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Institutional herding in financial markets:
New evidence through the lens of a simulated model

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Abstract

Due to data limitations and the absence of testable, model-based predictions, theory and evidence on herd behavior are only loosely connected. This paper contributes towards closing this gap in the herding literature. We use numerical simulations of a herd model to derive new, theory-based predictions for aggregate herding intensity. Using high-frequency, investor-specific trading data we confirm the predicted impact of information risk on herding. In contrast, the increase in buy herding measured for the financial crisis period cannot be explained by the herd model.

Keywords: Herd Behavior, Institutional Trading, Model Simulation
JEL classification: G11, G24

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1 Introduction

Herd behavior among investors is often viewed as a significant threat for the functioning of financial markets. The distorting effects of herding on financial markets range from informational inefficiency to increased stock price volatility, or even bubbles and crashes. While the herding phenomenon has been explored extensively in the literature, theory and evidence on herding are typically only loosely connected. The theoretical herding literature has greatly contributed to the understanding under which conditions herding may occur on an individual investor level for single stocks in a tick-by-tick trading context. Herd models, however, have not been exploited to provide insights on how such individual herding relates to herding intensity of an investor group aggregated across a set of heterogeneous stocks and over time. Due to data limitations, on the other hand, empirical researchers typically cannot assess herding in an investor-specific and high-frequency trading context. Instead they have to rely on estimates of aggregate herding intensity. As a consequence, the interpretation of estimation results is intuitive but typically not closely related to a particular herd model.¹ This paper proposes to interpret empirical herding measures through the lens of a simulated herd model, thereby contributing towards closing the gap between theoretical and empirical herding literature. Specifically, we simulate the herd model of Park and Sabourian (2011) for a broad range of parameters to derive testable, theory-based hypotheses on aggregate

¹For example, several empirical studies investigating the size effect of herding are based on the plausible but unproven hypothesis that herding intensity should be the larger the smaller the quantity and quality of available information, see e.g. Lakonishok et al. (1992), Wermers (1999), and Sias (2004). In the same vein, herding intensity is linked to the stage of the development of the financial market, see e.g. Walter and Weber (2006).
herding intensity. In a second step, these hypotheses are tested using a unique high-frequency, investor-specific data set.

The theoretical herding literature defines herd behavior as the switch in an agent’s opinion into the direction of the crowd, see e.g. Brunnermeier (2001). As herders ignore their own private information, herd behavior is always informationally inefficient and thus has the potential to distort prices and to destabilize markets. The main focus of the theoretical herding literature is the investigation of the microeconomic drivers for such informationally inefficient behavior. In the seminal work of Bikhchandani et al. (1992) and Banerjee (1992) herding stems from information externalities that an observable investment decision of one agent exhibits on subsequent agents’ expectation regarding the investment value. Other microeconomic foundations of herd behavior include reputational concerns (see e.g. Scharfstein and Stein (1990), Graham (1999) and Dasgupta et al. (2011)) as well as investigative herding (see Froot et al. (1992) and Hirshleifer et al. (1994)).

The herding concept was put into a financial market context by Avery and Zemsky (1998). Recently, their model was developed further to a more general setting by Park and Sabourian (2011). In line with the earlier literature, Park and Sabourian (2011) concentrate on indentifying microeconomic drivers for individual investor herding. In particular, they show that similar to Bikhchandani et al. (1992)
information externalities induce herd behavior in their model.\textsuperscript{3} It is this microeconomic focus of the theoretical herding literature that impedes an analytical approach to derive results for herding intensity aggregated across investors or time. This may explain why herd models have not been exploited to provide hypotheses on aggregate herding intensity that can be tested empirically.

Empirical researchers on the other hand typically do not have the means to assess herding on the same microlevel discussed in the theoretical literature. To be able to analyze investor herding and interpret it in the context of a particular herd model, high-frequency, investor-specific trading data would be needed. However, empirical studies typically have to rely on either investor-specific but low-frequency data as, e.g., in Lakonishok et al. (1992), Sias (2004), and Wermers (1999), or on high-frequency but anonymous transaction data, compare Barber et al. (2009) or Zhou and Lai (2009).\textsuperscript{4} Even with high-frequency, investor-specific trading data, empirical researchers would have to rely on proxies for aggregate informationally inefficient herding intensity as trading data by itself cannot fully reveal the drivers for a specific investor decision. This insight found its way into the work of Lakonishok et al. (1992) and Sias (2004), which have become trailblazers for the empirical herding literature. In Lakonishok et al.

\textsuperscript{3}Other financial market herd models such as Lee (1998), Chari and Kehoe (2004) and Cipriani and Guarino (2008) focus on more specific aspects of herd behavior. They investigate how investor herding is related to transaction costs, endogenous timing of trading decisions, informational spillovers between different assets respectively. Yet, in all these models information externalities remain the key driver for herd behavior among investors as in Park and Sabourian (2011).

\textsuperscript{4}An alternative approach in the empirical literature identifies herd behavior by analyzing the clustering of individual stock returns around a market consensus, see Chang et al. (2000) and Chiang and Zheng (2010). While this empirical approach does not require investor specific data, it is not directly connected to microeconomically founded herding theory.
herding of a group of investors is measured as a tendency to accumulate on one
side of the market. Specifically, they test whether the share of net buyers in individual
stocks significantly deviates from the average share of net buyers across all stocks.
Sias (2004) proposes a more dynamic approach to test for herding. He investigates
whether the accumulation of investors on one side of the market persists over time by
measuring the cross-sectional correlation of the share of net buyers over two adjacent
time periods. In contrast to the theoretical literature, both empirical herding measures
examine herding intensity on an aggregate level with respect to investors, stocks and
time. Hence, neither Lakonishok et al. (1992) nor Sias (2004) can tie their evidence
or its interpretation to a particular herd model leaving the theoretical and empirical
herding literature largely disconnected.

From a theoretical perspective, we contribute to close this gap in the literature by
deriving predictions for aggregate herding intensity from numerical simulations of the
Park and Sabourian (2011) model. Our analysis is based on the Park and Sabourian
(2011) model because its sequential trading structure allows for a quantitative analysis
of aggregate herding intensity in a financial market context. Moreover, herding in Park
and Sabourian (2011) is generated under rather weak assumptions and for a very rich set
of information structures. Therefore, it can be related to a wide range of microeconomic
drivers such as information asymmetries, reputational concerns and transaction costs
respectively.

