

Sacht, Stephen

Conference Paper

Analysis of Various Shocks within the High-Frequency Versions of the Baseline New-Keynesian Model

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2014: Evidenzbasierte Wirtschaftspolitik
- Session: Dynamic Macro Models, No. E10-V3

Provided in Cooperation with:

Verein für Socialpolitik / German Economic Association

Suggested Citation: Sacht, Stephen (2014) : Analysis of Various Shocks within the High-Frequency Versions of the Baseline New-Keynesian Model, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2014: Evidenzbasierte Wirtschaftspolitik - Session: Dynamic Macro Models, No. E10-V3, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft, Kiel und Hamburg

This Version is available at:

<https://hdl.handle.net/10419/100372>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Analysis of Various Shocks within the High-Frequency Versions of the Baseline New-Keynesian Model

Stephen Sacht*

July 13, 2014

Abstract

In this paper we analyze a hybrid small-scale New-Keynesian model with an arbitrary frequency of the agents' synchronized decision making. We study the impact of various demand and supply shocks on the dynamics of the model variables. We show that the corresponding impulse-response functions of high-frequency versions of the model can qualitatively as well as quantitatively be fairly dissimilar from their quarterly counterparts. This can be explained by the decrease in the effectiveness of monetary policy responses to these shocks and the overall increase of inertia in the model variables. In particular, different kinds of frequency-dependent persistence effects occur, which dampen the pass-through of output gap movements into inflation rate dynamics as the period length decreases. The main conclusion is that DSGE modelling may be more sensitive to its choice of the agents' decision interval.

Keywords: Hybrid New-Keynesian model; high-frequency modelling; monetary policy; frequency-dependent persistence.

JEL classification: C63, C68, E32, E52.

*Kiel Institute for the World Economy, Kiellinie 66, 24105 Kiel & Department of Economics, Christian-Albrechts-University Kiel, Olshausenstrasse 40, 24118 Kiel, Germany. Email: stephen.sacht@ifw-kiel.de or sacht@economics.uni-kiel.de

1 Introduction

In this paper we are concerned with an elementary methodological issue in the macroeconomic modelling of dynamic stochastic general equilibrium (DSGE) models. These models are usually formulated in discrete time. As an approximation to reality, they assume that decisions are taken discontinuously and that all transactions of a certain class occur in the same synchronized rhythm. There is, however, no argument that would allow one to identify a uniform natural decision period. If the models, especially in numerical studies, invoke a definite period, which is mostly a quarter, the choice is entirely determined by convention or the frequency of the available data.

However, over the last years researchers seek to develop new procedures in order to collect and analyze high-frequency data. Various research projects have benefited from rapid advances in information and communication technologies during the last two decades. As a prominent example, the Billion Prices Project at the MIT Sloan provides daily inflation data based on scraped price indices, i.e. indices calculated from prices which are available online. Ahrens and Sacht (2014) show that such kind of data can be used for the estimation of a New-Keynesian Phillips Curve (NKPC) since the risk of a small sample bias and structural breaks are reduced. The former is based on the fact that just a small amount of observations are available on a quarterly (or even annual) frequency. In order to circumvent this problem a long time span of several months (or years) must be considered with a high probability of structural breaks included. For instance, when estimating a model based on US data, the sample is split into two sub-samples, i.e. the GI and GM period, respectively. This can be motivated by a rapid decline in the volatility of inflation at the beginning of the 1980's. As Ahrens and Sacht (2014) show under consideration of daily data over an interval of roughly two years, they are able to provide significant parameter estimates for the degree of price stickiness (in Argentina).

As information on high-frequent adjustments in the economic indicators become available, this calls for a modification of the DSGE model with respect to the period length. In particular, a reduction of the period length is tantamount to more frequent decisions and transactions within a given calendar time unit. A model version with a shorter period than in the original formulation is thus a high-frequency economy. If the period of the benchmark model serves as the time unit, which is fixed, and a high-frequency economy is constituted by a period of length $0 < h < 1$, the latter will also be designated an h -economy. In this paper we are going to analyze various shocks in a hybrid baseline 3-equations NKM under explicit consideration of the period length. In particular, we compare IRFs based on these shocks across different frequencies of decision making. Furthermore, we give an explanation for how the transmission channel of monetary policy is affected in transition from one h -economy to the other. In order to transform the model into its high-frequency equivalent we apply specific transition rules on the frequency-dependent variables and parameters.

Modeling the impact of different period lengths on the dynamics of the current workhorse model used for monetary and fiscal policy evaluation, the NKM with sticky prices (and wages), has been done by Flaschel et al. (2008) and Anagnostopoulos and Giannitsarou (2010b) in the first place. These studies are mainly concerned with analyzing the impact of the change in the period length on the determinacy of the model. While Flaschel et al. (2008) indicate that determinacy turns out to pose no problem in a NKM, Anagnostopoulos and Giannitsarou (2010a, pp.11ff) show that determinacy may depend on the length of the adjustment period (jointly with a tax rate on labour). This is confirmed for high-frequency versions of RBC models (Benhabib and Farmer (1994)). Hintermaier (2005) discusses a set of numerical parameters such that, for instance, the quarterly economy is determinate but not its weekly version. Posch et al. (2011) present a numerical solution technique based on Chebychev polynomials in order solve a NKM formulated

entirely in continuous time. Besides theoretical work, empirical investigations on the role of the frequency in DSGE models is even more rare. For example, one might think of a calibration or estimation procedure, which takes explicit account of different decision intervals on the part of the economic agents and tries to let the data decide on an appropriate period. We only know of two examples of such work in the literature, namely Christiano (1985) and Aadland (2001).

Our work is quite closely related to Sacht and Wohltmann (2014), Sacht (2014) and Franke and Sacht (2014). The latter show that by diverging from the standard assumption of the baseline period length to be one quarter this leads to distinct changes in the dynamic properties of the model. In particular, the authors state that while determinacy of the model remains unaffected, the IRFs can differ in a quantitative and qualitative significant way just by increasing the frequency of decision making (i.e. assuming a monthly, weekly or daily length of the period). While Franke and Sacht (2014) also provide an elaborate analytical discussion of the polar case of a decision interval given in continuous-time (where h is approximately zero), in this paper we are concerned with the investigation of the IRFs based on different values of h only. Therefore, we also include a nominal interest rate shock in our study but also consider a technology as well as a cost-push shock. Most importantly, we fill the gap in the paper by Franke and Sacht (2014) by giving an explicit (economic) explanation for the observed changes in the IRFs across frequencies.

Sacht and Wohltmann (2014) study analytically and numerically the high-frequency optimal monetary policy responses to a (deterministic and stochastic) cost-push shock. They show that the welfare loss changes (in relation) across different optimal monetary policy regimes under variation of the period length. In particular, the monetary authority faces a decrease in their effectiveness to dampen the increase in the inflation rate. Overall, the welfare loss increases as the period length decreases together with an amplification of the policy trade-off in case of a cost-push, i.e. a more pronounced stagflation scenario occurs. While in their elaborate analytical work mainly the derivation of the loss functions under different optimal monetary policy regimes and frequencies are presented, an intensive investigation of the corresponding IRFs is also done in Sacht (2014). Here, the author also seeks to identify the economic rationale which leads to the results. Furthermore, we provide a sensitivity analysis with respect to the degree of price stickiness, the inflation targeting regime and the persistence in the autocorrelated shock process.

The remainder of this paper is structured as follows. In the next section we discuss the concept of a h -economy and provide transition rules for the frequency-dependent variables and parameters. In section 3 we provide economic explanations for the upcoming results based on the change in the (frequency-dependent) intrinsic, extrinsic and inherited persistence in the model. Section 4 contains the numerical analysis of a nominal interest rate, technology and cost-push shock based on IRFs under consideration of different frequencies. Section 5 concludes. The microfoundations of the h -economy model, the solution of the shock process on a higher frequency, the representation of the reduced-form solutions of the model variants as well as all Figures can be found in the Appendix.

2 The Concept of a High-Frequency Economy

In a NKM the underlying period length is assumed to be a quarter. Here, we stick to this convention by defining a quarter as the benchmark period length, although the latter is arbitrary and can be e.g. a month or a year instead. In particular, in order to compare two economies with different frequencies of the synchronized actions, we fix the time unit as a quarter and generally allow the agents to make their decisions and carry out the corresponding transactions every *subperiods*, i.e. h quarters. The period length relative to

a quarter is then defined by

$$0 < h = \frac{1}{f} < 1 \quad (1)$$

where f denotes the *frequency* of decision making. We assume that the frequency changes homogeneously. This means that all decisions and transactions of the representative household, firm and the central bank occur in the same, synchronized rhythm, e.g. on a quarterly ($f = 1$), monthly ($f = 3$), weekly ($f = 12$) or even daily ($f = 90$) frequency. As mentioned above, such a high-frequency economy is simply called an *h-economy*.

As a general assumption we claim that the transactions in quarterly magnitudes are spread symmetrically over the corresponding subperiods in an h-economy. For illustration, a nominal interest rate of, for example, 1.50% per quarter means that, in an h-economy, hundred dollars earn $h \cdot 1.50$ dollars over the period $[t, t+h]$. The time preference rate of the representative household is to be interpreted analogously.¹ This procedure can not only be found in Flaschel et. al (2008), Anagnostopoulos and Giannitsarou (2010a,b), Franke and Sacht (2014) and Sacht and Wohltmann (2014) with respect to standard NKMs, but also in the modelling of search and matching processes (Rogerson et al. (2005, p. 963)).

In order to transform a model under consideration of a benchmark period length into an h-economy, Franke and Sacht (2014) formulate specific transition rules. These rules can be summarized in the following three steps:

- (1) To start with, the *baseline* model with $h = 1$ must be formulated. Hence, given a discrete time specification, h is then considered explicitly. As a main assumption, we claim that the functional form of the model, i.e. the system equations, remain unchanged across different frequencies. As an example, the dynamic IS equation in a NKM describes the consumption decision of a representative household based on consumption smoothing. This holds independently on a low- or high-frequency as we show in the Appendix.
- (2) For the transformation into an h-economy framework, the *frequency-dependent* components of the model have to be suitably adjusted. In particular, the frequency-dependent variables and parameters are going to be changed under consideration of h . Regarding the parameters, e.g. the discount rates and probabilities, those differ across all subperiods relative to the benchmark period length as $0 < h < 1$ holds, while fractions remain unaffected. With respect to the variables, growth rates exhibit a certain time dimension and therefore have to be divided by h , while contemporaneous adjustment rates have no time dimension. Throughout this paper we will discuss these issues in greater detail.
- (3) For a direct comparison of several h-economies, the stock variables are uniformly expressed as *normalised*, e.g. quarterized (given the baseline model under $h = 1$ is formulated in quarterly) magnitudes. Given that the modified h-economy specification of the model after step (2) is applied, this means, that all frequency-dependent variables are divided by h in order to ensure conformity across different period lengths. In this respect, note that a variable without time-dimension like the output gap are *not* quarterized. Alternatively, normalization (quarterization) can be seen as nothing else than a specific aggregation technique applied on the high-frequency stock variables relative to the benchmark period length. In particular, different kinds of aggregate schemes exist in the (econometric) literature. In our study we apply the so-called *systematic/skip sampling aggregation* scheme based on the deviation of the corresponding variables by h . Once again, throughout this paper we will discuss this issue in greater detail.

¹While this treatment introduces a compound interest effect, it is confirmed numerically that this effect is certainly not strong enough to explain the differences over the first few quarters.

In this paper we consider the following standard NKM in its hybrid representation. In the *first step* we adopt the specifications presented in Smets and Wouters (2003), Christiano et. al (2005) and Galí (2008, his Chapter 3) in order to formulate the model, while the underlying period length is a *quarter* ($h = 1$):

$$y_t = \frac{1}{1+\chi} E_t y_{t+1} + \frac{\chi}{1+\chi} y_{t-1} - \delta_1 (i_t - E_t \pi_{t+1} - \bar{i}) + \delta_2 v_t^y \quad (2)$$

$$\pi_t = \frac{\beta}{1+\alpha\beta} E_t \pi_{t+1} + \frac{\alpha}{1+\alpha\beta} \pi_{t-1} + \kappa y_t + v_t^\pi \quad (3)$$

$$i_t = \bar{i} + \phi_\pi \pi_t + \phi_y y_t + v_t^i \quad (4)$$

$$v_t^z = \rho^z v_{t-1}^z + \varepsilon_t^z, \quad z = \{y, \pi, i\} \quad (5)$$

with

$$\delta_1 = \frac{1-\chi}{\sigma(1+\chi)} \quad (6)$$

$$\delta_2 = \frac{(1-\chi)(1+\eta)(\rho^y - 1)}{(1+\chi)(\sigma + \eta)} \quad (7)$$

$$\beta = \frac{1}{1+\nu} \quad (8)$$

$$\kappa = \frac{(1-\theta)(1-\theta\beta)}{\theta(1+\beta\alpha)} (\sigma + \eta) \quad (9)$$

E_t denotes the mathematical expectation operator conditional on information up to period t . Equation (2) describes a hybrid dynamic IS curve and results from the standard utility maximization approach of a representative household. In this case the current output gap depends negatively on the real interest rate ($i_t - E_t \pi_{t+1}$), i.e. it is stemming from intertemporal optimization of consumption and saving, which then results in consumption smoothing. The constant natural rate of interest is denoted by \bar{i} . The parameters $\sigma \geq 0$ and $\eta \geq 0$ denote the inverse intertemporal elasticity of substitution and the substitution elasticity of labor, respectively. Intrinsic persistence is imposed on this demand equation by the degree of the households' external habit formation denoted by $0 \leq \chi \leq 1$. Equation (3) is known as the hybrid NKPC where the output gap (y_t) is the driving force of inflation due to monopolistic competition and the Calvo price-setting scheme. The parameter β denotes the discount factor, where $\nu > 0$ serves as the time preference rate, i.e. $0 < \beta < 1$ holds. The price indexation parameter α displays the degree of *intrinsic persistence* in the inflation rate dynamics. The slope of the NKPC, which measures the degree of *inherited persistence* with respect to y_t , is given by the composite parameter $\kappa \geq 0$. The latter consists on the discount parameter, the degree of price indexation and the Calvo (1983) parameter of price setting, where the probability of resetting the price of a representative firm on a given frequency is denoted by $0 \leq \theta \leq 1$. According to the interest rate rule of Taylor (1993) type (cf. equation (4)), the monetary authority reacts directly to movements in the output gap ($\phi_y \geq 0$) and the inflation rate ($\phi_\pi \geq 0$). We assume that the exogenous driving forces in the model variables are given by a technology shock v_t^y , a cost-push shock v_t^π and a nominal interest shock v_t^i , respectively (cf. equation (5)). These shocks follow an autocorrelated process under consideration of an exogenous persistence parameter $0 \leq \rho^z < 1$ with $z = \{y, \pi, i\}$. Accordingly, the dynamics of the system are set in motion by one-time impulses denoted by $\varepsilon_{z,t}$ in time $t = 0$. Note that we consider a *deterministic* impulse only, i.e. the corresponding standard deviations of the shocks are assumed to be zero.²

²In direct comparison to the work of Franke and Sacht (2014), we allow also for a serially correlated technology and cost-push in addition to a nominal interest rate shock. Furthermore, backward-looking

In the *second step* we adjust the frequency-dependent variables and parameters of the NKM in quarterly magnitudes (described by the equations (2) to (9)) by h in order to transform the system in an h -economy. Note that now $0 < h < 1$ holds. It is not necessary to spell out the details here that lead from the microfoundations to the structural NKPC and dynamic IS equation in an h -economy.³ However, for completeness these microfoundations can be found in the Appendix. It turns out that, let's say, direct ad-hoc adjustments of the variables and parameters by h coincide with those obtained through microfoundation.

First, we discuss the representation of the NKPC in an h -economy, which is given by:

$$\Delta p_t = p_t - p_{t-h} = \frac{\beta(h)}{1 + \alpha\beta(h)} E_t \Delta p_{t+h} + \frac{\alpha}{1 + \alpha\beta(h)} \Delta p_{t-h} + \kappa(h) y_t + h v_t^\pi \quad (10)$$

with

$$\beta(h) = \frac{1}{1 + h\nu} \quad (11)$$

$$\kappa(h) = \frac{[1 - \theta(h)][1 - \theta(h)\beta(h)]}{\theta(h)[1 + \alpha\beta(h)]} (\sigma + \eta) = \frac{h^2(1 - \theta)(1 + \nu - \theta)}{(1 - h(1 - \theta))(1 + h\nu + \alpha)} (\sigma + \eta) \quad (12)$$

and $\Delta p_{t+h} = p_{t+h} - p_t$ as well as $\Delta p_{t-h} = p_{t-h} - p_{t-2h}$.⁴ The second part of equation (12) can be derived under consideration of the explicit expressions for $\theta(h)$ and $\beta(h)$. The values of two of the structural parameters, which enter the NKPC, are dependent on the frequency of decision-making. First, the time preference rate ν becomes $h\nu$ in an h -economy since the household is less discounting future changes in the utility over a subperiod with length $0 < h < 1$. From this it follows that the discount factor turns into $\beta = \beta(h) = 1/(1 + h\nu)$. Second, the price stickiness θ also has to be adjusted. As in a period of length h the probability for resetting the price of the firm will be $h(1 - \theta)$. We have $\theta(h) = 1 - h(1 - \theta)$ if the pure symbol θ is retained for the constituent stickiness parameter from the quarterly setting. The stickiness remains the same in the sense that on average a firm is allowed to reset the price every $1/[1 - \theta(h)]$ periods of length h , which, independently of h , means every

$$h/[1 - \theta(h)] = h/[1 - 1 + h(1 - \theta)] = 1/(1 - \theta)$$

quarters. Under consideration of $\beta(h)$ and $\theta(h)$, the slope of the NKPC turns into $\kappa(h)$. As we will discuss later, it can be shown that the slope becomes flatter on a higher frequency, i.e. as the period length decreases. Again, since the output gap exhibits no time dimension, $\kappa(h)$ is multiplied by h , therefore consider the term h^2 in equation (12). The parameters σ and η are not frequency-dependent since elasticities also exhibit no time-dimension. Note furthermore that according to the concept of price indexation, a fraction of firms show their willingness or ability to re-optimize their price in a *point in time*. Hence the parameter α , which denotes this fraction, is not frequency-dependent, either.

Making use of the previous expression for the high-frequency NKPC and the fact that the (natural) nominal interest rate is expressed by high-frequency magnitudes by

behavior in the inflation rate dynamics is incorporated by the assumption of price indexation (Christiano et. al (2005) instead of rule-of-thumb price-setting (Galí et. al (2001))). The reason for this is that we claim that price indexation is the dominant procedure in the modeling of a hybrid NKMs in the literature.

³The Calvo setting is helpful in this respect, whereas markup pricing together with Taylor's staggered wage contracts would be more difficult to treat (see Christiano (1985)).

