

Winter, Christoph; Röhrs, Sigrid

**Conference Paper**

## Public versus Private Provision of Liquidity: Is There a Trade-Off?

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2014: Evidenzbasierte Wirtschaftspolitik  
- Session: Sovereign Debt, No. F12-V4

**Provided in Cooperation with:**

Verein für Socialpolitik / German Economic Association

*Suggested Citation:* Winter, Christoph; Röhrs, Sigrid (2014) : Public versus Private Provision of Liquidity: Is There a Trade-Off?, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2014: Evidenzbasierte Wirtschaftspolitik - Session: Sovereign Debt, No. F12-V4, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft, Kiel und Hamburg

This Version is available at:

<https://hdl.handle.net/10419/100419>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# Public versus Private Provision of Liquidity: Is There a Trade-Off?\*

Sigrid Röhrs<sup>†</sup> and Christoph Winter<sup>‡</sup>

February 6, 2014

## Abstract

Many researchers have recommended to increase public debt in the aftermath of the financial crisis in order to relax borrowing constraints for private households. This advice is based on the common assumption that borrowing conditions of private agents are exogenous to public policy. We study the impact of government debt on the provision of private credit in an economy in which borrowing limits arise because of limited contract enforceability and are thus determined as equilibrium outcomes. As such, they also depend on public policy, in particular on the amount of public debt. Using an incomplete markets economy in which households are subject to uninsurable earnings shocks, we show that an increase in government debt crowds out the supply of private credit. We also find that government debt has significantly different implications for aggregate welfare and economic activity in general if borrowing constraints are endogenous.

**Key words:** Government Debt, Endogenous Borrowing Constraints, Limited Commitment, Incomplete Markets, Crowding Out

**JEL classification:** E2, E62, E44, D52,

---

\* *Acknowledgements:* We would like to thank Victor Tsyrennikov, Michael Golosov and Fabrizio Zilibotti for helpful comments. Winter gratefully acknowledges financial support from NCCR-Finrisk.

<sup>†</sup>Goethe University Frankfurt, Chair of Macroeconomics and Development, Office HOF 3.42, Grüneburgplatz 1, D-60323 Frankfurt am Main (Germany), +49 (069) 798 337 95, Sigrid.Roehrs@hof.uni-frankfurt.de

<sup>‡</sup>University of Zurich, Department of Economics, Office MUB-G405, Mühlebachstrasse 86, 8008 Zurich (Switzerland), +41 (044) 634 52 66, christoph.winter@econ.uzh.ch

# 1 Introduction

In the aftermath of the financial crisis, many countries, including the US, have seen a large increase in their government debt/GDP ratios. At the same time, many scholars have argued that the financial crisis has led to a tightening of credit constraints. Prominent examples include Hall (2011), Guerrieri and Lorenzoni (2011) as well as Eggertson and Krugman (2012). In order to understand the effects of government debt in the aftermath of the financial crisis, it is thus crucial to understand the interaction between public debt and borrowing constraints. This is our aim in this paper.

So far, the literature has identified two important channels through which private borrowing constraints and the supply of government debt interact. First, it is well known that the Ricardian Equivalence proposition Barro (1974) does not hold if there are binding borrowing constraints. As a consequence, changes in government debt affect economic activity and welfare. Second, government debt can help to relax borrowing constraints. This point was made, among others, by Woodford (1990), Aiyagari and McGrattan (1998), Holmstrom and Tirole (1998), Flodén (2001) and most recently by Azzimonti, de Francisco, and Quadrini (2012). By relaxing private borrowing constraints, an increase in government debt can thus help to restore Ricardian equivalence.

A common approach to model borrowing limits is to impose them in an ad-hoc fashion. In some cases, borrowing is ruled out altogether by imposing an ad-hoc limit of zero, see e.g. Aiyagari and McGrattan (1998). However, this approach ignores that in reality, borrowing constraints are likely to arise because of agency problems (e.g. adverse selection, limited commitment) or other type of frictions (e.g. transactions costs) in credit markets.

Ignoring the specific nature of the friction when studying the impact of public policy might be problematic, as public policy may affect the size of the friction and thus also the borrowing constraint. This point was first made by Yotsuzuka (1987), who discusses several agency problems and concludes that the degree to which government debt has real effects depends on the specific nature of the agency problem. Real effects are strongest if government debt can substitute for missing private credit. However, if the agency problem is such that government debt merely replaces private credit, Ricardian equivalence will continue to hold, even if there are liquidity constraints, and government debt has no real effects.

In this paper, our contribution is to study government debt in an environment in which borrowing limits emerge because private debt contracts are not enforceable. We aim at answering the following questions. How does public debt affect the provision of private liquidity (i.e. credit)? What does this imply for the effects of government debt on real activity, in particular the accumulation of private capital and the equilibrium prices of capital and labor? And, finally, how does the interaction between public debt and the provision of private credit influence the welfare effects of government debt?

The limited commitment environment is embedded in a production economy in which markets are incomplete, as in Aiyagari (1994). Households are subject to idiosyncratic income realizations. There is no aggregate risk, implying that government debt and private capital are perfect substitutes from the point of view of households who wish to transfer resources across periods.<sup>1</sup> Households can borrow and lend using an asset which pays off independent of the realization of the idiosyncratic income shock. Following Zhang (1997), Alvarez and Jermann (2000), Kehoe and Levine (2001), Krueger and Perri

---

<sup>1</sup>Gomes, Michaelides, and Polkovnichenko (2008) study an economy with aggregate risk, in which government debt and private capital are imperfect substitutes. They assume ad-hoc borrowing limits.

(2011) and Abraham and Cárceles-Poveda (2010), borrowers can default on their debt obligations. If they do, they are excluded from borrowing and lending forever. Borrowing limits are set in equilibrium such that borrowers always have an incentive to repay their debt, independently of the realization of their income process. The resulting borrowing limits are tighter than the natural borrowing limit defined by Aiyagari (1994), but looser than the popular ad-hoc limit of zero, which restricts private borrowing altogether. To the extent that the provision of government debt affects the incentive to default, it will also affect the borrowing limit.

Assuming market incompleteness is appealing in our context since it allows us to generate a realistic wealth distribution (see Cordoba 2008). This is because market incompleteness limits risk-sharing opportunities. A realistic degree of wealth inequality is important for our purpose, since a large fraction of US households are in debt and thus strongly affected by changes in the borrowing constraint. Moreover, the extremely unequal asset distribution observed in the US implies that a large fraction of the population receives mainly income from supplying labor. This, in turn, is important in order to evaluate the welfare effects of government debt arising from the changes in the equilibrium prices for capital and labor, which have a different impact on the wealth-rich and the wealth-poor (see Röhrs and Winter 2013). Moreover, Abraham and Cárceles-Poveda (2010) show that if markets are incomplete, assuming limited commitment implies that borrowing limits are monotonically increasing in income, a feature that is consistent with the data.

In our model, Ricardian equivalence does not hold because there are liquidity constraints and taxes are distortive. A higher public debt/GDP ratio thus crowds out private capital, and the equilibrium interest rate rises. Laubach (2009) empirically documents that an increase in US government debt/GDP ratio has indeed a significant positive impact on the real interest rate. In our model, a higher interest rate makes it more attractive for debtors to renege on their obligations. As a result, credit limits set by private lenders become tighter, the higher the government debt/GDP ratio. Our framework thus suggests that there is indeed a trade-off between public debt and the supply of private credit. As a result, government debt is less effective for providing liquidity if borrowing constraints react to changes in the debt/GDP ratio.

Our results have important implications in the light of the recent financial crisis. Many authors have suggested that during the financial crisis, financial frictions became larger and borrowing constraints tighter. Our model would predict that this effect is reinforced by the increase in government debt that we observe in the aftermath of the crisis. It is interesting to contrast this implication with the prominent policy recommendation that the government should expand its public debt if borrowing limits become tighter, in order to relax borrowing constraints (see e.g. Eggertson and Krugman (2012)). According to our results, an increase in government debt would force households to deleverage even more.

We show that in the presence of endogenous borrowing limits, government debt does not necessarily increase aggregate welfare, an observation that was also made by Yared (2013). In our model, the welfare effects of government debt are dominated by the development of equilibrium prices for capital and labor, as in Flodén (2001) or Röhrs and Winter (2013). Compared to the case in which borrowing limits are exogenously kept at their level corresponding to the long-run average debt/GDP in the US of 66 percent, the interest rate and the wage rate react less to a decrease in the debt/GDP ratio, and more to an increase in the debt/GDP ratio, relative to the long-run average. The different reaction of

aggregate prices is caused by the endogenous response of the endogenous borrowing limits, which are, by construction, looser than at the benchmark if we are at the debt/GDP ratio below the long-run average and tighter if the debt/GDP ratio is higher. As a consequence of the differences in aggregate prices, the long-run welfare effects of changes in the debt/GDP ratio are significantly smaller when borrowing limits adjust endogenously. The gap can be as large as 0.45 percentage points of lifetime consumption of the average household.

Our work contributes to several strands of the literature. The framework is closely related to the work by Ábrahám and Cárceles-Poveda (2010) who study a revenue neutral tax reform that eliminates capital income taxation. Since Ábrahám and Cárceles-Poveda (2010) abstract from government debt, they cannot analyze the trade-off between public debt and private credit, which is the main focus of our paper. The interactions between public and private insurance in models with limited commitment are also studied in other papers. Attanasio and Ríos-Rull (2000) study how certain types of insurance for aggregate shocks affect private allocations. Krueger and Perri (2011) as well as Broer (2011) analyze whether the government should provide public insurance against idiosyncratic income risk by implementing a progressive tax system. In line with these papers, our results also suggest that the provision of public insurance crowds out private insurance.