The focus of this paper is on the impact of market stress and information risk on
aggregate herding intensity because both concepts can be easily translated into the model and are of significant economic relevance. While herd behavior certainly has the potential to create times of market stress, it is less clear whether the reverse relationship holds, thereby creating vicious cycles of economic downturns and high volatility regimes. Information risk, defined as the probability of trading with a counterpart who holds private information about the asset (see Easley et al. (1996)), reflects the degree of asymmetric information in herd models. Information risk is thus a key determinant for herd behavior.

By simulating the Park and Sabourian (2011) herd model for a broad range of parameters generating about 2.6 billion trades to analyze, we obtain two testable hypotheses regarding the impact of information risk and market stress on aggregate herding intensity: First, an increase in information risk should result in an increase of both, buy and sell herding intensity. And second, increased market stress should have an asymmetric effect on herding intensity: it should imply a decrease in buy herding intensity but an increase in sell herding intensity. To the best of our knowledge, these findings are the first theory-founded comparative static results for herding intensity in a stock market.

From an empirical perspective, we contribute to the literature by testing model-based hypotheses using an intra-day, investor-specific data set provided by the German

\footnote{While Chiang and Zheng (2010) and Christie and Huang (1995) confirm that herding increases during times of market stress, Kremer and Nautz (2013a,b) find that herding in the German stock market even slightly decreased during the recent financial crisis. Similar results are provided by Hwang and Salmon (2004) for herding intensity during the Asian and the Russian crisis in the nineties.}
Federal Financial Supervisory Authority (BaFin). The data include all real-time transactions in the major German stock index DAX 30 carried out by banks and financial services institutions. In line with herding theory, the use of *intra-day* data is particularly appropriate for measuring herd behavior. Private information in financial markets is fast moving and the informational advantage from private signals can only be exploited for short time horizons. Measuring herding at lower frequencies may bias the results because new information might have reached the market in the meantime, establishing a new context for investor behavior. The use of *investor-specific* data is particularly important as we need to directly identify transactions by each trader in order to determine whether an investor follows the observed actions of other traders or her own trades. To assess herding empirically, we employ the herding measure proposed by Sias (2004). The dynamic nature of the Sias measure makes it particularly appropriate for the analysis of high-frequency data. It reflects the theoretical notion of herders’ switching behavior more accurately than the static measure of Lakonishok et al. (1992). Moreover, the Sias measure incorporates the intuition of the Park and Sabourian (2011) model that during periods of herding, high shares of e.g. net buy-

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6 This data set has already been used by two companion papers. Kremer and Nautz (2013b) demonstrate the importance of both data frequency and the possibility to identify traders for resulting herding measures. Kremer and Nautz (2013a) regress daily herding measures on e.g. size, volatility and other stock characteristics to analyze the causes of herding. The current paper builds on these studies in two important aspects. First, to the best of our knowledge, this paper is the first that analyzes *intra-day* herding intensity using investor-specific data. Second, similar to the bulk of the empirical literature, the empirical analyses of Kremer and Nautz (2013a,b) are not related to a particular herd model.

7 Recently, Cipriani and Guarino (2013) proposed a method to estimate a herd model with anonymous transaction data.

8 Note that like all empirical herding measures, the Sias measure is only an approximation of informationally inefficient herding defined in the theory. It is our view, however, that a comprehensive assessment of the accuracy of empirical herding measures with respect to theoretical herding should only be conducted after a methodological link between theory and evidence has been established and hence is beyond the scope of this study.
ers persist over time. The Sias measure also captures the second feature of our data: having access to investor-specific information, it allows differentiating between traders that indeed follow predecessors and traders that simply follow themselves, for example, because they split their trades. Interestingly, the Sias measure has not been applied to intra-day data before.

In accordance with our first theory-based hypothesis, our empirical results show that herding intensity increases with information risk. In particular, the analysis of half-hour trading intervals reveals a strong and significant co-movement of trading activity and the herding intensity of institutional traders. In contrast to our theory-based hypothesis on the effect of market stress on herding intensity, however, our results do not suggest an asymmetric impact of market stress on herding intensity. In fact, we find that both, sell as well as buy herding slightly increased in the German stock market during the financial crisis. Through the lens of the Park and Sabourian (2011) model, these results suggest that herding observed empirically during the financial crisis may only be unintentional or spurious.

The remainder of the paper is structured as follows: In Section 2 we review the model of Park and Sabourian (2011), which is the theoretical basis of our further analysis. We discuss how to define and measure herding intensity in the model and its simulation, and explain how information risk and the degree of market stress are reflected in the model. Section 3 introduces the simulation setup and derives the hypotheses on the role of information risk and market stress for herding intensity.
Section 4 introduces the empirical herding measure. Section 5 presents the data and shows the empirical results. Section 6 concludes.

2 Information risk and market stress in a herd model

2.1 The herd model

Park and Sabourian (2011) consider a sequential trading model à la Glosten and Milgrom (1985) consisting of a single asset, informed and noise traders, and a market maker. The model assumes rational expectations and common knowledge of its structure. Park and Sabourian (2011) not only investigate herd behavior but also contrarian behavior in their model. We focus only on herd behavior, however, since herd behavior is self-enforcing while contrarian behavior is self-defeating. Therefore, the destabilizing effects of contrarianism are limited and, thus, only of secondary importance for financial markets.

The asset: There is a single risky asset with unknown fundamental value \( V \in \{V_1, V_2, V_3\} \), where \( V_1 < V_2 < V_3 \). Its distribution is given by \( 0 < P(V = V_j) < 1 \) for \( j = 1, 2, 3 \) where \( \sum_{j=1}^{3} P(V = V_j) = 1 \). The asset is traded over \( t = 1, \ldots, T \) consecutive points in time. Thus, the trading period under consideration is \([0, T]\). In Section 3, we will choose \( T = 100 \) for simulating the model.

Cipriani and Guarino (2008) make a first attempt to theoretically study contagion and information cascades in a two asset model. Their model is heavily based on Avery and Zemsky (1998) and can thus be seen as a variation of Park and Sabourian (2011).
The traders: Traders arrive one at a time in a random exogenous order in the market and decide to buy, to sell, or not to trade one unit of the asset at the quoted bid and ask prices. Traders are either informed traders or noise traders. The fraction of informed traders is denoted by $\mu$. Informed traders base their decision to buy, sell or not to trade on their expectations regarding the asset’s true value. In addition to the publicly available information consisting of the history of trades $H_t$, i.e. all trades observed until period $t$, informed traders form their expectations according to a private signal $S \in \{S_1, S_2, S_3\}$ on the fundamental value of the asset. They will buy (sell) one unit of the asset if their expected value of the asset conditioned on their information set is strictly greater (smaller) than the ask (bid) price. Otherwise, informed traders choose not to trade. In the empirical herding literature, institutional investors are seen as a typical example for informed traders. Noise traders trade randomly, i.e. they decide to buy, sell or not to trade with equal probability of $1/3$.

