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Detecting financial contagion in a multivariate system

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Abstract

This paper proposes an original three-part sequential testing procedure (STP), with which to test for contagion using a multivariate model. First, it identifies structural breaks in the volatility of a given set of countries. Then a structural break test is applied to the correlation matrix to identify and date the potential contagion mechanism. As a third element, the STP tests for the distinctiveness of the break dates previously found. Compared to traditional contagion tests in a bivariate set-up, the STP has high testing power and is able to locate the dates of contagion more precisely. Monte Carlo simulations underline the importance of separating variance and correlation break testing, the endogenous dating of the breakpoints and the usage of multi-dimensional data. The procedure is applied for the 1997 Asian Financial Crisis, revealing the chronological order of the crisis events.

Keywords: Contagion, structural breaks, Asian crisis

JEL Classification: C32, G01, G15

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1 Introduction

A vast empirical literature describes the development of tests to evaluate, whether an unexpected negative shock in a particular country/market is propagating to other countries/markets. Many econometric strategies have been proposed to test for this feature.¹ Among those a popular approach, often labeled as shift-contagion, consists of testing, whether the correlation between countries/markets significantly increases during a crisis. A time varying correlation would support the contingent-crisis theory, for which multiple equilibria based on investor psychology, endogenous liquidity shocks causing a portfolio recomposition and/or political disturbances affecting the exchange rate regime lead to sudden increases in the link between countries/markets during crises. On the contrary, according to the non-crisis-contingent theories, the propagation of shocks is exclusively the continuation of existing (trade and/or financial) linkages. Since the seminal papers of King and Wadhwani (1990), Calvo and Mendoza (2000) and Baig and Goldfajn (1999), who recursively calculate the correlation between two countries' stock market indices and detect structural break(s), the empirical literature has offered more elaborate methodologies to test for contagion; see Dungey et al. (2005) for a literature survey. A key contribution to this literature is Forbes and Rigobon (2002) (hereafter FR), where it is shown that contagion is over-accepted, if one ignore the changes that happen to the variance when testing for changes in correlation. Consequently, the authors propose a correlation break test, which controls for potential volatility changes, and found much less support for shock transmission. Nevertheless, several shortcomings can be addressed to FR.

First, FR consider crisis dating as exogenous. In other words, the break date is not obtained using the data but imposed by the authors. Several procedures enable endogenous break date determination: Butler and Joaquin (2002) analyze extreme left-tail events and their characteristics paving the way for an extreme value theory approach of contagion (see Hartmann et al., 2004). Alternatively, Eichengreen et al. (1995, 1996) consider a dichotomous classification

¹Logit models in Bae et al. (2003) and Boyson et al. (2010), cointegration and common cycles analyses in Garcia Pascual (2003) and Candelon et al. (2005). Rodriguez (2007) applies different kinds of copulas to capture breaks in the interdependence of financial data. Extreme value models address the dependence in the tails of a distribution, as in Bae et al. (2003), Chan-Lau et al. (2004) or Pesaran and Pick (2007).

between crisis and non crisis periods, including contagion as an explicative exogenous variable. Favero and Giavazzi (2002) use also dummy variables to test for contagion. More recently, Candelon and Manner (2010) generalize the Favero and Giavazzi (2002) dummy approach and rely on structural break tests for correlation. It is interesting to notice that when the crisis dates are not imposed, contagion is supported most of the time and the location of the crisis is somewhat different from FR.

Second, FR assume that volatility and correlation breaks are simultaneous, occurring both at the beginning of the crisis. Candelon and Manner (2010) challenge this assumption. Using a copula based approach, they observe that during the Asian crisis, variance breaks have preceded correlation shifts in most cases. The economic motivation behind this stylized fact is that the transmission of the shocks is not immediate and takes place only when markets are already stressed. Hence, when analyzing the conditional correlation, purged from variance movements, one may actually observe a decrease in the dependence between markets/countries at the edge of the crisis. It then takes a certain amount of time until the conditional correlation increases beyond its initial value. Assuming simultaneity between volatility and correlation shifts would lead to a mix up of these two effects and thus to an underestimation of the presence of contagion. Testing for the distinctiveness between volatility and correlation breaks is thus crucial.

Third, FR exclusively analyze pairwise correlations. As noticed by Dungey et al. (2004), a multivariate approach is necessary to correctly apprehend contagion. Indeed a shock that originated in country/market i does not necessarily impact country/market j directly, but may indirectly transit via country/market k . In such cases, a bivariate analysis would not detect existing contagion. Furthermore, from a purely econometric perspective, Bai and Perron (1998) proved that the date of a break is more precisely detected and estimated in a multivariate system rather than in a univariate regression. Additional cross-sectional observations provide significant extra information for the detection of structural breaks (see Bai et al., 1998, Groen et al., 2011 and Qu and Perron, 2007).

This paper proposes to solve the three previous issues usually encountered in contagion tests. Relying on the theory developed in Qu and Perron (2007),

a sequential testing procedure is proposed to test for structural breaks in the covariance matrix of asset (market) returns. The covariance matrix is decomposed in order to separately evaluate variances and correlation coefficients, and test for breaks in variance and correlation. Furthermore, the procedure tests whether the inferred breaks are distinct from one another, employing the methodology developed in Perron and Oka (2011). The sequential procedure is performed in a multivariate dynamic set-up (of dimension larger than 2) in order to obtain more precise estimates of the break dates and thus to better evaluate the presence of contagion.

More precisely, in a first step the Qu and Perron (2007) test is implemented to test for structural changes in the variance within a large set of countries/market. Then, conditional on the estimated breaks in variance, structural break tests are applied to the correlation matrix to identify and date potential contagion. As a third element, we test for distinctiveness of the identified variance and correlation breaks, finally accepting (or rejecting) the presence of contagion and describing the chronology of events.

The asymptotic properties of the sequential testing procedure (STP hereafter) are reviewed and Monte-Carlo experiments confirm that considering large multivariate systems improves the quality of the estimated break dates. To illustrate our new sequential contagion test, we consider the Asian 1997 crisis. The results of our analysis clearly confirm the presence of contagion in this period, but they also offer new insights into the timing of the events.

The paper is structured as follows. Section 2 motivates and presents the procedure of the sequential testing procedure in a multivariate set-up. Section 3 presents the associated tests as well as their asymptotic distributions. The Monte Carlo simulations in Section 4 illustrate some advantages of applying our procedure, before Section 5 describes the empirical application of our method for the case of the 1997 Asian financial crisis. Conclusions can be found in Section 6.

2 Testing for contagion via Sequential Testing Procedure (STP)

2.1 The model

Consider a vector Y_t of n assets (market) returns for $t = 1, \dots, T$ and its stationary VAR(r) representation

$$\Gamma(L)Y_t = \varepsilon_t. \quad (1)$$

$\Gamma(L)$ is a lag polynomial with roots lying outside the unit circle to ensure stationarity. As we consider asset returns on a daily basis one can expect this simple model to be sufficient to describe the conditional mean. If necessary, exogenous regressors can be added to the conditional mean equation to encounter for systemic effects. The n -dimensional vector of error term $\varepsilon_t = [\varepsilon_{1,t}, \varepsilon_{2,t}, \dots, \varepsilon_{n,t}]'$ follows some (unknown) distribution with covariance matrix Σ . Technically, assumptions A4 and A5 of Qu and Perron (2007) are assumed to hold for the innovations. They are mild and allow for the typical features observed in financial returns, in particular conditional heteroscedasticity and autocorrelation. The assumptions can also be found in the appendix.

Shift-contagion is detected when the correlation between markets increases beyond its pre-crisis level. Since contemporaneous dependence is not part of the conditional mean model, it is captured by the covariance matrix Σ of the error term ε_t . Thus, testing for contagion boils down to test for an increase in the dependence among the residuals $\hat{\varepsilon}_t$. However, as noticed by Forbes and Rigobon (2002), a change in the covariance matrix Σ does not allow for the identification of contagion. The origin of a shift in a covariance term $\sigma_{ij} = \sigma_i \rho_{ij} \sigma_j$ would be unclear, as it could result from an increase in the correlation but as well as from a rise in the variance. Therefore we decompose the covariance matrix as

$$\Sigma = SRS. \quad (2)$$

R is the matrix of $n(n-1)/2$ different correlation coefficients ρ_{ij} and S is a diagonal matrix containing n standard deviations σ_i , for $i, j = 1, \dots, n$.

A test for contagion consists of detecting an increase in the elements of the

correlation matrix R , which measure interdependence only. However, during a financial crisis some elements of the S matrix are likely to increase due to increased market risk. Besides, there is no a priori reason to believe that the outbreak of a crisis in multiple countries occurs simultaneously, but that spillovers occur in a sequential manner. In fact, it is likely that these spillovers may occur several periods after the initial outbreak of the crisis. Therefore assuming the concordance between shifts in volatility and dependence is overly restrictive and can lead to imprecise or even biased estimates of the unknown time of structural changes. In our approach, breaks in volatility and correlation are not assumed to be simultaneous, but, instead, we test whether this is actually that case.