⁴Franke and Sacht (2014) show that the solution path of Δp_t (and y_t ; see below) turns into a specific *differential* equation in transition to a continuous time specification of the model. More precisely, it can be shown that, under our numerical parameter scenario given here, the IRFs of the model variables converge towards finite values if the period length h tends to zero. Therefore, we refer to the 'Observations' (or better: 'Lemmata') 1 to 3 (together with the associated Appendix) presented in Franke and Sacht (2014).

multiplying with h , the Taylor rule for the h -economy reads

$$hi_t = h\bar{i} + \phi_\pi \Delta p_t + h\phi_y y_t + hv_t^i. \quad (13)$$

It is worth mentioning that the monetary policy parameter with respect to the inflation rate (ϕ_π) remains unaffected by a change in the period length, while this does not hold for the policy reaction to the output gap denoted by ϕ_y . The reason is that the inflation rate is defined as a growth and the output gap as an adjustment rate. For the latter note that the output gap is defined as the difference in the actual level of output to the actual natural level of output (i.e. the level of output in the absence of nominal rigidities) and therefore has no time-dimension. Instead, ϕ_y is multiplied by h . This procedure can be found in an overwhelming amount of numerical studies, where the parameter ϕ_y is calibrated based on, e.g. an annual nominal interest rate. In this case ϕ_y is divided by 4 in order to express the monetary authorities reaction to changes in the output gap in quarterly magnitudes, while ϕ_π remains the same (an example is given in Galí (2008, p. 56)).

The output gap as a ratio of two flow magnitudes needs *not* to be transformed since, again, y_t is defined as a contemporaneous adjustment rate:

$$y_t = \frac{1}{1+\chi} E_t y_{t+h} + \frac{\chi}{1+\chi} y_{t-h} - \delta_1(h)(i_t - E_t \pi_{t+h} - \bar{i}) + \delta_2(h) v_t^y \quad (14)$$

with

$$\delta_1(h) = h \frac{1-\chi}{\sigma(1+\chi)} \quad (15)$$

$$\delta_2(h) = h \frac{(1-\chi)(1+\eta)(\rho(h)^y - 1)}{(1+\chi)(\sigma+\eta)}. \quad (16)$$

The only frequency dependent components of the dynamic IS equation are the variables i_t , $E_t \pi_{t+h}$ and \bar{i} , which are all multiplied by h and the persistence parameter ρ^y , which has to be reformulated into $\rho(h)^y = 1 - h(1 - \rho^y)$ (see the derivation below). $\rho(h)^y$ is part of the composite parameter $\delta_2(h)$ due to the existence of the technology shock v_t^y in the (hybrid) dynamic IS equation. Equivalent to the price indexation parameter α , the habit formation parameter χ stands for a fraction. In more detail, in the quarterly model current consumption c_t^k of household k and past aggregate consumption C_{t-1} (scaled to the household's consumption level) enter the utility function as the difference $c_t^k - \chi C_{t-1}$. This expression requires χ to be dimensionless, as in the h -economy it simply becomes $hc_t^k - \chi hC_{t-h}$, if the consumption variables are quarterized, too. Note that in a DSGE model, the goods market is cleared in every period, i.e. $y_t = c_t$ holds. Again, further details regarding the microfoundation of the high-frequency (hybrid) dynamic IS equation can be found in the Appendix.

Finally, we are going to transform the AR(1) shock processes (5). Therefore, we express the law of motion as a *contraction rate*

$$v_t^z - v_{t-1}^z = (\rho^z - 1)v_{t-1}^z + \varepsilon_t^z. \quad (17)$$

Accordingly, over the subperiod h we get

$$v_t^z - v_{t-h}^z = h(\rho^z - 1)v_{t-h}^z + \varepsilon_t^z \quad (18)$$

which, after simple re-arrangement, leads to

$$v_t^z = \rho(h)^z v_{t-h}^z + \varepsilon_t^z \quad (19)$$

where

$$\rho(h)^z = 1 - h(1 - \rho^z) \quad (20)$$

and $z = \{y, \pi, i\}$ holds (see also Franke and Sacht (2014) as well as Sacht and Wohltmann (2014)). Note that the term $(\rho^z - 1)$ can be interpreted as the *contraction persistence*. This contraction persistence refers to the nature of the AR(1) process itself since $0 \leq \rho^z < 1$ holds, i.e. the shock dies out over a finite amount of periods. By multiplying this contraction rate with h , the shock is spread over the corresponding amount of *subperiods*. In this case the contraction rate is given by equation (20) and, hence, now becomes frequency-dependent. A more elaborate discussion on the effects of a change in the shock process with respect to h is given in the next section. It goes without saying, that the deterministic impulse ε^z becomes part of the agents' information *after* the shock occurs. Due to the assumption of rational expectations, the shock hits the economy surprisingly on impact, while the dynamics of the variables are perfectly predictably afterwards.

In the *third and final step* we quarterize all frequency-dependent variables in order to ensure the comparability across different h -economies. Note that here we apply the *systematic/skip sampling aggregation* scheme in order to compare different h -economies. This means that we take into consideration the value of the inflation rate in the h -economy and its corresponding counterpart in the quarterly economy on the given quarter. For a direct comparison it is necessary to divide the associated variable(s) by h . This concept is taken directly from the econometrics literature. Silvestrini and Veredas (2008, p. 459) state that the systematic/skip sampling aggregation scheme has to be applied on *stock* variables like the inflation and the nominal interest rate. The reason is that decisions based on stocks depend on the recent realized values of these variables.⁵ Based on that, denoting the log price prevailing over the same time interval by p_t , the quarterized inflation rate is given by

$$\pi_t^{(h)} = \frac{\Delta p_t^{(h)}}{h} \quad (21)$$

with $\Delta p_t^{(h)} = p_t - p_{t-h}$. The superscript (h) indicates the difference in the variable of the baseline ($h = 1$) relative to the h -economy ($0 < h < 1$) model. Note that in the former case $\pi_t^{(1)} = \Delta p_t^{(1)}/1 = p_t - p_{t-1}$ holds. For clarification, in the following, we omit this superscript (except in cases where a direct comparison is necessary). Therefore the NKPC (10) and the Taylor rule (13) for $0 < h < 1$ are divided by h itself, respectively, while the dynamic IS equation remains unaffected. The *quarterized* h -economy version of the NKM is finally given by

$$y_t = \frac{1}{1+\chi} E_t y_{t+h} + \frac{\chi}{1+\chi} y_{t-h} - \delta_1(h)(i_t - E_t \pi_{t+h} - \bar{i}) + \delta_2(h) v_t^y \quad (22)$$

$$\pi_t = \frac{\beta(h)}{1+\alpha\beta(h)} E_t \pi_{t+h} + \frac{\alpha}{1+\alpha\beta(h)} \pi_{t-h} + \tilde{\kappa}(h) y_t + v_t^\pi \quad (23)$$

$$i_t = \phi_\pi \pi_t + \phi_y y_t + v_t^i. \quad (24)$$

$$v_t^z = \rho(h)^z v_{t-h}^z + \varepsilon_t^z \quad (25)$$

with (11), (15) and (16) as expressions for the composite parameters $\beta(h)$, $\delta_1(h)$ and $\delta_2(h)$,

⁵A large body of conflicting literature on this kind of time aggregation exists. For a survey see, again, Silvestrini and Veredas (2008). Hassler (2011) is being named as a representative of the literature, who points out that due to the existence of competing methods of aggregation, i.e. temporal aggregation (cumulation of flows), systematic/skip sampling (specific value at a point in time) or simply averaging, it is unclear which of these different schemes to choose. Drost and Nijman (1993) show that temporal aggregation and systematic/skip sampling might produce different outcomes. The reason is that information will be lost through aggregation - depending on what kind of aggregation method is applied. In Sacht and Wohltmann (2014) *temporal aggregation* is applied on the inflation rate, which is based on a specific economic interpretation. However, by direct comparison of the qualitative and quantitative observations in both studies, it turns out that there exist just minor quantitative differences in the results.

respectively. According to equation (12), $\kappa(h)$ is now redefined as

$$\tilde{\kappa}(h) = \frac{\kappa(h)}{h} = \frac{h(1-\theta)(1+\nu-\theta)}{(1-h(1-\theta))(1+h\nu+\alpha)}(\sigma+\eta). \quad (26)$$

Since we assume deterministic shocks only, we omit the expectation operator E_t . The dynamic system above is our point of departure for our analytical and numerical analysis undertaken in the following sections.

We close this section with a brief discussion on the stability of an h-economy, i.e. as $0 < h < 1$ holds. Therefore, we consider the purely forward-looking case ($\chi = \alpha = 0$) only. It is well-known (for $h = 1$) that within this model environment, the dynamic system is determinate and saddlepoint stable if the following condition is true:

$$(1 - \beta(h))\phi_y + \tilde{\kappa}(h)(\phi_\pi - 1) > 0. \quad (27)$$

Note that this expression is the same as shown in the seminal paper by Bullard and Mitra (2002, p. 1115) except for the frequency-dependent parameters β and κ , which have to be substituted by $\beta(h) = 1/(1 + \rho h)$ and $\tilde{\kappa}(h) = \kappa(h)/h$. It goes without saying that in the case $h = 1$, this stability condition is fulfilled if the Taylor principle holds, which requires $\phi_\pi > 1$ to be true since all (composite) parameters are assumed to be equal or greater than zero. It is obvious that in the purely forward-looking h-economy model variant, the Taylor principle also serves as a sufficient condition for the existence of a unique solution, which holds for any value of $0 \leq h < 1$. For a more elaborate discussion on determinacy — especially concerning the hybrid specification of the high-frequency model — we refer to Flaschel et. al (2008) and Franke and Sacht (2014).⁶

3 The Impacts of Frequency-Dependent Persistence

To explain the results presented here, it is important to analyze the underlying driving forces which cause the adjustments in the model under the variation of the period length h . To begin with, the structure of the shock and, in particular, the *exogenous* persistence process must be investigated. From equation (20) it can be seen that, when compared to its equivalent in the baseline model given by (5), the shock process under this specification for the high-frequency model differs from the one in the quarterly model due to the existence of h . In particular, $v_t^z = \rho^z v_{t-1}^z + \varepsilon_t^z$ is reformulated into $v_t^z = \rho(h)^z v_{t-h}^z + \varepsilon_t^z$ with $\rho(h)^z = 1 - h(1 - \rho^z)$ where z denotes the three different kinds of shocks. As a consequence, the history of the shock process is changed if the baseline model is reformulated into an h-economy framework. To see this, consider the expectation value of AR(1) process v_t^z given by the solution with respect to the initial shock in $t = 0$ (cf. equation (20); see the Appendix for a proof):

$$E_0 v_t^z = [\rho(h)^z]^t \varepsilon_0^z, \quad (28)$$

where E_0 denotes the expectation operator conditional on the information given in the initial period $t = 0$. As we turn to an investigation on a higher frequency relative to the baseline period length, the dynamics in the shock process are spread over all *subperiods*.⁷ This observation is mapped into an increase of the weight on the previous realization of the shock as h (v_{t-h}^z) declines. From this it follows that *less* of the impact of the initial

⁶Note that, given a hybrid specification of the NKM, a check on determinacy is not straightforward since an intensive analysis of the (in)stable Eigenvalues is strictly required in this case.

⁷Obviously, the shock process v_t^z displays the exogenous impact on the system in every subperiod up to t , e.g. for every month, week or day. In Sacht and Wohltmann (2014), $sh = t$ is assumed which leads to $s = 0, 1, \dots, t/h$. Here, v_t^z displays the value of the shock in the h-economy relative to every benchmark period (which is assumed also to be a quarter in their study). We consider this specification explicitly as we compute the value of the loss (relations) in Sacht (2014).

impulse (ε_0^z) has vanished relative to the baseline period length in the transition from one subperiod to another. More precisely, the difference in $v_t^z - v_{t-h}^z$ becomes *smaller* as h decreases since $\rho(h)^z > \rho^z$ holds for $0 < h < 1$ and all values of ρ^z . Due to the fact that the contraction persistence $\rho(h)^z$ is decomposed in a regular $(1 - \rho^z)$ and a frequency dependent (h) term, it can be denoted as the *frequency-dependent ‘contraction’ persistence* (FCP). According to equation (28) this kind of updating-process of the system’s exogenous driving force, applied in every period, must lead to higher persistence in an h-economy environment due to the increase in the ‘life span’ of the shock itself.

By investigating the dynamics in case of various shocks, *inherited* persistence must also be taken into account. This kind of persistence displays the cross-relationship between the movements in the output gap and the inflation rate. It is measured by the slope of the NKPC, i.e. the frequency-dependent composite parameter $\tilde{\kappa}(h)$. Note that this parameter is crucial for the implementation of a(n optimal) monetary policy strategy induced by the central bank. The latter has direct control over the nominal interest rate, which therefore serves as its primary policy instrument. However, the inflation rate can only be influenced *indirectly* by changing the output gap via consumption smoothing (see the next sections for a detailed analysis of the corresponding transmission channel(s)). Hence, the *effectiveness* of monetary policy relies entirely on $\tilde{\kappa}(h)$: the impact of the change in the output gap depends on the slope of the NKPC. A high value of the latter might lead to a stronger stabilization effect on the inflation rate as a shock occurs. For clarification we decompose $\tilde{\kappa}(h)$ into

$$\tilde{\kappa}(h) = \tilde{\lambda}(h)(\sigma + \eta) \quad (29)$$

with

$$\tilde{\lambda}(h) = \frac{h(1 - \theta)(1 + \nu - \theta)}{(1 - h(1 - \theta))(1 + h\nu + \alpha)}, \quad (30)$$

since the elasticities σ and η do not depend on h and account just for scale effects. Based on equation (30) we are able to discuss the change in the slope with respect to an increase in the degree of price stickiness and indexation as well as the period length. Applying the total differential on $\tilde{\lambda}(h)$ this leads to

$$d\tilde{\lambda}(h) = \frac{\partial \tilde{\lambda}(h)}{\partial \alpha} d\alpha + \frac{\partial \tilde{\lambda}(h)}{\partial \theta} d\theta + \frac{\partial \tilde{\lambda}(h)}{\partial h} dh.$$

Since all parameters in $\tilde{\lambda}(h)$ are bounded between 0 and 1, it can be easily checked that the following results regarding the sign of the partial derivatives hold, respectively:⁸

$$\frac{\partial \tilde{\lambda}(h)}{\partial \alpha} = \frac{h(\theta - 1)(1 - \theta + \nu)}{[1 + h(\theta - 1)](1 + \alpha + h\nu)^2} < 0 \quad (31)$$

$$\frac{\partial \tilde{\lambda}(h)}{\partial \theta} = \frac{h^2(\theta - 1) - h[2(1 - \theta) + \nu]}{[1 + h(\theta - 1)]^2(1 + \alpha + h\nu)} < 0 \quad (32)$$

$$\frac{\partial \tilde{\lambda}(h)}{\partial h} = -\frac{(\theta - 1)(1 - \theta + \nu)[1 + \alpha + h^2\nu(1 - \theta)]}{[1 + h(\theta - 1)]^2(1 + \alpha + h\nu)^2} > 0. \quad (33)$$

Note that the sign of these derivations remains unaltered in quarterly magnitudes, i.e. for $h = 1$ and $dh = 0$. According to the above expressions, the change in the slope is now decomposed into three terms, which measure the impact of the change in α , θ and h . Due to the consideration of h in $\tilde{\lambda}(h)$, equation (33) indicates a *frequency-dependent ‘inherited’ persistence* (FIP) effect.

⁸Note that for the Calvo parameter in quarterly magnitudes $0 \leq \theta \leq 1$ holds. Hence, this results in $(\theta - 1) \leq 0$ and $[1 + h(\theta - 1)] > 0$ for all values of h over its admissible range.

With respect to the expression (31), an increase in the fraction of firms, which set their actual price level equal to the previous one, strengthens the importance of backward-looking behaviour in the price adjustment process. This means that changes in the real marginal product (approximated by the output gap) become less considered the higher the non-frequency dependent *intrinsic* persistence will be. In other words, the more firms are indexing their prices to past realisations of the same, the more the impact of economic activity on the price-setting scheme is dampened.

Equivalently, according to the expression (32), the higher the probability of *not* setting the prices the lower the impact of the change in the output gap induced by the central bank. From an economic point of view, a high degree of price stickiness makes it difficult for the policy maker to influence inflation rate dynamics. The reason is that the price-setting scheme becomes increasingly degenerated from the movement in the real marginal cost the higher the likelihood for sticky prices will be. As a result the volatility in inflation increases with a higher degree of price stickiness.

An equivalent observation holds according to the expression (33): the higher the frequency of decision making, i.e. the *lower* the period length h , the *flatter* the slope of the NKPC will be. This result can be explained by the direct FIP effect on inflation relative to α and θ . Economically speaking the central bank must change its instrument variable more often to incorporate the higher frequency of reallocations and transactions. Once the monetary authority decided to influence economic activity they become bounded. This means that the central bank has to consider the circumstance that this intervention affects different fractions (about the same size) of agents over a corresponding amount of points in time (which depends directly on the period length). Hence, only a small fraction of firms is able to reset their prices on a single market day. As a result, ceteris paribus, the response of the inflation rate to a change in the output gap is lower on a higher frequency.

Given the expressions (31) to (33) it can be seen that the policy maker faces a (frequency-dependent) trade-off along several dimensions. For example, a decrease in the price stickiness can be offset by a decrease in the period length. This means that even under a high degree of flexible prices, the stabilization of the inflation rate might become hindered if the transactions are spread over more market days relative to the quarterly case. This effect is weakened as the degree of price indexation decreases. The reason is that the change in the real marginal costs (output gap) is more pronounced the more forward-looking the NKPC will be. In this case, the decrease in h can be offset or even over-compensated by the compound effect induced by the decrease in α and θ . More precisely, in this specific case the central bank faces a situation where its policy strategy is more effective the more the dynamics in inflation depend on economic activity – even on a higher frequency. The question arises if even in the polar case of a purely forward-looking NKM with almost perfect price flexibility, monetary policy can be effective on a nearly continuous time scale, i.e. as $\alpha = 0$, $\theta \approx 0$, $h \approx 0$ hold.⁹ We are going to investigate numerically related questions in the next section. Independently of the FIP effect, the volatility in the economic variables is always increasing as h decreases due to the FCP effect, i.e. the increase in the persistence of the shock process.

For a deeper understanding of both kinds of frequency-dependent persistence effects, we discuss this issue in greater detail. From an economic point of view both effects result from the change in economic activity depending on the period length, i.e. it reflects the higher frequencies in transactions and adjustments. In particular, households and firms are re-optimizing their plans automatically after a shock occurs in order to meet the basic assumption of general equilibria on the goods and labor market in every period.

⁹Note that such an scenario is superfluous if $\theta = h = 0$ holds due to model dichotomy. For $\theta = 0$ this means that monetary policy is of no use as price are *fully* flexible since the nominal variables adjust one to one with the price level as a shock occurs. For the analysis of h-economy model in continuous time ($h = 0$) see Franke and Sacht (2014) as well as Posch et al. (2011).