We also contribute to the debate on the importance of endogenous borrowing limits for the analysis of public policy. Mateos-Planas and Seccia (2006) study a change in social security that reduces income variability in an exchange economy with incomplete markets. They find that endogenous credit limits have almost no impact on the aggregate welfare effects of social security if the economy is closed. Andolfatto and Gervais (2008) as well as Rojas and Urrutia (2008) analyze the impact of social security in a life cycle model with incomplete markets. They conclude that social insurance has different welfare and distributional implications if debt constraints are endogenous. We find that endogenous borrowing constraints substantially alter the welfare effects of government debt.

Our paper is also related to the strand of the literature that analyzes the role of public debt in relaxing liquidity constraints in the production side of the economy, following the seminal work by Holmstrom and Tirole (1998). In many papers, borrowing constraints take the form of collateral constraints. Important recent examples include Angeletos et al. (2012) and Arai, Kunieda, and Nishida (2013). Since government bonds serve as collateral, an increase in the supply of bonds helps to relax borrowing constraints. The interbanking market is an important real-world example for a market in which government bonds are used as collateral, e.g. in repo transactions. In this paper, we focus on uncollateralized credit of households, in the spirit of Woodford (1990), Aiyagari and McGrattan (1998), Flodén (2001) or more recently Guerrieri and Lorenzoni (2011) and many others. We abstract from collateralized credit, because households typically use durables in order to secure their loans, (see for example Fernández-Villaverde and Krueger 2011), leaving no direct role for government bonds.

The remainder of the paper is structured as follows: In section 2 we describe the model. Section 3 shows how we calibrate the model to the US economy. Section 4 contains a discussion of our results. Section 5 concludes and contains suggestions for further research.

## 2 The Model

The economy we consider is a neoclassical growth model with incomplete markets where households face uninsurable income shocks, as in Aiyagari (1994). The economy consists of three sectors: households, firms and a government. In the following, we describe the three sectors in greater detail. We start by describing the bonds that households in our economy use to accumulate savings.

### 2.1 Supply and Demand for Bonds

Households self-insure against income fluctuations by saving in one-period risk-free bonds. Bonds are issued by firms and the government, as in Aiyagari and McGrattan (1998) and Flodén (2001). Bonds issued by firms are claims to physical capital. We abstract from aggregate risk, which implies that claims to physical capital and government bonds are perfect substitutes and thus yield the same return,  $r_t$ .<sup>2</sup> Differently from Aiyagari and McGrattan (1998) and Flodén (2001), we also allow households to borrow up to a certain limit. We view this as an important modification, given that the fraction of households that actually borrow in the data is substantial.<sup>3</sup>

### 2.2 Household Sector

The economy is populated by a continuum of ex-ante identical, infinitely lived households with total mass of one. Households maximize their expected utility by making a series of consumption, leisure and savings choices subject to a budget constraint and a borrowing limit on assets. In period  $t = 0$ , before any uncertainty has realized, their expected utility is given by

$$U(\{c_t, l_t\}_{t=1,2,\dots}) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

where  $\beta$  is the subjective discount factor. The per-period utility function,  $u(\cdot)$ , is assumed to be strictly increasing, strictly concave and continuously differentiable. Additionally, the first derivative is assumed to satisfy the following limiting (Inada) conditions:

$$\begin{aligned} \lim_{c \rightarrow 0} u_c(c, l) &= \infty, \quad \lim_{c \rightarrow \infty} u_c(c, l) = 0 \\ \lim_{l \rightarrow 0} u_l(c, l) &= \infty \end{aligned}$$

Household productivity is subject to a shock,  $\epsilon$ , that follows a Markov process with transition matrix  $\pi(\epsilon'|\epsilon)$ .

A household with bond holding (or wealth) state  $a$  today faces the following per-period budget constraint:

$$c_t + a_{t+1} = y_t + a$$

where  $a_{t+1}$  denotes the bond holdings of a household in the next period (savings).  $y_t$  is the household's (after-tax) income. Notice that  $a_{t+1}$  may also be negative, in which case the household borrows. Borrowing is restricted:

$$a_{t+1} \geq \underline{a}$$

---

<sup>2</sup>In Gomes, Michaelides, and Polkovnichenko (2008) and Gomes, Michaelides, and Polkovnichenko (2010), government bonds and private capital are imperfect substitutes due to aggregate uncertainty.

<sup>3</sup>Borrowing by households can be interpreted as bonds that are issued to other households ('IOUs') or to the government.

How exactly the borrowing limit will be determined assuming a limited commitment model will be discussed further below.

The government can tax labor income at some proportional tax rate,  $\tau_{l,t}$ , as well as financial income at some proportional tax rate,  $\tau_{a,t}$ , and can redistribute income via lump sum transfers,  $\chi$ . In our model we assume that the transfers stay constant over time and focus on the experiment of changing government debt and adjusting one of the two tax rates. We assume that only non-negative financial income is taxed or in other words there are no proportional subsidies in the face of financial losses. More precisely, we define the tax on financial income  $\tau_{a,t}$ , as follows:

$$\tau_a(a) = \begin{cases} \bar{\tau}_a & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases}$$

The after-tax interest rate is therefore given by  $\bar{r}_t = (1 - \tau_a(a))r_t$ . The after-tax wage rate is given by  $\bar{w}_t = (1 - \tau_{l,t})w_t$  where  $w_t$  is the price of labor in the economy. After-tax income is thus given by:

$$y_t = \bar{w}_t \epsilon_t (1 - l_t) + \bar{r}_t a + \chi$$

We follow the literature on limited commitment, for example Zhang (1997), Alvarez and Jermann (2000), Kehoe and Levine (2001), Krueger and Perri (2011) and Ábrahám and Cárceles-Poveda (2010), by assuming that households cannot commit to honor their debt contracts. Furthermore we assume that if households default, they are excluded from future borrowing and lending (autarky). The value of autarky can be expressed as follows:

$$\begin{aligned} V(\epsilon) &= \max_{c^{aut}, l^{aut}} \left\{ u(c^{aut}, l^{aut}) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) V(\epsilon'; \theta') \right\} \\ \text{s.t. } c^{aut} &= (1 + \lambda) \bar{w} \epsilon (1 - l^{aut}) + \chi \\ \theta' &= \Gamma[\theta] \end{aligned}$$

where  $\lambda$  is a parameter describing a consumption gain (if  $\lambda > 0$ ), which translates into a utility gain, resulting in autarky.  $\lambda$  captures, in reduced form, the differences between default regulation in reality and in our model. For example, a  $\lambda > 0$  could arise because in reality, exclusion from financial markets is temporary only, while exclusion is permanent in our model.<sup>4</sup> We calibrate the value of  $\lambda$  to match the number of people in debt.<sup>5</sup>

For households who do not default, the optimization problem can be stated as follows:

$$\begin{aligned} W(a, \epsilon) &= \max_{c, l, a'} \left\{ u(c, l) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) W(a', \epsilon') \right\} \\ \text{s.t. } c + a' &= \bar{w} \epsilon (1 - l) + (1 + \bar{r}) a + \chi \\ a' &\geq \xi(\epsilon; \theta) \text{ for all } \epsilon'|\epsilon \text{ with } \pi(\epsilon'|\epsilon) > 0 \\ \theta' &= \Gamma[\theta] \end{aligned} \tag{1}$$

<sup>4</sup>Assuming instead a temporary exclusion from financial markets would make the model more complicated and does not yield any qualitative value added to the analysis.

<sup>5</sup>There may also be additional costs related to default (e.g. social stigma) which are not modeled. This would make  $\lambda$  smaller. In the calibration section, we find that for our setting to be able to match the number of people in debt we need  $\lambda > 0$ .

Note that the household's borrowing limit  $\xi$  is a function of the distribution of assets ( $\theta$ ) and the realization of the income shock  $\epsilon$ .

More precisely, the borrowing limit is defined as follows:

$$\begin{aligned}\underline{a}(\epsilon; \theta) &= \{ \underline{a} : W(a = \underline{a}, \epsilon; \theta) = V(a = 0, \epsilon; \theta) \} \\ \xi(\epsilon; \theta) &\equiv \sup_{\epsilon' : \Pi(\epsilon' | \epsilon) > 0} \{ \underline{a}(\epsilon'; \Gamma(\theta)) \}\end{aligned}\tag{2}$$

We impose that the borrowing limits in equilibrium are set such that there is no default.<sup>6</sup>

Intuitively, the borrowing limit, denoted by  $\underline{a}$ , is defined such that the value of being in autarky is just equal to the value of keeping the debt and staying in the market, if a household is actually at the borrowing limit. Or, in other words, the borrowing limit is the lowest possible asset holdings of an agent so that he still prefers holding on to the debt contract and staying in the market rather than not repaying the debt but being excluded. Depending on the income state  $\epsilon$ , we have a different borrowing limit.

In particular, as shown by Ábrahám and Cárceles-Poveda (2010), for each income state today, the tightest possible borrowing limit is the relevant one. This is because the borrowing limit is set such that households do not have an incentive to default even in the worst possible state tomorrow (given the state today). Note that, depending on the transition matrix, some realizations of  $\epsilon$  might occur with zero probability, given today's state. In this case, there might be multiple borrowing limits, depending on the current realization of the income shock. As we will outline later, this will not be the case in our calibration. Hence, there is only one borrowing limit associated with each income state.<sup>7</sup>

Ábrahám and Cárceles-Poveda (2010) also show that if the period utility function is unbounded below, then equation (2) defines a unique and finite default threshold (see their proposition 2.1). Moreover, their arguments imply that  $\underline{a}_{nat} < \underline{a} \leq 0$ , where  $\underline{a}_{nat}$  is the natural borrowing limit defined such that households are able to repay their debt at every possible contingency without non-negative consumption. This relationship will become important in Section 4, where we analyze to what extent government debt leads to tighter borrowing constraints.