2.2 Contagion in a multivariate model

Given a particular set of countries under study, several scenarios of contagion can be depicted. For example, there could be a single ground zero country that initially crashes and there could be subsequent spillovers to countries that are close (geographically and/or economically). In that case, correlations between the initial crisis country and other countries would increase. However, some of the countries under study could remain unaffected by the crisis, so correlations between the original country and those countries would remain constant (or may even decrease). Alternatively, contagion could occur between all countries meaning that the entire correlation matrix changes. Apart from dependence changes, we also expect structural changes in the volatilities for some or all countries. Therefore, instability of the components of the covariance matrix can occur in several forms. Examples include (i) the classic case of a simultaneous break in all covariance parameters, (ii) a break in all covariance parameters at distinct times, (iii) the coincidence of break dates for certain subsets of parameters, or (iv) instability that is partial, i.e., affecting some parameters while other parameters remain stable. In fact, any combination of the mentioned examples is possible in financial data and can be detected by our procedure.

In terms of the model presented above, we assume that its conditional mean process is stable, but that the error distribution is split into $m + 1$ asymptotically distinct regimes. To simplify the presentation, we set the number of

breaks to at most a single break in each variance σ_i^2 , at individual dates k_i , and a single break in a set of correlation coefficients, at date k_ρ .² This covers case (ii) above, which is the least restrictive scenario, involving crisis outbreaks in all markets and a contagion event between all markets or a subset of them. The ordering of the breaks is not assumed to be known, so that a superscript (d) identifies the d -th regime and the d -th break date (regime end date) in the set. With break fractions $\lambda^{(d)} = k^{(d)}/T$, $\lambda^{(0)} = 0$, $\lambda^{(m+1)} = 1$, the error distribution is then given by $\varepsilon_t \sim (0, \Sigma^{(d)})$, for $[\lambda^{(d-1)}T] + 1 \leq t \leq [\lambda^{(d)}T]$, for $d = 1, \dots, m+1$. Note that, in our setting, there is at most one break date in each variance parameter and in the correlation matrix with the possibility that some break points are common, so the total number of breaks $m \leq n+1$, where n is the number of countries under study.

Thus, our aim is to test for multiple breakpoints in distinct (sets of) parameters in the covariance matrix of the error distribution of a multivariate regression model. The methodology and theory developed in Qu and Perron (2007) applies directly to our problem. In fact, their approach is much more general, in the sense that their results allow testing for multiple breakpoints in the regression parameters *and* in the covariance matrix. We suggest performing the test for structural breaks in the covariance matrix in two steps. In a first step, we perform univariate tests for a breakpoint in the volatility of each series, whereas in the second step we test, conditional on the change points in volatility, for a structural break in some or all correlation coefficients. The main reason for performing the test in two steps is that it is computationally very demanding to search for $n+1$ distinct breakpoints, which, for dimensions beyond two or three and reasonably large sample sizes can be a difficult task.

Case (ii), the fully unrestricted model, is the initially tested scenario of the sequential testing procedure (STP). However, if multiple parameters can be assumed to share a common breakpoint, it is beneficial to use that restriction. Such co-break restrictions improve the detection of breakpoints because new sources of instability information enhance the break signal, leading to higher power of the breakpoint tests and more precise estimation of the break location

²In principle it is straightforward to extend the test for multiple breakpoints in volatility and in the correlation matrix to allow for the possibility that contagion does not affect all countries simultaneously, but happens in multiple waves. However, this can also be achieved by separate contagion tests for a number of subsets of the countries under analysis.

as shown in Bai et al. (1998) and Groen et al. (2011). Thus, whenever possible, the tests are performed in multivariate systems. Since no prior knowledge about the break dates is assumed, the required restrictions have to be inferred from the data and our sequential procedure will allow for the identification of common break dates. Initially, the separate estimation of change points may produce dates that lie very close to each other. Therefore, the test for common breaks proposed in Perron and Oka (2011) is performed to test for suitable restrictions, implying scenarios (iii) or (i). Furthermore, the results of Qu and Perron (2007) show that the precision of the break date estimate can be further improved by adding series to the systems whose parameters are invariant across regimes, as additional stable series provide a further source of contrast between stability and instability. For this scenario (iv), the parameters must be restricted from breaking to keep the degrees-of-freedom unchanged.

2.3 The sequential testing procedure

Our STP exploits multidimensional data, potential restrictions, and delivers estimates of break dates in variances and correlations as follows:

1. Variance break tests: Test for a break in the variance σ_i^2 in every residual series $\hat{\varepsilon}_i$ for $i = 1, \dots, n$ individually. If stability is rejected, obtain regime-specific variance estimates and a break date estimate \hat{k}_i .
2. Distinctiveness tests: Test the distinctiveness of break dates for all cases, in which break dates k_i have been found to lie close together³ assuming a constant correlation matrix. If the null hypothesis of break coincidence cannot be rejected, the tested variance breaks are assumed to share the newly estimated common break date $\hat{k}^{(d)}$. New regime-specific variance estimates evolve. Assume that $m \leq n$ distinct break dates in the volatilities are identified.
3. Standardization: Clear each series with a variance break from its unconditional heteroscedasticity by dividing the residual data by the estimated

³There is no rule which difference in dates makes them lie “close together”, Perron and Oka (2011), who propose a test for the null that one or more parameters share a common break, offer no advice. We encourage the testing whenever the confidence intervals for the estimated breakpoints overlap.

- regime-specific standard deviations. This produces standardized residuals $\tilde{\varepsilon}_{it} = \hat{\varepsilon}_{it}/\hat{\sigma}_i^{(-)}$ for $t = 1, \dots, \hat{k}_i$, and $\tilde{\varepsilon}_{it} = \hat{\varepsilon}_{it}/\hat{\sigma}_i^{(+)}$ for $t = \hat{k}_i + 1, \dots, T$.
4. Correlation break test: Test the whole system of standardized residuals $\tilde{\varepsilon}$ for a break in a selected set of correlations. If stability is rejected, contagion is inferred to occur at the break date \hat{k}_ρ .
 5. Distinctiveness test: If the break in correlation coefficients has been estimated to lie close to one or more variance breaks, test the distinctiveness of the breaks. If coincidence is not rejected, a new common break date estimate is obtained along with parameter estimates.

The variance break tests of Step 1 evaluate the null hypothesis of stable unconditional variances against the alternative of an unknown breakpoint, k_i , with resulting pre- and post-break variances $\sigma_i^{2(-)}$ and $\sigma_i^{2(+)}$:

$$H_0 : \sigma_i^2 \equiv \sigma_{i,1}^2 = \sigma_{i,2}^2 = \dots = \sigma_{i,T}^2.$$

$$H_A : \sigma_i^{2(-)} \equiv \sigma_{i,1}^2 = \dots = \sigma_{i,k_i}^2 \neq \sigma_{i,k_i+1}^2 = \dots = \sigma_{i,T}^2 \equiv \sigma_i^{2(+)}.$$

The correlation break test of Step 4 needs the specification of correlation coefficients allowed to break. The user-given specification is incorporated through an $n \times n$ selection matrix V of logical variables, so that $V \circ R$ only contains correlation coefficients that are free to break, i.e., $V \circ R = R^f$, whereas $R^r = R - R^f$ defines the set of correlation coefficients restricted from breaking. Here \circ denotes the Hadamard product of element-by-element multiplication of two matrices. For example, if in a system of $n = 4$ time series the correlations between series 1-3 are tested for a break, the matrices are given as

$$V = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R^f = \begin{bmatrix} 0 & \rho_{12} & \rho_{13} & 0 \\ \rho_{12} & 0 & \rho_{23} & 0 \\ \rho_{13} & \rho_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R^r = \begin{bmatrix} 1 & 0 & 0 & \rho_{14} \\ 0 & 1 & 0 & \rho_{24} \\ 0 & 0 & 1 & \rho_{34} \\ \rho_{14} & \rho_{24} & \rho_{34} & 1 \end{bmatrix}.$$

With an unknown break date, k_ρ , the two hypotheses of a break in specified correlation coefficients are:

$$\begin{aligned}
H_0 : R_0^f &\equiv R_1^f = R_2^f = \dots = R_T^f, \\
R^r &\equiv R_1^r = R_2^r = \dots = R_T^r, \\
\text{and } S_1^{(1)} &= S_2^{(1)} = \dots \neq \dots = S_{k^{(d)}}^{(d)} \neq S_{k^{(d)}+1}^{(d+1)} = \dots \neq \dots = S_T^{(m)}. \\
H_A : R_A^{f(-)} &\equiv R_1^f = R_2^f = \dots = R_{k_\rho}^f \neq R_{k_\rho+1}^f = \dots = R_T^f \equiv R_A^{f(+)}, \\
R^r &\equiv R_1^r = R_2^r = \dots = R_T^r, \\
\text{and } S_1^{(1)} &= S_2^{(1)} = \dots \neq \dots = S_{k^{(d)}}^{(d)} \neq S_{k^{(d)}+1}^{(d+1)} = \dots \neq \dots = S_T^{(m)}.
\end{aligned}$$

Thus, we are testing for a structural break in R^f , conditional on the stability of R^r and conditional on $m \leq n$ distinct breakpoints in the matrix of standard deviations S .

