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# Human capital, basic research, and applied research: three dimensions of human knowledge and their differential growth effects

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## Abstract

We analyze the differential growth effects of basic research, applied research, and embodied human capital accumulation in an R&D-based growth model with endogenous fertility and endogenous education. In line with the empirical evidence, our model allows for i) a negative association between long-run economic growth and population growth, ii) a positive association between long-run economic growth and education, and iii) a positive association between the level of per capita GDP and expenditures for basic research. Our results also indicate that raising public investments in basic research reduces the growth rate of GDP in the short run because resources have to be drawn away from other productive sectors of the economy. These short-run costs of basic research might be an explanation for the reluctance of governments to increase public R&D expenditures notwithstanding the long-run benefits of such a policy.

**JEL classification:** H41, J11, J24, O32, O41

**Keywords:** basic vs. applied science, endogenous schooling decisions, endogenous fertility decisions, R&D-based growth, governmental research policies

*If we knew what it was we were doing,  
it would not be called research, would it?*  
(Albert Einstein)

## 1 Introduction

In the beginning of the 1990s a series of seminal papers (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992) managed to endogenously explain the hitherto unexplained evolution of technology over time. In so doing these papers provided new fundamentals for growth theory, which by then already acknowledged the central role that technology played in the development process (Solow, 1956), but was unable to explain its own driving forces. The new frameworks relied on the profit motive for research and development (R&D) in the sense that innovative firms can capture the rewards for designing new and/or better products by siphoning the monopolistic rents associated with the corresponding patent. With these tools at hand economists were increasingly able to analyze the impact of incentives, market structures, preferences, and policy measures on the R&D intensity and the pace of technological progress of industrialized countries (see Aghion and Howitt, 1999, 2005; Gancia and Zilibotti, 2005, for interesting overviews).<sup>1</sup>

Despite their huge impact and their invaluable insights, the standard R&D-based growth literature can be improved along the following lines. First, since human capital plays such a crucial role in the production process of new ideas, its accumulation should be endogenously explained. While empirical studies emphasize the importance of private education decisions and governmental education policies for economic prosperity (Krueger and Lindahl, 2001; de la Fuente and Doménéch, 2006; Cohen and Soto, 2007; Hanushek and Woessmann, 2012), they also find a negative association between population growth and economic growth in post World War II data (Brander and Dowrick, 1994; Kelley and Schmidt, 1995; Ahituv, 2001; Li and Zhang, 2007; Herzer et al., 2012). Since R&D-based growth frameworks abstain from a more detailed modeling of human capital, no distinction can be made as to whether increases in human capital are caused by faster population growth or by better education. As a consequence, these models counterfactually predict a positive effect of population growth on technological progress and economic prosperity. This unrealistic feature has first been addressed by Dalgaard and Kreiner (2001) and Strulik (2005), who show that an increase in the exogenous growth rate of uneducated labor mechanically decreases the average human capital level of a society and thereby hampers economic development. However, only very recent research by Strulik et al. (2013) and Hashimoto and Tabata (2013) endogenously explains the negative association between population growth and economic growth by acknowledging the existence of a quantity-quality trade-off effect in the spirit of Becker and Lewis (1973): parents with fewer children

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<sup>1</sup>Since these early endogenous growth models counterfactually implied hyper-exponential economic growth in the face of population growth and that larger countries would grow faster than smaller ones (scale effect), they were subsequently refined in the vein of semi-endogenous growth frameworks (Jones, 1995; Kortum, 1997; Segerström, 1998) and scale free Schumpeterian growth frameworks (Peretto, 1998; Young, 1998; Howitt, 1999). See Jones (1999), Li (2000), Li (2002), Jones (2002), Ha and Howitt (2007), and Madsen (2008) for a debate on the suitability of these two approaches.

tend to invest more in educating each child, while the converse holds true for parents with more children. On the aggregate, this leads to a situation where a slowdown of population growth fosters human capital accumulation and thereby R&D. Consequently, the economic growth rate rises with declining fertility, which is in line with the empirical evidence.

Second, as the introductory quote of Einstein plainly makes clear, apart from profit-driven purposeful applied research to design new and/or better products, there is another important but neglected dimension of R&D: Mokyr (2002), in particular, distinguishes between the *techniques* that a society can draw from, and the *propositional knowledge* it has at its disposal. The former can be interpreted as the result of profit-driven R&D, while the latter can be thought of as a society's knowledge of natural phenomena and regularities. This propositional knowledge is viewed as being a necessary input for the development of new techniques, and therefore, according to Mokyr (2002), acts as their *epistemic base*. Basic research to improve a society's understanding of natural phenomena and regularities is mostly carried out at Universities and other publicly funded research facilities. The reason is that there is a substantial difference between basic research and applied research with respect to excludability — while techniques are at least partially excludable due to the existence of a patent system, the knowledge of natural laws cannot be patented such that propositional knowledge is non-excludable. Consequently, there are barely any profits that basic research institutions (or individual scientists) are able to reap which implies that the public has to step in (see also Nelson, 1959; Shell, 1966; Mansfield, 1980; Park, 1998, for interesting discussions regarding the role of publicly funded research).<sup>2</sup> Without any public funding, systematic basic research could not be carried out, with all the negative repercussions that this has on an economy.<sup>3</sup> A distinction between basic research and applied research has been made by Park (1998), Morales (2004), Gersbach et al. (2009), Gersbach et al. (2012), Gersbach and Schneider (2013), and Akcigit et al. (2013). They assume that if societies want to develop new techniques, they also need basic knowledge to be able doing so. However, these approaches abstract from the endogenous explanation of human capital accumulation and population growth as described above. Furthermore, they assume that physical capital does not play a role in the production process with the consequence that transition phases (and therefore gestation lags of basic research) cannot be analyzed. Finally, they often apply restrictive assumptions on the size of intersectoral knowledge spillovers between basic and applied science.

In our contribution we aim to present a single tractable R&D-based growth framework with basic research, endogenous human capital investments, endogenous population growth, endogenous physical capital accumulation (which allows us to analyze transition phases), and a fairly general description of intersectoral knowledge spillovers between basic and applied science. In so doing, we integrate a child quality-quantity trade-off in the vein of Becker and Lewis (1973) as well as a publicly funded basic research sector in the vein of Park (1998) into a discrete time formulation of the generic growth framework of Romer (1990) and Jones (1995). Altogether, this allows us to analyze the differential growth effects of the three central dimensions of human knowledge: applied knowledge, basic knowledge, and embodied human capital. The resulting

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<sup>2</sup>The central role of governmentally funded R&D is also often acknowledged by recognizing that the social gains of R&D tend to be huge and outweigh the private gains to a large extent (Jones and Williams, 2000; Grossmann et al., 2010, 2013a,b).

<sup>3</sup>See Mansfield (1980) for empirical evidence.

qualitative implications are consistent with the empirical findings of a positive effect of education on growth (de la Fuente and Doménéch, 2006; Cohen and Soto, 2007; Hanushek and Woessmann, 2012), a negative effect of fertility on growth (cf. Li and Zhang, 2007; Herzer et al., 2012), a positive association between basic research investments and the level of per capita GDP (cf. Mansfield, 1980), and a gestation lag of basic research of around 20 years (cf. Adams, 1990).

The paper proceeds as follows. Section 2 describes the model that integrates endogenous fertility, endogenous education, and publicly funded basic research into an R&D-based economic growth framework. Section 3 contains our analytical results and propositions regarding the long-run growth effects of changing the household’s desire for fertility and education as well as changing governmental research policies. Section 4 contains a numerical illustration of the responses of the main endogenous variables to an increase in governmental expenditures for basic research with a particular emphasis on the transition phase and on its welfare implications. Section 5 is devoted to sensitivity analyses with respect to knowledge spillovers. Finally, Section 6 draws conclusions, outlines the policy implications of our framework, and identifies scope for further research.

## 2 The model

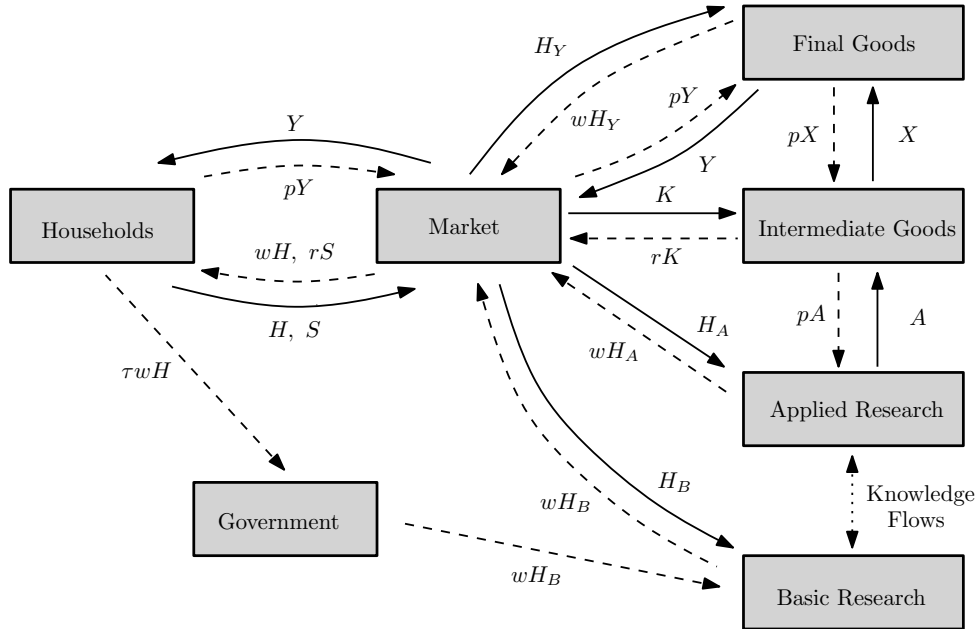
This section describes an R&D-based growth model in the vein of Romer (1990) and Jones (1995) that we augment with a basic research sector building upon Park (1998) and an endogenous fertility and education decision along the lines of Strulik et al. (2013). Note that we abstract from international knowledge diffusion by interpreting the model as the description of an aggregate of the economies that actively drive the knowledge frontier (cf. Keller, 2002, who shows that the vast majority of research expenditures are undertaken in the five most industrialized countries).

### 2.1 Basic assumptions

We consider an overlapping generations economy, in which people live for three time periods: childhood, adulthood, and retirement. Children do not face any economic decisions, while adults decide upon the family’s consumption level, its savings for retirement, the number of children, and the education expenditures for each child. They finance the related expenditures by supplying the time they do not spend on educating and raising their offspring on the labor market at the given wage rate determined in general equilibrium. The retirees in turn consume their savings carried over from adulthood.

There are four productive sectors: final goods production, intermediate goods production, basic science (which is governmentally financed), and applied science (which is profit-driven). Four production factors can be used in these sectors: i) the explanations of natural phenomena and regularities that are discovered in the basic science sector; ii) the blueprints for new technologies that are developed in the applied science sector; iii) physical capital which is accumulated due to household’s savings and is used for the production of machines in the intermediate goods sector; iv) embodied human capital which is used in four different forms: workers in the final goods sector denoted by  $L_{t,Y}$ , scientists in the basic research sector denoted by  $L_{t,B}$ , scientists in the profit-driven research sector denoted by  $L_{t,A}$  and human capital that parents spend on

child-care. The final goods sector employs workers and machines supplied by the intermediate goods sector to produce a consumption aggregate for a perfectly competitive market. The intermediate goods sector produces the machines for the final goods sector with physical capital as variable production factor and one machine-specific blueprint as fixed input. The blueprints for the machines are patented and sold by the profit-driven applied research sector. Finally, the basic research sector aims to discover and explain natural laws, the knowledge of which is a necessary input in the profit-driven research sector and, following Mokyr (2002), is interpreted as the propositional knowledge that a society has at its disposal. This propositional knowledge acts as the epistemic base for the techniques developed in the profit-driven research sector in the sense that the natural phenomena and regularities on which a technique relies have to be known in advance to design the associated blueprint. For example, the knowledge about nuclear fission has to be discovered before a society can design and build nuclear power plants.