The private signal: The distribution of signals is conditioned on the true value of the asset, i.e. $P(S = S_i \mid V = V_j) = p^{ij}$ with $0 \leq p^{ij} \leq 1$ and $\sum_{i=1}^{3} p^{ij} = 1$ for all $i, j = 1, 2, 3$. For each $i$, the shape of a private signal $S_i$ is given by $p^{ij}$, $j = 1, 2, 3$. In particular, Park and Sabourian (2011) define a signal $S_i$ to be

- monotonically decreasing iff $p^{i1} > p^{i2} > p^{i3}$.

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$^{10}$Chari and Kehoe (2004) show that endogenous timing of trading decisions enables herd behavior even if investors can choose to trade any real amount of stock at a unique frictionless market price.

$^{11}$Park and Sabourian (2011) derive upper bounds for $\mu$ that have to hold in order for herding to be possible. The parameterizations chosen for the model simulation all imply an upper bound of $\mu = 1$ for herding to be possible, compare section 3.
monotonically increasing iff $p^{i1} < p^{i2} < p^{i3}$,

u-shaped iff $p^{i1} > p^{i2}$ and $p^{i2} < p^{i3}$.

Park and Sabourian (2011) show that a necessary condition for herding is that there exists a u-shaped signal. In accordance with Park and Sabourian (2011), we consider the case where one signal is u-shaped and both, optimists and pessimists are present in the market, i.e. one signal is monotone increasing (optimist) and another signal is monotone decreasing (pessimist).

The market maker: Trading takes place in interaction with a market maker who quotes a bid and ask price. The market maker has access only to public information and is subject to perfect competition such that he makes zero-expected profit. Thus, he sets the ask (sell) price equal to his expected value of the asset given a buy (sell) order and the public information. Formally, he sets $\text{ask}_t = E[V|H_t \cup \{a_t = \text{buy}\}]$ and $\text{bid}_t = E[V|H_t \cup \{a_t = \text{sell}\}]$, where $a_t$ is the action of a trader in time $t$.

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12 An investor that receives a u-shaped signal can be regarded as an investor who has reputational concerns. Dasgupta et al. (2011), for instance, argue that positive externalities from reputation cause career-concerned money managers to pay premiums to trade into the direction of the crowd. A trader with a u-shaped signal follows the exact same pattern. As a u-shaped trader observes a tendency in the order flow, he discounts the possibility that the opposite extreme state could be true. Since his signal also causes him to weigh the middle state much lower than the tails, his price expectation jumps into the direction of the crowd potentially surpassing other investors’ expectations. In other words: a trader with a u-shaped signaled is prepared to pay a premium to trade with the crowd.

13 Park and Sabourian (2011) also introduce hill-shaped signals which are necessary for contrarian behavior. Since contrarian behavior is not in the focus of this study, we exclude hill-shaped signals from our simulation.

14 The existence of bid-ask spread implies the presence of transaction costs. Hence, the insights from e.g. Lee (1998) can also be generated in the Park and Sabourian (2011) framework.
2.2 Herding intensity

Park and Sabourian (2011) describe herding as a “history-induced switch of opinion [of a certain informed trader] in the direction of the crowd”. More precisely, in the model context, herding is defined as follows:

**Definition: Herding**

Let $b_t (s_t)$ be the number of buys (sells) observed until period $t$ at history $H_t$. A trader with signal $S$ **buy herds** in period $t$ at history $H_t$ if and only if

(i) $E[V|S] \leq \text{ask}_t$ (Informed trader with signal $S$ does not buy initially),

(ii) $E[V|S,H_t] > \text{ask}_t$ (Informed trader with signal $S$ buys in $t$)

(iii) $b_t > s_t$ (The history of trades contains more buys than sells, i.e. the crowd buys)

Analogously, a trader with signal $S$ **sell herds** in period $t$ at history $H_t$ if and only if

(i) $E[V|S] \geq \text{bid}_t$ (Informed trader with signal $S$ does not sell initially),

(ii) $E[V|S,H_t] < \text{bid}_t$ (Informed trader with signal $S$ sells in $t$)

(iii) $b_t < s_t$ (The history of trades contains more sells than buys, i.e. the crowd sells)

This definition is less restrictive than the one used in Park and Sabourian (2011). Above, herding refers to switches from *not buying* (not selling) to buying (selling), whereas Park and Sabourian (2011) define herding to be extreme switches from *selling*. 

11
to buying and vice versa. However, as Park and Sabourian (2011) already noted, allowing herd behavior to include switches from holding to selling or buying is a legitimate extension which they do not consider only to be consistent with some of the earlier theoretical work on herding. For our empirical application, including switches from holding to selling or buying is more appropriate because such switches also contribute to stock price movements.\textsuperscript{15} Item (iii) of the above definition also slightly differs from the one in Park and Sabourian (2011). There, (iii) reads \( E[V|H_t] > E[V] \) for buy herding (and analogously for sell herding) and is based on the idea that prices rise (fall) when there are more (less) buys than sells. However, for an empirical analysis of herd behavior based on trading data, it is more convenient to base the definition of herding more closely to the term “following the crowd”: While we can observe the number of buys and sells, the market’s expectation of the asset’s true value, \( E[V|H_t] \), can at best be approximated.

By definition, only informed traders can herd. Therefore, herding intensity is defined as the number of trades where traders engaged in herd behavior as a fraction of the total number of informed trades. In order to remain close to our empirical application we consider only trades from informed types and exclude holds, since we investigate institutional trading and our data does not cover holds. Specifically, for each trading

\textsuperscript{15}Note that it would also be possible to include switches from selling or buying to holding. However, we are mainly interested in herd behavior which potentially contributes to stock price volatility. Any switch to holding cannot amplify stock price movements or cause the stock price to move into the wrong direction. The only empirical effect would be a reduction in trading volume. By model assumption, however, liquidity is steadily provided by noise traders.
period \([0, T]\), sell herding intensity (SHI) in the model is measured as

\[
\text{Sell herding intensity} = \frac{\#\text{herding sells}}{\#\text{informed trades}}
\]

and the definition for buy herding intensity (BHI) follows analogously.

