	[0.462]
x Single x child $< 3$ years	0.567	[0.845]	-0.938	[1.205]	-0.151	[1.226]
Costs part time work	$2.576^{***}$	[0.131]	$2.514^{***}$	[0.145]	$1.729^{***}$	[0.152]
Rho (unobs. het. x inact.)					-0.470	[0.406]
Observations	3,877		3,858		3,858	
Wald chi2	2.26		3.65		5.70	
Log-likelihood	-3,847		-4,763		-4,718	
Positive 1st Derivates (in %)						
$U_c$ (consumption)	100.0		97.8		100.0	
$U_l$ (leisure)	87.0		92.2		92.1	

 Table 11: Estimation results: labor supply model

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*Notes:* Uncon. Model=unconstrained model, Con. Model=constrained model, Con. Model Het.=constrained model with unobserved heterogeneity (joint estimation), coeff.=regression coefficient, s.e.=standard error. *Source:* Own calculations based on SOEP, wave 2009; INKAR, wave 2010.