Note:  $Y$  refers to the final goods consumed by households and  $p_Y$  to their price;  $X$  refers to the intermediate goods used in final goods production with  $p_X$  being the price of these intermediates;  $A$  are the blueprints used for intermediate goods production with  $p_A$  being their price;  $K$  is the physical capital stock,  $S$  are aggregate savings, and  $r$  is the real interest rate;  $H$  is aggregate human capital in the production process (aggregate human capital net of time spent for child-care) with  $H_A$  being the human capital employed in the applied research sector,  $H_B$  referring to the human capital employed in the basic research sector, and  $H_Y$  denoting the human capital employed in the final goods sector;  $w$  refers to the wage rate and  $\tau$  denotes the income tax rate that the government sets to finance basic research.

Figure 1: Overview of the structure of the general equilibrium model

The wage bill of the scientists in the basic research sector is financed by the government through taxation of the wages of adult workers. The reason for governmental investments in basic research is that knowledge of natural phenomena and regularities cannot be patented and sold, that is, it is non-excludable. Consequently, there would be no compensation for basic scientists without governmental intervention implying that basic science is a pure public good.

By contrast, the blueprints designed in the applied research sector can be patented and sold such that they are at least *partially* excludable. Figure 1 summarizes the model structure.

## 2.2 Consumption side

Parents want to consume, save for retirement, have children, and educate these children. Assuming full depreciation of physical capital over the course of a generation allows us to conceptualize the associated utility function following Strulik et al. (2013) as

$$\max_{c_t, s_t, n_t, e_t} u_t = \log c_t + \beta \log[(R_{t+1} - 1) s_t] + \xi \log n_t + \theta \log e_t, \quad (1)$$

where  $c_t$  denotes consumption,  $s_t$  represents savings carried over to retirement,  $\beta = 1/(1 + \rho)$  refers to the discount factor with  $\rho > 0$  being the discount rate,  $R_{t+1}$  denotes the gross interest rate paid on assets between generation  $t$  and generation  $t+1$ ,  $n_t$  is the birth rate with  $\xi > 0$  being the utility weight that parents put on the number of their children,  $e_t$  refers to the education level of each child, and  $\theta > 0$  denotes the corresponding utility weight of children's education in the parent's utility function. In order to ensure non-negative consumption in retirement, we assume  $c_{t+1} = \max\{(R_{t+1} - 1)s_t, 0\}$ . The budget constraint of a young adult reads

$$(1 - \tau)(1 - \psi n_t - \eta e_t n_t) w_t h_t = c_t + s_t, \quad (2)$$

where  $\tau$  denotes the income tax rate,  $\psi$  are the time costs associated with each child irrespective of the child's education,  $\eta$  is the parent's time requirement for each unit of education of each child,  $h_t$  denotes the embodied human capital level of parents, and  $w_t$  represents the wage rate per unit of effective labor. Note that we assume that parents educate their children at home, which simplifies the exposition considerably. We could instead assume that there is a schooling sector in which teachers, who are financed by the education expenditures of households, educate the young. While being more realistic in industrialized countries, this alternative modeling strategy would not change our qualitative results. The solution of the parent's maximization problem is represented by the following optimal expenditures on consumption, savings, fertility, and education:

$$c_t = \frac{(1 - \tau)h_t w_t}{1 + \beta + \xi}, \quad s_t = \frac{\beta(1 - \tau)h_t w_t}{1 + \beta + \xi}, \quad n_t = \frac{\xi - \theta}{\psi(1 + \beta + \xi)}, \quad e_t = \frac{\theta\psi}{\eta(\xi - \theta)}. \quad (3)$$

The derivation of these optimal values can be found in the Appendix. These expressions imply that consumption and savings increase with income and decrease with the tax rate. Furthermore, if the discount factor  $\beta$  increases, people save more and consume less. Fertility and education do not depend on income and taxation because their costs are measured in time units: an increase in income also raises the opportunity costs of childcare and education to the same extent. This implies that childcare and education stay constant over time for growing income, a result that is justified by our focus on industrialized countries that already experienced the demographic transition in the past (see the Unified Growth Theory of Galor and Weil, 2000; Galor, 2005, 2011, for an appropriate description of the transition from stagnation to growth via the demographic transition). The trade-off between child quantity and quality according to Becker and Lewis

(1973) is clearly visible in (3) and can be summarized in the following lemma.

**Lemma 1.** *If parents want to have more children ( $\xi$  increases) they raise fertility and reduce education investments, while the converse holds true if parents want to have better educated children ( $\theta$  increases).*

*Proof.* See Appendix. □

Population growth is determined by the birth rate such that the population size at time  $t+1$  can be written as the product  $n_t L_t$ , that is,

$$L_{t+1} = \frac{\xi - \theta}{(1 + \beta + \xi)\psi} L_t. \quad (4)$$

Individual human capital increases with the time that parents spend educating their children multiplied by the parents' productivity of doing so,  $h_t$ . Furthermore, children obtain human capital by observing their parents and peers even in the absence of parental time investments in education as in Strulik et al. (2013). Overall we therefore have

$$h_{t+1} = e_t h_t + h_t = \left[ 1 + \frac{\theta\psi}{\eta(\xi - \theta)} \right] h_t. \quad (5)$$

The aggregate human capital stock ( $H_t$ ) of the economy is given by the product of the average embodied human capital ( $h_t$ ) and the population size ( $L_t$ ), that is,  $H_t = h_t L_t$ . Consequently, the human capital stock available for production and research ( $\bar{H}_t$ ) is given by the aggregate human capital stock adjusted for the time that parents spend raising and educating their children ( $\psi n_t + \eta e_t n_t$ )

$$\bar{H}_t := H_t(1 - \psi n_t - \eta e_t n_t) = \frac{L_t h_t (1 + \beta)}{1 + \beta + \xi}, \quad (6)$$

where  $(1 + \beta)/(1 + \beta + \xi) \in (0, 1)$  is the labor force participation rate of adults.

## 2.3 Production side

Final goods production, intermediate goods production, and applied research closely follow Romer (1990) and Jones (1995). We augment this structure to account for i) an income tax financed basic research sector that employs scientists to discover the natural laws the knowledge of which is needed for applied research, and ii) the endogenous evolution of aggregate human capital in the production process as determined by the household's fertility and education decisions, which in turn pin down labor force participation.

### 2.3.1 Final goods sector

Final output  $Y_t$ , which is tantamount to the gross domestic product (GDP), is produced according to the production function

$$Y_t = H_{t,Y}^{1-\alpha} \int_0^{A_t} x_{t,i}^\alpha di, \quad (7)$$

where  $H_{t,Y}$  is human capital employed in the final goods sector (workers),  $A_t$  is the technological base of a society, that is, it represents the most modern blueprint that has been developed in



the applied research sector,  $x_{t,i}$  is the amount of the blueprint-specific machine  $i$  used in final goods production, and  $\alpha$  is the elasticity of final output with respect to the machines of type  $i$ . Due to perfect competition in the final goods market, production factors are paid their marginal products implying that the wage rate per unit of human capital and prices of machines are given by, respectively,

$$w_{t,Y} = (1 - \alpha)H_{t,Y}^{-\alpha} \int_0^{A_t} x_{t,i}^\alpha di = (1 - \alpha) \frac{Y_t}{H_{t,Y}}, \quad (8)$$

$$p_{t,i} = \alpha H_{t,Y}^{1-\alpha} x_t^{\alpha-1}. \quad (9)$$

### 2.3.2 Intermediate goods sector

We assume that a single intermediate goods producer is able to convert capital  $k_{t,i}$  one-for-one into machines  $x_{t,i}$  after it has purchased the corresponding blueprint from the applied research sector. As a consequence, its operating profits are given by  $\pi_{t,i} = p_{t,i}k_{t,i} - R_t k_{t,i}$  and profit maximization leads to the familiar outcome of Dixit and Stiglitz (1977) that firms charge prices for machines that are a markup  $1/\alpha$  over marginal cost:

$$p_{t,i} = \frac{R_t}{\alpha}. \quad (10)$$

It is apparent that there is symmetry between firms implying that the index  $i$  can be dropped. As another consequence of symmetry, each firm employs  $k_t = K_t/A_t$  units of capital, where we denote the aggregate capital stock by  $K_t$ . Thus, we can rewrite the aggregate production function as

$$Y_t = (A_t H_{t,Y})^{1-\alpha} K_t^\alpha, \quad (11)$$

in which the technological base of a society appears as human capital augmenting.

### 2.3.3 Applied research sector

The applied research sector employs scientists with a human capital level  $H_{t,A}$  and productivity  $\delta$  to develop new blueprints that can be patented and sold to the intermediate goods sector. Knowledge spillovers can occur intertemporally in the applied research sector and intersectorally between basic research and applied research. Overall we would expect that the spillovers from basic research to applied research are larger than the other way round because patents are partially excludable, while the laws of nature, once discovered, can be exploited by every scientist without restriction. The production function of a firm in the applied research sector can be written as

$$A_{t+1} - A_t = \delta A_t^\phi B_t^\mu H_{t,A} \quad (12)$$

where  $B_t$  is a society's stock of propositional knowledge discovered by basic researchers, that is, the epistemic base for the techniques  $A_t$ , and  $\phi \in [0, 1]$  and  $\mu \in [0, 1]$  measure the extent of intertemporal knowledge spillovers in the applied research sector and intersectoral knowledge spillovers from basic research to applied research, respectively. Note that without any propositional knowledge  $B_t$ , no techniques can be developed. Of course we do not claim that our model is a suitable description of such a scenario (e.g. the stone age, or ancient Greece). Instead, we

assume that  $B_0 > 0$  and  $A_0 > 0$  to begin with and that the countries under consideration are already industrialized.<sup>4</sup>

**Remark 1.** For  $\tau = 0$ ,  $\theta = 0$ ,  $\xi > \psi(1 + \beta + \xi)$ ,  $\mu = 0$ , and  $\phi \in (0, 1)$ , our model nests the Jones (1995) framework, while for  $\tau = 0$ ,  $\theta = 0$ ,  $\xi = \psi(1 + \beta + \xi)$ ,  $\mu = 0$ , and  $\phi = 1$ , our model nests the Romer (1990) framework.

Firms in the applied research sector optimally choose human capital input  $H_{t,A}$  as to maximize their profits

$$\pi_{t,A} = p_{t,A} \delta A_t^\phi B_t^\mu H_{t,A} - w_{t,A} H_{t,A} \quad (13)$$

with  $p_{t,A}$  being the price of a blueprint and  $w_{t,A}$  being the wage rate of applied scientists. This leads to the optimality condition

$$w_{t,A} = p_{t,A} \delta A_t^\phi B_t^\mu, \quad (14)$$

such that the wages of applied scientists increase in their productivity, the price that can be charged for blueprints, the stock of techniques that are already available raised to the power of the intertemporal knowledge spillovers, and the stock of propositional knowledge that makes it easier to develop new blueprints raised to the power of intersectoral knowledge spillovers.

We assume that patent protection for a newly discovered blueprint lasts for one generation (cf. Aghion and Howitt, 2005), which is realistic given that patents usually expire after 20 years in industrialized countries (cf. The German Patent and Trade Mark Office, 2012; The United States Patent and Trademark Office, 2012). After a patent expired, the right to sell the blueprint is handed over to the government that can either consume or invest the associated proceeds. This assumption simplifies the exposition considerably, and, in contrast to standard endogenous and semi-endogenous growth models, enables us to trace the transitional dynamics because it allows us to calculate the present value of a patent even for a time-varying interest rate.