In a baseline NKM all reallocation processes are made by all agents within a quarter on a specific market day. However, as already stated, in an h-economy framework these reallocations are spread over the entire quarter, i.e. a h fraction (about the same size) of agents are re-optimizing on a specific market day within a month ($h = 1/3$), week ($h = 1/12$) or day ($h = 1/90$) respectively.¹⁰

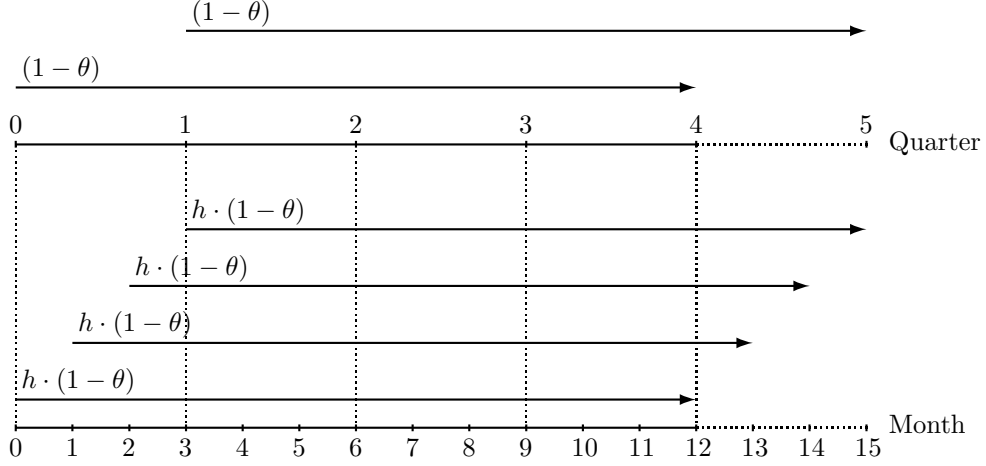


Figure 1: Frequency of price adjustments in the quarterly NKM versus an h-economy ($h = 1/3$).

Note: This graph shows an example for the relationship between the amount of market days and price adjustments. The arrows at the *top* denote the frequency of price adjustments in a *quarterly* economy and the arrows at the *bottom* denote the (selected) frequency of price adjustments in a *monthly* economy.

Figure 1 describes this issue in detail by considering the following example. Let us assume that the representative firm is able to adjust its nominal price level only every three quarters. Hence, if we consider only a quarterly model with $\theta = 2/3$ and $h = 1$, the firms are able to adjust their prices in January, April, July or October respectively (so only four market days exist at all), with probability $h(1 - \theta) = (1 - 2/3) = 1/3$. In a monthly economy the probability to adjust the price changes to $h(1 - \theta) = 1/3(1 - 2/3) = 1/9$, which means that the probability is lower in every month relative to a quarter as long as the shock process does not vanish. In the case of a weakly economy ($h = 1/12$) the probability declines further and so on.

This example shows how the FCP and FIP effects have an impact on the transition from a low to a high-frequency economy. For example, under consideration of the price stickiness some but not all firms are able to react to additional news which arrive during the period (e.g. at the beginning of the next month). Hence the corresponding proportion of firms which is able to adjust on each single market day (month) must obviously be smaller than in the baseline case (quarter). From that it follows, that the duration of price adjustments increases with a decrease in the period-length h . The same argument also holds for transactions on the goods market. The reason is that the volatility in the economy increases due to the optimization behavior of firms and households (expressed by the change in their optimization plans). Hence the corresponding time spans lead to a higher shock persistence, i.e. the change in the variables becomes smoother over time. In addition, later on, we might observe stronger impact effects at time $t = 0$ as the period length decreases. A general intuition for that is if the future is one month or one day ahead it has a greater impact on today's choices than if it is one quarter away (cf.

¹⁰It is assumed that the reallocation process takes place in a *point* of time. In this respect, it is irrelevant if we refer in particular to a begin- or end-of-the-period concept.

Anagnostopoulos and Giannitsarou (2010a, p. 12)).

It can be stated that the consideration of the period length sheds a different light on the discussion of monetary policy in (baseline) NKMs. In fact, a decrease in h might dampen the influence of the central bank to stabilize the economy. Our discussion in this section also fill the gap in the work by Franke and Sacht (2014) which lacks on an economic rationale for explaining the results reported in their paper. In order to examine the quantitative and qualitative observations with respect to a variation in the period length and the FCP as well as FIP effects, we are going to analyze IRFs in the next section. Furthermore, the FCP and FIP effects also play an important role in explaining the observations made with respect to *optimal* monetary policy in Sacht (2014).

4 Numerical Analysis of the Baseline NKM

In the following we are going to analyse the impact of all three shocks on the dynamics of the model for various kinds of h-economies. Therefore, we compare the corresponding IRFs with those of the baseline NKM for $h = 1$. Within this investigation we allow for two different specifications in which the underlying structure of the (high-frequency) model consists either on backward- and forward-looking elements ($0 \leq \chi \leq 1, 0 \leq \alpha \leq 1$) or forward-looking elements only in which we omit habit formation in consumption and price indexation ($\chi = \alpha = 0$). In the first case we refer to the *hybrid* and in the latter case to the *purely forward-looking* NKM. While it is well-known that the hybrid NKM accounts for persistence in the economic indicators, we show that inertia is indeed a significant characteristic of the IRFs in a high-frequency environment of the purely forward-looking NKM – even in the case of a non-autocorrelated shock ($\rho^z = 0$) due to the FCP effect. The state-space representation of the dynamic system (22) to (25) can be expressed by

$$0 = A X_t^{TR} + B X_{t+h}^{TR} + C X_{t-h}^{TR} + D V_t \quad (34)$$

$$V_t = N V_{t-h} + \Xi_t \quad (35)$$

with $X_t^{TR} = (y_t^{TR}, \pi_t^{TR})'$, $V_t = (v_t^i, v_t^y, v_t^\pi)'$ and $\Xi_t = (\varepsilon_t^i, \varepsilon_t^y, \varepsilon_t^\pi)'$. The system matrices A, B, C, D and N comprise all corresponding frequency-dependent (deep) parameters of the model.¹¹ The solution to (34) together with (35) is given by

$$X_t^{TR} = \Omega X_{t-h}^{TR} + \Phi V_t \quad (36)$$

where $\Omega \in \mathbb{R}^{3 \times 3}$ and $\Phi \in \mathbb{R}^{2 \times 3}$ are the solution matrices. In case of a purely forward-looking model (where $C = \mathbf{0}$ holds, which results in $\Omega = \mathbf{0}$) we obtain

$$X_t^{TR} = \Gamma V_t. \quad (37)$$

Hence, the dynamics in the output gap y_t^{TR} and the inflation rate π_t^{TR} for $0 < h < 1$ are described by the reduced-form solutions (36) and (37) for the hybrid and purely forward-looking case, respectively. A detailed description of the numerical derivation of the solution matrices Ω , Φ and Γ can be found in the Appendix. In particular, the solution matrices are computed numerically by applying the *method of undetermined coefficients* (also known as the “guess-and-verify” method; McCallum (1983)) and the *brute force iteration procedure* (Binder and Pesaran (1995, p. 155, fn. 26)). As an advantage over the analytical solution based on a the Schur- or Jordan decomposition method (for the hybrid NKM), we claim that the numerical solution exhibits a better manageability since the

¹¹The superscript *TR* indicates that here we study the dynamics under consideration of an ad-hoc nominal interest rule of Taylor type. We make this reference in order to distinguish this non-optimal monetary policy regime from the optimal ones discussed in the follow-up paper by Sacht (2014).

corresponding computer codes are easy to implement.¹² However, it is confirmed that the numerical and analytical solution procedures (presented in Sacht and Wohltmann (2014)) lead to the same results for Ω , Φ and Γ .¹³ All of the upcoming figures presented here also show the dynamics in the nominal interest rate, the real interest rate and the price level. Therefore, we plug the reduced-form solutions for y_t^{TR} and π_t^{TR} in the Taylor rule (24). The real interest rate is then given by the difference $i_t^{TR} - \pi_{t-h}^{TR}$ and the price level is simply computed by $p_t^{TR} = \sum_{k=0}^t \pi_k^{TR}$.

ν	Households' time preference rate	0.010
σ	Inverse intertemporal elasticity of substitution in consumption	1.000
η	Intertemporal elasticity of substitution of labour	1.000
θ	Calvo degree of price stickiness	0.667
α	Degree of Price Indexation	0.500
χ	Habit-formation parameter in consumption	0.500
\bar{i}	Natural Interest Rate	0.000
ϕ_π	Weight on inflation in the Taylor rule	1.500
ϕ_y	Weight on the output gap in the Taylor rule	0.125
ρ^z	Persistence in the shock process	0.500
ε^z	Impulse associated with the shock process	1.000

Table 1: Numerical parameter scenario.

Note: In the purely forward-looking NKM the parameters χ and α are set to 0. In case of a non-autocorrelated shock ρ^z is set to 0. The parameters of the shock process ρ^z and ε^z are identical across all shocks $z = \{y, \pi, i\}$.

The numerical setting in our simulations of the model relies on parameter values adopted from Franke and Sacht (2014). They are collected in Table 1, where the frequency-dependent coefficients are based on the quarterly time unit. A value of household's discount rate ν of about 1%, log utility ($\sigma = 1$), and a unitary Frisch elasticity of labour supply ($\eta = 1$) are standard. The policy coefficients ϕ_π and ϕ_y are the classical values which can be found in Taylor (1993). A standard price rigidity of $\theta = 0.667$ implies an average price duration of three quarters. These parameters are borrowed from Galí (2008, p. 52). The values for α and χ are in line with those in literature and, in particular, χ is close to the one taken from Smets and Wouters (2003, p. 1143). We simply assume that $\bar{i} = 0$ holds. Finally, the initial shock ε^z is calibrated on a moderate value of 1. Note that all different shocks (indicated by the superscript z) exhibit the same size of persistence and initial disturbance. Furthermore, it goes without saying that in the purely forward-looking NKM the parameters χ and α are set to zero. In the following we put emphasis on the analysis of one-off (impulse) shocks only, i.e. $\rho^z = 0$ holds. However, we briefly compare the results to the ones obtained in case of an autocorrelated shock $0 < \rho^z < 1$. Hereby, the value of $\rho^z = 0.5$ is associated with moderately persistent shocks. For a clear

¹²The MATLAB codes, which are used to produce the numerical results in this paper, can be downloaded from the authors' webpage at <http://www.makro-vwl.uni-kiel.de/de/team/dipl.-vw.-stephen-sacht>.

¹³As a drawback of the numerical solution approach presented here, this procedure might demand a specific amount of main memory and time for execution. While this problem does not appear when computing all IRFs presented here, for the evaluation of the (so-called) welfare functions in Sacht (2014) the associated analytical solutions being more appropriate to use in that case. For a detail discussion we refer directly to Sacht (2014).

arrangement, all corresponding Figures can be found in the Appendix. The analysis of all kind of shocks considered here have been heavily discussed for a quarterly economy ($h = 1$). We just name Galí (2008, his Chapter 3) and Walsh (2010, his Chapter 8) as being representative studies.

4.1 Nominal Interest Rate Shock

We start with analysing the impact of a nominal interest rate shock under variation of the period length. The corresponding IRFs for the purely forward-looking model are shown in Figure 2.¹⁴ On quarterly magnitudes ($h = 1$) an increase in the nominal interest rate by the central bank leads to an increase in the real interest rate due to the Taylor principle ($\phi_\pi > 1$). As a result the output gap and the inflation rate decline on impact, while a permanent decrease in the price level can be observed. The transmission channel can be explained as follows. In general, the household decides either to purchase consumption goods or bonds. The latter being held for one period and therefore can be indentured as a(n indirect) saving option. Since the household receives an interest payment when selling the bonds in the next period, an increase in the corresponding real interest rate increases the attractiveness for holding bonds. Hence, the demand for goods must decline at the same proportion as the amount of bonds increases. This effect is well-known as the intertemporal consumption smoothing effect, which can be explained by the Euler consumption equation (see equation (43) given in the Appendix). Under consideration of the Calvo (1983) price setting scheme, the firms which are able to set the price, decrease the latter as demand drops. The remaining firms face no price adjustment probability and, therefore, reduce the supply of goods. Hence, the goods market becomes cleared again after the shock occurs.

As we reduce the period length to a monthly ($h = 1/3$) and even a weekly ($h = 1/12$) magnitude, several observations are worth mentioning. First, although the shock is a non-autocorrelated one, all economic indicators exhibit a moderate degree of persistence. This is caused by the FCP effect, which leads to a smooth monotonic movement in the variables compared to a rapid jump from the initial period to the next in the case $h = 1$. Note that inflation inertia amplifies the pressure on the price level: the shorter the period length, the stronger the (negative) effect on the price level will be. Second, with respect to the impact effects, we observe a slightly stronger reaction in the real interest rate and the output gap, while the decrease in the inflation rate is less pronounced in an h-economy framework. However, the quantitative magnitude is quite small, i.e. the values of the inflation rate on impact are hard to distinguish in transition from a low to a high frequency. As we interpret a nominal interest rate shock as being a possible monetary policy strategy of the central bank, a nearly non-existing impact on the inflation rate seems to be appropriate for the monetary authority. It is meant by that the inflation rate becomes less destabilized on a higher frequency *on impact*. This observation is grounded on the FIP effect: the change in the output gap is transmitted less into inflation dynamics since the slope of the NKPC becomes flatter as h decreases. Nevertheless, the permanent drop in the price level is much stronger on a higher frequency - this can be seen as a severe problem of the central bank, i.e. as the prevention of permanent strong changes in the price level being one of its policy objectives.

¹⁴We have to indicate, that the time scale is adjusted for all values of h in order to allow for a graphical comparison of the IRFs for different frequencies. The interested reader might study the corresponding computer codes, used to reproduce the following Figures, for more information. Furthermore, it must be stated that possible intersections of different IRFs are hardly to be considered for an analysis. The reason is that we have the more observations, the lower h will be. In the polar case of $h = 1$ only quarterly values can be used for interpretation. Therefore, we do not draw a line in order to connect the quarterly values but use a square as a marker instead.

In Figure 3 the IRFs for a non-autocorrelated shock within the hybrid NKM are displayed. It can be seen that under the standard calibration of the parameters applied here, the dynamics of the output gap and the inflation rate in the h-economy models become hump-shaped. In particular, troughs can be observed. In the quarterly model we observe a monotonic movement of these variables back to the steady state. These results mimic those reported in Franke and Sacht (2014). The opposite holds for the nominal and real interest rate, where we observe an undershooting in both variables in the case $h = 1$ — and just slightly in a monthly but significantly not in a weekly economy. The real interest rate responses stronger on impact to a decrease in h , which leads to stronger decline in the output gap in period $t = 0$. However, the impact effects on the output gap are quantitatively less pronounced for all values of h relative to the forward-looking case. Again, the impact effects on the inflation rate are hard to distinguish across different frequencies. This can be once more explained by the FIP effect, where less of the change in the output gap is transmitted into the inflation rate on a higher frequency. Interestingly, the values of the nominal and real interest rate in the monthly economy lie below the ones given in the weekly economy, roughly before time $t = 1$, where the opposite holds after the first quarter. An exception are the dynamics in the output gap and the inflation rate, where the IRFs for the case $h = 1/12$ are significantly below the one for $h = 1/3$. This could probably be explained by the moderate degree of intrinsic persistence. As we had shown in the previous section, an increase in α reduces the effectiveness of monetary policy intervention, where this effect is amplified by the FIP one, i.e. as the period length decreases. Most likely, at a specific value of h , the FIP effect dominates the intrinsic persistence one and causes the, let's say, asymmetric movements as we go from a monthly to a weekly economy. Finally, the quantitative effects on the development in the price level are stronger for all values of h compared to the forward-looking case. Hence, the failure of the central bank for price level stabilization is more distinctive within a hybrid specification of the NKM.

Figures 4 and 5 represent the IRFs for an *autocorrelated* shock in the purely forward-looking and hybrid NKM, respectively. As we can see, the quantitative effects are not surprisingly more pronounced if a moderate persistence in the shock is observed. This holds especially for the price level (therefore consider the IRFs for $h = 1/12$) in both cases. In the forward-looking case, a convergence/overlapping in the IRFs can be identified as h decreases. This holds also in the hybrid NKM, where the now *amplified* FCP effects (note that $\rho(h)^i$ increases due to an increase in ρ^i) might offset the dominance of the FIP over the intrinsic persistence effect as described above for the non-autocorrelated case. Furthermore, we observe over- and undershooting behaviour in the IRFs across different values of h for the nominal and real interest rate. The result that no distinctive differences in the impact effects of the inflation rate occur for different values of h , prevails in the case $\rho^i > 0$. Both observations hold for both specifications of the model.

Hence, the ineffectiveness of monetary policy on inflation stabilization on impact is independent of the FCP effect. However, the latter amplifies the quantitative effect on the price level dramatically as discussed above. To sum up, in comparison to the non-autocorrelated shock we observe quantitatively stronger destabilization effects in the output gap and the inflation rate, while the results regarding the qualitative effects (due to the existence of troughs) remain. With respect to the latter observation it can be said that the impact of the intrinsic persistence together with the FCP effect strengthens the humped-shaped movements in the h-economies.

4.2 Technology Shock

Figure 6 depicts the adjustments in the model variables in case of a non-autocorrelated technology shock in the forward-looking NKM. As usual, first, we describe the transmis-

sion channel based on the case $h = 1$. An increase in ε^y leads to an increase in the efficiency of production. This affects potential output positively, while actual output remains unaffected, where the latter reacts one-to-one to changes in private consumption due to the assumption of a general equilibrium framework. As a result the output gap declines on impact. On the demand side, households face a decrease in the gross natural real interest rate. This means that the interest payment on bonds drops in the long run, i.e. the desired amount of bonds held in the steady state declines (to see this consider, again, the Euler consumption equation (43) given in the Appendix). Hence, the long-run demand for goods increases and coincides with the higher level of steady state (potential) output. Since the output gap drops, firms which are not able to set the price, lower the supply of good, while the remaining firms lower its price. The central bank responds to the decline in the output gap by lowering the nominal and, hence (due to the Taylor principle), the real interest rate in order to boost actual output based on the consumption smoothing effect.