In practice, one usually does not know a priori which elements of the correlation matrix are subject to structural change. We suggest a preliminary analysis with repeated tests for structural change, possibly based on bivariate data to identify the final testing specification.

Structural breaks that accompany financial contagion may lie so close together that they are associated with the same regime d from the set of asymptotically distinct regimes $d = 1, \dots, m+1$, as studied in Perron and Oka (2011). For a specified set of at least two break dates $k^{(d)} = \{k_i, k_j, \dots\}$, $n(k^{(d)}) \geq 2$, the two hypotheses involved in testing for local break date distinctiveness are

$$\begin{aligned}
H_0 : k_0^{(d)} &= k_i = k_j = \dots, \\
H_A : &\text{not } H_0.
\end{aligned} \tag{3}$$

The test statistics used to evaluate the hypotheses stated above and their asymptotic distributions are introduced in the next section. First, however, let us provide an illustration of a potential crisis scenario that can be identified using our procedure, illustrated in Table 1. Scenario (iii) of break coincidence in a parameter subset is combined with scenario (iv) of partial parameter instability. The crisis breaks out in market 1 first, which increases the variance σ_1^2 . Market 3 enters a high volatility crisis state afterwards and finally market 2. The last variance break occurs together with contagion between the first

$$\begin{aligned}
& \begin{bmatrix} \sigma_1^{(1)} & 0 & 0 & 0 \\ 0 & \sigma_2^{(1)} & 0 & 0 \\ 0 & 0 & \sigma_3^{(1)} & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix} \begin{bmatrix} 1 & \rho_{12}^{(1)} & \rho_{13}^{(1)} & \rho_{14} \\ \rho_{12}^{(1)} & 1 & \rho_{23}^{(1)} & \rho_{24} \\ \rho_{13}^{(1)} & \rho_{23}^{(1)} & 1 & \rho_{34} \\ \rho_{14} & \rho_{24} & \rho_{34} & 1 \end{bmatrix} \begin{bmatrix} \sigma_1^{(1)} & 0 & 0 & 0 \\ 0 & \sigma_2^{(1)} & 0 & 0 \\ 0 & 0 & \sigma_3^{(1)} & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix} = \begin{bmatrix} \sigma_1^{2(1)} & \sigma_{12}^{(1)} & \sigma_{13}^{(1)} & \sigma_{14}^{(1)} \\ \sigma_{12}^{(1)} & \sigma_2^{2(1)} & \sigma_{23}^{(1)} & \sigma_{24}^{(1)} \\ \sigma_{13}^{(1)} & \sigma_{23}^{(1)} & \sigma_3^{2(1)} & \sigma_{34}^{(1)} \\ \sigma_{14}^{(1)} & \sigma_{24}^{(1)} & \sigma_{34}^{(1)} & \sigma_4^2 \end{bmatrix} = \Sigma^{(1)} \\
& \text{Variance break in series 1} \\
& \begin{bmatrix} \sigma_1^{(2)} & 0 & 0 & 0 \\ 0 & \sigma_2^{(1)} & 0 & 0 \\ 0 & 0 & \sigma_3^{(1)} & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix} \begin{bmatrix} 1 & \rho_{12}^{(1)} & \rho_{13}^{(1)} & \rho_{14} \\ \rho_{12}^{(1)} & 1 & \rho_{23}^{(1)} & \rho_{24} \\ \rho_{13}^{(1)} & \rho_{23}^{(1)} & 1 & \rho_{34} \\ \rho_{14} & \rho_{24} & \rho_{34} & 1 \end{bmatrix} \begin{bmatrix} \sigma_1^{(2)} & 0 & 0 & 0 \\ 0 & \sigma_2^{(1)} & 0 & 0 \\ 0 & 0 & \sigma_3^{(1)} & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix} = \begin{bmatrix} \sigma_1^{2(2)} & \sigma_{12}^{(2)} & \sigma_{13}^{(2)} & \sigma_{14}^{(2)} \\ \sigma_{12}^{(2)} & \sigma_2^{2(1)} & \sigma_{23}^{(1)} & \sigma_{24}^{(1)} \\ \sigma_{13}^{(2)} & \sigma_{23}^{(1)} & \sigma_3^{2(1)} & \sigma_{34}^{(1)} \\ \sigma_{14}^{(2)} & \sigma_{24}^{(1)} & \sigma_{34}^{(1)} & \sigma_4^2 \end{bmatrix} = \Sigma^{(2)} \\
& \text{Variance break in series 3} \\
& \begin{bmatrix} \sigma_1^{(2)} & 0 & 0 & 0 \\ 0 & \sigma_2^{(1)} & 0 & 0 \\ 0 & 0 & \sigma_3^{(2)} & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix} \begin{bmatrix} 1 & \rho_{12}^{(1)} & \rho_{13}^{(1)} & \rho_{14} \\ \rho_{12}^{(1)} & 1 & \rho_{23}^{(1)} & \rho_{24} \\ \rho_{13}^{(1)} & \rho_{23}^{(1)} & 1 & \rho_{34} \\ \rho_{14} & \rho_{24} & \rho_{34} & 1 \end{bmatrix} \begin{bmatrix} \sigma_1^{(2)} & 0 & 0 & 0 \\ 0 & \sigma_2^{(1)} & 0 & 0 \\ 0 & 0 & \sigma_3^{(2)} & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix} = \begin{bmatrix} \sigma_1^{2(2)} & \sigma_{12}^{(2)} & \sigma_{13}^{(3)} & \sigma_{14}^{(2)} \\ \sigma_{12}^{(2)} & \sigma_2^{2(1)} & \sigma_{23}^{(2)} & \sigma_{24}^{(1)} \\ \sigma_{13}^{(3)} & \sigma_{23}^{(2)} & \sigma_3^{2(2)} & \sigma_{34}^{(2)} \\ \sigma_{14}^{(2)} & \sigma_{24}^{(1)} & \sigma_{34}^{(2)} & \sigma_4^2 \end{bmatrix} = \Sigma^{(3)} \\
& \text{Variance break in series 2 and correlation breaks between series 1, 2 and 3} \\
& \begin{bmatrix} \sigma_1^{(2)} & 0 & 0 & 0 \\ 0 & \sigma_2^{(2)} & 0 & 0 \\ 0 & 0 & \sigma_3^{(2)} & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix} \begin{bmatrix} 1 & \rho_{12}^{(2)} & \rho_{13}^{(2)} & \rho_{14} \\ \rho_{12}^{(2)} & 1 & \rho_{23}^{(2)} & \rho_{24} \\ \rho_{13}^{(2)} & \rho_{23}^{(2)} & 1 & \rho_{34} \\ \rho_{14} & \rho_{24} & \rho_{34} & 1 \end{bmatrix} \begin{bmatrix} \sigma_1^{(2)} & 0 & 0 & 0 \\ 0 & \sigma_2^{(2)} & 0 & 0 \\ 0 & 0 & \sigma_3^{(2)} & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix} = \begin{bmatrix} \sigma_1^{2(2)} & \sigma_{12}^{(3)} & \sigma_{13}^{(4)} & \sigma_{14}^{(2)} \\ \sigma_{12}^{(3)} & \sigma_2^{2(2)} & \sigma_{23}^{(3)} & \sigma_{24}^{(2)} \\ \sigma_{13}^{(4)} & \sigma_{23}^{(3)} & \sigma_3^{2(2)} & \sigma_{34}^{(2)} \\ \sigma_{14}^{(2)} & \sigma_{24}^{(2)} & \sigma_{34}^{(2)} & \sigma_4^2 \end{bmatrix} = \Sigma^{(4)}
\end{aligned}$$

Table 1: Example of crisis and contagion events resulting in four covariance matrix regimes $\Sigma^{(d)}$, $d = 1, \dots, 4$. The left side of the term shows the decomposed covariance matrix, $SRS = \Sigma$. All parameter changes are bold. Crisis breaks out in market 1 first, then in market 3 and reaches market 2 the last, while contagion occurs at the same time as the crisis outbreak in market 2. Market 4 stays completely unaffected.

three markets, which means that the three correlation coefficients ρ_{12} , ρ_{13} and ρ_{23} shift to a higher value. Finally, the fourth market is not affected by the crisis in any way, but still its covariance with the other markets changes. A test that sought instability in, e.g., the covariance $\sigma_{13} = \sigma_1 \rho_{13} \sigma_3$ would produce a biased break date estimate in-between the three distinct breaks. Instability of σ_{13} may be caused by changes in either standard deviation and by a change in correlation. A direct test on the covariance would not be able to identify the source of instability.