Applied research firms can charge prices for their blueprints that are equal to the operating profits of intermediate goods producers at time  $t$  — when patent protection is valid — because there is always a potential entrant willing to outbid a lower price. To put it differently, in case that blueprints were less expensive, firms would have an incentive to enter the market as intermediate goods producers and thereby increase demand for blueprints and drive up their price. We therefore have

$$p_{t,A} = (\alpha - \alpha^2) \frac{Y_t}{A_t}, \quad (15)$$

which follows from Equations (9) and (10) and the fact that  $x_i = k_i$  for all  $i$ .

### 2.3.4 Basic research sector

The government employs scientists to discover propositional knowledge in the basic research sector. In so doing it uses the proceeds of the income tax such that the governmental budget restriction reads

$$\frac{\tau(1 + \beta)w_t h_t L_t}{1 + \beta + \xi} = w_t h_t L_{t,B}, \quad (16)$$

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<sup>4</sup>R&D-based growth theory would clearly be the wrong framework to analyze the growth processes of historical societies; for this purpose see instead the Unified Growth Theory as pioneered by Galor and Weil (2000), Galor (2005), and Galor (2011).

where the left hand side represents total governmental revenues from the labor income tax and the right hand side refers to the total governmental expenditures for basic research. Note that the government is not allowed to run a budget deficit, a restriction that is necessary for analytical tractability and common in the literature (cf. Park, 1998; Grossmann et al., 2010, 2013a,b; Gersbach et al., 2012). Equation (16) can be solved for the amount of human capital employed in the basic research sector as

$$H_{t,B} = L_{t,B}h_t = \frac{\tau(1+\beta)}{1+\beta+\xi}H_t, \quad (17)$$

which is increasing in the tax rate ( $\tau$ ) and decreasing in the preference of parents for the number of children ( $\xi$ ) because higher fertility reduces the amount of time that a parent can spend supplying her skills on the labor market. Note that the preference for education of each child ( $\theta$ ) does not enter this expression because parents who want to educate their children better also choose to have fewer of them to the extent that their time budget keeps in balance. Propositional knowledge then evolves according to

$$B_{t+1} - B_t = \nu B_t^\omega A_t^\gamma H_{t,B} = \nu B_t^\omega A_t^\gamma \frac{\tau(1+\beta)}{1+\beta+\xi}H_t, \quad (18)$$

where  $\nu > 0$  is the productivity of scientists in the basic research sector,  $\omega \in [0, 1]$  refers to intertemporal knowledge spillovers in the basic research sector,  $\gamma \in [0, 1]$  denotes the intersectoral knowledge spillovers from applied research to basic research, and  $H_{t,B}$  is the amount of aggregate human capital that the state attracts to basic research by choosing a tax rate  $\tau$ .

## 2.4 Market clearing and the balanced growth path of the economy

Labor market clearing implies that the total amount of human capital net of the time spent for child care and education has either to be employed in the final goods sector, in the basic research sector, or in the applied research sector, that is, we have  $\bar{H}_t = h_t(L_{t,Y} + L_{t,B} + L_{t,A}) = H_{t,Y} + H_{t,B} + H_{t,A}$ . Furthermore, wages in all sectors have to equalize such that  $w_{t,Y} = w_{t,B} = w_{t,A}$ , otherwise one or more sectors would not be able to attract any workers and the economy ended up in a corner solution. Equalizing expressions (8) and (14), using Equation (15), and noting that employment in the basic research sector is given by Equation (17), yields demand for workers in the final goods sector and in the R&D sector as, respectively,

$$H_{t,Y} = \frac{A_t^{1-\phi} B_t^{-\mu}}{\alpha\delta}, \quad (19)$$

$$H_{t,A} = \bar{H}_t - H_{t,B} - H_{t,Y} = \frac{(\beta+1)(1-\tau)h_tL_t}{\beta+\xi+1} - \frac{A_t^{1-\phi} B_t^{-\mu}}{\alpha\delta}. \quad (20)$$

Altogether, the development of new blueprints can then be written as

$$A_{t+1} = \frac{(1+\beta)\delta(1-\tau)h_tL_tA_t^\phi B_t^\mu}{1+\beta+\xi} - \frac{(1-\alpha)A_t}{\alpha}, \quad (21)$$

where the main trade-off that public investments in basic research imply is the following: increasing taxes poaches labor from the applied research sector to the basic research sector. This means that the development of new blueprints slows down, while the discovery of propositional knowledge speeds up. Consequently, per capita GDP growth slows down after a tax increase, but it recovers in the medium run. The initial slowdown of growth is usually disregarded in the theoretical literature but it is in line with the empirical finding of a long gestation lag of basic research (cf. Adams, 1990) coupled with the fact that an increase in the tax rate draws labor from other productive uses into the basic research sector. The long-run impact of an increase in the tax rate on the *level* of per capita GDP is a priori not clear because the slowdown of growth in the short run and the increase of growth in the medium run exert their influence in opposite directions. However, we investigate this short-run vs. medium-run trade-off numerically in Section 4.

Full depreciation of physical capital and capital market clearing imply that the aggregate physical capital stock of an economy in generation  $t + 1$  is equal to aggregate savings (private savings of adults plus governmental savings). Furthermore, goods market clearing ensures that aggregate consumption (private consumption of adults and retirees plus governmental consumption) together with aggregate savings are equal to total output such that

$$K_{t+1} = s_t L_t = Y_t - c_t L_t - c_{2,t-1} \frac{L_t}{n_t} - G_t, \quad (22)$$

where  $c_{2,t-1}$  is second period's consumption of members of the generation that was born at time  $t - 1$  and  $G_t$  refers to governmental expenditures other than those for basic research. For simplicity we assume that these expenditures are unproductive, which has no bearing on the balanced growth path. Therefore, the aggregate physical capital stock of the next period is given by

$$K_{t+1} = \frac{(1 - \alpha)(1 - \tau)A_t \beta h_t L_t K_t^\alpha \left( \frac{A_t^{2-\phi} B_t^{-\mu}}{\alpha \delta} \right)^{-\alpha}}{1 + \beta + \xi}. \quad (23)$$

Putting all information together, the system fully describing the equilibrium dynamics of our model economy reads

$$A_{t+1} = \frac{(\alpha - 1)A_t}{\alpha} - \frac{(\beta + 1)\delta(\tau - 1)h_t L_t A_t^\phi B_t^\mu}{1 + \beta + \xi}, \quad (24)$$

$$h_{t+1} = \left[ \frac{\theta \psi}{\eta(\xi - \theta)} + 1 \right] h_t, \quad (25)$$

$$L_{t+1} = \frac{\xi - \theta}{\psi(1 + \beta + \xi)} L_t, \quad (26)$$

$$K_{t+1} = \frac{(1 - \alpha)(1 - \tau)A_t \beta h_t L_t K_t^\alpha \left( \frac{A_t^{2-\phi} B_t^{-\mu}}{\alpha \delta} \right)^{-\alpha}}{1 + \beta + \xi}, \quad (27)$$

$$B_{t+1} = \frac{1 + \beta}{1 + \beta + \xi} \nu \tau h_t L_t A_t^\gamma B_t^\omega + B_t, \quad (28)$$

with  $A_0 > 0$ ,  $h_0 > 0$ ,  $L_0 > 0$ ,  $K_0 > 0$ , and  $B_0 > 0$ . This system can be solved analytically for the balanced growth path and it can be analyzed numerically during the transition period.

### 3 Analytical results

We now derive the balanced growth rates of the central endogenous variables. To guarantee the existence of the balanced growth path and to rule out the empirically implausible situation of hyper-exponential growth, we restrict ourselves to the following specification of the intertemporal and intersectoral knowledge spillovers in this section.

**Assumption 1.** *The intertemporal and intersectoral knowledge spillovers are given by  $\omega \in [0, 1)$ ,  $\gamma \in [0, 1)$ ,  $\phi \in [0, 1)$ , and  $\mu \in [0, 1)$ . Furthermore, it holds that  $\mu + \phi < 1$  and  $\gamma + \omega < 1$ .*

The growth rates of the knowledge base ( $B_t$ ), the amount of blueprints ( $A_t$ ), and the aggregate physical capital stock ( $K_t$ ) are given by

$$g_{A,t} = \frac{A_{t+1} - A_t}{A_t} = -\frac{1}{\alpha} + \frac{(1 + \beta)\delta(1 - \tau)h_t N_t A_t^{\phi-1} B_t^\mu}{1 + \beta + \xi}, \quad (29)$$

$$g_{B,t} = \frac{B_{t+1} - B_t}{B_t} = \frac{1 + \beta}{1 + \beta + \xi} \nu \tau h_t N_t A_t^\gamma B_t^{\omega-1}, \quad (30)$$

$$g_{K,t} = \frac{K_{t+1} - K_t}{K_t} = \frac{(1 - \alpha)(1 - \tau)\beta A_t h_t N_t K_t^{\alpha-1} \left( \frac{A_t^{2-\phi} B_t^{-\mu}}{\alpha \delta} \right)^{-\alpha}}{1 + \beta + \xi}. \quad (31)$$

Furthermore, the balanced growth factors of the population ( $L_t$ ) and average individual human capital ( $h_t$ ) are

$$\tilde{h} := \frac{h_{t+1}}{h_t} = \frac{\theta\psi}{\eta(\xi - \theta)} + 1, \quad (32)$$

$$\tilde{L} := \frac{L_{t+1}}{L_t} = n = \frac{\xi - \theta}{\psi(1 + \beta + \xi)}. \quad (33)$$

Let  $\Omega$  be the product of the two growth factors  $\tilde{h}$  and  $\tilde{L}$ , that is, the growth factor of aggregate human capital as given by

$$\Omega := \tilde{h}\tilde{L} = \frac{\theta\psi + \eta(\xi - \theta)}{\eta\psi(1 + \beta + \xi)}. \quad (34)$$

Then we can state the following proposition.

**Proposition 1** (Balanced Growth Factors).

*i) The balanced growth factors of  $A$ ,  $B$ ,  $K$ , and  $Y$  are given by*

$$\begin{aligned} \tilde{A} &:= \frac{A_{t+1}}{A_t} = \Omega^{\frac{1+\mu-\omega}{(1-\phi)(1-\omega)-\gamma\mu}}, & \tilde{B} &:= \frac{B_{t+1}}{B_t} = \Omega^{\frac{1+\gamma-\phi}{(1-\phi)(1-\omega)-\gamma\mu}}, \\ \tilde{K} &:= \frac{K_{t+1}}{K_t} = \Omega^{\frac{(1-\gamma)\mu+(2-\phi)(1-\omega)}{(1-\phi)(1-\omega)-\gamma\mu}}, & \tilde{Y} &:= \frac{Y_{t+1}}{Y_t} = \Omega^{\frac{(1-\gamma)\mu+(2-\phi)(1-\omega)}{(1-\phi)(1-\omega)-\gamma\mu}}. \end{aligned}$$

*ii) These balanced growth factors are increasing in human capital accumulation and in the knowledge spillovers  $\mu$ ,  $\phi$ ,  $\omega$ , and  $\gamma$ .*

*iii) The balanced growth factors are independent of the tax rate  $\tau$ .*

*Proof.* See Appendix. □

These results imply that the central driving force of long-run economic growth is human capital accumulation, a result that is in line with Dalgaard and Kreiner (2001) and Strulik (2005) but more general because we can distinguish between the growth effects of changes in fertility and growth effects of changes in education (see Propositions 2 and 3 below). Furthermore, it is quite obvious that higher intertemporal and intersectoral knowledge spillovers lead to a higher balanced growth rate. An interesting result is that the long-run growth rate of the economy does not depend on governmental investments in basic research. The explanation for this result lies in a standard attribute of semi-endogenous growth theory: as long as intertemporal knowledge spillovers are smaller than unity, changes in policy variables will have transitory effects on growth rates and therefore only level effects on aggregate variables.

Next, we analyze the comparative statics of the balanced growth factors with respect to the determinants of human capital.