On a higher frequency we observe persistence in all variables due to the FCP effect. Furthermore, less pronounced effects in all variables can be observed in this case. Especially with respect to the real interest rate, this variable rather increases than decreases (for $h = 1/12$) on impact. The weaker impact effect on the output gap relative to the one observed in the case of the nominal interest rate shock, can be explained by the (inverse) elasticities of consumption σ and labour η . In order to see this, consider the relation in front of the expression for the shock process v_t^y in the dynamic IS curve (see equation (16)) given by

$$h[\rho(h)^y - 1] \frac{1 + \eta}{\sigma + \eta} \quad (38)$$

if no habit formation in the shock is assumed ($\chi = 0$). Under consideration of our parameter scenario, where $\sigma = \eta = 1$ ($\rho^y = 0$) holds, the values of previous equation being $|1|$, $|2/9|$ and $|11/144|$ in a quarterly, monthly and weekly economy, respectively. Hence, ceteris paribus, the shorter the period length, the weaker the impact of the shock on the output gap will be. The quantitative impact effects, therefore, depend on the corresponding elasticities and the *reinforced* FCP effect, which is displayed by the existence of $\rho(h)^y$ in the expression (38) and the shock process v_t^y itself. This reinforced FCP effect dominates the FIP one: the associated impact effect on the inflation rate is also weak on a higher relative to a lower frequency like for the output gap. In other words, although the effectiveness of monetary policy is weakened as h increases, the corresponding relative strong boost in technology feeds into inflation dynamics. The dominance of the (reinforced) FCP over the FIP effect also leads to a less pronounced drop in the steady state price level on a higher relative to a lower frequency. Due to the persistence in the inflation rate based on the FCP effect, the central bank is able to maintain a higher level in the long-run relative to the case $h = 1$.

As we turn to an analysis of the same shock within a hybrid specification of the NKM (see Figure 7), overall weaker impact effects relative to the purely forward-looking case can be observed. This can be explained by the distinctive troughs which display the interaction between intrinsic persistence and the (reinforced) FCP effect: due to backward-looking behaviour this induces rapid changes in economic activity. Hence, the future (non-)linear movements of the inflation rate are transmitted into the real interest rate and, therefore, dampen the decrease in the output gap. Note that the real interest rate in monthly magnitudes remains roughly unchanged on impact. This effect is more pronounced on a higher frequency because of $\rho(h)^y$ being considered in the expression for the composite parameter given by (38) and the shock process v_t^y . Due to the higher persistence in the inflation rate in the case $h = 1$ relative to both h-economies, the price level converges to a higher steady state value in quarterly magnitudes. This is just the opposite result as

we observed in the purely forward-looking model. This is most likely stemming from the dominance of the FIP over the (reinforced) FCP effect since the FIP one is now amplified by the increase in α . It follows that the central bank faces a greater destabilization of the price level on higher frequency in case of backward-looking behaviour of agents being considered.

Figures 8 and 9 present the IRFs for an *autocorrelated* shock in the purely forward-looking and hybrid NKM, respectively. While obviously the persistence in all variables increases, the qualitative results remain almost the same in comparison to the case $\rho^y = 0$; note that the real interest rate in the hybrid case is now also positive on impact. Two exceptions can be identified with respect to the steady state values of the price level. In the purely forward-looking case, all IRFs converge to the same price level in the long-run. Within the hybrid specification, the adjustments in the price level are of almost the same magnitude in early periods, while there is a divergence in the IRFs for later periods. It seems to be that the FCP and FIP effect are offsetting each other in the former case, while in the latter the FIP effect probably dominates the FCP one. However, a clear interpretation can be hardly obtained: while additional intrinsic persistence strengthens the FIP effect, the FCP effect is also boosted by the fact that $\rho^y > 0$ holds. As a result, the movements of the price level and, hence, a successful stabilisation policy of the monetary policy on a low relative to a high frequency depends on the interaction of α and $\rho(h)^y$ in this case.

4.3 Cost-Push Shock

Figure 10 depicts the adjustments in the model variables in case of a non-autocorrelated cost-push shock in the forward-looking NKM. For all values of h , a cost-push shock leads to an increase in the price level induced by firms, which are able to set the price. As a result the inflation rate increases on impact. Since the central bank can influence the inflation rate only *indirectly* via the output gap it faces a *trade-off*. In order to dampen the boost in the inflation rate, monetary authority must rise the nominal and — due to the Taylor principle — real interest rate. However, this leads to a decrease in the output gap since now it is more attractive to consume more goods instead of purchasing bonds. In other words, the households smooth their consumption due to a decrease in the interest payments on bonds. The trade-off is now described by the fact that the output gap and the inflation rate move into opposite directions. Note that in case of a cost-push shock in order to target a lower level of the inflation rate after the shock occurs, the central bank must allow for a negative output gap (on impact).

In transition to an h-economy environment, we observe stronger impact effects in all variables. The persistence also increases due to the FCP effect. The quantitative change in the impact effects implies an amplification in the policy trade-off. Since monetary policy becomes less effective due to the FIP effect, the central bank must allow for a more aggressive response of its policy instrument to the shock. More precisely, the real interest rate increases on higher magnitude the shorter the period length will be. It follows that a much stronger drop in the output gap is observed, which is needed to affect the inflation rate. In combination with the FCP, the FIP effect leads to a more destabilised scenario in an h-economy relative to a quarterly one. This can also be seen by considering the massive increase in the price level for $h = 1/12$.

As we turn to the hybrid specification of the NKM, which is depicted in Figure 11, we observe again that troughs in the dynamics of the variables occur. Besides these qualitative changes, the quantitative difference compared to the case $\rho(h)^y$ are distinctive. The reason is that due to the (moderate) degree of intrinsic persistence more rapid changes in the transactions on a higher frequency take place. Since the existence of backward-looking behaviour in price setting amplifies the FIP effect, monetary policy becomes now

less effective in stabilizing the inflation rate as h decreases. Under consideration of an autocorrelated shock, the reported qualitative and quantitative effects are strengthened – again, distinctive troughs emerge in the hybrid case. This can be seen by a brief inspection of the corresponding IRFs for an autocorrelated shock in the purely forward-looking and hybrid NKM, respectively, which are depicted in the Figures 12 and 13.

5 Conclusion

In this paper we study the differences in the purely forward-looking and hybrid specification of the baseline NKM based on various shocks and under different frequencies of decision making. Therefore, we show how the structural representation of the baseline NKM (which is given in quarterly magnitudes by convention) must be adjusted by the parameter which denotes the period length given by h . In order to explain our numerical results we identify two frequency-dependent persistence effects. The first one consists on the additional degree of persistence in the shock process, which comes into play as the dynamics in the shock are spread over all subperiods as the period length declines. We call this one the frequency-dependent contraction persistence (FCP) effect. It displays the impact of the increase in the amount of market days on the dynamics of the model. As a result the time of convergence to the steady state increases on a higher frequency since the frequency of transactions and adjustments increases. The second one has an impact on the pass-through of changes in the output gap into inflation dynamics via the slope of the NKPC. As a result the central bank must intervene more often to the smaller fraction of price setters on a higher frequency. We call this one the frequency-dependent inherited persistence (FIP) effect. This effect can be amplified or reduced by the non-frequency-dependent intrinsic persistence effect displayed by the degree of price stickiness as well as the effect stemming from the change in the price stickiness. As the period length decreases, the amount of transactions and reallocation processes are spread over all subperiods, which is equivalent to the FCP with respect to the price setting scheme. Hence, the driving force of the inflation rate, the output gap, explains less of the dynamics of the former on a higher frequency.

Within our numerical study, we observe an overall increase in the persistence in all variables – even in the case of a non-autocorrelated shock – due to the FCP effect. Across all shocks, the interaction of both persistence effects causes differences in the impact effects of the variables and more distinct hump-shaped adjustments in the hybrid NKM. This holds especially if an autocorrelated shock process is assumed. Furthermore, strong changes in the long-run steady state of the price level occur. The latter case indicates a severe problem as price stability is the main target of the central bank. This problem is much more obvious in case of a cost-push shock, where the monetary authority faces a trade-off between inflation rate and output gap stabilisation. This trade-off is strengthened on a higher frequency since the strong output gap reaction (on impact) does not feed into the NKPC due to the FIP effect, while the inflation rate increases dramatically. We are going to investigate this scenario in much more detail as we consider optimal monetary policy strategies in Sacht (2014).

The results of our numerical exercises resuscitate the question, “which interval best represents agents’ decision-making process?” (Aadland, 2001, p. 291). One answer is more practically oriented: use calibration or estimation methods to find out if variations of the period length can improve the matching of certain empirical moments or the value of an objective function in general. However, research in this direction seems to be rare.¹⁵

¹⁵Christiano (1985) and Aadland (2001) may be recalled, again, as the only two references regarding this question that we know. More modestly, in Ahrens and Sacht (2014), the authors estimate a high-frequency NKPC on daily inflation data for Argentina. In particular, the results for the Calvo price stickiness

Future theoretical and empirical investigations might also consider an asymmetric differentiation of the decision intervals of households and firms. Our numerical results show that the more information on prices and output is available, the increase in the amount of transactions and reallocation processes dampens the possibility of the central bank to stabilize the economic indicators. This can be seen as being equivalent to the results by Winkler and Wohltmann (2012), who show that an anticipated shock in an optimal monetary policy scenario causes a higher welfare loss (relative to a non-anticipated one) due of the adjustments of the economic indicators before the shock actually occurs. Due to this as the central bank faces rapid changes in the output gap and the inflation rate, the question arises, if this result prevails as we consider a heterogeneous approach regarding the period length. It means that e.g. the central bank might react on lower frequency compared to the other agents in the economy. We leave such kind of analysis to further research.

According to Franke and Sacht (2014), a theoretical answer returns to Foley’s principle and requires robustness of the period on which one decides to settle down.¹⁶ Accordingly, a period might be called robust if it is an upper-bound on the length of the decision intervals with, in our case, essentially similar IRFs. From this point of view, the main conclusion is that, in the present modelling framework, the conventional quarter cannot be claimed to be a robust period length. The more general message is that DSGE modelling might be more sensitive to the possible “pitfalls of timing misspecification” (Christiano (1985, p. 397)). In particular, it would be an important question how seriously the structure of optimal monetary policy rules might be affected by changes in the length of the period. As we stated above, we seek to answer this question in Sacht (2014).

6 Appendix

6.1 Microfoundations of the Baseline NKM in the Case $0 < h \leq 1$

The numerical results presented in this paper are derived under consideration of the analytic solutions by Woodford (2003, his Chapter 6) and Galí (2008, his Chapters 3 and 4), along with the specifications given in Smets and Wouters (2003) and Christiano et. al (2005), who all provide explicit microfoundations of the (hybrid) version of the standard NKM in *quarterly* magnitudes. Here, those results are adjusted towards the transition of the model’s structural equations into their h -economy counterparts. The definition of all variables and parameters are (if not done here) given in the main text.

Households:

We consider a standard utility function of the representative household subject to the underlying frequency given by¹⁷

$$U_t = E_t \sum_{k=0}^{\infty} \beta(h)^{hk} \left[\frac{1}{1-\sigma} (C_{t+hk} - \chi C_{t+hk-h})^{1-\sigma} - \frac{1}{1+\eta} N_{t+hk}^{1+\eta} \right] \quad (39)$$

parameter are quite in line with microeconomic evidence.

¹⁶Foley emphasises that “[n]o substantive prediction or explanation in a well-defined macroeconomic period model should depend on the real time length of the period” (1975, p. 310). While Franke and Sacht (2014) disprove this statement to be true based on an elaborate analytical and numerical analysis of a nominal interest rate shock in the baseline NKM, we show in this paper that this obviously also holds in the cases of a technology and a cost-push shock.

¹⁷Note that non-log-linearized variables like e.g. the aggregate price level P_t are denoted in capital letters. The logarithmic deviation of a variable from its steady state value, e.g. $\hat{c}_t = \log \left(\frac{c_t}{\bar{c}} \right)$, is denoted by lower case letters. Later on we simply consider $\hat{c}_t = c_t$ only.

where C_{t+hk} (C_{t+hk-h}) denotes the current (previous) level of (aggregate) consumption, where the difference $C_{t+hk} - \chi C_{t+hk-h}$ displays habit formation in consumption of the household. N_{t+hk} denotes employment in terms of hours worked. The Euler equation is the result of the first order condition with respect to the amount of bonds B_t subjected to the budget constraint

$$C_{t+hk} = -\frac{B_{t+hk}}{P_{t+hk}} + \frac{W_{t+hk}}{P_{t+hk}}N_{t+hk} + (1 + hi_{t+hk-h})\frac{B_{t+hk-h}}{P_{t+hk}} + \Pi_{t+hk}^r \quad (40)$$

where W_{t+hk} and Π_{t+hk}^r stand for the nominal wage and firms real profits, respectively. The first order condition is then given by

$$\begin{aligned} \frac{\partial U_t}{\partial C_{t+hk}} &= \frac{1}{1-\sigma}(C_t - \chi C_{t-h})^{-\sigma} \left(-\frac{1}{P_t} \right) + \beta(h)^h \frac{1}{1-\sigma} (E_t C_{t+h} - \chi C_t)^{-\sigma} \frac{1 + hi_t}{E_t P_{t+h}} = 0 \\ \Rightarrow \frac{1}{1-\sigma}(C_t - \chi C_{t-h})^{-\sigma} \frac{1}{P_t} &= \beta(h)^h \frac{1}{1-\sigma} (E_t C_{t+h} - \chi C_t)^{-\sigma} \frac{1 + hi_t}{E_t P_{t+h}} \\ \Rightarrow (C_t - \chi C_{t-h})^{-\sigma} &= \beta(h)^h \frac{1}{1-\sigma} (E_t C_{t+h} - \chi C_t)^{-\sigma} (1 + hi_t) E_t \left(\frac{P_t}{P_{t+h}} \right) \end{aligned} \quad (41)$$

for $k = 0$ and $k = 1$. Given the Fisher (1930) equation

$$(1 + hi_t) E_t \left(\frac{P_t}{P_{t+h}} \right) = 1 + hr_t = R_t, \quad (42)$$

equation (41) can be rewritten as

$$(C_t - \chi C_{t-h})^{-\sigma} = \beta(h)^h \frac{1}{1-\sigma} (E_t C_{t+h} - \chi C_t)^{-\sigma} R_t \quad (43)$$

where r_t is the real interest rate and R_t the gross real interest rate. Log-linearization of the previous equation leads to

$$-\sigma \log(C_t - \chi C_{t-h}) = h \log \beta(h) - \log \frac{1}{1-\sigma} + \sigma \log(E_t C_{t+h} - \chi C_t) + \log R_t \quad (44)$$

where $h \log \beta(h)$ and $\log \frac{1}{1-\sigma}$ have to be crossed out since h is constant. By applying the log-linearization technique (cf. Ascari and Ropele (2003), pp. 7) this leads to

$$-\sigma \frac{\bar{c}}{(1-\chi)\bar{c}} \hat{c}_t + \sigma \frac{\chi \bar{c}}{(1-\chi)\bar{c}} \hat{c}_{t-h} = -\sigma \frac{\bar{c}}{(1-\chi)\bar{c}} E_t \hat{c}_{t+h} + \sigma \frac{\chi \bar{c}}{(1-\chi)\bar{c}} \hat{c}_t + \frac{\bar{R}}{\bar{R}} \hat{r}_t \quad (45)$$

where \bar{c} (\bar{R}) denotes the steady state value in consumption (the gross real interest rate). Hence, the actual level of private consumption in relation to its steady state value (given in percent) is then expressed through

$$\begin{aligned} -\sigma \hat{c}_t + \sigma \chi \hat{c}_{t-h} &= -\sigma E_t \hat{c}_{t+h} + \sigma \chi \hat{c}_t + (1 - \chi) \hat{r}_t \\ \Rightarrow (1 + \chi) \hat{c}_t &= \chi \hat{c}_{t-h} + E_t \hat{c}_{t+h} - \frac{1 - \chi}{\sigma} \hat{r}_t \\ \Rightarrow \hat{c}_t &= \frac{1}{1 + \chi} E_t \hat{c}_{t+h} + \frac{\chi}{1 + \chi} \hat{c}_{t-h} - \frac{1 - \chi}{\sigma(1 + \chi)} \hat{r}_t. \end{aligned} \quad (46)$$

The rate of change \hat{r}_t is expressed through $h(i_t - E_t \pi_{t+h} - \bar{i})$. To see this consider (cf. equation (42))

$$\begin{aligned} \hat{r}_t &= \log \frac{R_t}{\bar{R}} \\ \Rightarrow \hat{r}_t &= \log \left(\frac{(1 + hi_t) E_t \left(\frac{P_t}{P_{t+h}} \right)}{(1 + h\bar{i})} \right) \\ \Rightarrow \hat{r}_t &= \log(1 + hi_t) + \log E_t \left(\frac{P_t}{P_{t+h}} \right) - \log(1 + h\bar{i}) \\ \Rightarrow \hat{r}_t &\approx hi_t - (E_t \log P_{t+h} - \log P_t) - h\bar{i}. \end{aligned} \quad (47)$$

Under the assumption of rational expectations $E_t P_t = P_t$ holds. We set $\log E_t P_{t+h} - \log P_t = h E_t \pi_{t+h}$ (therefore note that $\pi_t = (p_t - p_{t-h})/h$ holds). In the long-run the nominal interest rate equals the (non-time-varying) natural rate of interest, i.e. $h\bar{i}$ holds. Plugging (47) into the log-linearized Euler equation (46) under consideration of the goods market clearing condition and adding the natural level of output (which equals the steady state level of consumption) on both side leads to

$$y_t = \frac{1}{1+\chi} E_t y_{t+h} + \frac{\chi}{1+\chi} y_{t-h} - \frac{h(1-\chi)}{\sigma(1+\chi)} (i_t - E_t \pi_{t+h} - \tilde{r}_t) \quad (48)$$

where the natural rate of the *gross real* interest rate is given by (cf. Galí (2008), p. 49)

$$\tilde{r}_t = \bar{i} + \sigma \left(\frac{1+\eta}{\sigma+\eta} \right) E_t (v_{t+h}^y - v_t^y) \quad (49)$$

where v_t^y denotes a shock to technology, which measures the impact of an innovation on the efficiency of the production process (cf. equation (18)). The corresponding law of motion follows an AR(1) process with

$$v_t^y = \rho(h)^y v_{t-h}^y + \varepsilon_t^y \leftrightarrow v_{t+h}^y = \rho(h)^y v_t^y + \varepsilon_{t+h}^y \quad (50)$$

and $\rho(h)^y = 1 - h(1 - \rho^y)$. As we consider an one-time (deterministic) impulse only, $\varepsilon_{t+h}^y = 0$ holds. Plugging (50) into (49) leads to

$$\tilde{r}_t = \bar{i} + \sigma \left(\frac{1+\eta}{\sigma+\eta} \right) (\rho(h)^y - 1) v_t^y. \quad (51)$$

The hybrid dynamic IS equation for the h -economy is finally given by

$$y_t = \frac{1}{1+\chi} E_t y_{t+h} + \frac{\chi}{1+\chi} y_{t-h} - \delta_1(h) (i_t - E_t \pi_{t+h} - \bar{i}) + \delta_2(h) v_t^y \quad (52)$$

with

$$\begin{aligned} \delta_1(h) &= h \frac{1-\chi}{\sigma(1+\chi)} \\ \delta_2(h) &= h \frac{(1-\chi)(1+\eta)(\rho(h)^y - 1)}{(1+\chi)(\sigma+\eta)}. \end{aligned}$$