Usually, such a scenario could be identified by a sequence of bivariate stability tests. However, the presence of simultaneous breaks, and even the inclusion of market 4, will result in a more powerful test and will lead to a more precise dating of the break dates. Finally, a test of break date distinctiveness can determine that the variance of series 2 and the correlations between series 1, 2 and 3 break simultaneously.

3 Test statistics

All tests are based on (pseudo) likelihood ratio (LR) type statistics relying on the multivariate normal distribution, as in Qu and Perron (2007). Note that this does not mean that we assume a normal distribution or iid'ness for the data. Deviations from normality are accounted for by the asymptotic distribution of the results test statistics. Ignoring the irrelevant constant term, the Gaussian log-likelihood for a sample of length T is given by

$$l(\Sigma) = -\frac{T}{2} \log(|\Sigma|) - \frac{1}{2} \sum_{t=1}^T \varepsilon_t' \Sigma^{-1} \varepsilon_t.$$

Break tests involve the maximization of the restricted log-likelihood function l_0 with respect to a stable covariance matrix Σ_0 and the maximization of an unrestricted log-likelihood function l_A with respect to pre- and a post-break covariance matrices $\Sigma^{(-)}$ and $\Sigma^{(+)}$, respectively. The (unknown) break date k splits the sample into two regimes, and denoting maximized function values by an asterisk, we obtain:

$$l_0^* = \max_{\Sigma} l(\Sigma), \quad (4)$$

$$\begin{aligned} l_A^*(k) &= \max_{\Sigma^{(-)}, \Sigma^{(+)}} l(k, \Sigma^{(-)}, \Sigma^{(+)}) \\ &= \max_{\Sigma^{(-)}, \Sigma^{(+)}} -\frac{k}{2} \log(|\Sigma^{(-)}|) - \frac{1}{2} \sum_{t=1}^k \varepsilon_t' (\Sigma^{(-)})^{-1} \varepsilon_t \\ &\quad - \frac{(T-k)}{2} \log(|\Sigma^{(+)}|) - \frac{1}{2} \sum_{t=k+1}^T \varepsilon_t' (\Sigma^{(+)})^{-1} \varepsilon_t. \end{aligned} \quad (5)$$

Break detection tests

The variance break testing of STP Step 1 is univariate, so that equation (2) becomes $\Sigma = \sigma^2$. The log-likelihood functions are thus univariate and are

maximized with respect to the standard deviations, for $i = 1, \dots, n$.

$$l_0^* = \max_{\sigma_i} l(\sigma_i)$$

$$l_A^*(k_i) = \max_{\sigma_i^{(-)}, \sigma_i^{(+)}} l(k_i, \sigma_i^{(-)}, \sigma_i^{(+)}).$$

In Step 4 of the STP we look at the multivariate time series system as a whole. Following Step 3, the test for a change in the correlations is based on the standardized residuals $\tilde{\varepsilon}_i$ that, by construction, have unit variance. Hence, $S = I_n$ and, with $R = R^f + R^r$, equation (2) becomes $\Sigma = R$. The log-likelihood functions are then maximized with respect to the correlation matrices:

$$l_0^* = \max_R l(R)$$

$$l_A^*(k_\rho) = \max_{R^{(-)}, R^{(+)}} l(k_\rho, R^{(-)}, R^{(+)}).$$

The log-likelihood ratio statistics for a fixed breakpoint k is given by

$$LR_{BR}(k) = 2(l_A^*(k) - l_0^*).$$

For testing a single (known) change point and assuming correct specification of the model it follows that that $LR_{BR}(k) \sim \chi^2(d)$, with the degrees of freedom equal to the number of parameters free to break. In our setting we do not assume a correctly specified model and k is assumed to be unknown, so the break dating is established endogenously. Consequently, the asymptotic distribution of $LR_{BR}(k)$ is non-standard. Denote the set of sample dates that can potentially exhibit the structural break as T_κ . The test statistic is then given by the supremum of $LR_{BR}(k)$ over T_κ ,

$$\sup LR_{BR} = \sup_{k \in T_\kappa} LR_{BR}(k). \quad (6)$$

The most likely break date \hat{k} emerges from the highest LR statistic among T_κ .

$$\hat{k} = \arg \sup_{k \in T_\kappa} LR_{BR}(k). \quad (7)$$

In practice, the supremum is replaced by the maximum, in which case the statistic is usually denoted as $\max LR_{BR}$. An important manual setting concerns the trimming of data samples with respect to the range of possible break dates T_κ . Andrews (1993) shows that the $\max LR_{BR}$ statistic diverges to infinity at the edges of the sample, making it impossible for the test not to reject the null hypothesis of stability. We adopt his suggested trimming of the data by $\kappa = 0.15$ and obtain the interval of potential brake date choices $T_\kappa = [\kappa T, (1 - \kappa)T]$.

The maximum LR statistic (6) has critical values according to Andrews (2003) for normal data. Since we search for breaks in the elements of the covariance matrix of non-normal data, using these critical values would lead to size distortions, especially in small samples. Qu and Perron (2007) have determined the limit distribution of the relevant test statistic under the null hypothesis for more general data generating processes. In particular, the conditional heteroscedasticity typically encountered in financial data is permitted, as long as the generating process is of short memory with bounded fourth moments. For a single breakpoint in the elements of the covariance matrix selected by H , their results imply that

$$\sup_{k \in T_\kappa} LR_{BR}(k) \xrightarrow{d} \sup_{\pi \in \Pi} Q(\pi, b), \quad (8)$$

$$Q(\pi, b) = \frac{1}{2} \frac{(B_b(\pi) - \pi B_b(1))' H \Omega H' (B_b(\pi) - \pi B_b(1))}{\pi(1 - \pi)}.$$

Here, $\pi = k/T$, so that the interval Π denotes the set of possible break dates T_κ scaled to the interval $[0, 1]$. B_b is a b -vector of independent Brownian motions on $[0, 1]$, resulting in the Brownian bridge $B_b(\pi) - \pi B_b(1)$. In practice, the Brownian motion vectors must be simulated by partial sums of i.i.d. normal random variables. The number b gives the total number of elements of the covariance matrix Σ allowed to break under the alternative hypothesis.

H is a selection matrix that corresponds to those elements in Σ allowed to change. Specifically, H has to be a full row rank matrix of dimension $b \times n^2$, such that $H \text{vec}(\Sigma)$ is the b -dimensional vector of covariance elements allowed to change. When significance of a hypothesized break must be assessed in our STP, either only variances or only correlation coefficients will be free to

shift.⁴ In Step 1 of the STP, when testing for univariate variance breakpoints, $b = 1$ and $H = 1$. In Step 4 of the STP, the data consists of a multivariate system of standardized residuals with $\Sigma = R$. For example, considering $n = 4$ and a hypothesized break in $d = 3$ correlation coefficients ρ_{12} , ρ_{13} and ρ_{23} in correspondence with our earlier example, it follows that

$$vec(R) = (1, \rho_{12}, \rho_{13}, \rho_{14}, \rho_{21}, 1, \rho_{23}, \dots, 1)',$$

$$H = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Hvec(R) = (\rho_{12}, \rho_{13}, \rho_{21}, \rho_{23}, \rho_{31}, \rho_{32})'.$$

Note that $\rho_{ij} = \rho_{ji}$, which implies that $b = 2 \cdot d = 6$ equals twice the number of distinct correlation coefficients allowed to change, as these appear twice on the off-diagonal in $\Sigma = R$.

With the set of standardized residuals $\tilde{\varepsilon}$, let $\xi_t = \tilde{\varepsilon}_t \tilde{\varepsilon}_t' - I_n$ be an $n \times n$ matrix for all $t = 1, \dots, T$. Further, $vec(\xi)$ is a $T \times n^2$ matrix, where the t -th row contains the n^2 scaled products and cross-products of standardized residuals at date t . Then Ω is the $n^2 \times n^2$ matrix $\Omega = Var(vec(\xi))$. According to Qu and Perron (2007), the limit distribution (8) can be applied in our case of conditionally heteroscedastic data and sequential testing, if a robust estimator is used to estimate Ω . We propose the use of the covariance estimator presented in Newey and West (1986). The critical values derived from the limit distribution (8) are substantially larger than the ones described in Andrews (2003) when applied to financial return series subject to volatility clustering. The estimate for Ω is larger for leptokurtic residual distributions than under normality, preventing size distortions due to conditional heteroscedasticity.

Finally, note that the sequential application of our tests is justified by Theorem 6 in Qu and Perron (2007), which states that the distribution of a

⁴This is not to say that some breaks may turn out to simultaneously affect variances and correlations, but the original break significance is analyzed separately.

sequential break test is the same as given above in equation (8).