**Proposition 2** (Comparative Statics of Human Capital).

- i) *The balanced growth factor of individual human capital ( $\tilde{h}$ ) increases with the utility weight of children's education ( $\theta$ ), and decreases with the utility weight of the number of children ( $\xi$ ).*
- ii) *The balanced growth factor of the population ( $\tilde{L}$ ) increases with the utility weight of the number of children ( $\xi$ ), and decreases with the utility weight on children's education ( $\theta$ ).*
- iii) *The balanced growth factor of aggregate human capital ( $\Omega$ ) increases with the utility weight of children's education ( $\theta$ ) if and only if the time requirement for children's education is smaller than the time costs per child, that is, if  $\psi > \eta$ .*
- iv) *The balanced growth factor of aggregate human capital ( $\Omega$ ) increases with the utility weight of the number of children ( $\xi$ ) if and only if the time costs per child are smaller than the time requirement for children's education times a weighting factor, that is, if  $\psi < (1 + \beta + \xi)\eta/\theta$ .*

*Proof.* See Appendix. □

As a consequence of Proposition 2 and the fact that the growth factors of applied knowledge, basic knowledge, physical capital, and aggregate GDP increase in human capital accumulation (see Proposition 1), the comparative statics of  $\tilde{A}$ ,  $\tilde{B}$ ,  $\tilde{K}$ , and  $\tilde{Y}$  with respect to  $\xi$  and  $\theta$  lead to the same results.

**Proposition 3** (GDP per capita growth). *The steady-state growth factor of per capita GDP is given by*

$$\tilde{y} = \frac{\tilde{Y}}{\tilde{L}} = \left[ \frac{\theta\psi}{\eta(\xi - \theta)} + 1 \right]^\Lambda \left[ \frac{\xi - \theta}{\psi(1 + \beta + \xi)} \right]^{\Lambda-1} \quad (35)$$

with

$$\Lambda := \frac{(1 - \gamma)\mu + (2 - \phi)(1 - \omega)}{(1 - \phi)(1 - \omega) - \gamma\mu}.$$

We have that

a) *growth of per capita GDP decreases with the utility weight of the number of children ( $\xi$ ) if and only if the time costs for children are sufficiently high, that is,*

$$\psi > \eta \frac{(1 + \beta + \theta)(\Lambda - 1)(\xi - \theta)}{\theta[1 + \beta + \theta + \Lambda(\xi - \theta)]}. \quad (36)$$

b) *growth of per capita GDP increases with the utility weight for children's education ( $\theta$ ) if and only if the time requirement for each unit of children's education is sufficiently low, that is,*

$$\eta < \psi \frac{\theta + \Lambda(\xi - \theta)}{(\Lambda - 1)(\xi - \theta)}. \quad (37)$$

*Proof.* See Appendix. □

The intuitive explanation for part a) is the following: When the costs of children ( $\psi$ ) are high, an increase in the desire for the number of children reduces the labor supply of households and their education expenditures to a large extent [see Equation (3)]. The adverse effect of these reductions on next period's effective labor supply cannot be compensated for by the corresponding increases in fertility. As a consequence, there would be less aggregate human capital available for basic and applied research such that economic growth slows down. Conversely, when fixed costs of children are low, then the labor supply of households and their education expenditures only decrease slightly in response to an increase in the desire for the number of children, such that its overall effect is to increase next period's aggregate human capital stock. In this case there would be more aggregate human capital available for basic and applied research and economic growth gains momentum.

The intuitive explanation for part b) follows a similar line of reasoning: When the costs of education ( $\eta$ ) are high, then an increase in the desire for education of children only increases education expenditures slightly, while having a pronounced negative effect on labor supply. This negative quantity effect cannot be compensated by the positive quality effect of better education such that aggregate human capital available for basic and applied research declines and growth of per capita GDP slows down. Conversely, if the costs for education are low, education expenditures would rise substantially, while the labor supply of households would only decrease slightly. In this case the overall effect on next period's aggregate human capital available for basic and applied research would be positive and economic growth increases.

To summarize, our results indicate that the relationship between fertility, human capital accumulation, R&D, and economic growth is more general than suggested by earlier contributions. While Jones (1995) finds that population growth is positively related to R&D and economic growth, Strulik et al. (2013) suggest that the converse holds true. In our contribution we find that both can be true, that is, the overall growth effect of fertility and education can be positive and negative, depending on the underlying parameter values, in particular the relative time costs for child-rearing and education. When taking our results and the empirical findings of a negative association between economic growth and population growth seriously, the implication would be that in modern industrialized economies, the relative costs of education as compared to basic child-rearing costs are low. This can be justified by the presence of

public schooling and the high labor force participation, especially of women. While the former tends to decrease the private costs of education, the latter raises the private opportunity costs of child-care.

So far we investigated the long-run growth effects of human capital accumulation and basic and applied R&D. Our central result is that human capital accumulation promotes long-run economic development because it raises the central input factor in basic and applied research namely embodied human capital. However, our framework allows for the analysis of the differential growth effects of the two driving forces of human capital accumulation: population growth and education. Whether an increase in population growth or an increase education is growth-promoting depends on the underlying parameters. In modern industrialized countries with high labor force participation and high public schooling subsidies the relative costs of basic child care as compared to education tend to be lower than in case of low labor force participation and no public schooling. Consistent with the empirical findings, our model therefore suggests a negative effect of fertility on economic growth and a positive effect of education on economic growth.

We also established that subsidies for basic research are not effective in changing long-run growth. However, we have already seen that an increase in investments for basic research slows down economic growth in the short run because resources have to be drawn away from applied research and there is a gestation lag of basic research such that the increasing investments only pay off in the more distant future. To address the relative importance of basic vs. applied science in more detail, we now resort to numerical analyses of the transitional dynamics, where we also assess the effects of changing governmental basic research policies on household's utility levels.

## 4 Numerical illustration

To analyze the behavior of the economy during transition and to assess the differential effects of basic and applied science on medium-run growth, we simulate the dynamic system given by Equations (24)-(28) for an increase in public expenditures on basic science. The parameter values are summarized in Table 1 and are either taken from the literature (cf. Auerbach and Kotlikoff, 1987; Jones, 1995; Acemoglu, 2009) or otherwise adjusted such that the model's predictions are consistent with the increase in mean years of schooling of the adult population, the population growth rate, and the economic growth rate of the OECD countries over the years 2000-2010. The data stems from World Bank (2012), except the parameter for basic research expenditures ( $\tau$ ), which is inferred from data of the OECD (2012) on the fraction of GDP that the OECD spent on basic research between 2000-2009. At this point we want to stress, however, that we do not aim to calibrate the model to the observed pattern of a particular country or region, which would be futile given that a time period lasts for 20 years in our framework. The numerical example is rather intended to illustrate the adjustment processes during the transitional dynamics, the central mechanisms of our framework, and the differential timing of the response of the endogenous variables to the policy change. Furthermore, it allows us to get a glimpse on the welfare effects of investments in basic research.

The effects of an increase in basic research expenditures on the growth rates of basic knowledge, applied knowledge, the physical capital stock, and per capita GDP are shown in Figure



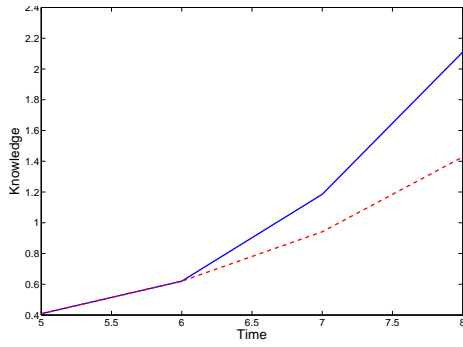
Table 1: Parameter values for simulation

Parameter	Value	Parameter	Value
$\beta$	0.6892	$\alpha$	1/3
$\xi$	0.5320	$\psi$	0.1900
$\eta$	0.1390	$\theta$	0.0500
$\tau$	0.0099	$\phi$	0.3000
$\mu$	0.2500	$\gamma$	0.0500
$\omega$	0.3000	$\nu$	1
$\delta$	1		

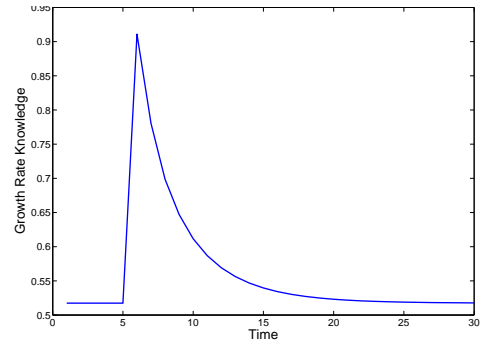
2 on the right hand side. We assume that the economy initially moves along the balanced growth path. At generation five, the government decides to increase public expenditures for basic research as a fraction of GDP by 0.5 percentage points. Afterwards the reaction of the four above-mentioned endogenous variables to this policy change is traced for another 25 generations. The increase in public expenditures at impact draws labor out of the applied research sector and into the basic research sector. This slows down growth in the number of blueprints [Panel d)] but spurs growth in propositional knowledge [Panel b)]. Since propositional knowledge is an important input for applied research, the accumulation of new blueprints speeds up in the medium run despite its slowdown in the short run. It then stays higher than without the increase in public expenditures for basic research for a considerable amount of time. The physical capital stock grows faster than in case of no policy change in the short- and medium run [Panel h)]. The overall consequence is a short-run slowdown of economic growth in terms of per capita GDP due to the temporary slowdown in the accumulation rate of patents [Panel f)], while growth of per capita GDP gains momentum in the medium run. In the long run, the positive growth effects of higher investments in basic research die out, which is exactly what we expect in light of Proposition 1.

The effects of an increase in basic research expenditures on the *levels* of basic knowledge, applied knowledge, the physical capital stock, and per capita GDP are shown in Figure 2 on the left hand side. Note that by focusing on a time window of only 3 generations, these figures provide a closer look at the level effects immediately after the increase in public education investments to identify the short-run costs as well as the long run benefits of basic research spending. The solid (blue) line refers to an economy that experienced an increase in public expenditures for basic research, while the dashed (red) line refers to an economy without such an increase. It becomes apparent that the adverse short-run effects of the increase in public spending on per capita GDP are outweighed by the associated long-run gains: the level of propositional knowledge [Panel a)], the number of patents [Panel c)], aggregate physical capital [Panel g)], and per capita GDP (Panel e)] are all higher in an economy that increases investments in basic research than in an economy without such an increase at generation 8.