Assuming that the economy is in a stationary equilibrium, we adopt the following definition of the natural borrowing limit from Aiyagari (1994) and Aiyagari and McGrattan (1998):<sup>8</sup>

$$\underline{a}_{nat} = - \frac{(1 - \tau_w)\epsilon_{low}w + tr}{r}\tag{3}$$

---

<sup>6</sup>The general mechanism - that borrowing conditions are tighter (looser) when the interest rate is higher (lower) - would also hold if we allowed for default in equilibrium. As our aim is to model the endogenous reaction of the borrowing conditions and not the default behaviour of households we thus assume a model of limited commitment with no default in equilibrium. See Livshits, MacGee, and Tertilt (2007) and Chatterjee et al. (2007) for a discussion of US bankruptcy laws and a quantitative model of consumer default.

<sup>7</sup>Ábrahám and Cárceles-Poveda (2010) also prove that the same borrowing limits would result by assuming perfectly competitive financial intermediaries.

<sup>8</sup>Notice that we slightly abuse notation here, since our stationarity assumption implies that all variables are detrended. See Appendix, Part A, for a detrended version of the households' problem in our economy.



### 2.3 Firm sector

We assume that the aggregate production technology which is operated by a representative firm to produce output,  $Y_t$ , using aggregate capital,  $K_t$ , and aggregate labor,  $L_t$ , as inputs is given as follows:

$$Y_t = F(K_t, X_t L_t)$$

where  $X_t$  denotes exogenous labor-augmenting technological progress. This technology is assumed to grow exogenously at a constant rate  $X_{t+1} = (1 + g)X_t$ . For simplicity we normalize initial technology to  $X_0 = 1$ , such that:

$$X_t = (1 + g)^t$$

The presence of technological progress implies that households, on average, become richer over time. Technological progress thus increases the propensity of households to borrow.<sup>9</sup>

The aggregate production function,  $F$ , is assumed to have the standard properties, in particular constant returns to scale. This ensures that in competitive equilibrium, the number of firms is indeterminate and we can assume the existence of a representative firm, without loss of generality.

### 2.4 Government sector

The government has to finance a time-invariant amount of government spending,  $G$ , and the total transfers to households,  $TR$ , by issuing new government bonds,  $B_{t+1}$ , and levying taxes on positive asset and labor income. Furthermore, the government services its debt,  $B_t$ , and makes interest payments,  $r_t B_t$ . The government budget constraint is thus given by:

$$G + r_t B_t + TR = B_{t+1} - B_t + \tau_l w_t L_t + \bar{\tau}_a r_t \hat{A}_t \quad (4)$$

where  $\hat{A}_t \geq A_t$  is the tax base for the asset income tax. As explained above taxes are only levied on positive financial income (no proportional transfers from the government for indebted people) and thus the tax base is defined as:

$$\hat{A}_t = \int_{a \geq 0} a d\theta(\epsilon, a)$$

where  $\theta(\epsilon, a)$  denotes the distribution of households over income and asset states. Aggregate transfers have to equal the sum of all individual transfers:

$$\int tr d\theta(\epsilon, a) = TR$$

### 2.5 Stationary equilibrium

Using the characterization of the three sectors we can now define the stationary equilibrium.

**Definition 1.** *Stationary Equilibrium: Given a transition matrix  $\pi$  and a government policy  $B, \tau_a(a), \tau_l, G$  a stationary equilibrium is defined by a stationary distribution of asset and income states  $\theta(a, \epsilon)$ , factor prices  $(r, w) = (r(K), w(K))$ , the value function  $W = W(a, \epsilon)$  and policy functions  $c(a, \epsilon), a'(a, \epsilon)$  such that*

---

<sup>9</sup>Technically, in the detrended version of the household problem which is presented in the appendix, technological progress reduces the discount factor. This reduction also decreases the propensity to save and thus increases borrowing, all other things equal.

1. Households' utility maximization problem is defined in equation (1).
2. Competitive firms maximize profits, such that factor prices are given by

$$w = F_L(K, XL) \quad (5)$$

$$r = F_K(K, XL) - \delta \quad (6)$$

3. The government budget constraint as defined in equation (4) holds.
4. Factor and goods markets have to clear:

- Labor market clearing:

$$N = \int \epsilon(1 - l)d\theta(\epsilon, a) = L$$

- Asset market clearing:

$$A' = \int a'd\theta(\epsilon, a) = K' + B'$$

- Goods market clearing:

$$\int cd\theta(\epsilon, a) + G + I = F(K, XL)$$

where investment  $I$  is given by

$$I \equiv K' - (1 - \delta)K$$

5. Rational expectations of households about the law of motion of the distribution of shocks and asset holdings,  $\Gamma$ , reflect the true law of motion, as given by

$$\theta' = \Gamma[\theta(a, \epsilon)] \quad (7)$$

where  $\theta(a, \epsilon)$  denotes the joint distribution of asset holdings and productivity shocks.

6. Borrowing limits are set such there is no default as given by equation (2).
7. The distribution of assets and income states is stationary  $\theta' = \theta$ .

## 2.6 Welfare Measure

In order to be able to compare the welfare effects of different government policies, we have to define a welfare criterion. We compute the aggregate value function:

$$\Omega = \int W(a, \epsilon; \theta)d\theta(a, \epsilon)$$

This criterion can either be interpreted as (1) a Utilitarian social welfare function where every individual has the same weight for the planner, (2) a steady-state ex ante welfare of an average consumer before realizing income shocks and initial asset holdings or (3) the probability limit of the utility of an infinitely lived dynasty where households utilities are altruistically linked to each other.

### 3 Calibration

We calibrate our model such that it is consistent with long run features of the US economy. Our calibration procedure is closely related to Ábrahám and Cárceles-Poveda (2010), and, in particular, to Röhrs and Winter (2013). We will refer to the resulting allocation as our benchmark economy.

The parameter values that result from our calibration procedure are shown in Table 1. Parameter values that are adopted from the existing literature are given in Table 2. In the following, we discuss the rationale behind our parameter choices in greater detail.

#### 3.1 Utility Function and Production Technology

We assume that preferences can be represented by a constant relative risk aversion utility function:

$$u(c) = \frac{(c^\eta l^{1-\eta})^{1-\mu}}{1-\mu}$$

Note that the coefficient of relative risk aversion, is equal to  $1 - \mu + \eta\mu = 1.3$ . This is well in the range (between 1 and 3) commonly chosen in the literature.  $\eta$  denotes the share of consumption in the utility function. We calibrate  $\eta$  such that the average share of time worked is 0.3. This results in  $\eta = 0.31$ . This choice implies an aggregate Frisch elasticity of 1.3.<sup>10</sup> This is broadly in line with the outcome of other macro models in which the Frisch elasticity of the overall population is considered, but an order of magnitude larger than the Frisch elasticity estimated using micro data from prime age workers.<sup>11</sup>

We assume that the aggregate technology is given by a Cobb-Douglas production function:

$$F(K, XL) = K^\alpha (XL)^{1-\alpha}$$

Initial technology is normalized to  $X_0 = 1$ , such that  $X_t = (1 + g)^t$ . We set  $g = 0.02$ , which implies that our economy grows at a rate of 2 percent per year. The parameter  $\alpha$ , which denotes the share of capital in total production, is set to 0.3. This implies a labor share of 0.7. The discount factor  $\beta$  is chosen such that the model reproduces a wealth-output ratio of 3.1 (cf. Cooley and Prescott (1995) or Ábrahám and Cárceles-Poveda (2010)). Since we do not model housing, wealth is defined as net financial assets excluding housing and other real estate. The resulting  $\beta$  is equal to 0.96. The annual depreciation rate  $\delta$  is set to 7 percent, which is a common value in the literature (see e.g. Trabandt and Uhlig (2011)).

#### 3.2 Taxes and Government Debt

Following Trabandt and Uhlig (2011), we set the labor income  $\tau_l$  to 0.28, the capital income tax rate  $\tau_k$  to 0.36, lump-sum transfers  $tr$  to 0.083 and the debt/GDP ratio to 0.67 in the benchmark.<sup>12</sup> Government spending  $G$  is set such that the government's budget constraint clears, given all other parameters.

<sup>10</sup>For our choice of the utility function, the Frisch elasticity is given by  $(1 - \mu + \eta\mu)/\mu \cdot (T - h)/h$ , where  $T$  denotes the time endowment (normalized to 1 in our case and  $h$  denotes the fraction of time spend at work, in our case 0.3).

<sup>11</sup>The debate on whether micro and macro elasticities are consistent is ongoing. See Keane and Rogerson (2011) for a summary.

<sup>12</sup>Similar values are also reported by Mendoza, Razin, and Tesar (1994).

Table 1: Calibrated Parameter Values

Parameter	Value	Target	Data	Model
Discount factor, $\beta$	0.96	Capital to output ratio	3.1	3.1
Weight of consumption in the utility function, $\eta$	0.31	Average labor supply	0.3	0.3
Borrowing constraint $\underline{a}$	-0.3	% of HH with no assets or debt	0.24	0.25
Gov. spending, $G$	0.15	gov. budget constraint clearing	-	-

Table 2: Parameters Set Exogenously

Parameter	Value
Capital's share, $\alpha$	0.3
Growth rate, $g$	0.02
Debt to GDP ratio, $b$	0.670
Labor tax, $\tau^l$	0.28
Capital tax, $\tau^k$	0.36
Transfers, $tr$	0.083
Risk Aversion $\mu$	2

### 3.3 Income process

We calibrate the vector of income states,  $s$ , and the transition matrix,  $\Pi$ , such that the distribution of earnings and net worth generated by the model are consistent with the data. Disciplining the model such that it is consistent with the skewed distribution of earnings and wealth observable in the US economy is key for assessing the effect of government debt.

We compute the distribution of earnings and net worth from the 2007 Survey of Consumer Finances (SCF) (see Table 3 and 4). Since we do not model housing or collateralized credit, we define net worth as net financial assets excluding housing and other real assets (see also Ábrahám and Cárceles-Poveda (2010)). Earnings are defined as labor earnings (wages and salaries) plus a fraction of business income before taxes, excluding government transfers.<sup>13</sup> This definition corresponds to the concept of earnings that is implied by our model.