Break date distinctiveness tests

Next, the break detection results are processed to further investigate whether a set of estimated breakpoints can be assumed to results from a common breakpoint that ends regime d . The test evaluates, whether removing the restriction of break simultaneity significantly increases the LR statistic

$$LR_{DI} = \max_{k^{(d)}, \Sigma^{(-)}, \Sigma^{(+)}} LR_{BR}(k^{(d)}, \Sigma^{(-)}, \Sigma^{(+)}) - \max_{k_0^{(d)}, \Sigma_0^{(-)}, \Sigma_0^{(+)}} LR_{BR}(k_0^{(d)}, \Sigma_0^{(-)}, \Sigma_0^{(+)}). \quad (9)$$

The distinct break dates $k^{(d)}$ under the alternative hypothesis have already been estimated in the break detection tests. The same holds for $\Sigma^{(-)}$ and $\Sigma^{(+)}$. The common break date $k_0^{(d)}$ under the null hypothesis has to be determined by using the statistic in (7) under the restriction of a common break. Pre-break and post-break covariance matrices result from equations (4) and (5).

In order to derive critical values, we perform the bootstrap simulations suggested in Perron and Oka (2011). The limit distribution of LR_{DI} in (9) evolves as

$$CB_\infty(s_g) = tr A(s_g) \xi(s_g) + \frac{|s_g|}{2} tr A(s_g)^2, \quad (10)$$

with

$$A(s_g) = \begin{cases} (\Sigma^{0(-)})^{-1/2} \Phi(\Sigma^{0(+)} - 1) (\Sigma^{0(-)})^{1/2}, & s_g \leq 0 \\ (\Sigma^{0(+)} - 1)^{-1/2} \Phi(\Sigma^{0(+)} - 1) (\Sigma^{0(+)})^{1/2}, & 0 < s_g. \end{cases} \quad (11)$$

Here, s_g is the largest fractional distance between the common break date estimate \hat{k}_0 and one of the elements in the set of break date estimates \hat{k}_A according to the alternative. $\xi(s)$ denotes a two-sided Brownian Motion process defined on $[-1, 1]$, which must be simulated by partial sums of i.i.d. normal random variables. $\Sigma^{0(-)}$ and $\Sigma^{0(+)}$ are true covariance matrices before and after the common break date, which are estimated from the data in the asymptotically stable regimes around \hat{k}_0 . Finally, we have $\hat{\Phi} = \hat{\Sigma}^{(+)} - \hat{\Sigma}^{(-)}$.

4 Monte Carlo study

In order to examine the performance of the presented method in small samples we ran various Monte Carlo simulations, with $s = 1,000$ repetitions each, and apply the endogenous maximum LR break test based on equation (6). We concentrate on three aspects of the testing problem; namely, increasing the dimension n of the tested time series system (Section 4.1), the need for endogenous break dating (Section 4.2), and the advantages of decomposing the covariance matrix and testing sequentially (Section 4.3).

The simulated time series have a Gaussian distribution and exhibit breaks in either variances, correlations, or both. Break tests are specified accordingly and applied to the data in order to detect the break. The parameters are set such that the unconditional variance, i.e., under the null hypothesis of no break, is equal to one, implying a standard normal distribution of the data. Accordingly, given the choice of a simulated variance shift of $\Delta\sigma^2$ and a break date k_σ , the simulated pre-break variance $\sigma^{2(-)}$ and post-break variance $\sigma^{2(+)} = \sigma^{2(-)} + \Delta\sigma^2$ satisfy $(k_\sigma/T)\sigma^{2(-)} + ((T - k_\sigma)/T)(\sigma^{2(-)} + \Delta\sigma^2) = 1$. The correlation coefficients of simulations that involve $n \geq 2$ time series are positive numbers, randomly generated in each simulation ensuring a positive-definite correlation matrix. If specified, a subset of the correlation coefficients exhibit a shift of $\Delta\rho$, resulting in pre- and post-break correlation matrices $R^{(-)}, R^{(+)}$.

If not stated differently, the length of the time series is $T = 500$ and the breaks are positioned in the middle the sample at $t = 250$. All test statistics assume a trimming of $\kappa = 0.15$, following the suggestion of Andrews (1993). Considering alternative settings (results for which are available upon request), the findings are robust to changes in the break location. If simulated data involves longer samples or larger shifts in the tested parameters, the power and efficiency of the tests improve.

4.1 Dimension of the time series system

The results in Bai et al. (1998) suggest that increasing the dimension n of a correlated time series system results in higher testing power and more precise break dating. In this section, we study these effects in the setting of testing for breakpoints in variances and correlations in small samples. An increase

in the dimension of the systems with additional correlated data is expected to increase the detectability of breaks, if they (a) breakpoints occur simultaneously in all parameters, since the instability signal is intensified, or if they (b) introduce additional stable parameters into the system, since the contrast between instability and stability is intensified. The data generating processes (DGP's) cover these two situations. The effects are studied separately for the cases of variance break tests and correlation break tests. The main question is whether increasing the dimension n always leads to a better performance of the test, or whether the increased degrees-of-freedom eventually may result in a diminishing performance.

For various n , we simulate *DGP-1*, namely multivariate Gaussian data with a simultaneous, small shift $\Delta\sigma^2 = 0.3$ in all variances, $X_t \sim \mathcal{N}(0, S^{(-)}RS^{(-)})$ for $t = 1, \dots, 250$, and $X_t \sim \mathcal{N}(0, S^{(+)}RS^{(+)})$ for $t = 251, \dots, 500$. Recall that the correlations are positive and random for each draw. The system as a whole is tested for a break in all variances, as reported in Table 2. We report the power of the tests for $\alpha = 0.1, 0.05, 0.01$, as well as the average of the estimated break location and the width of a 95% confidence interval of the location based on the 1,000 Monte Carlo simulations. Notably, the most precise and powerful variance break detection results from testing a co-break in $n = 4$ series. The findings support the asymptotic theory, but the positive impact of additional series in the system vanishes from medium-sized data sets on. The returns to additional dimensions diminish and turn negative, which suggest a saturation effect of evidence in variance break testing.

DGP-2 involves multivariate standard normal data of different dimensions, n , with a simultaneous, pure shift $\Delta\rho = 0.05$ in all correlation coefficients, $X_t \sim \mathcal{N}(0, R^{(-)})$ for $t = 1, \dots, 250$, and $X_t \sim \mathcal{N}(0, R^{(+)})$ for $t = 251, \dots, 500$. Table 3 lists the results of a test that is specified to find a break in all correlation coefficients at the same date. No concise saturation effect can be observed in the results. Notably, the dimension of the multivariate system, n , quadratically increases the number of changing correlation coefficients, $d_{\text{break}} = n(n-1)/2$. This apparently offsets the saturation effect observed for variance breaks, where the number of breaking parameters increases linearly in n . It is remarkable how sensitive the testing becomes to the small correlation shifts in high dimensions n , which is a very promising result for detecting contagion

$n = n_{\text{break}}$	\hat{k}_σ/T	Width	Power		
			$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0.51	0.660	0.674	0.567	0.297
2	0.51	0.564	0.792	0.706	0.512
3	0.50	0.520	0.908	0.828	0.672
4	0.51	0.384	0.958	0.916	0.720
5	0.51	0.534	0.831	0.804	0.707
6	0.51	0.524	0.774	0.742	0.667
7	0.50	0.590	0.775	0.749	0.700
8	0.51	0.581	0.720	0.688	0.606
9	0.50	0.602	0.683	0.660	0.593
10	0.51	0.610	0.631	0.608	0.575
15	0.51	0.644	0.640	0.618	0.589
20	0.50	0.650	0.480	0.458	0.411

Table 2: *DGP-1* of pure break at in all n variances of Gaussian data, $X_t \sim \mathcal{N}(0, S^{(-)}RS^{(-)})$ for $t = 1, \dots, 250$, $X_t \sim \mathcal{N}(0, S^{(+)}RS^{(+)})$ for $t = 251, \dots, 500$. Testing is accordingly specified. Shift in variances is $\Delta\sigma^2 = 0.3$. The table lists point estimate of variance break date k_σ , width of 95% empirical confidence interval of date estimation and testing power of sizes 10%, 5% and 1%.

d_{break}	d_{stable}	n	\hat{k}_ρ/T	Width	Power		
					$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0	2	0.49	0.653	0.716	0.666	0.566
3	0	3	0.50	0.443	0.909	0.869	0.813
6	0	4	0.50	0.186	0.974	0.969	0.935
10	0	5	0.50	0.081	0.994	0.990	0.980
15	0	6	0.50	0.053	1	1	0.999
21	0	7	0.50	0.047	1	1	1
28	0	8	0.50	0.043	1	1	1

Table 3: *DGP-2* of pure break in all $n(n-1)/2$ correlation coefficients of standard normal data, $X_t \sim \mathcal{N}(0, R^{(-)})$ for $t = 1, \dots, 250$, $X_t \sim \mathcal{N}(0, R^{(+)})$ for $t = 251, \dots, 500$. Testing is accordingly specified. Shift in correlations is $\Delta\rho = 0.05$. The table lists point estimate of correlation break date k_ρ , width of 95% empirical confidence interval of date estimation and testing power of sizes 10%, 5% and 1%.

between a large number of markets. However, non-normal financial data can be expected to reduce the level of power and efficiency (compare also Candelon and Manner, 2010).