Note that (52) equals its representation in the baseline (quarterly) economy if $h = 1$ holds:

$$y_t = \frac{1}{1+\chi} E_t y_{t+1} + \frac{\chi}{1+\chi} y_{t-1} - \delta_1 (i_t - E_t \pi_{t+1} - \bar{i}) + \delta_2 v_t^y \quad (53)$$

with

$$\begin{aligned} \delta_1 &= \frac{1-\chi}{\sigma(1+\chi)} \\ \delta_2 &= \frac{(1-\chi)(1+\eta)(\rho^y - 1)}{(1+\chi)(\sigma+\eta)}. \end{aligned}$$

Furthermore under the assumption of a Walrasian labor market

$$-\frac{\partial U_t}{\partial N_t} = \frac{W_t}{P_t} \quad (54)$$

holds. Under consideration of household's utility function (39) it follows (in log-linearized terms):

$$w_t - p_t = \sigma c_t + \eta n_t. \quad (55)$$

Firms:

In the following we will derive the purely forward-looking NKPC. We show that when applying the transformation procedure for the h-economy framework (in particular the steps (1) to (3) described in section 2), the adjustments of the frequency-dependent variables and parameters will lead directly to the final high-frequency representation of the NKPC. Hence, we will leave out the more time-consuming derivation of the hybrid NKPC (see e.g. Christiano et al. (2005)) and spell out the change in the frequency-dependent components for $0 < h \leq 1$ based on the solution for the purely forward-looking case only. According to Walsh (2010, Chapter 5) the following loss function of firms has to be minimized:¹⁸

$$V_t = \sum_{k=0}^{\infty} \beta(h)^{hk} E_t(z_{t+hk} - p_{t+hk}^*)^2. \quad (56)$$

It is the aim of a representative firm to minimize the expected price distortion (deviation of the future log-price level z_{t+hk} from the optimal log-price level p_{t+hk}^* , i.e. the price firms would set in period $t+hk$ if there were no price rigidity) given above by changing its own price z_{t+hk} . Under consideration of price stickiness, z_{t+hk} remains unchanged over time with a probability $\theta(h)$. In this case (56) is reformulated into

$$V_t = \sum_{k=0}^{\infty} [\theta(h)\beta(h)]^{hk} E_t(z_t - p_{t+hk}^*)^2. \quad (57)$$

Hence,

$$\begin{aligned} \frac{dV_t}{dz_t} &= 2 \sum_{k=0}^{\infty} [\theta(h)\beta(h)]^{hk} E_t(z_t - p_{t+hk}^*) = 0 \\ \Rightarrow \sum_{k=0}^{\infty} [\theta(h)\beta(h)]^{hk} E_t(z_t) &= \sum_{k=0}^{\infty} [\theta(h)\beta(h)]^{hk} E_t(p_{t+hk}^*). \end{aligned} \quad (58)$$

As we taking into account that $E_t(z_t) = z_t$ holds and that the corresponding term is given by the following expression of a geometric sum

$$\sum_{k=0}^{\infty} [\theta(h)\beta(h)]^{hk} = \frac{1}{1 - \theta(h)\beta(h)} \quad (59)$$

we get

$$z_t = [1 - \theta(h)\beta(h)] \sum_{k=0}^{\infty} [\theta(h)\beta(h)]^{hk} E_t(p_{t+hk}^*). \quad (60)$$

Equation (60) is the forward-solution of the dynamic equation for the optimal reset price z_t given by

$$z_t = [\theta(h)\beta(h)] E_t(z_{t+h}) + [1 - \theta(h)\beta(h)](\mu + mc_t) \quad (61)$$

where $p_t^* = \mu + mc_t$ denotes the optimal price set by the firm under monopolistic competition as the sum of the nominal marginal costs mc_t and the desired mark-up μ . The aggregate price level p_t is the weighted sum of the desired (lagged) price level under consideration of the probability of $h(1 - \theta)$ $[\theta(h)]$ that firms are [not] able to choose their price z_t :

$$p_t = \theta(h)p_{t-h} + h(1 - \theta)z_t. \quad (62)$$

¹⁸The time preference rate for a period of length h is $h\nu$. Hence the transformation of β leads to $\beta(h) = 1/(1 + h\nu)$. Furthermore recall that $\theta(h) = 1 - h(1 - \theta)$ holds.

Solving for z_t and $E_t(z_{t+h})$ respectively leads to

$$z_t = \frac{1}{h(1-\theta)}(p_t - \theta(h)p_{t-h}) \quad \text{and} \quad E_t(z_{t+h}) = \frac{1}{h(1-\theta)}(E_t p_{t+h} - \theta(h)p_t). \quad (63)$$

Substituting this into (61) gives

$$\frac{1}{h(1-\theta)}(p_t - \theta(h)p_{t-h}) = \frac{[\theta(h)\beta(h)]}{h(1-\theta)}(E_t p_{t+h} - \theta(h)p_t) + [1 - \theta(h)\beta(h)](\mu + mc_t). \quad (64)$$

By adding $(p_{t-h} - p_{t-h})$ on the left-hand side and $(p_t - p_t)$ on the right-hand side of the previous equation this leads to

$$\begin{aligned} \frac{1}{h(1-\theta)}(p_t - p_{t-h} - \theta(h)p_{t-h} + p_{t-h}) &= \frac{[\theta(h)\beta(h)]}{h(1-\theta)}(E_t p_{t+h} - p_t - \theta(h)p_t + p_t) \\ &+ [1 - \theta(h)\beta(h)](\mu + mc_t) \\ \Rightarrow \frac{1}{h(1-\theta)}(p_t - p_{t-h} + (1 - \theta(h))p_{t-h}) &= \frac{[\theta(h)\beta(h)]}{h(1-\theta)}(E_t p_{t+h} - p_t + (1 - \theta(h))p_t) \\ &+ [1 - \theta(h)\beta(h)](\mu + mc_t) \\ \Rightarrow \frac{1}{h(1-\theta)}(\Delta p_t + (1 - \theta(h))p_{t-h}) &= \frac{[\theta(h)\beta(h)]}{h(1-\theta)}(\Delta p_{t+h} + (1 - \theta(h))p_t) \\ &+ [1 - \theta(h)\beta(h)](\mu + mc_t) \\ \Rightarrow \frac{1}{h(1-\theta)}\Delta p_t + \frac{(1 - \theta(h))}{h(1-\theta)}p_{t-h} &= \frac{[\theta(h)\beta(h)]}{h(1-\theta)}\Delta p_{t+h} + \frac{[\theta(h)\beta(h)](1 - \theta(h))}{h(1-\theta)}p_t \\ &+ [1 - \theta(h)\beta(h)](\mu + mc_t) \\ \Rightarrow \frac{1}{(1 - \theta(h))}\Delta p_t + p_{t-h} &= \frac{[\theta(h)\beta(h)]}{(1 - \theta(h))}\Delta p_{t+h} + [\theta(h)\beta(h)]p_t \\ &+ \frac{h(1-\theta)[1 - \theta(h)\beta(h)]}{(1 - \theta(h))}(\mu + mc_t) \end{aligned} \quad (65)$$

where $\Delta p_t = p_t - p_{t-h}$ ($\Delta p_{t+h} = E_t p_{t+h} - p_t$) denotes the contemporaneous (one-period ahead) inflation rate in the h -economy. Subtracting p_t from both sides leads to

$$\begin{aligned} \frac{1}{(1 - \theta(h))}\Delta p_t + p_{t-h} - p_t &= \frac{[\theta(h)\beta(h)]}{(1 - \theta(h))}\Delta p_{t+h} + [\theta(h)\beta(h)]p_t - p_t \\ &+ \frac{h(1-\theta)[1 - \theta(h)\beta(h)]}{(1 - \theta(h))}(\mu + mc_t) \\ \Rightarrow \left(\frac{1}{(1 - \theta(h))} - 1 \right) \Delta p_t &= \frac{[\theta(h)\beta(h)]}{(1 - \theta(h))}\Delta p_{t+h} + ([\theta(h)\beta(h)] - 1)p_t \\ &+ \frac{h(1-\theta)[1 - \theta(h)\beta(h)]}{(1 - \theta(h))}(\mu + mc_t) \\ \Rightarrow \frac{\theta(h)}{(1 - \theta(h))}\Delta p_t &= \frac{[\theta(h)\beta(h)]}{(1 - \theta(h))}\Delta p_{t+h} - [1 - \theta(h)\beta(h)]p_t \\ &+ \frac{h(1-\theta)[1 - \theta(h)\beta(h)]}{(1 - \theta(h))}(\mu + mc_t). \end{aligned} \quad (66)$$

The expression of the inflation rate in the h-economy is then given by

$$\begin{aligned}
\Delta p_t &= \frac{[\theta(h)\beta(h)]}{\theta(h)}\Delta p_{t+h} - \frac{(1-\theta(h))[1-\theta(h)\beta(h)]}{\theta(h)}p_t \\
&\quad + \frac{h(1-\theta)[1-\theta(h)\beta(h)]}{\theta(h)}(\mu + mc_t) \\
\Rightarrow \Delta p_t &= \beta(h)\Delta p_{t+h} + \frac{h(1-\theta)[1-\theta(h)\beta(h)]}{\theta(h)}\left(\mu + mc_t - \frac{(1-\theta(h))}{h(1-\theta)}p_t\right) \\
\Rightarrow \Delta p_t &= \beta(h)\Delta p_{t+h} + \frac{[1-\theta(h)][1-\theta(h)\beta(h)]}{\theta(h)}(\mu + mc_t - p_t). \tag{67}
\end{aligned}$$

Note that $1 - \theta(h) = 1 - [1 - h(1 - \theta)] = h(1 - \theta)$ holds. To derive the NKPC in its h-economy representation it is necessary to introduce the output gap into the inflation equation above, what in fact does not change the structure of the NKPC concerning the period length because the corresponding elasticities σ and η are independent of h . To see this we claim that the representative firm uses labor N_t and technology A_t as input factors such that its production function (assuming constant return of scales) is simply given by

$$\hat{X}_t = A_t N_t \tag{68}$$

where \hat{X}_t denotes the actual level of output. Equation (68) in log-linearized terms equals to

$$y_t = a_t + n_t. \tag{69}$$

Real marginal costs can be expressed through

$$mc_t - p_t = mc_t^r = (w_t - p_t) - mpn_t \tag{70}$$

where mpn_t denotes the marginal product of labor as the result of deviating (68) with respect to N_t and linearizing it:

$$mpn_t = \frac{\partial Y_t}{\partial N_t} = a_t. \tag{71}$$

After applying some algebra, using (55), (69) and (71), real marginal costs are given by

$$mc_t^r = (\sigma + \eta)x_t - (1 + \eta)a_t \tag{72}$$

Using the following expression for real marginal cost in the steady state

$$\bar{mc}^r = -\mu = (\sigma + \eta)\bar{x}_t - (1 + \eta)a_t \tag{73}$$

leads to the equation for the (time-varying) natural level of output:¹⁹

$$\bar{x}_t = \frac{1 + \eta}{\sigma + \eta}a_t - \frac{\mu}{\sigma + \eta}. \tag{74}$$

Finally the deviation of (72) from (73) gives

$$\hat{mc}_t^r = mc_t^r - \bar{mc}^r = (\sigma + \eta)y_t. \tag{75}$$

where $y_t = x_t - \bar{x}_t$ denotes the output gap. Note that $mc_t^r = \hat{mc}_t^r + \bar{mc}^r = \hat{mc}_t^r - \mu$ holds. Hence (67) can be reformulated into

$$\Delta p_t = \beta(h)\Delta p_{t+h} + \frac{[1-\theta(h)][1-\theta(h)\beta(h)]}{\theta(h)}(\sigma + \eta)y_t \tag{76}$$

¹⁹Note that a_t can be interpreted as an autocorrelated technology shock and therefore equals v_t^y known from the derivation of the (high-frequency) hybrid dynamic IS equation above.

or more precisely after dividing equation (76) by h , the *quarterized* NKPC is finally given by

$$\pi_t^{(h)} = \beta(h)E_t\pi_{t+h}^{(h)} + \frac{[1 - \theta(h)][1 - \theta(h)\beta(h)]}{h\theta(h)}(\sigma + \eta)y_t. \quad (77)$$

It can be mentioned that in the benchmark case ($h = 1$) the previous equation will collapse into the more familiar expression

$$\pi_t^{(1)} = \beta E_t\pi_{t+1}^{(1)} + \frac{(1 - \theta)(1 - \theta\beta)}{\theta}(\sigma + \eta)y_t. \quad (78)$$

It can be seen from this, that in case of the NKPC (and, of course, for the dynamic IS equation) it is not necessary to spell out the details here, that lead from the micro-foundation of this equation in an h-economy. Instead — and as already mentioned — a high-frequency representation of the NKPC can be achieved by adjusting the frequency-dependent components directly with respect to h . We make use of this fact as we turn to the derivation the *hybrid* NKPC (e.g. Christiano et al. (2005)) for $0 < h \leq 1$ (where an exogenous shock is simply added under consideration of hv_t^π):

$$\Delta p_t = \frac{\beta(h)}{1 + \alpha\beta(h)}\Delta p_{t+h} + \frac{\alpha}{1 + \alpha\beta(h)}\Delta p_{t-h} + \kappa(h)y_t + hv_t^\pi \quad (79)$$

with

$$\beta(h) = \frac{1}{1 + h\nu} \quad (80)$$

$$\kappa(h) = \frac{[1 - \theta(h)][1 - \theta(h)\beta(h)]}{\theta(h)[1 + \alpha\beta(h)]}(\sigma + \eta) = \frac{h^2(1 - \theta)(1 + \nu - \theta)}{(1 - h(1 - \theta))(1 + h\nu + \alpha)}(\sigma + \eta) \quad (81)$$

and $\Delta p_{t-h} = p_{t-h} - p_{t-2h}$. Note that now the price indexation parameter α has to be taken into account. Since α denotes the *fraction* of firms which set its actual price level to the previous one, this parameter is *not* frequency-dependent. Furthermore, the *quarterized* hybrid NKPC is given by

$$\pi_t^{(h)} = \frac{\beta(h)}{1 + \alpha\beta(h)}E_t\pi_{t+h}^{(h)} + \frac{\alpha}{1 + \alpha\beta(h)}\pi_{t-h}^{(h)} + \tilde{\kappa}(h)y_t + v_t^\pi \quad (82)$$

where

$$\tilde{\kappa}(h) = \frac{\kappa(h)}{h} = \frac{h(1 - \theta)(1 + \nu - \theta)}{(1 - h(1 - \theta))(1 + h\nu + \alpha)}(\sigma + \eta)$$

holds. For the baseline quarterly model ($h = 1$) we get

$$\pi_t^{(1)} = \frac{\beta}{1 + \alpha\beta}E_t\pi_{t+1}^{(1)} + \frac{\alpha}{1 + \alpha\beta}\pi_{t-1}^{(1)} + \kappa y_t + v_t^\pi \quad (83)$$

with

$$\beta = \frac{1}{1 + \nu} \quad (84)$$

$$\kappa = \frac{(1 - \theta)(1 - \theta\beta)}{\theta(1 + \beta\alpha)}(\sigma + \eta). \quad (85)$$

6.2 Solution of the Shock Process in the Case $0 < h \leq 1$

According to the equations (17) to (19) given in section 2, the AR(1) shock process

$$v_t^z = \rho(h)^z v_{t-h}^z + \varepsilon_t^z \quad (86)$$

with

$$\rho(h)^z = 1 - h(1 - \rho^z)$$

and $z = \{i, y, \pi\}$, emerges when the updating of v_t^z in the AR(1) process is done every $0 < h \leq 1$ time units. The solution of (86) is then given by

$$E_0 v_t^z = [\rho(h)^z]^t \varepsilon_0^z. \quad (87)$$

Note that we assume a deterministic impulse in all of the shock processes z such that

$$\varepsilon_t^z = \begin{cases} \varepsilon_0^z & \text{for } t = 0 \\ 0 & \text{for } t > 0 \end{cases} \quad (88)$$

holds. According to Sacht and Wohltmann (2014), equation (87) can be derived as follows. For a clear arrangement, in the following we omit the superscript z . Based on (86) we get

$$\begin{aligned} v_{t-h} &= \rho(h)v_{t-2h} + \varepsilon_{t-h} \\ v_{t-2h} &= \rho(h)v_{t-3h} + \varepsilon_{t-2h} \\ &\vdots \\ v_{t-sh} &= \rho(h)v_{t-(s+1)h} + \varepsilon_{t-sh} \end{aligned}$$

with $s \in \{0, 1, 2, \dots, t\}$, where t denotes the subperiod with respect to h . The *backward-solution* is given recursively by

$$\begin{aligned} v_t &= \rho(h)[\rho(h)v_{t-2h} + \varepsilon_{t-h}] + \varepsilon_t \\ \Rightarrow v_t &= \rho(h)^2 v_{t-2h} + \rho(h)\varepsilon_{t-h} + \varepsilon_t \\ \Rightarrow v_t &= \rho(h)^2 [\rho(h)v_{t-3h} + \varepsilon_{t-2h}] + \rho(h)\varepsilon_{t-h} + \varepsilon_t \\ \Rightarrow v_t &= \rho(h)^3 v_{t-3h} + \rho(h)^2 \varepsilon_{t-2h} + \rho(h)\varepsilon_{t-h} + \varepsilon_t \\ &\vdots \\ \Rightarrow v_t &= \rho(h)^s v_{t-sh} + \rho(h)^{s-1} \varepsilon_{t-(s-1)h} + \rho(h)^{s-2} \varepsilon_{t-(s-2)h} + \dots \\ &\quad + \rho(h)^{s-(s-1)} \varepsilon_{t-(s-(s-1))h} + \rho(h)^{s-s} \varepsilon_{t-(s-s)h} \\ \Rightarrow v_t &= \rho(h)^s [\rho(h)v_{t-(s+1)h} + \varepsilon_{t-sh}] + \rho(h)^{s-1} \varepsilon_{t-(s-1)h} + \rho(h)^{s-2} \varepsilon_{t-(s-2)h} + \dots \\ &\quad + \rho(h)\varepsilon_{t-h} + \varepsilon_t \\ \Rightarrow v_t &= \varepsilon_t + \rho(h)\varepsilon_{t-h} + \rho(h)^2 \varepsilon_{t-2h} + \rho(h)^3 \varepsilon_{t-3h} + \dots + \rho(h)^s \varepsilon_{t-sh} + \dots \\ &\quad + \rho(h)^{t-1} \varepsilon_{t-(t-1)h} + \rho(h)^t \varepsilon_0 \\ \Rightarrow v_t &= \sum_{s=0}^t \rho(h)^s \varepsilon_{t-sh}. \end{aligned}$$

Finally, by taking expectations on the previous expression we get

$$E_0 v_t = \sum_{s=0}^t \rho(h)^s E_0 \varepsilon_{t-sh} = \rho(h)^t \varepsilon_0 \quad (89)$$

since $E_0 \varepsilon_s = 0$ for $s > 0$. Hereby, E_0 denotes the expectation operator conditional on the information given in the initial period $t = 0$.