Table 3 and 4 show that both earnings and net financial assets are very unequally distributed in the data. The richest 20 percent of the population hold more than 90 percent of all financial assets, net of debt. The distribution of earnings is less skewed. Households in the top quintile earn around 60 percent of the total earnings.

We find the following vector of income states:

$$s = \{0.055, 0.551, 1.195, 7.351\}$$

<sup>13</sup>The SCF does not specify the exact fraction of total business income that is attributable to labor and to capital. We define business income from sole proprietorship or a farm as labor earnings, whereas business income from other businesses or investments, net rent, trusts, or royalties is defined as capital income.

Table 3: Distributional Properties at Benchmark Stationary Economy

	Q1	Q2	Q3	Q4	Q5	Gini
Net financial assets						
Data	-1.60%	0.10%	1.64%	8.29%	91.57%	0.90
Benchmark Calibration	-1.57%	0.88%	3.92%	7.23%	89.54%	0.83
Earnings						
Data	-0.40%	3.19%	12.49%	23.33%	61.39%	0.62
Benchmark Calibration	0.00%	2.38%	12.58%	22.73%	62.31%	0.65

Remarks: Quintiles (Q1-Q5) denote net financial assets (resp. earnings) of a group in percent of total net financial assets (resp. earnings). The entries in 'data' are computed from the 2007 SCF. See main text for precise definitions. Notice that earnings can be negative due to the fact that labor earnings also contain part of the gains (or losses) of small enterprises.

Table 4: Upper Percentiles of Wealth Distribution at Benchmark

	upper 10%	upper 5%	upper 1%
Net financial assets			
Data	79.64%	66.83%	39.09%
Benchmark Calibration	70.58%	47.03%	13.53%

Remarks: The table shows the percent of net financial assets held by the wealthiest 10% (upper 10%), 5% (upper 5%) and 1% (upper 1%).

It should be noted that the highest income state is more than 130 times as high as the lowest income state.

Furthermore, we get the following transition matrix for the income states:

$$\Pi = \begin{bmatrix} 0.940 & 0.040 & 0.020 & 0.000 \\ 0.034 & 0.816 & 0.150 & 0.000 \\ 0.001 & 0.080 & 0.908 & 0.012 \\ 0.100 & 0.015 & 0.060 & 0.825 \end{bmatrix}$$

As can be seen from the transition matrix, there is a 10 percent probability of moving from the highest income state today to the lowest income state tomorrow. This generates a strong saving motive for income-rich households, leading to the high degree of wealth inequality that we also observe in the data. The same mechanism is also present in the transition matrix found by Castañeda, Díaz-Giménez, and Ríos-Rull (2003).

### 3.4 Borrowing limit

We calibrate the borrowing limit to match the percentage of households with negative or zero financial assets in the 2007 SCF (24 percent). We find a borrowing limit of  $\underline{a} = -0.3$ .

## 4 Results

It is well-known from the previous literature that government helps to relax binding borrowing limits if borrowing limits are ad-hoc and exogenous. In this section, we study to what extent the presence of endogenous borrowing limits changes the liquidity role that is played by government debt.

We find that endogenizing borrowing constraints makes an increase in government debt less effective as a way to inject liquidity in the economy. The reason is that private lenders become more restrictive when providing credit, the higher the debt/GDP ratio. This is because a higher debt/GDP ratio changes aggregate prices  $r$  and  $w$  such that households have a higher incentive to renege on their repayment obligations.

Another important result from the previous literature is that changes in government debt affect real activity if borrowing constraints are binding. We show that in the presence of endogenous borrowing limits, changes in the debt/GDP ratio have a smaller impact on the equilibrium rental prices for capital and labor. The reason is that the fraction of constrained households tends to be smaller under endogenous borrowing limits, compared to the case in which constraints are exogenous.

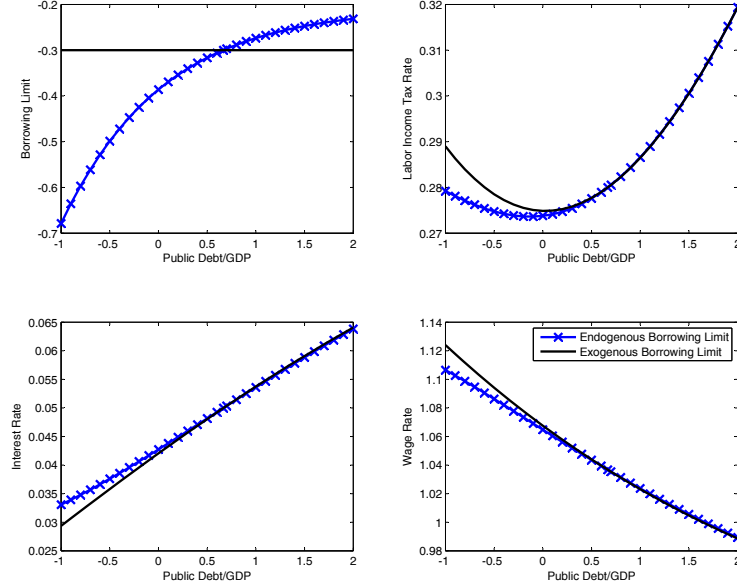
Interestingly, we show that endogenous borrowing constraints also affect the long-run welfare implications of changes in the debt/GDP ratio. Compared to the case in which the borrowing limits are exogenously fixed, higher debt/GDP ratios lead to a sharp decline in the welfare of poor households for which borrowing limits are more relevant. Instead, if we compare the welfare effects of government debt for the overall population, we find that government debt has much weaker effects on total welfare if we consider endogenous borrowing limits. The reason is that aggregate prices, which are the main driver of the welfare responses at the aggregate level, react much less to changes in the debt/GDP ratio if we allow borrowing limits to adjust.

Technically, we proceed as follows. We use the calibrated parameters of our benchmark economy. In our experiments where we keep the borrowing limit fixed, we use  $\underline{a} = -0.3$ , independently of the debt/GDP ratio. If we study the impact of endogenous borrowing limits, we calculate the borrowing limit as an equilibrium outcome of our model. For all debt/GDP ratios that are different from the benchmark value of 0.66, the borrowing limit that emerges endogenously could thus be different from  $\underline{a} = -0.3$ . It is important to note that at the benchmark, both economies exactly coincide. This allows us to study the impact of long-run changes in the debt/GDP ratio, relative to the benchmark scenario.

**Public debt as a means to relax private borrowing constraints.** We first analyze how borrowing limits responds to changes in the debt/GDP ratio. In a second step, we then analyze whether an increase in government debt implies that borrowing constraints become less binding in our model.

We find that borrowing limits become more restrictive if the government increases the amount of public debt, relative to GDP. This can be seen from Figure 4 (first panel), where we plot the borrowing

limits for various stationary equilibria, which are characterized by different debt/GDP ratios. Hence, public debt appears to crowd-out the supply of private credit. Notice that we plot the borrowing limit for households that are subject to the lowest income shock only, as this is the limit applied to all households.<sup>14</sup>



**Figure 1: Implied Borrowing Limits, Tax Rates, Interest Rates and Wage Rates for Different Stationary Equilibria.** In this exercise we plot the changes in the borrowing limits (first panel), tax rates (second panel), interest rates (third panel) and wage rates (fourth panel) implied by our model for different stationary equilibria that differ with respect to the public debt/GDP ratio. In the benchmark public debt amounts to 2/3 of GDP. The labor income tax is adjusted to balance the budget. Two cases: (1) exogenous (fixed) borrowing limit (black line); (2) endogenous borrowing limit (blue line with crosses).

In order to construct Figure 4, we adjust the capital income tax such that the government's budget constraint is satisfied for different public debt/GDP ratios. The results for the case in which the labor income tax is adjusted are similar, and are therefore relegated to the Appendix.

Our result that private lenders become more restrictive for higher the debt/GDP ratio can be explained as follows. As the second panel of Figure shows, a larger debt/GDP ratio increases the market-clearing interest rate. A higher interest rate in turn implies that borrowers need to pay more in order to service their debt. This makes it more likely that they renege on their obligations. As a response, lenders become more reluctant to provide credit.

The link between the level of the interest rate and the debt/GDP ratio stems from the fact that debt is non-neutral in our model. An increase in public debt crowds-out private capital, as households cannot respond optimally by reducing their saving because of binding borrowing constraints. Moreover, taxes are distortive. As a result, stationary equilibria with higher debt/GDP ratios are associated with higher interest rates. Interestingly, Laubach (2009) provides evidence for a strong positive link between government debt and interest rates in the data.

<sup>14</sup>This is due to the specification of the earnings transition matrix, which assigns positive probability of receiving the worst productivity shock in the following period, independently of the current productivity shock.

Given our assumption about technology, lower levels of private capital do not only imply a higher equilibrium interest rate, but also a lower equilibrium wage rate. All other things equal, a lower wage rate makes the autarky option less attractive for households. In autarky, households are excluded from financial markets, which means that they can rely only on adjustments in their labor supply in order to smooth their consumption across periods. Hence, a lower wage rate should increase the willingness of private lenders to provide credit in stationary equilibria with higher debt/GDP ratios. Therefore, the decrease in the wage rate and the increase in the interest rate work in opposite directions with respect to their impact on private borrowing limits.

However, the increase in the interest rate and the associated rise in the relative value of defaulting dominates the effect of a decrease in the wage rate. This is because households in the low-income group, whose behavior is relevant for determining the borrowing limit, have to work hard regardless of whether they are in autarky or not. Since our calibration implies that income shocks are very persistent, the change in the wage rate affects the autarky value and the value from obeying the contract in roughly the same way. In contrast, the change in the interest rate only affects the value of debt repayments.

