

Afanasyeva, Elena; Guentner, Jochen

Conference Paper

Bank Risk Taking, Credit Booms and Monetary Policy

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2014: Evidenzbasierte Wirtschaftspolitik
- Session: Financial Frictions, the Business Cycle, and Stabilization Policy, No. E07-V1

Provided in Cooperation with:

Verein für Socialpolitik / German Economic Association

Suggested Citation: Afanasyeva, Elena; Guentner, Jochen (2014) : Bank Risk Taking, Credit Booms and Monetary Policy, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2014: Evidenzbasierte Wirtschaftspolitik - Session: Financial Frictions, the Business Cycle, and Stabilization Policy, No. E07-V1, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft, Kiel und Hamburg

This Version is available at:

<https://hdl.handle.net/10419/100436>

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Bank Risk Taking, Credit Booms and Monetary Policy[☆]

[VERY PRELIMINARY AND INCOMPLETE]

This Version: February 28, 2014

Elena Afanasyeva*

Grueneburgplatz 1, House of Finance, 60323 Frankfurt am Main.

Phone: +49 69 798 33831, Fax: +49 69 798 33907, Email address: eafanasyeva@wiwi.uni-frankfurt.de.

Jochen Guentner**

Altenberger Strasse 69, 4040 Linz, Austria.

Phone: +43 732 2468 8360, Fax: +43 732 2468 9679, Email address: jochen.guentner@jku.at.

Abstract

This paper investigates the risk-taking channel of monetary policy on the asset side of banks' balance sheets. We use a factor-augmented vector autoregression (FAVAR) model to show that aggregate lending standards of U.S. banks, e.g. their collateral requirements for firms, are significantly loosened in response to an unexpected decrease in the Federal Funds rate. Based on this evidence, we reformulate the costly state verification (CSV) contract, embed it in a dynamic general equilibrium model, and show that – consistent with our empirical finding – a monetary easing implies an expansion of bank lending for a given amount of borrower collateral. The model also predicts a delayed increase in borrowers' default risk.

Keywords: Bank risk taking, Credit supply, Monetary policy, Risk channel

JEL classification: E44, E52

1. Introduction

One of the narrative explanations of the credit boom preceding the recent financial crisis and the Great Recession is the excessive risk taking of financial intermediaries due to monetary policy interest rates that have been “too low for too long” (see Taylor, 2007). On the one hand, loose monetary policy lowers the wholesale funding costs of banks and other financial intermediaries, which contributes to excessive leverage, i.e. risk taking on the liability side of their balance sheets. On the other hand, low monetary policy rates may also induce banks to take on more risk on the asset side, e.g. giving out more and riskier loans. While the liability-side risk-taking channel has received a lot of attention

[☆]We thank Tobias Adrian, Pooyan Amir Ahmadi, Hans Gersbach, Charles Kahn and Jean-Charles Rochet for helpful discussions.

*Elena Afanasyeva is Ph.D. student at Goethe University Frankfurt and the Institute for Monetary and Financial Stability.

**Jochen Güntner is Assistant Professor at the Department of Economics, Johannes Kepler University Linz.

in the recent macroeconomic literature (see, e.g., Gertler and Karadi, 2011; Gertler et al., 2012; Angeloni et al., 2013), much fewer studies have so far addressed the aggregate implications of the asset-side risk-taking channel of monetary policy. The intention of this paper is to fill this gap. First, we provide empirical evidence of an asset-side risk-taking channel in the aggregate lending behavior of U.S. banks. Motivated by this evidence, we develop a dynamic general equilibrium model with endogenous risk taking of financial intermediaries on the asset side.

In prior research, the empirical relevance of an asset-side risk-taking channel of monetary policy has shown based on microeconomic bank-level data, as in Jimenez et al. (forthcoming). When based on macroeconomic time series, however, the results have been ambiguous. For example, Angeloni et al. (2011) use a small-scale vector autoregression (VAR) model and find no significant evidence of risk taking of the U.S. banking sector on the asset side. Using a comprehensive data set of 140 time series, Buch et al. (2005) find evidence in favor of this channel, yet only for small U.S. banks.

The use of macroeconomic data in this context is complicated by a relatively short sample of time series observations due to the limited availability of adequate measures of asset-side risk taking. As a consequence, econometric models are easily prone to overfitting, i.e. an excessive number of parameters. To overcome the curse of dimensionality, we employ a factor-augmented vector autoregression (FAVAR) model (compare Bernanke et al., 2005) that allows us to parsimoniously extract information from a large set of macroeconomic time series, thereby mitigating the concern of overfitting. While the FAVAR specification is sufficiently parsimonious to be estimated even on a relatively short sample, it is less likely subject to “omitted-variable bias” than a small-scale VAR model. This is crucial, given that omitted-variable bias might invalidate the coefficient estimates and thus also the impulse response functions to a monetary policy shock (see, e.g., Bernanke et al., 2005). As a measure of the risk attitude of U.S. banks, we use the quantified qualitative measures from the Federal Reserve’s Senior Loan Officer Opinion Survey, which reflect changes in bank lending standards at quarterly frequency from 1991Q1 to 2012Q4. In contrast to the prior literature, we employ 19 different measures of bank lending standards, such as the net percentage of banks *increasing collateral requirements, tightening loan covenants*, etc. for various categories of loans and banks. Accordingly, we are able to extract a “lending standards factor” that explains the comovement in all underlying measures rather than relying on just one particular measure.

Our baseline specification contains three common factors, the Federal Funds rate as the only observable variable, and a lag order of two in quarterly data. We use the one-step Bayesian estimation approach by Gibbs sampling in Bernanke et al. (2005) and Amir Ahmadi and Uhlig (2009) and find that, together with the observable variable, a small number of common factors is sufficient to explain a substantial share of the variation in lending standards, ranging from 47% to 98% in terms of the *adjusted R*². Following Bernanke et al. (2005), monetary policy shocks are identified recursively, ordering the Federal Funds rate last in the FAVAR model. Given the extraordinary events at the beginning of our full sample, i.e. the Savings and Loans crisis and the early 1990s recession in the U.S., our baseline sample

period starts only in 1997Q1.¹ Moreover, our sample period ends in 2008Q4, given that the Federal Funds rate appears to be an inadequate measure of U.S. monetary policy after the bankruptcy of Lehman Brothers.

We find that all 19 measures of lending standards are eased in response to an expansionary monetary policy shock. The corresponding impulse response functions are both statistically and economically significant, suggesting a non-trivial role for monetary policy in asset-side risk taking of U.S. banks. Our findings are qualitatively robust to a change in the lag order, the number of unobservable factors, the number of observable variables, and the start of the sample period.²

Based on this empirical evidence, we develop a dynamic general equilibrium (DGE) model with a nontrivial role for financial intermediation through the banking sector. Our aim is to analyze the conditions which induce banks to endogenously raise their risk taking on the asset side in response to an expansionary monetary shock, i.e. an unexpected decrease in the policy interest rate that cannot be explained by economic conditions. The model builds on the quantitative analysis of the well-known financial accelerator in Bernanke et al. (1999) – hereafter BGG – and recent work on bank risk taking by Dell’Ariccia et al. (2010), Malherbe (2011), and Agur and Demertzis (2012). Our main theoretical contribution is that we allow for a risk-sharing contract between borrowers and lenders in financial capital markets, rather than assigning all production risk to lenders, as in Malherbe (2011), or to borrowers, as in Bernanke et al. (1999).

As in BGG, a continuum of competitive entrepreneurs is endowed with initial net worth that is, however, insufficient to fully finance production without seeking external finance. Entrepreneurs are subject to an idiosyncratic capital quality shock that affects the marginal return on their capital stock and determines whether the entrepreneur is able to repay the bank loan after production. Due to a costly state verification (CSV) problem in the spirit of Townsend (1979) between borrowers and lenders, the cost of external finance, i.e. is bank loans, to entrepreneurs depends on their own net worth. In contrast to BGG, however, the optimal debt contract cannot be made contingent on the aggregate state of the economy. As a consequence, any aggregate risk in the economy is borne by the risk-neutral banking sector, whereas the idiosyncratic, entrepreneur-specific risk can be diversified away.

As in Malherbe (2011) and Agur and Demertzis (2012), the continuum of risk-neutral competitive banks enters each period with accumulated net worth or bank equity. In addition, banks issue deposits to households and make loans to the entrepreneurial sector. Following the literature, we assume full deposit insurance. This simplifying institutional assumption allows us to abstract from any frictions between depositors and banks, because bank deposits always pay the riskless interest rate, and generates a moral hazard problem on the banks’ side. We show that, in the presence of

¹ As a robustness check, we shift the start of the sample backwards to 1994Q1 and 1991Q1, respectively. While our main results are qualitatively unchanged, the Gibbs sampler has difficulties to converge for the longer samples.

² We also verify that our results are not sensitive to the estimation approach and remain intact for the two-step estimation approach in Bernanke et al. (2005) based on principal components analysis.

limited liability, banks will generally lend more than is socially optimal, because they can default on their debt in case of an unfavorable aggregate shock (compare Malherbe, 2011). Without the assumption of limited liability, the optimal contract is similar to the one in BGG, except that the banking sector bears any aggregate risk.

How likely a bank is to default depends, among other things, on the riskless policy interest rate. An expansionary monetary policy shock implies lower funding costs for the banking sector and thus induces banks to expand their lending to entrepreneurs for a given stock of entrepreneurial net worth. In other words, in response to an expansionary monetary policy shock, banks lower their lending standards, as measured by the ratio of loans over borrower collateral. This theoretical finding replicates our empirical evidence and shows that it can be optimal for the banking sector as a whole to increase risk taking on the asset side in response to an expansionary monetary policy shock.

Our theoretical contribution is closely related to two recent papers. Piffer (2013) uses the New Keynesian DGE model of BGG to show that a monetary easing lowers the cost of external funding to entrepreneurs or “firms”. While this reduces the default probability on outstanding loans, it induces firms to borrow more and to leverage up. As a consequence, the monetary expansion is followed by a hump-shaped response of firm leverage and default rates. As in BGG, there is no active role for financial intermediaries in Piffer (2013). The state-contingent nature of the debt contract implies that risk-averse lenders always earn the policy rate on their deposits, while any risk is borne by the firm. More similar to our approach, Valencia (2011) assumes a CSV problem between depositors and financial intermediaries as well as between intermediaries and borrowers. Monopolistic banks collect deposits and make a “take-it-or-leave-it” loan offer to atomistic entrepreneurs. Limited liability allows entrepreneurs and banks to default on their loans and deposits, respectively. All agents are risk-neutral and the opportunity cost of savers and borrowers equals the risk-free policy rate. The latter assumption implies that a monetary easing lowers banks’ funding costs *and* raises their monopolistic rents, inducing them to increase their leverage and lending, and thus the risk of default.

The remainder of the paper is organized as follows. Section 2 outlines our econometric approach and presents the empirical evidence of an asset-side risk-taking channel of monetary policy for U.S. banks. Section 3 derives the optimal debt contract under CSV and incorporates it into a quantitative New Keynesian DSGE model. Section 4 concludes.

2. Empirical Evidence of Aggregate Bank Risk Taking

The empirical relevance of a risk-taking channel of monetary policy on the asset side has been shown mostly based on microeconomic banking-level data (see, e.g., Jimenez et al. forthcoming). When macroeconomic time series were used, however, the results have been less clear cut. Angeloni et al. (2011) set up a small-scale vector autoregression (VAR) model, including one of the measures from the Senior Loan Officer Opinion Survey of the U.S. Federal Reserve as their proxy for asset side risk-taking among the endogenous variables, and find no significant evidence

of aggregate risk taking of the U.S. banking sector on the asset side³. Using a rich data set of 140 time series and a factor-augmented vector autoregression (FAVAR) model, Buch et al. (2013) find evidence in favor of a risk-taking channel on the asset side only for small U.S. banks. Notably, Buch et al. (2013) use a different measure of asset risk – the riskiness of new loans provided in the Survey of Terms of Business Lending of the U.S. Federal Reserve, which restricts their sample to 1997Q2-2008Q2.

We take a similar approach, using the quantified qualitative measures capturing changes in banks' lending standards from the Federal Reserve's Senior Loan Officer Opinion Survey, which are available from 1991Q1. As in Buch et al. (2013), we employ a FAVAR model, which allows us to parsimoniously extract information from a large number of macroeconomic time series.⁴ Employing a FAVAR rather than a small-scale VAR model reduces the risk of omitted-variable bias that might contaminate the identification of unexpected monetary policy shocks (see also Bernanke et al., 2005). In order to be able to detect a possible risk-taking channel of monetary policy, we augment a large macroeconomic data set that is commonly used in the FAVAR literature by a comprehensive set of 19 lending standards measures, including the net percentage of banks *increasing collateral requirements, tightening loan covenants*, etc. for various types of loans and banks (see Appendix A1). This approach allows us to extract a common factor of all available measures rather than relying on one particular measure. Figure 1 illustrates that there is a lot of comovement among the different lending standard measures.

2.1. Econometric Specification

Suppose that the observation equation relating the $N \times 1$ vector of informational time series X_t to the $K \times 1$ vector of unobservable factors F_t and the $M \times 1$ vector of observable variables Y_t , where $K + M \ll N$, is

$$X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t, \quad (1)$$

where Λ^f is an $N \times K$ matrix of factor loadings of the unobservable factors, Λ^y is an $N \times M$ matrix of factor loadings of the observable variables, and e_t is an $N \times 1$ vector of error terms following a multivariate normal distribution with mean zero and covariance matrix R .

Suppose further that the joint dynamics of the unobserved factors in F_t and the observable variables in Y_t can be captured by the following transition law:

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + v_t, \quad (2)$$

³In particular, Angeloni et al. (2011) use the percentage of banks tightening credit standards on C&I loans to large and medium-sized firms.

⁴A detailed description of the data can be found in the Appendix A1.

where $\Phi(L)$ is a lag polynomial of order d and v_t is a $(K + M) \times 1$ vector of error terms following a multivariate normal distribution with mean zero and covariance matrix Q . Note that the error terms in e_t and v_t are assumed to be uncorrelated.