In *DGP-3* a subset of the series exhibits small co-breaks in their variances, so that we have a combined set of n_{break} series with $X_t \sim \mathcal{N}(0, S^{(-)}RS^{(-)})$ for $t = 1, \dots, 250$, and $X_t \sim \mathcal{N}(0, S^{(+)}RS^{(+)})$ for $t = 251, \dots, 500$, and n_{stable} stable series $X_t \sim \mathcal{N}(0, \Sigma)$ for $t = 1, \dots, 500$. Stable parameters must be restricted from breaking to avoid excessive degrees-of-freedom in testing and then a positive impact of additional series in the tested system should be observed. According to the results in Table 4, this is true for small to moderate system sizes. The results suggest successful detection of simultaneous crisis outbreaks, particularly for a balanced number of co- and non-breaking series. It

n_{break}	n_{stable}	n	\hat{k}_σ/T	Width	Power		
					$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
1	0	1	0.51	0.660	0.674	0.567	0.297
1	1	2	0.50	0.482	0.908	0.862	0.760
1	2	3	0.50	0.288	0.968	0.948	0.924
1	3	4	0.50	0.302	0.977	0.969	0.937
1	4	5	0.51	0.335	0.974	0.956	0.930
2	0	2	0.51	0.564	0.792	0.706	0.512
2	1	3	0.50	0.310	0.976	0.960	0.918
2	2	4	0.50	0.152	0.988	0.982	0.981
2	3	5	0.50	0.181	0.986	0.982	0.976
3	0	3	0.50	0.520	0.908	0.828	0.672
3	1	4	0.50	0.206	0.994	0.981	0.963
3	2	5	0.50	0.189	0.992	0.990	0.981
4	0	4	0.51	0.384	0.958	0.916	0.720
4	1	5	0.50	0.255	0.992	0.981	0.955
5	0	5	0.51	0.534	0.831	0.804	0.707

Table 4: *DGP-3*, with partial co-break in n_{break} variances, $X_t \sim \mathcal{N}(0, S^{(-)}RS^{(-)})$ for $t = 1, \dots, 250$, $X_t \sim \mathcal{N}(0, S^{(+)}RS^{(+)})$ for $t = 251, \dots, 500$, while n_{stable} variances stay stable, $X_t \sim \mathcal{N}(0, \Sigma)$ for $t = 1, \dots, 500$. Testing is accordingly specified. Shift in variances is $\Delta\sigma^2 = 0.3$. The table lists point estimate of variance break date k_σ , width of 95% empirical confidence interval of date estimation and testing power of sizes 10%, 5% and 1%.

is remarkable that the performance of the test in terms of power and estimation of the location improves greatly when stable series are added to the system.

Since there are several possibilities for contagion transmission, it is convenient and particularly interesting to consider scenarios of partial breaks with only a selection of correlations breaking. To this end, in *DGP-4* we have that $X_t \sim \mathcal{N}(0, R^{(-)})$ for $t = 1, \dots, 250$, and $X_t \sim \mathcal{N}(0, R^{(+)})$ for $t = 251, \dots, 500$. A simultaneous shift $\Delta\rho = 0.05$ is only simulated for those correlations that are part of the set R^f that is free to break. One plausible scenario involves a central market that transmits contagion to several others, while the correlation between the rest of the series remains stable. The second concerns two groups of markets, where contagion occurs within the groups, but not between them. The tests search for correlation instability in the respective sets accordingly. From Table 5 one can see that notably the scenario of grouped markets quickly produces promising results for an increase of the dimension n , with precise estimates of the break location and high power. The recommendation of a using a high-dimensional system in pure correlation break testing extends to grouped markets, while testing becomes less powerful and precise in the scenario of a central market for $n > 5$.

$d_{\rho,\text{break}}$	$d_{\rho,\text{stable}}$	n	\hat{k}_{ρ}/T	Width	Power			
					$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$	
Partial break scenario 1, central market								
1	0	2	0.50	0.653	0.918	0.883	0.810	
2	1	3	0.50	0.622	0.822	0.793	0.711	
3	3	4	0.50	0.593	0.854	0.812	0.764	
4	6	5	0.50	0.459	0.894	0.853	0.804	
5	10	6	0.50	0.491	0.915	0.895	0.826	
6	15	7	0.50	0.514	0.897	0.870	0.840	
7	21	8	0.50	0.508	0.887	0.869	0.827	
Partial break scenario 2, grouped markets								
1	0	2	0.50	0.404	0.918	0.883	0.810	
1	2	3	0.50	0.166	0.836	0.788	0.700	
2	4	4	0.50	0.030	0.961	0.953	0.936	
4	6	5	0.50	0.022	0.999	0.996	0.988	
6	9	6	0.50	0.015	1	1	1	
9	12	7	0.50	0.014	1	1	0.999	
12	16	8	0.50	0.012	1	1	1	

Table 5: Data follows $DGP-4$, $X_t \sim N(0, R^{(-)})$ for $t = 1, \dots, 250$, $X_t \sim N(0, R^{(+)})$ for $t = 251, \dots, 500$. First scenario: Central break involves only the $d_{\rho,\text{break}}$ correlation coefficients between series 1 and the rest of the series. Second scenario: Break affects the $d_{\rho,\text{break}}$ correlation coefficients between the series of group one and those between the series of group two. Testing is accordingly specified. Shift in correlations is $\Delta\rho = 0.05$. The table lists point estimate of correlation break date k_ρ , width of 95% empirical confidence interval of date estimation and testing power of sizes 10%, 5% and 1%.

4.2 Endogenous break dating

The simulations described in the following underline the choice of an endogenous break test sequence that determines breaks in variances and correlations separately. Intuition suggests that test inference is stronger, if a break date is endogenously determined and not arbitrarily chosen (except one has a priori knowledge about the break dates).

In $DGP-5$ we simulate univariate and bivariate Gaussian data with a moderate shift $\Delta\sigma^2 = 0.5$ in the variance(s) at date k_σ . $X_t \sim N(0, S^{(-)}RS^{(-)})$ for $t = 1, \dots, k_\sigma$, and $X_t \sim N(0, S^{(+)}RS^{(+)})$ for $t = k_\sigma + 1, \dots, 500$. The true values of the breakpoint k_σ lie close, but are mostly not identical, to date $k_{\text{ex}} = 250$, which is the hypothesized break date of an exogenous LR break test using equation (6). The results are compared to the usual endogenous break testing in Table 6. The power of the exogenous testing decreases with distance $k_{\text{ex}} - k_\sigma$, but that happens only slowly. We conclude that pre-determining the break dates is likely to date the breakpoint wrongly, even though statistical tests suggest the presence of a break. Consequently, endogenous break testing is superior as it does not suffer from this drawback. Furthermore, even if ex-

n	k_σ/T	Power			
		\hat{k}_σ/T	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Exogenous break determination					
1	0.4	0.5	0.939	0.890	0.724
	0.45	0.5	0.983	0.956	0.861
	0.49	0.5	0.988	0.975	0.925
	0.5	0.5	0.992	0.981	0.922
2	[0.4, 0.4]	0.5	0.992	0.982	0.942
	[0.4, 0.6]	0.5	0.945	0.917	0.851
	[0.5, 0.5]	0.5	1	0.999	0.996
Endogenous break determination					
1	0.4	0.41	0.954	0.910	0.784
	0.5	0.51	0.972	0.944	0.824
2	0.4	0.41	1	1	1
	0.5	0.50	1	1	1

Table 6: Break in all n variances at varying date k_σ in *DGP-5*, $X_t \sim \mathcal{N}(0, S^{(-)}RS^{(-)})$ for $t = 1, \dots, k_\sigma$, $X_t \sim \mathcal{N}(0, S^{(+)}RS^{(+)})$ for $t = k_\sigma + 1, \dots, 500$. Exogenous LR break testing assumes date $k_{\text{ex}} = 250$, whereas endogenous testing estimates k_σ/T . Variance shift is $\Delta\sigma^2 = 0.5$. Listed is the testing power with sizes 10%, 5% and 1% according to the $\chi^2(n)$ distribution and according to a bootstrap, respectively.

ogenous break testing happens to identify the correct break date ex ante, the power of endogenous break test is only slightly lower than for the case of a known break date.

4.3 Sequential testing

Here, we demonstrate that the instability in a specified parameter (like correlation) is more efficiently detected by evaluations of precisely that parameter, and not of a compound parameter (like covariance). To this end, consider *DGP-6* of bivariate normal data that exhibits breaks in both variances at $k_\sigma = 150$, before the correlation coefficient breaks at $k_\rho = 250$, so that $X_t \sim \mathcal{N}(0, S^{(-)}R^{(-)}S^{(-)})$ for $t = 1, \dots, 150$, and $X_t \sim \mathcal{N}(0, S^{(+)}R^{(-)}S^{(+)})$ for $t = 151, \dots, 250$, and $X_t \sim \mathcal{N}(0, S^{(+)}R^{(+)}S^{(+)})$ for $t = 251, \dots, 500$. Three tests of endogenous break detection to detect a change in the dependence, i.e. contagion, are compared in Table 7. The first method searches for a break in the covariance without decomposition, the second for a break in the correlation (ignoring changes in the variance), whereas the third method is a sequential testing of a variance co-break followed by a correlation break in the standardized data.