We now turn to the question to what extent changes in the supply for government bonds relax borrowing limits of households in our framework. In order to provide an answer, we use the natural borrowing limit, as defined in Section 2.2. Recall that the natural borrowing limit  $\underline{a}_{nat}$  is the maximum amount that a household in our economy would ever want to borrow. It is the amount of debt a household can service in the (very unlikely but possible event) of being hit by the worst income shock  $\epsilon_{low}$  forever, working full time and consuming zero. Since our specification of the households' utility function satisfies the Inada conditions,  $\underline{a}_{nat}$  is not binding. Therefore, the distance between  $\underline{a}_{nat}$  and  $\underline{a}$ , given by  $\Delta_{\underline{a}} \equiv |\underline{a}_{nat} - \underline{a}|$ , can be seen as a measure for how binding  $\underline{a}$  is: if  $\Delta_{\underline{a}}$  is close to zero, then  $\underline{a}$  is not very restrictive. If, however,  $\Delta_{\underline{a}}$  is close to  $\underline{a}_{nat}$ , this implies,  $\underline{a}$  is close to zero, and borrowing limits are relatively tight.<sup>15</sup>

As Figure 2 reveals,  $\Delta_{\underline{a}}$  is lower, the higher the stationary equilibrium ratio of debt/GDP ratio. This suggests that borrowing constraints become less binding as the government extends its supply of bonds. If we keep the borrowing limit fixed at its benchmark level of  $\underline{a} = 0.3$ , the difference solely reflects changes in the natural borrowing limit. Since the natural borrowing limit is an increasing function in  $r$  and a decreasing function in  $(1 - \tau_w)w$ ,  $\underline{a}_{nat}$  is lower for higher debt/GDP ratios, and the distance between  $\underline{a}_{nat}$  and  $\underline{a}$  shrinks. Government debt relaxes borrowing constraints, as predicted by the previous literature (see e.g. Woodford 1990 or Aiyagari and McGrattan 1998).

If the borrowing limit is endogenously determined by lenders such that borrowers have no incentive to default, we find that  $\underline{a}$  and  $\underline{a}_{nat}$  respond similarly to changes in the debt/GDP ratio. Hence, the difference between the two, given by  $\Delta_{\underline{a}}$ , responds much less to changes in the debt/GDP ratio, compared to the case in which the borrowing limit is exogenously fixed. This suggests that public debt is less effective in relaxing private borrowing constraints if they are linked to public debt as specified by our model.

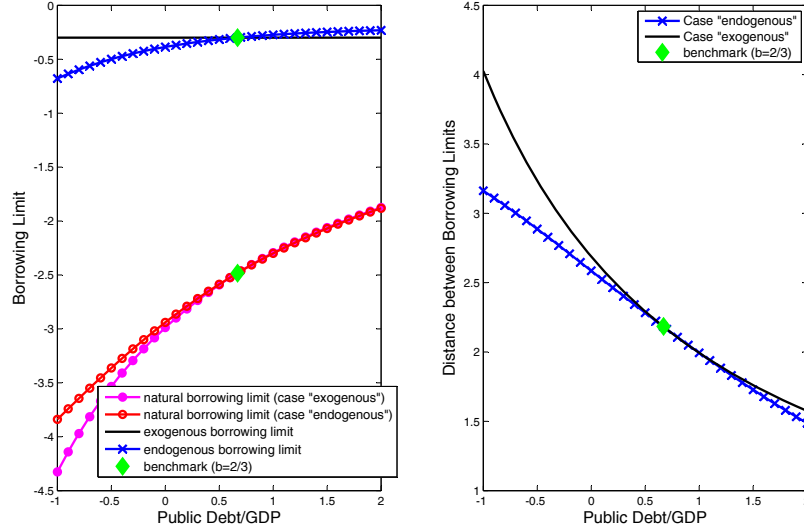
A similar picture emerges if we adopt a different measure for the bindingness of borrowing constraints. In Figure 3, we plot the fraction of constrained households, depending on the debt/GDP ratio. We find that for exogenous borrowing limits, the fraction of constrained households falls much faster if we

<sup>15</sup>Also recall from Section 2.2 that  $\underline{a}_{nat} < \underline{a} \leq 0$ .



increase the debt/GDP ratio. This confirms our finding that government debt is more useful in relaxing borrowing limits if these limits are exogenous to government debt.

In essence, our results suggests that ad-hoc borrowing constraints, which are the dominant choice in the literature, may lead researchers to overstate the liquidity role of government debt. In the next subsection, we analyze how the interaction between private borrowing limits and public debt changes the real effects of government debt.



**Figure 2: Natural Borrowing Limit, Endogenous Borrowing Limit and Distance Between them.** In the left-hand side panel we plot the changes in the natural borrowing limit case 'exogenous' (pink filled circles), the natural borrowing limit case 'endogenous' (red empty circles) the exogenous (fixed) borrowing limit (black line) and the endogenous borrowing limit (blue crosses) arising from limited commitment for stationary equilibria differing with respect to the public debt/GDP ratio. In the right-hand side panel we plot the distance between the two borrowing limits for the same set of stationary equilibria. At the benchmark public debt amounts to  $2/3$  of GDP (green diamond).

**Impact of government debt on economic activity.** The question of how public debt affects economic activity is closely linked to its impact on private borrowing constraints. If the Ricardian Equivalence proposition holds (see Barro 1974), government debt has no effects on private economic activity. However, binding borrowing constraints can lead to a break-down of the Ricardian Equivalence proposition.

It is thus interesting to analyze the role that the specification of the borrowing constraint plays for the real effects of government debt. As Figure 3 reveals, the fraction of constrained households is smaller for endogenous borrowing limits. Hence, we would expect that the effects of government debt on economic activity, summarized by the development of equilibrium prices for capital and labor, are smaller for endogenous borrowing limits as well. Recall from Figure 4 that this is indeed the case.

Finally, we would like to mention that in our economy, Ricardian equivalence does not only fail to hold because of borrowing constraints, but also because of distortionary taxation. However, we argue that borrowing constraints are the important driver behind the economic effects of government debt. In the Appendix, we present the results of an experiment in which we replace distortive tax rates by

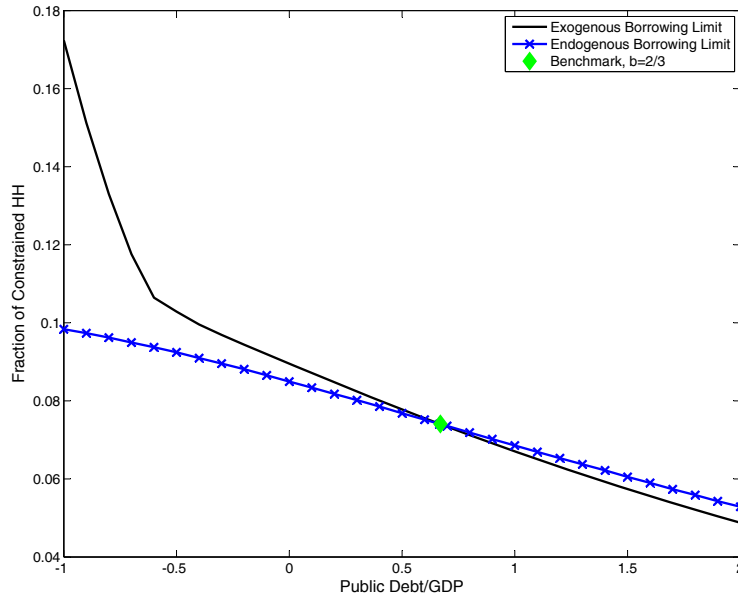


Figure 3: **Fraction of Constrained Households.** Here we plot the number of households sitting on the borrowing constraint for stationary equilibria differing with respect to the public debt/GDP ratio. At the benchmark public debt amounts to 2/3 of GDP (green diamond). Two cases: (1) exogenous borrowing limit (black line); (2) endogenous borrowing limit (blue line with crosses).

lump-sum taxation. We find that our results are almost unchanged, implying that the real effects of government debt are mainly driven by binding borrowing limits, and not by the fact that tax rates are distorting economic activity.

As we will see in the next paragraph, the interaction between aggregate prices and borrowing limits has important implications for the welfare effects of government debt.

**The Impact of Government Debt on Welfare.** Our previous analysis suggested that endogenous borrowing limits change the ability of government debt to provide liquidity to private households. We now examine to what extent this finding affects the long-run welfare effects of government debt.

We find that the welfare effects of government debt are significantly different if we introduce endogenous borrowing constraints. Consider Figure 3, where we plot the welfare effects of changes in the debt/GDP ratio for wealth-poor households, relative to the benchmark debt/GDP ratio. As we move to stationary equilibria associated with higher debt/GDP ratios, the welfare of the wealth-poor drops substantially. Strikingly, the decline is more pronounced if we consider the case of endogenous borrowing limits. Clearly, the welfare of the wealth-poor is strongly affected by changes in the borrowing limit, see Obiols-Homs (2011). Since government debt partly loses its ability to undo liquidity constraints if debt/GDP increase, we find that wealth-poor households are more affected if the constraint are endogenous.

The fact that welfare declines at all for this group is perhaps puzzling. After all, government debt still increases private liquidity. The reason for the decline is that the welfare of wealth-poor households depends strongly on the development of their labor income. Their labor income, however, is declining in the debt/GDP ratio, since higher debt/GDP ratios imply a lower wage rate.