Estimation of the FAVAR model in (1) and (2) requires that the data are transformed to induce stationarity of all variables.⁵ Our baseline sample contains quarterly observations for 1997Q1 through 2008Q4. Our motivation for starting only in 1997 is to exclude the period of the Savings and Loans crisis and the severe recession of the early 1990s. Given the relatively short-sample period and the linear specification of the FAVAR model in (1) and (2), including this period of likely non-linearity might pose problems for our estimation procedure. However, we conduct a sensitivity analysis based on an alternative sample starting in 1991Q1 and 1994Q1, respectively. Similarly, we exclude the period after 2008, given that U.S. monetary policy was effectively operating through the balance sheet of the Federal Reserve rather than through the Federal Funds rate, after the bankruptcy of Lehman Brothers. Note that the predominance of these unconventional measures would require a different strategy for identifying monetary policy shocks, as in Peersman (2011). Following Bernanke et al. (2005), the monetary policy shock is identified recursively, ordering the Federal Funds rate last in the state equation in (2).

Building on Bernanke et al. (2005) and Amir Ahmadi and Uhlig (2009), we estimate the FAVAR model in (1) and (2) by a one-step Bayesian approach.⁶ Since unobserved factors can only be estimated up to a rotation, due to the fundamental indeterminacy of factor models, we need to impose a set of restrictions on the observation equation. Following Bernanke et al. (2005), we impose standard restrictions in order to obtain a unique identification of the factors. The goal of this identification is to eliminate factors of the form: $F_t^* = AF_t + BY_t$. Solving this expression for F_t and plugging the result into the observation equation (2) yields

$$X_t = \Lambda^f A^{-1} F_t^* + (\Lambda^y + \Lambda^f A^{-1} B) Y_t. \quad (3)$$

Unique factor identification requires $A^{-1} F_t^* = F_t$ and $\Lambda^f A^{-1} B = \mathbf{0}$. Bernanke et al. (2005) suggest to impose sufficient (overidentifying) conditions by setting $A = I$ and $B = 0$.⁷

We apply multi-move Gibbs sampling in order to jointly sample the unobserved factors and the model parameters. Appendix A2 provides details on the prior distributions, the sampler, and how we monitor convergence of the Gibbs sampler. In the baseline model, we set the lag order of the state equation in (2) to two quarters and treat

⁵The transformation of each variable is detailed in Appendix A1. Note that the measures of lending standards enter the FAVAR in (standardized) levels, i.e. without being transformed, since they are stationary by construction.

⁶Alternatively, the FAVAR model could also be estimated using a two-step approach based on principal component analysis, as in the baseline of Bernanke et al. (2005). However, this method turns out to be more prone to overfitting, given our short sample, especially at higher lag orders. Using a lag order of one quarter, the results based on the two-step approach are very similar to the results based on the one-step Bayesian approach.

⁷Note that there are many other admissible identifying restrictions satisfying $A^{-1} F_t^* = F_t$ and $\Lambda^f A^{-1} B = \mathbf{0}$.

the Federal Funds rate as the only observable variable, i.e. $M = 1$.⁸ Given that our main interest is in explaining the fluctuations of lending standards, we determine the number of unobserved factors K in the model by the share of the total variance of these measures explained by F_t and the Y_t , i.e. the Federal Funds rate. Table 1 reports the adjusted R^2 for each of the 19 measures of lending standards based on one, two and three unobserved factors, respectively. It turns out that the first extracted factor exhibits a very high positive correlation with most of the lending standards measures, ranging from 47% to 98%, and can therefore be thought of as a “lending standards factor”, that captures a significant share of the common variability in the lending standards. Adding further unobserved factors improves the fit of the lending standards measures only marginally. As a consequence, in what follows we use the model with three unobserved factors as our baseline model.

2.2. Empirical Results

Figure 2 plots the impulse response functions of the Federal Funds rate and the 19 lending standards to an expansionary monetary policy shocks, i.e. an unexpected decrease in the Federal Funds rate by 25bps, based on the baseline model with $K = 3$. All impulse response functions are plotted in terms of standard deviations. The bands around the median response correspond to the 5-th and 95- th percentile, respectively, pointwise containing 90% of probability.⁹ We find that all measures of lending standards are loosened in response to an expansionary monetary policy shock. Moreover, the magnitude of this effect is both statistically and economically significant. The response of lending standards is gradual, reaching its peak after eight or nine quarters before returning to the steady state.

Figures 3 and 4 illustrate that these findings are fully robust to using a FAVAR specifications with only one or two unobserved factors. Moreover, we challenged our findings by extending the sample period backwards to 1994Q1 and 1991Q1, respectively, which had been excluded before due to the possible difficulty to capture possible non-linear dynamics of lending standards during the Savings and Loans crisis and the recession of the early 1990s. While the risk-taking effect is still present, it becomes increasingly harder to achieve a satisfactory convergence of the Gibbs sampler for our benchmark specification with three unobservable factors.¹⁰

To sum up, our findings provide strong empirical evidence for the existence of a risk-taking channel of monetary policy on the asset side of banks’ balance sheet. To the best of our knowledge, we are the first to establish the empirical relevance of this channel at the macro level for large domestic and foreign banks in the U.S.

⁸The results for lag orders of one and three quarters are very similar. Including CPI as an additional observable ($M = 2$) does not change the results substantially either.

⁹The responses are obtained based on a chain of effective length of 200000 iterations with burn-in of 50000 iterations.

¹⁰Qualitatively, the impulse response functions for both alternative samples are very similar to those for the sample starting in 1997Q1, if we include one and two factors, respectively. From three factors onwards, however, the speed of convergence is significantly reduced. For the sample starting in 1991Q1 and $K \geq 3$, for example, the Gibbs sample has trouble converging even for very long sampling chains, resulting in a widening of error bands. For example, we tried both single chains of 5 million iterations each (with thinning) from various starting points as well as 5 consecutive chains of 1 million draws each (also with thinning).

3. Theoretical Model of Bank Risk Taking

In the remainder of this paper, we develop a theoretical model that is capable of replicating the response of banks to a monetary easing identified in the previous empirical analysis. In particular, we want to show that it can be optimal for banks to increase the amount of lending per unit of borrower collateral in response to an expansionary monetary policy shock.

For this purpose, we draw on the CSV contract proposed by Townsend (1979) and first incorporated in a New Keynesian DSGE model by Bernanke et al. (1999). In contrast to the latter and most recent contributions (see, e.g., Piffer, 2013), however, we reformulate the optimal debt contract from the lender’s perspective. Recall that, in Bernanke et al. (1999), there is no active role for the so-called “financial intermediary”. The latter merely facilitates a perfect diversification of the borrowers’ idiosyncratic productivity risk and institutionalizes the participation constraint of the risk-averse depositors, along which the borrower moves when making its optimal capital decision.¹¹

Instead, we assume that the lender, i.e. the bank, chooses the volume of the loan, given the collateral of the borrower, i.e. the entrepreneur. Since the market power is in the hands of the banks, this corresponds to a “take-it-or-leave-it” contract, similar to the one in Valencia (2011). In order for a borrower to accept the bank’s contract offer, it must be at least as well off as without the loan. The details of the contract with and without aggregate risk are specified in the following.

3.1. The Optimal Loan Contract

Suppose that there is a continuum of entrepreneurs indexed i that, at time t , purchases capital $Q_t K_{t+1}^i$ for use at $t+1$, where K_{t+1}^i is the quantity of capital purchased and Q_t is the price of capital in period t . The gross return to capital of entrepreneur i , $\omega^i R_{t+1}^k$, depends on an idiosyncratic component ω^i and on the ex-post aggregate return to capital R_{t+1}^k . Following Bernanke et al. (1999), we assume that the random variable $\omega^i \in [0, \infty)$ is i.i.d. across entrepreneurs and across time, with a continuous and differentiable cumulative distribution function (*cdf*) $F(\omega)$ and an expected value of unity.

Entrepreneur i finances any capital purchases $Q_t K_{t+1}^i$ at the end of period t using accumulated net worth N_{t+1}^i as well as the borrowed amount B_{t+1}^i , such that

$$Q_t K_{t+1}^i = N_{t+1}^i + B_{t+1}^i. \quad (4)$$

As in Valencia (2011), entrepreneur i borrows the amount B_{t+1}^i from a monopolistic bank that is endowed with net worth or bank capital N_{t+1}^b at the end of period t and collects the remainder from households in the form of deposits

¹¹ Given the lenders’ passiveness, Brunnermeier et al. (2013) do *not* categorize Bernanke et al. (1999) as a model of financial intermediation.

D_{t+1} . Defining $B_{t+1} \equiv \int_0^1 B_{t+1}^i di$, the bank's *aggregate* balance sheet identity is

$$B_{t+1} \equiv N_{t+1}^b + D_{t+1}. \quad (5)$$

Following Bernanke et al. (1999), we motivate the importance of borrower collateral by the presence of a cost that the lender must pay in order to know the borrower's realization of ω^i , as in the CSV problem proposed by Townsend (1979). This cost can be interpreted as the cost of bankruptcy, such as accounting and legal costs or losses from asset liquidation (compare Bernanke et al., 1999). We follow the literature and assume that these costs are a fixed proportion μ of the entrepreneur's gross return on capital, $\omega^i R_{t+1}^k Q_t K_{t+1}^i$. Both the borrower and the lender are assumed to be risk-neutral and to care about expected returns only, whereas depositors are risk-averse. Hence, the bank promises to pay the risk-free gross rate of return R_{t+1} on deposits in each aggregate state.

Denote the gross non-default rate of return on the loan to entrepreneur i by Z_{t+1}^i . Given R_{t+1}^k , $Q_t K_{t+1}^i$, and N_{t+1}^i , the optimal contract specifies a relationship between Z_{t+1}^i and a cutoff value

$$\bar{\omega}^i \equiv \frac{Z_{t+1}^i B_{t+1}^i}{R_{t+1}^k Q_t K_{t+1}^i}, \quad (6)$$

such that the borrower pays the lender the fixed amount $\bar{\omega}^i R_{t+1}^k Q_t K_{t+1}^i$ and keeps the residual $(\omega^i - \bar{\omega}^i) R_{t+1}^k Q_t K_{t+1}^i$, if $\omega^i \geq \bar{\omega}^i$, while the lender monitors the borrower and extracts $(1 - \mu) \omega^i R_{t+1}^k Q_t K_{t+1}^i$ (net of default costs), if $\omega^i < \bar{\omega}^i$. In the latter case, the borrower defaults and receives nothing.

In contrast to Bernanke et al. (1999), we assume that the bank chooses the amount of lending to entrepreneur i , given the borrower's collateral N_{t+1}^i and subject to the participation constraint. The borrower will only accept the bank's offer if the corresponding expected return is at least as large as in "financial autarky", i.e. without the bank loan:

$$\int_{\bar{\omega}^i}^{\infty} (\omega - \bar{\omega}^i) R_{t+1}^k Q_t K_{t+1}^i f(\omega) d\omega \geq \int_0^{\infty} \omega R_{t+1}^k N_{t+1}^i f(\omega) d\omega = R_{t+1}^k N_{t+1}^i, \quad (7)$$

where the last equality uses $\int_0^{\infty} \omega f(\omega) d\omega = E(\omega) = 1$. Hence, the bank must promise the borrower an expected return no smaller than the expected return from investing just the net worth N_{t+1}^i .¹²

The expected gross return of the bank on a loan to entrepreneur i can be written as

$$\left\{ \bar{\omega}^i [1 - F(\bar{\omega}^i)] + (1 - \mu) \int_0^{\bar{\omega}^i} \omega f(\omega) d\omega \right\} R_{t+1}^k Q_t K_{t+1}^i.$$

¹²Note that our borrowers' participation constraint is thus different from that in Valencia (2011), where the opportunity cost of entrepreneurs is the risk-free rate R_{t+1} , implying that investment projects have a minimum size $> N_{t+1}^i$, whereas all entrepreneurs have unlimited access to the risk-free alternative opportunity.

Given that the bank pays the risk-free rate of return R_{t+1} on deposits, while we assume that no costs accrue on the bank's own net worth N_{t+1}^b , the bank's aggregate costs of funding are

$$R_{t+1}D_{t+1} = R_{t+1}(B_{t+1} - N_{t+1}^b) = R_{t+1}(Q_t K_{t+1} - N_{t+1} - N_{t+1}^b).$$

Suppose that the bank assigns a share $N_{t+1}^{b,i}$ of its total net worth N_{t+1}^b to the loan to entrepreneur i .¹³ Then, the bank's constrained profit maximization problem for the i th loan is given by

$$\begin{aligned} \max_{K_{t+1}^i, \bar{\omega}^i} \quad & \left\{ \bar{\omega}^i [1 - F(\bar{\omega}^i)] + (1 - \mu) \int_0^{\bar{\omega}^i} \omega^i f(\omega) d\omega \right\} R_{t+1}^k Q_t K_{t+1}^i - R_{t+1} (Q_t K_{t+1}^i - N_{t+1}^i - N_{t+1}^{b,i}), \\ \text{s.t.} \quad & \int_{\bar{\omega}^i}^{\infty} (\omega - \bar{\omega}^i) R_{t+1}^k Q_t K_{t+1}^i f(\omega) d\omega \geq R_{t+1}^k N_{t+1}^i. \end{aligned}$$

3.1.1. The Contract without Aggregate Risk

As a starting point, consider the case when there is no aggregate uncertainty and the aggregate return to capital R_{t+1}^k is known in advance. As a consequence, the only uncertainty in the loan contract between the bank and entrepreneur i arise from the idiosyncratic realization of ω^i .

The contract without aggregate uncertainty resembles standard risky debt (compare Bernanke et al., 1999). Entrepreneur i approaches the bank for a loan and brings its net worth to the counter. Given N_{t+1}^i , the bank decides how much to lend to entrepreneur i and thus also the size of the borrower's capital expenditure $Q_t K_{t+1}^i = N_{t+1}^i + B_{t+1}^i$.