6.3 Solution of the Baseline NKM in the Case $0 < h \leq 1$

In general, the state space representation of the NKM for $0 < h \leq 1$ is given by

$$0 = A X_t^{TR} + B X_{t+h}^{TR} + C X_{t-h}^{TR} + D V_t \quad (90)$$

$$V_t = N V_{t-h} + \Xi_t \quad (91)$$

with

$$\begin{aligned} A &= \begin{pmatrix} 1 + \delta_1(h)\phi_y & \delta_1(h)\phi_\pi \\ -\tilde{\kappa}(h) & 1 \end{pmatrix} \\ B &= \begin{pmatrix} -\frac{1}{1+\chi} & -\delta_1(h) \\ 0 & -\frac{\beta(h)}{1+\beta(h)\alpha} \end{pmatrix} \\ C &= \begin{pmatrix} -\frac{\chi}{1+\chi} & 0 \\ 0 & -\frac{\alpha}{1+\beta(h)\alpha} \end{pmatrix} \\ D &= \begin{pmatrix} \delta_1(h) & -\delta_2(h)(\rho(h)^y - 1) & 0 \\ 0 & 0 & -1 \end{pmatrix} \\ N &= \begin{pmatrix} \rho(h)^i & 0 & 0 \\ 0 & \rho(h)^y & 0 \\ 0 & 0 & \rho(h)^\pi \end{pmatrix} \end{aligned}$$

and $X_t^{TR} = (y_t^{TR}, \pi_t^{TR})'$, $V_t = (v_t^i, v_t^y, v_t^\pi)'$ and $\Xi_t = (\varepsilon_t^i, \varepsilon_t^y, \varepsilon_t^\pi)'$. The superscript TR indicates that we consider a (high-frequency) forward-looking/hybrid NKM, where an (non-optimal) ad-hoc Taylor rule is assumed.

Hybrid NKM:

Since the state space representation (90) exhibits forward-and backward-looking elements, as stated in the text we apply the method of undetermined coefficients in combination with the brute force iteration procedure in order to solve the model. First, we guess that the law of motion which describes the analytical ‘solution’ is given by

$$X_t^{TR} = \Omega X_{t-h}^{TR} + \Phi V_t \quad (92)$$

where $\Omega \in \mathbb{R}^{3 \times 3}$ and $\Phi \in \mathbb{R}^{2 \times 3}$ are the corresponding solution matrices. The former is a stable matrix as long as its determinant is not equal to zero, which ensures the invertibility of Ω . In particular, stability of Ω requires the sufficient condition $\phi_\pi > 1$, i.e. the ‘Taylor principle’, to be fulfilled (cf. the discussion with respect to equation (27)). We substitute (92) into (90), which leads to

$$A(\Omega X_{t-h}^{TR} + \Phi V_t) + B(\Omega X_t^{TR} + \Phi V_{t+h}) + C X_{t-h}^{TR} + D V_t = 0.$$

This is equivalent to

$$A(\Omega X_{t-h}^{TR} + \Phi V_t) + B[\Omega(\Omega X_{t-h}^{TR} + \Phi V_t) + \Phi(N V_t + \Xi_{t+h})] + C X_{t-h}^{TR} + D V_t = 0.$$

The reduced-form solution can be rewritten as

$$(A\Omega + B\Omega^2 + C)X_{t-h}^{TR} + (A\Phi + B\Omega\Phi + B\Phi N + D)V_t = 0. \quad (93)$$

Since Ξ_t consists on deterministic one-time impulse shocks, i.e. $\Xi_{t+h} = 0$ holds. Thus the solution matrix Ω can be (uniquely) determined by

$$B\Omega^2 + A\Omega + C = 0. \quad (94)$$

The previous expression turns out to be a quadratic matrix equation. Hence, an analytical solution for Ω can be hardly obtained since the solution reads

$$\Omega = -(B\Omega + A)^{-1}C. \quad (95)$$

Instead we are going to solve this quadratic matrix equation numerically. Therefore, we apply the brute force iteration procedure. Hence an equivalent recursive relation of (95) is given by

$$\Omega_n = -(B\Omega_{n-1} + A)^{-1}C \quad (96)$$

with an arbitrary number of iteration steps N , i.e. $n = \{1, 2, \dots, N\}$. We define as the initial value $\Omega_0 = \varsigma I$ with I being the identity matrix and $0 \leq \varsigma \leq 1$, where we set $\varsigma = 0.8$. The iteration process (96) proceeds until $\|\Omega_n - \Omega_{n-1}\| < \varrho$ holds, where ϱ is an arbitrarily small number (we set $\varrho = (0.1)^6$). Given the solution for Ω , for the computation of Φ applying the same recursive numerical solution method is required. This can be seen by the ‘solution’ for Φ :

$$\Phi = -(A + B\Omega)^{-1}(D + B\Phi N). \quad (97)$$

In order to check if both matrices serve as necessary and sufficient conditions for describing the model dynamics, Ω and Φ are plugged in the reduced-form solution (92), which then has to be equal to zero.²⁰ This is indeed true for all possible combination of the parameters, including the numerical parameter set assumed throughout this paper (cf. Table 1).

Purely forward-looking NKM:

For completeness, in the following we report the computation of the solution matrix in the purely forward-looking case, i.e. as $\chi = \alpha = 0$ holds. According to the equations (90) with $C = \mathbf{0}$ and (92) with $\Omega = \mathbf{0}$, respectively, we guess that the reduced-form solution is given by

$$X_t^{TR} = \Gamma V_t \quad (98)$$

where $\Gamma \in \mathbb{R}^{2 \times 3}$ is the corresponding solution matrix in the purely forward-looking case. Inserting (98) into (90) results in

$$\begin{aligned} A\Gamma V_t + B\Gamma V_{t+h} + D V_t &= 0 \\ \Rightarrow A\Gamma V_t + B\Gamma[N V_t + \Xi_{t+h}] + D V_t &= 0 \\ \Rightarrow (A\Gamma + B\Gamma N + D) V_t &= 0. \end{aligned}$$

Equivalent to Φ in the previous case, it is not possible to isolate Γ . Therefore, here we also apply the brute force iteration method. The solution for Γ can then being computed recursively by

$$\Gamma_n = -A^{-1}(D + B\Gamma_{n-1}N). \quad (99)$$

It is obvious that equation (97) collapsed into equation (99) as $\Omega = \mathbf{0}$ holds (and being substituted by Γ).²¹ According to equation (98), under consideration of the solution for the shock process for $0 < h \leq 1$ (cf. equation (89)), the reduced-form solution for the output gap and the inflation rate in the purely forward-looking case are then given by

$$X_t^{TR} = \Gamma V_t = \Gamma E_0 \begin{pmatrix} v_t^i \\ v_t^y \\ v_t^\pi \end{pmatrix} = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \end{pmatrix} \begin{pmatrix} [\rho(h)^i]^t \varepsilon_0^i \\ [\rho(h)^y]^t \varepsilon_0^y \\ [\rho(h)^\pi]^t \varepsilon_0^\pi \end{pmatrix}. \quad (100)$$

²⁰Furthermore, note that the solution by applying the brute force iteration procedure equals the one under the Schur decomposition method. This is confirmed when comparing the outcome for (95) and (97) to the ones, which are computed by using the numerical software programme MATLAB DYNARE (which makes use of the Schur decomposition).

²¹In particular, we search for the fix points of Ω_n and Γ_n according to (96) and (99), respectively, such that $\Omega_N = f(\Omega_N) = -(B\Omega_N + A)^{-1}C$ and $\Gamma_N = f(\Gamma_N) = -A^{-1}(D + B\Gamma_N N)$ hold.

6.4 Impulse Response Functions (TR) in the Case $0 < h \leq 1$

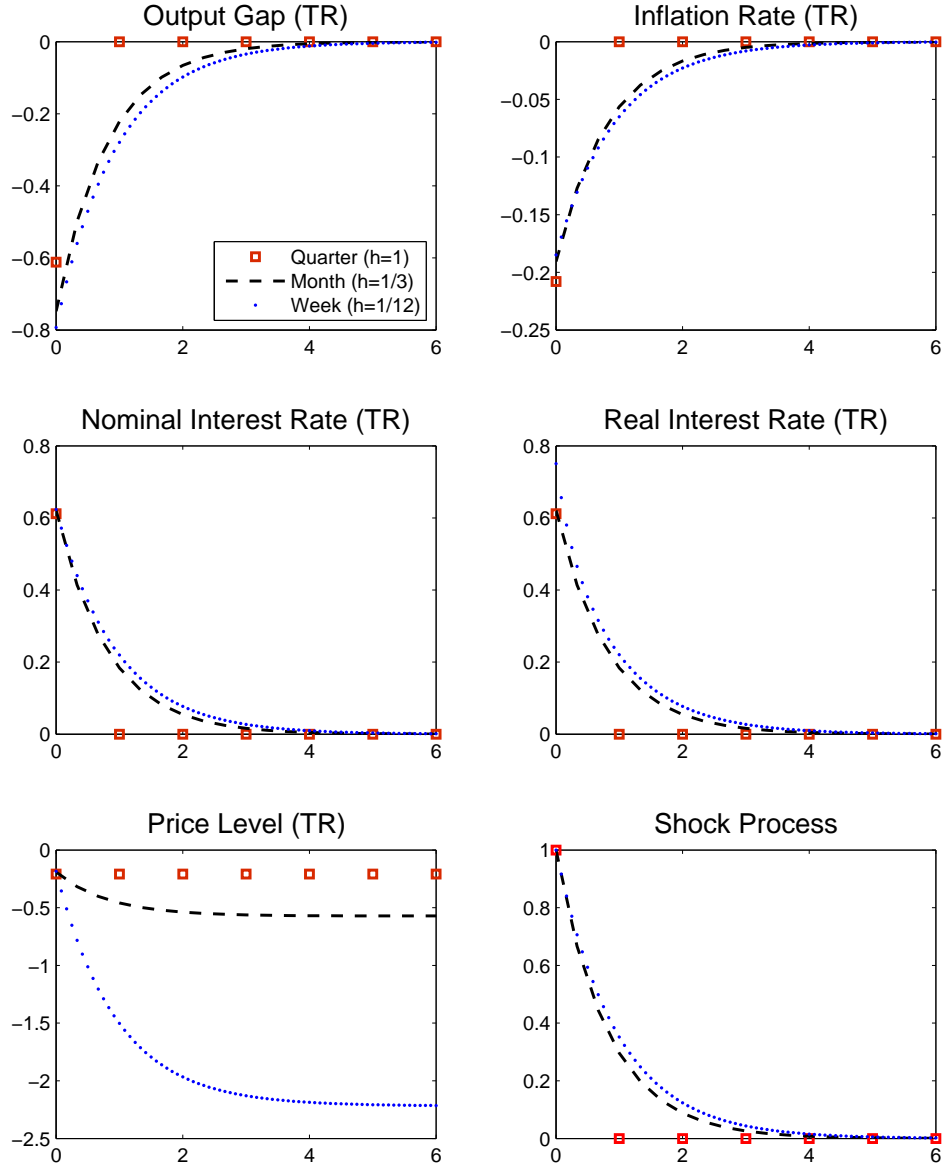


Figure 2: IRFs in case of a *non*-autocorrelated shock to the *nominal interest rate* in the *forward-looking* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

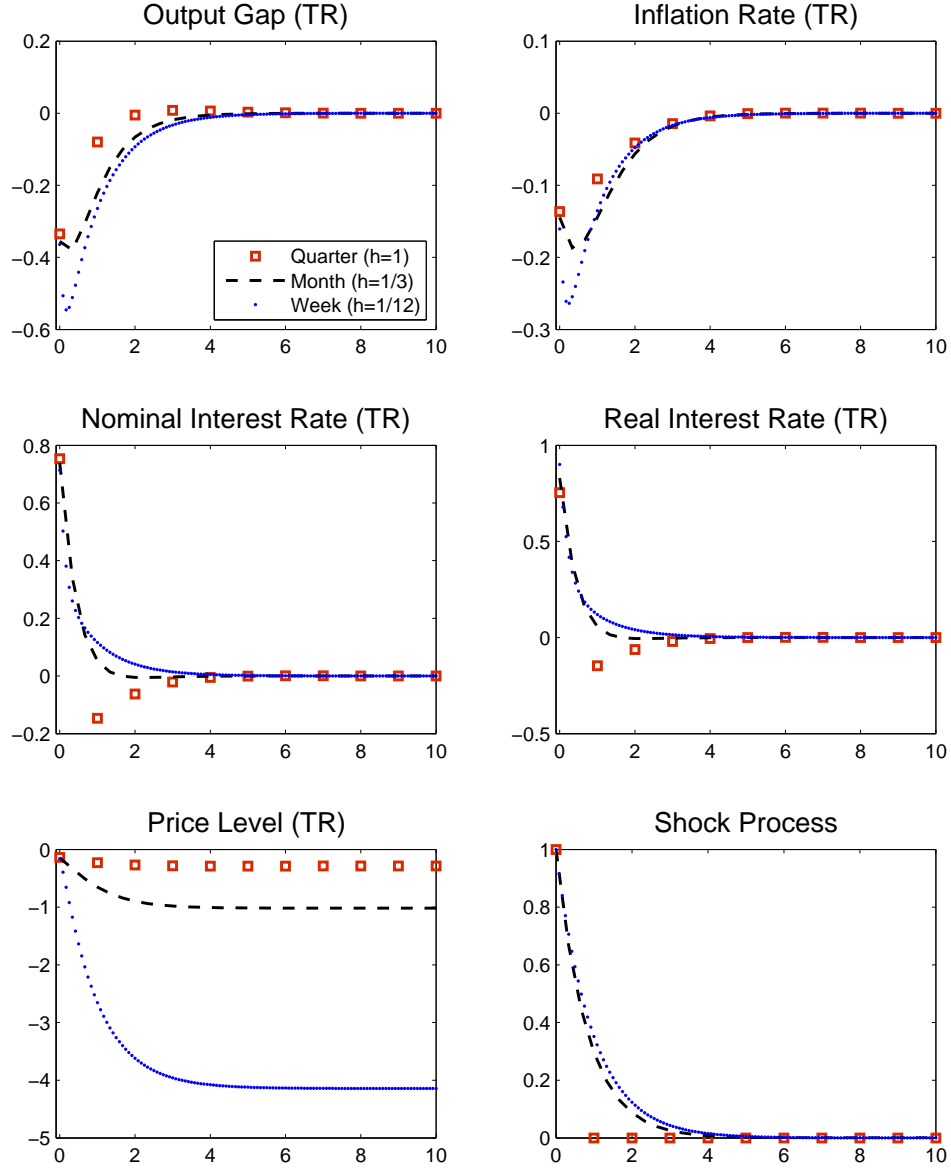


Figure 3: IRFs in case of a *non-autocorrelated* shock to the *nominal interest rate* in the *hybrid* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

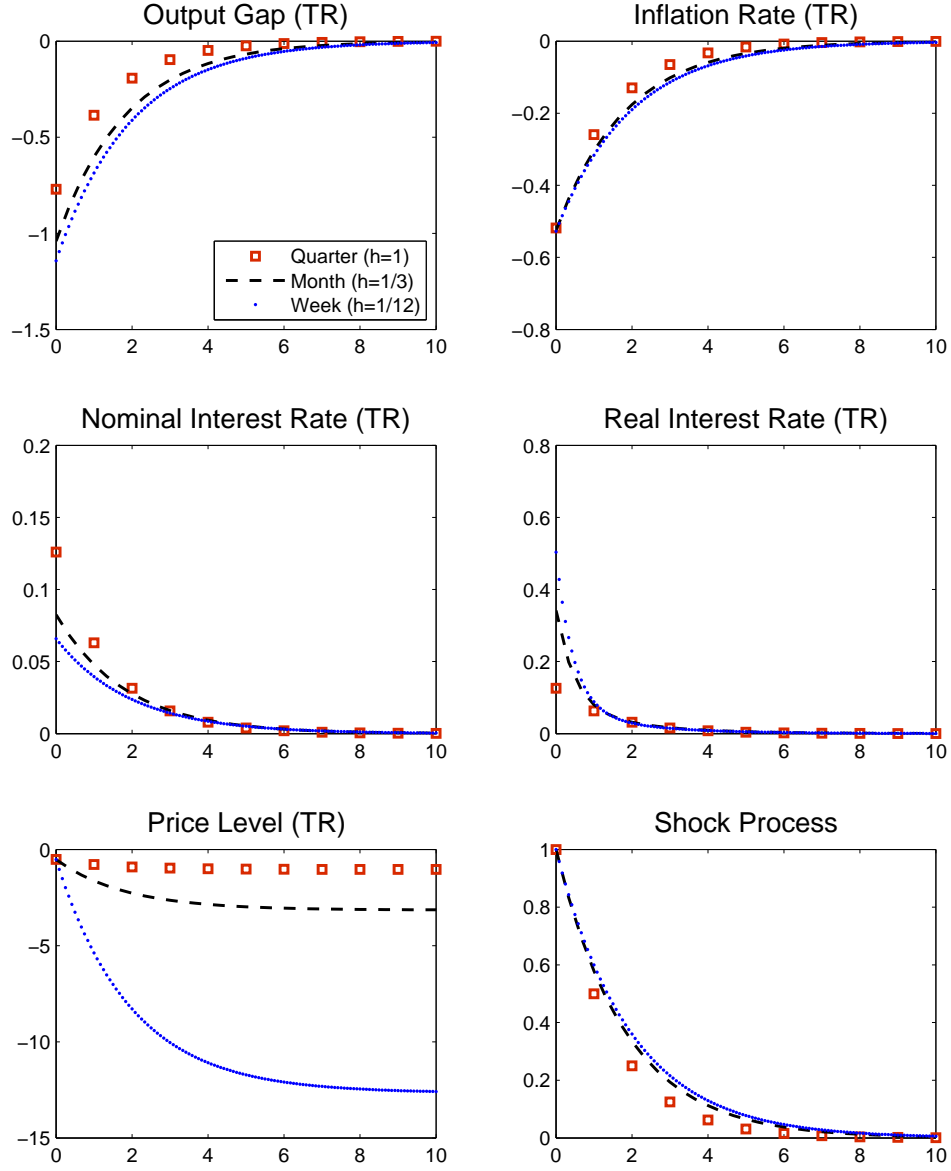


Figure 4: IRFs in case of an *autocorrelated* shock to the *nominal interest rate* in the *forward-looking* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