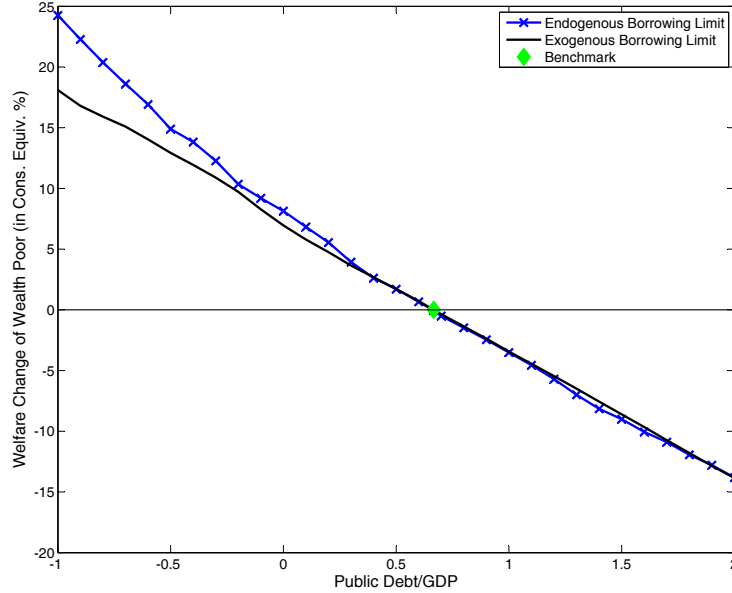


Figure 4: **Welfare of Wealth Poor.** Here we plot the change in consumption equivalent welfare of wealth poor agents holding zero or negative assets, for stationary equilibria differing with respect to the public debt/GDP ratio. At the benchmark public debt amounts to 2/3 of GDP (green diamond). Two cases: (1) exogenous borrowing limit (black line); (2) endogenous borrowing limit (blue line with crosses). For all cases, we assume that

As indicated by Figure 5, also aggregate welfare is declining with the debt/GDP ratio. In Röhrs and Winter (2013), we show that this can be explained by the response of the aggregate prices  $r$  and  $w$ , together with the unequal wealth and earnings distribution that is characteristic for the US economy. In essence, the skewed wealth distribution implies that only very few households profit from the higher interest rate associated with higher debt/GDP ratios. The majority of households mainly depends on labor income, and thus suffers from the decline in the wage rate.

Interestingly, Figure 5 indicates that the change in welfare - at the aggregate level - is much less pronounced for the case in which borrowing limits are endogenous. This result is perfectly consistent with our previous finding that aggregate prices respond much less if we introduce endogenous borrowing limits. Since aggregate welfare is mainly driven by the development of aggregate prices and the way income is redistributed between various subgroups of the population, this also explains why the welfare response is muted if borrowing limits are endogenous.

## 5 Conclusion and Further Research

In this paper, we study the role of government debt in providing liquidity to private households. Our contribution to the literature is to allow for borrowing limits that are endogenous. This is done by assuming that private loan contracts are not enforceable. As a consequence of this assumption, rational lenders set borrowing limits such that households do not have an incentive to default in equilibrium.

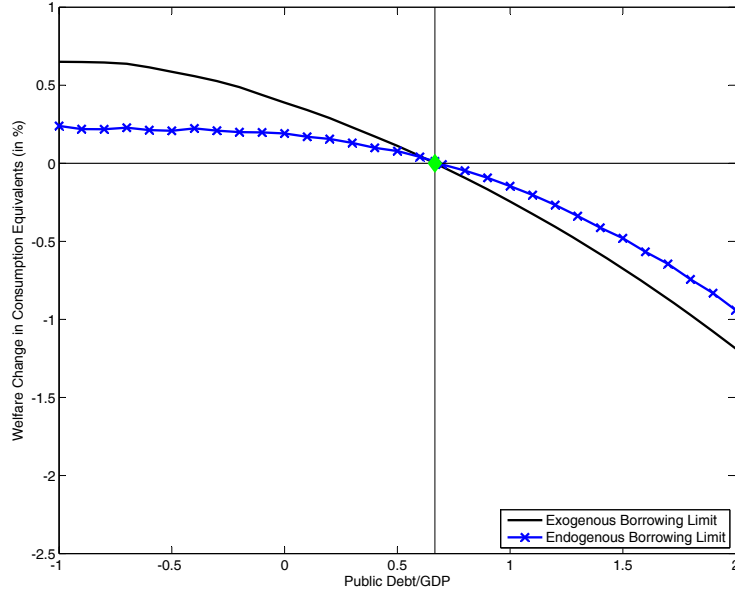


Figure 5: **Welfare of Total Population.** We plot the welfare change in consumption equivalent units (on the y-axis) for stationary equilibria differing in the public debt/GDP ratio (on the x-axis) for the total population. At the benchmark public debt amounts to 2/3 of GDP (green diamond). Two cases: (1) exogenous borrowing limit (black line);(2) endogenous borrowing limit (blue line with crosses).

Borrowing limits are thus determined by the households' incentive to default. We show that borrowing limits respond to changes in the debt/GDP ratio. As a response to an increase in the debt/GDP ratio, lenders become more reluctant to give credit. This is because higher debt/GDP ratios are associated with higher interest rates, a finding which is empirically confirmed by Laubach (2009). A higher interest rate, in turn, increases the cost of debt service and thus makes default more likely.

Compared to a model in which borrowing limits are exogenously fixed and thus invariant to public policy, an endogenous tightening of borrowing limits in response to an increase in the public debt/GDP ratio implies that government debt is less effective in providing liquidity to households. As a result of this, we show that public debt has very different effects on economic activity and on aggregate welfare in general, once the reaction of borrowing limits to changes in public debt is incorporated.

We would like to conclude by mentioning possible directions for further research. First of all, it would be interesting to study the interaction between government debt and borrowing limits in a quantitative model in which borrowing constraints arise because of other types of agency problems, e.g. asymmetric information. Moreover, as already mentioned in the introduction, our focus is exclusively on the household side. We abstract from the role that government bonds play as providing collateral in other sectors of the economy, importantly, the interbanking market (there in particular in repo transactions).

Furthermore, we concentrate on a closed economy set-up, assuming that aggregate prices are determined endogenously within the economy. Depending on the degree of which capital markets are integrated, the response of aggregate prices to changes in government might be substantially muted. Several facts (e.g. the home bias in portfolio holdings, which applies to both bonds and equity, see Tesar and Werner 1995) suggest that capital markets are far from being perfectly integrated. Moreover, as

mentioned above, there is empirical evidence that the real interest rate in the US responds to changes in the debt/GDP ratio.

Also, when computing the welfare effects, we focus only on comparisons of stationary equilibria, ignoring the transition between them. As we show in Röhrs and Winter (2013), within a framework in which borrowing constraints are exogenous, the transitional welfare effects can be sizable, and, more importantly, also of the opposite sign. We leave these important extensions for future research.

## References

- Ábrahám, Árpád, and Eva Cárceles-Poveda. 2010. “Endogenous Trading Constraints with Incomplete Asset Markets.” *Journal of Economic Theory* 145 (3): 974–1004.
- Aiyagari, S. Rao, and Ellen R. McGrattan. 1998. “The Optimum Quantity of Debt.” *Journal of Monetary Economics* 42 (3): 447–469.
- Aiyagari, S Rao. 1994. “Uninsured Idiosyncratic Risk and Aggregate Saving.” *The Quarterly Journal of Economics* 109 (3): 659–84.
- Alvarez, Fernando, and Urban J. Jermann. 2000. “Efficiency, Equilibrium, and Asset Pricing with Risk of Default.” *Econometrica* 68 (4): 775–798.
- Andolfatto, David, and Martin Gervais. 2008. “Endogenous debt constraints in a life-cycle model with an application to social security.” *Journal of Economic Dynamics and Control* 32 (12): 3745–3759 (December).
- Angeletos, George-Marios, Fabrice Collard, Harris Dellas, and Behzad Diba. 2012. “Optimal Public Debt Management and Liquidity Provision.” Technical Report, Massachusetts Institute of Technology.
- Arai, Real, Takuma Kunieda, and Keigo Nishida. 2013. “Is Public Debt Growth-Enhancing or Growth-Reducing?” Technical Report, City University of Hong Kong.
- Attanasio, Orazio, and José-Víctor Ríos-Rull. 2000. “Consumption Smoothing in Island Economies: Can Public Insurance Reduce Welfare?” *European Economic Review* 44 (7): 1225–1258 (June).
- Azzimonti, Marina, Eva de Francisco, and Vincenzo Quadrini. 2012. “Financial globalization, inequality, and the raising of public debt.” Working papers 12-6, Federal Reserve Bank of Philadelphia.
- Barro, R.J. 1974. “Are Government Bonds Net Wealth?” *The Journal of Political Economy* 82:1095–1117.
- Broer, Tobias. 2011. “Crowding out and crowding in: When does redistribution improve risk-sharing in limited commitment economies?” *Journal of Economic Theory* 146 (3): 957–975 (May).
- Castañeda, Ana, Javier Díaz-Giménez, and José-Víctor Ríos-Rull. 2003. “Accounting for the U.S. Earnings and Wealth Inequality.” *Journal of Political Economy* 111 (4): 818–857 (August).
- Chatterjee, Satyajit, Dean Corbae, Makoto Nakajima, and Jos-Vctor Ros-Rull. 2007. “A Quantitative Theory Of Unsecured Consumer Credit With Risk Of Default.” *Econometrica* 75:1525–1689.
- Cooley, Thomas F., and Edward C. Prescott. 1995. “Economic Growth and Business Cycles.” In *Frontiers of Buesiness Cycle Research*, edited by Thomas F. Cooley. Princeton University Press.
- Cordoba, Juan-Carlos. 2008. “U.S. Inequality: Debt Constraints or Incomplete Asset Markets?” *Journal of Monetary Economics* 55 (2): 350–364 (March).
- Eggertson, Gauti B., and Paul Krugman. 2012. “Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach.” *Quarterly Journal of Economics*, pp. 1469–1513.
- Fernández-Villaverde, Jesús, and Dirk Krueger. 2011. “Consumption and Saving over the Life Cycle: How Important are Consumer Durables?” *Macroeconomic Dynamics* 15:725–770.