Following Bernanke et al. (1999), define the expected share of total profits (before default costs) of the lender as

$$\Gamma(\bar{\omega}^i) \equiv \bar{\omega}^i [1 - F(\bar{\omega}^i)] + \int_0^{\bar{\omega}^i} \omega^i f(\omega) d\omega,$$

where $0 < \Gamma(\bar{\omega}^i) < 1$ by definition, and the expected monitoring costs of the lender as

$$\mu G(\bar{\omega}^i) \equiv \mu \int_0^{\bar{\omega}^i} \omega^i f(\omega) d\omega,$$

and note that

$$\Gamma'(\bar{\omega}^i) = 1 - F(\bar{\omega}^i) > 0, \quad \Gamma''(\bar{\omega}^i) = -f(\bar{\omega}^i) < 0, \quad \mu G'(\bar{\omega}^i) \equiv \mu \bar{\omega}^i f(\bar{\omega}^i) > 0.$$

Then the expected share of total profits net of default costs going to the lender and the expected share going to the

¹³So far, we consider only cases where aggregate shocks are small enough so that the bank never defaults. As a consequence, the assignment of bank capital to a particular loan i is without loss of generality and mainly for notational convenience.

entrepreneur is $\Gamma(\bar{\omega}^i) - \mu G(\bar{\omega}^i)$ and $1 - \Gamma(\bar{\omega}^i)$, respectively.

Using the above definitions and suppressing time subscripts and index superscripts for notational convenience, the bank's constrained profit maximization problem can be written as

$$\max_{k, \bar{\omega}} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] R^k QK - R(QK - N - N^b) \quad \text{s.t.} \quad [1 - \Gamma(\bar{\omega})] R^k QK = R^k N. \quad (8)$$

Furthermore defining $s \equiv R^k/R$, $k \equiv QK/N$, and $n \equiv N^b/N$, we obtain

$$\max_{k, \bar{\omega}} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] sk - (k - 1 - n) \quad \text{s.t.} \quad [1 - \Gamma(\bar{\omega})] sk = s.$$

The corresponding first-order conditions with respect to k , $\bar{\omega}$, and λ are

$$\begin{aligned} k : \quad & [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] s - 1 + \lambda [1 - \Gamma(\bar{\omega})] s = 0, \\ \bar{\omega} : \quad & [\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})] sk - \lambda \Gamma'(\bar{\omega}) sk = 0, \\ \lambda : \quad & [1 - \Gamma(\bar{\omega})] sk - s = 0. \end{aligned}$$

Appendix B.1 shows that the optimal contract implies a relation $k = \psi(s)$, where $\psi'(s) > 0$, between the *external finance premium* (EFP) $s \equiv R^k/R$ and the optimal capital/net worth ratio $k \equiv QK/N$. Although the bank rather than the entrepreneur chooses the amount of lending given collateral, the same qualitative relationship as in Bernanke et al. (1999) arises from the optimal contract without aggregate risk. As a result, an increase in the EFP, e.g. due to an exogenous reduction in the risk-free interest rate R , induces the bank to lend more to entrepreneurs for a given amount of net worth.

Figure 5 illustrates the mechanism driving this result. First note that the lender's iso-profit curves (IPC) and the borrower's participation constraint (PC) can be plotted in $(k, \bar{\omega})$ -space and that the constrained profit maximum of the lender is determined by the tangential point between the IPC and the PC.¹⁴ The corresponding expressions for the lender's IPC and the borrower's PC, respectively, are

$$k_{IPC} = \frac{\pi^b - 1 - n}{[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] s - 1}, \quad (9)$$

$$k_{PC} \geq \frac{1}{1 - \Gamma(\bar{\omega})}, \quad (10)$$

where π^b denotes an arbitrary level of bank profits.

From equation (10), it is clear that the borrower's PC is not affected by the EFP s . In the absence of aggregate risk,

¹⁴Appendix B.1 proves that the optimal contract yields a *unique interior* solution.

the borrower's expected share of total profits, $1 - \Gamma(\bar{\omega})$, must be at least as large as the borrower's "skin in the game", $1/k \equiv N/QK$. For any given value of $\bar{\omega}$ and thus an expected distribution of total profits, the borrower's PC determines a minimum for k and thus for the bank's "skin in the game", below which the entrepreneur would not accept the loan contract.

Note that the bank's IPC accounts for expected default costs as well as for funding costs. The bank maximizes expected profits by choosing the tangential point between the borrower's PC and its lowest IPC in $(k, \bar{\omega})$ -space. I.e., the bank tries to minimize its "skin in the game" for an expected share of total profits $\Gamma(\bar{\omega})$.

The first panel illustrates the tangential point between the borrower's PC and the lender's IPC for the calibration in Bernanke et al. (1999).¹⁵ Note that for $QK = N$, the borrower is fully self-financed, will never default ($\bar{\omega} = 0$), and retains any profits ($\Gamma(0) = 0$).

Now consider the effects of expansionary monetary policy when R^k is known in advance, i.e. a decrease in R and thus an increase in $s \equiv R^k/R$. While the borrower's PC remains unaffected, the lender's IPC are tilted upwards, as illustrated by the second panel of Figure 5. Although the borrower would accept any point above its PC on the new IPC, this would no longer be optimal from the lender's perspective.

Instead, the bank can move to a lower IPC with a higher gross profit share, as indicated in the third panel. In doing so, however, it must satisfy the borrower's PC, as in the new optimal contract $(k_{new}^*, \omega_{new}^*)$, where both the bank's expected share of total profits and its "skin in the game" have increased.

The previous discussion illustrates a convenient feature of our formulation of the optimal loan contract in Bernanke et al. (1999). The bank responds to an increase in the EFP, e.g. due to a monetary expansion, by expanding the amount of a loan for a given amount of borrower net worth. While this result is based on comparative statics in the model without aggregate risk, it squares nicely with our empirical finding that U.S. banks *lower their collateral requirements* in response to a decrease in the Federal Funds rate.

3.1.2. The Contract with Aggregate Risk

When there is aggregate uncertainty, the aggregate return to capital R_{t+1}^k is determined ex post. As a consequence, the default threshold $\bar{\omega}^i$ characterizing a loan contract between the bank and entrepreneur i would generally be a function of R_{t+1}^k . We circumvent the complications arising from this state contingency by making a simplifying assumption about the risk-sharing agreement between the borrower and the lender. In a modification of Bernanke et al. (1999) we assume that the borrower's PC must be satisfied *ex post* and that the bank absorbs any aggregate risk. Note that this assumption is only viable, if the bank has a capital buffer N_{t+1}^b . Moreover, we assume that the fluctuations in R_{t+1}^k are small enough to be absorbed without the bank defaulting. The depositors are thus shielded from any uncertainty about

¹⁵While this is hard to see, the tangential point is indeed interior and unique.

the rate of return on their savings.¹⁶

Since the borrower's capital expenditure $Q_t K_{t+1}^i$ and net worth N_{t+1}^i are predetermined in period $t + 1$, equation (10) implies that the ex-post share of total profits, $1 - \Gamma(\bar{\omega})$, and the corresponding default threshold, $\bar{\omega}^i$, can *not* be made contingent on the aggregate state of the economy. From the definition of cutoff in (6), however, this implies that the non-default rate of return Z_{t+1}^i must vary to absorb any changes in R_{t+1}^k .

In contrast to Bernanke et al. (1999), where both $\bar{\omega}^i$ and Z_{t+1}^i are *countercyclical* (e.g., a higher than expected realization of R_{t+1}^k lowers the default threshold and thus also the non-default rate of return required by the lender), here $\bar{\omega}^i$ is *acyclical*, while Z_{t+1}^i is *procyclical*. For example, a higher than expected realization of R_{t+1}^k raises Z_{t+1}^i , whereas the borrower's and the lender's expected profit shares are still determined by their "skin in the game", i.e. by the relative shares of N_{t+1}^i and B_{t+1}^i in $Q_t K_{t+1}^i$.¹⁷ In other words, we assume that borrowers default due to idiosyncratic risk, only, whereas they do not default due to aggregate risk.

Appendix B.2 shows that the optimal financial contract between the bank and entrepreneur i implies a positive relation between the capital/net worth ratio $Q_t K_{t+1}^i / N_{t+1}^i$ and the expected EFP $s_t \equiv E_t(R_{t+1}^k / R_{t+1})$, i.e.

$$Q_t K_{t+1}^i = \psi(s_t) N_{t+1}^i, \quad \psi'(s_t) > 0. \quad (11)$$

3.2. The Dynamic General Equilibrium Model

TBD

4. Concluding Remarks

TBD

¹⁶Recall that both the lender and the borrower are assumed to be risk-neutral, whereas depositors are risk-averse. Hence, a different risk-sharing agreement in the loan contract would be equally conceivable, while the absence of frictions between the bank and its depositors is for simplicity.

¹⁷Note that the aggregate uncertainty discussed in this section is orthogonal to the effect of a monetary expansion illustrated in Figure 5, which works through a change in the capital/net worth ratio.

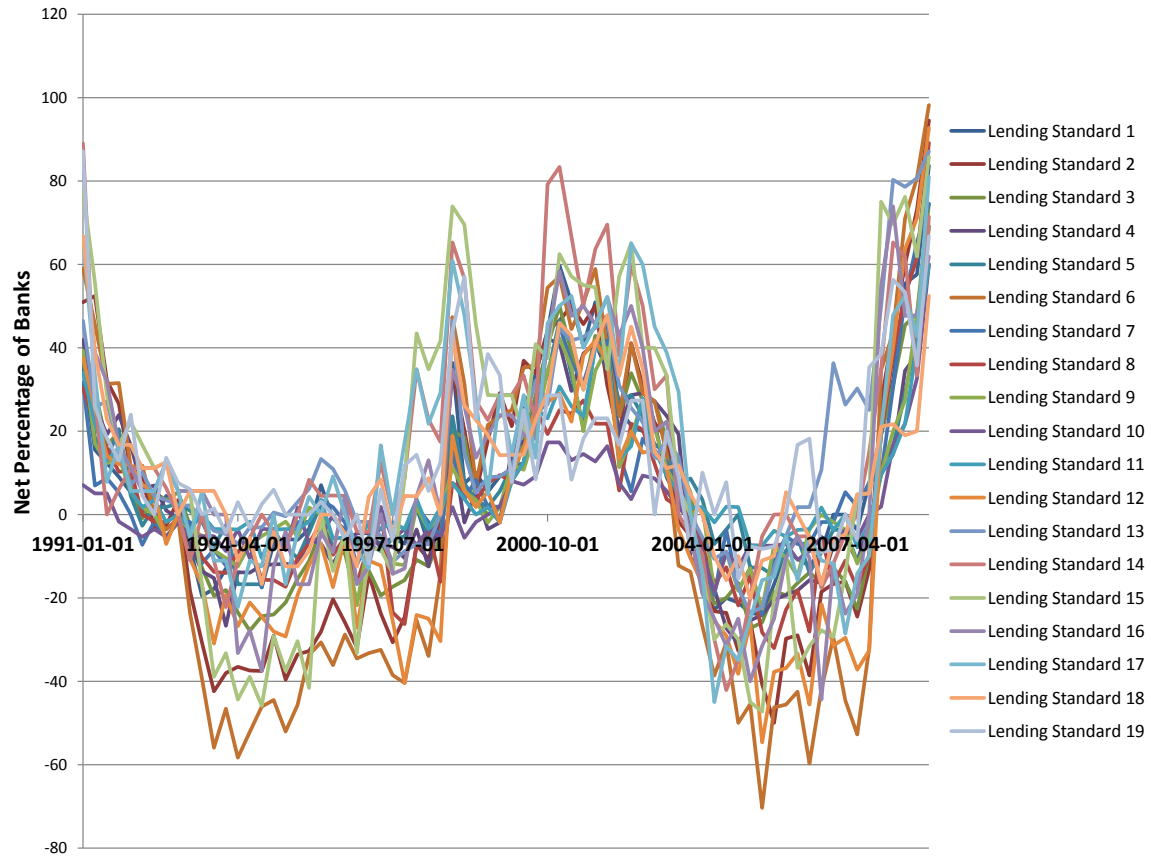
Tables

Table 1: Adjusted R^2 for the Measures of Lending Standards, sample 1997Q1 - 2008Q4

No.	Lending Standard Description	1 factor	2 factors	3 factors
1	domestic banks tightening standards on C&I loans to large and middle firms	0,95	0,95	0,95
2	domestic banks increasing the costs of credit lines to large and middle firms	0,98	0,97	0,97
3	domestic banks tightening loan covenants for large and middle firms	0,92	0,92	0,92
4	domestic banks reducing the maximum size of credit lines to large and middle firms	0,93	0,93	0,93
5	domestic banks increasing collateral requirements for large and middle firms	0,93	0,94	0,93
6	domestic banks increasing spreads of loan rates over banks' cost of funds to large and middle firms	0,89	0,90	0,90
7	domestic banks tightening standards for C&I loans to small firms	0,88	0,88	0,88
8	domestic banks increasing the cost of credit lines to small firms	0,93	0,92	0,92
9	domestic banks tightening loan covenants for small firms	0,84	0,84	0,85
10	domestic banks reducing the maximum size of credit lines to small firms	0,87	0,87	0,87
11	domestic banks increasing collateral requirements for small firms	0,93	0,93	0,93
12	domestic banks increasing spreads of loan rates over banks' cost of funds to small firms	0,73	0,86	0,87
13	domestic banks tightening standards for commercial real estate loans	0,80	0,80	0,80
14	foreign banks tightening standards for approving C&I loans	0,75	0,74	0,74
15	foreign banks increasing costs of credit lines	0,84	0,83	0,83
16	foreign banks tightening loan covenants	0,73	0,74	0,74
17	foreign banks reducing the maximum size of credit lines	0,77	0,80	0,79
18	foreign banks increasing collateral requirements	0,48	0,49	0,49
19	foreign banks tightening standards for commercial real estate loans	0,47	0,48	0,47