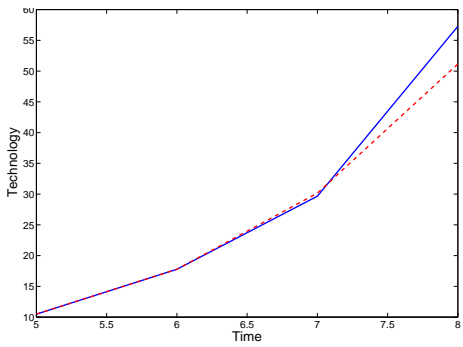
While the effect of an increase in basic research expenditures on the level of per capita GDP is negative in the short run and positive in the medium and long-run, we do not yet know whether the policy is welfare-improving, and, if yes, which tax rate would maximize welfare over a certain time horizon. Recall that the individuals value consumption and not production or



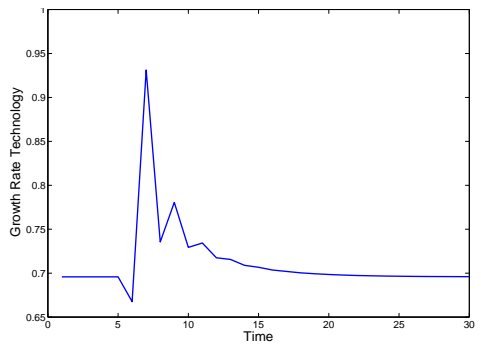
(a) Knowledge



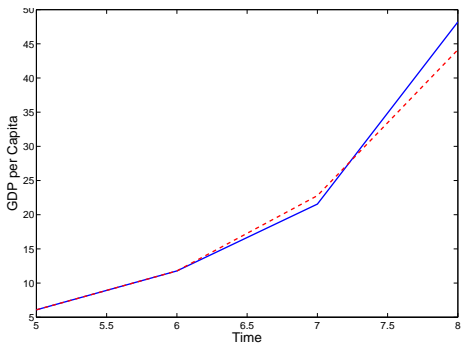
(b) Knowledge Growth



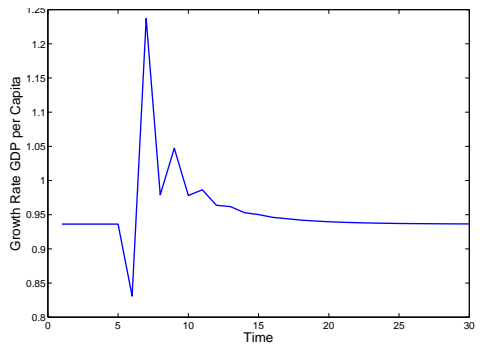
(c) Technology



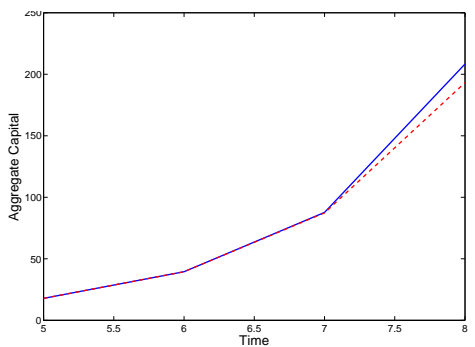
(d) Technology Growth



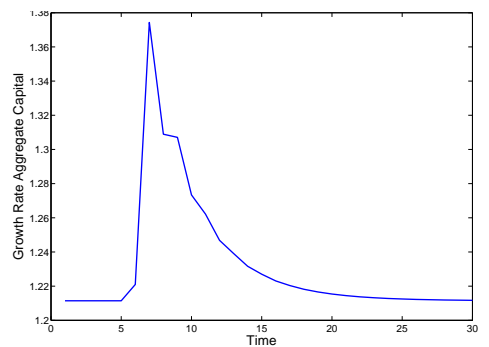
(e) GDP per capita



(f) Growth of GDP per Capita



(g) Aggregate Capital



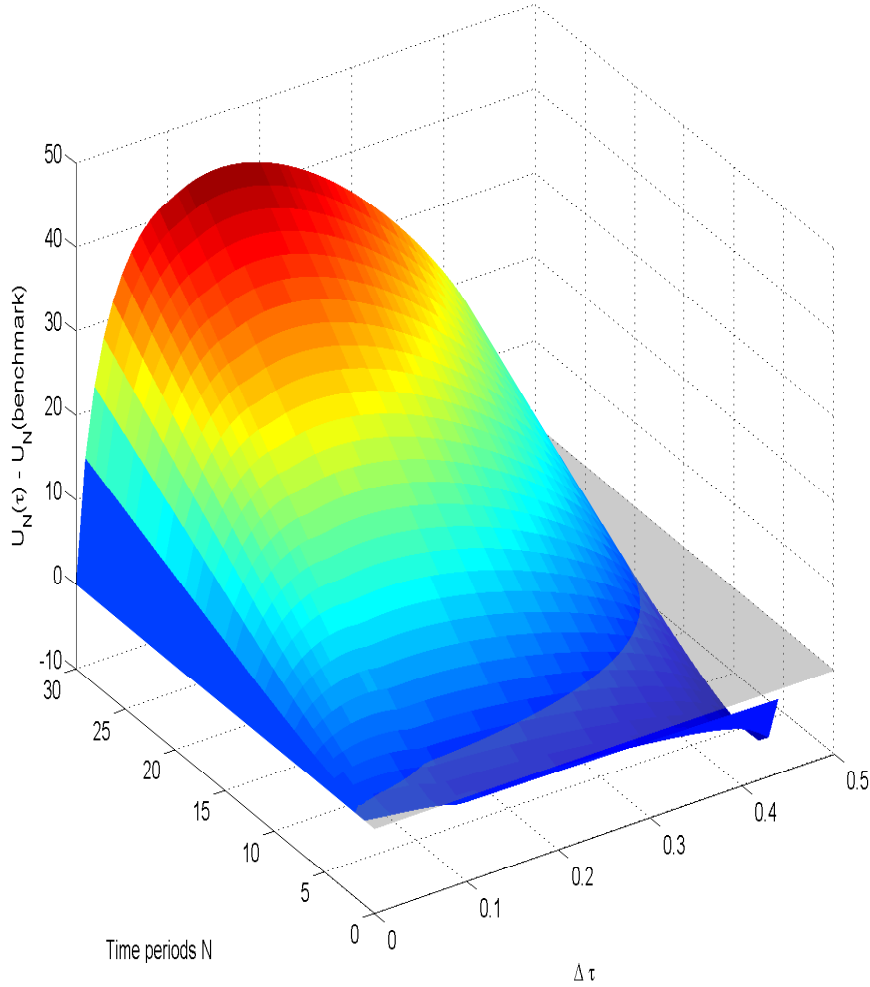
(h) Growth of Aggregate Capital

Note: The solid (blue) line refers to an economy that initially moves along its balanced growth path. After 5 generations, an 0.5 percentage point increase in public expenditures for basic research as a fraction of GDP occurs. The dashed (red) line refers to an economy where no increase in public expenditures for basic research occurs.

Figure 2: Simulation of an increase in public expenditures for basic research

income per capita. Furthermore, they also care for the number of children they have and the education level they can provide them with [see Equation (1)]. Therefore we also calculate the effects of changes in basic research expenditures on utility levels over different time horizons and different changes in basic research investments ( $\tau$ ). The result is shown in Figure 3, which displays the difference in the aggregate utility level between inhabitants of an economy that changes its research policy and inhabitants of an economy without such a change ( $\Delta\tau \equiv 0$ ). Aggregate utility is calculated as the sum of the lifetime utilities up to time horizon  $N$

$$U_N := \sum_{j=1}^N u_j(c_j, c_{j+1}, e, n). \quad (38)$$



Note: The figure displays the difference in aggregate utility levels between inhabitants of an economy that changes its research policies and inhabitants of an economy without such a change. The time horizon is displayed on the  $x$ -axis, while the change in  $\tau$  is displayed on the  $y$ -axis. In case that the difference is positive, the inhabitants of the economy with the corresponding change in the research policy are better off in the relevant time period. The shaded plane corresponds to case where inhabitants of both economies are equally well off, that is, the difference equals zero.

Figure 3: Changes in lifetime utility for changes in basic research expenditures  $\tau$  for different time horizons ( $x$ -axis) and different changes in  $\tau$  ( $y$ -axis)

In Figure 3 the time horizon is displayed on the  $x$ -axis. Initially (at  $N = 0$ ), the economy moves along a balanced growth path and then faces a change in the research policy ( $\tau$ ), the extent of which is displayed on the  $y$ -axis. The associated change in aggregate utility with respect to the benchmark case (without a policy change) is displayed on the  $z$ -axis. The figure reveals that an increase in  $\tau$  is welfare-reducing in the years after the impact and welfare-improving over longer time horizons. The larger the extent of the increase in  $\tau$ , the more pronounced is the initial decrease in welfare and the longer it takes until the welfare gains materialize. The reaction of long-run welfare levels (after 30 generations) to an increase in  $\tau$  is positive for small increases in basic research expenditures, but turns negative after a certain level of basic research expenditures has been reached. This implies that there is an interior welfare-maximizing rate of basic research investments for each increase of  $\tau$  and each time horizon. After 30 generations, the maximum level of welfare would be reached by increasing  $\tau$  to around 16%, which corresponds to 10.67% of GDP, a level that exceeds the current expenditure levels by a large extent.

Altogether, our result that investments in basic research are associated with a slowdown of economic growth and a decrease in utility in the short run<sup>5</sup> provides an explanation for the reluctance of policy-makers to invest in basic research despite that the benefits of doing so are substantial in the long run. If the government and voters care primarily for the near future, that is, a period of around 20 years according to our framework, their preferred policy would be to oppose spending increases for basic research.

## 5 Sensitivity analysis with respect to knowledge spillovers

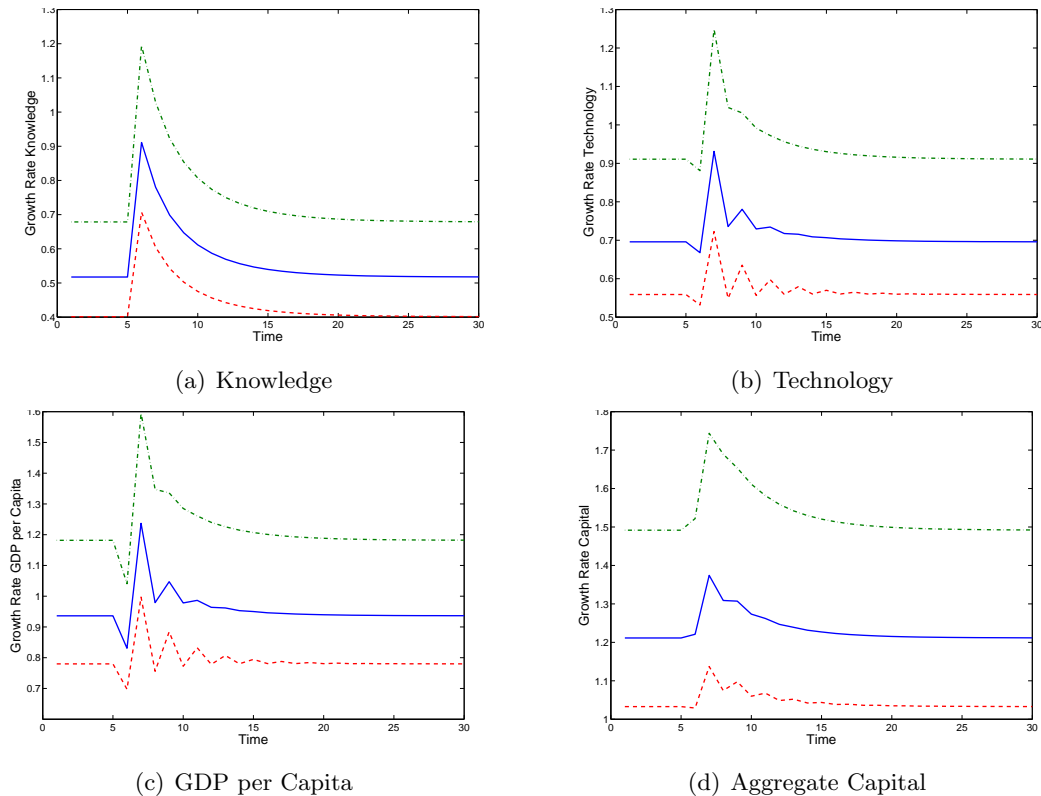
The choice of parameter values for the illustrative simulation (see Table 1) relied on the requirement that the model's predictions either fit to observables for the OECD over the years 2000-2010 or that the parameter values reflect a common sense inferred from empirical observations, that is, that they are widely used in other studies. However, it is notoriously difficult to estimate intertemporal and intersectoral knowledge spillovers. Most studies therefore calibrate the corresponding parameters such that the model's predicted growth series match the observed ones (which was the approach that we followed as well). In this section we aim to illustrate that our qualitative findings are fairly robust to changes in the spillover parameters  $\phi$ ,  $\mu$ ,  $\omega$ , and  $\gamma$ . In so doing we repeat the simulation of growth in basic knowledge, applied knowledge, physical capital, and per capita GDP by distinguishing three different cases i) a low spillover case with  $\phi = 0.25$ ,  $\mu = 0.2$ ,  $\omega = 0.25$ , and  $\gamma = 0.01$ ; ii) a high spillover case with  $\phi = 0.35$ ,  $\mu = 0.3$ ,  $\omega = 0.4$ , and  $\gamma = 0.07$ ; iii) an intermediate spillover case corresponding to our baseline specification with  $\phi = 0.3$ ,  $\mu = 0.25$ ,  $\omega = 0.3$ , and  $\gamma = 0.05$ . The growth and level effects are depicted in Figure 4 with the red dashed line referring to the low spillover case, the blue solid line referring to the intermediate spillover case and the green dash-dotted line referring to the high spillover

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<sup>5</sup>Note that the initial decrease in utility would even occur in case that the government was allowed to finance the additional basic research expenditures by issuing bonds and taxing future generations (the governmental budget would not need to be balanced in this case). The reason is the associated slowdown of applied research in the face of higher investments in basic research (because employment of human capital increases in the latter sector and decreases in the former). Since there is a gestation lag of basic research, the initial effect is always a slowdown of economic growth such that changes in governmental basic research policies cannot be Pareto-improving for the given tax scheme.

case. Altogether, our general results that an increase in basic research expenditures leads to a medium-run increase in growth of basic knowledge, applied knowledge, physical capital, and per capita GDP, to a short-run slowdown in growth of applied knowledge and per capita GDP, and no long-run effects on growth rates still hold true for all of these specifications.