- Flodén, Martin. 2001. “The Effectiveness of Government Debt and Transfers as Insurance.” *Journal of Monetary Economics* 48 (1): 81–108 (August).
- Gomes, Francisco, Alexander Michaelides, and Valery Polkovnichenko. 2008. Fiscal Policy in an Incomplete Markets Economy. Mimeo, London School of Economics.
- . 2010. “Quantifying the Distortionary Fiscal Cost of ‘The Bailout’.” Cepr discussion paper DP7941, Centre for Economic Policy Research.
- Guerrieri, Veronica, and Guido Lorenzoni. 2011, November. “Credit Crises, Precautionary Savings, and the Liquidity Trap.” Working paper 17583, National Bureau of Economic Research.
- Hall, Robert E. 2011. “The High Sensitivity of Economic Activity to Financial Frictions.” *The Economic Journal* 121:351–378.
- Holmstrom, Bengt, and Jean Tirole. 1998. “Private and Public Supply of Liquidity.” *Journal of Political Economy* 106:1–40.
- Keane, Michael P., and Richard Rogerson. 2011, September. “Reconciling Micro and Macro Labor Supply Elasticities: A Structural Perspective.” Nber working papers 17430, National Bureau of Economic Research, Inc.
- Kehoe, Timothy J, and David K Levine. 2001. “Liquidity Constrained Markets versus Debt Constrained Markets.” *Econometrica* 69 (3): 575–98 (May).
- Krueger, Dirk, and Fabrizio Perri. 2011. “Public versus Private Risk Sharing.” *Journal of Economic Theory* 146 (3): 920–956.
- Laubach, Thomas. 2009. “New Evidence on the Interest Rate Effects of Budget Deficits and Debt.” *Journal of the European Economic Association* 7 (4): 858–885 (06).
- Livshits, Igor, James MacGee, and Michle Tertilt. 2007, September. “Accounting for the Rise in Consumer Bankruptcies.” Nber working papers 13363, National Bureau of Economic Research, Inc.
- Mateos-Planas, Xavier, and Giulio Seccia. 2006. “Welfare implications of endogenous credit limits with bankruptcy.” *Journal of Economic Dynamics and Control* 30 (11): 2081–2115 (November).
- Mendoza, Enrique G., Assaf Razin, and Linda L. Tesar. 1994. “Effective Tax Rates in Macroeconomics: Cross-Country Estimates of Tax Rates on Factor Incomes and Consumption.” *Journal of Monetary Economics* 34 (3): 297–323 (December).
- Obiols-Homs, Francesc. 2011. “On borrowing limits and welfare.” *Review of Economic Dynamics* 14:279–294.
- Röhrs, Sigrid, and Christoph Winter. 2013. Reducing Government Debt in the Presence of Inequality. Mimeo.
- Rojas, Juan A., and Carlos Urrutia. 2008. “Social Security with Uninsurable Income Risk and Endogenous Borrowing Constraints.” *Review of Economic Dynamics* 11 (1): 83–103 (January).
- Tesar, Linda L., and Ingrid M. Werner. 1995. “Home bias and high turnover.” *Journal of International Money and Finance* 14:467–492.

- Trabandt, Mathias, and Harald Uhlig. 2011. "The Laffer Curve Revisited." *Journal of Monetary Economics* 58 (15343): 305–327 (September).
- Woodford, Michael. 1990. "Public Debt as Private Liquidity." *American Economic Review* 80 (2): 382–88 (May).
- Yared, Pierre. 2013. "Public Debt under Limited Private Credit." *Journal of the European Economic Association* 11:229–245.
- Yotsuzuka, Toshiki. 1987. "Ricardian Equivalence in the Presence of Capital Market Imperfections." *Journal of Monetary Economics* 20:411–436.
- Zhang, Harold H. 1997. "Endogenous Borrowing Constraints with Incomplete Markets." *Journal of Finance* 52 (5): 2187–2209 (December).



## Appendix

### A: Detrended Formulation of the Households' Maximization Problem

In our model, there is a balanced growth path along which variables will be growing at the rate of technology growth. To find the stationary equilibrium of the model or to compute the transition from one stationary equilibrium to another it is useful to first detrend variables with respect to this exogenous productivity growth component to obtain a formulation where variables are constant in the balanced growth equilibrium. (This procedure was also used in the earlier literature, for example by Aiyagari and McGrattan, 1998 and Flodén, 2001). Denote a detrended variable by "tilde":  $\tilde{x} = \frac{x}{Y}$ . The present value of lifetime utility can then be denoted as follows:

$$U(\{\tilde{c}_t\}_{t=1,2,\dots}, \{1-l_t\}_{t=1,2,\dots}) = E_0 \sum_{t=0}^{\infty} \beta^t Y_t^{\eta(1-\mu)} u(\tilde{c}_t, 1-l_t)$$

Now using the fact that  $Y_t = Y_0(1+g)^t$ , where  $Y_0$  is output in period 0, we can write:

$$\begin{aligned} U(\{\tilde{c}_t\}_{t=1,2,\dots}, \{1-l_t\}_{t=1,2,\dots}) &= Y_0^{\eta(1-\mu)} E_0 \sum_{t=0}^{\infty} \beta^t (1+g)^{t\eta(1-\mu)} u(\tilde{c}_t, 1-l_t) \\ &= Y_0^{\eta(1-\mu)} E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t u(\tilde{c}_t, 1-l_t) \end{aligned}$$

where  $\tilde{\beta} = \beta \cdot (1+g)^{\eta(1-\mu)}$ .

Similarly, we can find a detrended version of the household budget constraint by dividing it by  $Y_t$ :

$$\begin{aligned} \frac{c_t}{Y_t} + \frac{Y_{t+1}}{Y_t} \frac{a_{t+1}}{Y_{t+1}} &= \frac{\bar{w}_t}{Y_t} \epsilon l_t + (1 + \bar{r}_t) \frac{a}{Y_t} + \chi \\ \tilde{c}_t + (1+g)\tilde{a}_{t+1} &= \tilde{w}_t \epsilon l_t + (1 + \bar{r}_t) \tilde{a}_t + \tilde{\chi} \end{aligned}$$

Also the borrowing constraint can be detrended:

$$\tilde{a}_{t+1} \geq \tilde{a}_t$$

The resulting recursive formulation in detrended variables is given by:

$$\begin{aligned} W_t(\tilde{a}, \epsilon) &= \max_{\tilde{a}', \tilde{c}, l} Y_0^{\eta(1-\mu)} u(\tilde{c}, 1-l) + \tilde{\beta} \sum_{\epsilon'} \pi(\epsilon'|\epsilon) W_{t+1}(\tilde{a}', \epsilon') \\ \text{s.t. } \tilde{c} + (1+g)\tilde{a}' &= \tilde{w}\epsilon l + (1 + \bar{r})\tilde{a} + \tilde{\chi} \\ \tilde{a}' &\geq \tilde{a} \\ \theta' &= \Gamma[\theta] \end{aligned}$$

### B: The Impact of Distortive Taxation

In this section we want to address the concern that our results about crowding out and welfare for stationary equilibria with different debt/GDP ratios are solely due to the distortiveness of taxation instead of the effect of government debt itself. To do this we analyze the following additional experiment:

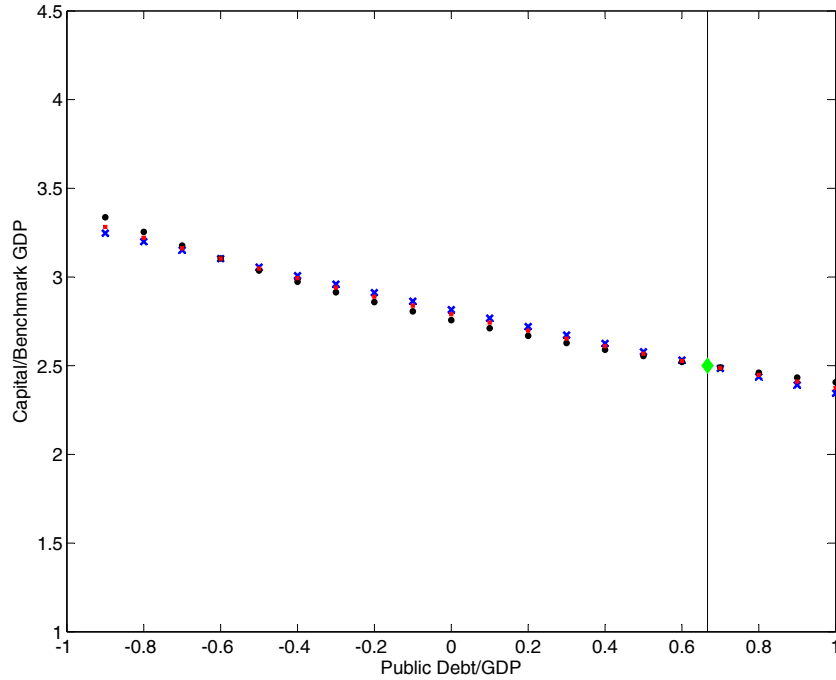


Figure 6: **Welfare Analysis and Crowding Out with Lump Sum Tax.** In this exercise we plot Capital divided by benchmark GDP (on the ordinate) for different stationary equilibria that differ with respect to the public debt/GDP ratio (on the abscissa), relative to the benchmark in which public debt amounts to 2/3 of GDP (green diamond and vertical line). In the stationary equilibrium to equalize the government budget with a different tax base and different debt/GDP ratio either the capital income tax (blue crosses), the labor income tax (red squares) or the transfer/lump sum component (black circles) has to be adapted.