### 2.3 Information risk and market stress in the model

Easley et al. (1996) introduce information risk as the probability that an observed trade was executed by an informed trader. Thus, information risk coincides with the parameter \(\mu\), the fraction of informed traders, in the model of Park and Sabourian (2011). Therefore, we derive our theoretical prediction for the effect of information risk on herding intensity by conducting comparative static analysis for herding intensity with respect to changes in \(\mu\).

Times of market stress are typically understood as times of deteriorated economic outlook and increased risk, when markets become more pessimistic and more uncertain. In the model of Park and Sabourian (2011), these changes in the distribution of the fundamental value of the asset are reflected in lower \(E[V]\) and higher \(\text{Var}(V)\). Both effects can be summarized using the coefficient of variation, \(VC(V) := \sqrt{\text{Var}(V)}/E[V]\), as a measure of market stress. The higher \(VC(V)\), the higher the degree of market stress.
3 Simulating the herd model for a heterogenous stock market

Empirical studies on herd behavior typically derive results for herding intensity as an average for a large set of stocks. These stocks are likely to differ in their characteristics, which in terms of the herd model means that each stock is described by a distinct parameterization for the fraction of informed traders, the prior distribution of the asset, and the distribution of the private signals. Moreover, these characteristics cannot be expected to be constant over time. In accordance with the empirical literature, we are therefore particularly interested in the comparative statics of herding intensity as an average over a broad range of parameterizations. Yet, the model of Park and Sabourian (2011) is not designed to allow the derivation of a tractable closed form solution for the average herding intensity expected for a broad range of model parameterizations. In fact, even for a single parameterization, comparative static results cannot be obtained analytically, see the appendix. As a consequence, we derive comparative static results on the role of information risk and market stress on average herding intensity by means of numerical model simulations. In the following, we explain the choice of parameter values and the simulation setup.
3.1 Simulation setup

In our simulations, we assume that the fraction of informed traders, $\mu$, is taken from $M = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$, i.e. $|M| = 9$. Note that values $\{0.2, ..., 0.7\}$ correspond to the range of market shares of institutional investors observed for our sample period, compare Kremer and Nautz (2013a).

The prior distribution for an asset, $P(V)$, is taken from the set

$$\mathcal{P} = \{P(V) : P(V_j) \in \{0.1, 0.2, \ldots, 0.8\} \text{ for } j = 1, 2, 3 \text{ and } \sum_{j=1}^{3} P(V_j) = 1\}.$$  

Thereby, we consider only situations where the risky asset $V$ takes each value $V_1, V_2, V_3$ with positive probability. This parametrization produces $|\mathcal{P}| = 36$ different asset distributions.

The conditional signal distribution, $P(S|V)$ is chosen from

$$\tilde{\mathcal{C}} = \{P(S|V) : p^{ij} \in \{0.1, 0.2, \ldots, 0.8\} \text{ for } i, j = 1, 2, 3\}$$

where we consider only those signal structures $\mathcal{C} \subset \tilde{\mathcal{C}}$ which imply more optimists in “good times”, i.e. $p^{13} < p^{23} < p^{33}$, and more pessimists in “bad times”, i.e. $p^{11} > p^{21} > p^{31}$. As a result, the simulation accounts for $|\mathcal{C}| = 41$ different signal structures.\textsuperscript{16}

\textsuperscript{16}Note that the concept of e.g. more pessimists in bad times is different from the earlier introduced concept of the signals of informed traders. While $(p^{i1}, p^{i2}, p^{i3})$ constitute the $i$-th column of $P(S|V)$, the signal $S_j$ is described by the $j$-th row of $P(S|V)$. That is, our parameterization contains monotone increasing, monotone decreasing, and u-shaped signals while in good times, i.e. when $V_3$ realizes, there is a higher likelihood for an informed trader to receive an increasing signal than a decreasing signal, whereas in bad times, i.e. when $V_1$ realizes, the opposite holds.
Considering all possible combinations of the above parameters we obtain \( \Omega := \mathcal{M} \times \mathcal{P} \times \mathcal{C} \), where \( |\Omega| = 9 \times 36 \times 41 = 13284 \). Each element \( \omega = (\mu, P(V), P(S|V)) \in \Omega \) represents a specific stock. For each stock, sell or buy herding (or both) are possible in principle, i.e. the upper boundaries for \( \mu \) are never binding (compare Park and Sabourian (2011), pp. 991-992 and pp. 1011-1012). Each stock is traded over \( T = 100 \) points of time. For each model parameterization, the simulation is repeated 2000 times which produces more than 2.6 billion simulated trades to analyze.

The results of these model simulations are used to derive predictions on the effect of changes in information risk on average herding intensity as follows: In a first step, we fix \( \mu \in \mathcal{M} \) and calculate average herding intensity as the average across all parameterizations in \( \{\mu\} \times \mathcal{P} \times \mathcal{C} \). In a second step, we evaluate how average herding intensity varies with \( \mu \). Correspondingly, to analyze the effect of market stress on average herding intensity, we fix \( P(V) \in \mathcal{P} \) and calculate average herding intensity across all parameterizations in \( \mathcal{M} \times \{P(V)\} \times \mathcal{C} \). Next, we evaluate how average herding intensity varies with the distribution of the asset, \( P(V) \), where the degree of market stress implied by \( P(V) \) is given by its coefficient of variation, \( VC(V) \).

### 3.2 Simulation results

Figure 1 shows boxplots for average herding intensity for sell and buy herding, respectively, over 2000 simulations for parameterizations of the model that differ only in the fraction of informed traders. The simulation results clearly indicate that both, average
buy and sell herding intensity increase in the fraction of informed traders in a symmetric way. Intuitively, private information may be easier dominated by the information contained in the history of trades as each preceding trade is more likely to be carried out by an informed type. The simulation results further suggest a weaker increase in herding intensity as well as an increase in the variance of herding intensity when $\mu$ approaches one. This could be explained by the increased bid-ask spread induced by an increase in the fraction of informed traders, making a switch from not buying (not selling) to buying (selling) less likely. Note that for our empirically relevant range of $\mu \in [0.2, 0.7]$ the increase in herding intensity is steep and each set of parameterizations exhibits only small variations across the 2000 simulations.