Furthermore, we also investigated the sensitivity of welfare with respect to changes in the spillovers. The result that there is an interior welfare-maximizing level of basic research for each generation that stands to benefit from an increase in public basic research expenditures remains unaffected. This also holds true for the adverse short-run effects of increasing basic research expenditures. However, as expected, the optimal public research expenditures for generations that stand to benefit from a policy change are sensitive to changes in the spillovers. Figure 5 displays the effects of changes in  $\tau$  on aggregate utility up to generation  $N = 30$  for low spillovers (red dashed line), intermediate spillovers (blue solid line), and high spillovers (green dash-dotted line). The maximum of the change in utility with respect to a change in the basic research expenditures increases with an increase in spillovers. In particular, for the low spillover case the maximum occurs at  $\tau = 0.11$ , which corresponds to 7.33% of GDP and for the high spillover case the maximum occurs at  $\tau = 0.22$ , which corresponds to 14.67% of GDP. However, the general result still holds true that the optimal public research expenditures from the view of future generations are substantially higher than the levels that we currently observe (in the



Note: The solid (blue) line refers to the intermediate spillover case, the dash-dotted (green) line refers to the high spillover case, and the dashed (red) line refers to the low spillover case.

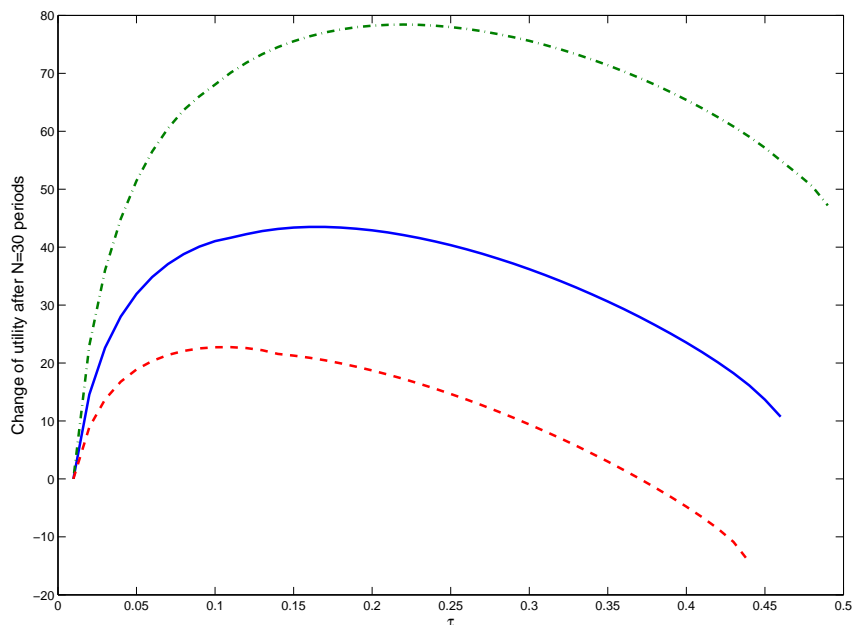
Figure 4: Sensitivity check with respect to intertemporal and intersectoral knowledge spillovers

OECD around 1% of GDP).

## 6 Conclusions

We set up an R&D-based economic growth framework with endogenous fertility, endogenous education, and a distinction between publicly funded basic research and profit-driven applied research. The model allows to distinguish between the most important dimensions of human knowledge: embodied human capital, disembodied knowledge of natural phenomena, and disembodied knowledge of techniques that can be applied in the production process. Furthermore, it features a fairly general structure of intertemporal and intersectoral knowledge spillovers such that our framework encompasses some well-known growth models like the ones of Romer (1990) and Jones (1995) as special cases.

Our main insights are that a) the interrelations between economic growth and population growth depend on the relative costs of child-rearing vs. education. In industrialized countries with substantial public schooling investments and a high labor force participation of women, the relative costs of education tend to be low such that an increase in fertility is negatively associated with economic growth. This result conforms with the empirical findings of Brander and Dowrick (1994), Kelley and Schmidt (1995), Ahituv (2001), Li and Zhang (2007), and Herzer et al. (2012); b) increasing governmental expenditures on basic science raises per capita GDP in the long run, which is in line with the findings on the growth-promoting effects of basic



Note: The solid (blue) line refers to the intermediate spillover case, the dash-dotted (green) line refers to the high spillover case, and the dashed (red) line refers to the low spillover case.

Figure 5: Changes in lifetime utility for changes in basic research expenditures  $\tau$  for the time horizon  $N = 30$

science reported by Mansfield (1980); c) there is an initial slowdown of economic growth after growth-promoting increases in expenditures for basic research are implemented, which is in line with the gestation lag of basic science found in the empirical analysis of Adams (1990); d) the welfare-effects of changes in basic research investments are negative in the short run and positive in the long run, with the negative short run effects being more pronounced and longer-lasting for larger increases in basic research investments. This could be an explanation for the reluctance of governments to raise expenditures for basic science, despite the substantial long-run gains that are often reported for R&D investments (cf. Jones and Williams, 2000; Grossmann et al., 2010, 2013a,b); e) there exists an interior welfare-maximizing level of basic research expenditures for each generation that stands to benefit from such a policy. This level exceeds current levels of basic research spending in the OECD by a large amount.

Naturally, we had to rely on a number of simplifying assumptions to keep the model analytically tractable. We believe that most of these assumptions do not impact upon the generality of our results. For example: a) including a desire of individuals for knowing how nature works by augmenting their utility function with a term that features the stock of basic knowledge ( $B_t$ ) would not change the growth-effects of basic research expenditures and would only add a positive amount to the utility levels of individuals after a policy change; b) introducing an education sector, in which teachers are employed to educate the young would lead to the same results with respect to fertility and education, only labor force participation would change because an activity that happened informally before would move to the formal economy with no bearing on the growth and welfare effects; c) allowing for a more nuanced distinction between basic and applied science than in our scenario that features only the two polar cases of pure basic research and pure applied research would, of course, add realism to the framework but would not change the basic trade-offs that are involved and would render the model much more complicated to analyze; d) allowing for a governmental budget deficit under the given tax scheme would not change the finding that there cannot be a Pareto improvement in the wake of increases in public research expenditures because the first generations always stand to lose from such a policy due to its adverse short-run growth effects.

We think that there is clearly scope for future research in assessing the effects of different types of taxation (e.g. capital gains taxes, consumption taxes, labor taxes in case of endogenous labor supply etc.) within our framework. In addition, analyzing international knowledge flows between countries of different size (e.g. along the lines of Gersbach et al., 2012; Gersbach and Schneider, 2013) and also analyzing the growth and welfare effects of the migration of skilled workers are surely promising avenues for further investigation. Finally, designing a framework for explaining the emergence of universities as institutions that integrate basic research and education is on top of our research agenda.

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## Appendix

### A Derivation of the optimal values $c_t, s_t, n_t, e_t$

The optimization problem of the household is given by Equations (1) and (2). The Lagrangian of this problem is

$$\mathcal{L} = \log c_t + \beta \log[(R_{t+1} - 1) s_t] + \xi \log n_t + \theta \log e_t + \lambda [(1 - \tau)(1 - \psi n_t - \eta e_t n_t) w_t h_t - c_t - s_t].$$

Therefore, the first order conditions are given by

$$\frac{\partial \mathcal{L}}{\partial c_t} = \frac{1}{c_t} - \lambda \stackrel{!}{=} 0, \quad (39)$$

$$\frac{\partial \mathcal{L}}{\partial s_t} = \frac{\beta}{s_t} - \lambda \stackrel{!}{=} 0, \quad (40)$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = \frac{\xi}{n_t} + \lambda(1 - \tau)(-\psi - \eta e_t) h_t w_t \stackrel{!}{=} 0, \quad (41)$$

$$\frac{\partial \mathcal{L}}{\partial e_t} = \frac{\theta}{e_t} + \lambda(1 - \tau)(-\eta n_t) h_t w_t \stackrel{!}{=} 0, \quad (42)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = (1 - \tau)(1 - \psi n_t - \eta e_t n_t) w_t h_t - c_t - s_t \stackrel{!}{=} 0. \quad (43)$$

Rearranging Equation (41) yields

$$\frac{\xi}{n_t(\psi + \eta e_t)} = \lambda(1 - \tau) h_t w_t.$$

Inserting the result into Equation (42), we obtain the optimal level of education

$$\frac{\theta}{e_t \eta n_t} = \frac{\xi}{n_t(\psi + \eta e_t)} \Leftrightarrow e_t = \frac{\theta \psi}{\eta(\xi - \theta)}.$$

Rearranging Equation (39) and using Equation (40) yields

$$c_t = \frac{1}{\beta} s_t. \quad (44)$$

Rearranging Equation (41) in combination with Equation (39) yields

$$n_t(\psi + \eta e_t)(1 - \tau) h_t w_t = \frac{\xi}{\lambda} = \xi c_t. \quad (45)$$

Optimal consumption  $c_t$  can then be obtained by inserting Equations (44) and (45) into the budget constraint (43) such that

$$\begin{aligned} (1 - \tau)(1 - \psi n_t - \eta e_t n_t) w_t h_t - c_t - s_t &= 0 \\ \Leftrightarrow (1 - \tau)(1 - \psi n_t - \eta e_t n_t) w_t h_t &= (1 + \beta) c_t \\ \Leftrightarrow (1 - \tau) h_t w_t + \xi c_t &= (1 + \beta) c_t \\ \Leftrightarrow c_t &= \frac{(1 - \tau) h_t w_t}{1 + \beta + \xi}. \end{aligned}$$



As a consequence, optimal savings are given by

$$s_t = \beta c_t = \beta \frac{(1 - \tau)h_t w_t}{1 + \beta + \xi}.$$

The optimal birth rate can be calculated by inserting the optimal values for  $c_t$ ,  $s_t$ , and  $e_t$  into the budget constraint (43). This yields

$$\begin{aligned} \frac{1}{(\psi + \eta e_t)} - \frac{c_t + s_t}{(1 - \tau)h_t w_t(\psi + \eta e_t)} &= n_t \\ \Leftrightarrow \frac{1}{(\psi + \eta e_t)} \left( \frac{\xi}{1 + \beta + \xi} \right) &= n_t \\ \Leftrightarrow n_t &= \frac{\xi - \theta}{\psi(1 + \beta + \xi)}. \end{aligned}$$

## B Proofs

*Proof of Lemma 1.* The partial derivatives of fertility  $n_t$  and education  $e_t$  with respect to  $\xi$  and  $\theta$  are

$$\begin{aligned} \frac{\partial n_t}{\partial \xi} &= \frac{1 + \beta + \theta}{(1 + \beta + \theta)^2 \psi} > 0, & \frac{\partial n_t}{\partial \theta} &= -\frac{1}{(1 + \beta + \theta)\psi} < 0 \\ \frac{\partial e_t}{\partial \xi} &= -\frac{\theta \psi}{\eta(\theta - \xi)^2} < 0, & \frac{\partial e_t}{\partial \theta} &= \frac{\xi \psi}{\eta(\theta - \xi)^2} > 0. \end{aligned}$$