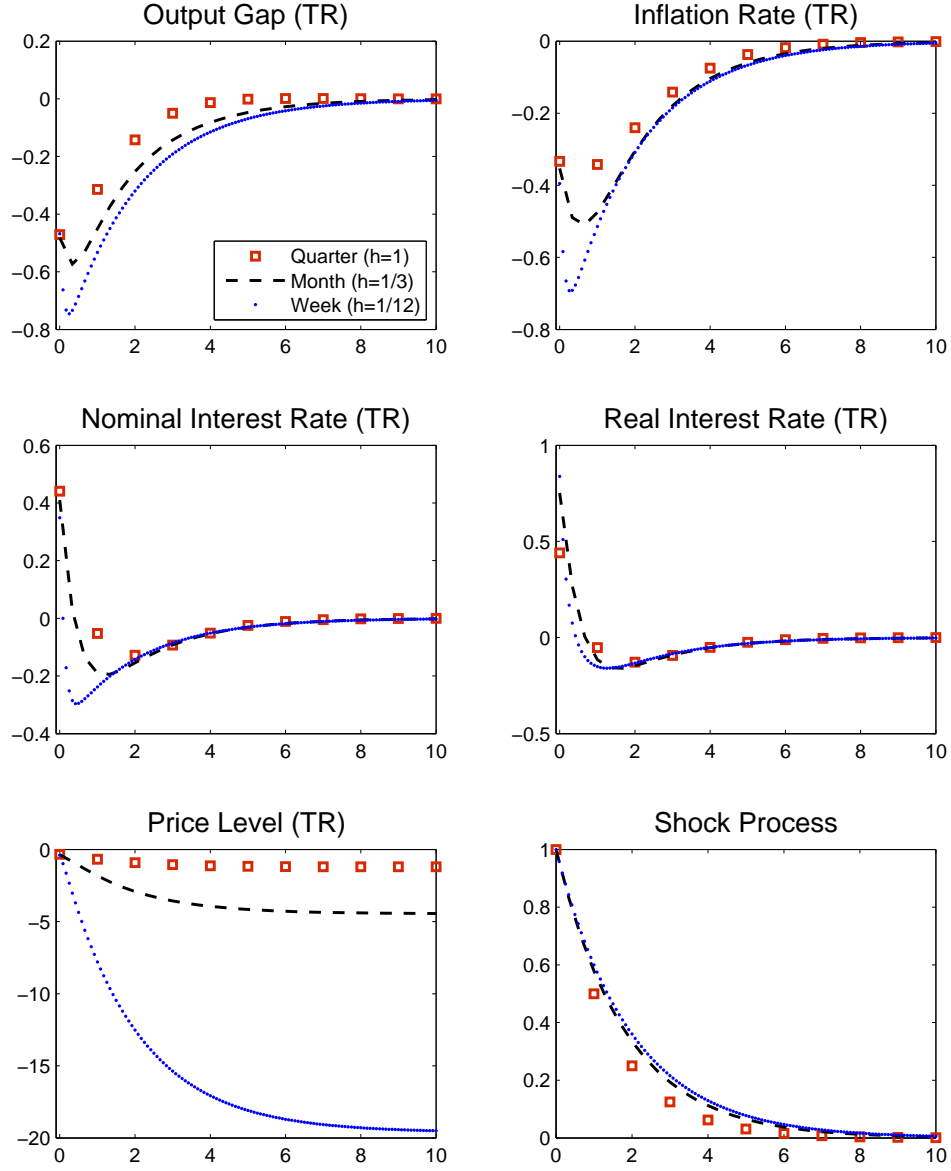


Figure 5: IRFs in case of an *autocorrelated* shock to the *nominal interest rate* in the *hybrid NKM* (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

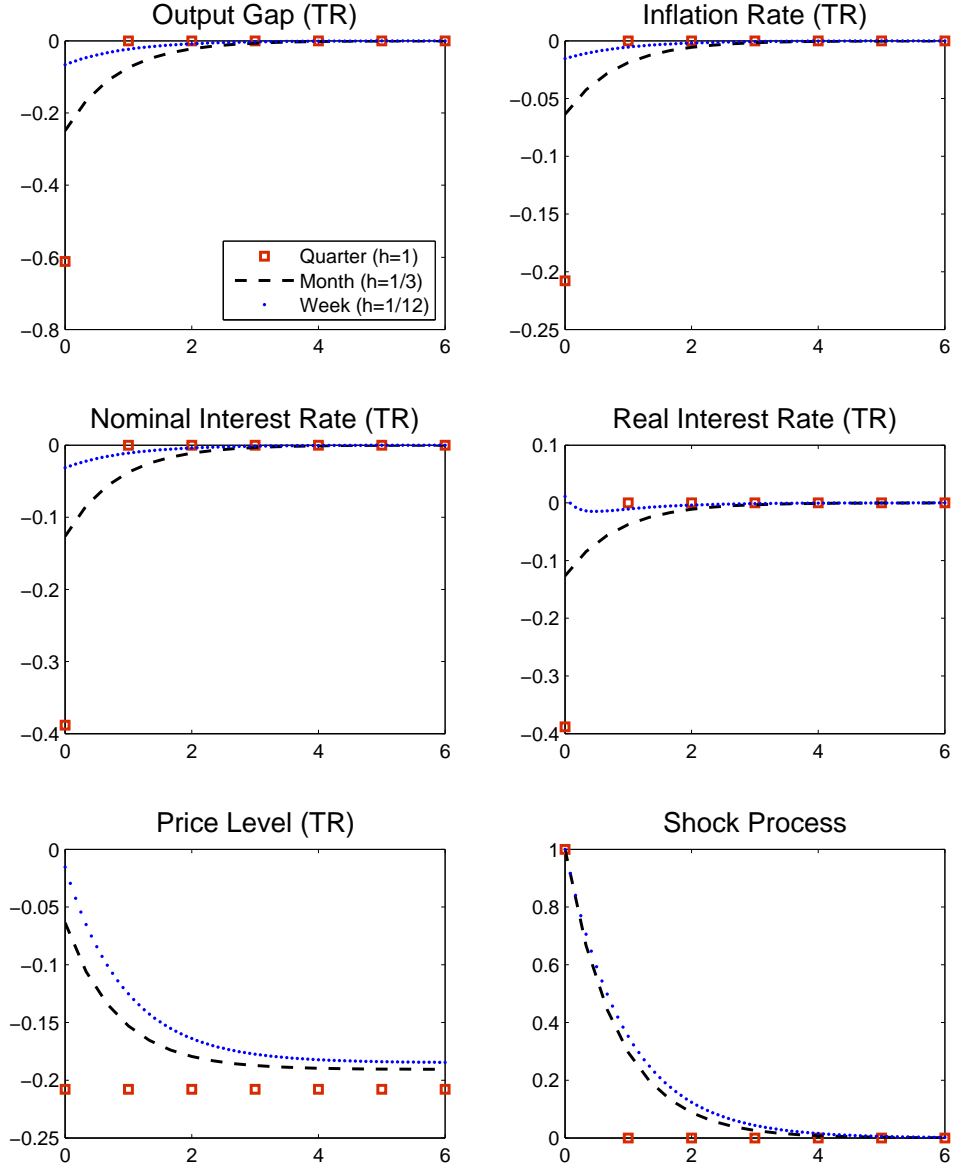


Figure 6: IRFs in case of a *non-autocorrelated technology* shock in the *forward-looking* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* line depicts the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

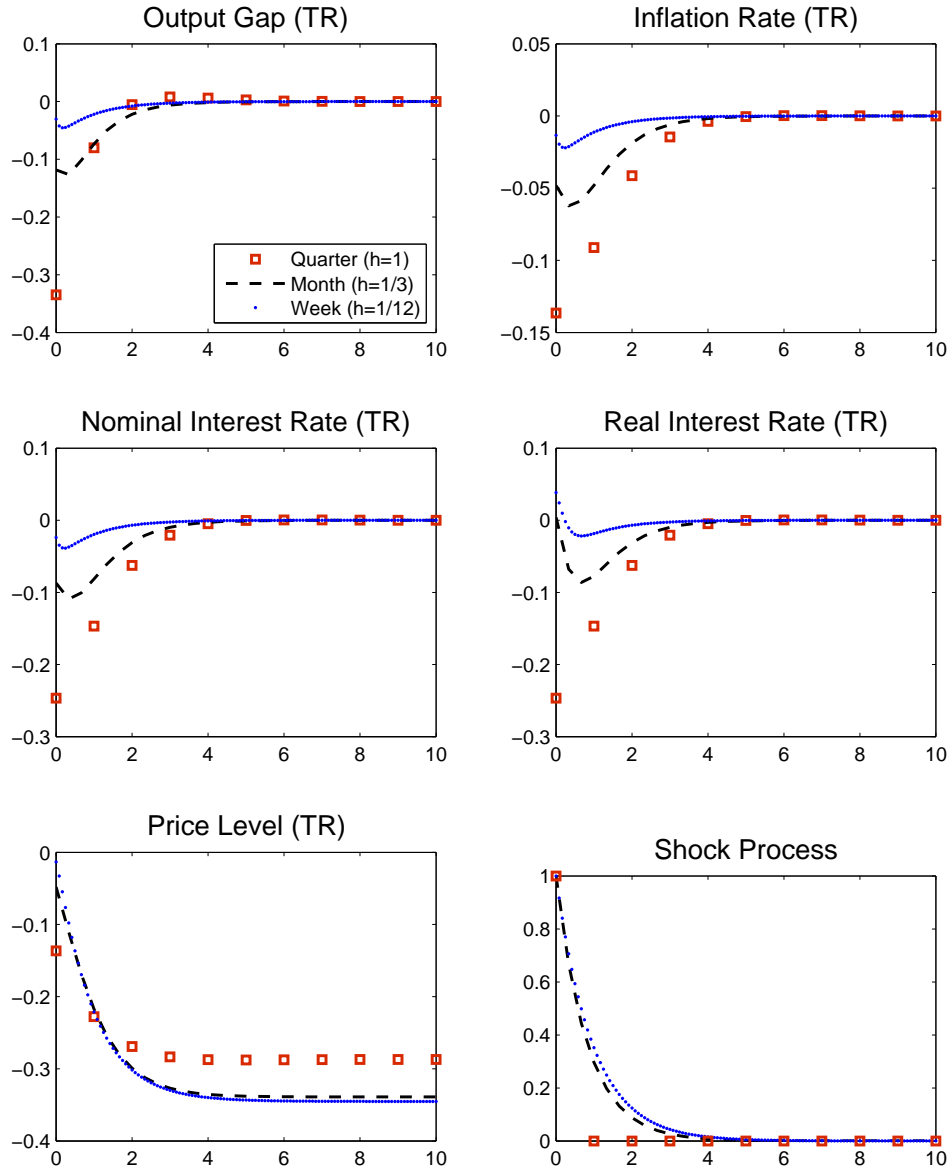


Figure 7: IRFs in case of a *non-autocorrelated technology* shock in the *hybrid* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

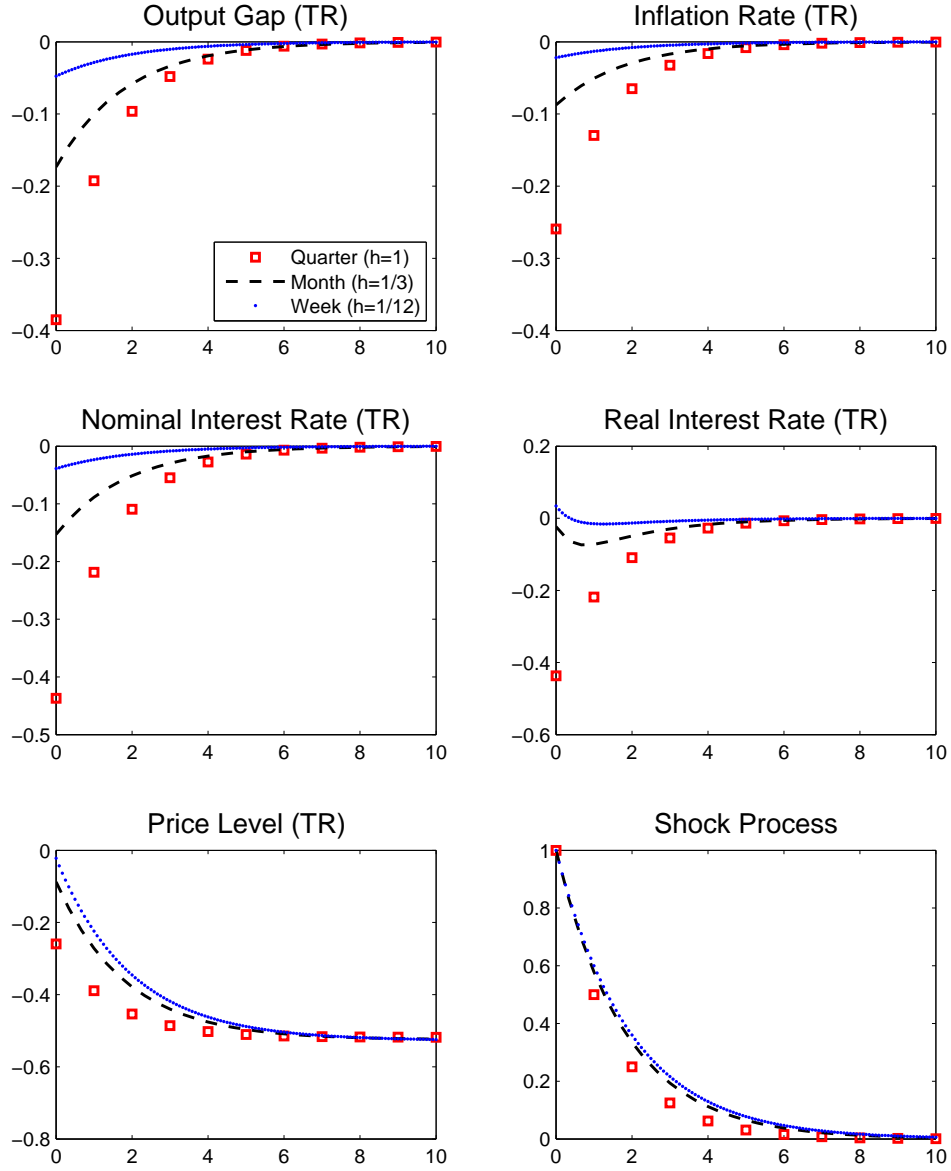


Figure 8: IRFs in case of an *autocorrelated technology* shock in the *forward-looking NKM* (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

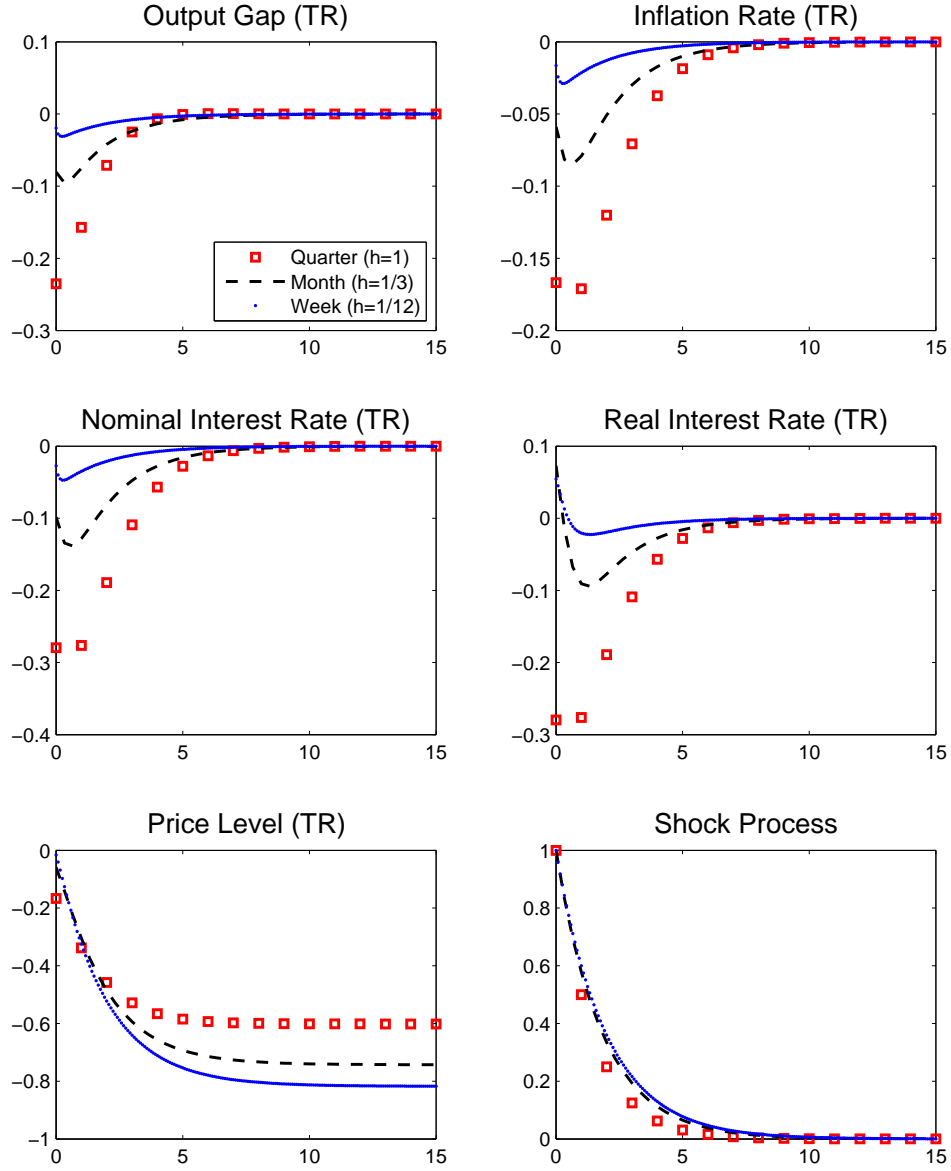


Figure 9: IRFs in case of an *autocorrelated technology* shock in the *hybrid* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