The fraction of informed traders determines the probability for the market maker to encounter an informed trader and, thus, the information risk in the market. Therefore, the simulation results shown in Figure 1 can be summarized as follows:

**Hypothesis 1** (Information Risk and Herding Intensity). *Average sell and buy herding intensity increase in information risk.*

Figure 2 shows sell and buy herding intensity for parameterizations that differ only in the degree of market stress as it is reflected by the variation coefficient, $\sqrt{\text{Var}(V)/E[V]}$, of the fundamental value.$^{17}$ The higher the variation coefficient, the more severe the market stress. In contrast to information risk, the impact of market stress on herding

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$^{17}$Unlike in Figure 1 we plot the average herding intensity across 2000 simulations instead of boxplots, for the sake of readability. The variation of herding intensity across 2000 simulation is, however, comparable to the variations in Figure 1.
Figure 1: Information risk and herding intensity

(a) Sell Herding
(b) Buy Herding

Notes: Sell and buy herding intensity, respectively, are plotted against the fraction of informed traders. The boxplots show the variation across 2000 simulations of herding intensity for parameterization \(\{\mu\} \times P \times C\), where the fraction of informed traders, \(\mu\), is plotted along the horizontal. On the ordinate we plot herding intensity as a fraction of informed traders that engaged in herd behavior. The central mark of each box is the median, the edges of the boxes are the 25th and 75th percentiles, the whiskers are the most extreme data points.

is highly asymmetrical. For sell herding intensity, the simulation results demonstrate a strong positive relationship of average herding intensity and the variation coefficient. Therefore, the higher the degree of market stress, the higher the average sell herding intensity to be expected in a heterogenous stock market. For buy herding intensity, however, the higher the variation coefficient, the smaller the average herding intensity, although the relationship is clearly less pronounced.\(^{18}\) We summarize our simulation

\(^{18}\)Note that the more disperse pattern in the response of buy herding intensity is solely due to our measure of market stress. To see this, consider an increase of the variation coefficient that is mainly driven by a decrease of the expected value of the asset \(E[V]\). In this case, a greater variation coefficient should clearly increase sell herding while buy herding should be expected to occur less frequently. Note that simulation results for buy herding were similar to those obtained for sell herding, if we plotted
Figure 2: Market stress and herding intensity

Notes: Sell and buy herding intensity, respectively, are plotted against the variation coefficient. Each dot shows the herding intensity averaged across 2000 simulations for parameterization $\mathcal{M} \times \{P(V)\} \times \mathcal{C}$, where the variation coefficient, $VC(V)$, induced by the asset’s distribution, $P(V)$, is plotted along the horizontal. On the ordinate we plot herding intensity as a fraction of informed traders that engaged in herd behavior across 2000 simulations.

results obtained for the relationship between our proxy for market stress and average herding intensity as follows:

**Hypothesis 2** (Herding Intensity and Market Stress). *Average buy herding intensity decreases with market stress, whereas sell herding intensity increases.*

average buy herding intensity against $\sqrt{\text{Var}(V)E[V]}$. 
4 Empirical herding measure

Simulating a herd model allows us to determine for each trade whether herding actually occurred. As a result, the exact herding intensity can be calculated for each model simulation. In an empirical application, it is much more difficult to decide whether a trader herds or not since researchers have no access to private signals.

The dynamic herding measure proposed by Sias (2004) is designed to explore whether (institutional) investors follow each others’ trades by examining the correlation between the traders’ buying tendency over time. The Sias herding measure is, therefore, particularly appropriate for high-frequency data. Similar to the static herding measure proposed by Lakonishok et al. (1992), the starting point of the Sias measure is the number of buyers as a fraction of all traders. Specifically, consider a number of \( N_{it} \) institutions trading in stock \( i \) at time \( t \). Out of these \( N_{it} \) institutions, a number of \( b_{it} \) institutions are net buyers of stock \( i \) at time \( t \). The buyer ratio \( br_{it} \) is then defined as \( br_{it} = \frac{b_{it}}{N_{it}} \). According to Sias (2004), the ratio is standardized to have zero mean and unit variance:

\[
\Delta_{it} = \frac{br_{it} - \bar{br}_t}{\sigma(br_{it})},
\]

(1)

where \( \sigma(br_{it}) \) is the cross sectional standard deviation of buyer ratios across \( I \) stocks at time \( t \). The Sias herding measure is based on the correlation between the standardized...
buyer ratios in consecutive periods:

$$\Delta_{it} = \beta_t \Delta_{i,t-1} + \epsilon_{it}.$$ \hfill (2)

The cross-sectional regression is estimated for each time $t$ and then the Sias measure for herding intensity is calculated as the time-series average of the estimated coefficients:

$$Sias = \frac{\sum_{t=2}^{T} \beta_t}{T-1}.$$ It is worth emphasizing that this kind of averaging is very much in line with the way we calculate average herding intensity in the model simulation.

The Sias methodology further differentiates between investors who follow the trades of others (i.e., *true herding* according to Sias (2004)) and those who follow their own trades. For this purpose, the correlation is decomposed into two components:

$$\beta_t = \rho(\Delta_{it}, \Delta_{i,t-1}) = \left[ \frac{1}{(I-1)\sigma(br_{it})\sigma(br_{i,t-1})} \right] \sum_{i=1}^{I} \left[ \sum_{n=1}^{N_{it}} \frac{(D_{nit} - \bar{br}_t)(D_{ni,t-1} - \bar{br}_{t-1})}{N_{it}N_{i,t-1}} \right]$$

$$+ \left[ \frac{1}{(I-1)\sigma(br_{it})\sigma(br_{i,t-1})} \right] \sum_{i=1}^{I} \sum_{n=1}^{N_{it}} \sum_{m=1, m \neq n}^{N_{i,t-1}} \frac{(D_{nit} - \bar{br}_t)(D_{mi,t-1} - \bar{br}_{t-1})}{N_{it}N_{i,t-1}}, \hfill (3)$$

where $I$ is the number of stocks traded. $D_{nit}$ is a dummy variable that equals one if institution $n$ is a buyer in $i$ at time $t$ and zero otherwise. $D_{mi,t-1}$ is a dummy variable that equals one if trader $m$ (who is different from trader $n$) is a buyer at time $t - 1$. Therefore, the first part of the measure represents the component of the cross-sectional inter-temporal correlation that results from institutions following their own
strategies when buying or selling the same stocks over adjacent time intervals. The second part indicates the portion of correlation resulting from institutions following the trades of others over adjacent time intervals. According to Sias (2004), a positive correlation that results from institutions following other institutions, i.e., the latter part of the decomposed correlation, can be regarded as evidence for herd behavior. In the subsequent empirical analysis, we shall therefore focus on the latter term of equation (3) which we denote by $Sias$. According to Choi and Sias (2009), Equation (3) can be further decomposed to distinguish between the correlations associated with “buy herding” ($br_{i,t-1} > 0.5$) and “sell herding” ($br_{i,t-1} < 0.5$).