□

*Proof of Proposition 1.* i) Along the balanced growth path, the growth rates of the endogenous variables have to be constant. Equation (29) therefore leads to

$$\begin{aligned} \frac{g_{A,t} - g_{A,t-1}}{g_{A,t-1}} \stackrel{!}{=} 0 &\Leftrightarrow \frac{-\frac{1}{\alpha} + \frac{(1+\beta)\delta(1-\tau)h_t L_t A_t^{\phi-1} B_t^\mu}{1+\beta+\xi} + \frac{1}{\alpha} - \frac{(1+\beta)\delta(1-\tau)h_{t-1} L_{t-1} A_{t-1}^{\phi-1} B_{t-1}^\mu}{1+\beta+\xi}}{-\frac{1}{\alpha} + \frac{(1+\beta)\delta(1-\tau)h_{t-1} L_{t-1} A_{t-1}^{\phi-1} B_{t-1}^\mu}{1+\beta+\xi}} = 0 \\ &\Leftrightarrow (1 + \beta)\delta(1 - \tau)h_t L_t A_t^{\phi-1} B_t^\mu = (1 + \beta)\delta(1 - \tau)h_{t-1} L_{t-1} A_{t-1}^{\phi-1} B_{t-1}^\mu \\ &\Leftrightarrow \frac{h_t}{h_{t-1}} \frac{L_t}{L_{t-1}} \left( \frac{A_t}{A_{t-1}} \right)^{\phi-1} \left( \frac{B_t}{B_{t-1}} \right)^\mu = \Omega \left( \frac{A_t}{A_{t-1}} \right)^{\phi-1} \left( \frac{B_t}{B_{t-1}} \right)^\mu = 1. \end{aligned} \quad (46)$$

Furthermore, for the knowledge base  $B$  it follows from Equation (30) that

$$\begin{aligned}
\frac{g_{B,t} - g_{B,t-1}}{g_{B,t-1}} \stackrel{!}{=} 0 &\Leftrightarrow \frac{\frac{1+\beta}{1+\beta+\xi} \nu \tau h_t L_t A_t^\gamma B_t^{\omega-1} - \frac{1+\beta}{1+\beta+\xi} \nu \tau h_{t-1} L_{t-1} A_{t-1}^\gamma B_{t-1}^{\omega-1}}{\frac{1+\beta}{1+\beta+\xi} \nu \tau h_{t-1} L_{t-1} A_{t-1}^\gamma B_{t-1}^{\omega-1}} = 0 \\
&\Leftrightarrow \frac{1+\beta}{1+\beta+\xi} \nu \tau h_t L_t A_t^\gamma B_t^{\omega-1} = \frac{1+\beta}{1+\beta+\xi} \nu \tau h_{t-1} L_{t-1} A_{t-1}^\gamma B_{t-1}^{\omega-1} \\
&\Leftrightarrow \frac{h_t}{h_{t-1}} \frac{L_t}{L_{t-1}} \left( \frac{A_t}{A_{t-1}} \right)^\gamma \left( \frac{B_t}{B_{t-1}} \right)^{\omega-1} = 1 \\
&\Leftrightarrow \left( \frac{B_t}{B_{t-1}} \right)^{1-\omega} = \Omega \left( \frac{A_t}{A_{t-1}} \right)^\gamma \\
&\Leftrightarrow \left( \frac{B_t}{B_{t-1}} \right) = \Omega^{\frac{1}{1-\omega}} \left( \frac{A_t}{A_{t-1}} \right)^{\frac{\gamma}{1-\omega}}. \tag{47}
\end{aligned}$$

Insert Equation (47) into Equation (46) to obtain the balanced growth factor for applied research

$$\Omega \left( \frac{A_t}{A_{t-1}} \right)^{\phi-1} \left[ \Omega^{\frac{1}{1-\omega}} \left( \frac{A_t}{A_{t-1}} \right)^{\frac{\gamma}{1-\omega}} \right]^\mu = 1 \Leftrightarrow \tilde{A} := \frac{A_t}{A_{t-1}} = \Omega^{\frac{1+\mu-\omega}{(1-\phi)(1-\omega)-\gamma\mu}}. \tag{48}$$

Hence, the balanced growth factor for basic research can be obtained by inserting Equation (48) into Equation (47)

$$\tilde{B} := \frac{B_t}{B_{t-1}} = \Omega^{\frac{1}{1-\omega}} \tilde{A}^{\frac{\gamma}{1-\omega}} = \Omega^{\frac{1+\gamma-\phi}{(1-\phi)(1-\omega)-\gamma\mu}}. \tag{49}$$

The balanced growth factor for aggregate physical capital can be calculated using Equations (31), (48), and (49)

$$\begin{aligned}
\frac{g_{K,t} - g_{K,t-1}}{g_{K,t-1}} &= 0 \\
\Leftrightarrow A_t h_t L_t K_t^{\alpha-1} \left( \frac{A_t^{2-\phi} B_t^{-\mu}}{\alpha \delta} \right)^{-\alpha} &= A_{t-1} h_{t-1} L_{t-1} K_{t-1}^{\alpha-1} \left( \frac{A_{t-1}^{2-\phi} B_{t-1}^{-\mu}}{\alpha \delta} \right)^{-\alpha} \\
\Leftrightarrow \tilde{A}^{1-\alpha(2-\phi)} \tilde{B}^{\mu\alpha} \Omega &= \left( \frac{K_t}{K_{t-1}} \right)^{1-\alpha} = \tilde{K}^{1-\alpha} \\
\tilde{K} &= \Omega^{\frac{(1-\gamma)\mu+(2-\phi)(1-\omega)}{(1-\phi)(1-\omega)-\gamma\mu}}. \tag{50}
\end{aligned}$$

Aggregate output is given by

$$Y_t = (A_t H_{t,Y})^{1-\alpha} K_t^\alpha = \left( \frac{A_t^{2-\phi} B_t^{-\mu}}{\alpha \delta} \right)^{1-\alpha} K_t^\alpha.$$

The balanced growth factor of aggregate output follows from Equations (48), (49), and (50) and

rearranging:

$$\begin{aligned}
\tilde{Y} &:= \frac{Y_t}{Y_{t-1}} \\
&= \left( \frac{A_t}{A_{t-1}} \right)^{(2-\phi)(1-\alpha)} \left( \frac{B_t}{B_{t-1}} \right)^{(-\mu)(1-\alpha)} \left( \frac{K_t}{K_{t-1}} \right)^\alpha \\
&= \Omega^{\frac{1+\mu-\omega}{(1-\phi)(1-\omega)-\gamma\mu}(2-\phi)(1-\alpha)} \cdot \Omega^{\frac{1+\gamma-\phi}{(1-\phi)(1-\omega)-\gamma\mu}(-\mu)(1-\alpha)} \cdot \Omega^{\frac{(1-\gamma)\mu+(2-\phi)(1-\omega)}{(1-\phi)(1-\omega)-\gamma\mu}\alpha} \\
&= \Omega^{\frac{(1-\gamma)\mu+(2-\phi)(1-\omega)}{(1-\phi)(1-\omega)-\gamma\mu}} \\
&= \tilde{K}.
\end{aligned} \tag{51}$$

ii) The fact that the growth factors are increasing with human capital accumulation is obvious because the exponents are all larger than 1.

For the second part we calculate the partial derivatives of  $\tilde{A}$  with respect to the spillover parameters  $\mu$ ,  $\phi$ ,  $\omega$ , and  $\gamma$  as

$$\begin{aligned}
\frac{\partial \tilde{A}}{\partial \mu} &= \tilde{A} \log(\Omega) \frac{(1-\omega)(1+\gamma-\phi)}{[(1-\phi)(1-\omega)-\gamma\mu]^2} > 0, & \frac{\partial \tilde{A}}{\partial \phi} &= \tilde{A} \log(\Omega) \frac{(1-\omega)(1+\mu-\omega)}{[(1-\phi)(1-\omega)-\gamma\mu]^2} > 0, \\
\frac{\partial \tilde{A}}{\partial \omega} &= \tilde{A} \log(\Omega) \frac{\mu(1+\gamma-\phi)}{[(1-\phi)(1-\omega)-\gamma\mu]^2} \geq 0, & \frac{\partial \tilde{A}}{\partial \gamma} &= \tilde{A} \log(\Omega) \frac{\mu(1+\mu-\omega)}{[(1-\phi)(1-\omega)-\gamma\mu]^2} \geq 0.
\end{aligned}$$

The partial derivatives of  $\tilde{B}$  with respect to the spillover parameters  $\mu$ ,  $\phi$ ,  $\omega$ , and  $\gamma$  are given by

$$\begin{aligned}
\frac{\partial \tilde{B}}{\partial \mu} &= \tilde{B} \log(\Omega) \gamma \frac{(1+\gamma-\phi)}{[(1-\phi)(1-\omega)-\gamma\mu]^2} \geq 0, & \frac{\partial \tilde{B}}{\partial \phi} &= \tilde{B} \log(\Omega) \gamma \frac{(1+\mu-\omega)}{[(1-\phi)(1-\omega)-\gamma\mu]^2} \geq 0, \\
\frac{\partial \tilde{B}}{\partial \omega} &= \tilde{B} \log(\Omega) \frac{(1-\phi)(1+\gamma-\phi)}{[(1-\phi)(1-\omega)-\gamma\mu]^2} > 0, & \frac{\partial \tilde{B}}{\partial \gamma} &= \tilde{B} \log(\Omega) \frac{(1-\phi)(1+\mu-\omega)}{[(1-\phi)(1-\omega)-\gamma\mu]^2} > 0.
\end{aligned}$$

The partial derivatives of  $\tilde{K}$  with respect to the spillover parameters  $\mu$ ,  $\phi$ ,  $\omega$ , and  $\gamma$  are given by

$$\begin{aligned}
\frac{\partial \tilde{K}}{\partial \mu} &= \tilde{K} \log(\Omega) \frac{(1+\gamma-\phi)(1-\omega)}{[(1-\phi)(1-\omega)-\gamma\mu]^2} > 0, & \frac{\partial \tilde{K}}{\partial \phi} &= \tilde{K} \log(\Omega) \frac{(1+\mu-\omega)(1-\omega)}{[(1-\phi)(1-\omega)-\gamma\mu]^2} > 0, \\
\frac{\partial \tilde{K}}{\partial \omega} &= \tilde{K} \log(\Omega) \frac{\mu(1+\gamma-\phi)}{[(1-\phi)(1-\omega)-\gamma\mu]^2} \geq 0, & \frac{\partial \tilde{K}}{\partial \gamma} &= \tilde{K} \log(\Omega) \frac{\mu(1+\mu-\omega)}{[(1-\phi)(1-\omega)-\gamma\mu]^2} \geq 0.
\end{aligned}$$

The partial derivatives of  $\tilde{Y}$  with respect to the spillover parameters are the same like the partial derivatives of  $\tilde{K}$ .

iii) This follows immediately from inspecting the growth factors. □

*Proof of Proposition 2.* The partial derivatives of the growth factor of individual human capital  $\tilde{h}$  with respect to  $\xi$  and  $\theta$  are

$$\frac{\partial \tilde{h}}{\partial \theta} = \frac{\xi\psi}{\eta(\theta-\xi)^2} > 0, \quad \frac{\partial \tilde{h}}{\partial \xi} = -\frac{\theta\psi}{\eta(1+\beta+\xi)} < 0.$$

The partial derivatives of the growth factor of the total population  $\tilde{L}$  with respect to  $\xi$  and  $\theta$  are

$$\frac{\partial \tilde{L}}{\partial \theta} = -\frac{1}{(1 + \beta + \xi)\psi} < 0, \quad \frac{\partial \tilde{L}}{\partial \xi} = \frac{1 + \beta + \theta}{(1 + \beta + \xi)^2 \psi} > 0.$$