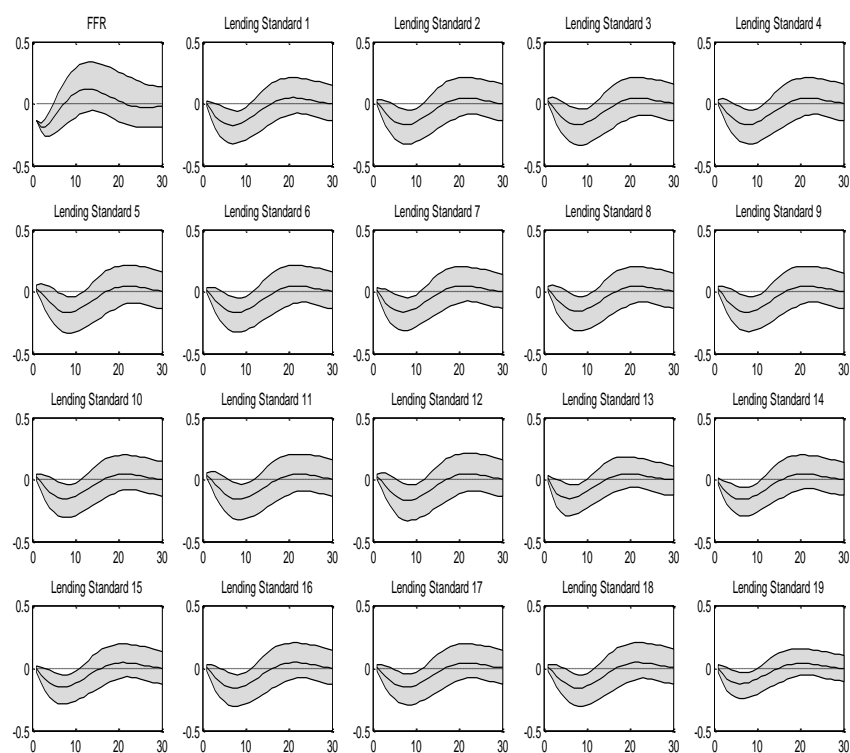
Figures

Figure 1: Lending Standard Measures, 1991Q1 - 2008Q4



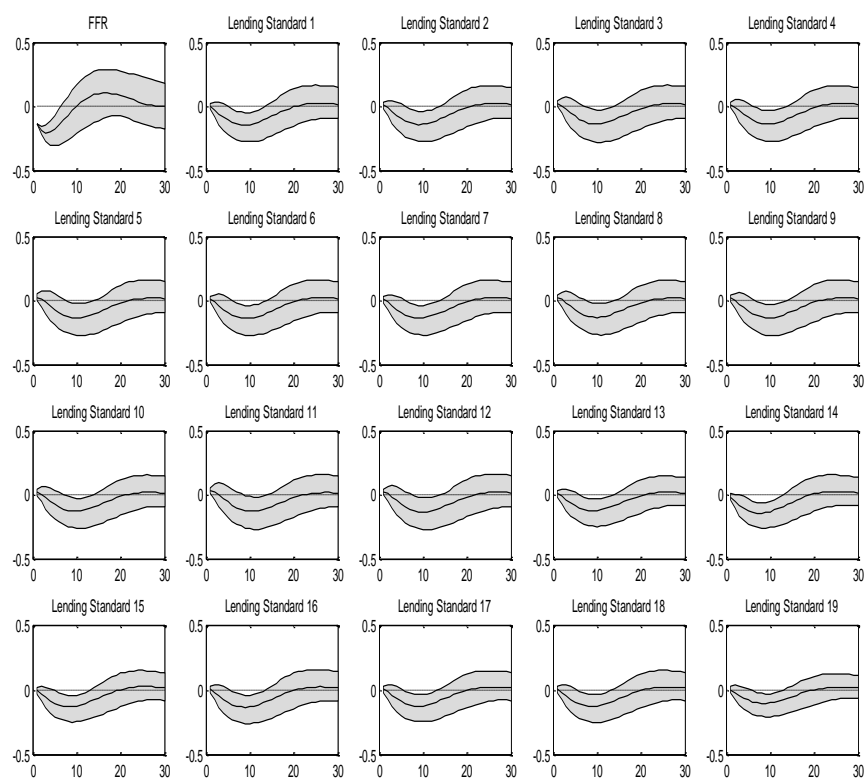
Notes: see Appendix A1 for a detailed description of lending standard variables.

Figure 2: Impulse Responses Lending Standard Measures to a 25bps Expansionary Monetary Policy Shock in the Model with Three Unobserved Factors.



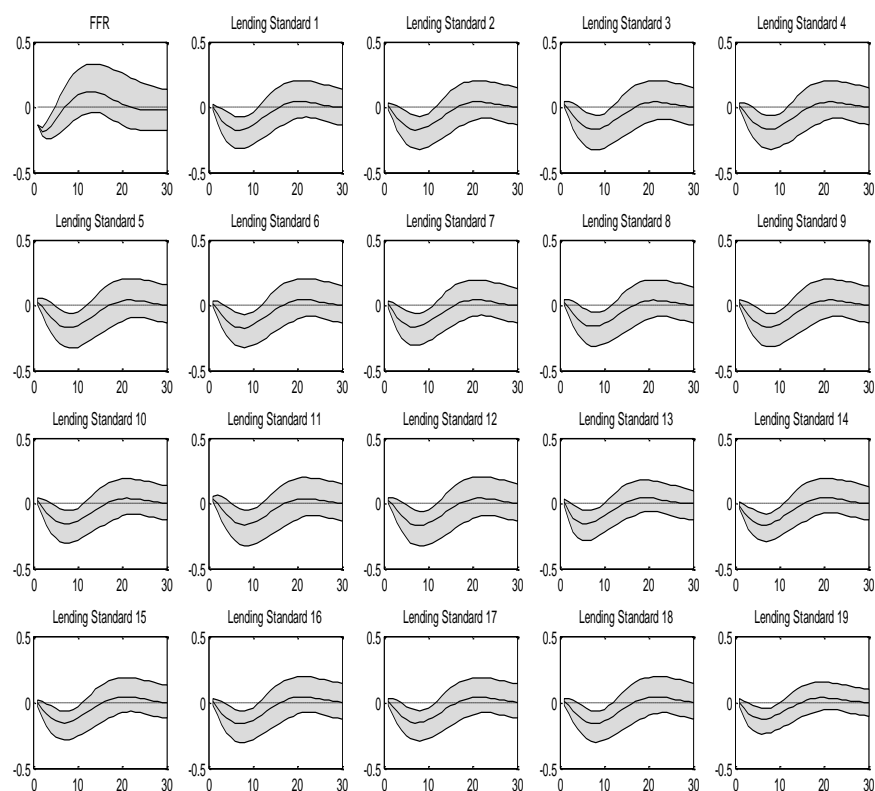
Notes: see Appendix A1 for a detailed description of lending standard variables.

Figure 3: Impulse Responses Lending Standard Measures to a 25bps Expansionary Monetary Policy Shock in the Model with One Unobserved Factor.



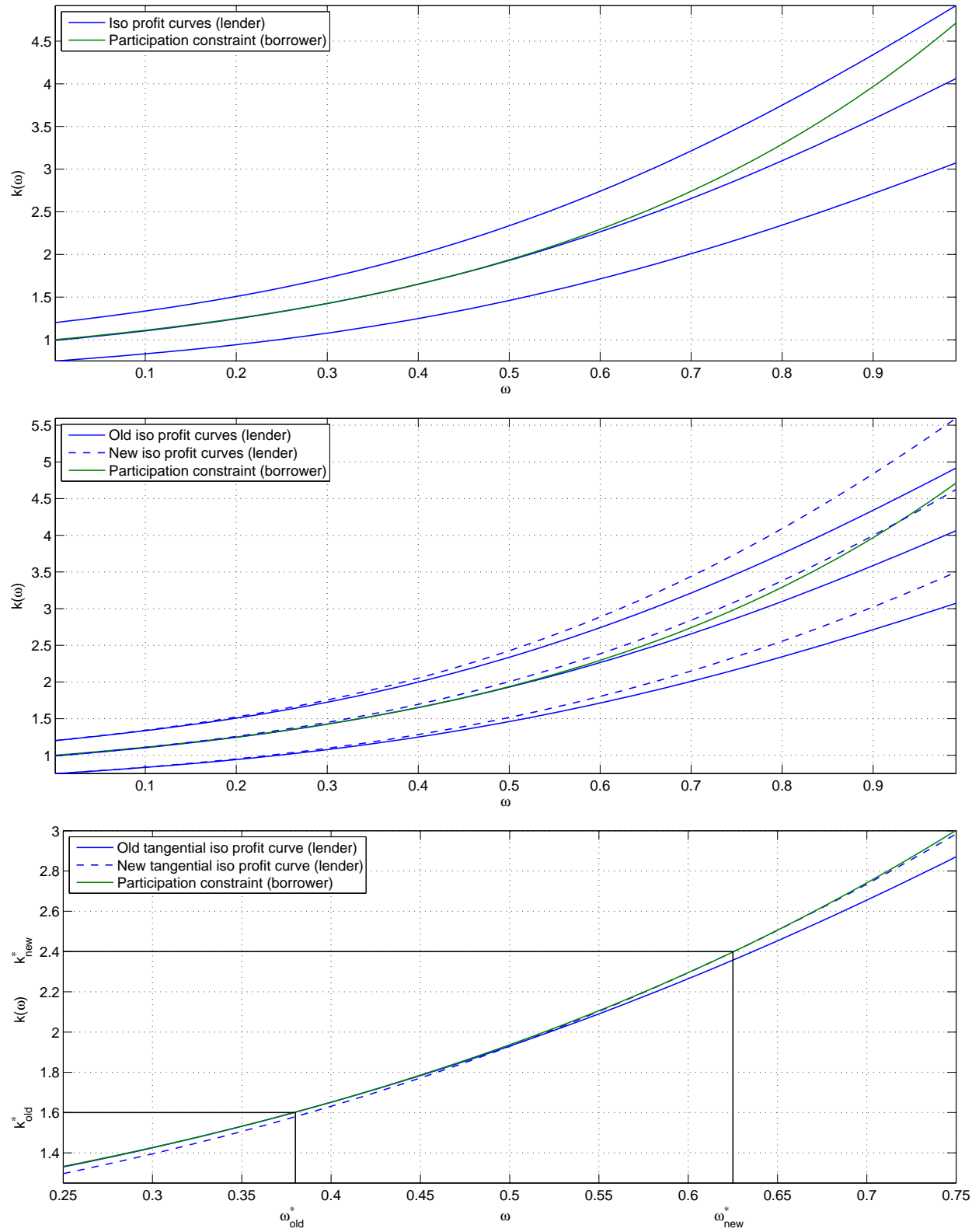
Notes: see Appendix A1 for a detailed description of lending standard variables.

Figure 4: Impulse Responses Lending Standard Measures to a 25bps Expansionary Monetary Policy Shock in the Model with Two Unobserved Factors.



Notes: see Appendix A1 for a detailed description of lending standard variables.

Figure 5: Illustration of the Optimal CSV Contract without Aggregate Risk and the Effects of Expansionary Monetary Policy.



Appendix A. Econometric Methodology

Appendix A.1. Data

Table 2: Data and Data Transformation in the FAVAR

Overall No.	No. in Block	Series ID ^a	Slow ^b	Transformation ^c	Description
1	1	INDPRO	yes	5	Industrial Production Index: Total (2007=100, SA)
2	2	IPBUSEQ	yes	5	Industrial Production: Business Equipment (2007=100, SA)
3	3	IPCONGD	yes	5	Industrial Production: Consumer Goods (2007=100, SA)
4	4	IPDCONGD	yes	5	Industrial Production: Durable Consumer Goods (2007=100, SA)
5	5	IPDMAN	yes	5	Industrial Production: Durable Manufacturing (NAICS) (2007=100, SA)
6	6	IPDMAT	yes	5	Industrial Production: Durable Materials (2007=100, SA)
7	7	IPFINAL	yes	5	Industrial Production: Final Products (Market Group) (2007=100, SA)
8	8	IPMAN	yes	5	Industrial Production: Manufacturing (NAICS) (2007=100, SA)
9	9	IPMAT	yes	5	Industrial Production: Materials (2007=100, SA)
10	10	IPMINE	yes	5	Industrial Production: Mining (2007=100, SA)
11	11	IPNCONGD	yes	5	Industrial Production: Nondurable Consumer Goods (2007=100, SA)
12	12	IPNMAN	yes	5	Industrial Production: Non-durable Manufacturing (NAICS) (2007=100, SA)
13	13	IPNMAT	yes	5	Industrial Production: nondurable Materials (2007=100, SA)
14	14	IPUTIL	yes	5	Industrial Production: Electric and Gas Utilities (2007=100, SA)
15	15	BSCURT02USM160S	yes	1	Business Tendency Surveys for Manufacturing: Rate of Capacity Utilization (% of Capacity), SA

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Table 2 – Continued from previous page

Overall No.	No. in Block	Series ID ^a	Slow ^b	Transformation ^c	Description
16	16	RPI	yes	5	Real personal income, Billions of 2009 chained USD, SAAR
17	17	PIECTR	yes	5	Real personal income excluding current transfer receipts, Billions of 2009 chained USD, SAAR
18	18	GDPC1	yes	5	Real Gross Domestic Product, Billions of 2009 USD chained , SAAR
19	1	CE16OV	yes	5	Civilian Employment (thous., SA)
20	2	DMANEMP	yes	5	All Employees: Durable Goods (thous., SA)
21	3	EMRATIO	yes	4	Employment-Population Ratio (Percent, SA)
22	4	MANEMP	yes	5	All Employees: Manufacturing (thous., SA)
23	5	PAYEMS	yes	5	All Employees: Total Nonfarm (thous., SA)
24	6	SRVPRD	yes	5	All Employees: Service Providing Industries (thous., SA)
25	7	USCONS	yes	5	All Employees: Construction (thous., SA)
26	8	USGOVT	yes	5	All Employees: Government (thous., SA)
27	9	USINFO	yes	5	All Employees: Information Services (thous., SA)
28	10	USMINE	yes	5	All Employees: Mining and Logging (thous., SA)
29	11	USPRIV	yes	5	All Employees: Total Private Industries (thous., SA)
30	12	CES0600000007	yes	1	Average Weekly Hours of Production and Nonsupervisory Employees (SA)
31	13	CES0800000007	yes	1	Average Weekly Hours of Production and Nonsupervisory Employees: Mining and Logging (SA)
32	14	CES1000000007	yes	1	Average Weekly Hours of Production and Nonsupervisory Employees: Private Service Providing, (SA)