The covariance test has high power, but produces a severe estimation bias concerning the location of the break. This could be expected, as the

$\Delta\rho$	$\Delta\sigma^2$	\hat{t}/T	Width	Power		
				$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
Method 1: Covariance break detection						
0.55	0.9	0.42	0.216	1	1	1
0.35	0.9	0.41	0.328	1	1	1
0.35	0.5	0.48	0.262	1	1	0.996
0.15	0.5	0.43	0.302	0.996	0.994	0.962
0.05	0.5	0.38	0.372	0.858	0.796	0.710
Method 2: Correlation break detection						
0.55	0.9	0.49	0.236	1	0.998	0.994
0.35	0.9	0.47	0.328	0.996	0.992	0.918
0.35	0.5	0.50	0.108	0.994	0.984	0.960
0.15	0.5	0.48	0.552	0.812	0.772	0.712
0.05	0.5	0.45	0.688	0.601	0.511	0.389
Method 3: Sequential break detection						
0.55	0.9	0.50	0.060	1	1	1
0.35	0.9	0.50	0.130	1	1	1
0.35	0.5	0.50	0.092	1	1	1
0.15	0.5	0.49	0.362	1	1	1
0.05	0.5	0.49	0.663	0.714	0.656	0.544

Table 7: *DGP-6* of bivariate data, $X_t \sim N(0, S^{(-)}R^{(-)}S^{(-)})$ for $t = 1, \dots, 150$, $X_t \sim N(0, S^{(+)}R^{(-)}S^{(+)})$ for $t = 151, \dots, 250$, $X_t \sim N(0, S^{(+)}R^{(+)}S^{(+)})$ for $t = 251, \dots, 500$. One break occurs in both variances at $k_\sigma = 150$, second break in correlation coefficient at $k_\rho = 250$. Method 1: Testing for a break in the covariance. Method 2: Testing for a break in the correlation coefficient. Method 3: Testing for breaks in the variances and then for a break in the correlation. The table lists point estimate of interdependence break date k , width of 95% empirical confidence interval of date estimation and testing power of sizes 10%, 5% and 1%.

signals from the changes in volatility and correlation are mixed in this case. Ignoring the variances altogether produces the results of method 2. The power is slightly worse, but the bias in the estimates of the location are significantly smaller. Finally, the sequential approach reliably detects the correct location with great precision, especially for strong correlation shifts. The results are notably alike to unreported simulations in the case correlation shifts alone, i.e. with stable variances.⁵ This indicates that the estimation of variance breaks does not interfere with the later correlation break detection, as long as the variance breaks are handled in an appropriate way. This supports the asymptotic result in Theorem 6 of Qu and Perron (2007) that justifies the sequential testing approach, in contrast to a joint test for multiple breakpoint.

All the simulation results taken jointly motivate the implementation of our STP procedure, as it is robust to a number of potential situations without losing much efficiency when its full flexibility is not needed.

⁵Compare the last row of Table 7 with the first row of Table 3.

	Variance break	
Japan	12-04-96***	[11-27-96,01-23-97]
Hong Kong	08-13-97***	[07-31-97,08-27-97]
Thailand	05-12-97***	[04-03-97,07-02-97]
Indonesia	08-13-97***	[07-31-97,08-27-97]
Taiwan	07-21-97*	[08-06-96,01-27-98]
Malaysia	08-01-97***	[07-23-97,08-11-97]
Korea	10-14-97***	[10-13-97,10-31-97]
Philippines	04-21-97***	[04-08-97,06-18-97]

Table 8: Break date point estimates and 95% confidence intervals for variance breaks in the residuals of a VARX(1,1) model for the return series of eight East Asian countries according to the STP. *** denotes 99%, ** 95% and * 90% significance.

5 Contagion during the Asian crisis

The east Asian markets in 1997 constitute a well-known and much disputed example of financial contagion. The results from previous studies are mixed. Bae et al. (2003) finds only little evidence for contagion within Asia or from Asia to other world regions in 1997, and Dungey et al. (2005) report inconsistent findings for contagion within the region. Contagious events are identified by Bekaert et al. (2005), Corsetti et al. (2005), Rodriguez (2007) and Cho and Parhizgari (2008), whereas Candelon et al. (2005) can only find shock transmission via the continuation of preexisting transmission channels. Dungey et al. (2004) find no contagion from East Asia to the Pacific Region.

We apply the STP to eight series of daily stock index returns between January 07, 1996 and June 30, 1998 of Japan (JA), Hong Kong (HK), Thailand (TH), Indonesia (IN), Taiwan (TA), Malaysia (MA), Korea (KO) and the Philippines (PH). The data is taken from Datastream, labeled in US\$ and has been estimated by a VARX(1,1) model, with the BIC determining the optimal lag structure. U.S. stock index returns from the previous period serve as an exogenous input X_{t-1} that introduces a common shock factor into the regression, which is why the model deviates from a simple VAR(r) regression. The STP results contain variance break dates and correlation break date point estimates along with confidence intervals. The confidence intervals have been determined by a block bootstrap of 1,000 repetitions with blocks of 20 days in order to preserve the volatility clustering in the data.

The first results from the STP are the univariate variance break dates, listed in Table 8. The significant results indicate that all countries were af-

	Hong Kong	Thailand	Indonesia	Correlation break with			Korea	Philippines
				Taiwan	Malaysia			
Japan	02-02-98 n.s.	02-05-98*** [09-19-97,02-12-98]	02-03-98** [05-27-97,02-11-98]	12-09-97*** [07-22-97,01-30-98]	02-12-98*** [09-17-97,02-12-98]		02-10-98*** [07-08-97,02-12-98]	02-12-98 n.s.
Hong Kong	-	12-02-97*** [07-21-97,01-27-98]	02-12-98 n.s.	12-01-97*** [10-03-97,12-30-97]	02-12-98 n.s.		02-02-98*** [12-23-97,02-12-98]	- 01-30-98 n.s.
Thailand			-	10-21-97*** [08-20-97,01-05-98]	-		02-12-98*** [08-20-97,02-12-98]	- 09-25-97**
Indonesia			02-05-98*** [08-14-97,02-11-98]	10-24-97** [07-24-96,01-21-98]	02-05-98*** [10-03-97,02-10-98]		02-10-98*** [08-02-96,02-12-98]	[09-26-96,12-03-97]
Taiwan					-		02-11-98*** [04-21-97,02-12-98]	02-11-98 n.s. 10-23-97 n.s.
Malaysia					10-23-97*** [09-30-96,01-29-98]		02-12-98*** [12-26-97,02-12-98]	- 02-11-98 n.s.
Korea								- 01-08-98* [04-28-97,02-11-98]

Table 9: Break date point estimates and 95% confidence intervals for correlation breaks in the residuals of a VARX(1,1) model according to the STP. *** denotes 99%, ** 95% and * 90% significance.

Scenario	Correlation break	
(sc1) simultaneous contagion between all countries	02-11-98 n.s.	-
(sc2) simultaneous contagion between those country pairs that indicated contagion between 01-30-98 and 02-12-98 in bivariate testing	02-05-98***	[10-15-97,02-12-98]
(sc3) simultaneous contagion between those country pairs that indicated contagion between 01-30-98 and 02-05-98 in bivariate testing	02-02-98***	[12-02-97,02-02-98]
(sc4) simultaneous contagion between those country pairs that indicated contagion between 02-10-98 and 02-12-98 in bivariate testing	02-11-98***	[12-08-97,02-12-98]
(sc5) simultaneous contagion in pairings HK-TH and HK-TA	12-02-97***	[10-28-97,12-30-97]
(sc6) simultaneous contagion between central market TW and IN, MA, TH, PH respectively	10-23-97***	[09-19-97,01-27-98]

Table 10: Break date point estimates and 95% confidence intervals for important scenarios of simultaneous correlation breaks in the residuals of a VARX(1,1) model for the return series of eight East Asian countries according to the STP. *** denotes 99%, ** 95% and * 90% significance.

ected by crisis. Applying the tests for breakpoint distinctiveness, we found that the variance breaks of Hong Kong and Indonesia occur at the same time. Continuing the steps of the STP, correlation coefficients are examined next in order to detect contagion. Initially, correlation breaks are studied individually, hence in bivariate time series systems, with results listed in Table 9. Complementing the results of Candelon and Manner (2010), we find that financial contagion between several East Asian countries is significant. Notably, the very first countries affected by contagion are Thailand and the Philippines, on September 25, 1997. This fits the widely accepted notion of Thailand’s role in the initial spillover after the devaluation of the Thai Bath on July 2, 1997 (see, for example, Dungey et al., 2006). We further confirm that contagion occurred later than the outbreak of crisis in the individual countries. This is evident, as hardly any confidence intervals of the date estimates for variance breaks overlap with those of correlation coefficients.