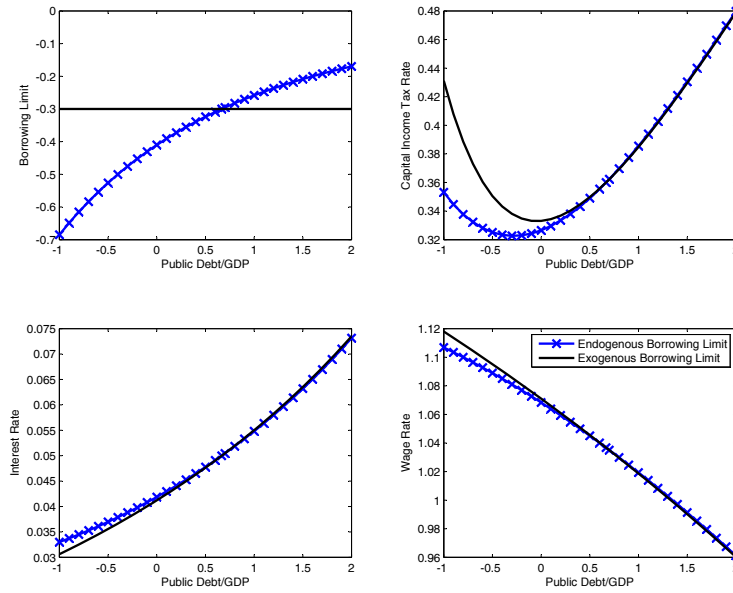
We fix the labor and the capital tax rate at their benchmark levels. When we adjust the debt/GDP ratio, we modify only the lump sum transfer in order to keep the government's budget balanced.

Figure 6 compares the three possible tax adjustment policies in terms of their crowding out effect. Clearly, the difference between tax policies is not very pronounced. We thus conclude that crowding out is mainly due to the change in the level of government debt and thus only marginally changes for different tax policies.

### C: Additional results: capital income tax instead of labor income tax is adjusted in order to balance the government's budget constraint (not for publication)

In this section, we present the results of a robustness exercise, in which we adjust the capital income tax in order to balance the government's budget constraint. In the main text, we kept the capital income tax constant and instead changed the labor income tax. It turns out that our main results are robust to a change in the tax instrument.

In order to facilitate a comparison between the results, we present the Figures in the same order as in the main text.



**Figure 7: Implied Borrowing Limits, Tax Rates, Interest Rates and Wage Rates for Different Stationary Equilibria.** In this exercise we plot the changes in the borrowing limits (first panel), tax rates (second panel), interest rates (third panel) and wage rates (fourth panel) implied by our model for different stationary equilibria that differ with respect to the public debt/GDP ratio. In the benchmark public debt amounts to 2/3 of GDP. The capital income tax is adjusted to balance the budget. Two cases: (1) exogenous (fixed) borrowing limit (black line); (2) endogenous borrowing limit (blue line with crosses).

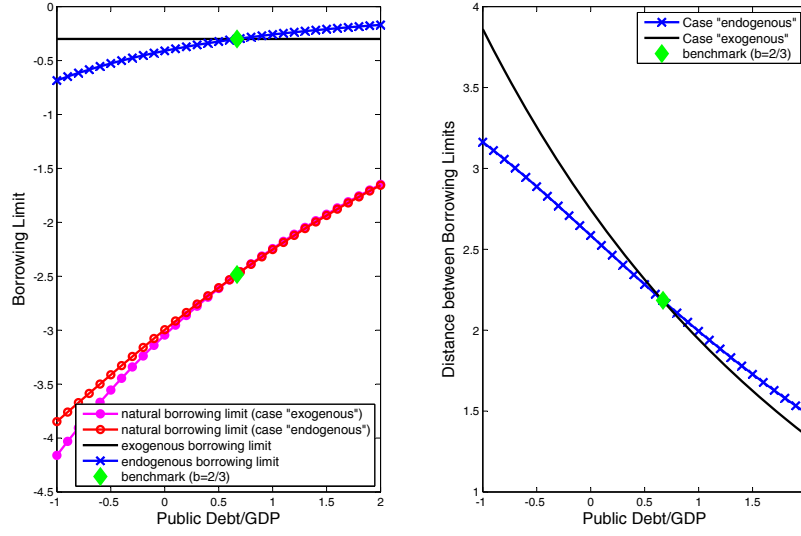


Figure 8: **Natural Borrowing Limit, Endogenous Borrowing Limit and Distance Between them.** In the left-hand side panel we plot the changes in the natural borrowing limit case 'exogenous' (pink filled circles), the natural borrowing limit case 'endogenous' (red empty circles) the exogenous (fixed) borrowing limit (black line) and the endogenous borrowing limit (blue crosses) arising from limited commitment for stationary equilibria differing with respect to the public debt/GDP ratio. In the right-hand side panel we plot the distance between the two borrowing limits for the same set of stationary equilibria. At the benchmark public debt amounts to  $2/3$  of GDP (green diamond).

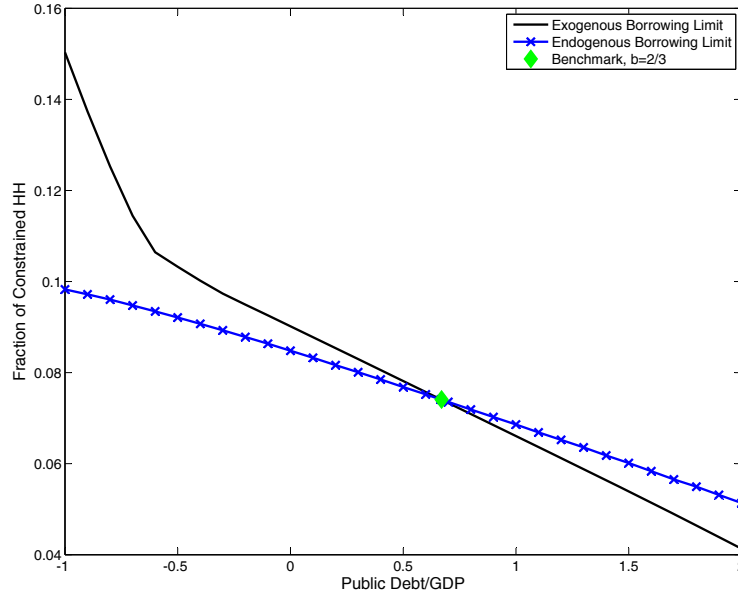


Figure 9: **Fraction of Constrained Households.** Here we plot the number of households sitting on the borrowing constraint for stationary equilibria differing with respect to the public debt/GDP ratio. At the benchmark public debt amounts to  $2/3$  of GDP (green diamond). Two cases: (1) exogenous borrowing limit (black line); (2) endogenous borrowing limit (blue line with crosses).

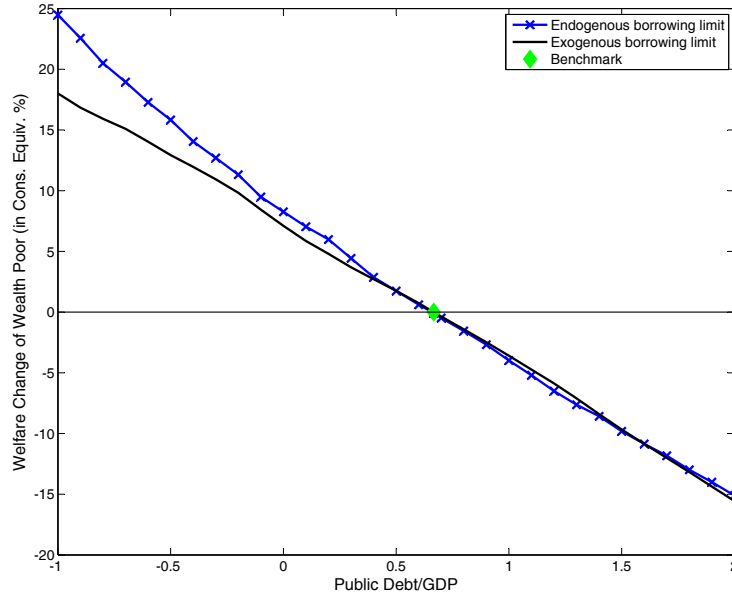


Figure 10: **Welfare of Wealth Poor.** Here we plot the change in consumption equivalent welfare of wealth poor agents holding zero or negative assets, for stationary equilibria differing with respect to the public debt/GDP ratio. At the benchmark public debt amounts to  $2/3$  of GDP (green diamond). Two cases: (1) exogenous borrowing limit (black line); (2) endogenous borrowing limit (blue line with crosses). For all cases, we assume that

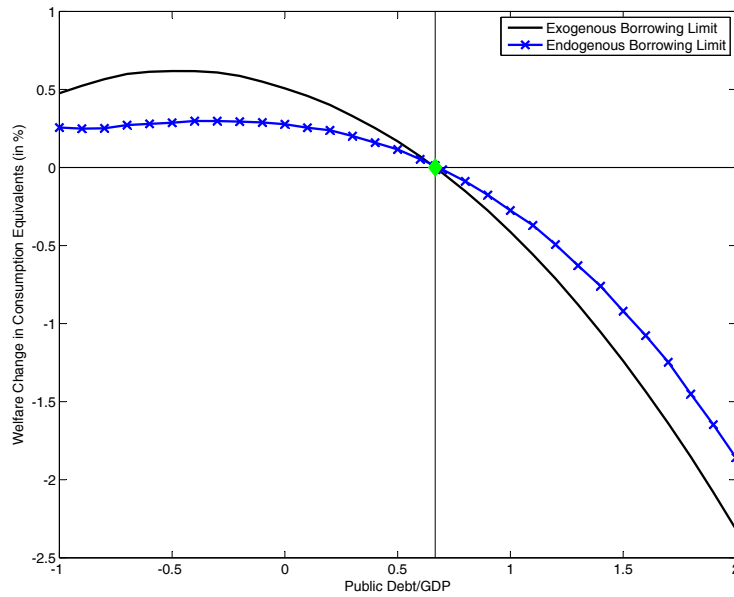


Figure 11: **Welfare of Total Population.** We plot the welfare change in consumption equivalent units (on the y-axis) for stationary equilibria differing in the public debt/GDP ratio (on the x-axis) for the total population. At the benchmark public debt amounts to  $2/3$  of GDP (green diamond). Two cases: (1) exogenous borrowing limit (black line); (2) endogenous borrowing limit (blue line with crosses).