The partial derivatives of the growth factor of aggregate human capital  $\Omega$  with respect to  $\xi$  and  $\theta$  are

$$\begin{aligned} \frac{\partial \Omega}{\partial \theta} &= \frac{\psi - \eta}{\eta\psi(1 + \beta + \psi)} > 0 \Leftrightarrow \psi > \eta\kappa, \\ \frac{\partial \Omega}{\partial \xi} &= \frac{\eta(1 + \beta + \theta) - \theta\psi}{\eta\psi(1 + \beta + \xi)^2} > 0 \Leftrightarrow \eta(1 + \beta + \theta) - \theta\psi > 0 \Leftrightarrow \frac{\psi}{\eta} < \frac{1 + \beta + \theta}{\theta}. \end{aligned}$$

□

*Proof of Proposition 3.* The partial derivative of the balanced growth rate of per capita GDP with respect to  $\xi$  is given by

$$\begin{aligned} \frac{\partial \tilde{y}}{\partial \xi} &= - \left[ \frac{\eta(\xi - \theta) + \theta\psi}{\eta\psi(1 + \beta + \xi)} \right]^{\Lambda - 1} \frac{\eta(1 + \beta + \theta)(1 - \Lambda)(\xi - \theta) + \theta\psi[1 + \beta + \theta + \Lambda(\xi - \theta)]}{\eta(\theta - \xi)^2(1 + \beta + \xi)} < 0 \\ &\Leftrightarrow \theta\psi(1 + \beta + \theta + \Lambda(\xi - \theta)) > \eta(1 + \beta + \theta)(\Lambda - 1)(\xi - \theta) \\ &\Leftrightarrow \psi > \eta \frac{(1 + \beta + \theta)(\Lambda - 1)(\xi - \theta)}{\theta[1 + \beta + \theta + \Lambda(\xi - \theta)]}, \end{aligned}$$

since  $\Lambda > 1$ . The partial derivative of per capita GDP with respect to  $\theta$  is given by

$$\begin{aligned} \frac{\partial \tilde{y}}{\partial \theta} &= \left[ \frac{\eta(\xi - \theta) + \theta\psi}{\eta\psi(1 + \beta + \xi)} \right]^{\Lambda - 1} \frac{\eta(1 - \Lambda)(\xi - \theta) + \psi[\theta + \Lambda(\xi - \theta)]}{\eta(\xi - \theta)^2} > 0 \\ &\Leftrightarrow \psi(\theta + \Lambda(\xi - \theta)) > \eta(\Lambda - 1)(\xi - \theta) \\ &\Leftrightarrow \eta < \psi \frac{\theta + \Lambda(\xi - \theta)}{(\Lambda - 1)(\xi - \theta)}. \end{aligned}$$

□

## References

- Acemoglu, D. (2009). *Introduction to Modern Economic Growth*. Princeton University Press.
- Adams, J. D. (1990). Fundamental stocks of knowledge and productivity growth. *Journal of Political Economy*, Vol. 98(No. 4):673–702.
- Aghion, P. and Howitt, P. (1992). A model of growth through creative destruction. *Econometrica*, Vol. 60(No. 2):323–351.
- Aghion, P. and Howitt, P. (1999). *Endogenous Economic Growth*. The MIT Press.
- Aghion, P. and Howitt, P. (2005). *Handbook of Economic Growth, Volume 1A*, chapter 2: “Growth with Quality-Improving Innovations: An Integrated Framework”, pages 68–110.

- Ahituv, A. (2001). Be fruitful or multiply: On the interplay between fertility and economic development. *Journal of Population Economics*, Vol. 14:51–71.
- Akcigit, U., Hanley, D., and Serrano-Velarde, N. (2013). Back to Basic: Basic Research Spillovers, Innovation Policy and Growth. NBER Working Paper No. 19473.
- Auerbach, A. J. and Kotlikoff, L. J. (1987). *Dynamic Fiscal Policy*. Cambridge University Press.
- Becker, G. S. and Lewis, H. G. (1973). On the interaction between the quantity and quality of children. *Journal of Political Economy*, Vol. 81:279–288.
- Brander, J. A. and Dowrick, S. (1994). The role of fertility and population in economic growth. *Journal of Population Economics*, Vol. 7(No. 1):1–25.
- Cohen, D. and Soto, M. (2007). Growth and human capital: good data, good results. *Journal of Economic Growth*, Vol. 12:51–76.
- Dalgaard, C. and Kreiner, C. (2001). Is declining productivity inevitable? *Journal of Economic Growth*, Vol. 6(No. 3):187–203.
- de la Fuente, A. and Domenéch, R. (2006). Human capital in growth regressions: How much difference does data quality make? *Journal of the European Economic Association*, Vol. 4(No. 1):1–36.
- Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity. *American Economic Review*, Vol. 67(No. 3):297–308.
- Galor, O. (2005). *Handbook of Economic Growth*, chapter 4. “From Stagnation to Growth: Unified Growth Theory”, pages 171–293.
- Galor, O. (2011). *Unified Growth Theory*. Princeton University Press.
- Galor, O. and Weil, D. (2000). Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond. *The American Economic Review*, Vol. 90(No. 4):806–828.
- Gancia, G. and Zilibotti, F. (2005). *Handbook of Economic Growth*, chapter 3: “Horizontal Innovation in the Theory of Growth and Development”, pages 112–170. Elsevier.
- Gersbach, H. and Schneider, M. T. (2013). On the Global Supply of Basic Research. CER-ETH Working Paper 13/175.
- Gersbach, H., Schneider, M. T., and Schneller, O. (2012). Basic research, openness, and convergence. *Journal of Economic Growth*, Vol. 18:33–68.
- Gersbach, H., Sorger, G., and Amon, C. (2009). Hierarchical Growth: Basic and Applied Research. Department of Economics, University of Vienna, Working Paper No: 0912.
- Grossman, G. M. and Helpman, E. (1991). Quality ladders in the theory of economic growth. *Review of Economic Studies*, Vol. 58(No. 1):43–61.

- Grossmann, V., Steger, T. M., and Trimborn, T. (2010). Quantifying Optimal Growth Policy. CESifo Working Paper No. 3092.
- Grossmann, V., Steger, T. M., and Trimborn, T. (2013a). Dynamically optimal R&D subsidization. *Journal of Economic Dynamics and Control*, Vol. 37:516–534.
- Grossmann, V., Steger, T. M., and Trimborn, T. (2013b). The macroeconomics of TANSTAAFL. *Journal of Macroeconomics*. Forthcoming.
- Ha and Howitt (2007). Accounting for Trends in Productivity and R&D: A Schumpeterian Critique of Semi-Endogenous Growth Theory. *Journal of Money, Credit and Banking*, Vol. 39(No. 4):733–774.
- Hanushek, E. A. and Woessmann, L. (2012). Do better schools lead to more growth? Cognitive skills, economic outcomes, and causation. *Journal of Economic Growth*, Vol. 17:267–321.
- Hashimoto, K. and Tabata, K. (2013). Rising Longevity, Human Capital and Fertility in Overlapping Generations Version of an R&D-based Growth Model. Discussion Paper Series 104, School of Economics, Kwansai Gakuin University.
- Herzer, D., Strulik, H., and Vollmer, S. (2012). The long-run determinants of fertility: one century of demographic change 1900-1999. *Journal of Economic Growth*, Vol. 17(No. 4):357–385.
- Howitt, P. (1999). Steady endogenous growth with population and R&D inputs growing. *Journal of Political Economy*, Vol. 107(No. 4):715–730.
- Jones, C. I. (1995). R&D-based models of economic growth. *Journal of Political Economy*, Vol. 103(No. 4):759–783.
- Jones, C. I. (1999). Growth: With or Without Scale Effects? *American Economic Review*, Vol. 89(No. 2):139–144.
- Jones, C. I. (2002). Sources of U.S. Economic Growth in a World of Ideas. *American Economic Review*, Vol. 92(No. 1):220–239.
- Jones, C. I. and Williams, J. C. (2000). Too Much of a Good Thing? The Economics of Investment in R&D. *Journal of Economic Growth*, Vol. 5:65–85.
- Keller, W. (2002). Geographic Localization of International Technology Diffusion. *The American Economic Review*, Vol. 92(No. 1):120–142.
- Kelley, A. C. and Schmidt, R. M. (1995). Aggregate population and economic growth correlations: the role of the components of demographic change. *Demography*, Vol. 32(No. 4):543–555.
- Kortum, S. (1997). Research, patenting and technological change. *Econometrica*, Vol. 65(No. 6):1389–1419.
- Krueger, A. B. and Lindahl, M. (2001). Education for Growth: Why and for Whom? *Journal of Economic Literature*, Vol. 39(No. 4):1101–1136.

- Li, C.-W. (2000). Endogenous vs. semi-endogenous growth in a two-r&d-sector model. *The Economic Journal*, Vol. 110(No. 462):C109–C122.
- Li, C.-W. (2002). Growth and scale effects: the role of knowledge spillovers. *Economics Letters*, Vol. 74:177–185.
- Li, H. and Zhang, J. (2007). Do high birth rates hamper economic growth? *Review of Economics and Statistics*, Vol. 89:110–117.
- Madsen, J. B. (2008). Semi-endogenous versus Schumpeterian growth models: testing the knowledge production function using international data. *Journal of Economic Growth*, Vol. 13:1–26.
- Mansfield, E. (1980). Basic Research and Productivity Increase in Manufacturing. *The American Economic Review*, Vol. 70(No. 5):863–873.
- Mokyr, J. (2002). *The Gifts of Athena*. Princeton University Press.
- Morales, M. F. (2004). Research policy and endogenous growth. *Spanish Economic Review*, Vol. 6:179–209.
- Nelson, R. R. (1959). The simple economics of basic scientific research. *The Journal of Political Economy*, Vol. 67(No. 3):297–306.
- OECD (2012). Research and Development Statistics (RDS). URL: [http://www.oecd.org/document/52/0,3746,en\\_2649\\_34273\\_34537140\\_1\\_1\\_1\\_1,00.html](http://www.oecd.org/document/52/0,3746,en_2649_34273_34537140_1_1_1_1,00.html).
- Park, W. G. (1998). A theoretical model of government research and growth. *Journal of Economic Behavior & Organization*, Vol. 34:69–85.
- Peretto, P. F. (1998). Technological change and population growth. *Journal of Economic Growth*, Vol. 3(No. 4):283–311.
- Romer, P. (1990). Endogenous technological change. *Journal of Political Economy*, Vol. 98(No. 5):71–102.
- Segerström, P. S. (1998). Endogenous growth without scale effects. *American Economic Review*, Vol. 88(No. 5):1290–1310.
- Shell, K. (1966). Toward A Theory of Inventive Activity and Capital Accumulation. *The American Economic Review*, Vol. 56(No. 1/2):62–68.
- Solow, R. M. (1956). A contribution to the theory of economic growth. *The Quarterly Journal of Economics*, Vol. 70(No. 1):65–94.
- Strulik, H. (2005). The role of human capital and population growth in R&D-based models of economic growth. *Review of International Economics*, Vol. 13(No. 1):129–145.
- Strulik, H., Pretzner, K., and Prskawetz, A. (2013). The past and future of knowledge-based growth. *Journal of Economic Growth*, Vol. 18(No. 4). 411-437.

- The German Patent and Trade Mark Office (2012). Url: <http://www.dpma.de/english/patent/index.html> [accessed on 12/11/2012].
- The United States Patent and Trademark Office (2012). Url: <http://www.uspto.gov> [accessed on 12/11/2012].
- World Bank (2012). World Development Indicators & Global Development Finance Database. <http://databank.worldbank.org/ddp/home.do?Step=12&id=4&CNO=2>.
- Young, A. (1998). Growth without scale effects. *Journal of Political Economy*, Vol. 106(No. 5):41–63.