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Overall No.	No. in Block	Series ID ^a	Slow ^b	Transformation ^c	Description
33	15	CES2000000007	yes	1	Average Weekly Hours of Production and Nonsupervisory Employees: Durables (SA)
34	16	CES3100000007	yes	1	Average Weekly Hours of Production and Nonsupervisory Employees: Construction (SA)
35	17	CES4000000007	yes	1	Average Weekly Hours of Production and Nonsupervisory Employees: Information (SA)
36	18	CES5000000007	yes	1	Average Weekly Hours of Production and Nonsupervisory Employees: Trade, Transportation, Utilities (SA)
37	19	CES6000000007	yes	1	Average Weekly Hours of Production and Nonsupervisory Employees: Professional and Business Services (SA)
38	1	PCECC96	yes	5	Real Personal consumption expenditure, SAAR, chained 2009 BIL USD
39	1	HOUST	no	4	Housing Starts: Total: New Privately Owned Housing Units Started (thsd. of units) SAAR
40	2	HOUSTMW	no	4	Housing Starts:Midwest: New Privately Owned Housing Units Started (thsd. of units) SAAR
41	3	HOUSTNE	no	4	Housing Starts: Northeast: New Privately Owned Housing Units Started (thsd. of units) SAAR
42	4	HOUSTS	no	4	Housing Starts: South: New Privately Owned Housing Units Started (thsd. of units) SAAR
43	5	HOUSTW	no	4	Housing Starts: West: New Privately Owned Housing Units Started (thsd. of units) SAAR
44	6	PERMIT	no	4	New Private Housing Units Authorized by Building Permits, (thsd. of units) SAAR

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Overall No.	No. in Block	Series ID ^a	Slow ^b	Transformation ^c	Description
45	1	S&P 500	no	5	S&P 500 Stock Price Index, NSA, end of period
46	1	EXCAUS	no	5	Canadian Dollars to One U.S. Dollar, NSA
47	2	EXJPUS	no	5	Japanese Yen to One U.S. Dollar, NSA
48	3	EXSZUS	no	5	Swiss Francs to One U.S. Dollar, NSA
49	4	EXUSUK	no	5	U.S. Dollars to One British Pound, NSA
50	1	AAA	no	1	Moody's Seasoned Aaa Corporate Bond Yield, Percent, NSA
51	2	BAA	no	1	Moody's Seasoned Baa Corporate Bond Yield, Percent, NSA
52	3	FEDFUNDS	no	1	Effective FFR, Percent, NSA
53	4	GS1	no	1	1-Year Treasury Constant Maturity Rate, Percent, NSA
54	5	GS10	no	1	10-Year Treasury Constant Maturity Rate, Percent, NSA
55	6	GS3	no	1	3-Year Treasury Constant Maturity Rate, Percent, NSA
56	7	GS3M	no	1	3-Month Treasury Constant Maturity Rate, Percent, NSA
57	8	GS5	no	1	5-Year Treasury Constant Maturity Rate, Percent, NSA
58	9	AAA_FFR	no	1	Spread: AAA-FFR
59	10	BAA_FFR	no	1	Spread: BAA-FFR
60	11	GS1_FFR	no	1	Spread: GS1-FFR
61	12	GS10_FFR	no	1	Spread: GS10-FFR
62	13	GS3_FFR	no	1	Spread: GS3-FFR
63	14	GS3M_FFR	no	1	Spread: GS3M-FFR
64	15	GS5_FFR	no	1	Spread:GS5-FFR
65	1	BOGNONBR	no	5	Non-Borrowed Reserves of Depository Institutions, Mill USD, SA
66	2	AMBSL	no	5	Monetary Base, Bill USD, SA
67	3	M1	no	5	M1, Bill USD, SA
68	4	M2	no	5	M2, Bill USD, SA
69	5	MZM	no	5	MZM, Bill USD, SA

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Overall No.	No. in Block	Series ID ^a	Slow ^b	Transformation ^c	Description
70	6	LOANS	no	5	Total Loans and Leases, Bill USD, SA
71	7	REALLN	no	5	Real estate loans, Bill USD, SA
72	8	BUSLOANS	no	5	C&I loans, Bill USD; SA
73	9	CONSUMER	no	5	Consumer loans, Bill USD, SA
74	1	CPALTT	yes	5	Consumer Price Index: Total All Items for the United States, 2005=100, SA
75	2	CPIAUCSL	yes	5	Consumer Price Index for All Urban Consumers: All Items, 1982-84=100, SA
76	3	CPIFABSL	yes	5	Consumer Price Index for All Urban Consumers: Food and Beverages, 1982-84=100, SA
77	4	CPILFESL	yes	5	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy, 1982-84=100, SA
78	5	CPIMEDSL	yes	5	Consumer Price Index for All Urban Consumers: Medical Care, 1982-84=100, SA
79	6	DNRGRG3M086SBEA	yes	5	Personal consumption expenditures: Energy goods and services, chain-type index, 2009=100
80	7	DPCXRG3M086SBEA	yes	5	Personal consumption expenditures: Market-based PCE excluding food and energy , chain-type index, 2009=100
81	8	PPICRM	no	5	Producer Price Index: Crude Materials for Further Processing, 1982=100, SA
82	9	PPIFCG	yes	5	Producer Price Index: Finished Consumer Goods, 1982=100, SA
83	10	PPIFGS	yes	5	Producer Price Index: Finished Goods, 1982=100, SA
84	11	PPIIEG	yes	5	Producer Price Index: Intermediate Energy Goods, 1982=100, SA
85	12	PPIITM	yes	5	Producer Price Index: Intermediate Materials: Supplies & Components, 1982=100, SA

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Overall No.	No. in Block	Series ID^a	Slow^b	Transformation^c	Description
86	1	CSCICP02USM661S	no	1	Consumer Opinion Surveys: Confidence Indicators: Composite Indicator, 2005=1.00, SA, end of period
87	1	SUBLPDCILS_N.Q	no	1	Net percentage of domestic banks tightening standards for C&I loans to large and middle-market firms, Percentage
88	2	SUBLPDCILTC_N.Q	no	1	Net percentage of domestic banks increasing the cost of credit lines to large and middle-market firms, Percentage
89	3	SUBLPDCILTL_N.Q	no	1	Net percentage of domestic banks tightening loan covenants for large and middle-market firms, Percentage
90	4	SUBLPDCILTM_N.Q	no	1	Net percentage of domestic banks reducing the maximum size of credit lines for large and middle-market firms, Percentage
91	5	SUBLPDCILTQ_N.Q	no	1	Net percentage of domestic banks increasing collateral requirements for large and middle-market firms, Percentage
92	6	SUBLPDCILTS_N.Q	no	1	Net percentage of domestic banks increasing spreads of loan rates over banks' cost of funds to large and middle-market firms, Percentage
93	7	SUBLPDCISS_N.Q	no	1	Net percentage of domestic banks tightening standards for C&I loans to small firms, Percentage
94	8	SUBLPDCISTC_N.Q	no	1	Net percentage of domestic banks increasing the cost of credit lines to small firms, Percentage
95	9	SUBLPDCISTL_N.Q	no	1	Net percentage of domestic banks tightening loan covenants for small firms, Percentage

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Table 2 – *Continued from previous page*

Overall No.	No. in Block	Series ID^a	Slow^b	Transformation^c	Description
96	10	SUBLPDCISTM_N.Q	no	1	Net percentage of domestic banks reducing the maximum size credit lines for small firms, Percentage
97	11	SUBLPDCISTQ_N.Q	no	1	Net percentage of domestic banks increasing collateral requirements for small firms, Percentage
98	12	SUBLPDCISTS_N.Q	no	1	Net percentage of domestic banks increasing spreads of loan rates over banks' cost of funds to small firms, Percentage
99	13	SUBLPDRCS_N.Q	no	1	Net percentage of domestic banks tightening standards for commercial real estate loans, Percentage
100	14	SUBLPFCIS_N.Q	no	1	Net percentage of foreign banks tightening standards for approving C&I loans, Percentage
101	15	SUBLPFCITC_N.Q	no	1	Net percentage of foreign banks increasing costs of credit lines, Percentage
102	16	SUBLPFCITL_N.Q	no	1	Net percentage of foreign banks tightening loan covenants, Percentage
103	17	SUBLPFCITM_N.Q	no	1	Net percentage of foreign banks reducing the maximum size of credit lines, Percentage
104	18	SUBLPFCITQ_N.Q	no	1	Net percentage of foreign banks increasing collateralization requirements, Percentage
105	19	SUBLPFRCS_N.Q	no	1	Net percentage of foreign banks tightening standards for commercial real estate loans, Percentage
106	1	AHETPI	yes	5	Average Hourly Earnings of Production and Nonsupervisory Employees: Total Private, USD per Hour, SA
107	2	CES0600000008	yes	5	Average Hourly Earnings of Production and Nonsupervisory Employees: Goods producing, USD per hour, SA

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Table 2 – *Continued from previous page*

Overall No.	No. in Block	Series ID^a	Slow^b	Transformation^c	Description
108	3	CES0800000008	yes	5	Average Hourly Earnings of Production and Nonsupervisory Employees: Private Service Producing, USD per Hour, SA
109	4	CES1000000008	yes	5	Average Hourly Earnings of Production and Nonsupervisory Employees: Mining and Logging, USD per Hour, SA
110	5	CES2000000008	yes	5	Average Hourly Earnings of Production and Nonsupervisory Employees: Construction, USD per Hour, SA
111	6	CES3000000008	yes	5	Average Hourly Earnings of Production and Nonsupervisory Employees: Manufacturing, USD per Hour, SA
112	1	B015RX1Q020SBEA	no	1	Change in real private inventories: Nonfarm, Billions of 2009 chained USD, SAAR
113	2	B018RX1Q020SBEA	no	1	Change in real private inventories: Farm, Billions of 2009 chained USD, SAAR
114	3	NAPMNOI	no	1	ISM Manufacturing: New Orders Index, SA

^a Macroeconomic time series are taken from the FRED database, lending standards measures are taken from the Senior Loan Officer Opinion Survey of the Federal Reserve.

^b If yes, a variable is assumed to be slow-moving.

^c Variable transformations codes are as follows: 1 - no transformation, 2 - difference, 4 - logarithm, 5 - log-difference.

Appendix A.2. Bayesian Estimation

In order to jointly estimate equations (1) and (2) with Bayesian methods it is convenient to rewrite the model in state-space form:

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \Lambda^f & \Lambda^y \\ 0 & I \end{bmatrix} \begin{bmatrix} F_t \\ Y_t \end{bmatrix} + \begin{bmatrix} e_t \\ 0 \end{bmatrix} \quad (12)$$

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + \nu_t, \quad (13)$$

where Y_t is vector of observables of dimension $[M \times 1]$, F_t is vector of unobserved factors of dimension $[K \times 1]$ and X_t are informational time series of dimension $[N \times 1]$. The loading coefficient matrices Λ^f of dimension $[N \times K]$ and Λ^y of dimension $[N \times M]$ are restricted in order to achieve factor identification. The error vectors are assumed to be normally distributed and uncorrelated: $e_t \sim N(0, R)$ and $\nu_t \sim N(0, Q)$, where R is diagonal matrix.

In a one-step Bayesian estimation all parameters are treated as random variables. The parameter vector θ includes the factor loadings and variance-covariance matrix of the observation equation (3) as well as VAR coefficients and variance-covariance matrix of the transition (state) equation (4), i.e. $\theta = (\Lambda^f, \Lambda^y, R, \text{vec}(\Phi), Q)$. Furthermore, unobserved factors are also treated as random variables and will be sampled. Observation and transition equations can be rewritten as:

$$X_t = \Lambda F_t + e_t \quad (14)$$

$$F_t = \Phi(L)F_{t-1} + \nu_t, \quad (15)$$

where Λ is the loading matrix, $X_t = (X'_t, Y'_t)$, $e_t = (e'_t, 0)$ and $F_t = (F'_t, Y'_t)$. Denote $\tilde{X}_t = (X_1, X_2, \dots, X_T)$ and $\tilde{F}_t = (F_1, F_2, \dots, F_T)$ as the respective histories from 1 to T. Our goal is to obtain marginal densities of parameters and factors, which can be integrated out of the joint posterior density $p(\theta, \tilde{F}_T)$. In other words, we are interested in the following objects:

$$p(\tilde{F}_T) = \int p(\theta, \tilde{F}_T) d\theta \quad (16)$$

$$p(\theta) = \int p(\theta, \tilde{F}_T) d\tilde{F}_T \quad (17)$$

Gibbs Sampling

The sampling is a multi-move Gibbs sampling of Carter and Kohn (1994), which alternately samples from the parameters and factors as follows:

Step 1. Choose a starting value θ_0

Step 2. Draw $\tilde{F}_T^{(1)}$ from the conditional density $p(\tilde{F}_T | \tilde{X}_T, \theta_0)$

Step 3. Draw $\theta^{(1)}$ from the conditional density $p(\theta|\tilde{X}_T, \tilde{F}_T^{(1)})$

Repeat Steps 2 and 3 until convergence.