Clearly, however, several correlation breaks are dated within a few days of one another, with overlapping confidence intervals. For example, contagion is significantly inferred in 5 pairs of countries between February 2, 1998 and February 5, 1998 and in another 6 pairs between February 10, 1998 and February 12, 1998. Other contagion clusters are detected in October and December of the previous year. This is an indication that correlation breaks are simultaneous in many cases, which motivates a multivariate testing of correlation breaks to improve the precision of the break date estimation. Table 10 reports

the correlation break results when using multivariate systems. Each of the selected scenarios investigates a set of at least $n = 3$ series, in which some correlations can be restricted from breaking, while others may break simultaneously. In the previous bivariate testing Taiwan appeared in several early correlation break dates at the end of October 1997. Thus, Taiwan, but also Malaysia or Indonesia, appear to contribute to the early diffusion of the Asian crisis according to bivariate testing. Scenario (sc6), a multivariate correlation break with central spillover country Taiwan between Malaysia and Indonesia, results in a contagion date on October 23, 1997.

When in scenario (sc5) Hong Kong is considered as a central contagion market to Thailand and Taiwan, contagion is detected on December 2, 1997. The confidence intervals of bivariate testing suggest a possibility of overlapping contagion between country pairs of the (sc5) set with pairs of early February countries from scenario (sc3), for example when looking at the pair of Hong Kong and Thailand and at the pair of Japan and Thailand. Again, distinctiveness tests were employed and reject the null hypothesis of simultaneous breaks. Even the pair Japan and Taiwan, estimated to have a correlation break on December 9 1997, cannot be assumed to share a common break with the (sc5) set earlier in December.

If all correlation coefficients are allowed to break in scenario (sc1), then the most likely contagion date is on February 11 1998, although the test statistic is not significant. The number of non-significant bivariate results indicates that this scenario may have too many degrees of freedom. Also, the variety in break dates found in smaller systems encourages the splitting of the group. The results in scenarios (sc3) and (sc4) best capture the contagion diffusion in the countries affected last. The outcome is an early February 1998 group and one affected one week later. Allowing for this time lag between the two sets of scenarios (sc3) and (sc4) as opposed to the common break hypothesis of scenario (sc2) results in a significant distinctiveness result at 99% confidence. In contrast, splitting countries into groups within the scenarios (sc3) and (sc4) groups does not yield significant distinctiveness results, which reinforces the cluster choice.

Summarizing the results, the application of the STP along with the variety of grouping possibilities enables a differentiated contagion analysis in the

Date	Pairs of affected countries
September 25 1997	Thailand-Philippines
October 23 1997	Taiwan-Thailand, Taiwan-Malaysia, Taiwan-Philippines, Taiwan-Indonesia
December 2 1997	Hong Kong-Thailand, Hong Kong-Taiwan
December 9 1997	Taiwan-Japan
January 8 1998	Korea-Philippines
February 2 1998	Japan-Hong Kong, Japan-Thailand, Japan-Indonesia, Hong Kong-Korea, Hong Kong-Philippines, Thailand-Indonesia, Thailand-Malaysia, Indonesia-Malaysia
February 11 1998	Japan-Malaysia, Japan-Korea, Japan-Philippines, Hong-Kong-Indonesia, Hong Kong-Malaysia, Thailand-Korea, Indonesia-Korea, Indonesia-Philippines, Taiwan-Korea, Malaysia-Korea, Malaysia-Philippines

Table 11: Time line of correlation break date estimates, which indicate the existence of contagion events between the stock markets of the listed countries.

Asian Financial Crisis case. The findings of Candelon and Manner (2010) are confirmed: The crisis has hit South East Asian countries first, before contagion affected the countries of the region several months later. The dating of the contagion events is different from the findings of Candelon and Manner (2010), possibly because of the changed VARX modeling, and certainly because larger sets of correlation coefficients were clustered. The results are collected in Table 11. First, increases in the interdependence between stock returns can be detected between Thailand and the Philippines on September 25, 1997, followed by spillovers on October 23, 1997 between central Taiwan and Thailand, Malaysia, the Philippines and Indonesia, respectively. The event precedes the news of a more global crisis impact, as evidenced by the drop of the Dow Jones industrial average by 7.2 percent on October 27, 1997, see Walsh (1998). A wave of contagion events occurred in early December affecting the countries of Hong Kong, Thailand, Taiwan and Japan. This result is somehow in line with the focus on Thailand and Hong Kong as contagion transfer countries in previous Asian crisis studies. Finally a large group of markets experienced increase interdependence in early February, up to 99 trading days after the first contagion episode. This lateness indicates that contagion may have been preventable between these countries, if policy action had been differently conducted after the first crisis and spillover impacts.

6 Conclusions and outlook

In this paper we propose a new approach to test for shift-contagion in financial markets. The approach has two distinct features. First of all, it separates the outbreak of a crisis, represented by structural break in volatility, and contagion, an increase in the interdependence between markets measured by correlation. This is done in a sequential way, simplifying the computations, but also the interpretation of the results. Second, the approach makes use of multivariate data, instead of relying on bivariate tests for all country pairs. This leads to very good power of the test and results in more precise estimates of the break dates. The tests are valid under mild assumptions, in particular heteroscedasticity and autocorrelation, and rely on the asymptotic theory developed in Qu and Perron (2007). Furthermore, while in general structural breaks are assumed to occur at distinct dates, we test whether some parameters share a common breakpoint using the test proposed in Perron and Oka (2011). This can lead to a simpler model and to more reliable testing results.

Monte Carlo simulations show favorable properties of our approach. First of all, using multivariate systems when testing for contagion will lead to higher power and a more reliable identification of the break dates. However, it is also found that beyond a certain number of time series, somewhere around 5-10, increasing the size of the system will not necessarily lead to better results. Second, our simulations illustrate that separately treating variance and correlation breaks will prevent biased estimation of the contagion date when the two do not break simultaneously, while the procedure is still valid without any drawbacks when the increased generality is not needed.

Our application to the Asian crisis in 1997 offers some new insight into the way the crisis spread over the region. First, we confirm the occurrence of contagion in general. We also find clear evidence that changes in volatility took place before changes in correlation, justifying the separate treatment of the two and the sequential approach to contagion testing. Next, subsequent episodes of contagion are evidenced in the winter months of 1997/1998, each efficiently dated by multivariate testing, since a number correlation coefficients can be assumed to break at common dates.

The question of determining the origin and causality of contagion events

is one aspect that our presented approach does not directly regard. In contrast, Khalid and Kawai (2003) employ the concept of Granger causality and the frequency domain approach of Bodart and Candelon (2009) recognizes a directional dimension as well. A test that distinguishes between sudden contagion as opposed to slow mutation can further be considered, see Candelon et al. (2008) and their analysis of structural changes in the synchronization of financial bear and bull markets or Brière et al. (2012) and their discrimination between contagion and globalization. Such issues could further be addressed in future research.

A Appendix

Assumptions

We make the following assumptions about the error term. They are formulated with respect to regimes, as $\varepsilon_t \sim (0, \Sigma_j)$, for $[\lambda_{j-1}T] + 1 \leq t \leq [\lambda_j T]$, with the break fractions $\lambda_j = t_j/T$, $\lambda_0 = 0$, $\lambda_{m+1} = 1$, for $j = 1, \dots, m+1$. The assumptions cover the typical features observed in financial returns. In particular, the presence of conditional heteroscedasticity is allowed. Assumption A3 guarantees asymptotic distinctiveness of stability regimes.

- Assumption A1: Let $\mathcal{F}_t = \sigma - \text{field}\{\dots, \varepsilon_{t-2}, \varepsilon_{t-1}\}$. ε_t is weakly stationary within each segment and (a) $\{\varepsilon_t, \mathcal{F}_t\}$ forms a strongly mixing (α -mixing) sequence with size $-4r/(r-2)$ for some $8 > r > 2$, (b) $E(\varepsilon_t) = 0$ and $\sup_t \|\varepsilon_t\|_{2r+\delta} < M < \infty$ for some $\delta > 0$ and $M > 0$, where $\|X\|_r = (\sum_i \sum_j E|X_{i,j}|^r)^{1/r}$, for $r \geq 1$, is the L_r -norm of a random matrix X , (c) let $S_{k,j}(l) = \sum_{t_{j-1}^0+l+1}^{T_{j-1}^0+l+k} (\varepsilon_t \varepsilon'_t)$, $j = 1, \dots, m+1$, for each $e \in \mathbb{R}^n$ of length 1, $\text{var}(\langle e, S_{k,l}(0) \rangle) \geq v(k)$ for some function $v(k) \rightarrow \infty$ as $k \rightarrow \infty$ (with $\langle \cdot \rangle$, the usual inner product).
- Assumption A2: $\{\varepsilon_t \varepsilon'_t - \Sigma_j^0\}$ satisfies the conditions stated for ε_t in Assumption A1.
- Assumption A3: $0 < \lambda_1^0 < \dots < \lambda_m^0 < 1$, where $t_j^0 = [T\lambda_j^0]$, $j = 1, \dots, m+1$.

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