5 Information risk, market stress and herding intensity:

Empirical results

5.1 Data

The data are provided by the German Federal Financial Supervisory Authority (BaFin). Under Section 9 of the German Securities Trading Act, all credit institutions and financial services institutions are required to report to BaFin any transaction in securities or derivatives which are admitted to trading on an organized market. These records make it possible to identify all relevant trade characteristics, including the trader (the institution), the particular stock, time, number of traded shares, price, and the volume of the transaction. Moreover, the records specify on whose behalf the trade was executed,
i.e., whether the institution traded for its own account or on behalf of a client that is not a financial institution. Since this study is concerned with institutional trades, particularly those of financial institutions, we focus on the trading of own accounts, i.e., those cases when a bank or a financial services institution is clearly the originator of the trade. We exclude institutions trading exclusively for the purpose of market making. We also exclude institutions that are formally mandated as designated sponsors, i.e., liquidity providers, for a specific stock. Following the herding literature, we are particularly interested in the herding behavior of institutional investors because they are more likely to be informed compared to e.g. retail investors. Moreover, institutional investors are the predominant class in the stock market with the power to move the market and impact prices, particularly if they herd.

The analysis focuses on shares listed on the DAX 30 (the index of the 30 largest and most liquid stocks), where stocks are selected according to the index compositions at the end of the observation period on March 31, 2009. Following the empirical literature, we require that at least five institutions were active in the market at each day. Using data from July 2006 to March 2009 (698 trading days), we are able to investigate whether trading behavior has changed during the financial crisis. Over the sample period, there are 1120 institutions performing proprietary transactions. Among those 1120 traders, 1044 trade on the DAX 30 stocks.

19 For each stock, there are usually about two institutions formally mandated as market maker. The institutions are not completely dropped from the sample (unless they have already been dropped due to purely engaging in market maker business), but only for those stocks for which they act as designated sponsors. The designated sponsors for each stock are published at http://www.deutsche-boerse.com. For more detailed information about the data, see Kremer and Nautz (2013a,b).
5.2 Information risk and herding intensity

The more informed traders are active in a market, the higher the probability of informed trading and, thus, information risk. According to Hypothesis 1, average herding intensity increases with information risk reflected in the parameter $\mu$, the fraction of informed traders. In the following, we use two empirical proxies for the level of information risk: i) the number of active institutional traders and ii) the share of the institutional trading volume.

According to e.g. Foster and Viswanathan (1993) and Tannous et al. (2013), the fraction of informed traders and, thus, information risk cannot be expected to be constant over a trading day. In order to account for intra-day trading patterns in the German stock market, we divide each trading day into 17 half-hour intervals. A trading day is defined as the opening hours of the trading platform Xetra (9 a.m. to 5:30 p.m.), on which the bulk of trades occur. The use of half-hour intervals ensures that the number of active institutions is sufficiently high for calculating intra-day herding measures.\(^{20}\) The first two columns of Table 1 show how both empirical proxies for information risk are distributed within a day. Apparently, institutional traders are more active at the opening and closing intervals, irrespective of the measure of trading activity.

In order to investigate the intra-day pattern of herding intensity, we calculate the Sias herding measure for each half-hour time interval separately. The results of this

\(^{20}\)For sake of robustness, we also divided the trading day into 9 one-hour intervals but our main results do not depend on this choice. For brevity, results are not shown but are available on request.
<table>
<thead>
<tr>
<th>Time</th>
<th>Traders</th>
<th>Trading Volume</th>
<th>Sias</th>
<th>Sias</th>
</tr>
</thead>
<tbody>
<tr>
<td>09:00 - 09:30</td>
<td>25.33</td>
<td>6.73</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>09:30 - 10:00</td>
<td>21.05</td>
<td>5.34</td>
<td>25.92</td>
<td>9.92</td>
</tr>
<tr>
<td>10:00 - 10:30</td>
<td>15.75</td>
<td>2.57</td>
<td>28.59</td>
<td>7.54</td>
</tr>
<tr>
<td>10:30 - 11:00</td>
<td>22.88</td>
<td>6.73</td>
<td>30.43</td>
<td>7.85</td>
</tr>
<tr>
<td>11:00 - 11:30</td>
<td>19.58</td>
<td>4.51</td>
<td>34.30</td>
<td>9.98</td>
</tr>
<tr>
<td>11:30 - 12:00</td>
<td>18.72</td>
<td>4.15</td>
<td>33.98</td>
<td>8.24</td>
</tr>
<tr>
<td>12:00 - 12:30</td>
<td>17.96</td>
<td>3.77</td>
<td>33.91</td>
<td>7.83</td>
</tr>
<tr>
<td>12:30 - 01:00</td>
<td>17.08</td>
<td>3.39</td>
<td>33.81</td>
<td>6.96</td>
</tr>
<tr>
<td>01:00 - 01:30</td>
<td>17.36</td>
<td>4.31</td>
<td>33.28</td>
<td>7.84</td>
</tr>
<tr>
<td>01:30 - 02:00</td>
<td>16.57</td>
<td>3.28</td>
<td>34.00</td>
<td>8.56</td>
</tr>
<tr>
<td>02:00 - 02:30</td>
<td>17.85</td>
<td>3.96</td>
<td>34.74</td>
<td>8.60</td>
</tr>
<tr>
<td>02:30 - 03:00</td>
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<td>4.63</td>
<td>33.38</td>
<td>8.29</td>
</tr>
<tr>
<td>03:00 - 03:30</td>
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<td>4.42</td>
<td>34.21</td>
<td>9.31</td>
</tr>
<tr>
<td>03:30 - 04:00</td>
<td>20.42</td>
<td>6.43</td>
<td>34.19</td>
<td>10.60</td>
</tr>
<tr>
<td>04:00 - 04:30</td>
<td>20.70</td>
<td>6.98</td>
<td>35.65</td>
<td>12.86</td>
</tr>
<tr>
<td>04:30 - 05:00</td>
<td>20.74</td>
<td>7.64</td>
<td>34.62</td>
<td>11.90</td>
</tr>
<tr>
<td>05:00 - 05:30</td>
<td>22.50</td>
<td>10.13</td>
<td>32.94</td>
<td>12.53</td>
</tr>
</tbody>
</table>

Notes: The table shows how information risk and herding intensity evolves over the trading day. *Traders* denotes the average number of active institutional traders, *Trading Volume* refers to the average percentage share of the daily trading volume of institutional investors. For instance, on average, 6.73% of the daily institutional trading volume appeared from 9 a.m. to 9:30 a.m. The column do not add to one because we focus on the predominant German platform Xetra®, where trading takes place from 9 a.m. till 5.30 p.m. CET, while the opening period for the German stock exchange at the floor ends at 8 p.m. *Sias* and *Sias* represent the overall and the adjusted Sias herding measure (in percent), where the latter only considers institutions that follow the trades of others, see Equation (3). Standard errors are given in parentheses.
exercise are also shown in Table 1. The third column shows for each interval the overall Sias measure (Sias) which is based on the average correlation of buy ratios between two intervals, see Equation (2) in Section 4. Following Sias (2004), this correlation may overstate the true herding intensity because it does not account for correlation which results from traders who follow themselves. It is a distinguishing feature of our investor-specific data that it allows to address that problem even on an intra-day basis. In particular, column four reports the correlation due to investors following the trades of others (Sias), see Equation (3).