Choice of Starting Values

One of the choices is to start from the solution implied by the principal components analysis (Bernanke et al., 2005). It was our baseline choice for the most runs. However, starting the chains (even very long ones) from the same point may not be enough to achieve the target distribution in practice, even if the chain appears to have converged. Therefore, we also tried "agnostic" starting values: $vec(\Phi) = 0$, $Q = I$, $\Lambda^f = 0$, $\Lambda^y = \text{OLS of the regression of } X \text{ on } Y$ and $R = \text{fitted residual covariance matrix from the OLS regression of } X \text{ on } Y$. It did not affect the results. Furthermore, we ran multiple consecutive chains, setting the starting values of the subsequent chain to the values obtained in the last iteration of the previous chain. Each chain had a length of 1 million draws and we kept every fifth draw (thinning), as the chains were highly autocorrelated along some of the parameters.

Conditional Densities and Priors

In order to draw from $p(\tilde{F}_T|\tilde{X}_T, \theta)$, we need to resort to Kalman filtering (Kim and Nelson, 1999). Due to the Markov (memoryless) property of F_t , the conditional distribution of the history of factors can be expressed as a product of conditional distributions of factors at date t:

$$p(\tilde{F}_T|\tilde{X}_T, \theta) = p(F_T|\tilde{X}_T, \theta) \prod_{t=1}^{T-1} p(F_t|F_{t+1}, \tilde{X}_t, \theta) \quad (18)$$

The original model is linear-Gaussian, which implies:

$$F_T|\tilde{X}_T, \theta \sim N(F_{T|T}, P_{T|T}) \quad (19)$$

$$F_t|F_{t+1}, \tilde{X}_t, \theta \sim N(F_{t|t, F_{t+1}}, P_{t|t, F_{t+1}}), \quad (20)$$

where

$$F_{T|T} = E(F_T|\tilde{X}_T, \theta) \quad (21)$$

$$P_{T|T} = cov(F_T|\tilde{X}_T, \theta) \quad (22)$$

$$F_{t|t, F_{t+1}} = E(F_t|F_{t+1}, \tilde{X}_t, \theta) = E(F_t|F_{t+1}, F_{t|t}, \theta) \quad (23)$$

$$P_{t|t, F_{t+1}} = cov(F_t|F_{t+1}, \tilde{X}_t, \theta) = cov(F_t|F_{t+1}, F_{t|t}, \theta) \quad (24)$$

$$(25)$$

$F_{t|t}$ and $P_{t|t}$ are calculated by the Kalman filter for $t = 1, \dots, T$ conditional on θ and the respective data history \tilde{X}_t . Starting values for the Kalman filter are zeros for the factors and the identity matrix for the covariance matrix. Further, a Kalman smoother is applied to obtain the updated values of $F_{T-1|T-1, F_T}$ and $P_{T-1|T-1, F_T}$.

The prior on the parameters from the observation equation Λ and variance-covariance matrix R is as follows. Since R is assumed to be diagonal, estimates of Λ and diagonal elements of R (R_{ii}) can be obtained with OLS equation by equation. Conjugate priors are assumed to have the form:

$$R_{ii} \sim iG(\delta_0/2, \eta_0/2) \quad (26)$$

$$\Lambda_i | R_{ii} \sim N(0, R_{ii} M_0^{-1}), \quad (27)$$

where, following (Bernanke et al., 2005), we set $\delta_0 = 6$, $\eta_0 = 2 * 10^{-3}$ and $M_0 = I$. Then the posterior can be shown to have the form:

$$R_{ii} | \tilde{X}_T, \tilde{F}_T \sim iG(\delta_i/2, \eta/2) \quad (28)$$

$$\Lambda_i | R_{ii}, \tilde{X}_T, \tilde{F}_T \sim N(\bar{\Lambda}_i, R_{ii} \bar{M}_i^{-1}), \quad (29)$$

where

$$\delta_i = \delta_0/2 + \hat{e}_i' \hat{e}_i + \hat{\Lambda}_i' [M_0^{-1} + (\tilde{F}_T'^i \tilde{F}_T^i)^{-1}]^{-1} \hat{\Lambda}_i \quad (30)$$

$$\eta = \eta_0/2 + T \quad (31)$$

$$\bar{\Lambda}_i = \bar{M}_i^{-1} (\tilde{F}_T'^i \tilde{F}_T^i) \hat{\Lambda}_i \quad (32)$$

$$\bar{M}_i = M_0 + \tilde{F}_T'^i \tilde{F}_T^i, \quad (33)$$

where \tilde{F}_T^i are the regressors of the i th equation.

The prior on the transition (state) equation is as follows. The transition equation has a standard VAR, we estimate it equation by equation to obtain $vec(\hat{\Phi})$ and \hat{Q} . A conjugate Normal-Inverse-Wishart-prior is imposed:

$$Q \sim iW(Q_0, K + M + 2) \quad (34)$$

$$vec(\Phi) | Q \sim N(0, Q \otimes \Omega_0), \quad (35)$$

where diagonal elements of Q_0 are set to the residual variances of the corresponding univariate equations, $\hat{\sigma}_i^2$ (as in Kadiyala and Karlsson, 1997). Diagonal elements of Ω_0 are set in the spirit of the Minnesota prior, i.e. prior variance

of parameter on k lagged variable j in equation i is $\sigma_i^2/k\sigma_j^2$. This prior yields the following conjugate posterior:

$$Q|\tilde{\mathbf{X}}_T, \tilde{\mathbf{F}}_T \sim iW(\bar{Q}, T + K + M + 2) \quad (36)$$

$$vec(\Phi)|Q, \tilde{\mathbf{X}}_T, \tilde{\mathbf{F}}_T \sim N(vec(\bar{\Phi}), Q \otimes \bar{\Omega}), \quad (37)$$

where

$$\bar{Q} = Q_0 + \hat{V}'\hat{V} + \hat{\Phi}'[\Omega_0 + (\mathbf{F}_{T-1}'\tilde{\mathbf{F}}_{T-1})^{-1}]^{-1}\hat{\Phi} \quad (38)$$

$$\bar{\Phi} = \bar{\Omega}(\mathbf{F}_{T-1}'\tilde{\mathbf{F}}_{T-1})\hat{\Phi} \quad (39)$$

$$\bar{\Omega} = (\Omega_0^{-1} + \mathbf{F}_{T-1}'\tilde{\mathbf{F}}_{T-1})^{-1} \quad (40)$$

with \hat{V} being the matrix of OLS residuals. Following Bernanke et al. (2005) and Amir Ahmadi and Uhlig (2009), we enforce stationarity of the system by truncating the draws of Φ where the largest eigenvalue is larger than 1 in absolute value.

Monitoring Convergence

Geman and Geman (1984) show that both joint and marginal distributions will converge to their target distributions at an exponential rate as number of replications approaches infinity. In practice, however, the convergence may be slow and requires careful monitoring. We monitor convergence by various means: 1) plotting the coefficients against the number of replications (level shifts and trends should not occur); 2) comparing the medians and means of the coefficients at different parts of the chain (large differences should not occur); 3) plotting and comparing the medians of the factors obtained from first and second half of the chain (large and frequent deviations should not occur): the figures from the baseline model with 3 factors are reported below. For our application, it turns out that the convergence is quite slow and becomes substantially more problematic the more factors are added.

Figure 6: Median Monitoring for the First Factor: First and Second Half of Draws after Burn-in, Baseline Model

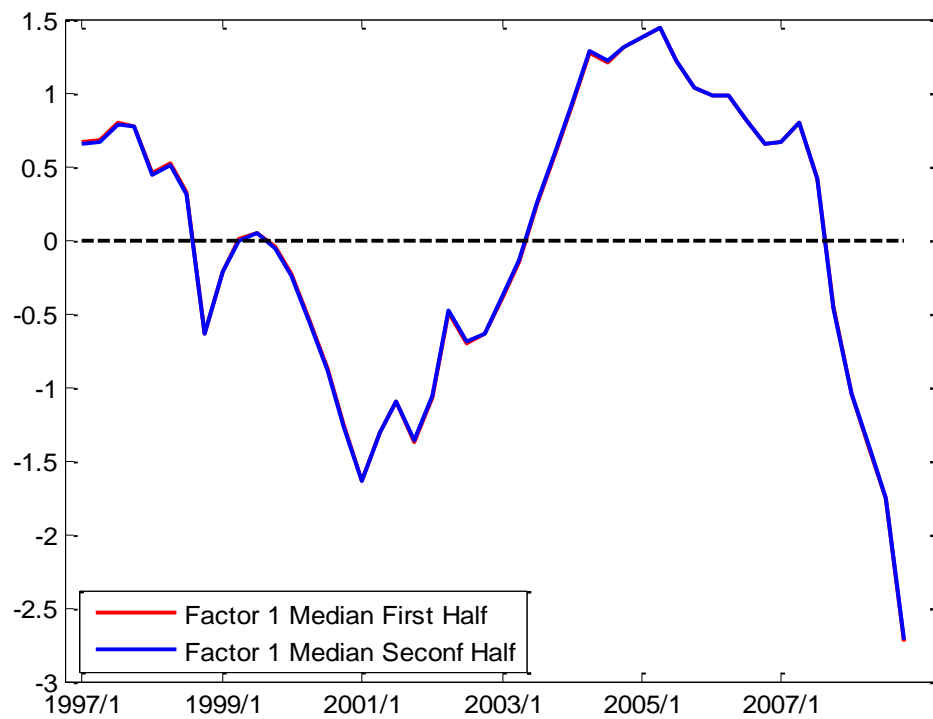


Figure 7: Median Monitoring for the Second Factor: First and Second Half of Draws after Burn-in, Baseline Model

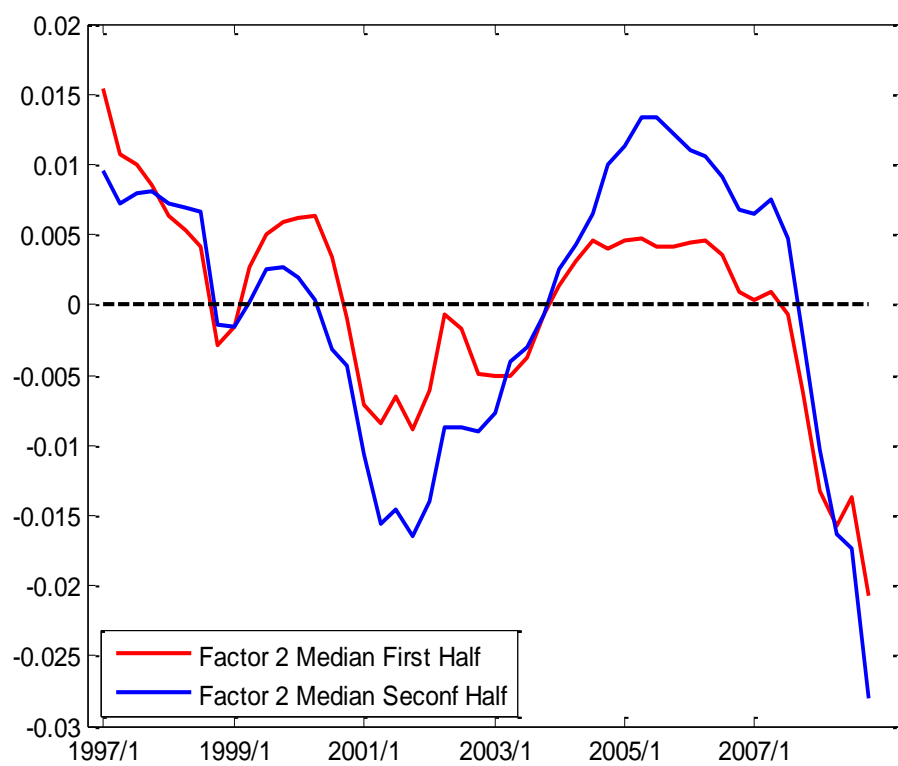
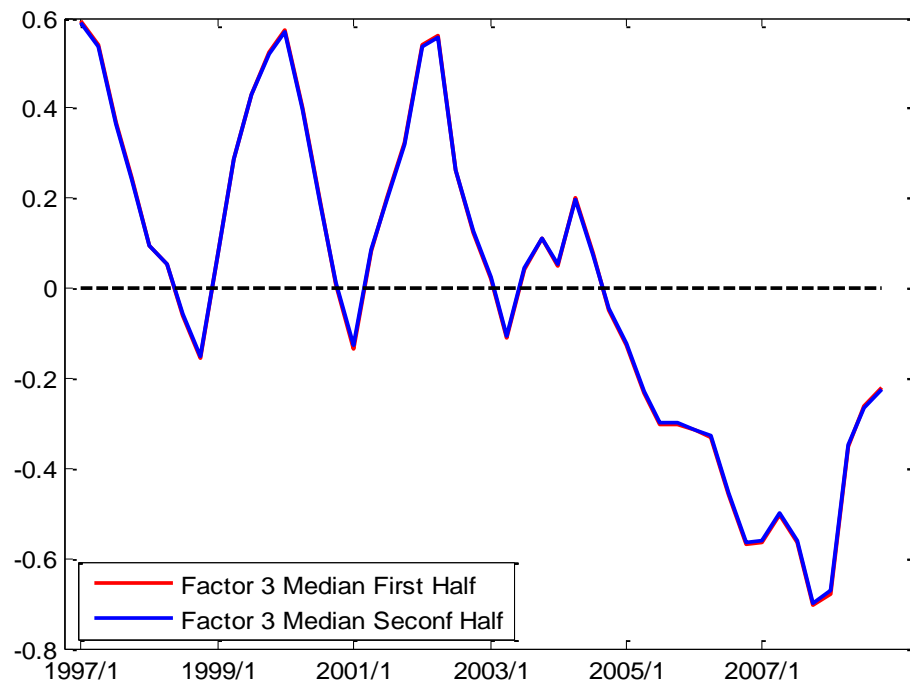


Figure 8: Median Monitoring for the Third Factor: First and Second Half of Draws after Burn-in, Baseline Model



Appendix B. Optimal Loan Contract

This appendix provides details on the optimal financial contract, following the logic in Bernanke et al. (1999). Given the different assumptions about the roles of borrowers and lenders, however, we deviate from the latter, where this is necessary.