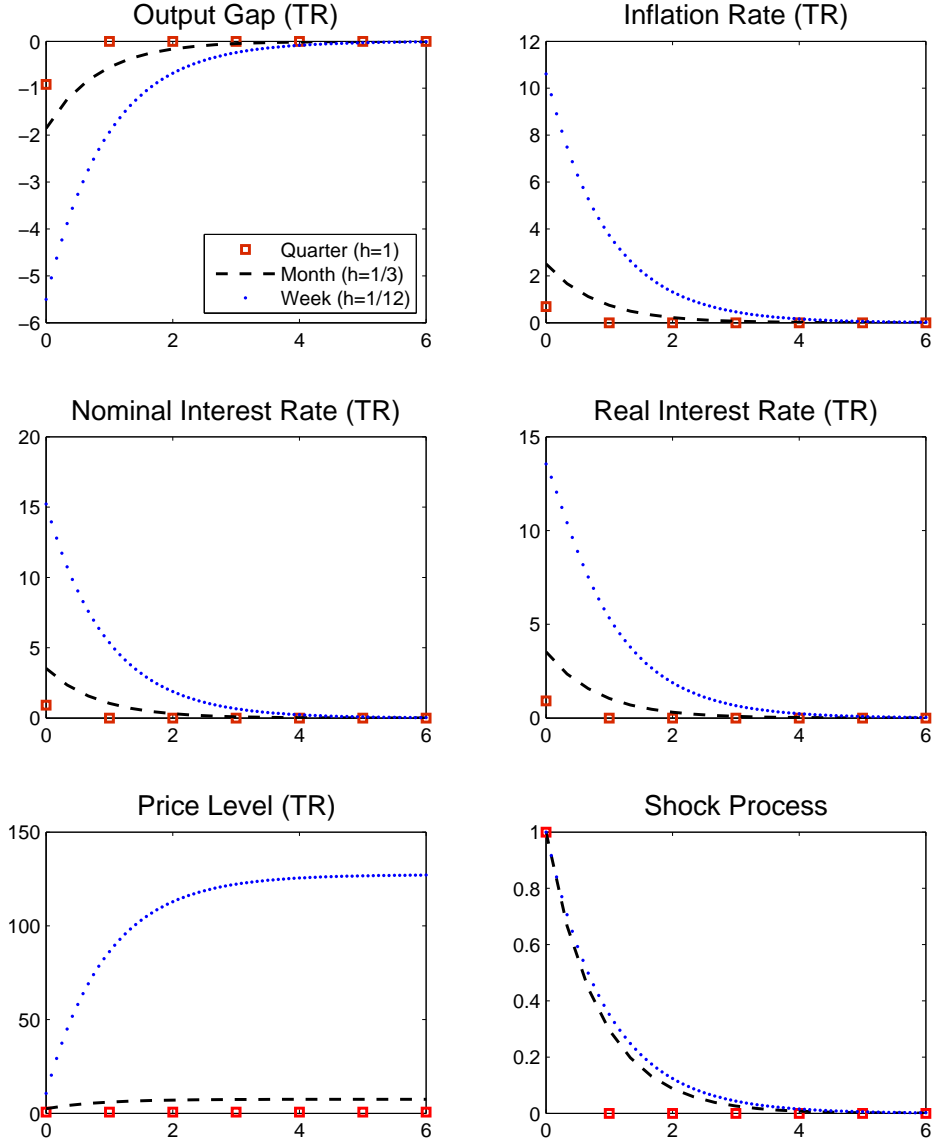


Figure 10: IRFs in case of a *non*-autocorrelated *cost-push* shock in the *forward-looking* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

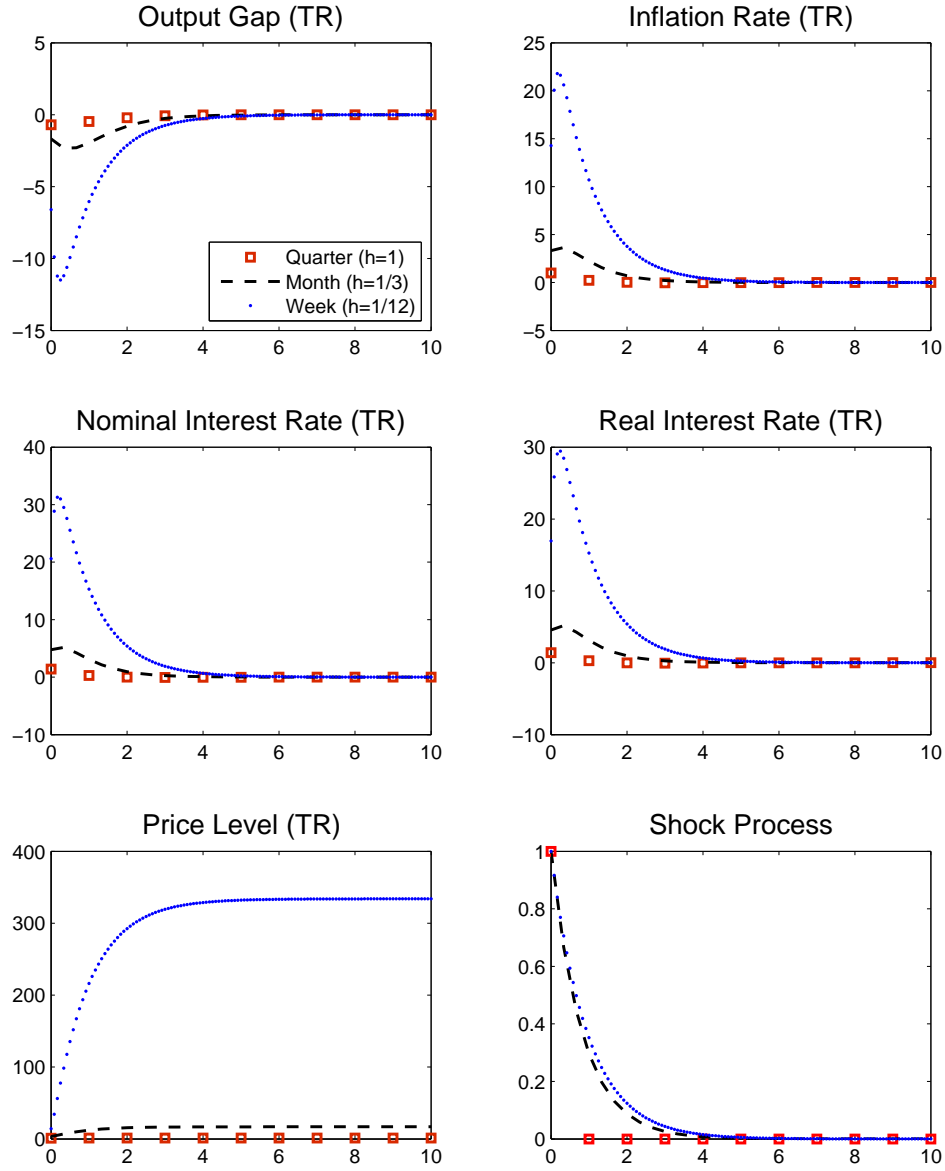


Figure 11: IRFs in case of a *non-autocorrelated cost-push* shock in the *hybrid* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

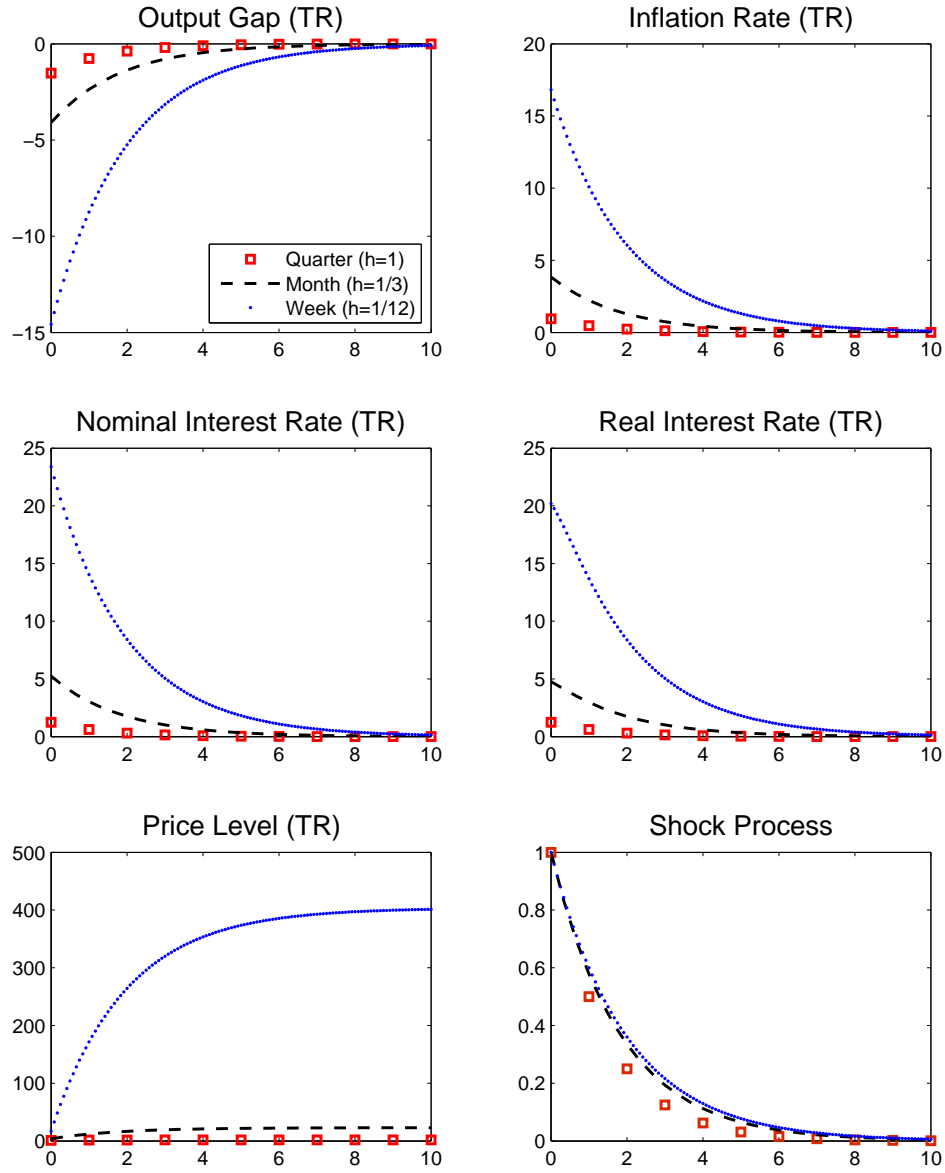


Figure 12: IRFs in case of an *autocorrelated cost-push* shock in the *forward-looking* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

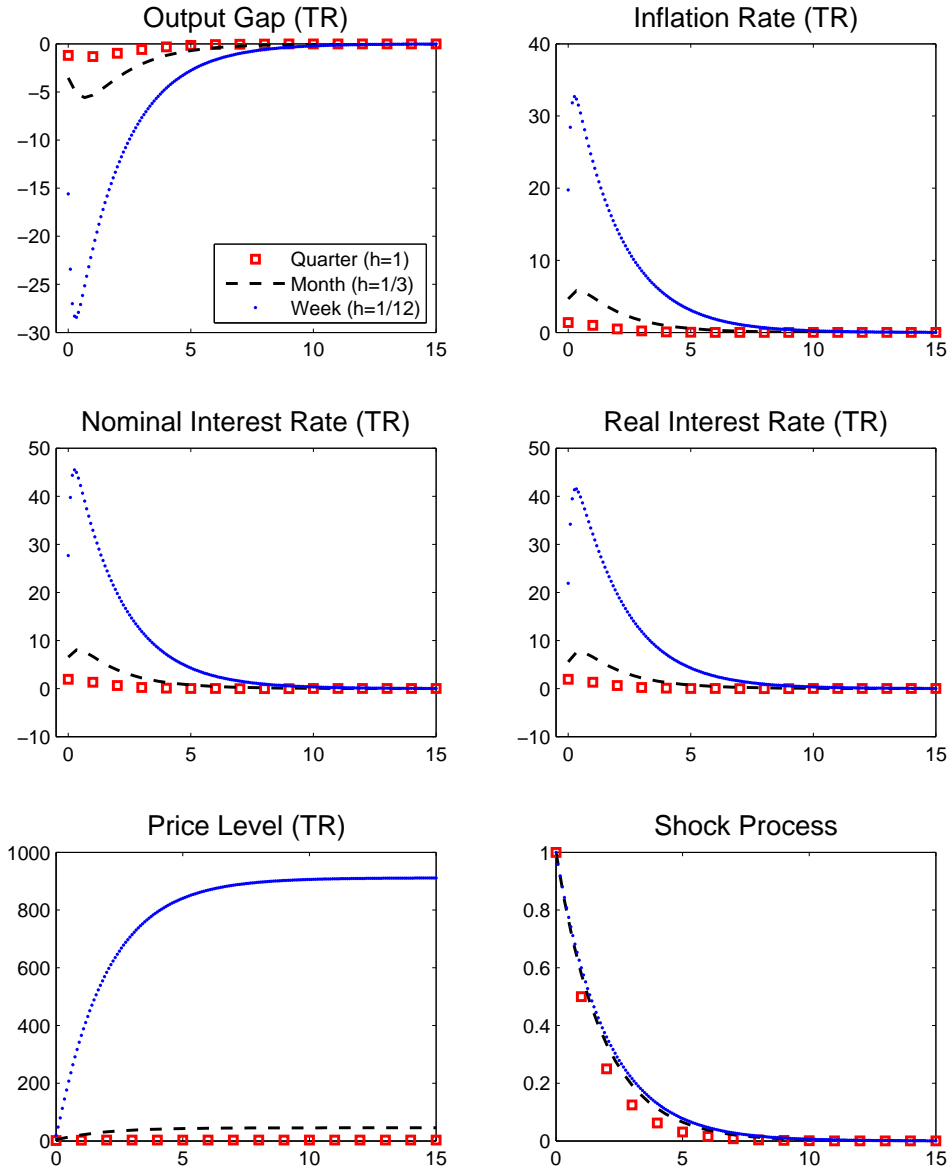


Figure 13: IRFs in case of an *autocorrelated cost-push* shock in the *hybrid* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

Literature

- AHRENS, S. AND SACT, S. (2014): Estimating A High-Frequency New-Keynesian Phillips Curve. *Empirical Economics*, Vol. 46(2), pp. 607-628.
- AADLAND, D. (2001): High Frequency Real Business Cycles. *Journal of Monetary Economics*, Vol. 48(2), pp. 271-292.
- ANAGNOSTOPOULOS, A. AND GIANNITSAROU, C. (2010a): *Indeterminacy And Period Length*. Working Paper, Stony Brook and University of Cambridge (UK).
- ANAGNOSTOPOULOS, A. AND GIANNITSAROU, C. (2010b): Modeling Time And Macroeconomic Dynamics. *CEPR Discussion Paper* No. 8050, Centre for Economic Policy Research, London.
- ASCARI, G. AND ROPELE, T. (2003): *Notes On Real Business Cycle*. Mimeo, University of Pavia.
- BENHABIB, J. AND FARMER, R.E.A. (1994): Indeterminacy And Increasing Returns. *Journal of Economic Theory*, Vol. 63(1), pp. 19-41.
- BINDER, M. AND PESARAN, M.H. (1995): Multivariate Rational Expectations Models And Macroeconomic Modeling: A Review And Some New Results. in: PESARAN, M.H. AND WICKENS, M. (Eds.) (1995): *Handbook Of Applied Econometrics: Macroeconomics*, Basil Blackwell, pp. 139-187.
- BULLARD, J. AND MITRA, K. (2002): Learning About Monetary Policy Rules. *Journal of Monetary Economics*, Vol. 49(6), pp. 1105-1129.
- CALVO, G.A. (1983): Staggered Prices In A Utility-Maximizing Framework. *Journal of Monetary Economics*, Vol. 12(3), pp. 383-398.
- CHRISTIANO, L.J. (1985): A Method For Estimating The Timing Interval In A Linear Econometric Model With An Application To Taylor's Model of Staggered Contracts. *Journal of Economic Dynamics and Control*, Vol. 9(4), pp. 363-404.
- CHRISTIANO, L.J., EICHENBAUM, M. AND EVANS, C.L. (2005): Nominal Rigidities And The Dynamic Effects Of A Shock To Monetary Policy. *Journal of Political Economy*, Vol. 113(1), pp. 1-45.
- DROST F.C. AND NIJMAN T.E. (1993): Temporal Aggregation Of GARCH Processes. *Econometrica*, Vol. 61(4), pp. 909-927.
- FISHER, I. (1930): *The Theory Of Interest*. Macmillan, New York.
- FLASCHEL, P., FRANKE, R. AND PROAÑO, C.R. (2008): On Equilibrium Determinacy In New Keynesian Models With Staggered Wage And Price Setting. *The B.E. Journal of Macroeconomics*, Vol. 8(1), Topics, Article 31.
- FOLEY, D.K. (1975): On Two Specifications Of Asset Equilibrium In Macroeconomic Models. *Journal of Political Economy*, Vol. 83(2), pp. 303-324.

- FRANKE, R. AND SACT, S. (2014): Some Observations In The High-Frequency Versions Of A Standard New-Keynesian Model. *Bulletin of Economic Research*, Vol. 66(1), pp. 72-94.
- GALÍ, J., GERTLER, M. AND LÓPEZ-SALIDO, J.D. (2001): European Inflation Dynamics. *European Economic Review*, Vol. 45(7), pp. 1237-1270.
- GALÍ, J. (2008): *Monetary Policy, Inflation, And The Business Cycle*. Princeton University Press, Princeton.
- HASSLER, U. (2011): *Temporal Aggregation Of Multiple Time Series In The Frequency Domain*. Working Paper, Goethe University Frankfurt.
- HINTERMAIER, T. (2005): A Sunspot Paradox. *Economics Letters*, Vol. 87(2), pp. 285-290.
- MCCALLUM, B.T. (1983): On Non-Uniqueness In Rational Expectations Models: An Attempt At Perspective. *Journal of Monetary Economics*, Vol. 11(2), pp. 139-168.
- POSCH, O. RUBIO-RAMÍREZ, J.F. AND FERNÁNDEZ-VILLAVÉRDE, J. (2011): *Solving The New Keynesian Model In Continuous Time*. 2011 Meeting Papers No. 829, Society for Economic Dynamics.
- ROGERSON, R., SHIMER, R. AND WRIGHT, R. (2005): Search-Theoretic Models Of The Labor Market: A Survey. *Journal of Economic Literature*, Vol. 43(4), pp. 959-988.
- SACT, S. AND WOHLTMANN, H.-W. (2014): *Inflation Targeting And Welfare Analysis In High-Frequency New-Keynesian Models*. Mimeo, Department of Economics, Christian Albrechts University Kiel, July 2014.
- SACT, S. (2014): *Optimal Monetary Policy Responses and Welfare Analysis*. Economics Working Paper No. 2014-03, Department of Economics, Christian Albrechts University Kiel, January 2014.
- SILVESTRINI, A. AND VEREDAS, D. (2008): Temporal Aggregation Of Univariate And Multivariate Time Series Models: A Survey. *Journal of Economic Surveys*, Vol. 22(3), pp. 458-497.
- SMETS, F. AND WOUTERS, R. (2003): An Estimated Dynamic Stochastic General Equilibrium Model Of The Euro Area. *Journal of the European Economic Association*, Vol. 1(5), pp. 1123-1175.
- TAYLOR, J.B. (1993): Discretion Versus Policy Rules In Practice. *Carnegie-Rochester Conference Series on Public Policy*, Vol. 39(1), pp. 195-214.
- WALSH, C. (2010): *Monetary Theory And Policy*. 3rd Edition, MIT Press, Cambridge (US).
- WINKLER, R.C. AND WOHLTMANN, H.-W. (2012): On The (De)Stabilizing Effects of News Shocks. *Economics Letters*, Vol. 114(3), pp. 256-258.