Table 1 offers several insights concerning the intra-day pattern of institutional herding. First of all, both Sias measures provide strong evidence for the presence of herding for each half-hour interval of the trading day. Second, intra-day herding measures are significantly larger than those obtained for data with lower-frequency, compare Kremer and Nautz (2013a,b). Third, the sizable differences between Sias and Sias highlights the importance of using investor-specific data.

How is the observed intra-day variation of information risk related to the intra-day herding intensity of institutional investors? The Sias herding measure depends on the trading behavior of two subsequent time periods. Therefore, for each time interval herding intensity is compared with the average information risk of the corresponding time intervals.\footnote{Note that this is line with the intuition from the herd model of Park and Sabourian (2011). On the one hand, high information risk in \( t - 1 \) leads institutional investors to believe that there is a high degree of information contained in previously observed trades. On the other hand, high information risk in \( t \) ensures that there is a high number of potential herders active in the market. Both effects contribute positively to herding intensity in period \( t \).} Figure 3 reveals a strong intra-day co-movement between both proxies.
of information risk and Sias. In fact, we find overwhelming evidence in favor of Hypothesis 1: the null-hypothesis of zero correlation between information risk and herding intensity can be rejected irrespective of the underlying proxy of information risk. For example, the rank-correlation coefficient between the average trading volume and the corresponding Sias measure is 0.80, which is significantly above zero at the 1% level.\textsuperscript{22}

Note that the peaks in Sias at market opening and following the opening of the

\textsuperscript{22}More precisely, the associated p-value of the rank-test is 0.0003. Pearson’s correlation coefficient is 0.91 and significant at all conventional levels. Note that a rank correlation coefficient might be more appropriate than Pearson’s correlation coefficient, since it accounts for the potentially non-linear relation between information risk and herding intensity suggested by the numerical simulation of the herd model, see Figure 1.
US market at 3:30 p.m. – 4 p.m. correspond with high activity of informed traders suggesting that at market openings there is a lot of information contained in observed trades on which subsequent traders herd. This confirms the experimental findings of Park and Sgroi (2012) who observe that traders with relatively strong signals trade first, while potential herders delay.

5.3 Herding in times of market stress

According to Hypothesis 2, sell herding should increase in times of market stress when uncertainty increases and markets become more pessimistic about the value of the asset. In contrast, buy herding intensity should decline in a crisis. In our application, a natural candidate to test this hypothesis is the outbreak of the financial crisis. In order to investigate the effect of the crisis on herding intensity, we calculate sell and buy herding measures for the crisis and the pre-crisis period separately. The pre-crisis period ends on August 9, 2007 as this is widely considered as the starting date of the financial crisis in Europe, see e.g. European Central Bank (2007) and Abbassi and Linzert (2012).

Herding measures obtained before and during the crisis are displayed in Table 2. The results are hardly compatible with the predictions of the simulated model. At first sight, the statistically significant yet small increase in sell herding (5.74 > 5.41) is in line with theoretical expectations. However, buy herding intensity has definitely not decreased in the crisis period. In fact, buy herding has even increased (5.09 > 4.10).
Table 2: Herding intensity - before and during the financial crisis

<table>
<thead>
<tr>
<th></th>
<th>Sias</th>
<th>Sias</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-crisis period</td>
<td>Crisis period</td>
</tr>
<tr>
<td><strong>Buy Herding</strong></td>
<td>14.37 (0.37)</td>
<td>13.87 (0.35)</td>
</tr>
<tr>
<td></td>
<td>4.10 (0.10)</td>
<td>5.09 (0.11)</td>
</tr>
<tr>
<td><strong>Sell Herding</strong></td>
<td>18.87 (0.23)</td>
<td>15.65 (0.25)</td>
</tr>
<tr>
<td></td>
<td>5.41 (0.09)</td>
<td>5.74 (0.08)</td>
</tr>
</tbody>
</table>

Notes: This table reports adjusted ($S_{i,a}$) and unadjusted ($S_{i}$) herding measures based on half-hour intervals estimated separately for the pre-crisis and the crisis period. The Sias measures are further decomposed into its buy and sell herding components, compare Section 4. Standard errors are given in parentheses.
How can this contradicting evidence be explained? Probably, the effects claimed by Hypothesis 2 hold but are overshadowed by counteracting factors. For example, Kremer and Nautz (2013b) show that the market share of institutional investors has dropped sharply since the outbreak of the financial crisis. If this drop in trading activity of financial institutions can be interpreted as a decline in information risk, then a crisis-driven increase in sell herding could be ameliorated by a decrease of sell herding due to lower information risk. However, in this case, a potential drop in information risk makes the observed increase in buy herding even more puzzling. Another explanation could be that the deterioration in the economic outlook induced by the financial crisis was relatively small compared to the increase in uncertainty. In this case, our simulation exercise shows that both buy and sell herding intensity should increase where sell herding should increase slightly stronger.\(^{23}\) Still, the evidence shows that buy herding increased slightly more contradicting the simulation-based prediction.

Ederington and Goh (1998) and Jorion et al. (2005) argue that firms have an incentive to withhold bad news from investors, but release good news voluntarily. Such incentives may increase during times of market stress, as positive news help to separate the firm from its poorly performing peers and, thereby, to shield it from negative spillover effects. In the framework of Park and Sabourian (2011), a large share of informed traders might translate those positive news into the same monotone increasing signal advising them to buy. Since the resulting increase in buys stems from investors’ correlated signals rather than from investors inferring information of the trades of oth-

\(^{23}\)Results are not shown here, but are available upon request.
ers, the model predicts higher buy ratios but not increased buy herding. As a result, the observed increase in the buy herding measure during the financial crisis may be only spurious and unrelated to the herding behavior considered in the theoretical herding literature.