Appendix B.1. Without Aggregate Risk

In the absence of aggregate uncertainty, the loan contract between the bank and entrepreneur i is only affected by the entrepreneur's idiosyncratic risk ω^i . Consequently, the bank's constrained profit maximization problem can be formulated as in equation (8), where all terms are defined in the main text.

Given the borrower's net worth, the bank chooses the volume of the loan and thus k . For any value of k , the entrepreneur's participation constraint (PC) pins down the default threshold $\bar{\omega}^i$, which splits the expected total profits from the investment project between the borrower and the lender. Given $\bar{\omega}^i$, the non-default rate of return on the loan to entrepreneur i , Z_{t+1}^i , will then be determined by (6).

For notational convenience, we suppress any time subscripts and index superscripts throughout the appendix, while our aim remains to derive the properties of the optimal contract between the bank and entrepreneur i .

The EFP and Loan Supply

In what follows, we establish a positive relation $k = \psi(s)$, $\psi'(s) > 0$, between the *external finance premium* (EFP) $s \equiv R^k/R$ and the bank's optimal choice of the capital/net worth ratio $k \equiv R^k/$. The Lagrangian corresponding to the problem in (8) is given by

$$\mathcal{L} = [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] sk - (k - 1 - n) + \lambda \{[1 - \Gamma(\bar{\omega})] sk - s\},$$

where $n \equiv N^b/N$ and λ is the Lagrangian multiplier on the borrower's PC. The first-order conditions (FOC) are

$$k : [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] s - 1 + \lambda [1 - \Gamma(\bar{\omega})] s = 0,$$

$$\bar{\omega} : [\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})] sk - \lambda \Gamma'(\bar{\omega}) sk = 0,$$

$$\lambda : [1 - \Gamma(\bar{\omega})] sk - s = 0.$$

Note that the assumptions made about $\Gamma(\bar{\omega})$ and $\mu G(\bar{\omega})$ imply that the bank's expected profit share net of expected default costs satisfies

$$\Gamma(\bar{\omega}) - \mu G(\bar{\omega}) > 0 \quad \text{for} \quad \bar{\omega} \in (0, \infty)$$

and

$$\lim_{\bar{\omega} \rightarrow 0} \Gamma(\bar{\omega}) - \mu G(\bar{\omega}) = 0, \quad \lim_{\bar{\omega} \rightarrow \infty} \Gamma(\bar{\omega}) - \mu G(\bar{\omega}) = 1 - \mu.$$

In order for the bank's profits to be bounded in the case where the borrower defaults with probability one, we therefore assume that $s < 1/(1 - \mu)$ (compare Bernanke et al., 1999).

We further assume that $\bar{\omega}h(\bar{\omega})$ is increasing in $\bar{\omega}$, where $h(\omega)$ denotes the *hazard rate* $f(\bar{\omega})/[1 - F(\bar{\omega})]$.¹⁸ Hence, there exists an $\bar{\omega}^*$ such that

$$\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega}) = [1 - F(\bar{\omega})][1 - \mu \bar{\omega}h(\bar{\omega})] \gtrless 0 \quad \text{for} \quad \bar{\omega} \gtrless \bar{\omega}^*,$$

i.e., the bank's expected net profit share reaches a global maximum at $\bar{\omega}^*$. Moreover, the above assumption implies

$$\Gamma'(\bar{\omega})G''(\bar{\omega}) - \Gamma''(\bar{\omega})G'(\bar{\omega}) = \frac{d[\bar{\omega}h(\bar{\omega})]}{d\bar{\omega}}[1 - F(\bar{\omega})]^2 > 0 \quad \forall \bar{\omega}.$$

Consider first the FOC w.r.t. $\bar{\omega}$, which implies that

$$\lambda(\bar{\omega}) = \frac{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})}{\Gamma'(\bar{\omega})}.$$

Taking the partial derivative w.r.t. $\bar{\omega}$, we obtain

$$\begin{aligned} \lambda'(\bar{\omega}) &= \frac{\Gamma'(\bar{\omega})[\Gamma''(\bar{\omega}) - \mu G''(\bar{\omega})] - \Gamma''(\bar{\omega})[\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})]}{[\Gamma'(\bar{\omega})]^2} \\ &= \frac{\mu[\Gamma''(\bar{\omega})G'(\bar{\omega}) - \Gamma'(\bar{\omega})G''(\bar{\omega})]}{[\Gamma'(\bar{\omega})]^2} < 0, \end{aligned}$$

because $\Gamma'(\bar{\omega}) = 1 - F(\bar{\omega}) > 0$ and $\Gamma''(\bar{\omega})G'(\bar{\omega}) - \Gamma'(\bar{\omega})G''(\bar{\omega}) < 0$ for all $\bar{\omega}$. Taking limits,

$$\lim_{\bar{\omega} \rightarrow 0} \lambda(\bar{\omega}) = 1, \quad \lim_{\bar{\omega} \rightarrow \bar{\omega}^*} \lambda(\bar{\omega}) = 0.$$

Hence, in contrast to Bernanke et al. (1999), $\lambda(\bar{\omega})$ is a decreasing function of the cutoff. This is a logical consequence of the borrower's PC, since the borrower's expected share of total profits is a decreasing function of $\bar{\omega}$.

From the FOC w.r.t. k , we can furthermore define a function

$$\rho(\bar{\omega}) \equiv \frac{1}{\Gamma(\bar{\omega}) - \mu G(\bar{\omega}) + \lambda[1 - \Gamma(\bar{\omega})]} = s.$$

¹⁸Given that we borrow the definitions of $\Gamma(\bar{\omega})$ and $\Gamma(\bar{\omega}) - \mu G(\bar{\omega})$ from Bernanke et al. (1999), our assumption about the hazard rate and its implications are identical to those in their Appendix A.

Taking the partial derivative w.r.t. $\bar{\omega}$, we obtain

$$\begin{aligned}\rho'(\bar{\omega}) &= -\rho(\bar{\omega})^2 \{ \Gamma'(\bar{\omega}) - \mu G'(\bar{\omega}) + \lambda'(\bar{\omega}) [1 - \Gamma(\bar{\omega})] - \lambda(\bar{\omega}) \Gamma'(\bar{\omega}) \} \\ &= -\rho(\bar{\omega})^2 \{ \lambda(\bar{\omega}) \Gamma'(\bar{\omega}) + \lambda'(\bar{\omega}) [1 - \Gamma(\bar{\omega})] - \lambda(\bar{\omega}) \Gamma'(\bar{\omega}) \} \\ &= \underbrace{-\rho(\bar{\omega})^2}_{<0} \underbrace{\lambda'(\bar{\omega})}_{<0} \underbrace{[1 - \Gamma(\bar{\omega})]}_{>0} > 0,\end{aligned}$$

where the second equality uses the FOC w.r.t. $\bar{\omega}$. Taking limits,

$$\begin{aligned}\lim_{\bar{\omega} \rightarrow 0} \rho(\bar{\omega}) &= 1 \quad (\text{due to } \lim_{\bar{\omega} \rightarrow 0} \lambda(\bar{\omega}) = 1 \text{ and } \lim_{\bar{\omega} \rightarrow 0} G(\bar{\omega}) = 0), \\ \lim_{\bar{\omega} \rightarrow \bar{\omega}^*} \rho(\bar{\omega}) &= \frac{1}{\Gamma(\bar{\omega}^*) - \mu G(\bar{\omega}^*)} \equiv s^* \quad (\text{due to } \lim_{\bar{\omega} \rightarrow \bar{\omega}^*} \lambda(\bar{\omega}) = 0).\end{aligned}$$

Accordingly, there is a one-to-one mapping between the optimal cutoff $\bar{\omega}$ and the premium on external funds s , as in Bernanke et al. (1999). Inverting the function $s = \rho(\bar{\omega})$, we can therefore express the cutoff as $\bar{\omega} = \bar{\omega}(s)$, where $\bar{\omega}'(s) > 0$ for $s \in (1, s^*)$.

From the FOC w.r.t. λ , i.e. the borrower's PC, we finally define

$$\Psi(\bar{\omega}) = \frac{1}{1 - \Gamma(\bar{\omega})} = k.$$

Taking the partial derivative w.r.t. $\bar{\omega}$, we obtain

$$\begin{aligned}\Psi'(\bar{\omega}) &= -\Psi(\bar{\omega})^2 [-\Gamma'(\bar{\omega})] \\ &= \underbrace{\Psi(\bar{\omega})^2}_{>0} \underbrace{[1 - \Gamma'(\bar{\omega})]}_{>0} > 0.\end{aligned}$$

Hence, the qualitative implications are the same as in Bernanke et al. (1999). Taking limits,

$$\lim_{\bar{\omega} \rightarrow 0} \Psi(\bar{\omega}) = 1, \quad \lim_{\bar{\omega} \rightarrow \bar{\omega}^*} \Psi(\bar{\omega}) = [1 - \lambda(\bar{\omega}^*)]^{-1} < \infty.$$

Combining $k = \Psi(\bar{\omega})$ and $\bar{\omega} = \bar{\omega}(s)$, where $\Psi'(\bar{\omega}) > 0$ and $\bar{\omega}'(s) > 0$, we can thus express the capital/net worth ratio $k = QK/N$ as a function $k = \psi(s)$, where $\psi'(s) > 0$ for $s \in (1, s^*)$.

Proof of an Interior Solution

Bernanke et al. (1999) use a general equilibrium argument to justify the assumption of an interior solution, i.e. an optimal contract where $\bar{\omega} < \bar{\omega}^*$ and $s < s^*$. In particular, they argue that, “as s approaches s^* from below, the capital

stock becomes unbounded. In equilibrium this will lower the excess return s .” (compare Bernanke et al., 1999, p. 1384).

Here, we follow an analytical argument instead. Recall that the lender’s iso-profit curves (IPC) and the borrower’s PC in $(k, \bar{\omega})$ -space can be written as

$$k_{IPC} = \frac{\pi^b - 1 - n}{[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]s - 1},$$

$$k_{PC} \geq \frac{1}{1 - \Gamma(\bar{\omega})},$$

where π^b denotes an arbitrary level of bank profits.

Recall further that, in $(k, \bar{\omega})$ -space, the optimal contract is determined by the tangential point of the lender’s IPC (from below) with the borrower’s PC. Consider first the borrower’s PC. Since $\Gamma'(\bar{\omega}) > 0$, k_{PC} is a strictly increasing function for $\bar{\omega} \in [0, \infty)$, i.e., the borrower’s PC has a positive slope everywhere in $(k, \bar{\omega})$ -space.

Consider next the lender’s IPC. Taking the partial derivative of k_{IPC} w.r.t. $\bar{\omega}$,

$$\left. \frac{\partial k}{\partial \bar{\omega}} \right|_{IPC} = (1 - \pi^b + n) \frac{[\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})]s}{\{[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]s - 1\}^2} \begin{cases} > 0 & \text{for } \bar{\omega} \in [0, \bar{\omega}^*) \\ = 0 & \text{for } \bar{\omega} = \bar{\omega}^* \\ < 0 & \text{for } \bar{\omega} \in (\bar{\omega}^*, \infty) \end{cases},$$

i.e., the lender’s IPC has a positive slope in $(k, \bar{\omega})$ -space *left of* $\bar{\omega}^*$ but a negative slope *right of* $\bar{\omega}^*$. Since the optimal contract requires that

$$\left. \frac{\partial k}{\partial \bar{\omega}} \right|_{IPC} = \left. \frac{\partial k}{\partial \bar{\omega}} \right|_{PC},$$

at the tangential point, and we already know that

$$\left. \frac{\partial k}{\partial \bar{\omega}} \right|_{PC} = \frac{\Gamma'(\bar{\omega})}{[1 - \Gamma(\bar{\omega})]^2} > 0 \quad \text{for } \bar{\omega} \in [0, \infty),$$

the optimal contract can only be obtained for $\bar{\omega} < \bar{\omega}^*$, which implies an *interior solution* to the bank’s constrained profit maximization problem.¹⁹

Proof of Uniqueness

As was shown above, the tangential point of the borrower’s participation constraint (PC) and the lender’s iso-profit curve (IPC) is located on the interval $[0, \bar{\omega}^*)$. To show uniqueness, we proceed in two steps. First, we show that at the

¹⁹Note that this argument can be applied to the formulation of the financial contract in Bernanke et al. (1999), likewise implying an interior solution.

tangency point the curvature of the participation constraint is higher than the curvature of the iso-profit curve. Second, we discuss under which conditions the convexity (concavity) of PC and IPC are warranted on the interval $[0, \bar{\omega}^*)$. Given the differences in curvature at the tangency point shown in step 1, convexity implies a unique solution at $\bar{\omega} > 0$, whereas concavity implies a unique solution at $\bar{\omega} = 0$.