6 Concluding remarks

Due to data limitations and the absence of testable, model-based predictions, the theoretical and the empirical herding literature are only loosely connected. This paper proposes an approach that contributes towards closing this gap. To obtain theory-founded results, we conduct numerical simulations of the financial market herd model of Park and Sabourian (2011). These theory-based hypotheses are tested empirically applying the herding measure of Sias (2004) to investor-specific and high-frequency trading data from the German stock market DAX. In particular, we derive and test hypotheses on how information risk and market stress affect herding intensity.

In accordance with our simulation results, we find that aggregate herding intensity increases with information risk. The empirical evidence regarding the impact of market stress on herding intensity, however, is only mixed. In particular, the estimated increase in buy herding during the recent financial crisis is not consistent with the simulation-based model prediction.

The results provided in this paper demonstrate that more research is needed to
further close the gap between theory and evidence. For example, during crises periods


correlation across assets and contagious effects may play a particular role in explaining


investors’ behavior. Herd models, however, are typically single asset models and are not
designed to provide insights about herd behavior in a context of correlated assets and
informational spillovers. To improve the interpretation of evidence based on aggregate


erming measures, an extension of herd models to a multiple asset setting would be an


interesting avenue for future research. Empirical herding measures, on the other hand,


assess correlated trade behavior (see, e.g., Lakonishok et al. (1992), Sias (2004), Chang
et al. (2000) or Patterson and Sharma (2010)) and are, thus, very good in detecting


situations where investors accumulate on one side of the market. They can hardly


reveal, however, to what extent this correlation is actually due to traders neglecting


their private information and following the actions of others. Therefore, empirical


herding measures cannot distinguish between true (or informationally inefficient) and


spurious (or unintentional) herd behavior. In the case of spurious herding, correlated


trading is not necessarily a sign of inefficiency but could be due to a common reaction
to fundamentals or similar risk models, see Kremer and Nautz (2013a).


The current paper showed that the pattern of trading correlation and information
risk can be related to true herding, which raises worries about market efficiency in
times of high information risk. By contrast, our results suggest that the increase in the
correlation of buys estimated for the crisis period is more convincingly explained by
unintentional herding.
References


A Appendix

A.1 Analytical results on herding intensity

We will now present an analytical formula for theoretical sell herding intensity in the context of the model of Park and Sabourian (2011). Investigating this formula more closely, we will see that the relationship between herding intensity and probability of informed trading (= $\mu$) as well as market turbulence (= $\sqrt{Var[V]/E[V]}$) is too complex to develop comparative statics analytically.

It can be shown that the expected number of herding sells $E[s_{T,M}^h]$ is given by

$$E[s_{T,M}^h] = \sum_{i=1}^{3} P(V_i) \left\{ \sum_{j=1}^{T} \left( \frac{\mu P(S_2|V_i)}{\mu(P(S_2|V_i) + P(S_3|V_i) - \frac{1}{3}) + \frac{1}{3}} \right)^j \left[ \sum_{k=j}^{T} P(S_{T,M} = k|V_i) \left( \frac{\mu(P(S_3|V_i) - \frac{1}{3}) + \frac{1}{3}}{\mu(P(S_2|V_i) + P(S_3|V_i) - \frac{1}{3}) + \frac{1}{3}} \right)^{k-j} \right] \right\},$$

where $M := \{\mu, P(V), P(S|V)\}$ be the parametrization of the model, $s_{T,M}^h$ denotes the actual number of sell herds and $S_{T,M}$ is the number of sells that occur while $S_2$ engages in sell herding.\(^{24}\) The formula is mainly derived via application of Bayes’ rule and the law of iterated expectations. To develop some intuition behind it, consider first only the term $\sum_{i=1}^{3} P(V_i) \{ \cdot \}$. The factor $\{ \cdot \}$ contains the estimated number of sell herds given a realization of the risky asset $V = V_i$. The probability weighted sum, thus is

\(^{24}\)The proof for this formula is provided on request.
the expected number of sell herds over all possible states of the risky asset $V$. Now, consider the terms within the curly brackets, i.e. $\sum_{j=1}^{T} j \left( \frac{\mu P(S_2|V_i)}{\mu (P(S_3|V_i)^{-1} + 1)} \right)^j$. The number $j$ stands for the number of herding sells in some history $H_t$. The factor $(\cdot)^j$ stands for the probability that the u-shaped informed trader $S_2$ arrives on the market $j$ times and each time decides to sell, given that history $H_t$ contains $k \geq j$ sells under which a herding sell can occur. The sum in brackets finally, describes the probability that $k - j$ sells stem from either noise traders or $S_3$ for all $k \geq j$ and given that $k$ sells occur under which $S_2$ would engage in sell herding.

The important thing to take away from this formula is that it is not feasible to conduct comparative statics of herding intensity analytically. First note that there is a lot of complexity hidden in $P(\bar{S}_{T,M}|V_i)$. This probability is impossible to compute analytically since we would need to calculate the probabilities of all history paths $H_T$. Depending on the model parameterization, we would need to calculate the probabilities of at least $6^T$ history paths, where $6$ amounts to the number of different possible states of the model, we need to consider in each step. Moreover, the above formula only yields results for the expected number of herding sells for a given model parameterization. If wanted to generalize our assessment to arbitrary model parameterizations or the average number of herding sells for different model parameterizations, the tractability of expected herding sells would be reduced even further. Finally note, that (4) only provides the value for the number of herding sells. SHI, however, is defined as the number of herding sells divided by the number of informed trades. Consequently, the expected sell herding intensity would be given by the expectation of that ratio. Since
the number of informed trades is also random variable that is not independent of the number of herding sells, $E\left(\frac{\#\text{ herding sells}}{\#\text{ informed trades}}\right)$ is even harder to compute.

But even if we were to agree that (4) is a good proxy to base our analytical discussion upon, comparative statics of the expected number of herding sells with respect to changes in $\mu$ and $P(V)$ would not be fruitful. For the latter simply note that the complexity of the sum makes it impossible to isolate $E[V]$ or $\text{Var}[V]$ on the right hand side of (4).\textsuperscript{25}

\textsuperscript{25}Regarding the probability of informed trading, it seems at first glance possible to differentiate the right hand side of equation (4) with respect to $\mu$. The sign of the derivative, however, will depend on the signal structures for informed traders $S_2$ and $S_3$ as well as the distribution $P(V)$ of the risky asset which will prevent us from establishing general analytical results.