Step 1. At the tangency point it holds:

$$\frac{1}{1 - \Gamma(\bar{\omega})} = \frac{\pi^b - 1 - n}{[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]s - 1},$$

i.e., the levels of k implied by PC and IPC are equal. Furthermore, it holds:

$$\frac{\Gamma'(\bar{\omega})}{(1 - \Gamma(\bar{\omega}))^2} = \frac{(1 - \pi^b + n)s(\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega}))}{([\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]s - 1)^2},$$

i.e. $\frac{\partial k}{\partial \bar{\omega}}|_{PC} = \frac{\partial k}{\partial \bar{\omega}}|_{IPC}$ at the tangency point. Denote $A(\bar{\omega}) = (\partial^2 k / \partial \bar{\omega}^2)|_{PC}$ and $B(\bar{\omega}) = (\partial^2 k / \partial \bar{\omega}^2)|_{IPC}$. In what follows, we suppress dependence of Γ and G on the argument $\bar{\omega}$ to simplify notation:

$$A(\bar{\omega}) = \frac{\Gamma''(1 - \Gamma)^2 + 2(1 - \Gamma)(\Gamma')^2}{(1 - \Gamma)^4}$$

$$B(\bar{\omega}) = (1 - \pi^b + n)s \frac{(\Gamma'' - \mu G'')([\Gamma - \mu G]s - 1)^2 - 2s(\Gamma' - \mu G')^2([\Gamma - \mu G]s - 1)}{([\Gamma - \mu G]s - 1)^4}$$

We need to know $A(\bar{\omega}) \leq B(\bar{\omega})$. After some algebra and using the two relations holding at the tangency point stated above, we get:

$$\begin{aligned} A(\bar{\omega}) \leq B(\bar{\omega}) &\Leftrightarrow \Gamma'' + \frac{2(\Gamma')^2}{1 - \Gamma} \leq \frac{\Gamma'(\Gamma'' - \mu G'')}{\Gamma' - \mu G'} - \frac{2s(\Gamma' - \mu G')\Gamma'}{(\Gamma - \mu G)s - 1} \Leftrightarrow \\ &\Leftrightarrow \frac{\mu(G''\Gamma' - \Gamma''G')}{\Gamma' - \mu G'} + 2 \left[\frac{(\Gamma')^2(\pi^b - 1 - n) - (\Gamma' - \mu G')s\Gamma'}{(\Gamma - \mu G)s - 1} \right] \leq 0 \Leftrightarrow \\ &\Leftrightarrow \frac{\mu(G''\Gamma' - \Gamma''G')}{\Gamma' - \mu G'} + \frac{2(\Gamma')^2(\pi^b - 1 - n)}{(\Gamma - \mu G)s - 1} + \frac{2(\Gamma' - \mu G')s\Gamma'}{1 - (\Gamma - \mu G)s} > 0 \Leftrightarrow A(\bar{\omega}) > B(\bar{\omega}) \end{aligned}$$

Note in particular that:

$$\frac{\mu(G''\Gamma' - \Gamma''G')}{\Gamma' - \mu G'} > 0,$$

since $G''\Gamma' - \Gamma''G' > 0 \forall \bar{\omega}^{20}$ and $\Gamma' - \mu G > 0$ for $\bar{\omega} \in [0, \bar{\omega}^*)$. Furthermore,

$$\frac{2(\Gamma')^2(\pi^b - 1 - n)}{(\Gamma - \mu G)s - 1} > 0,$$

since $(\pi^b - 1 - n) < 0$ and $([\Gamma - \mu G]s - 1) < 0$. And finally:

$$\frac{2(\Gamma' - \mu G')s\Gamma'}{1 - (\Gamma - \mu G)s} > 0,$$

since $(\Gamma' - \mu G') > 0$ on $[0, \bar{\omega}^*)$, $(1 - [\Gamma - \mu G]s) > 0$ and $\Gamma' = 1 - F > 0$. This proves step 1, i.e. at the tangency point the participation constraint has a higher curvature than the iso-profit curve.

Step 2. Note that the sign of second partial derivatives $A(\bar{\omega})$ and $B(\bar{\omega})$ defined above is generally dependent on the parameters of the log-normal distribution assumed for $\bar{\omega}$. In particular, the sign of $A(\bar{\omega})$ on $[0, \bar{\omega}^*)$ is determined by the sign of the following expression²¹:

$$\Gamma''(1 - \Gamma) + 2(\Gamma')^2 \leq 0 \Leftrightarrow -f(\bar{\omega})(1 - \Gamma(\bar{\omega})) + 2(1 - F(\bar{\omega}))^2 \leq 0 \Leftrightarrow f(\bar{\omega})(1 - \Gamma(\bar{\omega})) \leq 2(1 - F(\bar{\omega}))^2$$

While $0 < (1 - \Gamma(\bar{\omega})) < 1$ and $0 < (1 - F(\bar{\omega})) < 1$ for all distributional parameters of $F(\bar{\omega})$, the size of $f(\bar{\omega})$ can vary substantially depending on the mean and variance of $F(\bar{\omega})$. Given the distributional assumptions of Bernanke et al. (1999) ($\ln \bar{\omega} \sim N(-0.5\sigma^2, \sigma^2)$) one can show that there is a threshold $\bar{\sigma}$ such that:

$$f(\bar{\omega})(1 - \Gamma(\bar{\omega})) > 2(1 - F(\bar{\omega}))^2 \quad \Leftrightarrow \quad A(\bar{\omega}) < 0 \quad \text{for } \sigma > \bar{\sigma}$$

$$f(\bar{\omega})(1 - \Gamma(\bar{\omega})) < 2(1 - F(\bar{\omega}))^2 \quad \Leftrightarrow \quad A(\bar{\omega}) > 0 \quad \text{for } \sigma < \bar{\sigma}$$

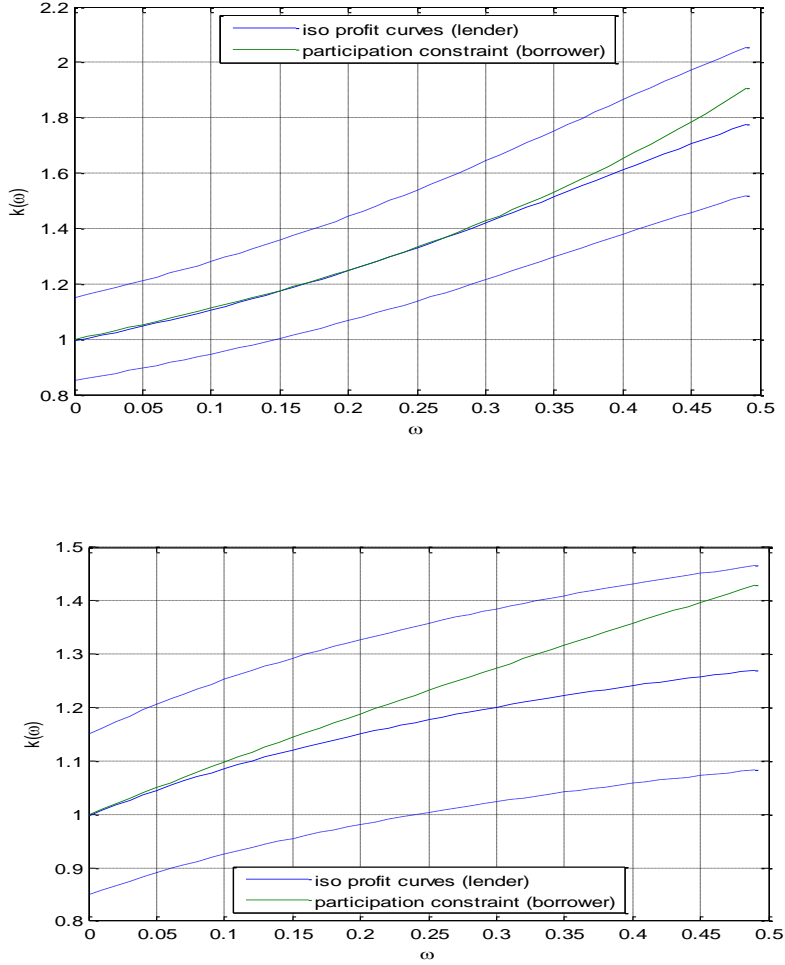
In other words, for $\sigma > \bar{\sigma}$ the participation constraint is concave and for $\sigma < \bar{\sigma}$ the participation constraint is convex. As the Figure 9 below illustrates, in the first case the solution is unique and $\bar{\omega} > 0$, whereas in the second case the solution is unique at $\bar{\omega} = 0$.

For realistic parameterization of σ (like the one of Bernanke et al. (1999), who set $\sigma^2 = 0.28$), the convexity case applies, i.e. there is a unique solution on $(0, \bar{\omega}^*)$.

²⁰This follows from the assumption of Bernanke et al. (1999) that the product of the default threshold and the hazard rate $\bar{\omega}h(\bar{\omega})$ is increasing in $\bar{\omega}$.

²¹The analysis for the IPC would be very similar, as the sign of $B(\bar{\omega})$ depends on the sign of the following iso-morphic expression: $(\Gamma'' - \mu G'')([\Gamma - \mu G]s - 1) - 2s(\Gamma' - \mu G')^2 \leq 0$.

Figure 9: Illustration of the Optimal CSV Contract without Aggregate Risk and the Effects of σ .



Notes: parameterization of σ in the upper panel follows Bernanke et al. (1999), i.e. $\sigma^2 = 0.28$, in the lower panel $\sigma^2 = 2.28$.

Appendix B.2. With Aggregate Risk

In the presence of aggregate uncertainty, the loan contract between the bank and entrepreneur i is affected both by the entrepreneur's idiosyncratic risk ω^i and by the ex-post realization of R_{t+1}^k . In this appendix, we establish a positive relation between the capital/net worth ratio $Q_t K_{t+1}^i / N_{t+1}^i$ and the ex ante (expected) EFP $s_t \equiv E_t(R_{t+1}^k / R_{t+1})$. Again, we suppress any time subscripts and index superscripts.

For this purpose, it is convenient to write the profits per unit of capital expenditures as $\tilde{u}\omega R^k$, where \tilde{u} denotes an aggregate shock to the gross rate of return on capital, and ω continues to denote the idiosyncratic shock, with $E(\tilde{u}) = E(\omega) = 1$. Using the definitions from the main text and Appendix B.1, we can then rewrite the bank's

constrained profit maximization problem in equation (8) as

$$\max_{k, \bar{\omega}} E \{ [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] \tilde{u}sk - (k - 1 - n) \} \quad \text{s.t.} \quad E \{ [1 - \Gamma(\bar{\omega})] \tilde{u}sk - \tilde{u}s \} \geq 0$$

The corresponding Lagrangian,

$$\mathcal{L} = E \{ [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] \tilde{u}sk - (k - 1 - n) + \lambda ([1 - \Gamma(\bar{\omega})] \tilde{u}sk - \tilde{u}s) \},$$

yields the first-order conditions (FOC)

$$\begin{aligned} k : \quad & E \{ [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] \tilde{u}s - 1 + \lambda [1 - \Gamma(\bar{\omega})] \tilde{u}s \} = 0, \\ \bar{\omega} : \quad & E \{ [\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})] \tilde{u}sk - \lambda \Gamma'(\bar{\omega}) \tilde{u}sk \} = 0, \\ \lambda : \quad & E \{ [1 - \Gamma(\bar{\omega})] \tilde{u}sk - \tilde{u}s \} = 0. \end{aligned}$$

As discussed in the main text, we assume that the borrower's PC must be satisfied *ex post*, i.e. conditional on the realization of \tilde{u} . As a consequence, $\bar{\omega}$ and all functions thereof, such as $\Gamma(\bar{\omega})$ and $\Gamma'(\bar{\omega})$, are independent of \tilde{u} . Using this assumption, we the FOCs simplify to

$$\begin{aligned} k : \quad & E \{ [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] \tilde{u}s + \lambda [1 - \Gamma(\bar{\omega})] \tilde{u}s \} = 1, \\ \bar{\omega} : \quad & [\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})] = \lambda \Gamma'(\bar{\omega}), \\ \lambda : \quad & [1 - \Gamma(\bar{\omega})] k = 1. \end{aligned}$$

Taking partial derivatives of the borrower's *ex-post* PC w.r.t. k and $\bar{\omega}$, we obtain

$$\frac{\partial}{\partial k} = 1 - \Gamma(\bar{\omega}) - \Gamma'(\bar{\omega}) k \frac{\partial \bar{\omega}}{\partial k} = 0 \quad \Rightarrow \quad \frac{\partial \bar{\omega}}{\partial k} = \frac{1 - \Gamma(\bar{\omega})}{\Gamma'(\bar{\omega}) k} > 0$$

and

$$\frac{\partial}{\partial s} = -\Gamma'(\bar{\omega}) k \frac{\partial \bar{\omega}}{\partial s} = 0 \quad \Rightarrow \quad \frac{\partial \bar{\omega}}{\partial s} = 0.$$

Following Bernanke et al. (1999), define $\Upsilon(\bar{\omega}) \equiv \Gamma(\bar{\omega}) - \mu G(\bar{\omega}) + \lambda [1 - \Gamma(\bar{\omega})]$. Then totally differentiating the FOC

w.r.t. k ,

$$\begin{aligned}
& E \left\{ \tilde{u}' Y(\bar{\omega}) + \tilde{u}' s Y'(\bar{\omega}) \left(\frac{\partial \bar{\omega}}{\partial s} ds + \frac{\partial \bar{\omega}}{\partial k} dk \right) \right\} = 0 \\
\Leftrightarrow & E \left\{ \tilde{u}' s Y'(\bar{\omega}) \frac{\partial \bar{\omega}}{\partial k} \right\} dk = -E \left\{ \tilde{u}' Y(\bar{\omega}) + \tilde{u}' s Y'(\bar{\omega}) \frac{\partial \bar{\omega}}{\partial s} \right\} ds \\
\Rightarrow & \frac{dk}{ds} = - \frac{E \left\{ \tilde{u}' Y(\bar{\omega}) + \tilde{u}' s Y'(\bar{\omega}) \frac{\partial \bar{\omega}}{\partial s} \right\}}{E \left\{ \tilde{u}' s Y'(\bar{\omega}) \frac{\partial \bar{\omega}}{\partial k} \right\}} = - \frac{E \left\{ \tilde{u}' Y(\bar{\omega}) \right\}}{E \left\{ \tilde{u}' s Y'(\bar{\omega}) \frac{\partial \bar{\omega}}{\partial k} \right\}} > 0,
\end{aligned}$$

where we use the previous findings that $\partial \bar{\omega} / \partial k > 0$, $\partial \bar{\omega} / \partial s = 0$, and

$$Y'(\bar{\omega}) = \underbrace{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega}) - \lambda(\bar{\omega}) \Gamma'(\bar{\omega})}_{= 0 \text{ from the FOC w.r.t. } \bar{\omega}} + \lambda'(\bar{\omega}) [1 - \Gamma(\omega)] = \lambda'(\bar{\omega}) k^{-1} < 0.$$

Hence, the optimal contract implies a positive relation between the capital/net worth ratio k and the ex-ante EFP s also in the case with aggregate risk, similar to Bernanke et al. (1999).

5. References

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